Using Writing in Mathematics to Deepen Student Learning
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"Writing in mathematics gives me a window into my students’ thoughts that I don’t normally get when they just compute problems. It shows me their roadblocks, and it also gives me, as a teacher, a road map."

—Maggie Johnston
9th-grade mathematics teacher, Denver, Colorado
INTRODUCTION

Writing is the ability to compose text effectively for different purposes and audiences. When many of us reflect on our own school experiences, we recall writing in English and history classes, but not in mathematics. Math classes previously relied on skill-building and conceptual understanding activities. Today, teachers are realizing that writing during a math lesson is more than just a way to document information; it is a way to deepen student learning and a tool for helping students gain new perspectives.

They recognize, too, that students whose strengths are language-based—and many are—use writing as the key to understanding other disciplines, especially mathematics. For example, Dr. David K. Pugalee (2004) conducted a study with 9th-grade algebra students to determine if journal writing can be an effective instructional tool in mathematics education and found that it may have a positive effect on problem solving because the writer must organize and describe internal thoughts.

Like most things, learning to write well requires instruction and practice. In this booklet, I aim to nudge secondary math teachers who are thinking about using writing in their classrooms more extensively and to encourage those who want to begin. Perhaps you will come to the same conclusion as mathematics educator Marilyn Burns, who said, “I can no longer imagine teaching math without making writing an integral aspect of students’ learning” (p. 30).

Section One gives a brief background that answers the question you may be wondering: Why write in mathematics? Section Two describes the existing role of writing in the mathematics curriculum, and Section Three provides strategies and ideas to put into practice right away.
Researchers agree that, like reading, improving student’s writing skills improves their capacity to learn (National Institute for Literacy, 2007). We also know that writing fosters community in a classroom and, because writing is a social act, it is a vehicle for students to learn more about themselves and others. Researcher Donna Alvermann (2002), an expert in adolescent literacy, studies students’ self-efficacy and engagement. She urges all teachers, despite their content area expertise, to encourage students to read and write in many different ways. She does so because she believes that writing raises the “cognitive bar,” challenging students to problem solve and think critically. What other single action requires students to be so grounded in a viewpoint that they can convince others, to know a process so thoroughly that they can explain it to someone else, or to grasp the nuances of an idea so deeply that they can convey it in a way that provokes thought and sparks discussion?

I know that writing does this for me each time I draft an article or prepare a workshop presentation. Until I read what I have written, I don’t see the holes in my logic, the missing steps, or the rambling thoughts. Writing informs me that I only have a cursory knowledge of the content when I need a deep one. Simply put, it doesn’t let me cut corners. Whenever I have conducted workshops on using writing in mathematics, math teachers, quick to see the parallels with problem solving, have rallied to the idea. Happily, teachers who want to begin infusing writing in their instruction or who want to increase their use of it don’t have to start from scratch. They can begin with their own well-crafted lessons and add a writing activity that will enhance student engagement and heighten cognitive demands.

**Why are we writing in math class?**

David Pugalee (2005), who researches the relationship between language and mathematics learning, asserts that writing supports mathematical reasoning and problem solving and helps students internalize the characteristics of effective communication. He suggests that teachers read student writing for evidence of logical conclusions, justification of answers and processes, and the use of facts to explain their thinking.
In *Writing Next*, researchers Graham and Perin (2007) identify the following 11 elements of current writing instruction that help young people learn to write well and to use writing as a tool for learning.

1. Teach students strategies for planning, revising, and editing.
2. Explicitly and systematically teach students how to summarize texts.
3. Use instructional arrangements in which students work together on writing.
4. Assign students specific, reachable goals for their writing tasks.
5. Use computers and word processors as instructional supports for writing assignments.
6. Teach sentence combining as a way to construct more complex, sophisticated sentences.
7. Engage students in prewriting activities to generate ideas for composing.
8. Use inquiry activities where students analyze immediate, concrete data to develop ideas and content for a particular writing task.
9. Use the writing process to provide extended writing opportunities.
10. Provide opportunities to read, analyze, and emulate models of good writing.
11. Use writing as a tool for learning content material.

There are many formal and informal ways to make these elements actionable in schools. For example, a school undertaking a writing initiative might adopt one element a month for a year. Or, if teachers already meet regularly for book studies, they could meet to share their ideas, concerns, and successes about implementing one of these writing elements. I encourage you to try, share, and try again. You will get to know your students in ways you never have before, and you will most certainly know who is and isn’t learning mathematics content.
SECTION TWO: Writing as part of the mathematics curriculum

Writing is not a separate entity from the mathematics curriculum; it is part of it. Among the learning goals that the National Council of Teachers of Mathematics (NCTM) has set for all students is to communicate their mathematical thinking.

