

MOBILE LEARNING AND EARLY AGE MATHEMATICS

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ABSTRACT

The ability to develop engaging simulations and constructive learning experiences using mobile devices is unprecedented, presenting a disruption in educational practices of historical proportions. In this paper we describe some of the unique virtues that mobile learning hold for early age mathematics education. In particular, we describe how object-based learning, any place / anywhere learning, collaborative learning, gamified learning, customized learning, and adaptive learning, come to play in our work on *SlateMath*. *SlateMath* is a richly indexed portfolio of hundreds of instructional units, designed to support the teaching and learning of mathematics at the elementary school level (kindergarten through sixth grade) using mobile devices. *SlateMath* is cross-platform, multi-lingual, and freely available for schools and teachers world-wide.

KEYWORDS

Mathematics education, tablets in education, mobile learning, adaptive learning.

1. INTRODUCTION

SlateMath is a large scale portfolio of immersive math education apps for tablets and personal computers. The minimal learning object in *SlateMath* is called *episode*: a bite-sized interactive progression of several related and increasingly challenging hands-on activities, presented in a playful and animated user experience. A typical interaction with a *SlateMath* episode lasts between ten to thirty minutes, and seeks to address a single, well-defined, curricular-driven mathematical concept, skill, or insight. In addition to these hands-on activities, *SlateMath* also features numerous interactive worksheets that last as long as it takes to complete the given problem sets.

The hands-on episodes focus on building an intuitive understanding of the subject matter, e.g. understanding, hands-on, the rationale behind the distributive rule $a(b + c) = ab + ac$; this is done via games and interactions that use graphical imaging to drive home the nature of the distributive rule and how it comes to play in solving algebraic problems. The worksheets, on the other hand, are designed to build a mastery of the related skill set, e.g. solving algebraic problems that involve different manifestations of the distributive rule. Both the episodes and the worksheets are adaptive, as we describe shortly.

The *SlateMath* portfolio is a structured collection of about 700 such episodes and worksheets, designed to support math education from kindergarten through sixth grade. The verb "support" is important. Unlike various instructional technology approaches that seek to replace traditional frontal teaching with on-line video lectures (Thompson, 2011), *SlateMath* is not intended to teach math, but rather to help teachers teach math. It provides an abundance of richly-indexed activities and exercises that can light up a lecture and turn a classroom into a lively math laboratory. Teachers normally use between one to three *SlateMath* episodes in each class meeting, in a blended fashion that alternates between frontal teaching and self-paced student practice.

Perhaps the biggest challenge in early-age math education is that children gradually develop math deficiencies that go unnoticed and uncorrected. These hidden deficiencies linger into the program, causing mounting frustration and accelerated incompetence. With *SlateMath*, the child's performance is continuously monitored. When the child's work indicates that he or she is remiss of some requisite skill, *SlateMath* prescribes remedial work, using relevant, pre-requisite episodes. This student diagnostics is carried out by the software continuously and in real-time, while the errors are still fresh and unsettled.

SlateMath comes with student authentication, tracking, and analytics capabilities. A teacher dashboard provides both on-line and periodical status reports about each student's performance. The system is cross-platform and multi-lingual, and is freely available to schools and teachers world-wide. The iOS version of SlateMath can be downloaded from the Apple app store, and an on-line web version is also available (www.slateMath.com). An Android version will be released soon.

The paper starts with a description of the SlateMath approach to math education. This sets the stage to a more general discussion of the virtues of mobile learning in early age math education.

2. EPISODE EXAMPLE

Much of early-age math education revolves around understanding and mastering the four basic mathematical operations: *addition*, *subtraction*, *multiplication*, and *division*. Every one of these operations entails an elaborate body of evolving knowledge, which is best acquired over a period of several years (Lee and Ginsburg, 2009, Kline, 1998). For example, according to the American math curriculum, also known as the *Common Core State Standard*, children should start being exposed to simple forms of division in grade 3. That said, nothing should prevent educators from sowing the seeds of understanding division much earlier in the program. To illustrate, one way to set the stage for understanding division is to introduce children to the notion of *parity*, and to the related concepts of *odd* and *even*. Parity is based on a simple form of division – division by two. This section describes a SlateMath episode designed to introduce the notion of parity in a hands-on, constructive fashion. We note in passing that there is nothing special about this particular episode, and we present it in order to introduce readers to the spirit of SlateMath-based learning.

