Abstract Title Page

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Title:
Perceptual Learning in Early Mathematics: Interacting with Problem Structure Improves Mapping, Solving and Fluency

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Abstract Body

Limit 4 pages single-spaced.

Background / Context: *Description of prior research and its intellectual context.*
Mathematics is the study of structure but students think of math as solving problems according to rules. Given a word problem, students might launch into calculating the correct answer – but mathematicians, given the same problem, may consider finding out classes of similar problems or representing such problems in an efficient and principled way. Students' perception that math is about computing actually reflects a regularity present throughout their math experiences; they receive much instruction and practice on solving problems with immediately presented rules (Schoenfeld, 1989; Stigler & Hiebert, 2009). To solve, however, students must (at least implicitly) understand structure (Kellman & Massey, 2013). Repetitive practice with solving structurally identical problems reduces the need to understand structure.

Students can learn procedures, but they often have trouble knowing *when* to apply learned procedures, especially to problems unlike those they trained with (e.g., Gick and Holyoak, 1983). Here we rely on the psychological mechanism of *perceptual learning* (PL), our natural ability to extract invariant information across multiple learning experiences (Gibson, 1969; Kellman & Garrigan, 2009), to bring about this early appreciation of structure.

In two studies, we examined the effectiveness of a PL approach to word problems, and to explore the relationship between understanding structure and solving. We designed and tested a PL-based intervention called Math Problem Insight (MPI). In MPI, elementary school students practice solving and representing the structure of many word problems that varied along contextual and mathematical dimensions. The goal of MPI is to encourage the learner's own attentional system to highlight and extract invariant structure from varied contexts, by providing variations of verbal structures that map onto related abstract representations.

Purpose / Objective / Research Question / Focus of Study: *Description of the focus of the research.*
The main research questions were: (1) Can PL interventions in which representation is the goal (rather than computing) help students make better generalization and use symbols to represent situations? (2) Are students who are proficient at solving are equally proficient in structure representation?

In Study 1, we measured students' improvements in solving and representing the structure of word problems from solving and mapping practice with a PL intervention. The study also examined whether students were equally adept at solving and representing equivalent problems.

In Study 2, we randomly assigned students to have only either representing experience (the Mapping condition) or solving experience (the Solving condition) during the PL intervention, to examine whether these particular short exercises led to task specific or general improvements with word problems.

Setting: *Description of the research location.*
In Study 1, third-grade homeschoolers enrolled in an online school in Idaho completed the study from home, as part of typical online math assignments. Each student was individually assigned login names and passwords so that they could complete the assignment at their own pace.

In Study 2, third-graders from an elementary school in a Midwestern town logged in from their school computers to complete the study. This school expressed interest in the MPI intervention because they had a decrease in the percentage of students passing mathematics (as measured by their state's standardized test) from 2011 to 2012. The teachers varied in how much class time was reserved for MPI. Most of the participants who completed the study (finished pretest, MPI learning trials, and posttest) came from a single class (18/21). The remaining three classes had fewer students complete the experiment (5/26, 2/26, 1/13).

Population / Participants / Subjects:
Description of the participants in the study: who, how many, key features, or characteristics.
In Study 1, 61 third-graders participated (31 in the experimental group, 30 in the comparison group. In Study 2, we included 26 third-graders who completed all portions of the study (10 in the Mapping condition, 16 in the Solving condition).

Intervention / Program / Practice:
Description of the intervention, program, or practice, including details of administration and duration.
The Math Problem Insight (MPI) was designed to promote PL of one-step word problems that can be solved with addition, subtraction, multiplication, or division of integers. Consistent with the principles of PL, MPI consists of many brief interactive trials incorporating systematic variations. Students were asked to respond to these trials in one of two ways: (1) by solving the presented word problem (Solve trials; Figure 1A shows an example) or (2) by choosing the symbolic expression or short sentence that correctly represented the situation out of four presented options (Map trials; Figure 2A shows an example). In Study 1, the MPI consisted of both Solve and Map trials. In Study 2, students were randomly assigned to a special version of the MPI that included only one type of learning trial (Solve or Map).

