The Significance of Recursion Using the TI-84 Sequence Editor.

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Abstract

Patterns are a ubiquitous phenomenon. They exist in nature's plant species emerging as a complete order from truncated matter. Patterns are also present in the fine arts where the aesthetic properties of form exist and conjugate through paintings and sculptures. Mathematical patterns, the focus of this position paper, dominate financial scenarios, growth and decay problems, as well as population rates. These topics can be articulated through the language of recursion where a present numerical value occurs in the sequential context of prior ones. Many college students have little or no knowledge of this recursion concept, and traditional analytic pedagogy is often characterized by tedium. The TI-84's designated sequence mode is modeled and implemented here as a paragon for students to define, investigate and visually perceive recursively defined problem solving tasks in terms of the natural value, *n.* The sequence mode structurally delineates the recursion function by translating present and antecedent iterations associated with the following conceptions: 1. a computational sequence. 2. a financial scenario and 3. a medication problem. These three tasks are conveyed by applying *n* as an independent variable, a table of integer or discrete outcome values, and a graphic representation of a disconnected set of points. The TI-84 sequence graphing mode edifies the cognition of advanced algebraic topics for students.

Introduction

The concept of recursion distinguishes other algebraic concepts in its myriad of applications relating numerical relationships within an interdependent operational structure. Mathematics texts often allude to the classic sequence conjecture discovered by Fibonacci regarding

the reproduction of rabbits: The number of rabbits produced in one year is precipitated by a single pair, which then multiplies to a new pair monthly. Fibonacci's work is the prototype of the recursion definition where the current nth term is contingent upon the preceding term or terms. This abstraction is translated into the sequence F_n (a generic Fibonacci value)= F_{n-1}(the preceding Fibonacci value) +F_{n-2} (the Fibonacci value two positions before it). Wolff (2006) reiterates this recursion process as a "discrete dynamic system" synonymous with the "recurrence" and "difference" equation illustrating a mathematical induction procedure of indefinite continuation. The following simplistic sequence depicts inductive reasoning.

$1=1^2$

$1+3=2^2$

$1+3+5=3^2$

Therefore, $1+3+5+7=4^2$ since inductively the sum of the consecutive odd counting numbers beginning with 1 is equivalent to the square of the number of terms on the left side of the quantitative equation.

Freudenthal (1973, 1982) emphasized a "dependency principle" from Euhler's original definition of functions when characterizing relations among quantities: "Our world is not a calcified relational system but a realm of change, a realm of variable objects depending on each other." Shuard and Neill (1977) extend this line of thinking by viewing functions as a relationship between members of two sets whereby "each member of the domain has only one image." The recursion function contrasts the explicit representation with respect to the semantics of "rule," "starting term," "next term" and "previous term." The explicit interpretation procedurally substitutes input values to create output values that balance the equation. Fundamentally, however, sequences involve a constant or a common difference that is arithmetic or a common ratio that is geometric. Recursion is

synonymous with iterative patterns: patterns derived from nature, linguistic patterns in sentence structure, artistic patterns that juxtapose image and color, computer programming, and, of course, mathematical models that evolve from financial growth and decay, population statistics as well as chemical reactions to cite the most basic examples.

Verstappen (1982) discerns three conditions for depicting functions through the application of mathematical language: a. geometric-schemes, diagrams, graphs, histograms, drawings, b. arithmetical-numbers, tables, ordered pairs, and c. algebraic- symbols, formulas and mappings. The TI-84 expounds on Verstappen's observations as a mandatory resource for students to study mathematical models and representation in courses including College Algebra, Pre-Calculus and Calculus. Three graphing calculator components that exemplify algebraic abstractions in an interconnected network are 1. Function 2.Table and 3. Graph. In the TI-84 function mode, students translate x and y values into explicit notation expressing an output (y) in terms of an input (x). The principle of equivalence is enhanced by the table feature which displays a list of numerical values that balance. A graphic representation is subsequent upon creating an appropriate viewing window of x and y minimum and maximum values. Similarly, the recursive sequence mode establishes function, table and graph, however, with additional variables that denote the process where successive steps of a pattern are dependent on the steps that precede them.

The Process

Model Problem #1: Solving a computational sequence.

