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The Leadership Role of STATE SUPERVISORS of MATHEMATICS

Report of a Conference
Under the Auspices of the
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U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
Abraham Ribicoff, *Secretary*
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Foreword

TO HELP State Supervisors of Mathematics develop constructive leadership programs suitable for immediate adoption, under varying State and local conditions, the U.S. Office of Education sponsored a National Conference of State Supervisors of Mathematics at Washington, D.C., from June 19 to 23, 1961. The proceedings of the Conference are reported in this bulletin.

The Conference had four main objectives:

1. To focus attention on the evolving mathematics curriculum in order to determine the most effective things the State Supervisor can do to help strengthen mathematics teaching.
2. To consider how the State Supervisor can help improve inservice and preservice education for mathematics teachers.
3. To identify what types of research are needed in the field of mathematics teaching and to consider the implications of present research findings.
4. To determine how the State Supervisor can evaluate his State's program for strengthening mathematics teaching and how he can help school systems evaluate their programs.

The planning committee for the Conference was composed of the following personnel from the Office: J. Dan Hull, Kenneth E. Brown, Marjorie C. Johnston, Marguerite Kluttz, Daniel W. Snader, and Mary K. Tulock. Franklin Padgett assisted in compiling this final report of the Conference.

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IRVING ADLER

1. Contemporary mathematics is distinguished by the following characteristics:

It is classical mathematics grown mature.

It is classical mathematics grown self-conscious and self-critical.

It is modern mathematics developed as a more efficient way of dealing with the content of classical mathematics.

It is mathematics related more and more intimately to man's activities in industry, social life, science, and philosophy.

2. Classical mathematics, in its growth to maturity, has done the following:

Changed from a number system to number systems.

Changed from geometry to geometries.

Unified the concepts of number and space.

Developed analysis.

Evolved the concept of function.

Developed the theories of functionals.

3. Modern mathematics is to classical mathematics as elementary algebra is to elementary arithmetics. Modern mathematics is necessarily axiomatic, deductive, and abstract. It does not replace classical mathematics. It generalizes it, supplements it, unifies it, and deepens our understanding of it. But classical mathematics in the form of arithmetic, analysis, and geometry is as important as it ever was.

4. At one time the geometry of Euclid was unique in being axiomatic and deductive. Now all mathematical structures are axiomatic and deductive. To introduce young people to deductive systems, we now have many such systems from which to choose.

5. Even if we could do without intuition in creating mathematics, it would be folly to try to do without it in teaching mathematics. Although a deductive argument may show us where one tree stands in relation to other trees, an intuitive argument is often the best way of seeing the woods that are made up of all those trees.

6. The spread of industrial automation tends to reduce the number of unskilled and semiskilled workers and to increase the need for

technically trained personnel with a knowledge of mathematics. Hence, the importance of mathematics in vocational training will continue to grow in the future.

7. While mathematics in the form of pure mathematics has reached dizzying heights of abstraction, it has kept its feet on the ground by multiplying and extending its applications.

Implications of the Changes in Mathematics for the Mathematics Supervisor

CARL B. ALLENDOERFER

1. The State supervisor of mathematics should be:

   A missionary who exudes inspiration.

   An organizer who can arrange inservice education.

   A counselor who influences his constituents through persuasion rather than by authority.

2. In support of the new mathematics programs one may point to the professional standing and caliber of the key people and to the general agreement of mathematicians on the basic principles.

3. Unless a teacher has had rather extensive preparation in the use of the new teaching materials, it is unrealistic to expect him to use them adequately.

4. The best approach to supervision is to be an advisor and a friend, make no claim to have all the answers, offer help and support where needed, but never tell a school district what it must do.

New Media of Instruction

ARTHUR LALIME

1. Although the basic elements of the new media of instruction have been with us for many years, implementing these devices and techniques in the educational program is a recent development.

2. The new media of communication can be used more efficiently and with greater educational impact in a school system that is organized for team teaching.

3. New schools, in Norwalk (Connecticut) and elsewhere, are being designed to provide suitable facilities for team teaching and for effective use of the new media of instruction.

4. The following principles are basic to the team-teaching program in Norwalk:

The size of the instructional group is determined by what is to be learned.

New careers having increased prestige and salaries are possible through team teaching.

The plan of instruction should provide for the most effective use of each teacher's capabilities.

Some learning can be effectively acquired through automated devices.

All teachers need not engage in certain administrative and clerical duties.

The Role of the Supervisor in Developing Curriculum Materials

VERYL SCHULT

1. Although the published literature assigns little responsibility to supervisors for curriculum development, they are as a matter of fact deeply involved in this work.

2. Suggestions for developing a mathematics curriculum are:

Set up a general advisory committee.
Create an awareness of the need for curriculum work.
Organize a planning or steering committee.
Coordinate the work of all committees concerned with the mathematics curriculum.
Appoint a production committee.
Reproduce and distribute copies of the tentative course.
Arrange for inservice education in connection with the tentative course.
Plan to evaluate pupil achievement.
Revise the production committee's tentative course.
Print the new curriculum guide, and in the process use an artist's services.
Distribute the new curriculum guide.
Promote good public relations for the new curriculum.
Actually use the new curriculum guide in the classrooms.

Evaluation of the New Mathematics Programs

DONOVAN A. JOHNSON

1. In order to render value judgments on curricular proposals, certain criteria are needed as guidelines. Criteria based on the following disciplines are suggested: mathematics, psychology, pedagogy, philosophy, and measurement.

2. Since the students' future needs cannot be known with certainty, mathematics instruction should emphasize flexibility, procedures, and broad principles, rather than specific facts.

3. If learning is to take place, the students must be ready, willing, and able to learn; and they will learn only if they react, respond, or participate.

4. A danger in the current revolution in mathematics is that it may go too far, confronting students with a degree of abstraction beyond their mathematical maturity and resulting in bewilderment and revulsion against mathematics.

5. Curriculum groups should identify the specific competencies needed in business, government, and citizen activities, as well as in science and mathematics A list of these competencies would provide a partial basis for judging the adequacy of a given program.

6. True evaluation of a mathematics curriculum is a synthesis of many factors, including the judgment of specialists, a comprehensive testing program, and research.

How the State Supervisor Can Stimulate Local Leadership

JAMES H. ZANT

1. The classroom teacher is the key person in bringing about real improvement in mathematics teaching.

2. State supervisors of mathematics have a twofold obligation: to cause teachers to want improvement in mathematics education and to provide opportunities for them to lean the necessary new materials.

3. The National Science Foundation Summer Institutes might give participants an opportunity to plan new ways of presenting the new content in their teaching the following year.

4. An often overlooked source of personnel for staffing inservice programs is the large number of teachers who attend NSF Academic Year Institutes.

5. Appropriate recommendations from a highly respected Statewide committee on mathematics may be effective in improving the preservice education of mathematics teachers.

6. New textbooks must be examined critically to make sure that new concepts are an integral part of the content rather than just a superficial addition to the old text.

7. The State supervisor must be a competent professional person: he must know the materials available, and he must be able to command the respect of teachers, administrators, and college mathematicians in his State.

How the State Supervisor Can Promote Preservice and Inservice Education for Mathematics Teachers

BERNARD H. GUNDLACH

1. The State supervisor of mathematics should make available new texts and teaching materials to educational personnel concerned with mathematics programs, lay the groundwork for the introduction of well-organized inservice and preservice education programs for all mathematics teachers, K through 12, and organize a permanent pool of key resource personnel who will be capable of lending professional assistance to every classroom under their jurisdiction.
2. The essential features of a revised mathematics curriculum are its content and teaching approaches. Although the preparation of teachers in the new content is difficult, the preparation of various teaching methods is equally challenging.
3. Because technology and mathematics are changing rapidly, we no longer can be sure that the present mathematical tools will be adequate to solve the problems of the future. Instead of teaching our students only how to *use* mathematical tools, we must also teach them how to *make* their own mathematical tools for the problem situations of the future.
4. Rote learning and the drudgery of memorization are likely consequences of the typical teaching of algorisms in arithmetic; they often lead to frustration and boredom, both dreadful enemies of inspired learning.
5. Competent personnel are essential for staffing initial inservice programs.
6. Since teachers should always be well informed and up to date concerning new developments, some form of inservice education will most likely become a permanently established feature of all modern school systems.

The Role of the State Supervisor in Encouraging Research and Implementing Research Findings

JOHN KINSELLA

1. Several practices, all too common in educational research, need to be changed:

The assumption that representativeness implies randomness of the sample.

The use of sensitive high-powered statistical methods on measures of very low reliability.

The assumption that if a difference is statistically significant, it is important.

2. Some questions about mathematics teaching to which answers may be found through kinds of research yet to be devised are the following:

What effects do new instructional media have on mathematics teaching?

What methods most effectively predict success in mathematics?

What factors create enduring interest in mathematics? What factors discourage and/or destroy such interest?

What are the leadership potentialities in using institute participants as resource persons for inservice education programs?

What are the State-to-State and national results of teaching the evolving new methods to students who vary greatly in ability?

What are the best procedures for preparing teachers to teach the evolving new mathematics?

3. Some selected research findings on important areas of mathematics education are the following:

Classes taught with emphasis on meaning, understanding, and discovery showed better retention and transfer of learning than did classes taught by more traditional methods. Early reports on the results from some of the new programs in mathematics, indicate: (a) certain mathematical concepts can be taught much earlier than usually considered possible; (b) some of the subject matter of the new mathematics can be learned by a high percent of students who have taken it. (Most studies show that students acquire about the same amount of knowledge from television instruction as from traditional instruction.)

The Papers

Although the U.S. Office of Education is responsible for editing the following papers, the conclusions and interpretations are those of the authors and do not necessarily represent the views of the Office.

