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Developmental predictors of fraction concepts and procedures

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A B S T R A C T

Developmental predictors of children's fraction concepts and procedures at the end of fourth grade were investigated in a 2-year longitudinal study. Participants were 357 children who started the study in third grade. Attentive behavior, language, nonverbal reasoning, number line estimation, calculation fluency, and reading fluency each contributed uniquely to later conceptual understanding of fractions. Number line estimation, attentive behavior, calculation fluency, and working memory made unique contributions to acquisition of fraction arithmetic procedures. Notably, number line estimation made the largest independent contribution in both models. The results suggest that although there is considerable shared variance among the predictors, both general and number-related competencies are uniquely important for explaining why some children struggle with fractions.

Introduction

Fractions are central to elementary and middle school mathematics. Their importance is reflected in their emphasis within the U.S. Common Core State Standards (Council of Chief State School Officers & National Governors Association Center for Best Practices, 2010). Between fourth and sixth grades, children are expected to acquire knowledge of fraction equivalence and ordering (e.g., comparison of fractions with different numerators and denominators such as 3/4 and 1/2), learn fraction

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arithmetic procedures (e.g., addition and subtraction of mixed numbers with unlike denominators), and solve word problems involving fractions.

Weak understanding of fractions has serious long-term consequences. Not only are fractions essential for learning algebra and more advanced mathematics (National Mathematics Advisory Panel [NMAP], 2008), they also are important for daily life functioning such as managing personal finances and doing home repairs. Knowledge of fractions is key for understanding rate of change, an integral part of algebra. Elementary school students' fraction knowledge predicts their algebraic knowledge in high school even after controlling for family education and income, intellectual capacity, and knowledge of whole number arithmetic (Siegler et al., 2012). Students who do not succeed in algebra are less likely to graduate from college than are higher achieving students and have few career opportunities in STEM disciplines (science, technology, engineering, and mathematics; NMAP, 2008; Sadler & Tai, 2007).

In the first few grades, math instruction focuses on whole numbers. Children come to see that whole numbers have magnitudes that can be assigned locations on number lines and that each whole number has a unique predecessor and successor. These insights are supported by children's understanding of the cardinal meanings of number words and competency with counting (Baroody, Eiland, & Thompson, 2009; Muldoon, Towse, Simms, Perra, & Menzies, 2013). With practice, young children develop facility with whole number operations. Although whole number sense can support understanding of fractions, learning fractions poses additional conceptual challenges for young learners (Hecht, Vagi, & Torgesen, 2007); multiplying fractions can yield answers smaller than either multiplicand, dividing fractions can yield answers larger than either dividend, identifying the predecessor and successor of a fraction is impossible, and so on. Decimal fractions pose similar difficulties; for example, the number with more digits is not necessarily larger, unlike with whole numbers.

Although the importance of fraction knowledge for understanding of most aspects of mathematics is clear, little is known about why some children master fractions quickly but others struggle even as adults. Therefore, in the current study, we identified predictors of fourth graders' knowledge of fraction concepts and procedures. Our goals were to advance understanding of mathematical development and to pinpoint predictors of potential learning problems early so that educators can address the difficulties before they become entrenched.

Our conceptual framework for identifying potential predictors of fraction learning grew out of Geary's (2004) model of mathematics learning. At the most general level, Geary's model distinguishes between mathematics concepts and mathematics procedures. Acquisition of mathematical knowledge in any given area requires both accurate and fluent execution of procedures and concepts. Conceptual knowledge and procedural knowledge are mutually supportive, with increasing competence of each type contributing to increasing competence in the other (Hecht & Vagi, 2010, 2012; NMAP, 2008; Rittle-Johnson & Siegler, 1998).

Within Geary's (2004) model, general cognitive processes, such as working memory and attention, support learning of both math concepts and procedures. The central executive controls the cognitive processes that are needed for learning and executing procedures. Geary also identified both symbolic and nonverbal cognitive systems as important for representing and manipulating mathematical information. Language systems are important for learning number names and the verbal count sequence; nonverbal reasoning is involved in representing and comparing numerical magnitudes. This model guided the processes that were assessed in the current study.

Fraction concepts and procedures

Fraction concepts include understanding that fractions represent parts of an object or parts of a set of objects, that they can be represented by fraction symbols (e.g., 1/3), and that fractions are numbers that reflect magnitudes (e.g., 2/5, 2/4, and 2/3 can be ranked from smallest to largest). Procedures involve computation with fractions. Conceptual knowledge appears to play a particularly important role in learning fraction procedures (Hallett, Nunes, & Bryant, 2010; Hecht & Vagi, 2010; NMAP, 2008; Seethaler, Fuchs, Star, & Bryant, 2011; Siegler, Thompson, & Schneider, 2011; Vamvakoussi & Vosniadou, 2010), although there is some evidence that fraction concepts and procedures are independently related to general fraction performance (Hallett, Nunes, Bryant, & Thorpe, 2012). One reason is that if students understand why the procedures are appropriate, they are less likely to forget fraction procedures such as the rule that common denominators are required for addition and subtraction but not for multiplication or division (Hecht, 1998; Hecht, Close, & Santisi, 2003). Consistent with this analysis, conceptual knowledge of fractions accounts for variance in students' mathematics achievement test scores above and beyond that accounted for by fraction computation skill; the reverse is not the case (Siegler & Pyke, 2012; Siegler et al., 2011).

General predictors

A variety of general processes are related to mathematics learning in general and to fraction learning in particular. For example, mathematics performance is positively correlated with classroom attention (e.g., Fuchs et al., 2005, 2006) and working memory (e.g., Kail & Hall, 1999; Lee, Ng, & Ng, 2009; Locuniak & Jordan, 2008; Passolunghi & Siegel, 2001; Swanson, 2006). However, these skills predict different types of mathematical knowledge. Fuchs and colleagues (2010) found that the ability to solve addition and subtraction fact problems in first grade was predicted by classroom attention (as measured by a teacher rating scale) over and above domain-specific number representation and estimation performance; the central executive component of working memory was not uniquely related to knowledge of addition and subtraction facts. In contrast, the same central executive components were unique predictors of problem solving. Similarly, among fifth graders, nonverbal reasoning, concept formation, and working memory have been found to be unique predictors for both whole and rational number computation, but language was a unique predictor only for rational number computation (Seethaler et al., 2011). Hecht and Vagi (2010) found that fourth graders' working memory and classroom attention uniquely predicted fifth graders' fraction skills after controlling for several domain-specific abilities.

