How Partnerships are Core to a Linking Research and Practice Agenda

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Acknowledgements
Support for the preparation of this paper was provided by the U.S. Dept. of Education-IES Research Training Programs in the Education Sciences under grant no. R305B130007, and as part of the Wisconsin Center for Education Research Postdoctoral Training Program in Mathematical Thinking, Learning, and Instruction at the University of Wisconsin-Madison. I thank Jon Davis, Mary Lou Rohwer, and the mathematics education community at UW-Madison for their support in advancing the ideas discussed in this paper.
Abstract

Partnerships between researchers and teachers are central to stimulating advancements in a linking research and practice agenda. This paper addresses two key aims. First, research on supporting students’ representational fluency in technology-rich algebra learning environments is used to illustrate a linking research and practice agenda. Three themes are addressed as outcomes of this researcher-practitioner partnership: (a) the importance of addressing a shared problem of practice, (b) the role of theoretical lenses in supporting both practice and research, and (c) how to design for collaboration using both practice-based and research-based methods. Second, emerging issues in establishing productive researcher-practitioner partnerships are introduced from the perspective of both researchers and practitioners. Viable next steps for bridging the experiences of researchers and practitioners are discussed.

Keywords: linking research and practice, partnerships, design-based research, representational fluency, technology
One facet of the current climate of mathematics education is an explicit emphasis and call to take action on issues that serve both communities of researchers and practitioners (Heid et al., 2006; Langrall, 2014). The National Council of Teacher’s of Mathematics (NCMT) has advanced a Linking Research and Practice agenda (Arbaugh et al., 2010) that emphasizes the bi-directional relationship between activities of researchers and of practitioners. This climate to address shared problems of practice—issues that are of concern to both researchers and practitioners—is a major impetus that guides my program of research.

The purpose of this paper is two fold. First, I address how research can be designed and conducted to support a linking research and practice agenda. The case of a collaborative teaching experiment designed to support change in algebra students’ representational fluency is discussed. Second, some emerging issues in researcher-practitioner partnerships are introduced, together with possible next steps that may be fruitful for bridging the experiences of researchers and practitioners.

**Part I: A Research Agenda Built on Shared Problems of Practice**

This section addresses how a research study is situated in a larger agenda aimed at advancing the field of mathematics education by targeting shared problems of practice that advance both practical and theoretical aims.

**Motivation: A Shared Problem of Practice**

Core to a linking research and practice agenda, this study was motivated by a problem of practice: “How do we help students to be facile in moving among representations, including those created by technology?” (Arbaugh et al., 2010, p. 21), such as computer algebra systems (CAS). The construct of representational fluency—the ability to create, interpret, and connect multiple representations in doing and communicating about mathematics—was chosen to focus
the investigation into this issue. CAS are a representational toolkit (Dick & Edwards, 2008) that afford access to creating representations and moving among symbolic, graphic, numeric, and verbal (i.e., written) representation types.

We are just now starting to see evidence of CAS as a prominent emerging technology in several mainstream U.S. curricula (Davis & Fonger, 2014) and standards for mathematical practice (National Governors Association Center for Best Practices, & Council of Chief State School Offices, 2010). Thus it is pressing to understand productive supports for students’ engagement in a general mathematical process, such as representational fluency, in the context of this emerging technology.

**Research Design**

The research study was conducted from a design-based research paradigm in which the main goal was to engineer forms of learning and study that learning in the context in which it was supported (Cobb et al., 2003; Gravemeijer & Cobb, 2006). With support from Texas Instruments and the Department of Mathematics at Western Michigan University, the author-researcher and an interested high school teacher partnered to conduct a collaborative teaching experiment (Cobb, 2000) in a ninth-grade algebra classroom. In this collaborative endeavor we formed a partnership around the core issues of creating opportunities for students to develop representational fluency in solving equations in a combined CAS and paper-and-pencil environment. The researcher also conducted semi-structured task-based interviews (Goldin, 2000) to capture select student’s development of representational fluency in solving equations from the beginning to the end of the teaching experiment.

On a daily basis during the experiment, the researcher and teacher were engaged in supportive roles. The author-researcher was a participant observer (Patton, 2002) while the
teacher taught all lessons. As a team, the pair met daily to plan and design future lessons, and to debrief enacted lessons. While the researcher and practitioner took on different roles, they were complementary roles. Much of the partnership was established through collaborative planning and reflections on practice with a focus on supporting students’ representational fluency in a CAS/paper-and-pencil environment.