The Changes Taking Place in Mathematics

IRVING ADLER

THIS CONFERENCE is part of the vigorous movement now under way to reexamine and modify the teaching of mathematics. The avowed purpose of this movement is to bring the teaching of mathematics up to date by taking into account the changes that have taken place in mathematics. It is necessary to begin, then, by surveying the nature and significance of these changes. This is not the first such survey, nor will it be the last. The improvement of mathematics instruction is a continuing process. Those who participate in this process must keep turning their heads to look repeatedly, now at mathematics, and now at the schools. I hope that in taking a new look at the new mathematics today we may have the basis for some fresh insights into what must be done to make the teaching of mathematics more effective.

What are the distinguishing characteristics of *contemporary mathematics*? It seems to me that there are four. I would describe them as follows: (1) Contemporary mathematics is classical mathematics grown mature. (2) Contemporary mathematics is classical mathematics grown self-conscious and self-critical. (3) It is also modern mathematics, which developed as a more efficient way of dealing with the content of classical mathematics. (4) Finally, contemporary mathematics is mathematics that is more and more intimately related to man's activities in industry, social life, science, and philosophy. We shall examine each of these four characteristics in turn.

Classical Mathematics Grown Mature

Classical mathematics may be described as the study of number and space. The study of number became arithmetic, algebra, and analysis. The study of space became geometry. We shall note and comment on six aspects of the growth to maturity of classical mathematics.

1. Change from Number System to Number Systems

We no longer have just one number system. We have many number systems. The pluralization of the concept of number system developed in two ways.

First, there was a steady expansion of the original number system of everyday use. At first the only numbers known were the natural numbers, used for counting. The requirements of measurement led to the introduction of rational numbers. Geometric theory led to the introduction of irrational numbers, and algebraic theory led to the introduction of negative numbers and complex numbers. If we recognize as a number system any set of numbers that is closed under addition and multiplication, subject to the usual associative, commutative, and distributive laws, then we see that the expansion of the number system has given us five number systems, one within the other. We have the system of natural numbers within the system of integers, within the system of rational numbers, within the system of real numbers, within the system of complex numbers.

Second, more number systems were introduced in the course of the development of arithmetic and algebra. Initially, the concept of number systems was modified and elaborated through the introduction of the concepts of group, ring, field, and vector space. Also, many new structures were discovered, similar to number systems in the sense that they too were groups, rings, fields, or vector spaces. For example, the set of all permutations of n objects is a group, where permutations are "multiplied" by performing them one after the other. The set of numbers $a+b\sqrt{2}$, where a and b are rational, is a field. The set of residue classes, modulo m (where m is an integer), is a ring. It is a field if m is a prime number. The set of all polynomials with real coefficients is a ring. If p is a prime, and q is a power of p, then the set of all roots, modulo p, of the equation $x^q-x=0$, is a field. The set of n-by-n matrices, whose terms are complex numbers, is a vector space over the complex field, etc.

Additional systems, something like number systems, were also discovered in other areas of study. In the study of logic, an algebra of propositions was developed in which disjunction played the role of addition and conjunction the role of multiplication. In the study of the theory of sets, an algebra of subsets of a set was developed in which union played the role of addition and intersection the role of multiplication.

An incidental, but important, result of the refinement and pluralization of the concept of number system was that we finally learned how to ask a question sensibly in algebra. We had formerly looked for roots of an algebraic equation without knowing exactly what we were looking for. Consequently, we weren't quite sure whether we could believe in the results. For example, some mathematicians, like Descartes, used to reject complex roots as meaningless. Now we are very specific in formulating our questions unambiguously. We specify a domain in which a problem is defined and in which an answer is sought. For example, we may say, "Given an algebraic equation with coefficients in the real field, does it have roots in the real field? Does it have roots in some extension field of the real field?"

2. Change from Geometry to Geometries

We no longer have just one geometry. We have many geometries. The pluralization of the concept of geometry developed in two separate ways. We found that there are other spaces besides the traditional three-dimensional Euclidean space. We also found that there are many geometries within any given space.

The attempts to prove the parallel postulate led to the recognition that there is a space (Lobatschewskian) that satisfies all the axioms of Euclid except the parallel postulate. Thus, we got two geometric spaces, Euclidean and non-Euclidean, instead of one. Then Riemann gave us an infinite number of spaces when he demonstrated that a space can be constructed by using any one of an infinite number of quadratic forms to define a metric on a manifold. More spaces were obtained by recognizing the validity of spaces of n-dimensions, where n is any positive integer.

In the study of Euclidean space of three dimensions many separate geometries emerged. One is the geometry of congruence and a second the geometry of similarity. In addition: the geometry of incidence relations (projective geometry), the geometry based on concepts of nearness and connectedness (topology) and many others, such as inversion geometry and affine geometry.

3. Unification of the Concepts of Number and Space

The separate disciplines of algebra and geometry converged towards each other and joined in Cartesian geometry. This resulted in a fusion of the notions of number and space. The real number system is nothing but a Euclidean plane. Algebraists, of course, would prefer to say it the other way around.

4. Development of Analysis

Various concepts and techniques based on the idea of *limit* were introduced into mathematics. These include such notions as continuity, the sum of an infinite series, and derivative (which is a limit of a sum). The development of analysis led to a new dichotomy in the subject-matter of mathematics to replace the old one of number versus space. We recognized that the number system (real or complex) has an algebraic aspect, found in the properties of the operations addition and multiplication, and a topological aspect based on the concept of nearness and which underlies all considerations of convergence. These two aspects of the number system can be studied separately or in combination.

5. Evolution of the Concept of Function

A more and more general concept of function has developed gradually. The first functions employed were algebraic functions. The study of trigonometry led to the introduction of the circular functions. Calculus contributed the logarithmic function (the integral of dx/x) and its inverse, the exponential function. The pendulum problem contributed elliptic integrals and functions. The theory of heat contributed functions defined by Fourier series. Experience with many functions ultimately led to recognition of the fact that a function need not have a derivative, and does not even have to be continuous.

6. Development of Theories of Functionals

A functional is a special kind of function: Its domain is a set of subsets of a space; its range is a set of numbers. A typical example of a functional is a line integral. Our common measures of length, area, and volume are also functionals that assign a number to a set of points in Euclidean space. The study of physics introduces other functionals such as mass, electrical charge, etc.

A special branch of mathematics, the calculus of variations, deals with the problem of maximizing or minimizing the value of a functional. It tells us, for example, that a plane figure whose perimeter has a given length has the maximum possible area if its perimeter is a circle.

Classical Mathematics Grown Self-Conscious and Self-Critical

The tumultuous growth of mathematics during a period of over two thousand years produced a great superstructure resting on a very shaky foundation. During the second half of the 19th century mathematicians undertook a systematic analysis of the foundations of mathematics in order to plug the holes that had developed. We note some of the questions that they raised and answered. The answers to these questions have helped to give contemporary mathematics its characteristic flavor.

1. What Is a Number?

Weird numbers like the square root of 2, or the square root of -1 had been introduced as a convenience. What did they really mean, and why were they legitimately called numbers? The answer to this question was supplied by what we may call "Operation Bootstrap", the constructive definition of larger and more adequate number systems with the help of the smaller and less adequate ones. The natural numbers are used to construct the integers; the integers are used to construct the rationals; the rationals are used to construct the reals; and the reals are used to construct the complex number system. At each stage of the construction, it is proved as a theorem in the smaller system that the larger system that has been constructed really exists. This removes once and for all the mystery that lurked in the shadows of our earlier vague notions of the irrational and the imaginary.

2. What Is a Continuum?

This question had plagued mathematics since the days of Zeno and his famous paradoxes. The Dedekind-Cantor theory of irrationals supplied the answer. Developing this theory required excursions into the theory of point sets.

3. What Is Infinity?

The concept of infinity crept into mathematics through many doors. It entered in the form of the infinite divisibility of the Euclidean line. It also entered in the form of the infinite extensibility of the Euclidean line. It popped up again in infinite series and the definite integral. Riemann cleared up one confusion by making the distinction between unbounded and infinite. Cantor finally brought infinity under control by developing the theory of sets and of transfinite numbers. The theory of sets had its own troubles in the form of apparent contradictions. But these were ultimately eliminated by excluding from consideration such unmanageable ideas as "the set of all sets."

4. What Is a Variable?

For some time mathematicians relied on the rather meaningless answer that a variable was a number that changed while it was under discussion. A better and more meaningful answer came from the further development of the science of logic: A variable is merely a dummy that may be replaced by any element of a given set. Closely related to the concept of variable is that of an open sentence. An open sentence is a sort of printing press for printing a lot of statements that have the same form. A statement is produced by the open sentence through replacing each variable in it by some element selected from a specified set.

5. What Is a Function?

In the happy days of the past all the functions people ever had to use were represented by well-behaved analytic expressions. This led to naive notions of what a function is when based on the use of formulas or graphs. But then pathological cases began to arise, such as a continuous function that had no derivatives or a continuous function whose graph filled a square. The concept of function was finally clarified when it was defined in its most general form on the basis of the theory of sets: A function is a mapping of one set of elements into another set of elements. Or, alternatively, it is defined as a particular kind of subset of the Cartesian product of the domain and range. An incidental result of the perfected concept of function is that *many-valued functions* have been banished from the realm of legitimate ideas. By definition, every function is single-valued.

6. What Is an Integral?

The first integrals were integrals of continuous functions over a finite interval. It soon became necessary to extend the concept of integrability to make room for integrals of discontinuous functions, such as step functions, and for integrals over the infinite real line. Lebesgue raised the question of how far we can generalize the concept of integrability and of integral, and answered it by developing the theory of measure and the Lebesgue integral. Underlying the theory of measure is the concept of a *measurable space* defined by means of a δ-ring of subsets of the space (a ring closed under differences and denumerable unions). Measure theory now provides a rigorous foundation for the theory of probability.

7. What Is a Geometry?

Felix Klein answered this question with the help of the concept of a group of transformations. A given space can be mapped into itself in many ways. Of particular interest are those mappings or transformations that are one-to-one and have the entire space as range. The set of all such one-to-one into transformations is a group. For each subgroup of this group it is possible to consider questions such as these: What figures in this space are mapped onto each other by the transformations of the subgroup? What properties of these figures remain unchanged when the figures are transformed?