How can you tell if a given number, say 28, is odd or even? The simplest way to go about it is to look at the number's rightmost digit; if it's divisible by two, the number is even; otherwise, it's odd. This rule works well, yet it provides no algebraic insight whatsoever about "being odd" or "being even" – a property commonly known in mathematics as *parity*. Importantly, our goal is not to just teach children how to tell if a given number is odd or even – we wish children to "feel" parity in a natural and constructive manner. With that in mind, SlateMath features a dedicated episode and a teaching guide that focus on solving parity questions using an interactive, hands-on "parity lab".

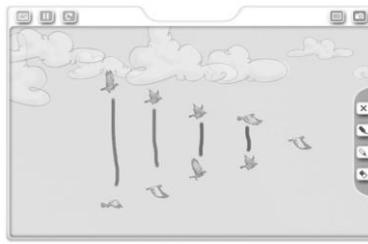
As the Figure 1 shows, the child is invited to play with different flocks of birds, each having a different formation and symmetry. The teacher explains that there are two ways to determine if the number of birds in a given flock is odd or even: we can either try to divide the birds into pairs, or we can try to partition them into two subsets containing an equal number of birds. If we succeed to do so, and no bird is left out of the division, we say that the number of birds in the flock is even. If one bird is left out of the division, we say that the number of birds is odd.

Using the touch user interface of mobile devices, the software encourages the child to rearrange the flock into pairs or into equal subsets, and observe how many birds were left out of the division. This hands-on manipulation, when applied to different flocks of birds, leads to many interesting mathematical insights. For example, we quickly discover that irrespective of how many birds a flock contains, either one bird is left out of the division, or no birds are left out. And, because the software encourages moving the birds around and rearranging them in groups, the child discovers that "being odd" or "being even" is not a property of individual birds, but rather a property of the entire flock. Some of these learned insights run deep. For example, by playing with certain flocks, children discover that the sum of two even numbers is always an even number, and that the sum of two odd numbers is, surprisingly, also an even number. They use these insights to reason about the parity of subsequent complex flock formations without having to count the birds. In short, the children are exposed not only to mathematical concepts, but also to how mathematicians think. Importantly, each SlateMath episode is accompanied by a teaching guide that suggests a set of activities, insights, and discussion points. For example, teachers are encouraged to point out that when we pair up any number of birds, the number of birds that are left out of the division is at most one. This plants the seeds for understanding the subsequent notions of a *remainder* and *modular arithmetic*.

The parity episode is just one out of hundreds of episodes contained in SlateMath. This elaborate body of instructional materials, and the pedagogy behind it, constitute an approach to early-age math education that can be described as *object-based*, *mobile*, *collaborative*, *gamified*, *customized*, and *adaptive*. The rest of the paper describes each one of these virtues in some detail.



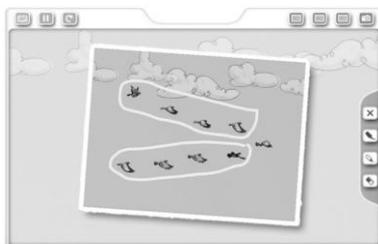
Is the number of birds odd or even?
Can you answer the question without counting?



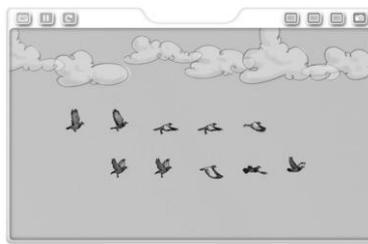
Using a retractable overlay screen and virtual crayons, we can arrange the birds in pairs. Note that one bird is left out.



Creating two groups containing an equal number of birds. Once again, one bird is left out of the division.



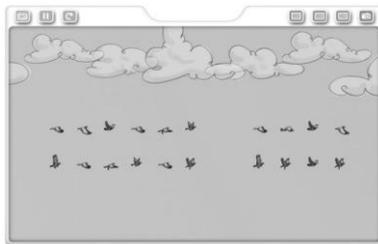
A camera tool can be used to take and save snapshots of one's work.



Another flock formation. The leading and trailing birds can be moved around, creating a pair.



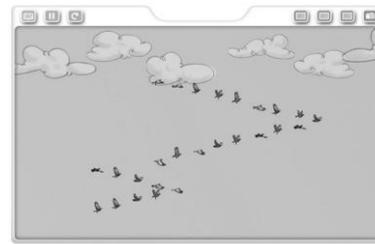
Another flock formation, out of many more flocks of varying symmetry.



We use this formation to reason that the sum of two even numbers is an even number.



We use this formation to reason that the sum of two odd numbers is also an even number.