Bi-directional links between a conjectured local instructional theory informed and were informed by ongoing experimentation and analysis in the classroom (see Fonger, 2013 for details). For example, we used the teacher’s formative assessment tools, such as warm-up exercises and exit tickets to inform revision and creation of subsequent activities. Daily ongoing reflections were directed toward several theoretical frames, including both social and psychological frames on learning in a classroom environment, elaborated next.

**Theoretical Frameworks**

The interplay of psychological and social lenses on learning was approached from an emergent perspective (Cobb & Yackel, 1996). This lens afforded a flexible view on classroom mathematical practices and individual student activity and cognition that was necessary for accomplishing the goal of both supporting and characterizing students’ representational fluency.

Social (classroom) supports for students’ representational fluency were framed by (a) a learning progression—a sequence of successively more sophisticated ideas and how they evolve over time as supported by curriculum and instruction; and (b) an activity structure for coordinated tool use—predict, act, reflect, connect, reconcile (e.g., see Fonger, [2014] for elaboration). The construct of representational fluency guided investigation into social supports for learning and psychological activity for characterizing students’ movement within and among tool-based representations. A rule of four model (Figure 1) and SOLO taxonomy (Table 1;
adapted from Biggs & Collis, [1982]) were adopted as analytic lenses to model sophistication in students’ representational fluency (Fonger, submitted).

Figure 1. A rule of four web of representation types (Huntley, Marcus, Kahan, & Miller, 2007).

Table 1
Characterizing sophistication in representational fluency

<table>
<thead>
<tr>
<th>Types</th>
<th>SOLO Level</th>
<th>Indicators of Representational Fluency</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Representation Type</td>
<td>Prestructural</td>
<td>• Incorrect create, incorrect interpret</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Limited understanding of meaning</td>
</tr>
<tr>
<td></td>
<td>Unistructural</td>
<td>• Incorrect correct and/or incorrect interpret (not both)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Meaning within, no/limited connection to other types</td>
</tr>
<tr>
<td>More than One</td>
<td>Multistructural</td>
<td>• Correct create or correct interpret (not both)</td>
</tr>
<tr>
<td>Representation Type</td>
<td></td>
<td>• No/limited meaning of relationship</td>
</tr>
<tr>
<td></td>
<td>Relational</td>
<td>• Correct create and correct interpret</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Expresses meaning of relationship across types</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• A connection if interprets an invariant feature across representation types</td>
</tr>
</tbody>
</table>

In summary, two primary theoretical lenses informed ongoing and retrospective analyses of the classroom teaching experiment (social lenses): a learning progression, and an activity structure for the coordination of tools. At the individual (psychological) level, sophistication in
representational fluency was analyzed from a coordinated lens of the rule of four and an adapted solo taxonomy. Table 2 organizes these perspectives accordingly.

Table 2.
Analytic lenses on classroom and individual activity

<table>
<thead>
<tr>
<th>Classroom (Social)</th>
<th>Individual (Psychological)</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Comparing expressions to related by “=” to solve equations from a functions approach</td>
<td></td>
</tr>
<tr>
<td>• Predict, act, reflect, connect, reconcile tool-based representations</td>
<td></td>
</tr>
<tr>
<td>Representational fluency in solving equations with CAS/paper-and-pencil:</td>
<td></td>
</tr>
<tr>
<td>• Rule of Four model</td>
<td></td>
</tr>
<tr>
<td>• SOLO Taxonomy</td>
<td></td>
</tr>
</tbody>
</table>

Select Findings

This section addresses three key findings. First, the role of a functions approach to solving equations is posited to be supportive of students’ development of representational fluency. Second, a predict, act, reflect, connect, reconcile activity structure is conjectured to play an important role in students’ coordination of tool use in solving equations, supportive of students’ representational fluency. A vignette demonstrating one student’s development of representational fluency is introduced third to complement these two classroom-based findings.