Klein proposed the fruitful idea that the study of such questions for any subgroup constitutes a geometry. Thus, there is a geometry belonging to every one of these subgroups of the full transformation group. On the basis of this idea it is possible to set up hierarchies of geometries. When the group of one geometry is a subgroup of the group of another geometry, the latter geometry is more general than the former. The theorems of the geometry determined by the larger group become theorems of the geometry determined by the smaller group. For example, the geometry of similarity in Euclidean geometry belongs to a particular group of transformations of vector space. The geometry of congruences belongs to a subgroup of that group. The theorems about similar figures also apply to congruent figures. That is, congruence is a special case of similarity. One of the most beautiful results of this approach to the study of geometry was Klein's demonstration of the existence of both Euclidean geometry and non-Euclidean geometry within projective geometry.

8. Axiomatization of Euclidean Geometry

Euclid's Elements undertook the task of developing geometry as a deductive system, in which all theorems are derived from explicitly stated postulates. Euclid did his work so well that the Elements became the model and the inspiration for other postulational systems, even outside the domain of mathematics. For example, Newton wrote his *Principia*, and Spinoza tried to write his *Ethics* as a sequence of theorems derived from explicitly stated assumptions and appropriate definitions. Nevertheless, the model itself is imperfect as a postulational structure. Euclid injected circular reasoning in the form of proofs by superposition, and he left a big gap in the structure when he failed to deal with relations of order. These defects were corrected in Hilbert's axioms for Euclidean geometry. Many other ways of axiomatizing Euclidean geometry are possible. Birkhoff has prepared a set of axioms that uses *length of a line* and *angle measure* as primitive notions. Prenowitz has prepared another set of axioms built around the notion of a *convex set*. The latter two sets of axioms have been offered as having the advantage of leading to a rigorous development of Euclidean geometry that is simple enough to be within the grasp of high school students.

I have described eight areas in which mathematics has subjected itself to searching self-criticism in order to build a rigorous foundation for its elaborate superstructure. If you reexamine what I have said about each of these eight areas you will find that I had to use the word "set" in order to describe what was done. All avenues of investigation of the foundations of mathematics converge towards set theory.

In a systematic deductive development of mathematics all constructions radiate out from set theory like the spokes of a wheel from a hub. Set theoretic concepts are as necessary to mathematical discourse today as common nouns are to ordinary discourse. The analogy invoked by this remark, by the way, is not a superficial one, in view of the fact that every common noun defines a set.

Modern Mathematics

1. Classical Versus Modern Approach

Modern mathematics is the direct result of the multiplicity of spaces and geometries and the multiplicity of algebraic structures developed by classical mathematics. Modern mathematics is to classical mathematics as elementary algebra is to elementary arithmetic. Elementary arithmetic deals with many numbers in the real number system, but its statements are always assertions about relationships connecting specific numbers. Elementary algebra, on the other hand, by using variables, has a way of making assertions that are valid for many numbers or even for all numbers in the real number system. For example, whereas arithmetic may say $2+3=3+2$, algebra will say $x+y=y+x$. Similarly, while classical mathematics deals with many different mathematical structures, its typical approach is to study relationships in one structure at a time. Modern mathematics, on the other hand, studies at one stroke the properties of all structures of a particular type. Thus, while classical mathematics may study the real number system, which happens to be an ordered field, modern mathematics will study ordered fields in general. Naturally, whatever is discovered about ordered fields in general will apply to the real number system in particular.

2. Structure: Axiomatic, Deductive, Abstract

The purpose of modern mathematics in dealing with many structures at once dictates the form that it takes. Modern mathematics is necessarily axiomatic, deductive, and abstract. It defines a type of structure, such as a field, as a set of elements and relations satisfying certain axioms. As in Euclid's axiomatic treatment of geometry, it deduces theorems from axioms. The treatment is abstract in the sense that no meanings are attached to the terms and the relations used in the axioms other than those expressed by the axioms themselves. Thus, in a topological space, a point should not be understood as something that can be represented by a dot on the blackboard.

It is merely an element in a set having special subsets known as open sets that satisfy certain axioms.

The advantage of the abstract approach is that one abstract structure may have many concrete representations. By not being tied to one interpretation of the terms, we are free to use many interpretations. Thus, some topological spaces have points that are *points* of elementary geometry, others have points that are the *lines* of elementary geometry, and still others have points that are *functions*. Similarly, there are many possible interpretations for the points of a projective plane: They may be interpreted as the points of a Euclidean plane to which the ideal points on the "line at infinity" have been added, or as the lines in a bundle of lines through a point in Euclidean 3-space, or as certain equivalence classes of triples of numbers.

3. Algebraic Structures Versus Spaces

Some of the abstract structures of modern mathematics are referred to as algebraic structures. Others are referred to as spaces. What is the distinction between them? An algebraic structure is defined by means of *operations* analogous to addition and multiplication, satisfying certain axioms. Groups, for example, are defined in terms of one operation, and rings and fields are defined in terms of two operations. A space, however, is defined by means of certain *distinguished subsets* that satisfy particular axioms. Thus, a projective space is defined in terms of subsets called lines, a topological space in terms of the subsets called open sets, and a measurable space in terms of subsets called measurable sets.

4. Many Deductive Systems

At one time the geometry of Euclid was unique in being axiomatic and deductive. Now all mathematical structures are axiomatic and deductive. One of the arguments for teaching Euclid in the high schools used to be that it was the best vehicle for introducing young people to a deductive system. This argument is no longer valid. To introduce young people to deductive systems, we now have many deductive systems to choose from. We don't have to wait for the 10th year to do it, and we don't have to restrict deductive reasoning to the study of geometry.

5. From Few Assumptions to Many Deductions

A characteristic of modern mathematics is its attempt to be as general as possible. A modern mathematician is not content with proving a theorem by means of given axioms. As soon as he has proved the theorem, he tries to see how many axioms he can leave out and still prove the theorem. He tries to find out how much he can weaken his axioms and still be able to prove the theorem. In short, he is never happy merely with finding sufficient conditions for his theorem. He wants to know which conditions are also necessary. By proving a theorem with the weakest possible set of assumptions he finds the broadest domain to which the theorem applies.

6. Structure-Preserving Mappings

The modern mathematician is concerned with whole structures rather than individual members of the structure. His typical technique for studying a structure is to map one structure into another. Naturally, a central role is played by the structure-preserving mappings known in general as "homomorphisms," and as "isomorphisms" if they are one-to-one and onto. Thus, the algebraist uses mappings that put sums into sums and products into products. The student of projective geometry uses mappings that put lines into lines (collineations). The student of topology uses mappings that put open sets into open sets (open mappings), or more frequently, mappings for which the inverse image of an open set is an open set (continuous mappings). The student of measure theory uses measurable functions, for which the inverse image of a measurable set is a measurable set.

7. Properties of the Real Number System

The modern mathematician often uses a classical mathematical structure as the starting point of his investigations. He analyzes the structure to isolate from it certain separate qualities. He studies these qualities in abstraction in separate structures. Then he puts the structures together in various combinations. Thus, he finds that the real number system may be viewed as a group, a ring, a field, an ordered set, a topological space, etc. So he studies abstractly groups, rings, fields, ordered sets, topological spaces, etc. Then he studies such things as topological groups, topological rings, topological fields, ordered fields, etc., in which two structures are combined into one.

After he has found many qualities in a given structure, the modern mathematician is often particularly interested in finding out how many of these qualities he must put together to obtain the original structure. For example, the real number system has these properties: (a) It is a number system in the sense that it has two operations (called addition and multiplication) that are associative and commutative and the multiplication is distributive with respect to addition. (*b*) It is an extension of the rational number system. (*c*) It is ordered. (*d*) It has the property that any nonempty set in it having an upper bound has a least upper bound. The interesting thing about these properties is that they characterize the real number system. Any system that has these four properties must be the real number system (that is, is isomorphic to it).

8. The Role of Intuition

Since we have stressed the advantages of the formal deductive approach of modern mathematics, it is necessary to say something about its limitations. Not all mathematics is formal and deductive. In the first place, mathematical discovery is not deductive. The research mathematician who gropes his way towards new theorems is guided by analogy, hunches, trial and error, and flashes of intuitive insight. It is only after he has made his discoveries that he uses hindsight and shows how he could have arrived at his conclusions most economically by deductive reasoning.

In the second place, a deductive system is incapable of supplying the justification for its acceptance as a legitimate mathematical system. A mathematical system is legitimate only if it is consistent, that is, free of contradiction. A proof of the consistency of a system may be developed as a theorem of metamathematics (reasoning *about* the mathematical system, but not *in* that system). But the validity of the proof depends on assuming the consistency of the system of axioms underlying the metamathematical argument. In the case of a system inclusive enough to contain the natural numbers of arithmetic, this assumption means taking more for granted than does the assumtion that arithmetic itself is consistent. This fact follows from Gödel's theorem that such a consistency proof requires rules of inference even stronger than those of arithmetic. Thus the problem of proving the consistency of arithmetic is not solved, but merely shifted to other ground. For this reason, it is common practice to follow two procedures to establish the consistency of a deductive system. One procedure is to prove only *relative* consistency. For example, it is possible to prove that non-Euclidean geometry is consistent if Euclidean geometry is consistent. The second procedure is to prove the absolute consistency of a system by producing a concrete representation of it. But this, of course, involves an appeal to intuition. In constructing mathematical systems we cannot exclude intuition entirely. We can only restrict the area in which we must lean on intuition.

Even if we could do without intuition in creating mathematics, it would be folly to try to do without it in teaching mathematics. I am sure that all of you must have had the experience that I have often had of following a deductive argument, step by step, and ending up not knowing what it was that had been proved. While a deductive argument may show us where one tree stands in relation to another, an intuitive argument is often the best way of seeing the woods that are made up of all those trees.