In MPI, there were 14 categories of addition/subtraction (AS) and multiplication/division (MD) word problems represented in the database of almost 2500 problems, defined by their isomorphic structures using the classification system developed by the Cognitively Guided Instruction (CGI) group (Carpenter et al., 1999). Within each AS and MD category, word problems were constructed from a variety of contexts (Please see Tables 1 & 2 for examples).

There was no explicit instruction. Instead, variant and invariant information were present across the trials. For a given mathematical relationship, multiple classification trials revealed structural invariance in changing contexts. Across different types of trials, students had to interact with different structures, allowing contrasts and relations to emerge. Incorrectly answered trials (of all types) were followed with animated feedback showing a horizontal bar visualization that modeled the situation (shown in Figures 1B and 2B). Corrective feedback was designed to highlight structural components in a similar way across trials.

Problems were presented in an mixed fashion using the ARTS adaptive learning system (Mettler, Massey & Kellman, 2011), in which accuracy and speed of response to instances of each category were used to space and sequence subsequent learning trials uniquely for each learner. ARTS also implements objective learning criteria involving accuracy, speed, and maintenance of proficiency across spacing intervals. Students finished the MPI learning trials when they correctly answer at least 4 out of 6 trials of a particular category and in roughly 30 seconds per trial (speed criteria varied depending on the text length in the word problem).

Research Design: *Description of the research design.*
Study 1: pretest– posttest design with a matched comparison group. The comparison group received conventional mathematics lessons that overlapped with the MPI content in terms of using basic mathematical operations in problem solving, and took the online assessment only at the time of posttest.

Study 2: random assignment experimental design. One group participated in the MPI with only Solve trials. The other group participated in the MPI with only Map trials.

Data Collection and Analysis: *Description of the methods for collecting and analyzing data.*
In both studies, the pretest and posttest were a representative sampling of problems from all categories, incorporated seamlessly as part of the MPI experience. The pretest was administered prior to learning trials, and the posttest was presented after the participant retired their last category. No feedback was given on pretest and posttest trials. The trials on the pretest, posttest, and MPI consisted of Solve, Unknown-Left Map (when the unknowns are depicted on the left of the equal sign), and Unknown-Right Map problems (when the unknowns are on the right of the equal sign). All posttest problems were novel instances of the structures experienced during MPI. We recorded and analyzed the accuracy and response time of each trial in both assessments and during the MPI.

Study 1: (1) Can we train students to accurately solve and represent word problems using principles of PL? We compared pretest and posttest performance of the experimental group using 2 (Test: pretest, posttest) x 3 (Trial-type: Solve, Unknown-Left Map, Unknown-Right Map) repeated measures ANOVAs on accuracy and response times, and compared posttest performance between the comparison and the experimental groups using 2 (Condition: Solving, Mapping) x 3 (Trial-type) mixed ANOVAs. (2) To examine whether students' solving proficiency relates to their ability to represent mathematical structure, we compared their accuracies in Solve trials and in the two types of Map trials during the MPI learning trials with a 3 (Trial-type) x 2 (Operations: AS, MD) repeated-measures ANOVA on accuracy.

Study 2: Because of the limited time allotted for MPI during class time, our data may have included a greater proportion of highly proficient students who were able to complete the MPI. Because of this possibility, we included performance on the MPI learning trials (proportion of correct trials) as a covariate. To determine which version of MPI improved students' performance, accuracy was examined with a 2 (Condition) x 2 (Test) x 3 (Trial-type) mixed repeated measures ANCOVA. Condition was a between-subjects variable while Test and Trial-type were within-subjects variables.

Findings / Results: *Description of the main findings with specific details.*
Table 3 shows the results of Study 1, and Table 4 shows the results of Study 2.

Study 1: (1) Mapping of word problem to its algebraic representation is very difficult for students, but after MPI training, students improved in their ability to solve and represent word problems, especially more so in Solve trials than in both Unknown-Left and Unknown-Right Map trials, all $t(15) > 3.79$, $p < .003$, $d > .97$. There were also large changes in fluency from