Number-related cognitive predictors

Both nonverbal approximate and symbolic exact numerical representations are associated with mathematics development (Hanich, Jordan, Kaplan, & Dick, 2001). Core nonverbal components of number representations (the approximate number system or ANS) are seen during infancy—years before formal instruction (Feigenson, Dehaene, & Spelke, 2004). Individual differences in both preschoolers' and school-age children's nonverbal approximate number representations are associated with conventional mathematics learning during elementary school (Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2011). For example, Halberda and colleagues (2008) showed that poor math achievement in elementary school is associated with high school students' difficulty in reliably discriminating which of two dot collections is more numerous when the ratio of dots in the two collections is small. However, the extent to which the ANS is associated with early fraction knowledge is unclear.

The ability to translate nonsymbolic magnitude representations to symbolic ones also influences mathematical development. Supporting the importance of magnitude representations, accurate estimation of the location of numerals on a number line correlates strongly with mathematics achievement (e.g., Booth & Siegler, 2006; Booth & Siegler, 2008; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Geary, Nugent, Hoard, & Byrd-Craven, 2007; Siegler et al., 2011). Learning arithmetic combinations involves more than rote memorization, as reflected by the finding that the large majority of children's errors are close in magnitude to the correct answer to the problem (Geary & Wiley, 1991; LeFevre et al., 1996; Siegler, 1988). Moreover, core deficits in symbolic magnitude processing are a hallmark of learning disabilities in mathematics such as dyscalculia (Butterworth, Varma, & Laurillard, 2011); that is, children with dyscalculia have trouble in seeing the value of a numeral relative to other numerals even though they can differentiate between quantities represented nonverbally (Rouselle & Noelle, 2007). Again, however, with the exception of Siegler and colleagues (2011) and Siegler and Pyke (2012), all of the evidence comes from studies of math achievement with whole numbers.

A basic hypothesis of the current study was that accurate representations of numerical magnitudes are essential for acquisition of both conceptual and procedural knowledge of fractions, just as they are with whole numbers. To test this hypothesis, we examined the degree to which a wide variety of cognitive processes in third grade predicted fraction conceptual and procedural knowledge at the end of fourth grade. For comparison, we also examined predictors of performance on a general mathematics achievement measure that primarily involved whole numbers. The general cognitive predictors were attentive behavior, working memory, language, and nonverbal reasoning; the number-specific cognitive predictors were ANS acuity and number line estimation. We also controlled for third-grade reading and mathematics fluency in the models. Multiple regression analyses allowed us to examine the relative importance of these variables in predicting fraction concepts and procedures. The analyses also permitted us to examine how much of the explained variance was unique to each variable rather than shared.

Such information is essential for theoretical understanding of development of fraction knowledge and also can inform instruction. Some predictors may be malleable—and, thus, potential targets for instructional design. For example, representations of numerical magnitudes are highly responsive to interventions; such interventions have been shown to improve a wide range of numerical knowledge in young children (Booth & Siegler, 2008; Ramani & Siegler, 2008; Siegler & Ramani, 2009; Whyte & Bull, 2008). Similarly, Fuchs and colleagues (2008) found that instruction designed to support executive control helps students with math difficulties to compensate for deficiencies in inattentive behavior and improves their problem-solving performance. Predictors that account for unique variance in fraction learning and that seem likely to be malleable can inform the development of interventions for students who are struggling to learn fractions.

Method

Participants

Children were drawn from nine elementary schools in two public school districts in Delaware (on the U.S. East Coast) that serve families of diverse socioeconomic status. Informed consent forms were distributed to all third graders in the target schools. The initial sample size was 481. Only children who completed all of the measures were included in the longitudinal analyses ($N = 357$).[1] As shown in Table 1, demographics were similar for participants and nonparticipants. Income status was determined by participation in the free or reduced price lunch program at school. Both school districts followed the Common Core State Standards for math instruction in fourth grade and introduced fraction instruction at roughly the same time.

Measures

Predictor measures

Language. The Peabody Picture Vocabulary Test–fourth edition (PPVT; Dunn & Dunn, 2007) was used to assess language ability. On this test, participants are shown four pictures and are asked to point to the one that corresponds to the word spoken by the assessor. The internal reliability of the measure is high ($\alpha > .96$). The correlation between the PPVT and a validated measure of verbal IQ is .89.

Nonverbal reasoning. The Matrix Reasoning subtest of the Wechsler Abbreviated Scale of Intelligence (WASI; Wechsler, 1999) was used to assess nonverbal reasoning. Children are shown a series of grids with pictures in all but one cell and are asked to choose one of five choices to complete the pattern. Both the internal reliability of the measure ($\alpha > .90$) and the correlation between it and overall WASI performance IQ ($r = .87$) are high.

[1] The number of participants was slightly larger ($N = 364$) for the regression analyses with math achievement and number line estimation as dependent variables because 7 children had left the study by the end of fourth grade, when the fractions measures were administered.

Table 1
Background information for study participants ($N = 357$) and nonparticipants ($N = 124$).

Characteristic	Participants (mean age = 105.97 ± 5.35 months)[a]		Nonparticipants (mean age = 106.10 ± 5.70 months)[a]	
	n	%	*n*	%
Gender				
Male	169	47.3	66	53.2
Female	188	52.7	58	46.8
Race				
White	191	53.5	63	50.8
Black	133	37.3	52	41.9
Asian/Pacific Island	21	5.9	7	5.6
American Indian/Alaskan Native	12	3.4	2	1.6
Hispanic	56	15.7	24	19.4
Low income[b]	210	58.8	83	66.9
English language learner	41	11.5	11	8.9
Special education[c]	39	10.9	16	12.9

[a] Mean age was determined at the beginning of the study.
[b] Low income refers to students receiving free or reduced price lunch.
[c] Special education indicates that the child is receiving special education services in school.