While we stress the limitations of deductive argument, let us not forget, meanwhile, the limitations of intuition. Intuition is a useful guide, but sometimes an unreliable one. Let us remember that intuition misled some 18th-century mathematicians into saying that the infinite series $1-1+1-1+\ldots$ has a sum equal to $\frac{1}{2}$. What is "discovered" by intuition has to be checked by rigorous deductive argument.

8. Mathematics as a Game

Mathematics in the form of an axiomatic deductive system has sometimes been described as a game. The mathematician, it is said, makes up the rules of the game in the form of axioms. Then he proceeds to play the game according to the rules. Some people have gone beyond this assertion to say that mathematics is *only* a game, played without regard to any possible applications. This assertion is not correct. It is true that the pure mathematician should be free to explore in any direction his curiosity carries him. But his choice of the rules of his game is not entirely arbitrary. He tries to select rules that he and his colleagues will judge to be significant. And the measure of significance is usually the extent to which the rules relate to existing mathematical structures and to practical applications.

Pure mathematics has benefited greatly by growing up in intimate contact with practical applications. To see the truth of this statement we need only recall how the theory of elliptic integrals and functions grew out of the pendulum problem, how Fourier series grew out of the study of heat, how the study of Riemannian geometry was stimulated by relativity theory, and how the study of Hilbert spaces was encouraged by quantum-mechanics. The greatest mathematicians have always combined dedication to pure mathematics with a strong interest in its applications. This is shown for example in the work of Gauss, Klein, Poincare, Weyl, and Von Neumann, to mention only a few.

10. Growth Through Specialization

The growth of modern mathematics has led to increased specialization. Specialization leads to a fragmentation by subdivision of subject matter. At the same time, however, there is a growing unification of method. Set-theoretic methods permeate all of modern mathematics. Algebraic methods reach out into topology. Topological methods are used to deal with problems in analysis, etc. For example, in algebraic topology, essentially geometric configurations are studied by the examination of associated groups. The typical technique is to set up a sequence of groups such that each group is mapped homomorphically into the next one in the sequence. Now this algebraic technique, developed originally for the study of topology, has been generalized and axiomatized in homological algebra and has become a technique of abstract algebra. Another interesting example of the trend towards unification is found in the way an existence proof in the theory of differential equations can be derived from a fixed-point theorem in topology.

11. Classical Mathematics: Extended and Clarified

One fact, implicit in what I have said about modern mathematics, should nevertheless be stated explicitly. Modern mathematics does not replace classical mathematics. It generalizes it, supplements it, unifies it, and deepens our understanding of it. But classical mathematics in the form of arithmetic, analysis, and geometry is as important as it ever was.

12. Strategy of Contemporary Mathematics

I would like to comment at this point on some aspects of the strategy of contemporary mathematics that are important even though not at all new. The mathematician uses some of his typical methods in classical, as well as in modern, mathematics. These methods are worth noting here because the mathematician uses them in elementary, as well as in advanced, mathematics.

a. After defining a structure or a configuration of which there may be many examples, the mathematician frequently sets himself the task of identifying and classifying all the possible examples. For instance, algebraists are trying to identify and classify all finite groups. They have solved the classification problem for finite commutative groups. They have only scratched the surface of the problem for finite groups in general. Classification is often accomplished by picking out a set of simple cases and then expressing all other cases as combinations of the simple cases. For example, finite commutative groups are factored into products of cyclic groups. In arithmetic, integers are expressed as products of primes. In plane geometry, polygons are decomposed into triangles.

b. Classification is often accomplished by dividing the set of things being studied into equivalence classes and then showing that each equivalence class contains one and only one element of a special type. This is the technique of *reducing to canonical form*. In elementary algebra, for example, we reduce polynomials to the canonical form $a_0x^n + \ldots + a$. In arithmetic we reduce fractions to the canonical form called "lowest terms".

c. Sometimes we pick out a set of simple cases and show that although other cases cannot be reduced to these simple cases, at least they can be approximated by them. Approximation theorems occur in many branches of mathematics. In the theory of numbers for example, there is a theorem that every real number can be approximated as closely as we please by a rational number whose denominator is not too big. Specifically, it says that if a is real and if g is a positive integer, we can find integers x, y such that $|x-ay|<\frac{1}{g}$, with $1 \leq y \leq g$. In the theory of functions of a real variable we have the Weierstrass theorem that any continuous function on the closed segment from 0 to 1 can be approximated by polynomials. In algebraic topology, compact metric spaces are approximated by finite polyhedra. In elementary calculus and in plane geometry we often approximate a rectifiable curve by an inscribed polygon.

d. Sometimes we study a structure by seeing what happens when we map it into one of a set of simple and well-known examples of that structure. For example, a group is often studied by mapping it into a matrix group. A ring may be studied by mapping it into the ring of endomorphisms of a group (homomorphisms of the group into itself). If we map each polynomial with real coefficients into its residue class modulo x^2+1, we have a homomorphism that can be used to prove that the complex number system exists, that it is a field, that it contains the real number system as a subfield, and, of course, that it contains a square root of -1.

e. We have just described four ways in which the simplest cases of a structure or configuration are used to study the general cased. There is one technique which uses the opposite approach. It studies the general cases by paying particular attention to the exceptional, pathological case. It is analogous to the way in which psychologists learn about normal behavior by studying neuroses and psychoses. In the theory of functions of a complex variable, for example, we encounter the singular points of a function. These are the points where the function doesn't behave itself at all, and in fact, may not even be defined.

Far from ignoring the singular points, we pay particular attention to them, because they have a great influence on the behavior of the function at the points near the singularity. The function behaves in one way near a pole, and in a different way near an essential singularity. Similarly, in the theory of systems of ordinary linear differential equations, we find it to our advantage to examine the critical point of the system (the place where all components of the derivative vanish), because the nature of the solutions to the system of equations depends on what happens in the neighborhood of the critical point. In elementary calculus we use a similar approach to help us to curve tracing by first identifying the pathological cases—the discontinuities, the maxima and minima, and the points of inflection.

f. An interesting feature of many branches of mathematics is the existence of *duality*. Wherever duality occurs, a theorem can immediately be translated into a dual theorem without any further proof. In projective plane geometry, point and line are dual to each other. In projective 3-space, point and plane are dual to each other. In linear algebra, a vector space over a scalar field and the set of linear mappings of the space into the scalar field are dual to each other. In elementary plane geometry there is an imperfect duality of point and line, and a related duality of side and angle in a polygon. In spherical geometry there is the duality of great circle and pole. Conscious use of duality, where it exists, makes possible the discovery of new relationships, and affords a deeper insight into the relationships that hold a structure together.

Mathematics More and More Intimately Related to Man's Activities

While mathematics, in the form of pure mathematics, has reached dizzying heights of abstraction, it has kept its feet on the ground by multiplying and extending its applications.

1. Handmaiden of the Sciences

Now, more than ever, it is true that mathematics is the handmaiden of the sciences. Before this century the science of physics had already made abundant use of mathematics in mechanics, optics, the theory of heat, and electromagnetic theory. Analysis used to be the chief mathematical tool of the physicist. Now, with the development of relativity theory and quantum-mechanics, he has had to learn Riemannian geometry and modern algebra. The increasing role that chance processes play in physical theory compels him to learn probability theory as well.

No area of science today can avoid using mathematical methods. For example, mathematics has spilled over from physics into the physical sciences of chemistry and geology. It has invaded the life sciences of biology and psychology, and has expanded into the social sciences too.

2. Industry, Commerce, and Defense

With the growing complexity of industrial and business life, industry and commerce have raised more and more questions that can be answered adequately only by the use of mathematical methods. Insurance and pension systems use actuarial mathematics. Industries undertaking quality control use statistical methods. Commercial establishments choose from among alternative courses of action with the help of linear programming. Military strategists plan their moves with the help of the theory of games. Communications engineers use information theory and Boolean algebra.

3. Automatic Electronic Computers

To deal with complicated problems at high speed, automatic electronic computers have been constructed. Mathematicians share in the designing of the computers and in setting up their programs. Because of the widespread use of computers, logarithms have lost their importance as a computational tool.

4. Expansion in New Directions

To meet the new demands made by science, industry, and government, mathematics has had to grow in new directions. There is a vigorous development of the theory of differential equations, and methods of solving them by means of computers. There has been a tremendous growth in the theories of probability and statistics. New areas of study like linear programming, the theory of games, and information theory have sprung up over night.

5. Importance in Vocational Training

The spread of automation in industry is changing the character of the labor force. The tendency is to reduce the amount of unskilled and semi-skilled labor that is used, and to increase the need for technically trained personnel with a knowledge of mathematics. The importance of mathematics in vocational training will continue to grow in the future.

6. Essential Ingredient of a Liberal Education

The production of food, clothing, and shelter has never been man's sole concern. He has always found it necessary to ponder over deep questions about man's relation to man and about his place in the universe. Even these questions have now taken on a mathematical character. I shall refer to only two obvious examples.

For thousands of years philosophers have had endless debates about the meaning of infinity. It is impossible to talk sense on this subject without taking into account what mathematics has contributed through the theory of the continuum and transfinite numbers. In the past, philosophers have speculated about the nature of space. Is it finite or infinite? Is it bounded or unbounded? It is impossible to talk sense about these questions too without taking into account what Riemannian geometry has contributed through the theory of relativity. Mathematics is increasingly important as an essential ingredient of a liberal education.

In this conference that opens today, you will be discussing ways to adjust mathematics teaching to all the changes that have taken place within mathematics as a pure science and the many uses of mathematics in the world of work and the world of thought. There can be no doubt that because of your efforts here this week our schools will make considerable progress in this direction.