A functions approach to solving equations. A functions approach to solving equations is linked to students’ development of representational fluency (see Fonger, Rohwer, & Davis, in preparation). For example, to solve a linear equations such as 4x – 9 = -7x + 13 for x, students compared the expressions related by the equal sign using graphs and tables of f1(x) = 4x - 9 and f2(x) = -7x + 13. They also performed symbolic transpositions on the equation. The teacher, Ms. L., encouraged reflection on the representation types according to a rule of four model created for classroom use. To exemplify these ideas, consider Vignette I, a typical approach used to solve equations.
Vignette I
Ms. L: Now let's look at our other representations [CAS and paper-and-pencil] At \( x = 2 \), [points to \( x \)-value of 2 in table] that was my solution here [points to values of \( f1(2) \) and \( f2(2) \) in table], where these lines crossed [points to \( (2, -1) \) in the graph], and we did it symbolically.

Ms. L: We did it all three ways. So we got the graph, we got the table and we can see from the rule, that's the same thing as our equation [points to Classroom Model of Rule of Four, as below].

An activity structure for the coordination of tools. An activity structure of predict, act, reflect, connect, and reconcile is posited to support the coordination of CAS and paper-and-pencil in examining equivalence of expressions (Fonger, 2014) and equation solving. For example, in solving a linear equation with no solutions, students were encouraged to first predict the nature of the solutions using graphs, tables, or by otherwise comparing the equivalence of the expressions related by an equal sign. Consider Vignette II.

Vignette II
Ms. L encouraged the class to use their CAS to predict the nature of the solution to the equation \( 6x = 5x + 7 + x \) by comparing \( f1(x) = 6x \) and \( f2(x) = 5x + 7 + x \). Annie compared the functions by creating a CAS graph and table [at right].
The teacher then presented another students’ symbolic transposition work at the board [reproduced at right] and reflected on both to conclude “that means there’s no solution.”

**Development of representational fluency.** Following the analytic techniques detailed by Fonger (submitted), Annie’s development of representational fluency shifted from prestructural to unistructural representational fluency within the symbolic representation type, and from multistructural to relational connection across symbolic, graphic, numeric, and CAS-verbal representation types. This development in Annie’s sophistication is modeled in Table 3, where the Rule of Four web is modeled as a diagraph between vertices for symbolic (S), verbal (V), graphic (G), and numeric (N) representation types. Also see Fonger et al. (in preparation) for an elaboration of these results.
Table 3
Annie’s development of representational fluency in solving an equation with infinite solutions

<table>
<thead>
<tr>
<th></th>
<th>Initial</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td></td>
</tr>
</tbody>
</table>

**Prestructural**
- Annie demonstrated inconsistent and incorrect strategies for combining like terms in symbolic equations.
- In the initial attempt, common terms on either side of the equal sign were added together to yield an expression.
- In the final attempt, she added \(-2 + 2\) to get \(-4\), and concluded this is the solution.

**Unistructural**
- Annie was able to see how the case of infinite solutions could be signified by a symbolic identity equation in which the expressions related by the equal sign were “the same.”

**Multistructural**
- Annie did not consider the use of multiple representations on her own, when prompted to graph two specified equations, she used her graphing calculator to view a table, from which she anticipated a correct graph.
- The “sameness” in numeric and graphic types was not meaningful with respect to the solution to the equation, nor within the symbolic equation itself.

**Relational Connection**
- Annie self-prompted the use of a graphic representation to overcome barriers she had encountered in her first approach within the symbolic representation type.
- Annie coordinated the solutions of \(x = 0\) and \(x = 2\) from symbolic to verbal and symbolic to numeric table.

To illustrate Annie’s development of representational fluency, consider an example from a final interview in which Annie treated the equation \(2 - 2x = -2x + 2\) as a relationship between two expressions. As given in Vignette III, Annie graphed and viewed a function table for \(f1(x) = 2 - 2x\) and \(f2(x) = -2x + 2\) to conclude “they have infinite solutions.”

**Vignette III**

*Annie: But if you take (trails off…) Cause I know, hold on, I need a graph for this.*

*(Presses Home, Add Graph page)*

*Researcher: Why do you need a graph? Can you tell me about that?*
Annie: Because if I take both of them (points to $2 - 2x$ and $-2x + 2$) which is probably going to be equal in the graph, it may help.
Annie: (Types $2 - 2x$ into $f_6(x)$, Enter, Tab, types $-2x + 2$ into $f_7(x)$, Enter, mumbles to self as she types).

[Graph image]

Annie: [Looking at graph above] (Deep inhalation of breath) That's my problem! Researcher: What do you mean?
Annie: Well, they have (presses control T) infinite solutions (cursors in table from $f_6(0) = 2$ to $f_7(0) = 2$ [see screen below]).