Since the parity of a symmetric arrowhead is odd (top left example), and odd plus odd gives even (previous example), we conclude that the number of birds in this flock is even.

Figure 1: Screenshots from the parity episode. different flocks of birds fly into the screen; the task is to determine if the number of birds in each flock is odd or even. to do so, the child is encouraged to move the birds around and rearrange them in groups. this is done using a retractable and transparent overlay screen, a set of crayons, an eraser, and a camera – all software artifacts. (the figures are far more attractive in their original colors)

3. OBJECT-BASED LEARNING

We believe that early-age mathematical skills are best acquired through direct manipulation of familiar objects in familiar settings: counting animals, sorting cookie jars, cutting and pasting geometric figures, and so on. Indeed, child developmental psychologists, most notably Jean Piaget, observed that tactile manipulations of concrete objects help children form math concepts and operations using their own cognitive devices (Wadsworth, 1996). This innate learning process can be significantly accelerated by teachers who can prepare, monitor, and guide such hands-on explorations in the classroom.

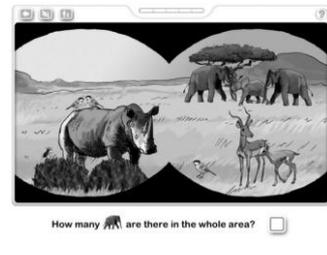
Unfortunately, it is unrealistic to expect teachers to arrange, and bring to the classroom, thousands of physical objects ranging from birds to beads to boats. Fortunately, mobile devices can come to the rescue. For example, our SlateMath software employs a state-of-art graphical user interface and a physics engine to simulate the manipulation of numerous familiar objects in an accurate and engaging manner. These guided interactions are carefully designed to facilitate hands-on exploration of math concepts and insights within a playful and supportive learning environment. Figure 2 gives some examples.



Counting, from objects to number:
How many fish do you see?
(when a fish is touched, it glows)



Counting, from number to objects:
Create a necklace using n beads.



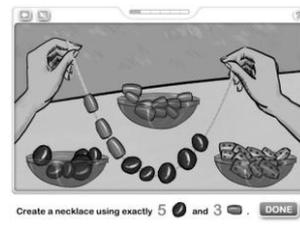
Forward counting, over space:
how many elephants in the entire scene? (the scene continues, and can be scrolled, sideways)



Forward counting, over time:
How many birds flew over the roof?
(snapshot in mid action)



Speed counting:
How many meerkats did you spy?
(the meerkats pop in and out)



Addition episode: the next question is "*how many beads are in the necklace altogether?*"

Figure 2. Examples of some slatemath episodes that focus on different flavors of *counting*. the software features an optional voice-over channel that helps overcome reading difficulties. children answer the questions using three possible means: choosing numbers from a number wheel, using the keyboard, or writing the number on the screen, using a stylus or one's finger.

The impact of "touch interface" on learning has been studied by Ketamo (2002), Dede (2009), and Savila (2010), among others. Likewise, the importance of playing with "virtual manipulatives" (what we call "object-based learning") has been described by Reimer and Moyer (2005), Steen et al (2006), and Bolyard (2006). The judicious combination of these two modalities help us expose children to numerous real life yet simulated scenarios that involve mathematical reasoning, and to help them gain a hands-on understanding of the underlying mathematical concepts and techniques.

4. ANYWHERE / ANYPLACE LEARNING

The abundance of hardware and software platforms that characterizes today's mobile space is not a curse, but rather a blessing. This technological diversity provides the foundation on which competition thrives, incentivizing companies to constantly innovate and come up with better hardware platforms, operating systems, programming languages and software libraries. Thus, multi-platform is here to stay; instead of trying to ignore or undermine it, we recommend coping head-on with the challenges that it presents. Below is an example of one such a coping strategy.

Which tablets should our school adopt? iPads? Kindle Fires? Touch laptops? The desired answer should obviously be – use whatever devices are available in the classroom. Indeed, educators are increasingly embracing a BYOD, or *Bring Your Own Device* policy (Sharples et al, 2010): teachers and students should be encouraged to use different tablets and smartphones, sourced by schools, homes, governments, and philanthropists. For example, consider a teacher who wishes her students to use a SlateMath app to practice their math skills. The BYOD commitment requires that SlateMath will run as-is on all the mobile devices that happen to be present in the teacher's classroom, irrespective of make and operating system differences.