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Implications of the Changes in Mathematics for the Role of the Mathematics Supervisor

CARL B. ALLENDOERFER

ONE of my most serious professional weaknesses is a willingness to speak in public on subjects with which I have had relatively little personal experience. Although I am quite close to the changing scene in school mathematics, I know hardly anything about State supervisors of mathematics. I have met some of you at other conferences in an informal way, but I know very little about your actual duties and activities. We have no State supervisor of mathematics in Washington from whom I could get a briefing in preparing this paper.

On the other hand, I have been in close touch with many of the school districts in the State of Washington, for their people have a right to look to the mathematics department of their State University for leadership and assistance. I have met with school boards, superintendents, principals, teachers, parents, and pupils, and I have a reasonably good idea of their problems. Probably the State of Washington does not present precisely the same problems as those which you meet in your own States, and consequently some of my remarks may not fit your particular situation. You should listen to me then as if the title of my paper were "How I would like to see a supervisor of mathematics function in the State of Washington." I hope that a paper with this more limited scope may still be of some use to all of you.

The functions of such a State supervisor may be indicated by four words: information, inspiration, inservice education, and humility. The first three of these imply duties and the fourth is an attitude. Let us discuss these in turn.

Information

When I meet with groups of mathematicians on the national level, I can easily get the impression that the revolution in mathematics is an accomplished fact. Everyone at these meetings knows all about the Commission on Mathematics, SMSG, UICSM, the Ball State project, the Madison project, and so forth. We are prepared to discuss the differences among these programs, their relative merits, possible advances beyond them, and how well our students are doing in them. When we get back home, however, we will probably find the situation quite different. In the first place, many members of university faculties are not fully informed about this movement. With certain outstanding exceptions, school districts have only heard rumors about a change and are badly confused In some of these districts there is a teacher who has attended a summer institute, or a principal who has got the word, or some parents who have heard of Cuisinaire, or a curriculum supervisor who has attempted to get things off the ground. But there is often a lack of the deep understanding which must be spread through every level of administration and instruction of the school system.

When I am called upon to meet with representatives of such a school system, it is clear that my first job is to give them a very large amount of factual information. The objectives and the nature of the reform movement must be explained in words that carry meaning to the particular audience involved. I have quite different speeches on these matters for administrators, teachers and parents. The administrators and teachers need a detailed summary of the existing sets of recommendations pointing out differences as well as similarities; they are anxious about the future and should be told what further plans these curricular groups are making. They are especially concerned about the relative merits of the new commercial textbooks now coming on the market. They are concerned about acceleration, calculus, integrated curricula, university entrance requirements, and advanced placement. The list is endless.

Surely, then, you must be extremely well informed on all of these matters. You must be prepared to explain the current movement to everyone in sight. Moreover, you must have materials to distribute which they can read and in turn pass on to others in the school system.

Our first problem is to arrange for you to obtain this information as quickly as possible and in a form suitable for distribution in the field. There are many present channels, such as meetings of this kind, meetings of the National Council of Teachers of Mathematics, *The Mathematics Teacher*, newsletters from various curricular groups, salesmen from publishing companies, and visiting speakers. These must all be exploited, but the flow of information could be much better organized. I suggest that one of the discussion groups at this conference deal with this problem, and possibly that this is a field where cooperation and national effort would be to the advantage of all. Is there a possibility that the Office of Education could help you assemble this information and see that it is published and made available in a convenient form?

Inspiration

In spite of all the ferment for reform in informed quarters, I find that some school people are resistant to change and cool to this movement. It is easy to understand their point of view, for change often means trouble and frequent criticism from parents who do not understand what is going on. Some teachers and principals fear that they cannot handle the new materials. Parents are nervous and fear that their children may not be able to get into college or find employment if the old-fashioned drill is not maintained.

Clearly, your second job is to be a salesman and to inspire these conservatives to join the rest of the profession. There are various kinds of weapons which you can use. First, you can emphasize the professional standing of the people behind the movement and become acquainted with the prominent names, their institutional connections, and their publications. Second, you can speak of the very general agreement of every competent mathematician who has studied this matter, for we are agreed on these ideas with very few exceptions.

Most convincing, however, is a careful analysis of standard textbooks pointing out the serious errors contained in them, not to mention the many sources of confusion. I have broken down quite a few recalcitrant citizens by posing the following problem which I extracted from a best-selling algebra book.

Simplify: $$\sqrt{x^2+2x+1}+\sqrt{x^2-2x+1}$$

The usual answer from the uninitiated is $2x$. Then I ask them to check the result by substituting $x=0$ in the original expression and in their answer. The bewildering result is that $2=0$. After these people have realized that there really is something concrete and useful to learn, they are much more willing to listen to what I have to say.

A more difficult question to answer is whether the results of the new forms of instruction are actually better than the old. So far as I am aware, there have been few, if any, serious comparative studies which would stand up under critical analysis. The best answer that can be given at present is to quote the reactions of teachers and students who have had experience with the new programs. A collection of such comments would be most helpful.

A problem related to that of inspiration is that of dealing with those school people who think that conversion to the new program is an easy matter. I have had requests for a series of six lectures which would "modernize" all the mathematics teachers in a school system. Other people are looking for a gimmick, such as Cuisinaire rods, television, programmed learning, or the like; and they assume that if they make a command decision to adopt some such new device their problems will be solved. In such situations there is clearly a basic lack of understanding of the nature of the new program, and additional information and inspiration must be forthcoming.

On the other hand, there is the reverse problem of handling the over-eager, "let's-do-it-all-tomorrow" type of administrator. His ardor must not be cooled, but he must be brought to realize that he is in for a long-range program of education and continuing change. The sudden introduction of new materials throughout a school system without proper preparation of the teaching staff can result in horrible chaos and disillusionment with the whole idea. The proper approach to such zealots is to outline a proper step-by-step procedure full of information, inspiration, and inservice education.

Inservice Education

After a school system is fully aware of the nature of the proposed changes, and after all portions of the community including parents, administrators, and teachers are agreed that they are ready to embark upon the adventure, then comes the hard work of getting down to business.

It is sheer madness to give the ordinary teacher any of the new teaching materials and tell him to start using them, unless he has had rather extensive preparation. If he has been to one, or preferably, several, well-organized summer institutes, or has just graduated from a proper undergraduate curriculum, there will probably be no problem. Most of our teachers, however, do not meet these qualifications, and some sort of inservice education is absolutely necessary.

The most serious problem of inservice education is to obtain competent lecturers for the courses. If a school system is within commuting range of a college which has the right sort of mathematics faculty, professors from the college can sometimes be prevailed upon to give the lectures. In large centers, requests for inservice courses can swamp the college people; we at the University of Washington can meet only a small fraction of the requests of this kind from within our own area alone. It is a serious blunder to take just any college man, for the mathematics situation in many colleges is at least as bad as it is in the schools. College people can help out, but they can make only a small dent in the total problem.

More and more I am advising school systems that in this matter they must lift themselves by their own boot straps. A superintendent's first reaction to such a proposal is usually unfavorable, but with patience administrators can be brought around. There are many aspects of this boot-strap operation, and each school district must find the method which suits its own conditions. My own preference in the State of Washington goes something like this: At the University of Washington we offer a good selection of modern courses for school teachers. For example, this summer we have work in arithmetic, junior high school algebra and geometry, senior high school geometry, 9th- and 11th-grade algebra, statistics, and calculus. We recommend that a school district support one or more key teachers and send them to our summer session. On the basis of his study there, this teacher can then help lead other teachers in the district in after-school seminars on the modern material. If he runs into difficulties, he can contact the university; and we can help him out during the year by correspondence or by occasional visits. When this kind of program has been repeated several years, the school district will begin to stand on its own feet.

Although the inservice problem is serious for senior high teachers, it is more acute for the junior high and elementary people. Many teachers, not having strong backgrounds in mathematics, require the most careful and extensive retraining. Relatively few of them are able to attend summer school or summer institutes, and so inservice education is all that is left. I recommend that their retraining be conducted by a competent senior high school mathematics teacher, whose regular teaching duties are appropriately reduced. If a local mathematics supervisor is up to date, he could easily be giving instruction of this kind as one of his main functions.

All of this requires organization and outside help, for the school districts are not experienced in such a program. I suggest that a major function of a State mathematics supervisor is to conduct the field work necessary to get such programs organized. This will take repeated visits to the districts—it cannot be done from a desk in the State capitol. And so I believe that a major topic for discussion at this conference should be ways and means of organizing inservice programs in your respective States.

Humility

Finally, I should like to turn to the proper attitude of a State supervisor toward the teachers in the field. On this point I have had two kinds of experiences. During the last war, I held a Government assignment as supervisor of training programs sponsored by the Air Force and approximately 25 colleges. With the aid of highly competent advisors I prepared the curriculum and sent regulations out to the field. Soon after the students were in their classes I took a trip to Haverford College, my own institution and one of the participating colleges, to see how things were going. To my great shock I found that the regulations I had written, with my own college in mind, were unsuitable in a great many ways for the problems actually being faced. This was a sobering experience which has affected my attitude toward the central administration of education ever since.

More recently I have been working with school districts in my area and have experimented with different approaches. I am convinced beyond any doubt that the very worst way to supervise education is to issue regulations from a central authority. In a great many instances the regulations do not fit local conditions, and moreover the imposition of such regulations stifles creative work on the part of the local people.

I have found that the best approach is to take the position of advisor and friend, to make no claim to have all the answers, to offer help and support where needed, but never to *tell* a school district what it *must* do. Let us concentrate on making our supervisors expert consultants, rather than minor-league dictators.

Conclusion

As I view the situation, the job of the State supervisor has been greatly transformed by the introduction of the new ideas for mathematics education. You are becoming the most important figure in your State's program in mathematics, and unless you are prepared to do your job, your State is likely to fall behind the rest of the country in this effort. You must be a well-informed source of information, a missionary who exudes inspiration, an organizer who can arrange for inservice education and consultation of many kinds, and a counselor and friend who influences your constituents through persuasion rather than by means of regulations and authority. You have a big job, and I hope that the discussions in this conference will help each of you to prepare yourself for your manifold tasks.