NCTM recommends that writing about mathematics be nurtured across grades. NCTM suggests that students write explanations about how they solved a problem, solutions to exercises as if they were writing a textbook, essays about what it means to prove something, or reports describing the significant contributions of well-known mathematicians (2000). Joan Countryman (1992), who explores the relationship between mathematics and writing, offers the following four benefits of writing in mathematics class:

Students write to keep ongoing records about what they’re doing and learning.

While they're writing, students can restate new material in their own words, identify computations that are easy or difficult, or reflect on aspects that confuse them, as in this note from a student to his math teacher.

**Dear Mr. Kuhn:**

Today we talked a lot about equations. I understand what an equation is. I have used them many times. But I don't really understand how an equation can be turned into a sentence. I know how to do the math but not how to explain it. Would you please spend some more time tomorrow teaching us how to make sentences out of equations?

**Thanks,**

**Sammy**

**Principles and Standards for School Mathematics**

“Reflection and communication are intertwined processes in mathematics learning...Writing in mathematics can also help students consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas” NCTM (2000), p.61.
**Students write in order to solve math problems.**

Students can write the facts they need to answer a question beforehand and afterwards, then check their computations against their written facts. This also helps them see different ways to arrive at an answer. When doing a unit on slope, one of Maggie’s students wrote down her thought process this way:

*In order to solve the problem, I need to find the rate of change in the sale of blue jeans by subtracting the amount sold in 1992 from 1996 and then add the costs on top of the cost in 1992 until it reaches this year.*

Another student wrote:

*I need to find the slope, which is the rate of change, and then next find B and finally solve the problem. The rate of change is the jeans.*

**Students write to explain mathematical ideas.**

When students write explanations of their work and give examples, teachers can better assess student understanding and progress throughout time. Writing is an ideal vehicle for formative assessment, providing teachers with the information they need to adjust their instruction.

Three kinds of writing prompts reflect three aspects of learning mathematics: 1) content, 2) process, and 3) affective. Content prompts deal with mathematical concepts and relationships, process prompts focus on algorithms and problem solving, and affective prompts center on students’ attitudes and feelings. Content prompt examples include these:

- Define parallel in your own words.
- How would you describe a number line?
- Write a paragraph about the graph in the news.
- Write one sentence that describes an equation.
- How do you know that 1/4 is greater than 1/5? Explain your thinking.
- Write as many examples of a square that you can think of in five minutes.
- What properties do triangles have?
- Write everything you know about money.
- What were the key points in today’s lesson?
Students write to describe learning processes.

Writing about problem solving requires students to monitor and reflect on the strategies and processes they select. Maggie explains why she uses writing to enhance the metacognitive aspect of learning in mathematics: “If there’s no writing in math class, all they’re doing is the evaluation-execution portion of learning. Orientation and organization come before execution, and that’s what writing gets at. That’s the most valuable piece of writing in mathematics class,” she said.

Maggie develops the reflective writing prompts she uses with a learning objective in mind. Frequently, it includes a metacognitive piece, which will require her students to organize a problem, do the problem, and rethink it. This is how it looks in her classroom:

Maggie shows the class an example problem from her Algebra 2 class: \(2 \log_4 x - 5 \log\) and asks, “Does anyone know how to do this problem?”

She waits for a few minutes, sensing students’ hesitancy to respond. Maggie has anticipated this, so she removes the problem from the overhead, replacing it with a writing prompt she has prepared.

“Here’s the writing prompt: When I see this problem, my first reaction is. . . Here are your options:

1. I realize why I hate math.
2. I’m a little afraid, but I know that condense means to make something smaller.
3. I’m not exactly sure how to do this problem, but we’ve been learning about logs.”

Maggie has set the stage for learning by providing students a vehicle for describing their learning processes. The rest of the class period includes whole-class brainstorming about what students already know about solving similar problems, time to work alone and with a partner, and sharing learning processes and outcomes. To keep the workload down, Maggie uses only two or three writing prompts with each unit and tries to connect big ideas from each unit through writing. “Once they write about those big ideas, it really enhances their understanding, especially at the high school level,” said Maggie.
SECTION THREE:  
Strategies and ideas for writing in mathematics

Not only should teachers be aware of the instructional strategies that are most effective in improving student writing (National Institute for Literacy, 2007; Graham & Perin, 2007); they also need ideas for implementation. In this section, I address that need by returning to the work of Joan Countryman (1992), who observes that writing in the mathematics classroom generally looks like one of six types: Free writes; biography and autobiography; learning logs, blogs, and journals; summaries; word problems; and formal writing. With an understanding of these forms, strategies for implementation, and practical ideas for integrating technology tools, all teachers—regardless of their own comfort levels with writing—can enhance their mathematics lessons.