Cross-platform compatibility is also essential to support the increasingly popular any-time, any-place learning mode. Consider a student who has just used SlateMath on a school-supplied iPad. On her way home in the school bus, the intrigued student may wish to continue playing with SlateMath on her Android smartphone. And, once back at home, she may wish to switch to a larger screen, using a laptop that runs SlateMath in a web browser. The SlateMath app runs as-is on all these devices, giving the student a consistent and continuous user experience. The software operates exactly the same on each device for the simple reason that it is the same software running everywhere. And, as the student hops from one device to another, her academic performance and usage history are updated seamlessly on the cloud, feeding the teacher's dashboard.

The ability to execute the same code base on multiple devices and operating systems is made possible thanks to an elaborate cross-platform software architecture and authoring system. This architecture, which allows executing the same software in both browser and native code environments, will be described in a separate paper.

5. COLLABORATIVE LEARNING

When we started running pilots using SlateMath in classrooms, we noticed that many teachers preferred having two students interact with a single tablet, rather than using a one-to-one setting. At the beginning, we thought that the two-to-one setting was chosen because of lack of tablets scarcity; as the year progressed, though, we realize that putting two children on one tablet has many virtues on educational grounds.

When two students use a single device, they must take turns: one student solves a problem, or interacts with a game stage. When the next problem or game stage starts, the second student takes over. This mode of interaction has three main benefits. First, it teaches patience and delayed gratification. Second, the "passive" child can observe the work of the "active" child and reflect on the educational contents without the pressure of actual interaction. Third, in many cases, the "passive" child acts like a guide, helping the other child with tips and advice. Taken together, all these behavioral patterns contribute greatly to the development of critically important social skills.

When it comes to sharing, tablets have a significant tactile advantage over laptops. Since they lie flat on the table's top, they can be easily swiveled from one side to another. As a result, several children can share the same device without having to move from the chairs.

6. GAMIFIED LEARNING

For many children and adults alike, desktop and laptop computers symbolize "work", whereas tablets symbolize "play". Indeed, developers who build tablet software strive to create a fresh and engaging user experience. The GUI, terminology, and "lore" of tablet software draw heavily from the world of gaming.

Indeed, "gamification" is a hugely important concept in tablet software, as many developers use gaming ideas to make learning more fun and addictive (Kapp, 2012).

That said, as educators, we must make sure that the gamification will not compromise our educational mission. For example, many math teaching programs and textbooks sugarcoat the "boring math" with gamification elements like number-eating cows and similar contraptions, each following one developer's fantasy on the best way to explain some mathematical subject. We don't believe in number-eating cows, for two reasons. First, cows don't eat numbers. Second, math is sufficiently attractive in its own right, and there is no need to decorate it with infantile metaphors. Our experience shows that SlateMath endears math on children as young as 4 year-olds by helping them master common tasks that unfold in common settings: counting animals, hanging balloons, decorating cakes, and so on. There is no need to have a dinosaur slap its tail on the smaller of two numbers when there are many interesting ordinary scenarios in which order comparisons come to play. Moreover, identifying which mathematical procedures are applicable to which everyday situations is in itself an important math literacy skill.

We do gamify the SlateMath working environment with peripheral motivators like avatars, badges, and a quest-like context. And yet, although we want children to feel engaged and rewarded, we don't believe in shortcuts and instant learning. Analytic reasoning, like any worthwhile intellectual art, takes time to develop in one's mind. Therefore, we want the magic of math to grow on children gradually, as they complete more SlateMath episodes and become increasingly more competent and confident in their work.

7. CUSTOMIZED LEARNING

Individual SlateMath episodes are short and self-contained. Each year in the K-6 math program is supported by a portfolio of about 120 SlateMath episodes. Each one of these episodes entails a series of related tasks, resulting in thousands of different interactive activities.

The resulting SlateMath portfolio is highly modular, richly indexed, and recombinant. Every week, teachers select the SlateMath episodes that best support their weekly teaching program, and assemble them into a customized, weekly math learning bundle (we provide default weekly bundles, designed to cover the entire year). This way, teachers control the selection and order in which the episodes become available to their students. And, in a BYOD (Bring Your Own Device) setting, teachers can control the episodes availability and staging outside school as well. All these tasks are done through drag-and-drop operations on the teacher's dashboard.

The pre-determined contents of the weekly math bundle can be easily overridden. If the teacher feels that the class dynamics requires access to additional SlateMath episodes, he or she can easily make them available to the students. Episode selection is done by queries like "I need episodes that practice ordering objects according to their length" or "I need episodes that support the Common Core 1.MD.2 topic". Once again, all these tasks are done on the teacher's dashboard.

