Abstract Title Page

Title: Integrating Non-Mathematical Domains into Mathematical Development: Key Factors to Consider in Constructing Effective Interventions

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Abstract Body

Background:

The successful acquisition and development of mathematics skills and concepts is a critical aspect of children's early academic growth (Baroody, Lai, & Mix, 2006; Jordan, Hanich, & Uberti, 2003). The cumulative nature of early mathematics development—later competencies building on earlier ones—underscores the need for early prevention and intervention with children at-risk of developing mathematics difficulties. *To best identify children at risk of later mathematics difficulties and design effective interventions that provide maximal benefit to these children, it is critical to link components from diverse braches of the educational sciences in understanding mathematical development. Specifically, identifying malleable non-mathematical factors that impact mathematics development and understanding their individual relations to the acquisition of mathematical knowledge is a necessary component to conduct before developing interventions.* Although early mathematical knowledge is the critical basis for developing more advanced knowledge (De Smedt, Verschaffel, & Ghesquiere, 2009; Jordan, Kaplan, Locuniak, & Ramineni, 2007), it is evident that a range of both mathematical and non-mathematical factors (e.g., working memory and language) impact children's early mathematical development (Fuchs et al., 2005; 2008; 2010; Gathercole, Pickering, Knight, & Stegmann, 2004; Jarvis & Gathercole, 2006; Purpura, Hume, Sims, & Lonigan, 2011; Raghubar, Barnes, & Hecht, 2010); however, there is a dearth of research investigating the links of working memory and language to *specific* aspects of informal mathematics skills at the preschool and kindergarten ages. Identifying whether these two domains have specific or general relations to early mathematical skills and concepts will provide a foundation by which to base the construction of interventions targeting mathematical growth.

Research Question:

The purpose of this study was to systematically evaluate the unique relations of working memory and language to a range of specific early mathematics skills in a sample of preschool and kindergarten age children. It was hypothesized that language skills have a general relation to all (or most) of the early mathematics skills and concepts because children at this age are in the process of linking number-words, quantities, and symbols. Further, it was also hypothesized that the relation of working memory and early mathematics will only be found in those skills or concepts that require multiple steps or the integration of multiple earlier skills and concepts.

Setting:

Data were collected in 45 public and private preschools and elementary schools. Children were assessed on all tasks in the spring of the academic year. Individuals who had either completed or were working toward completion of a Bachelor's degree in psychology or education conducted the assessments. Assessments took place in the local preschools or kindergarten classrooms during non-instructional time in a quiet room designated by the individual school directors or teachers.

Participants:

The 199 children who completed all assessments were evenly split by sex (51.8% female) and approximately representative of the demographics of the area (59.8% Caucasian, 28.6% African-American, and 11.6% other race/ethnicity). Approximately half the children (*n* = 106) were in kindergarten and the others (*n* = 93) were in their second year of preschool. Children ranged in age from 4.05 years to 6.83 years (*M* = 5.54 years, *SD* = .75 years), were primarily English speaking, and had no known developmental disorders.

Data Collection and Analysis:

Data Collection: Children were assessed on 10 measures of early mathematics skills/concepts. The specific tasks selected assess skills from the different levels of informal knowledge noted earlier and/or other commonly used measures of early mathematics skills that have been found to be strong predictors of later mathematics. These tasks include: verbal counting, one-to-one counting, cardinality, subitizing, number comparison, set comparison, number order, numeral identification, set-to-numerals, and story problems. Children were also assessed on the Expressive One-Word Picture Vocabulary Task – Third Edition (EOWPVT; Brownell, 2000), the Last-Word Verbal Working Memory Test, and the Woodcock Johnson-III Calculation Subtest (Woodcock et al., 2001). All tasks had acceptable reliability.

Data Analysis: To evaluate the relations of language and working memory with specific early mathematics skills, a series of separate mixed-effects regression analyses (Raudenbush & Bryk, 2002) were conducted. In each of the analyses, school was included as a random effect to account for variance across schools. There were a total of 45 schools with an average of 4.4 children per school. Age, grade (kindergarten or preschool), gender, and the WJ-III Calculation Subtest were included as fixed effects covariates. Working memory and language tasks were included in the model as fixed effect predictors. Benjamini-Hochberg corrections were applied within each regression analysis to correct for multiple comparisons.

Results:

Means, standard deviations, skew, and kurtosis are presented in Table 1. All tasks were normally distributed and no tasks exhibited significant skew or kurtosis. Correlations between tasks are presented in Table 2. Ten separate mixed-effect regression analyses were conducted. In Table 3, a summary of the significance values for predicting each early mathematics skills is presented. All analyses were conducted using raw scores. When analyses were conducted with age-regressed standardized scores, the results were comparable to those presented here. Further, when analyses were conducted without controlling for broad calculation ability the results were also consistent with the presented results.