New Media of Instruction*

ARTHUR W. LALIME

THE BASIC ELEMENTS OF the new media of instruction have been with us for many years. The implementation of these devices and techniques in the educational program is long overdue, but breakthroughs are occurring throughout the Nation on two related fronts:

1. Schools are being designed and school programs modified to make use of the new techniques.
2. Classroom teachers, college research departments, and industrial corporations are developing promising teaching techniques for classroom use.

These developments are not isolated factors in our national educational complex. They are part of the intensified interest in our search to find ways to improve instruction at all levels.

Communication between teacher and pupil always has been and always will be the basis of education as we understand it. New tools of classroom communication are being adopted slowly. Compared with our dynamic industrial development, classroom use of these proven tools of communication reveals education as something of an old-fashioned locomotive in the roundhouse, lighting up the boilers, while industry is taking off by jet.

We all recognize that American education is a highly complex social institution that will slowly but surely respond to the needs and ideas of the times. We stand on the edge of an exciting era with many major breakthroughs on the horizon—breakthroughs holding the high promise of more and better education for the young people of this Nation.

Assaults are being made on the age old problem of *how we learn.* For years, work has been going on in our universities to solve the many ramifications of this psychological riddle. Under the provisions of the National Defense Education Act and other federal and privately sponsored research projects, programs are being developed which may change the educational pattern during the next 10 years. Our national institutions have made significant contributions in science

*Comments accompanying and explaining a demonstration of new instructional media.

education, modern foreign-language instruction, and in the teaching of elementary and secondary mathematics.

The results of this work are being force-fed to the public. Scholarly texts are written and research papers are published in our educational journals, but for the first time newspapers, magazines, and television writers are reporting these results to the public in a highly dramatic fashion.

Parents are asking educators—

What about community colleges for our city?

What must we do to have advanced placement classes in our secondary schools?

What provisions are we making for the gifted?

Why must we wait so long for the new mathematics programs to be instituted in our schools?

Must our children wear out the present books before we buy the new?

Or must we wait till the teachers wear out?

Why do we not have team teaching in our schools?

As coordinators and supervisors in this worthy field of human endeavor, you are well aware of these impending changes.

I would like to relate this concept of educational metamorphosis to the educational activities occurring in Norwalk (Connecticut).

Norwalk is a small city with 13,000 children in the public schools. Our yearly per-pupil expenditure is close to the average of the State of Connecticut and we are now suffering from the chronic educational pains of expansion brought about by increasing school population and a shrinking educational dollar. Exciting educational experiments are now happening in ciites both rich and poor. Those developed in Lexington and Newton (Massachusetts) and Norwalk (Connecticut) have some of the desirable qualities which I believe will improve education

Norwalk has completed its third year of a team-teaching program. Now past the initial testing stage, that program is operational and growing. Last year, in 75 classrooms 20 percent of the pupils participated full time in the program. Next year it will expand, particularly at the junior and senior high school levels. The success of the communication techniques associated with team teaching has encouraged many regular classroom teachers to adopt and adapt these techniques in the self-contained classroom.

I am not alone in holding the conviction that the new media of communication can be used more efficiently and with greater educational impact in a school system organized along team-teaching lines.

Advantages of the Overhead Projector

Let me give you some of the advantages in using one of these new media—the overhead projector.

1. *The projector is used in front of the group.*
   The presenter faces the group, maintaining eye contact at all times.
   He can observe reactions and adjust his program accordingly.
2. *A bright image is projected in a fully lighted room.*
   The room need not be darkened, thus reducing inattention and drowsiness.
   Audience and presenter are fully visable at all times.
   It is not necessary to keep the image on screen at all times or to turn lights on and off several times, as with projection in a darkened room.
3. *The horizontal stage provides for flexibility in presentation.*
   The presenter can:
   Write or draw extemporaneously.
   Project transparent objects, animated devices, or fluids.
   Use a pointer to call attention to details of transparencies.
   Use transparencies with several layers of film, unmasking them in progressive disclosure or building them up to form a composite image.
4. *A separate projectionist is not required.*
   The overhead projector complements the presenter; it does not replace him.
   The presenter controls the projector at all times and takes a prominent part in the presentation.
5. *Transparencies up to 10" x 10" can be used.*
   The large size simplifies preparation of artwork for transparencies.
   In most cases, photographic reduction of original artwork is not necessary for production of transparencies.
6. *Transparencies can be home-made.*
   Rudimentary art skills can produce dramatic, professional-looking transparencies.
   Transparency-producing equipment is simple and inexpensive.
7. *Color can be used effectively and economically.*
   Diazo-color films, in a wide range of colors, make it possible to prepare multi-color transparencies at a fraction of the cost involved in colored photographic transparencies.
8. *A group's individual work can be demonstrated on projector.*
   No special skills are required to operate the projector.
   Any member of the group can use the projector to work out a problem or offer a suggestion.

Transparencies for teaching elementary arithmetic and secondary mathematics can be made cheaply and easily with simple graphic equipment and supplies, available from your stationery store or audiovisual dealer.

Plant Facilities for Team Teaching

Team teaching requires no more space than self-contained classroom teaching, but the shape and functional requirements should be modified to meet new demands. There is a need for large group-instruction spaces, a nonteaching study hall or independent workroom areas, and small conference-type rooms. The need for flexibility in classroom size can be met by acoustically sound-proof, quick-folding partitions.

Large-group instruction is a most important feature of team teaching and good teacher-pupil communication is the key to successful presentations. Each pupil must be able to see clearly and hear well, and be comfortably seated in a well-ventilated room. It is helpful if each pupil has before him a working surface upon which he can take notes.

These are the minimum requirements of any classroom. The same conditions of seeing, hearing, and being comfortable apply to any teaching situation, with a large group or a small one.

Conclusion

Classroom instruction at the elementary level has traditionally followed these concepts:

1. The status of all teachers is the same.
2. The quality of learning is determined by class size.
3. Each teacher has to do all the things that need to be done.
4. Effective learning must be in face-to-face situation with the teachers.
5. All teachers must perform the same roles.

The self-contained classroom hinders the optimum effectiveness of new instructional materials and limits the use of educational television, films, and other devices.

Team teaching in Norwalk has been planned with the following concepts in mind:

1. New careers have been created with increased prestige and salaries.
2. What is to be learned determines the size of the group.
3. Teaching strengths of teachers should be capitalized upon.
4. Some learning can be effective when acquired through automation.
5. All teachers do not necessarily have to be administrators and clerks.

The Role of the Mathematics Supervisor in Developing Curriculum Materials

VERYL SCHULT

THE WORD "CURRICULUM" is a rather recent addition to our language. Webster's early editions do not mention it. In the 1856 edition of Webster's *An American Dictionary of the English Language*, however, the word appeared with two definitions: "(1) A race course; a place for running; a chariot. (2) A course, in general; applied particularly to the course of study in a university." In recent editions, "a chariot" has been dropped, and the word is used in the sense in which we now use it. The meaning has broadened from specific course guides to all activities and materials related to the pupils' school work.

In recent years, significant changes have taken place in curriculum construction. A more thorough study of the goals of education and the contribution of particular subjects is being made, scientific methods and findings are providing valuable background for curriculum construction, and the importance of how we learn, as well as what we learn, is being recognized.

The present problems of what to teach are not new. Aristotle is said to have remarked that all people do not agree on what a child should learn, that we cannot determine whether to instruct a child in what will be useful to him in life, or what tends to virtue, or what is excellent, for, says he, all of these things have their separate defenders.

I think Aristotle would be pleased if he could step in today and see the great amount of thought and effort spent on developing curriculums on the basis of a thorough study of "what will be useful, what tends to virtue, and what is excellent."

The U.S. Office of Education publication, *Offerings and Enrollments in Science and Mathematics in Public High Schools—1958* includes a report on trends in curriculum revision. In reply to the question to principals, "Is the mathematics curriculum in your school being revised this year?", 40 percent of the answers were affirmative. Activity in this area increased notably from 1952 to 1957 and there are good reasons to assume that it is still increasing. During the

10-year period 1949–1959, while the school-population age group increased 22 percent and the number of high school pupils 45 percent, the mathematics enrollments increased 75 percent. Thus, there is not only a larger mathematics population to provide for but also a great range in abilities to consider.

Last year 200,000 elementry and high school pupils studied new programs in mathematics. The new mathematics is related to every phase of the technological and social order in which we live—hence, the great activity in curriculum development in mathematics.

The present curriculum in secondary school mathematics is about one hundred years old. Mathematicians until recently have ignored the secondary school, while colleges continued to teach traditional courses and expect a traditional preparation from secondary schools. Colleges taught modern mathematics only at the graduate level. Secondary school administrators and teachers could not modernize the curriculum without college cooperation. Fortunately, the groups now attempting to modernize the curriculum combine the efforts of mathematicians, teachers, and psychologists in order to develop the best possible program. Also, these groups are preparing materials for students (both for classroom and for supplementary study), providing inservice work for teachers (on an individual basis through readings, and for groups), making careful evaluations, and, last but not least, providing the publicity necessary for general acceptance of a new program.

Where does the mathematics supervisor fit into this general curriculum picture? My discussion of the role of the mathematics supervisor in developing curriculum materials is based on three main sources of information: (1) The literature on the subject, (2) my personal experience working in my own school system and talking and working with people in positions similar to mine, and (3) answers to questionnaires sent to a State supervisor in every State.

The literature is surprisingly meager concerning the role of supervisors in curriculum development. The role of teachers, principals, and superintendents is written up at great length. In examining the recent books on curriculum and curriculum yearbooks of the American Association of School Administrators and the Association for Supervision and Curriculum Development in our library, I find that the word "supervisor" is not even mentioned in any index. Perhaps our place in the curriculum setup is still to be found. The pamphlet, *The Supervisor of Mathematics: His Role in the Improvement of Mathematics Instruction* [2], probably gives the most complete summary of

[2] Published by the National Council of Teachers of Mathematics, 1959. 10 p.

supervisory and curriculum activities in mathematics that is available.