MPI practice, as indicated by changes in RT of correct responses in all trials, $t(30) = 4.37$, $p < .001$, $d = .79$. Students initially took an average of 33 seconds per problem. After MPI, the average RT reduced to 20 seconds, a 38% drop in solution time. Compared to the comparison group, the MPI showed limited benefit of accuracy, but a resounding improvement in fluency with word problems, all $t(59) > 4.81$, $p < .001$, $d > 1.23$. On average, the MPI group only needed 19 seconds per problem while the comparison group spent 49 seconds on the same problems. (2) In the MPI learning trials, students were significantly more accurate in Unknown-Right than Unknown-Left Map trials, $F(1, 30) = 14.62$, $p < .002$, $\eta^2 = .33$; also, they were more accurate on Solve trials than both types of Map trials, $F(1, 30) > 28.27$, $p < .001$, $\eta^2 > .49$. It is not surprising that students generally struggle on Unknown-Left Map trials, but the difference between Solve and Unknown-Right Map trials is surprising: students can identify and compute an operation more easily than simply identifying it.

Study 2: Both solving and mapping activities contributed to learning. In particular, the Mapping condition improved in the Solve trials, $t(9) = 2.28$, $p = .049$, $d = .82$, and in Unknown-Right Map trials, $t(9) = 2.93$, $p = .017$, $d = .93$. The Solving condition also demonstrated improvements, namely on Unknown-Left Map trials, $t(15) = 2.75$, $p = .015$, $d = .71$. Solving practice contributed to improved mapping performance and mapping performance also fostered better solving.

Conclusions: *Description of conclusions, recommendations, and limitations based on findings.*
For elementary math students, identifying the operation for a word problem is a distinctly different task than computing the solution to the word problem. Training that emphasizes perceptual learning through mapping across multiple isomorphic representations improves students' pick up of abstract structure. Additionally, PL training with a particular task generalizes to other tasks that involve related mathematical structures. This replicates findings of other PL interventions in mathematics (e.g., Kellman et al., 2008; Kellman, Massey, & Son, 2009). Our results suggest a subtle advantage for mapping activities over simply solving. This is notable because mapping across representations are uncommon in elementary math pedagogy. An important difference between the two studies was that the first demonstrated general fluency gains from pretest to posttest while Study 2 did not (instead there was evidence of learning gains for specific types of trials). This may be due to the oversampling of highly proficient students (pretest RTs in Study 2 were similar to posttest RTs in Study 1). Additionally, these differences may be attributable to the characteristics of the population sampled in the two studies. These differences raise questions about the potential generalizability of online learning interventions (with minimal teacher/parent scrutiny) for a variety of populations.

Finally, although Studies 1 and 2 are difficult to compare directly due to different participant groups, the results were consistent with the possibility that the strongest PLM intervention effects occurred when mapping problems, requiring interactions with problem structures across representational formats, were mixed with solving practice also specifically designed to enhance PL. This result may have important implications for instructional interventions and deserves further investigation.

Appendices

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Appendix A. References

References are to be in APA version 6 format.

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Gick, M. L., & Holyoak, K. J. (1983). Schema induction and analogical transfer. *Cognitive Psychology, 15*, 1–38.

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Kellman, P. J., & Massey, C. M. (2013). Perceptual Learning, Cognition, and Expertise. In: B. H. Ross (Ed.), *The Psychology of Learning and Motivation* (pp. 117–165), Academic Press, Elsevier Inc.

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Mettler, E., Massey, C. M., & Kellman, P. J. (2011). Improving adaptive learning technology through the use of response times. In L. Carlson, C. Holscher, & T. Shipley (Eds.), *Proceedings of the 33rd Annual Conference of the Cognitive Science Society*. Boston, MA: Cognitive Science Society, 2532–2537.

Schoenfeld, A. H. (1989). Explorations of students' mathematical beliefs and behavior. *Journal for Research in Mathematics Education, 20*, 338.

Stigler, J. W., & Hiebert, J. (2009). *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom*. Free Press.

Appendix B. Tables and Figures

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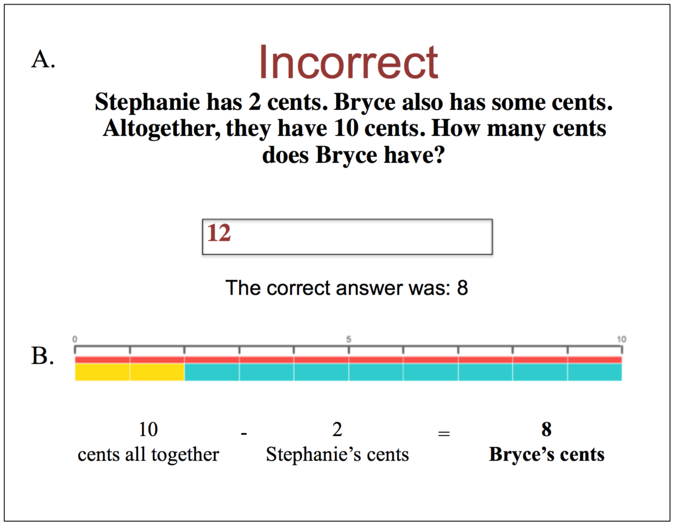