8. ADAPTIVE LEARNING

When serving an episode that requires adding up integers, SlateMath can detect that the child's counting skills are not sufficiently developed; when serving an episode that involves fraction arithmetic, SlateMath can detect that the child has trouble computing common denominators; when serving an episode that involves calculating triangle areas using geometric manipulations, SlateMath can detect that the child does not know how to calculate rectangular areas. SlateMath continuously monitors the child's actions in each episode, and accumulates reams of data about the child's evolving math abilities. Importantly, this continuous monitoring is done in vivo, while the child interacts with the actual episodes, rather than in subsequent and out of context quizzes. The latter check if the child can pass math exams; the former check if the child understands math.

The diagnostic data that SlateMath collects is continuously mined, summarized, and presented on the teacher's dashboard. Teachers can get real-time information about each child's current performance, as well as tallied information about the child's cumulative progress. If the child's work indicates that he or she is remiss of some math topic, SlateMath recommends remedial action, using relevant episodes. The teacher can either accept these recommendations, or modify them to come up with another corrective procedure.

The fact that different children learn in different paces is one of the biggest challenges in mathematics education. In less tightly layered fields of study, missing a class or a concept can be easily rectified, or even side-stepped. Yet in math, if children fail to grasp the notion of a common denominator, they will be at loss when learning fraction arithmetic. They may be able to solve equations, following supplied algorithms, but they will not understand why the algorithms work. Such are the early fault lines from which math anxiety begins to emerge, setting the stage for a life-long math phobia that typically takes hold sometimes between elementary and middle school.

9. CONCLUSION

In the era of tablets and smart phones, learning can occur anytime, anyplace. That said, we believe that no technology can replace human touch, especially in early-age math education. In particular, the teacher's role in staging and guiding the learning process is indispensable. That's why SlateMath does not pretend to teach math, but rather to help teachers teach math.

According to Bertrand Russel, "Mathematics, rightly viewed, possesses not only truth, but supreme beauty — sublimely pure, and capable of a stern perfection such as only the greatest art can show." With that in mind, our goal is not limited to developing reasoning skills and math literacy. We are equally interested to endear math on children, and to kindle an affinity to quantitative reasoning and to science in general. Math proficiency provides a foundation for becoming a rational, productive, and self-reliant person, and opens numerous career possibilities; helping children build this foundation is our overriding passion and commitment. We feel fortunate that tablets enable us to fulfill this mission with unprecedented effectiveness.

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REFERENCES

- Bolyard, J.J., 2006. A Comparison of the Impact of Two Virtual Manipulatives on Student Achievement and Conceptual Understanding of Integer Addition and Subtraction. *Ph.D. thesis, George Mason University*.
- Dede, C. Immersive Interfaces for Engagement and Learning, *Science*, January 2009: 323 (5910), pp. 66-69.
- Kapp, K.M., 2012. The Gamification of Learning and Instruction: Game-based Methods and Strategies for Training and Education, John Wiley & Sons.
- Ketamo, H., 2002. mLearning for Kindergarten's Mathematics Teaching. *Proceedings of the IEEE International Workshop on Wireless and Mobile Technologies in Education (WMTE'02)*, pp.167-168. 2002.
- Kline, K., 1998. Kindergarten is More Than Counting, *Teaching Children Mathematics*, v5 n2, pp. 84-87.
- Lee, J.S. and Ginsburg, H.P., 2009. Early Childhood Teachers' Misconceptions about Mathematics Education for Young Children in the United States, *Australasian Journal of Early Childhood*, v34 n4 pp. 37-45.
- Reimer, K. and Moyer, P.S., 2005. Third-Graders Learn About Fractions Using Virtual Manipulatives: A Classroom Study. *Journal of Computers in Mathematics and Science Teaching*, 24(1), Norfolk, VA: AACE. pp. 5-25.
- Savilla B., 2010. Integrating the iPod Touch in K–12 Education: Visions and Vices. *Computers in the Schools*, Vol. 27, Iss. 2.
- Sharples, M., Taylor, J., and Vavoula, G., 2010. *A Theory of Learning for the Mobile Age*, VS Verlag für Sozialwissenschaften.
- Steen, K., Brooks, D. and Lyon, T., 2006. The Impact of Virtual Manipulatives on First Grade Geometry Instruction and Learning. *Journal of Computers in Mathematics and Science Teaching*, 25(4), Chesapeake, VA: AACE. pp. 373-391.
- Thomson, C., 2011. How Khan Academy is Changing the Rules of Education, *Wired Magazine*, July, 2011.
- Wadsworth, B.J., 1996. *Piaget's theory of cognitive and affective development*: Longman: White Plains, NY.