Working Memory. Working memory was only a significant predictor of three of the ten early mathematics skills and concepts assessed: cardinality (counting a subset), set comparison, and number order. Working memory was a marginally significant predictor of subitizing.

Expressive Language. Language was a significant predictor of nearly all mathematics skills and concepts with the exception of one-to-one counting and subitizing. However, it was a marginally significant predictor of one-to-one counting.

Covariates. Age was a significant or marginally significant predictor of nearly all early mathematics skills indicating that older children performed better than younger children. Grade was not a significant predictor of any early mathematics skills indicating that grade and level of instructional focus at school did not significant alter the results. After the application of the Benjamini-Hochberg correction for multiple comparisons, sex was only a marginally significant predictor of verbal counting, one-to-one counting, cardinal knowledge, and story problems. Finally, the calculation task was significantly related to all early mathematics skills with the exception of numeral identification, set-to-numerals, and one-to-one counting.

Conclusions and Educational Implications:

The results of this study suggest that both language skills and working memory are related to a range of early mathematics skills. Although both domains are correlated with all early mathematics skills that were assessed, the nature of these relations differs when accounting for the other domain and controlling for background variables. Notably, language skills appear to have a *general* relation to early mathematics skills because it was significantly related to nearly all early mathematics skills that were assessed. In contrast, working memory has a *specific* relation to aspects of early mathematics because it was significantly related to only a few individual mathematics skills. These differential relations are important to consider in the broader development of models of early mathematics development as well as in the development of curricula and instructional techniques. Further, these findings underscore the importance of evaluating relations between individual mathematics skills and non-mathematical domains at a relatively fine-grained level.

In building a learning trajectory of mathematics development, it is important to understand how different non-mathematical factors integrate into the model. The general relation of children's language skills with all of the early mathematics skills was not unexpected, but is critical to understand. The finding was not unexpected because, for each early mathematics skill, children must either (a) know number names (word knowledge which is inherently a vocabulary task), (b) connect the number names with specific quantities (connecting word knowledge with their "definition" or "meaning"), (c) connect the number names with numerals, (d) both b and c, or (e) understand the meaning of comparative terms. This relation between language and mathematics is critical to understand because language skills could play a key role in the acquisition of new knowledge and the integration of that knowledge with prior knowledge. As language is a strong predictor of general early mathematics development (Lefevre et al., 2010), and language appears to underlie each of the aspects of early mathematics, when language is underdeveloped, it is likely to be an impediment in the successful acquisition of early mathematical knowledge. As such, it may be necessary to account for children's language skills when developing and individualizing interventions and general instruction. However, further work needs to be conducted to determine if it is general language skills that account for this relation or if there are specific aspects of *mathematical* language for which general language is acting as a proxy.

The relation of working memory to mathematical learning trajectories appears to be more specific than the relation of language. Similar to prior findings with children in elementary school (Mannamaa et al., 2011; Nyroos & Wiklund-Hornqvist, 2012), working memory skills are only related to a few early mathematics skills. Interestingly, the specific early mathematics skills that are related to working memory are typically viewed as some of the strongest predictors of later mathematics success (Kroesbergen, Van Luit, Van Lieshout, Loosbroek, & Van de Rijt, 2009; Palmer & Baroody, 2011; Sarnecka & Carey, 2008). Each of these tasks also requires children to complete multiple steps before solving. For example, to complete the cardinality task, children must not only hold the requested set size (or the type of items to be counted in the second part of that task) in their memory, but they must perform the act of counting out a set until they reach the specified, or cardinal, number. Similarly, in the set comparison task children needed to be able to enumerate each of the four sets, hold the total set sizes in their memory, then identify which set was the largest. Even though the set comparison and the numeral comparison tasks are similar in overall task structure (e.g., identifying "most" or "fewest"), working memory did not significantly predict the numeral comparison task. This difference is likely because the working memory load is lessened in the numeral comparison task because children do not have to enumerate the quantity of each number because it is already provided. The relation of working memory to these specific early mathematics skills may indicate that the instructional methodology utilized for these skills may need to be distinct from the instructional methodology utilized for the other skills that were not related to working memory. Given the significant working memory demands for specific skills, it may be important to account for children's working memory skills in the instruction of these specific domains by either (a) modifying classroom curricula or individualized instructional activities (by reducing task demands or including instruction aids) to minimize the impact of working memory deficits during learning, or (b) intervening on children's working memory skills separate from the mathematics instruction.