Figure 1. (A) Screenshot of a Solve trial showing a blank for the solution of the given word problem. (B) The animated feedback shown if the chosen option is incorrect.

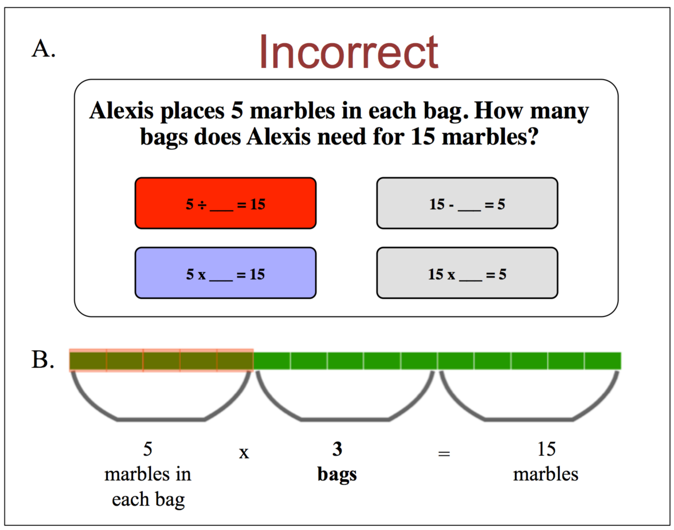

Figure 2. (A) Screenshot of a Map trial showing four possible options. (B) The animated feedback shown if the chosen option is incorrect.

Note: To help students make connections across problems, the structural components were consistently depicted. These structural components are given context specific labels relevant to each problem (i.e., instead of *units per group,* students would see *marbles per bag*). For instance, in the MD feedback animations (see Figure 2B), *units* were shown as small green squares; *groups* were shown as simple gray brackets that group the squares, and *units per group* were shown as a highlighted group of green squares in the left-most bracketed group. In the AD feedback animations (see Figure 1B for an example), the *start* value was depicted as a yellow bar, the *change* value was depicted in green, and the *result* was depicted in red.

Table 1. Example AS problems based on CGI classifications (Carpenter et al., 1999)

	Start Unknown (subtraction)	***Change Unknown (subtraction)***	***Result Unknown (addition)***
Join	Jesus has animal crackers. Brittney gave him 5 more. Now he has 13 animal crackers. How many did Jesus have to start with?	Karen has 8 crayons. Darin added some more crayons. Now Karen has 13 crayons. How many crayons did Darin add?	Evan has 8 stickers and gets 5 new stickers. How many stickers does Evan have now?
Separate	Alvin has 13 dollars and gives away 5 dollars. How many dollars does Alvin have now?	Nina had 13 brownies and gave away some of them. Now, she has 8 brownies. How many brownies did Nina give away?	Monica gives away 5 cents and now has 8 cents left. How many cents did Monica start with?
Part-Part-Whole		*[Part unknown]* Wyatt had 8 walnuts and some hazelnuts. In total, he has 13 nuts. How many hazelnuts does Wyatt have?	*[Whole unknown]* Ming has 8 sculpture and 5 paintings. How many art pieces does Ming have total?
Compare	*[Quantity 1]* Olivia became 5 weeks older. Olivia is now 13 weeks old. How old, in weeks, was Olivia before?	*[Difference]* Leslie has 8 roses. Arianna has 13 roses. How many more roses does Leslie have than Arianna?	*[Quantity 2]* Michael walked 8 kilometers. Chloe walked 5 more kilometers than Michael. How far, in kilometers, did Chloe walk?