1. Free writing

Free writing involves writing nonstop for a fixed amount of time, usually just a few minutes. Free writes don’t allow time for students to agonize over grammar or spelling; rather, they encourage students to think freely and raise questions about a topic or idea. When you first ask students to free write, give them high-interest topics, so they can begin to write immediately. For example, students usually have strong opinions about whether or not they will ever use advanced mathematics in “real life,” why they should use calculators during high-stakes assessments, or whether an additional year of math should be required in the high school curriculum. Once they are used to writing on broad topics, ask them to write on math-related topics, such as predicting the effect of one rotating object on another or comparing methods of analyzing data sets. Finally, develop prompts that relate specifically to the content you are covering in class by asking students to summarize their learning or describing steps in solving a problem.

Implementation Idea: The Journalists’ Questions¹

Once students have become comfortable writing for short periods, introduce “The Journalists’ Questions” (who? what? when? where? why? how?). Explain that this traditional reporters’ style of writing is a way of identifying essential information.

When studying units of measure, for instance, ask students to write the general topic “the metric system” on their papers and list the six questions, leaving room for a response to each. Give them a few minutes at the beginning or end of class to answer as many as they can. Collect and look over responses, and you will quickly discern what information or misinformation students already have. Here’s how the six questions might apply to the topic “the metric system.”

The next day, have a class discussion to clear up any misconceptions. Give students the option to write a paragraph describing the metric system or defending its use in science and mathematics. If you have access to technology, a fun way to apply this strategy is to pair it with a Web site that you can check out ahead of time. Assign a topic, ask students to complete as many of the Journalists’ Questions as they can, and write a summary paragraph on the topic.

2. Biography & autobiography

This type of narrative non-fiction writing encourages students to write descriptively, and to identify significant events, personality traits, turning points, and impacts on a person’s life. The life stories of important mathematicians will intrigue some of your students, and there are many Web sites with biographies, such as Omnibiography.com; allmath.com/biography.php; or even Simpsonsmath.com, where students can read the math biographies of the show’s writers.

To extend this writing opportunity, encourage students to write letters or e-mails to the mathematicians (or the Simpson creators) they learned about. Alternatively, ask students to illustrate in the form of a pie chart the percentage of the person’s life devoted to the activity that made him or her famous. You also can use a variation of autobiography called a “mathography” to learn more about your students’ attitudes, perceptions of their effort and abilities, and to know the kind of support they think they need.
Implementation Idea: Write Your Mathography

At the beginning of a year, ask students to write about their experiences learning mathematics, describing the strategies their instructors used to help them learn. Or, provide a choice of writing prompts, such as these:

- What early math accomplishments do you remember? (e.g., When and how did you learn to count? Who taught you? How did he or she teach you? Did you “show off” this new talent to others?)
- When you were in elementary school, what did you like (or not like) about math?
- What do you remember about learning to add and subtract? Which did you think was more fun? Why?
- Was math ever your favorite subject? If so, why? If not, why?

3. Learning logs, blogs, & journals

Over time, the lines distinguishing learning logs and journals have blurred, but students still can use these writing vehicles to respond to class discussions, make connections between real-life and the content, and to reflect on themselves as learners. Learning logs should focus on content, whereas journals might focus on students’ ideas and questions about a broad range of general topics. Students use each differently. While journaling, they might reflect on anything they consider relevant, carry on conversations with their teachers, or both. Learning logs, on the other hand, are less about “feeling” and more about understanding content. Similarly, blogs allow students to share ideas and solutions in real time with each other, other classes, and the teacher.

Although both kinds of insights are valuable, teachers should assign learning logs, blogs, and journals with different intent. Students should write frequently in their logs and blogs for several minutes at a time at least once a week and should use an established template for entering information. Journal writing can be less frequent, perhaps every two weeks. Here is one example of a learning log template that you can adapt and provide students.