I shall now describe what I think are the important steps in curriculum development and point out possible functions of the mathematics supervisor in connection with each one of them.

Step 1

A general advisory committee does the following: (*a*) Constantly surveys the whole curriculum of the schools. (*b*) Works with school personnel in developing a philosophy of education and deciding what is important. (*c*) Studies movements and trends in all fields. (*d*) Surveys the work of special committees such as those on textbooks, instructional aids, televised teaching, and libraries.

The mathematics supervisor might—

- Serve on the general advisory committee.
- Keep this committee informed about experiments and about the work of national committees.
- Recommend the formation of such a committee if one does not exist.

Step 2

School personnel are made aware of the needs for curriculum work in mathematics because of (*a*) changing school populations, (*b*) results of research, (*c*) obsolescence of courses of study, (*d*) emergency differentiated curricula set up in various schools to meet special needs, and (*e*) results of tests.

The mathematics supervisor might—

- Make school personnel aware of these needs through information in newsletters, and thus answer, directly or indirectly, the question: *Why is revision needed?*
- Confer with administrators and teachers about activities in mathematics and work through superintendents of schools at the local level.
- Report results of research and experimentation through consultants, conferences, and bulletins.
- Encourage careful experimentation by teachers or school systems equipped to carry through with dependable results.
- Arrange for extra time and/or extra pay for teachers doing experimentation.
- Work in an advisory capacity with local committees who are planning for special curriculum needs.
- Speak at meetings of administrators or teachers and let them know that the supervisor is available for help.
- Circulate information about work and reports of important committees such as the Commission on Mathematics of the College Entrance Examination Board.

- ✦Arrange for speakers at meetings to let teachers know, for instance, how college entrance examinations are changing.
- ✦Inform teachers about changes in standardized tests, mathematics contests, etc.
- ✦Publicize opportunities for teachers to study more mathematics through such means as National Science Foundation Institutes, institutes sponsored by industry, and Continental Classroom (TV).
- ✦Secure college and university cooperation in offering timely courses that fulfill the teachers' needs.

Step 3

The State Department of Education or a State organization in mathematics sets up a planning (or steering) committee which will: (*a*) Study ways for pupils to attain, in mathematics, the objectives set up by the Advisory Committee for all pupils. (*b*) Study the curriculum needs in mathematics. (*c*) Become familiar with reports of national committees concerned with curriculum. (*d*) Survey the experimentation in mathematics. (*e*) Visit schools where unusually fine work is going on. (*f*) Develop outlines of the new work. (*g*) Submit the outlines to superior teachers for reactions. (*h*) Prepare interim bulletin(s) to meet special needs.

The mathematics supervisor might—

- ✦Recommend personnel for the committee to the State Department of Education.
- ✦Serve on this committee either as chairman or as a member.
- ✦Arrange with authorities for some released time for the Planning Committee to work.
- ✦Provide wide representation on the committee, including teachers (on several school levels), subject-matter specialists, lay specialists, administrators, members of the State Department Staff, educational specialists who are authorities on the learning process.
- ✦Supply information to the committee through professional magazines and through such reports as those of the Committee on the Undergraduate Program in Mathematics and the Commission on Mathematics of College Entrance Examination Board, the NCTM Curriculum Committee Report, and the yearbooks of NCTM.
- ✦Plan to get teachers' and principals' reactions concerning the recommendations of the Planning Committee.
- ✦Make available to the committee pertinent facts about the school population for which it is planning, such as results of statewide testing, statistics concerning study beyond the high school, dropouts, population changes, and occupational changes.
- ✦Make available information concerning the preparation of teachers who will be teaching the new courses.

Step 4

The activities of all committees concerned with mathematics curriculums are coordinated and information is mutually exchanged. Potential committees are those concerned with (*a*) television teaching, (*b*) inservice education, (*c*) textbooks, (*d*) learning aids, (*e*) testing.

The mathematics supervisor might—

✦Serve on all such committees as a member or in an advisory capacity.

✦Serve as the liaison.

✦Keep the committees informed of important activities in mathematics.

Step 5

A Production Committee is appointed which will: (*a*) Consider best ways to effect changes in classroom procedures and subject content. (*b*) Examine other courses of study to find ideas that look promising. (*c*) Obtain teachers' opinions on the most useful kinds of publications. (*d*) Write up the new courses in detail.

The mathematics supervisor might—

✦Serve with the Production Committee either as a member or in an advisory capacity.

✦Prepare bibliographies of books and publications for this committee.

✦Make summaries of important reports.

✦Make available either learning aids (such as those supplied under the National Defense Education Act) or good descriptions of them.

✦Contribute helpful ideas as a result of wide experience and opportunities of having seen other curriculums at work.

✦Get important reports of national commissions, etc. for the use of the committee.

✦Make available professional books and magazines for the committee to use in writing up the courses.

✦Get sufficient supplies and clerical help for the committee.

✦Get editorial assistance and mathematics experts for help in editing.

✦Get ideas and results of firsthand research from successful, experienced teachers.

Step 6

New and tentative courses are mimeographed and distributed for trial use in (*a*) several experimental centers, (*b*) one school system, or (*c*) all schools throughout the State.

The mathematics supervisor might—

✦Arrange with schools and teachers to try out the courses.

✦Prepare evaluation forms for teacher reports on the new work units.

- Present the courses at a general meeting, when members of the Planning Committee and the Production Committee would discuss the new work.
- Prepare for teachers a good bibliography of books, magazines, films, and instructional aids to help them secure the most valuable results from the new courses.

Step 7

Inservice education is arranged in connection with the new course of study through such means as (a) meetings of teachers in single school systems, (b) regional meetings, (c) television teaching, (d) college inservice courses, (e) guides for individual or group study.

The mathematics supervisor might—

- Plan programs in which one or more members of the Planning or Production Committee could discuss phases of the course of study or in which the subcommittee which wrote a particular unit could discuss that unit.
- Get services of consultants who would speak at individual meetings, put on workshops, or give a series of lectures.
- Get the cooperation of colleges to provide inservice courses (under NDEA or otherwise) in which teachers (often with little or no expense to themselves) could study mathematics that would help them teach the new work better.
- Inform administrators of the importance of inservice work and ask them to consider including it in their budgets.
- Acquaint teachers with publications that are helpful for individual or group study (such as the study guides prepared by the School Mathematics Study Group and the chapter "Promoting the Continuous Growth of Mathematical Concepts" in the 24th yearbook of NCTM).
- Teach or arrange for demonstration lessons to illustrate some new concepts or topics.
- Prepare tape recordings of demonstration lessons or outstanding speeches, announce that the tapes are available, and lend them to school systems.
- Arrange for teachers to visit classes doing new and interesting work.
- Enlist the services of a small group of interested individuals to program one or more of the units as a self-teaching device or "teaching machine."
- Arrange inservice courses or lessons on television; have them kinescoped and then lent or sold.
- Set up committees to work on special problems such as advanced placement, slow learners, etc.
- Use every opportunity to get the cooperation of college teachers as consultants, teachers of inservice courses, or speakers.
- Encourage teachers to undertake research projects.
- Bring in foreign exchange teachers to learn about schools and curriculums in their countries.

Step 8

The evaluation of pupil achievement is planned to include tests prepared by one of the following groups: (*a*) The Planning Committee, (*b*) the Production Committee, (*c*) committees of teachers set up for evaluation purposes, (*d*) the supervisor, with teacher assistance.

The mathematics supervisor might—

- Arrange with principals to have pupils using the new course tested by standardized tests and new tests.
- Summarize and interpret results of the testing for administrators and teachers.

Step 9

Production Committee revises and rewrites courses of study on the basis of (*a*) evaluation of units by teachers who taught them, (*b*) reactions of pupils who studied them, (*c*) new information and knowledge gained by the committee in its further study.

The mathematics supervisor might—

- Serve in an advisory capacity.
- Summarize evaluations for the committee.
- Report on testing.
- Supply recent publications.
- Help in careful editing.

Step 10

Artists help with production of printed curriculum.

The mathematics supervisor might—

- Supply mathematical ideas to be expressed by the artist in art form.

Step 11

The new curriculum is distributed.

The mathematics supervisor might—

- Furnish lists of schools, principals, and teachers from information in his office.
- Help plan the distribution.
- Supply schools with appropriate information concerning the use of the new courses.

Step 12

Good public relations concerning the new mathematics are promoted through (*a*) newspaper publicity, (*b*) reports in professional magazines, (*c*) reports at meetings of administrators, (*d*) discussions at meetings of lay groups, (*e*) exchange of courses with other states.

The mathematics supervisor might—

✦Send information to administrators concerning the need for the new curriculum, its purposes, suggestions for its use, preparation needed by teachers, and answers to questions which teachers and principals have submitted.

✦Write articles for professional publications describing the new mathematics work.

✦When invited, speak at parents' meetings on the new mathematics.

✦Acquaint college professors of mathematics methods courses with what their graduates will be expected to teach.

Step 13

The new curriculum is being used in the classroom.

The mathematics supervisor might—

✦Arrange meetings where the new bulletin or course of study will be discussed for the benefit of new teachers who are unfamiliar with the preliminary work.

✦Set up a committee to work on tests based on the new courses.

✦Sponsor conferences to discuss grading of students in the new work.

✦Call attention to the fact that although curriculum development is a continuous process, an agreed-upon body of content at any particular time is needed.

Conclusion

The recent Report of the President's Commission on National Goals emphasized the importance of guarding the rights of the individual, ensuring his development, and enlarging his opportunities. In mathematics we have a very special responsibility here, and I feel that some of the professional activities I have outlined can indeed help guard the pupil's rights, help ensure his development, and help enlarge his opportunities.

