Abstract Title Page
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Title:

Middle-grade students' misconceptions about the graphical representation of simple fractions: An Assessment from the Eliciting Mathematical Misconceptions Project (EM^2)

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Abstract Body

Limit 4 pages single-spaced.

Background / Context:

Description of prior research and its intellectual context.

Rational number concepts underpin many topics in advanced mathematics and understanding these concepts is a prerequisite for students' success in high-school level courses. Students with rational number misconceptions that are not diagnosed and remediated in the middle grades are likely to encounter difficulty in high-school mathematics courses (Bozick, Ingels, & Owings, 2002; Lesh, Post, & Behr, 1988; National Mathematics Advisory Panel, 2008; National Research Council, 2001). In spite of the importance of rational number concepts, many students and adults in the United States have considerable difficulty with fractions and decimals (Hecht, Vagi, & Torgeson, 2007; Mazzocco & Devlin, 2008; Moss, 2005; National Assessment of Educational Progress, 2007). The goal of the Eliciting Mathematical Concepts (EM^2) project is to develop diagnostic assessments that target rational number misconceptions that research indicates are common among middle-grades students.

Purpose / Objective / Research Question / Focus of Study:

Description of the focus of the research.

EM^2 is a measurement project funded by the U.S. Department of Education's Institute of Education Sciences. The project is developing short, highly focused cognitive diagnostic assessments that will be available to educators as an open-source online assessment system. EM^2 assessments target grades 5-7 rational number concepts related to fractions, decimals, and operations with fractions and decimals. While other diagnostic assessment systems exist, few focus on the identification and categorization of specific underlying student conceptions—including misconceptions, over-generalizations, and systematic sets of errors—that are the focus of the EM^2 Project.

The proposed poster presentation will focus on the EM^2 assessments that target student misconceptions about how fractions can be represented as two-dimensional images. Representing fractions is one of the significant mathematical concepts the National Council of Teachers of Mathematics (NCTM, 2006) identifies in its publication *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*.

EM^2 team members have diverse backgrounds in data analysis and psychometrics, K-12 mathematics education, professional development for K-12 mathematics teachers, assessment development, and the use of educational technology to support teaching and learning in the classroom. The EM^2 members will develop an interactive poster session in which we present the cognitive and educational research underlying the project, the assessments, and current findings that support our hypotheses about the appropriate structure for these assessments. We will also present our findings regarding the prevalence of the targeted misconceptions in the sample of students who have completed the assessments to date. The session will be designed to engage those who view the poster online and in-person in a conversation about the goals of the EM2 project using the *representing fractions* assessments as examples of the work thus far.

Setting:

Description of the research location.

EM2 is working with multiple school districts in New England to administer the assessments in classroom settings.

Population / Participants / Subjects:
Description of the participants in the study: who, how many, key features, or characteristics.

To date, EM2 has administered versions of the *representing fractions* assessments to over 700 fifth-grade students attending five public schools in a New England school district. The poster session will present findings from the most recent wave of data collection, which includes 132 fifth-grade students. [A large-scale pilot test of the assessment is scheduled for May 2013. At present, the authors assume that the poster will present findings from this wave of data collection and analysis. However, the information presented in this structured abstract is limited to the analysis of data collection and analysis to date.]

Intervention / Program / Practice:
Description of the intervention, program, or practice, including details of administration and duration.

Each EM^2 assessment is designed to identify 1-3 misconceptions based on the pattern of students' selected responses, not simply on the number of correct or incorrect answers. The *representing fractions* assessments present students with items that include a simple fraction and the image of a shape with a shaded region. To complete the assessment, students first provide a selected response by answering "yes" or "no" to the question "Is the shaded part X?", as well as a constructed response explaining their answer using words or pictures. Figures 1 and 2 are current versions of the two *representing fractions* assessments (please insert Figure 1 here).

The *representing fractions* assessments are developed to identify students who apply the following misconceptions about the graphical representation of fractions:

> Misconception 1: Viewing the numerator as the number of pieces shaded and the denominator as the number of total pieces without attending to whether or not the shape is divided into equal size regions or pieces.
>
> Misconception 2: Not recognizing the size of a region relative to the whole when (a) the regions are different shapes, but are the same size or (b) one or more partitions distinguishing regions are added or taken away.