[Function table image]

In this example, Annie drew meaning about solving an equation with infinite solutions ($2 - 2x = -2x + 2$) by creating a graph and function table. Her correct translation from symbolic to graphic and numeric representation types informed her correct conclusion about the nature of this equation having infinite solutions, relational representational fluency.

**Linking social and psychological analyses.** Based on the select findings presented above, I argue that a functions approach to solving equations and an activity structure for the coordination of tools supported Annie’s development of representational fluency. In particular, when solving an equation in the final interview, Annie compared symbolic expressions related
by an equal sign and translated to a function graph and table to discern the nature of the solution to the equation (Vignette III). This activity is compatible with the classroom instructional supports for a functions approach to solving (Vignette I). Annie’s use of her CAS was also consistent with the activity structure evident in classroom activity in which graphical representations were often used as a means to predict the nature of a solution to an equation (Vignette II). In Annie’s activity during the final interview, she created and interpreted graphs and function tables to overcome a sticking point she was not otherwise able to overcome from the symbolic representation type only (Vignette III). Taken together, these select findings demonstrate how social lenses on classroom activity and psychological lenses on individual activity can illustrate the reflexive nature of learning in classroom research.

**A Foundation for a Research Agenda**

This report of Fonger’s research activity here-to-fore highlights several aspects that support a strong foundation for a research agenda rooted in linking research and practice. Three themes are considered here: (a) address a problem of practice through disciplined inquiry, (b) explicate theoretical lenses to allow for synergy in practice and theory, (c) design for collaboration using both practice-based and research-based methods.

First, a key motivation behind the research reported herein was the following problem of practice: “How do we help students to be facile in moving among representations, including those created by technology?” (Arbaugh et al., 2010, p. 21). It was critical to the design and conduct of this research that both the researcher and collaborating teacher valued inquiry into this shared problem. Second, several analytic lenses were adopted to guide the research and experimentation in the classroom. For the case of the “Rule of Four,” both the researcher and teacher adopted this lens as a tool to inform this work. The teacher used this tool during
classroom experimentation to highlight the use of multiple representations in a functions approach to solving equations. The researcher used this tool together with the SOLO taxonomy to create visual models of Annie’s representational fluency. This example showcases how an analytic lens can be supportive of a linking research and practice agenda. Third, from a design-based research perspective, a major goal of this research was to engineer forms of learning and study that learning in the context in which it was supported (Gravemeijer & Cobb, 2006). It was critical that the author-researcher and participating teacher formed a partnership that built on each of their respective areas of expertise. An important example of how the teacher’s practice informed the research was the use of formative assessment tools such as warm-ups and exit tickets. These formative assessment practices informed our daily reflection on the classroom experimentation and often guided our decisions for how to support students in future instructional experimentation. Thus overall it was important that our research was collaborative and flexible to account for revisions based on both practice and theory.

In close, this research addresses two of NCTM’s strategic priorities (NCTM Board of Directors, 2013): (a) linking research and practice, and (b) promoting strategic use of technology in the classroom. Future research along this line of inquiry is needed to target and strengthen ties between classroom experimentation and individual cognition and activity in support of students’ representational fluency. The investigation of supports for students’ representational fluency in the context of technology-rich classroom settings continues to be a rich arena for addressing the needs of classroom practice and theory building, central to a linking research and practice agenda.
Part II: Emerging Issues in Bridging Experiences of Researchers and Practitioners

“From a [learning sciences] perspective, the persistent divide between research and practice is unacceptable. It hampers efforts to develop a robust science of learning, and it also interferes with the development of educational innovations that are compatible with educational institutions, organizations, and participants” (Nathan & Alibali, 2010, p. 3). Further supported by continued calls to bridge experiences of researchers and practitioners (Arbaugh et al, 2010; Heid et al., 2006; Langrall, 2014), the remainder of this paper focuses on the following questions: What are emerging issues in establishing researcher-practitioner partnerships? What are viable next steps in addressing these issues? These questions will be considered in turn from the lens of teachers, researchers, and partnerships. An elaboration on these ideas will also be explored in a series of forthcoming papers (e.g., Fonger, Strachota, Reiten, & Ozgur, in preparation; Lord, Stephens, Isler, & Fonger, in preparation). Note that in raising these potential emerging issues my intent is to raise awareness to the need to be more open and supportive of research in applied settings such as in classrooms or school districts.