Attentive behavior. To measure attention, the inattention subscale of the SWAN Rating Scale (Swanson et al., 2006) was used. This nine-item rating scale is based on the criteria for attention deficit hyperactivity disorder for inattention from the fourth edition of the *Diagnostic and Statistical Manual of Mental Disorders* (American Psychiatric Association, l994). Teachers use a rating scale of 1 to 7 for each item. In previous work at first to fifth grades (Fuchs et al., 2006, 2010), this instrument proved to be highly reliable ($\alpha > .97$). Teachers were instructed to rate children's attention during math classes.

Working memory. Working memory was assessed with the Counting Recall subtest of the Working Memory Test Battery for Children (WMTB-C; Pickering & Gathercole, 2001). This subtest has six items that vary in the span that they demand. On each item, children count a set of between four and seven dots on each of a series of cards. The number of cards whose dots need to be counted on a given item varies with the span level. At the end of an item, children must recall, in order, the number of counted dots that appeared on each card. Passing four items at a level allows children to go to the next level, where the number of items that must be remembered increases by one item. Counting recall has been demonstrated to be predictive of mathematics outcomes in the elementary grades (Fuchs et al., 2010).

Approximate number system. Participants were assessed using Panamath software Version 1.21 (Halberda et al., 2008). The task was administered on a laptop computer using settings for 10-year-olds at medium difficulty for 5 min. On each trial, children were shown 5 to 21 blue circles and 5 to 21 yellow circles for 1382 ms; the number of circles of each color varied semi-randomly over items. The dots were not interspersed; that is, the yellow dots appeared on the left half of the screen and the blue dots appeared on the right half of the screen. The task was to indicate the more numerous set of dots by pressing keys labeled with either a blue or yellow circle. Children completed a total of 120 trials. The ratio of one color of circles to the other included the following four ratio ranges (30 trials/range): 1.15–1.28, 1.28–1.43, 1.48–1.65, and 2.43–2.71. The most difficult ratio (1.15) involved a comparison of 15 versus 13 circles; the easiest ratio (2.71) involved a comparison of 8 versus 3 circles. A random half of the trials involved displays in which the total area covered by blue dots was approximately equal to the total area covered by yellow dots. For all trials, the sizes of the individual dots of each color varied. Percentage correct, response time, and Weber fraction were recorded electronically to ensure reliability.

Number line estimation. Children estimated where whole numbers (0–1000) should be placed on 25-cm number lines (Siegler & Opfer, 2003). Students were presented with 22 whole number estimation problems (56, 606, 179, 122, 34, 78, 150, 938, 100, 163, 754, 5, 725, 18, 246, 722, 818, 738, 366, 2, 486, and 147). Prior to beginning the task, children were asked to show the assessor where 0 and 1000 go on the number line. Feedback was provided for incorrect responses (Opfer & Siegler, 2007). Next, one practice trial in which children received feedback was presented. The assessor asked children to mark where 150 went on the number line. The assessor then marked the correct location and wrote the number corresponding to children's mark. The assessor verbally explained, "You told me that 150 would go here [pointing to children's mark], but actually this is where 150 goes [pointing to correct mark]. The line that you marked is where X actually goes." After this feedback trial, the remaining 0 to 1000 number line problems were administered.

The score was calculated as the distance from the correct placement for each estimate averaged across the 22 items. Higher percentages of absolute error indicated poorer performance. Internal reliability on this task was .89.

Calculation fluency. On the Addition Fluency subtest of the Wechsler Individual Achievement Test (WIAT; Psychological Corporation, 1992), students have 1 min to solve 48 addition problems in which the addends are between 0 and 10. Test–retest reliability in third grade is .87.

Reading fluency. In the Sight Word Efficiency subtest of the Test of Word Reading Efficiency (TOWRE; Torgesen, Wagner, & Rashotte, 1999), students are presented with a list of words and asked to read aloud as many words as possible within 45 s. The number of words read correctly is the score. Both test–retest reliability and alternate form reliability are higher than .90.

Outcome measures

Fraction concepts. Fraction concepts were assessed using 6 shaded fraction items (labeled as "fraction concepts" in Hecht et al., 2003) and 18 released fraction items from recent National Assessments of Educational Progress (NAEPs; U.S. Department of Education, 2007, 2009). For the shaded fraction items, symbol–picture tasks were presented to students. A polygon figure or set of figures was next to each fraction symbol. Students were instructed to shade the figure or set of figures to indicate the amount represented by the fraction symbol. The NAEP items assess concepts such as part–whole understanding (e.g., "Shade 1/3 of the rectangle above"), fraction comparison and equivalence (e.g., "Which picture shows that 3/4 is the same as 6/8?"), and number line accuracy (e.g., "On the portion of the number line above, a dot shows where 1/2 is. Use another dot to show where 3/4 is."). Response formats included multiple choice, short answer, shading fractional parts of a whole, and labeling a fraction on a number line. The maximum score was 24; reliability was .81 at the end of fourth grade.

Fraction procedures. The fraction procedures assessment, adapted from Hecht (1998), used eight fraction computation items and eight corresponding fraction word problems. Each set of eight items contained four addition and four subtraction items, and three of each set of eight items involved mixed numbers. All problems involved fractions with the same denominators. The number sentences that corresponded to the word problems were identical to those for the computation items. For example, one addition word problem was, "Sara painted 2/5 of a picture on Monday and 1/5 of the picture on Tuesday. How much of the picture did she paint during both days?" The corresponding computation item was, "2/5 + 1/5 = ___." Coefficient alpha was .95 during the spring of fourth grade.