Graph the first 10 terms of the following sequence, and evaluate the 25^{th} term. $u(n)=u(n-1)-7$; $u(0)=4$

1. **The calculator must be in Sequence Mode.**

```
NORMAL  SCI  ENG
FLOAT  0 1 2 3 4 5 6 7 8 9
RADIAN  DEGREE
FUNC  PAR  POL  SEQ
CONNECTED  DOT
SEQUENTIAL  SIMUL
REAL  a+bi  re^θi
FULL  HORIZ  G-T
SET CLOCK 01/01/01 12:15AM
```

2. **Display the y= editor and insert the following.**

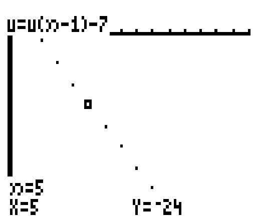
```
Plot1  Plot2  Plot3
 nMin=1
 u(n)=u(n-1)-7
 u(nMin)={4}
 v(n)=
 v(nMin)=
 w(n)=
 w(nMin)=
```

- **The *n* min is set to 1, which is the starting term. The letter *n* is used instead of function x since the representation reflects a natural number.**
- **u(*n*) is the given recursion equation.**
- **u(*n* Min) is the value of the starting term.**

3. **To graph the sequence, set the following window.**

```
WINDOW               WINDOW
 nMin=1              ↑PlotStep=1
 nMax=10              Xmin=0
 PlotStart=1          Xmax=15
 PlotStep=1           Xscl=1
 Xmin=0               Ymin=-60
 Xmax=15              Ymax=6
↓Xscl=1               Yscl=1
```

- **The x-axis will represents the values associated with *n*, while the y-axis represents the values associated with u(*n*).**
- **Plot Step refers to the increment value of plotted terms.**



4. **Determining the 25th term can be solved in two ways:**
   1. Table   2. The Home Screen
   The TBLSET Feature (2nd Window) will determine the independent starting value of the table of values along with the Delta Table.

```
TABLE SETUP
 TblStart=25
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

5. 2^{nd} Graph will produce the table of recursive values.

n	u(n)
25	-164
26	-171
27	-178
28	-185
29	-192
30	-199
31	-206

n=25

6. On the Home Screen, 2^{nd} function of 7 will also reproduce the 25^{th} term.

```
u(25)
                -164
```

Model Problem #2: Solving a Finance Problem.

Joni deposits $100 in the bank offering 8% annual interest compounded monthly. Use the TI-84 sequence mode to find out her account balance at the end of one quarter and the end of one year.

- **Determine a recursively defined function.**
- **Graph the relationship.**
- **Use the Table feature to observe her bank balance patterns.**

1. **Variables for interest rate and monthly compounding rate must be assigned on the Home Screen.**

```
.08→R
              .08
12→N
               12
```

2. **Enter the following recursion values in the y=editor.**

```
Plot1 Plot2 Plot3
 nMin=0
\u(n)=█(n-1)+(R/
N)u(n-1)
 u(nMin)=(100)
\v(n)=
 v(nMin)=
\w(n)=
```

3. The lowercase *n* is obtained from the variable sequence mode. The uppercase N is obtained from ALPHA N.

The lowercase *n* represents interest time periods while u(*n*) represents the balance at the end of the previous month plus the interest earned. The *n* minimum starts at 0 (before interest is obtained and u *(n* Min) is the initial $100 deposit). R represents the annual interest rate while N represents monthly compounding.

4. The equation itself represents the balance at the end of the previous month u (n-1) plus the previous month's balance u(n-1) multiplied by the monthly interest rate R/N (interest earned).

5. The table provides the ending balance for the months listed *n.* u*(n)* represents the total balance at the end of each month.

n	u(n)
0	100
1	100.67
2	101.34
3	102.01
4	102.69
5	103.38
6	104.07

n=0

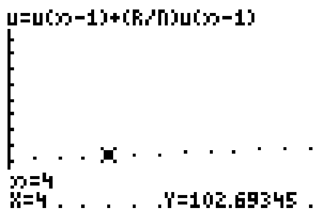
7. Set the Window as follows:

```
WINDOW              WINDOW
 nMin=0             ↑PlotStep=1
 nMax=12             Xmin=0
 PlotStart=1         Xmax=12
 PlotStep=1          Xscl=1
 Xmin=0              Ymin=70
 Xmax=12             Ymax=200
↓Xscl=1              Yscl=█
```

- ***n* min and *n* max represent the monthly compounding periods. (*n* min is the smallest *n* value to evaluate while *n* max is the largest n value to be deciphered).**

- **X min and X max also represent the monthly compounding periods. (the smallest and the largest *x* value to be deciphered).**
- **Y min and Y max represent the account balance at the end of each month. (the smallest and largest *y* value to be deciphered).**
- **Plot Start- the first term number to be plotted.**
- **Plot Step- the incremental *n* value.**

8. The graph displays a shallow incremental growth.



9. The Table feature reiterates the slow growth.

n	u(n)
0	100
1	100.67
2	101.34
3	102.01
4	102.69
5	103.38
6	104.07

n=0

Model Problem #3: Solving a Medication Problem.

Mr. Dylan takes 20 milligrams of a blood pressure medication every four hours. Thirty percent of the drug is removed from the bloodstream every four hours.

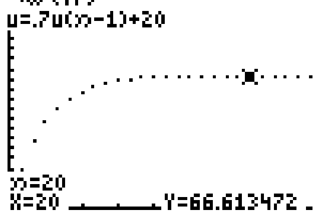
- **Write a recursively defined function for the amount of medication in Mr. Dylan's bloodstream after *n* hours.**
- **Represent Mr. Dylan's "maintenance level" of this drug given the dosage of 20 milligrams both graphically and via Table.**
- **How does doubling Mr. Dylan's dosage affect his "maintenance level." Represent the doubling via function, table and graph.**

1. **The initial 20 milligram representation, and graphic display of the maintenance level.**