Table 2. MD example problems from four contexts based on CGI classifications (Carpenter et al., 1999)

	Units per Group (partitive division)	***Groups (measurement division)***	***Units (multiplication)***
Objects/ Groups	Jasmine places 24 marbles in 6 bags, with the same number in each. How many marbles are in each bag?	Nicole puts 4 apples in each barrel. How many barrels does Nicole need for 24 apples?	Wei observes 4 sculptures in each museum. How many total sculptures does Wei observe in 6 museums?
Distance/ Time	Giovanni bikes 24 yards in 6 seconds. How many yards per second did Giovanni bike?	Emily travels 4 miles per hour. How many hours will it take for Emily to travel 24 miles?	Taylor drives 4 kilometers per week. How many kilometers does Taylor drive in 6 weeks?
Money (Exchange)	Joseph buys 6 pies for 24 dollars. If each pie cost the same amount, how much does one pie cost?	Logan earns 4 dollars per hour. How many hours does Logan need to earn 24 dollars?	Maya trades 4 coupons per doll. How many coupons does Maya trade for 6 dolls?
Compare	*[Quantity1 missing]* Misaki's plant is 4 times as tall as Holly's plant, which is 6 feet tall. How tall, in feet, is Misaki's plant?	*[Scalar multiplier]* Luke's fence is 6 meters tall. Martina's fence is 24 meters tall. Martina's fence is how many times as tall as Luke's fence?	*[Quantity2]* Sergey's beard is 6 inches long, and is 4 times as long as Howard's beard. How long is Howard's beard?

Table 3. Study 1's mean proportion correct and RTs of correct responses on pre- and posttest measures. Standard deviations are shown in parentheses.

	ACCURACY		RESPONSE TIME (in seconds)	
	Pretest	**Posttest**	**Pretest**	**Posttest**
ALL STUDENTS				
MPI GROUP (n = 31)				
Solve	**.81** (.21)	**.89** (.13)	**29.7** (14.4)	**18.3** (13.5)
Map, Unknown Left	**.60** (.28)	**.77** (.23)	**34.3** (19.3)	**20.6** (14.0)
Map, Unknown Right	**.67** (.24)	**.76** (.22)	**33.5** (16.1)	**20.0** (12.2)
COMPARISON GROUP (n = 30)				
Solve	**--**	**.86** (.17)	**--**	**44.6** (25.5)
Map, Unknown Left	**--**	**.62** (.26)	**--**	**61.8** (45.6)
Map, Unknown Right	**--**	**.73** (.25)	**--**	**46.7** (26.3)
PRETEST ACCURACY ≤ .80				
MPI GROUP (n = 16)				
Solve	**.69** (.21)	**.87** (.15)	**32.4** (15.4)	**22.0** (17.8)
Map, Unknown Left	**.43** (.25)	**.76** (.23)	**33.5** (23.6)	**24.8** (18.1)
Map, Unknown Right	**.53** (.23)	**.70** (.25)	**38.9** (18.0)	**22.5** (16.0)
COMPARISON GROUP (n = 14)				
Solve	**--**	**.75** (.19)	**--**	**47.6** (23.3)
Map, Unknown Left	**--**	**.47** (.25)	**--**	**62.9** (60.8)
Map, Unknown Right	**--**	**.56** (.25)	**--**	**39.4** (20.9)

Table 4. Study 2's mean proportion correct and RTs on pre- and posttest measures. Standard deviations are shown in parentheses.

	ACCURACY		RESPONSE TIME (sec)	
	Pretest	**Posttest**	**Pretest**	**Posttest**
SOLVING CONDITION (n = 16)				
Solve	**.80** (.14)	**.83** (.17)	**17.9** (7.9)	**15.9** (11.2)
Map, Unknown Left	**.53** (.27)	**.70** (.19)	**22.1** (10.0)	**21.7** (9.5)
Map, Unknown Right	**.66** (.20)	**.67** (.26)	**19.7** (6.9)	**21.1** (11.8)
MAPPING CONDITION (n = 10)				
Solve	**.73** (.18)	**.83** (.26)	**15.9** (11.2)	**19.8** (6.7)
Map, Unknown Left	**.61** (.28)	**.66** (.24)	**21.7** (9.5)	**17.1** (7.4)
Map, Unknown Right	**.63** (.21)	**.81** (.22)	**21.1** (11.8)	**19.6** (13.0)