Mathematics achievement. To measure general mathematics achievement, the Wide Range Achievement Test–fourth edition in math (WRAT; Wilkinson & Robertson, 2006) was used. This test measures basic computation in addition, subtraction, multiplication, and division primarily with whole numbers. (Very few items involving fractions were attempted at the third- and fourth-grade levels.) Internal reliability of the WRAT is high ($\alpha > .90$). The WRAT is highly correlated with other broad mathematics achievement measures (Wilkinson & Robertson, 2006).

Procedure

Language (PPVT), nonverbal reasoning (WASI Matrix Reasoning), attention (SWAN), reading fluency (TOWRE), number line estimation, and calculation fluency (WIAT) were assessed during the winter of third grade, and working memory (WMTB-C) was assessed during the spring of third grade. The ANS task and mathematics achievement (WRAT) were assessed during the winter of fourth grade, and fraction concepts and procedures were assessed during the spring of fourth grade, after all students had received formal fraction instruction in school. With two exceptions, all predictor tests were administered individually and all outcome measures were group administered. The exceptions were that one predictor, calculation fluency, was given in a group and one outcome measure, fraction procedures, was given individually. Instructions to all tests were read verbatim.

Results

As indicated in Table 2, the mean scores on all measures for both nonparticipants (children who did not complete all study measures) and participants were very similar. Little's (1988) Missing Completely at Random (MCAR) test results were not significant, indicating that the data were missing completely at random. Because accuracy was near ceiling on the overall ANS task ($M = .90$), analysis of accuracy on this task was limited to the 30 items in the most difficult ratio range (1.15–1.28), where the mean accuracy was 81% ($SD = 8.7$).

Significant bivariate relations were present between most predictors and outcome variables (Table 3). However, multivariate associations better capture the full network of relations among predictors and criteria (Stevens, 2002; Tabachnick & Fidell, 2013). Therefore, the data were analyzed using three direct entry (standard) multiple regression analyses (MRAs). Predictor variables were language (PPVT), nonverbal reasoning (WASI matrices), attention (SWAN), working memory (WMTB-C), number line estimation, reading fluency (TOWRE), and calculation fluency (WIAT); dependent measures were fraction procedures, fraction concepts, and fourth-grade mathematics achievement (WRAT). Number line estimation also was included as a dependent measure in the final model. Because we were interested in potentially malleable knowledge, we did not include background variables (age, special education status, income status, and gender) in the reported regression models. Preliminary results showed that the pattern of results concerning unique variance did not change when the demographics were included. Preliminary analyses also revealed that the ANS did not make significant contributions to any of the models and, thus, was not included in the MRAs.

The MRA predicting fourth-grade fraction concepts was significant, $R^2 = .562$, $F(7,349) = 63.853$, $p = .001$. See Table 4 for the unstandardized and standardized beta coefficients, standard errors, and R-square change for each predictor variable, with all others controlled. Six predictors made unique contributions to understanding of fraction concepts: number line estimation with whole numbers,

Table 2
Means of measures for study participants ($N = 357$) and nonparticipants ($N = 124$).

Predictor	Participants		Nonparticipants	
	M	*SD*	*M*	*SD*
Language (percentile)	46.85	28.60	48.15	28.72
Nonverbal reasoning (scaled; $M = 10$)	9.75	3.24	10.02	3.31
Attention (raw)	36.98	11.82	35.96	12.64
Working memory (percentile)	30.67	28.91	33.85	29.96
Number line estimation (% absolute error)	10.82	6.70	11.35	6.81
ANS (% correct)	80.91	8.68	82.44	9.34
Reading fluency (percentile)	64.51	25.20	63.31	23.86
Calculation fluency (percentile)	51.62	27.19	50.24	24.85
Math achievement, third grade (percentile)	58.26	27.45	53.60	26.95
Math achievement, fourth grade (percentile)	50.12	31.05	52.38	30.54
Fractions concepts (raw)	17.47	4.27	17.17	4.03
Fractions procedures (raw)	10.85	5.90	10.40	5.86

Table 3
Correlations among all measures.

Measure	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1. Age[a]	–																	
2. Gender[b]	−.068	–																
3. Minority[c]	−.047	.018	–															
4. English language learner	.015	−.028	.016	–														
5. Low income[d]	.146**	.016	.209**	.141**	–													
6. Special education[e]	.222**	−.064	.070	−.013	.092	–												
7. Language	−.088	−.063	−.237**	−.229**	−.408**	−.194**	–											
8. Nonverbal reasoning	−.178**	.107*	−.109*	.043	−.251**	−.145**	.465**	–										
9. Attention	−.221**	.147**	−.146**	.004	−.224**	−.343**	.332**	.392**	–									
10. Working memory	−.137**	−.021	−.131*	−.055	−.063	−.229**	.231**	.322**	.315**	–								
11. Number line estimation[f]	.191**	.193**	.228**	.071	.209**	.333**	−.412**	−.371**	−.399**	−.344**	–							
12. ANS	−.121*	.073	.009	.037	−.040	−.114*	.104	.106*	.242**	.129*	−.120*	–						
13. Reading fluency	−.260**	−.039	−.110*	−.074	−.212**	−.432**	.345**	.206**	.449**	.196**	−.362**	.126*	–					
14. Calculation fluency	−.052	−.117*	−.100	−.045	−.134*	−.245**	.183**	.186**	.336**	.228**	−.348**	.051	.380**	–				
15. Fraction concepts	−.256**	.042	−.170**	−.089	−.293**	−.373**	.504**	.453**	.552**	.348**	−.590**	.155**	.451**	.439**	–			
16. Fraction procedures	−.188**	.046	−.072	.012	−.109*	−.250**	.310**	.334**	.364**	.335**	−.439**	.102	.234**	.305**	.622**	–		
17. Math achievement, third grade	−.131*	−.071	−.257**	.065	−.253**	−.292**	.413**	.429**	.490**	.354**	−.433**	.120*	.451**	.531**	.597**	.406**	–	
18. Math achievement, fourth grade	−.230**	−.040	−.160**	.053	−.281**	−.342**	.411**	.434**	.570**	.341**	−.492**	.192**	.471**	.496**	.704**	.472**	.739**	–

[a] Age was determined at the beginning of the study.
[b] Gender was coded 1 = male, 0 = female.
[c] Minority status was coded 1 = minority, 0 = nonminority.
[d] Low income refers to students receiving free or reduced price lunch.
[e] Special education indicates that the child is receiving special education services in school and was coded 1 = receives special education services.
[f] On the number line estimation task, lower scores indicate higher performance.
* $p < .05$.
** $p < .01$.