It is also important to note specific patterns that were found regarding the covariates. To begin, grade was not a significant covariate for any of the measures. Second, the general mathematics measure was a significant or marginally significant predictor of all variables except for one-to-one counting, numeral identification, and set to numerals. Other than age, language skills was the only variable to significantly predict the numeral identification and set-to-numerals tasks—suggesting that these skills may actually be better defined as a distinct language-based skill rather than "mathematical" skills. Prior research has indicated that these two domains—together termed numeral knowledge—mediate the relation between informal and formal mathematics skills (Purpura, Baroody, & Lonigan, 2013). Such a distinction may provide one specific connection between mathematics and reading development—particularly in identifying potential pathways linking mathematics and reading difficulties.

Overall, the findings from this study provide a unique framework for the evaluation of early numeracy skills that can be utilized to enhance both research and teaching of early mathematics development. By understanding the connections between mathematical and non-mathematical constructs, a model learning trajectory for mathematical development can be delineated—and such a model can be used to guide instruction through the identification of which mathematical and non-mathematical skills have causal relations in their development. The links of working memory and language to specific mathematics skills set forth a targeted set of future research that can help address important developmental and instructional issues—particularly those related to instructional methodology and sequencing in preschool and kindergarten.

Appendices

Not included in page count.

Appendix A. References

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Appendix B. Tables and Figures

Table 1

Means, Standard Deviations, Range, Skewness, and Kurtosis of the sum scores of the mathematics, language, and working memory tasks

Task	Mean	SD	Range[a]	Skew	Kurtosis
Mathematics					
Verbal Counting	5.18	1.88	0 - 7	-.83	-.60
One-to-one Counting	4.32	1.07	0 – 5	-1.76	2.76
Cardinality	6.17	1.89	0 – 8	-1.12	.88
Subitizing	4.80	1.31	0 – 7	-.20	.00
Number Comparison	3.78	2.03	0 – 6	-.43	-1.17
Set Comparison	4.99	1.51	0 – 6	-1.51	1.34
Number Order	4.46	1.96	0 – 6	-1.14	.16
Number Identification	7.48	2.13	0 – 9	-1.39	1.21
Set to Numerals	4.00	1.24	0 – 5	-1.29	1.12
Story Problems	4.67	2.01	0 – 7	-.57	-.71
WJ-III Calculation	3.51	3.70	0 – 16	.95	.36
Non-Mathematics					
One-Word Picture Vocabulary	49.21	17.06	0 – 91	-.20	-.59
Working Memory	15.24	7.56	0 – 28	-.66	-.70

Note. N = 199.

[a] The range indicates both the possible and actual range of scores for all tasks except the WJ-III Calculation task, the One-Word Picture Vocabulary task, and the Working Memory task. For these three tasks, only the actual range is presented because the tasks are designed for wide age-ranges of individuals.

Table 2

Correlations between the sum scores of all tasks

	1	2	3	4	5	6	7	8	9	10	11	12	13
1. Verbal Counting	-												
2. One-to-one Counting	.62	-											
3. Cardinality	.70	.63	-										
4. Subitizing	.38	.44	.45	-									
5. Number Comparison	.63	.64	.62	.49	-								
6. Set Comparison	.46	.53	.55	.42	.62	-							
7. Number Order	.64	.63	.76	.50	.66	.64	-						
8. Number Identification	.75	.59	.72	.41	.63	.51	.76	-					
9. Set-to-Numerals	.52	.54	.66	.42	.57	.57	.89	.61	-				
10. Story Problems	.59	.59	.62	.49	.62	.63	.65	.58	.55	-			
11. Working Memory	.37	.32	.50	.37	.40	.45	.55	.42	.40	.42	-		
12. Language	.52	.62	.56	.44	.63	.52	.62	.57	.52	.57	.52	-	
13. Calculation	.55	.57	.52	.44	.60	.48	.60	.51	.42	.59	.41	.58	-

Note. N = 199. All correlations were significant at $p < .001$.

Table 3
Significance of predictor variables and covariates in the mixed-effect regressions for each early mathematics skill.

Task	Age	Grade	Sex	Calculation	Working Memory	Language
Verbal Counting		+	+	*		+
One-to-one Counting	+		+			+
Cardinal Knowledge	*		+	*	*	*
Subitizing	+			+	+	
Numeral Comparison	*			*		*
Set Comparison	+			+	*	*
Number Order	*	+		*	*	*
Numeral Identification	+					*
Set to Numerals	*					*
Story Problems	*		+	*		*

Note. N = 199. All *p*-values not included in the table were greater than $p = .100$.
*indicates a significant predictor after applying the Benjamini-Hochberg correction
+indicates a marginally significant predictor after applying the Benjamini-Hochberg correction.