The hypothesized pattern of answers identifying the targeted misconceptions for the two assessments are presented in Table 1 and Table 2 (please insert Table 1 and Table 2 here).

The EM^2 assessments are designed to classify students as being likely to have (or not have) a targeted misconception based on their pattern of selected responses. While the EM^2 study team is currently using students' constructed responses as part of the validation process, the reporting feature of the planned online assessment system will include students' open responses so that teachers can refer to students' explanations to confirm this classification prior to providing instruction to remediate the misconception.

Research Design:
Description of the research design.

Not applicable.

Data Collection and Analysis:
Description of the methods for collecting and analyzing data.

There are two *representing fractions* assessments. Assessment 1 is designed to identify misconception 1 and Assessment 2 is designed to identify misconception 2. Teachers administer the assessments to students on separate days; and students complete each assessment in under 20 minutes. Assessment data for this presentation was collected using paper-and-pencil assessments; the online assessment system is in production.

The two *representing fractions* assessments are designed to be administered in conjunction with each other. Students complete the second assessment only if the results from assessment 1 indicate that they do not have misconception 1. The reason for this is that a student with misconception 1 focuses only on the number of shaded regions without attending to whether the regions are of equal size. As a result, any student with misconception 1 will also have misconception 2 and will not recognize that it is the size of the shaded region that is relevant, not whether the shaded regions have different shapes or are missing partitions.

The project is using diagnostic cognitive modeling (DCM) methods described by Rupp, Templin, and Henson's book on diagnostic measurement (2010). While the longer-term goal of the project is to use more sophisticated DCM analysis to empirically confirm the hypothesized structure of the assessments, analyses conducted to date have focused on qualitative scoring conducted by expert coders and item-level descriptive statistics (including the Kullback-Liebler Information index).

Findings / Results:
Description of the main findings with specific details.

The poster session will present information about the EM2 project's overall goals for developing diagnostic assessments, as well as findings from data collected to date. Qualitative coding conducted by trained coders indicates that 18.9% of the participating students (25 of 132) have misconception 1. Of the remaining, 18% had misconception 2.

Conclusions:
Description of conclusions, recommendations, and limitations based on findings.

Our goal for the poster session will be to introduce attendees to the goals of the EM2 project and to engage with them in a discussion about the structure of the *representing fractions* assessment and the current findings, as well as the process through which this interdisciplinary team is working to develop diagnostic assessments.

Appendices

Not included in page count.

Appendix A. References

References are to be in APA version 6 format.

Bozick, R., Ingels, S. J., & Owings, J. A. (2002). *Mathematics Coursetaking and Achievement at the End of High School: Evidence from the education longitudinal study of 2002* (NCES 2008-319): National Center for Education Statistics USDOE.

Hecht, S. A., Vagi, K. J., & Torgesen, J. K. (2007). Fraction skills and proportional reasoning. In D. B. Berch & M. M. M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 121–132). Baltimore: Paul H. Brookes Publishing Co.

Lesh, R., Post, T. R., & Behr, M. (1988). Proportional reasoning. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 93–118). Reston, VA: National Council of Teachers of Mathematics.

Mazzocco, M. M. M., & Devlin, K. T. (2008). Parts and holes: Gaps in rational number sense among children with vs. without mathematical learning disabilities. *Developmental Science, 11*(5), 681-691.

Moss, J. (2005). Pipes, tubes, and beakers: New approaches to teaching the rational-number stem. In S. Donovan & J. Bransford (Eds.), *How students learn: Mathematics in the classroom* (pp. 309-349). Washington, D.C.: The National Academies Press.

National Assessment of Educational Progress. (2007). Main NAEP Data Explorer. Retrieved from: http://nces.ed.gov/nationsreportcard/naepdata/

National Council of Teachers of Mathematics. (2006). *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A quest for coherence*. Reston, VA: National Council of Teachers of Mathematics.

National Mathematics Advisory Panel. (2008). *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*. Washington, D.C.: U.S. Department of Education.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, and B.Findell (Eds.), Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Rupp, A. A., Templin, J., & Henson, R. J. (2010). *Diagnostic measurement: Theory, methods, and applications.* New York: Guilford Press.