Table 4
Results of multiple regression for third-grade predictors of fraction concepts during spring of fourth grade.

Variable	B	$SE\ B$	β	(ΔR^2)[a]	Bivariate correlation
Language	.043	.009	.196***	(.026)	.504
Nonverbal reasoning	.078	.030	.111**	(.008)	.453
Attention	.081	.016	.225***	(.033)	.552
Working memory	.036	.033	.044	(.002)	.348
Number line estimation	−.173	.028	−.271***	(.049)	−.590
Reading fluency	.034	.017	.086*	(.005)	.451
Calculation fluency	.106	.025	.169***	(.022)	.439
Constant	3.691	1.659			

Note. $R = .749$, $R^2 = .562$ ($N = 357$, $p < .001$).
* $p < .05$.
** $p < .01$.
*** $p < .001$.
[a] R^2 change after all other variables have been entered.

Table 5
Results of multiple regression for third-grade predictors of fraction procedures during spring of fourth grade.

Variable	B	$SE\ B$	β	(ΔR^2)[a]	Bivariate correlation
Language	.025	.017	.081	(.005)	.310
Nonverbal reasoning	.093	.053	.097	(.006)	.334
Attention	.068	.028	.135*	(.012)	.364
Working memory	.158	.058	.137**	(.015)	.335
Number line estimation	−.208	.049	−.237***	(.037)	−.439
Reading fluency	−.021	.030	−.037	(.001)	.234
Calculation fluency	.110	.044	.126*	(.012)	.305
Constant	1.244	2.923			

Note. $R = .535$, $R^2 = .287$ ($N = 357$, $p < .001$).
* $p < .05$.
** $p < .01$.
*** $p < .001$.
[a] R^2 change after all other variables have been entered.

attention, language, calculation fluency, nonverbal reasoning, and (to a lesser extent) reading fluency. The relative unique contribution of the independent variables was evaluated through the interpretation of standardized beta coefficients (Cohen, Cohen, West, & Aiken, 2003; Keith, 2006; Pedhazur, 1997). Importantly, number line estimation made the largest unique contribution; its predictive value was 1.6 times as large as that of calculation fluency (|−.271/.169|) and 1.4 times as large as that of language (|−.271/.196|).

The overall association of the MRA predicting fourth-grade fraction procedures was statistically significant but accounted for less variance, $R^2 = .287$, $F(7,349) = 20.030$, $p = .001$. Table 5 presents the unstandardized and standardized beta coefficients, standard errors, and R-square change for each predictor variable, with all others controlled. Four predictors made unique contributions to proficiency with fraction arithmetic procedures: number line estimation, attention, working memory, and calculation fluency. Number line estimation with whole numbers again made the largest unique contribution; its predictive value was 1.9 times as large as that of calculation fluency (|−.237/.126|) and approximately 1.7 times as large as that of working memory (|−.237/.137|).

The MRA predicting fourth-grade mathematics achievement also was significant, $R^2 = .529$, $F(7,356) = 57.143$, $p = .001$. See Table 6 for the unstandardized and standardized beta coefficients, standard errors, and R-square change for each predictor variable, with all others controlled. Calculation fluency and attention made the largest unique contributions. The predictive value of calculation fluency was 1.8 times as large as that of nonverbal ability (|−.254/.142|) and 1.85 times as large as that of number line estimation (|−.254/−.137|). Similarly, the predictive value of attentive behavior was

Table 6
Results of multiple regression for third-grade predictors of mathematics achievement (WRAT) during winter of fourth grade.

Variable	*B*	*SE B*	β	(ΔR^2)[a]	Bivariate correlation
Language	.023	.011	.094*	(.006)	.411
Nonverbal reasoning	.113	.035	.142**	(.014)	.434
Attention	.109	.018	.265***	(.046)	.570
Working memory	.057	.038	.060	(.003)	.341
Number line estimation	−.100	.032	−.137**	(.012)	−.492
Reading fluency	.062	.020	.137**	(.013)	.471
Calculation fluency	.183	.029	.254***	(.051)	.496
Constant	11.374	1.935			

Note. $R = .727$, $R^2 = .529$ ($N = 364$, $p < .001$).
* $p < .05$.
** $p < .01$.
*** $p < .001$.
[a] R^2 change after all other variables have been entered.

Table 7
Results of multiple regression for third-grade predictors of number line estimation during winter of third grade.

Variable	B	*SE B*	β	(ΔR^2)[a]	Bivariate correlation
Language	−.071	.018	−.208***	(.031)	−.412
Nonverbal reasoning	−.131	.057	−.120*	(.010)	−.371
Attention	−.071	.030	−.125*	(.010)	−.399
Working memory	−.207	.061	−.159***	(.021)	−.344
Reading fluency	−.073	.032	−.118*	(.010)	−.362
Calculation fluency	−.165	.047	−.168***	(.023)	−.348
Constant	37.972	2.431			

Note. $R = .579$, $R^2 = .336$ ($N = 364$, $p < .001$).
* $p < .05$.
*** $p < .001$.
[a] R^2 change after all other variables have been entered.

approximately 1.9 times as large as that of nonverbal ability (|.265/.142|) and 1.9 times as large as that of number line estimation (|.265/−.137|).

Finally, an analysis was performed to determine predictors of number line estimation accuracy, $R^2 = .336$, $F(6, 357) = 30.066$, $p = .001$. Table 7 shows the unstandardized and standardized beta coefficients, standard errors, and *R*-square change for each predictor variable, with all others controlled. All of the variables were uniquely but moderately predictive. Language (−.208) was the strongest predictor.