```
Plot1 Plot2 Plot3
 nMin=1
·u(n)=.7u(n-1)+2
0
 u(nMin)={20}
·v(n)=
 v(nMin)=
·w(n)=
```

```
WINDOW
 nMin=1
 nMax=25
 PlotStart=1
 PlotStep=1
 Xmin=0
 Xmax=25
↓Xscl=3
```

```
WINDOW
↑PlotStep=1
 Xmin=0
 Xmax=25
 Xscl=3
 Ymin=0
 Ymax=100
 Yscl=5
```




```
TABLE SETUP
 TblStart=20
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

n	u(n)
20	66.613
21	66.629
22	66.641
23	66.648
24	66.654
25	66.658
26	66.66

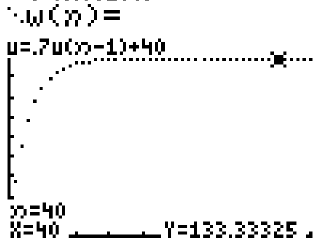
n=20

2. **The doubling representation, and graphic display of the maintenance level.**

```
Plot1 Plot2 Plot3
 nMin=1
·u(n)=.7u(n-1)+4
0
 u(nMin)={40}
·v(n)=
 v(nMin)=
·w(n)=
```

```
WINDOW
 nMin=1
 nMax=45
 PlotStart=1
 PlotStep=1
 Xmin=0
 Xmax=45
↓Xscl=5
```

```
WINDOW
↑PlotStep=1
 Xmin=0
 Xmax=45
 Xscl=5
 Ymin=0
 Ymax=150
 Yscl=15
```

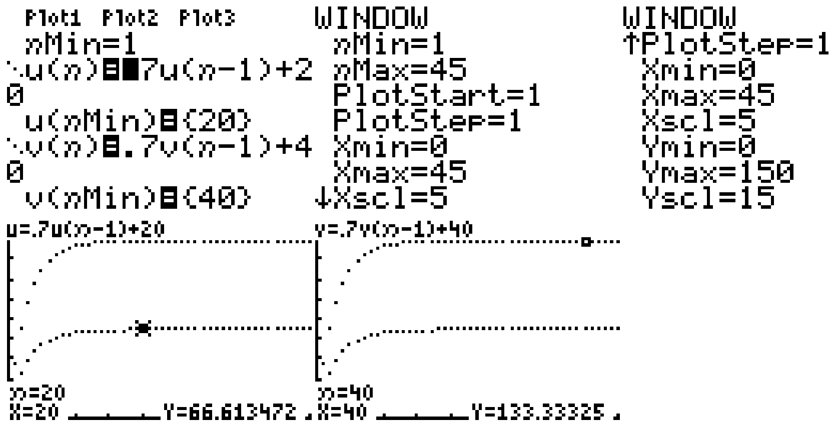



```
TABLE SETUP
 TblStart=40
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

n	u(n)
40	133.33
41	133.33
42	133.33
43	133.33
44	133.33
45	133.33
46	133.33

n=40

3. **Both functions displayed on the same plot.**



Conclusion

The TI-84 Sequence Editor is an effective and efficient mathematical tool used to enhance the comprehension and application of the recursion principle. All three problem solving tasks cited here developed inductive reasoning within the context of a graphing calculator language; an expression of algebraic variables and constants extended by a function and its comparative graph. The computation problem #1 analyzed patterns produced by the preceding term notation of u (*n*-1). This established the foundation for modeling the recurrent systems of finance in task #2, and the medication absorption problem of #3. The compound interest problem involved the assigning and storing of the necessary variables N and R. The function recognized that the ending balance for each year is essentially the sum of the previous period, and the subsequent interest earned. The final task was multifaceted since it involved an interpretation of the 30% dissemination of the drug throughout the bloodstream. This is pivotal to the function's coefficient of u and the constant. Relevant conjectures about the medication's potency can be concluded based

on the graph's curve and where it stabilizes as a constant. Finally, the TI-84 abets pedagogy, and unravels the most abstract of mathematical concepts.

References

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