Appendix B. Tables and Figures

Not included in page count.

Figure 1.

Please do NOT write your name on this paper.

Representing Fractions I

MARKING INSTRUCTIONS

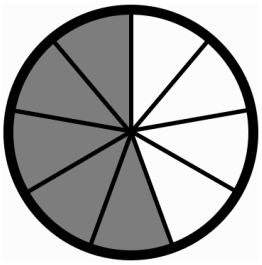
CORRECT: ● **INCORRECT:** ⊘ ⊠ ◒ ◔

- **Use a No. 2 pencil or a blue or black ink pen only.**
- **Do not use pens with ink that soaks through the paper.**
- **Make solid marks that fill the response completely.**
- **Write darkly in "Explain" areas so it can be easily read.**

In each problem below there is a bold outline around the whole figure.

Decide if the shaded part is equal to the fraction given.

1. Is the shaded part $\frac{5}{9}$?

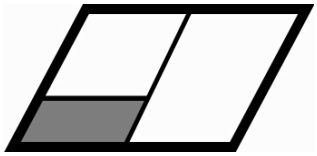


○ Yes

○ No

Explain your choice using words and/or pictures.

2. Is the shaded part $\frac{1}{3}$?

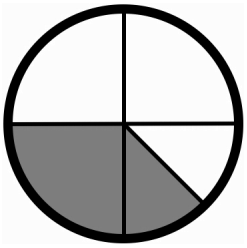


○ Yes

○ No

Explain your choice using words and/or pictures.

3. Is the shaded part $\frac{2}{5}$?

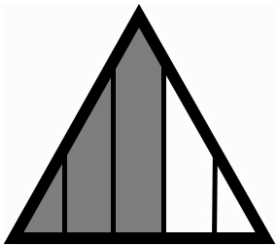


○ Yes

○ No

Explain your choice using words and/or pictures.

4. Is the shaded part $\frac{3}{5}$?



○ Yes

○ No

Explain your choice using words and/or pictures.

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Figure 1 (continued).

Representing Fractions I

2 of 2 pages

MARKING INSTRUCTIONS	
CORRECT: ●	INCORRECT: ⊘ ⊗ ◒ ◔

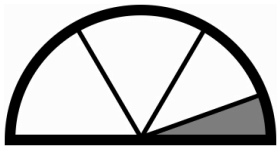
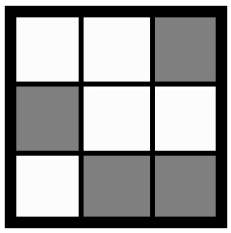
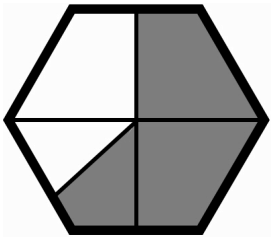
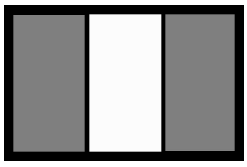
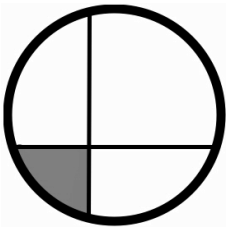
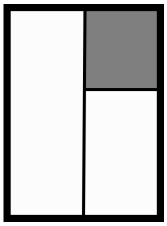
5. Is the shaded part $\frac{1}{4}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.
6. Is the shaded part $\frac{4}{5}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.
7. Is the shaded part $\frac{3}{5}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.
8. Is the shaded part $\frac{2}{3}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.
9. Is the shaded part $\frac{1}{4}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.
10. Is the shaded part $\frac{1}{3}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.

Figure 2.

Please do NOT write your name on this paper.

Representing Fractions II

MARKING INSTRUCTIONS

CORRECT: ● **INCORRECT:** ⊘ ⊠ ◒ ◔

- **Use a No. 2 pencil or a blue or black ink pen only.**
- **Do not use pens with ink that soaks through the paper.**
- **Make solid marks that fill the response completely.**
- **Write darkly in "Explain" areas so it can be easily read.**

In each problem below there is a bold outline around the whole figure.

Decide if the shaded part is equal to the fraction given.