Discussion

We assessed predictive relations between third graders' performance on cognitive, behavioral, and achievement measures and the same children's fraction concepts and procedures and mathematics achievement in fourth grade. Significant bivariate correlations were present between most predictors and outcome measures.

The ANS task produced near ceiling levels of accuracy. However, on the most difficult magnitude comparisons, where the ratios of the two sets of dots were smallest, small but generally significant correlations with our outcome measures were present. Mazzocco, Feigenson, and Halberda (2011) reported much stronger correlations between ninth-grade ANS performance and mathematics achievement in elementary school than were found in the current study. However, in that study, the ANS did not uniquely predict performance on rank ordering of fractions and decimals over and above a symbolic number identification task. Mazzocco and colleagues observed that mapping precision with number symbols was more predictive than nonsymbolic approximation for tasks that are "significantly removed from intuitive computations" (p. 12). Our results support this interpretation.

The number line estimation task, which requires translation between symbolically and nonsymbolically represented whole number magnitudes, was much more strongly correlated with fraction concepts and fraction procedures than was the ANS task. Previous work with younger children also has shown that symbolic magnitude comparison tasks are more closely linked with conventional mathematics outcomes than are nonsymbolic ANS-type tasks (Holloway & Ansari, 2009; Soltesz, Szucs, & Szucs, 2010).

The contribution of our predictors varied somewhat for the fraction outcome measures. For fraction procedures, the predictors accounted for approximately 30% of the variance in performance, with number line estimation, working memory, attentive behavior, and calculation fluency each making unique contributions. Number line estimation made the largest unique contribution; its predictive value was nearly twice as large as that calculation fluency. For fraction concepts, the predictors accounted for a much larger portion of the variance (56%), with number line estimation, calculation fluency, language, nonverbal reasoning, and attentive behavior making unique contributions; number line estimation and attentive behavior made the largest unique contributions. These results suggest that although there is considerable shared variance among the predictors, both number-specific and more general competencies are uniquely important for explaining why some children struggle with fractions. The relatively small amount of variance accounted for by the set of predictors for fraction procedures may reflect the influence of instruction. That is, some children may have learned to add and subtract fractions in a relatively rote fashion. However, it should be noted that the fraction procedures assessed in the current study involved only simple calculations with like denominators. The skills used to solve more difficult procedural problems with unlike denominators may show a different pattern of findings. For example, working memory, attention, or reasoning abilities might become increasingly important when students must convert 6/10 to 3/5 and so forth.

Although our number line estimation task involved whole numbers, it nevertheless was a highly important predictor of fraction outcomes. Siegler and colleagues (2011) observed, "Numerical development involves coming to understand that all real numbers have magnitudes that can be ordered and assigned specific locations on number lines" (p. 274). Both whole numbers and fractions have magnitudes and are interspersed on number lines; students who acquire this insight with whole numbers seem to have an advantage in learning fraction concepts as well. In addition, thinking about parts and wholes may well facilitate number line estimation with whole numbers; for example, to estimate 754 on a 0-to-1000 number line, it would be adaptive to mentally divide the line into quarters to get close to 750 or 3/4 of 1000.

Attention and language comprehension were distinctive domain-general predictors of fraction concepts over and above the contributions of reading, nonverbal reasoning, working memory, and the mathematics predictors. The special relation to language might reflect the verbal nature of many fraction concept items (e.g., "These three fractions are equivalent. Write two more fractions that are equivalent to them.") The relation might also reflect the importance of vocabulary knowledge for acquiring fraction understanding, which might in turn affect children's later procedural competence with fractions. Seethaler and colleagues (2011) found that language was uniquely predictive of fraction procedures in fifth grade (they did not focus on fraction concepts), which we did not find in fourth grade. The later influence of language on fractions procedures might be associated with the mutually supportive relation between fraction concepts and procedures (Hecht & Vagi, 2010).

In addition to expanding our understanding of the acquisition of fraction conceptual and procedural knowledge, the current findings are useful for thinking about instruction. Fraction conceptual knowledge drives the learning of more advanced mathematics such as algebra (NMAP, 2008). The development of fraction reasoning, as measured by our fraction concepts instrument, is related to a mix of cognitive, behavioral, and numerical abilities. However, knowledge of numerical magnitudes, as indicated by accuracy of number line estimation, appears to be an especially strong early predictor of performance after children's first year of formal fraction instruction. It is more strongly predictive than proficiency in whole number computational fluency or attentional capacity.

The current findings suggest that to prepare children for the challenging fourth-grade curriculum in fractions recommended in the Common Core State Standards, primary-grade math instruction should target children's number competence related to their understanding of numerical magnitudes. Recent randomized studies have revealed that understanding of numerical magnitudes is malleable in most children (e.g., Jordan, Glutting, Dyson, Hassinger-Das, & Irwin, 2012; Ramani & Siegler, 2008) and that children who have learned numbers in a meaningful way become fluent with number combinations more easily than those with weaker number sense (Jordan, Kaplan, Ramineni, & Locuniak, 2008).

Although performance on our counting recall working memory task was significantly correlated with outcome measures, it was a unique predictor of performance only on fraction procedures. It is possible that working memory makes a larger contribution to fraction performance in later grades when the demands for holding and manipulating numerical information increase. It should also be noted that working memory is not a unitary construct (Baddeley, 1986); other processes related to the phonological loop and the visual sketchpad might make additional contributions to fraction learning.

The challenge for fraction instruction is to help children understand that magnitudes are a property not only of whole numbers but of all real numbers; that is, children need to learn that any real number can be placed on a number line—positive and negative numbers; whole numbers and fractions; common fractions, decimals, and percentages (Siegler et al., 2011). Although fractions pose unique challenges, understanding of whole number magnitudes provides children with a foundation for acquiring corresponding knowledge about fractions. Similarly, understanding fraction magnitudes provides them with a solid base for learning fraction procedures. A recent large randomized control trial (Fuchs et al., 2012) showed that a fourth-grade intervention that focused primarily on fraction concepts, especially understanding of fraction magnitudes, enhanced at-risk learners' fraction procedures as well as concepts more than standard classroom instruction. This was the case even though the classroom instruction allocated much more instructional time to fraction procedures than did the intervention.