Teachers Engaged in Research Partnerships

Potential issues. One potential issues that practitioners may face in engaging in research is that traditional school structures may not allow for sustained collaboration and community building even within the same school, let alone across schools in a district, or across institutions such as a college or university partnership. A second potential (or perceived) issue involves a concern for resources. Teacher’s time is precious, and there are high demands in the school culture and climate including standards, assessments, and accountability, which may hamper efforts to engage in research-based practices. In conversations with a current high school teacher, he expressed an interest in collaboration, yet described the struggle in achieving this: collaboration is certainly valued, but in
reality, it is hard to achieve … things are always being added to our plates, but nothing is being taken off (paraphrased personal communication, March, 2015).

**Viable next steps.** The teacher’s role in research has transitioned from the *impetus* of research, to a *consumer* of research, to *producers* of research (emphasis in original) (Van Zoest, 2006). Thus in positioning teachers as producers of research, it helps us to reframe the practices of teaching (listening, observing, and analyzing) from the lens of research, recognizing the commonalities across experiences of professionals in varied roles. Teaching can be described as authentic inquiry, yet both the process of engaging in inquiry and the product of that inquiry (e.g., one’s professional knowledge of curriculum and instructional supports for student learning) may remain tacit (Van Zoest, 2006). As researchers, we can learn from this professional knowledge and help to make it more explicit.

Another possible next step in supporting teachers’ engagement in research in the context of high demands and accountability is to reframe perceived roles, time commitments, and resources. A practitioner’s role in research can be very diverse and flexible from the principle or co-research, as in the study discussed by Fonger (2012, 2013, 2014), to a consumer of research or research participant. This level of participation may also vary based on the time scale from small-scale studies over the course of a few hours or days, to larger scale projects that span several years.

**Researchers Engaged in Partnerships with Practitioners**

**Potential issues.** One potential barrier to researchers engaging in partnerships with practitioners is administrative access to conducting research in schools. For example, a researcher may have goals to conduct disciplined inquiry within a certain context of the school system, yet may face real issues in finding a good fit in the local context of their institution. Another potential barrier to researchers engaging in linking research and practice is the value given to such research endeavors in their local context. For example, engaging in teacher-researcher
collaborations may not be valued in all research climates, especially under high accountability for gaining promotion and tenure.

**Viable next steps.** There are several viable next steps in addressing some of the aforementioned issues that researchers may face in pursuing productive partnerships with practitioners. First, there should be a balance in what motivates research; theory based concerns are important, but should not overshadow the importance of problems of practice. That is, school contexts may motivate research. I argue that the goals of theory-building and productive classroom practices are not diametrically opposed but are mutually supportive (e.g., see Fonger, 2012, 2013, 2014). Second, the choice of research methodology can be formal yet adaptive. Productive directions might include action research, lesson study, and design-based research or design experiments.

**Partnerships to Bridge Experiences of Researchers and Practitioners**

**Potential issues.** A researcher-practitioner partnership is a collaborative relationship. Like any relationship, collaboration takes time, is built on trust, and clear communication. The time-intensive nature of establishing a productive partnership may be a perceived barrier from the perspectives of both researchers and practitioners in an era of high accountability. Second, researcher partnerships may be messy and unpredictable, another potential barrier.

**Viable next steps.** One potential next step to addressing issues in forming productive researcher-practitioner partnerships is to view them as a form of linking research and practice that is a two-way street. Teachers and students may benefit from engaging in research-based practice, and researchers may benefit from teachers’ engagement in order to advance authentic inquiry into core issues. Second, in establishing partnerships, researchers and practitioners may find it productive to build from shared problems of practice. For example, the report by Arbaugh et al. (2010) may serve useful in considering the variety of questions that are important to practitioners.
Third, focusing on common practices might help to curtail surprises as learning opportunities, not setbacks. For example, some common aspects of the work of teachers and the work of researchers include the practices of listening, observing, analyzing, and reporting to various stakeholder audiences.

**Conclusion**

In this paper I gave examples of how a research agenda can be built around partnerships that address shared problems of practice, and framed these in the context of emerging issues in the field. Some possible next steps for practitioners, researchers, and partnerships have been identified to address these issues. Sustained efforts to bridge the experiences of mathematics education researchers and practitioners are advocated. Seeking out common ground to form researcher-practitioner partnerships is a productive way to advance a linking research and practice agenda.
References


CAS and paper-and-pencil