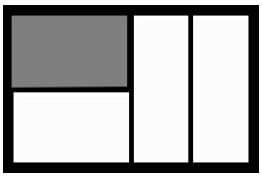
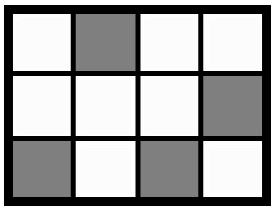
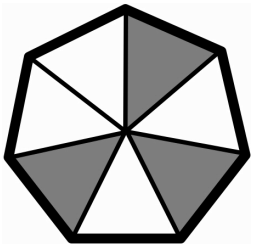
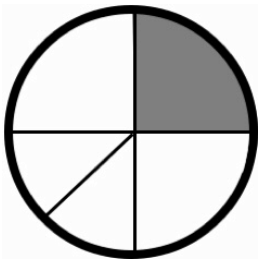
1. Is the shaded part $\frac{1}{4}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.
2. Is the shaded part $\frac{1}{3}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.
3. Is the shaded part $\frac{3}{7}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.
4. Is the shaded part $\frac{1}{4}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.

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Figure 2 (continued).

Representing Fractions II

MARKING INSTRUCTIONS
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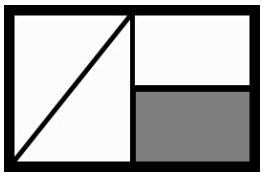
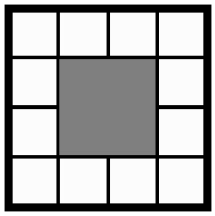
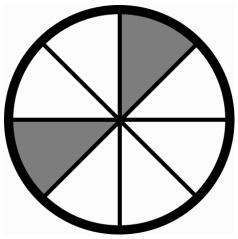
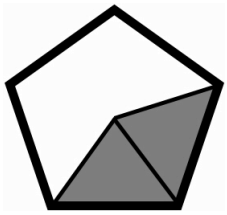
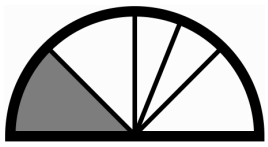
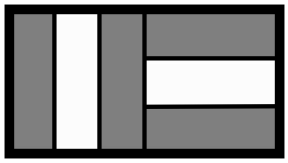
5. Is the shaded part $\frac{1}{4}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.
6. Is the shaded part $\frac{1}{4}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.
7. Is the shaded part $\frac{1}{4}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.
8. Is the shaded part $\frac{2}{5}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.
9. Is the shaded part $\frac{1}{4}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.
10. Is the shaded part $\frac{2}{3}$? ○ Yes ○ No 	Explain your choice using words and/or pictures.

Table 1 Representing Fractions Pattern Table for Misconception 1

Correct Response	Y	N	N	N	N	N	N	Y	N	N
Item #	**1.**	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
Response Pattern Misconception 1: Part 1) Views the numerator as the number of pieces shaded and the denominator as the number of total pieces- doesn't pay attention to equal size pieces (regions)	**<u>Y</u>**	Y	**Y**	**Y**	**Y**	**<u>N</u>**	**Y**	**<u>Y</u>**	Y	Y
Item Characteristics										
• Baseline understanding part to whole, equal size pieces (continuous)	√									
• Baseline understanding part to whole, equal size pieces (discontinuous)						√		√		
• Misconception -Different sized parts (unit fraction)		√			√				√	√
• Misconception -Different sized parts (continuous)			√	√			√			

Table 2. Representing Fractions Pattern Table for Misconception 2

Correct Response	Y	Y	Y	Y	Y (1/4)	Y	Y (1/4)	Y	Y (1/4)	Y (2/3)
Item #	**1.**	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
			7 sided 2 shaded				Circle 2/8 shaded		Semicircle	
Response Pattern Misconception 2: Does NOT recognize size of region (area) compared to the whole	N	N	Y	N	N	N	N	N	N	
Item Characteristics										
• Baseline understanding – discontinuous, equal sized part			√				√			
• M2 - Lacks recognition of equivalent fractions		√				√	√			√
• M2 - Does not recognize size of region when looking at different shape, same size	√				√					√
• M2 -Does not realize you can add or take away partitions – look at area compared to the whole				√				√	√	