The current results raise numerous questions for future investigations. Does understanding of numerical magnitudes continue to be strongly predictive of acquisition of fraction arithmetic procedures in later grades, or do other domain-specific and domain-general processes become more important as children progress to middle and high school? Is early fraction knowledge predictive of mathematics achievement in middle and high school? Will instruction in fraction concepts related to magnitude understanding increase later mastery of fraction arithmetic procedures, or are the two correlated but not causally related? Answering these and other questions raised by the current study promises to contribute to both mathematics education and theoretical understanding of mathematical development.

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References

American Psychiatric Association (1994). *Diagnostic and statistical manual of mental disorders* (4th ed.). Washington, DC: Author.

Baddeley, A. D. (1986). *Working memory*. Oxford, UK: Oxford University Press.

Baroody, A. J., Eiland, M., & Thompson, B. (2009). Fostering at-risk preschoolers' number sense. *Early Education and Development, 20*, 80–120.

Booth, J. L., & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology, 41*, 189–201.

Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development, 79*, 1016–1031.

Butterworth, B., Varma, S., & Laurillard, D. (2011). Dyscalculia: From brain to education. *Science, 332*, 1049–1053.

Cohen, J., Cohen, P., West, S. G., & Aiken, L. (2003). *Applied multiple regression/correlation analysis for the behavioral sciences* (3rd ed.). Mahwah, NJ: Lawrence Erlbaum.

Council of Chief State School Officers & National Governors Association Center for Best Practices (2010). *Common Core State Standards for mathematics: Common Core State Standards Initiative*. Retrieved from <http://www.corestandards.org/assets/CCSSI_Mathematics%20Standards.pdf>.

Dunn, L., & Dunn, D. (2007). *PPVT-4: Peabody picture vocabulary test manual* (4th ed.). Minneapolis, MN: Pearson Assessments.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*, 307–314.

Fuchs, L. S., Compton, D. L., Fuchs, D., Paulsen, K., Bryant, J. D., & Hamlett, C. L. (2005). The prevention, identification, and cognitive determinants of mathematics difficulty. *Journal of Educational Psychology, 97*, 493–513.

Fuchs, L. S., Schumacher, R. F., Long, J., Namkung, J., Hamlett, C. L., Cirino, P. T., et al. (2012). *Improving at-risk learners' understanding of fractions*. Unpublished manuscript.

Fuchs, L. S., Fuchs, D., Compton, D. L., Powell, S. R., Seethaler, P. M., Capizzi, A. M., et al (2006). The cognitive correlates of third-grade skill in arithmetic, algorithmic computation, and arithmetic word problems. *Journal of Educational Psychology, 98*, 29–43.

Fuchs, L. S., Fuchs, D., Powell, S. R., Seethaler, P. M., Cirino, P. T., & Fletcher, J. M. (2008). Intensive intervention for students with mathematics disabilities: Seven principles of effective practice. *Learning Disability Quarterly, 31*, 79–92.

Fuchs, L. S., Geary, D. C., Compton, D. L., Fuchs, D., Hamlett, C. L., & Bryant, J. V. (2010). The contributions of numerosity and domain-general abilities to school readiness. *Child Development, 81*, 1520–1533.

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37*, 4–15.

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007a). Cognitive mechanisms underlying achievement deficits in children with mathematics learning disability. *Child Development, 78*, 1343–1359.

Geary, D. C., Nugent, L., Hoard, M. K., & Byrd-Craven, J. (2007b). Strategy use, long-term memory, and working memory capacity. In D. B. Berch & M. M. Mazzocco (Eds.), *Why is mathematics so hard for some children? The nature and origins of mathematics learning difficulties and disabilities* (pp. 83–105). Baltimore, MD: Paul H. Brookes.

Geary, D. C., & Wiley, J. G. (1991). Cognitive addition: Strategy choice and speed-of-processing differences in young and elderly adults. *Psychology and Aging, 6*, 474–483.

Halberda, J., Mazzocco, M., & Feigenson, L. (2008). Individual differences in nonverbal number acuity correlate with math achievement. *Nature, 455*, 665–668.

Hallett, D., Nunes, T., & Bryant, P. (2010). Individual differences in conceptual and procedural knowledge when learning fractions. *Journal of Educational Psychology, 102*, 395–406.

Hallett, D., Nunes, T., Bryant, P., & Thorpe, C. M. (2012). Individual differences in conceptual and procedural fraction understanding: The role of abilities and school experience. *Journal of Experimental Child Psychology, 113*, 469–486.

Hanich, L., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematics cognition in children with learning difficulties. *Journal of Educational Psychology, 93*, 615–626.

Hecht, S. (1998). Toward an information-processing account of individual differences in fraction skills. *Journal of Educational Psychology, 90*, 545–559.

Hecht, S., Close, L., & Santisi, M. (2003). Sources of individual differences in fraction skills. *Journal of Experimental Child Psychology, 86*, 277–302.

Hecht, S. A., & Vagi, K. J. (2010). Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology, 102*, 843–859.

Hecht, S. A., & Vagi, K. J. (2012). Patterns of strengths and weaknesses in children's knowledge about fractions. *Journal of Experimental Child Psychology, 111*, 212–229.

Hecht, S. A., Vagi, K. J., & Torgesen, J. K. (2007). Fraction skills and proportional reasoning. In D. B. Berch & M. M. M. Mazzocco (Eds.), *Why is mathematics so hard for some children? The nature and origins of mathematics learning difficulties and disabilities* (pp. 121–132). Baltimore, MD: Paul H. Brookes.

Holloway, I. D., & Ansari, D. (2009). Mapping numerical magnitudes onto symbols: The numerical distance effect and individual differences in children's mathematics achievement. *Journal of Experimental Child Psychology, 103*, 17–29.