### Algebra I Learning Log

- Date:
- Name of topic, presenter, chapter title, program/video:
- Pages read, length of discussion/explanation or program/video:
- Main points:
- Summary of main points:
Implementation Idea: Climbing and Diving

This is a good strategy to use when students are writing in their learning logs. As students move back and forth between climbing and diving (into a topic), they internalize the process and the content. For example, following a unit on the area of polygons, students spend 10 minutes writing everything they learned, including formulas and descriptions. They then read over their writing and select one idea to explore further, such as why the formula to determine the area of a triangle works for all triangles, regardless of type. The second 10-minute writing allows them to justify their thinking or reflect on their understanding. If you have access to technology, ask students to journal electronically or keep a class e-learning log or blog. (For more on Climbing and Diving, see Teaching Writing in the Content Areas by Urquhart & McIver.)

Implementation Idea: Double-entry Journals

A double-entry journal is a long-standing popular activity for any content area. Frequently, teachers pair it with a reading assignment where students summarize on the left, which represents literal information from the text, and respond to the summary on the right, which represents inferential and critical thinking. One way to use a double-entry journal focusing on problem solving is to have students write the problem in the left column and the details for solving it in the right column. To adapt the journal, ask students to create a third column and write a personal reflection that describes what was frustrating or easy, or to draw a horizontal line two-thirds of the way down the paper to provide a section for summarizing the process.

4. Summaries

Because students often find summary writing difficult, teachers should explicitly teach summary skills and provide plenty of opportunities for practice. Journals and learning logs are ideal places for students to practice summarizing. Students should understand they must identify main ideas, discriminating between information that is essential and information that is merely interesting. Here are some practical suggestions for using summary writing:

- Teach students how to organize key ideas into logical patterns using webs, charts, or diagrams, such as a Venn Diagram, which visually represents common elements, or a concept map, which presents relationship among a set of connected ideas.
- Use summary writing when students are learning large amounts of information and vocabulary terms.
- Allow students to write collaboratively or summarize together as a whole class.
Types of Numbers

Prime
17 10 3 3 11 2 3
Even
2 4 18 6 600

See how many things you have to begin with. Then take away some. Count how many things you have left, and that is your answer.

A bird found 9 beetles. He ate 4. At the same time, 5 beetles hid under a rock. Then the bird flew away.

Subtraction
How does it work?
Equation
Drawing
Story

Concept Map

9 - 4 = 5
Implementation Idea: Magnet Summaries

A great way to explicitly teach summary writing is with the “Magnet Summary” strategy.

Sample steps for using Magnet Summaries

- When studying equations, provide students a written selection (e.g., a textbook section, Web site, or teacher-created document) that explains how to solve an equation.
- Read the selection ahead of time and develop a list of key vocabulary words that appear, such as inverse, variable, logarithm, exponent, etc.
- Provide students with the same number of index cards as total number of words on the list.
- Write the magnet word list on the board, overhead transparency, or PowerPoint, and ask students to write each word on an index card.
- Distribute the text selection, and ask students to read it. Working on their own, they should reread and look for the magnet words, identifying key words or phrases in the text that help explain or directly relate to the magnet word. Explain that these are the words or phrases that the magnet words “attract,” and students can highlight them with a marker, if it will help them remember the meaning of the words.
- On the back of each index card, students then write a summary sentence in their own words that defines, describes, or expresses the main idea of the magnet word.
- Students read their sentences and organize the cards in a logical order.
- Students write a summary paragraph, adding transitional words and phrases.
- Students read their summaries to others, and revise for clarity and cohesion, using peer feedback.

5. Word problems

Words are tools for thinking in mathematics, just as they are in other disciplines. Good word problems promote thinking and encourage students to use their own language, thus owning their ideas. Pugalee cites the research of Winograd (1992), suggesting that students, when asked to write original word problems, will write more interesting and challenging ones than textbook problems.
Implementation Idea: Explicitly Teach Relevant Vocabulary

Whether writing their own word problems or preparing to write constructed responses, students need to be comfortable with certain words, know their definitions, and be able to use them in writing tasks. There are six concrete steps for learning a new word (Paynter, Bodrova, & Doty, 2005):

1. Identify the new word and elicit students’ background knowledge (e.g., the term “probability model”).

2. Explain the meaning of the new word (e.g., tell students that a probability model is a technique for representing the chances of something occurring, and that they will use a model, or something they can manipulate, to determine the chances of two students each drawing a red chip from a bag of five blue and five red chips. Show them two colored charts, one representing each student’s chances, and a third chart representing the chances of both of the students drawing a red chip.

3. Monitor students as they work in small groups to generate their own explanations. If they have difficulty, you might need to provide a sentence stem or ask questions or provide tools, such as dictionaries or thesauruses.

4. Ask students to create a visual representation of the new word. If you have access to technology and the Internet, encourage students to find clip art, photos, or interactive models (see a selection of mathematics and science interactives at http://www.explorelarning.com).