Jordan, N. C., Glutting, J., Dyson, N., Hassinger-Das, B., & Irwin, C. (2012). Building kindergartners' number sense: A randomized controlled study. *Journal of Educational Psychology, 104*, 646–660.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2008). Development of number combination skill in the early school years: When do fingers help? *Developmental Science, 11*, 662–668.

Kail, R., & Hall, L. K. (1999). Sources of developmental change in children's word-problem performance. *Journal of Educational Psychology, 91*, 660–668.

Keith, T. Z. (2006). *Multiple regression and beyond*. Boston: Allyn & Bacon.

Lee, K., Ng, L., & Ng, S. F. (2009). The contributions of working memory and executive functioning to problem representation and solution generation in algebraic word problems. *Journal of Educational Psychology, 101*, 373–387.

LeFevre, J. A., Bisanz, J., Daley, K. E., Buffone, L., Greenham, S. L., & Sadesky, G. S. (1996). Multiple routes to solution of single-digit multiplication problems. *Journal of Experimental Psychology: General, 125*, 284–306.

Libertus, M. E., Feigenson, L., & Halberda, J. (2011). Preschool acuity of the approximate number system correlates with school math ability. *Developmental Science, 14*, 1292–1300.

Little, R. J. A. (1988). A test of missing completely at random for multivariate data with missing values. *Journal of the American Statistical Association, 83*, 1198–1202.

Locuniak, M. N., & Jordan, N. C. (2008). Using kindergarten number sense to predict calculation fluency in second grade. *Journal of Learning Disabilities, 41*, 451–459.

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011). Preschoolers' precision of the approximate number system predicts later school mathematics performance. *PLoS One, 6*, e23749.

Muldoon, K., Towse, J., Simms, V., Perra, O., & Menzies, V. (2013). A longitudinal analysis of estimation, counting skills, and mathematics ability across the first school year. *Developmental Psychology, 49*, 250–257.

National Mathematics Advisory Panel (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: US Department of Education.

Opfer, J., & Siegler, R. S. (2007). Representational change and children's numerical estimation. *Cognitive Psychology, 55*, 169–195.

Passolunghi, M. C., & Siegel, L. S. (2001). Short-term memory, working memory, and inhibitory control in children with difficulties in arithmetic problem solving. *Journal of Experimental Child Psychology, 80*, 44–57.

Pedhazur, E. J. (1997). *Multiple regression in behavioral research: Explanation and prediction* (3rd ed.). Orlando, FL: Harcourt Brace.

Pickering, S., & Gathercole, S. (2001). *Working memory test battery for children*. London: Psychological Corporation.

Psychological Corporation (1992). *Wechsler individual achievement test – Manual*. San Antonio, TX: Author.
Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development, 79*, 375–394.
Rittle-Johnson, B., & Siegler, R. S. (1998). The relation between conceptual and procedural knowledge in learning mathematics: A review. In C. Donlan (Ed.), *The development of mathematical skills* (pp. 75–110). Hove, UK: Psychology Press.
Rousselle, L., & Noelle, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs. non-symbolic number magnitude processing. *Cognition, 102*, 361–395.
Sadler, P. M., & Tai, R. H. (2007). The two high-school pillars supporting college science. *Science, 317*, 457–458.
Seethaler, P. M., Fuchs, L. S., Star, J. R., & Bryant, J. R. (2011). The cognitive predictors of computational skill with whole versus rational numbers: An exploratory study. *Learning and Individual Differences, 21*, 536–542.
Siegler, R. S. (1988). Strategy choice procedures and the development of multiplication skill. *Journal of Experimental Psychology: General, 117*, 258–275.
Siegler, R. S., & Pyke, A. A. (2012). Developmental and Individual Differences in Understanding of Fractions. Developmental Psychology. http://dx.doi.org/10.1037/a0031200.
Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., et al (2012). Early predictors of high school mathematics achievement. *Psychological Science, 23*, 691–697.
Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science, 14*, 237–243.
Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games—but not circular ones—improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology, 101*, 545–560.
Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology, 62*, 273–296.
Soltesz, F., Szucs, D., & Szucs, L. (2010). Relationships between magnitude representation counting and memory in 4- to 7-year-old children: A developmental study. *Behavioral and Brain Functions, 6*. http://dx.doi.org/10.1186/1744-9081-6-13.
Stevens, J. P. (2002). *Applied multivariate statistics for the social sciences* (4th ed.). Mahwah, NJ: Lawrence Erlbaum.
Swanson, H. L. (2006). Cross-sectional and incremental changes in working memory and mathematics problem solving. *Journal of Educational Psychology, 98*, 265–281.
Swanson, J., Schuck, S., Mann, M., Carlson, C., Hartman, K., Sergeant, J., et al. (2006). *Categorical and dimensional definitions and evaluations of symptoms of ADHD: The SNAP and SWAN rating scales*. Unpublished manuscript. Irvine: University of California.
Tabachnick, B. G., & Fidell, L. S. (2013). *Using multivariate statistics* (6th ed.). Boston: Allyn & Bacon.
Torgesen, J. K., Wagner, R. K., & Rashotte, C. A. (1999). *TOWRE: Test of word reading efficiency*. Austin, TX: Pro-Ed.
U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (2007). *2007 Mathematics assessment.* Retrieved from <http://nces.ed.gov/nationsreportcard> July 2011.
U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress. (2009). *2009 Mathematics assessment.* Retrieved from <http://nces.ed.gov/nationsreportcard> July 2011.
Vamvakoussi, X., & Vosniadou, S. (2010). Understanding the structure of the set of rational numbers: A conceptual change approach. *Learning and Instruction, 14*, 453–467.
Wechsler, D. (1999). *Wechsler Abbreviated Scale of Intelligence (WASI)*. San Antonio, TX: Harcourt Assessment.
Whyte, J. C., & Bull, R. (2008). Number games, magnitude representation, and basic number skills in preschoolers. *Developmental Psychology, 44*, 588–596.
Wilkinson, G. S., & Robertson, G. J. (2006). *Wide Range Achievement Test 4 professional manual*. Lutz, FL: Psychological Assessment Resources.