5. Provide an experience to use the new word (e.g., discuss the probability that students will win the state lottery).

6. Engage students in activities (e.g., vocabulary baseball or vocabulary bingo) to help them remember the word and its meaning.
In addition to teaching mathematics-specific vocabulary, teach students terms they will see on state tests. Several of the most-used terms on constructed response items appear in the chart that follows. You can assign the writing task on the chart or adapt it to use with your students.

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Mathematics Writing Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analyze</td>
<td>Break down a topic into its parts and examine how each part functions in relation to the whole.</td>
<td>Examine a set of economic statistics and write an editorial on the cause and effect relationship between high wages and inflation.</td>
</tr>
<tr>
<td>Describe</td>
<td>Represent a person, object, or idea in words.</td>
<td>Write a detailed description of a model so that someone else can re-create it.</td>
</tr>
<tr>
<td>Evaluate</td>
<td>General: Determine the significance, worth, or quality of objects or ideas.</td>
<td>Write an article for a student newspaper about the benefits of taking extra math courses in high school.</td>
</tr>
<tr>
<td></td>
<td>Specific to mathematics: Evaluate an expression for a given amount.</td>
<td>Evaluate the expression for $x = 5, y = 3$. Work it out, and write an explanation for someone else to follow.</td>
</tr>
<tr>
<td>Narrate</td>
<td>Write a sequence of actions occurring over time.</td>
<td>Relate the story of the evolution of the abacus through ancient, middle, and modern times.</td>
</tr>
<tr>
<td>Reflect/Question</td>
<td>Present to readers the same questions you ask yourself.</td>
<td>Think about a different method of solving the same problem that still results in an accurate answer, and explain it.</td>
</tr>
<tr>
<td>Summarize</td>
<td>Briefly restate others’ ideas in your own words.</td>
<td>Write a report compiling highlights of the research on a mathematical question that intrigues you.</td>
</tr>
<tr>
<td>Synthesize</td>
<td>Combine information and ideas from multiple sources.</td>
<td>Write the final rubric criteria for a math writing assignment.</td>
</tr>
</tbody>
</table>
6. Formal writing

Formal writing should always be graded. When students write formally, they go beyond the kinds of short writings they might have been doing in learning logs, blogs, journals, or free writes to write research reports and essays. Formal writing can still be done collaboratively, however.

Implementation Idea: Conduct a Research Project and Write a Report

Research topics can cover a wide range of students’ interests—Aristotle’s contributions to the field of mathematics, the reason gas prices go up or down just before an election, or differences in attitudes towards mathematics in other countries. Whatever students’ interests, when it is time to write the report, explain that formal mathematics research papers include several sections, although not all papers contain all sections. A basic format to adapt and share follows.

1. Title: Use it to catch the attention of the reader and reflect the content of the paper.
2. Abstract: This is a summary paragraph that explains the basic purpose of the paper, states the question(s) answered, and tells the reader what was proved.
3. Literature Review: When researchers consult existing literature and extend the ideas they find there, they include a literature review. To complete this section, ask questions such as, “What kind of relevant studies or techniques should I know about in order to do the project?” “How have others gone about trying to solve the problem, and how is my approach different?”
4. Statement of the Problem: This “sets the stage” for the paper. A brief introduction gives the reader some context. What inspired you to explore this problem? Is it a modification of some other question? Why is this problem important or interesting to you?
5. Body of the Report: This takes the reader on a trip through the research project. Using your learning log, write the body of the report as narrative non-fiction, including your initial ideas. Include nonlinguistic representations and examples as support.
6. Ideas for Further Research: State any questions that surfaced that are beyond the scope of the project.
7. References. You must credit the people whose work you use. The Internet is a great resource for this. Writing labs at colleges or universities, in particular, offer online user-friendly guidance.
FINAL THOUGHTS

Standards have long been a kind of road map for teachers. For Maggie Johnston, the mathematics teacher quoted at the beginning of this booklet, writing also is a road map, and perhaps a richer, more detailed one. Teachers who include writing experiences in their classrooms set the stage for active problem solving, invention and discovery, increased reading, and improved content learning. Students get a chance to express new knowledge and skills in their own words, organize their thinking about the content, share their ideas, experience a creative side of mathematics, and learn to value the act of writing; teachers get a tool that can motivate and engage students, instant evaluations about students’ learning, and a way to participate in interdisciplinary collaboration. Whatever the purpose, writing should be as much at home in a mathematics class as in an English class.
REFERENCES


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