Does the task seem formidable? I hope you do not feel like Ophelia when she replied to her brother in *Hamlet*, act I, scene 3:

> . . . But, good my brother,
> Do not, as some ungracious pastors do,
> Show me the steep and thorny way to heaven;
> Whiles, like a puff'd and reckless libertine
> Himself the primrose path of dalliance treads . . . .

Since I speak largely from my own experience, I can assure you that I have tried to do most of the things that I recommend. I realize that circumstances differ greatly, depending on the size of a State and the staff. I trust, though, that some of the elements I have pointed out are common to every situation.

Evaluating a School Mathematics Curriculum

DONOVAN A. JOHNSON

IN VIEW of the current ferment in mathematics education we are all interested in ways to improve our judgment as we make decisions regarding new programs. How are we going to decide what mathematics to teach? How can we determine the best way to attain a given objective? How can we decide when a given concept should be taught?

It is going to be extremely difficult to make valid judgments of the relative effectiveness of different curricula. Many of our decisions will be based on subjective value judgments. Many decisions will have to wait until long-range effects are determined. Other decisions will need the cooperative judgment of specialists in several fields.

In discussing mathematics programs, I am confining myself to the specific curriculum proposals now being made by various groups and individuals. The evaluation of the mathematics program of a specific school often includes many factors beyond the curriculum. For example, the Evaluative Criteria used by secondary school administrators take into account the school's facilities, staff, registrations in given courses, and length of period, as well as courses and course content.

Evaluation is more than measurement, although it is usually based on measurement. Evaluation involves the use of judgment. In evaluation, we usually decide whether certain measurements, actions, or materials are good, bad, or indifferent. In evaluation we compare and then render judgment regarding the order relation of the items involved.

If we are to render value judgments regarding curricula proposals, we need certain criteria as guide lines. What should these criteria be? For our discussion today, I would like to suggest criteria of the following four types: mathematical, psychological, pedagogical, and philosophical.

Mathematical Criteria

There are several aspects of the mathematical content of a given curriculum to consider. The first of these is that the mathematical content of an acceptable curriculum must be *good mathematics*. By this, I do not necessarily mean modern mathematics or classical mathematics. Good mathematics is mathematics that is correct, precise, and elegant. Good mathematics uses the terminology, the processes, the sequence, and the symbolism that are as correct, clear, and complete as possible at the level for which the mathematics is prepared. To attain the proper degree of rigor for a given level is one of our most difficult problems.

Evaluating the mathematical content of a given program requires mathematical sophistication beyond that of many elementary or secondary teachers. In the current national experimental programs, however, attaining the proper degree of rigor for a given level is not much of a problem since the materials usually have been produced by groups composed of competent mathematicians.

However, the mathematical content must not only be good mathematics. It must also be appropriate mathematics. To be appropriate mathematics, it needs to be adapted to the students involved. It should meet the needs of the students, currently and in the future. In view of the uncertainty of what mathematics our students will need in the future, this choice is difficult. Thus, it would seem that the mathematics should emphasize flexibility, procedures, and broad principles rather than specific facts. It should provide experiences in applying concepts and skills as well as present the esthetic aspects of mathematics. It should provide experiences that develop good learning habits as well as a desire to learn.

The National Council of Teachers of Mathematics has appointed a committee on the Analysis of Experimental Programs, with Philip Peak of Indiana University as chairman. This committee will visit experimental projects and collect information relative to nine questions.

1. Placement

Various topics are being introduced at a number of different grade levels. The question is: *At a particular grade level, what topics can be developed most effectively?*

2. New Topics

Many topics hitherto not commonly included in mathematics programs are now being taught. The question is: *Which of these topics should become an integral part of the school mathematics program and at what level?*

3. Structure

There has been much discussion recently on the need for studying mathematical structures. The question is: *What emphasis should be placed on the study of mathematical structures to result in a better understanding and use of mathematics?*

4. Social Applications

Social applications of mathematics are being discussed by those concerned with better mathematics instruction. The question is: *How much emphasis should be placed on the social applications of mathematics and what should be the purpose and nature of these applications?*

5. Vocabulary

An individual's language develops as he develops his ideas and has relevant experiences. The person developing a language of mathematics proceeds in this same way. The question is: *How rapidly can an individual be led from the use of a general unsophisticated mathematical language to the use of a very precise, sophisticated mathematical language?*

6. Concepts Versus Skills

Mathematics has value as a tool to be used by other disciplines, but it also involves abstract ideas or concepts. The question is: *What relationship shall exist in the mathematics programs between the function of developing concepts and that of developing the manipulation of symbols?*

7. Proof

There are many possible grade levels and degrees of rigor at which proof might be introduced into the mathematics program. The question is: *At what grade level should proof be introduced, and with what degree of rigor; and how rapidly should the learner be led to the position where he recognizes and appreciates rigorous proof?*

8. Organization

Topics and areas of study may be organized in a number of ways. The question is: *Is there a principle of organization that supersedes* *other principles by providing a better learning pattern, more retention, or more efficiency?*

9. Correctness of the Mathematics

There is no question as to whether or not one should use correct mathematics, but the question here is: *What constitutes correct mathematics as determined by either logic or acceptable authority, and why is the particular authority the acceptable one?*

Psychological Criteria

Since our real concern is learning the mathematics we have selected, we must next consider criteria based on principles of learning. Can the selected mathematical concepts, skills, habits, and attitudes be learned?

Let us remind ourselves that we can't force a student to learn mathematics. If he is to learn, the student must be ready, willing, and able to learn the ideas we propose to teach. Furthermore, he will learn only if he reacts, responds, or participates.

Our first psychological criterion should be this matter of readiness. When should a concept be taught? From experiences with mathematical concepts such as those of Professor Suppes (Stanford University) in teaching geometry in first grade and those of Professor Davis (Syracuse University) in teaching quadratic equations in fifth grade, it appears that young children learn anything more readily than adults do. Thus, the conclusions reached nearly half a century ago by the Committee of Seven concerning a fixed time schedule for presenting topics in arithmetic now seem to have been in error. It appears that at this time we cannot render judgment, *on the basis of level of difficulty*, what is the proper time to present a given arithmetical topic.

A second requirement for learning is that the learner be motivated so that he is willing to learn. This seems to be a major strength of new proposals. Here is what Ferguson reported at the regional conferences sponsored by the National Council of Teachers of Mathematics last fall.

> Some of us who have taught the old traditional mathematics feel it is a miracle that some of our students became mathematicians considering the way we taught them. The evidence we have to date is simply this: teachers are much more enthusiastic about the new mathematics programs and the new techniques of teaching them. The students show more interest and enthusiasm for mathematics than ever before. Almost all (if not all) teachers who have tried teaching the new programs (the University of Illinois Com-

mittee on School Mathematics—UICSM—the School Mathematics Study Group—SMSG—and the University of Maryland Mathematics Program—UMMaP, etc.) do not want to return to teaching the old traditional texts.

The mathematics in the new programs is not easier; it is not watered down, but it is more interesting and challenging to students and teachers alike. On traditional tests students taking the new programs have performed, so far as we can judge, about as well as the students taking the traditional program. If this is the case, the students taking the new programs have the same mathematical knowledge as the students taught in the traditional manner, plus many new ideas and topics. On power tests, such as the Advanced Mathematics Examination of the College Entrance Examination Board and the Contest Examination of the Mathematical Association of America, it is a different story. In schools that have used the UICSM Program for three or more years, the students in the UICSM Program have done significantly better than students of comparable ability who have had only the traditional courses.

However, in evaluating these enthusiastic results we must keep several factors in mind:

1. The courses are new and different.
2. The teachers and students are participating in an experiment.
3. The teachers are spending much time in preparation of lessons.

Much in this revolution is to be welcomed, and undoubtedly the mathematics teaching will be greatly modified and improved during the next decade. Many advantages will be gained thereby. But certain dangers need to be borne in mind. Perhaps the greatest is the danger that the revolution may go too far and confront students with courses so abstract that they exceed the youngsters' mathematical maturity, and thus result in bewilderment and revulsion against mathematics rather than increased knowledge. The community may expect that the skillful mathematicians pushing this revolution will bear such dangers in mind and make haste slowly in carrying out the generally useful changes they plan.

A requirement for learning concepts is that the student be *able* to learn the ideas being taught. Observe this editorial from the New York *Times* of November 28, 1960:

To add meaning to what is taught requires intuition, participation, illustration, application, and proper language. Many of the new programs place emphasis on this aspect by having students discover the principle or idea involved. This is certainly an effective method of instruction, at least for reasonable size classes. On the other hand, some programs seem to lose interest in concrete representations, visualizations, or applications. It is unfortunate that very few scientists, economists, statisticians, or computer programmers have been involved in writing the new courses. Likewise, few experts in materials of instruction or psychologists have been consulted during the writing of experimental programs.

Pedagogical Criteria

Concerning methods of teaching the new mathematics programs, we should consider the following questions:

1. Do the teachers have the necessary background for the program? If not, can they be given it through inservice education?
2. Is sufficient time available for adequate presentation of the topics outlined? If too much is outlined can these topics be postponed or some eliminated? Can additional time be made available?
3. Does the curriculum have adequate materials, such as texts, units, and teacher guides?
4. Does the school have the resources to provide the teachers and the students with the necessary text material or facilities? (A given course is usually not teachable if text material is not available.)
5. Has the material been tried out experimentally? (This is one of the strengths of most current proposals, even some commercial programs. In evaluating the experiments, however, we must be realistic.) What kind of teachers were involved? How reliable are the teacher reports? What was the nature of the schools or students who participated?

Following is a preview of a typical SMSG experiment:

Twenty elementary teachers in the Twin Cities area will be selected to participate in this experiment. Ten of these will have taught the SMSG 4th-grade course in the academic year 1960–61. These teachers will teach the SMSG 4th-grade course to their pupils. The other 10 teachers will be selected to match the first 10 as closely as possible in regard to teaching experience and qualifications, but will have had no experience with SMSG materials. They will teach a conventional mathematics course.

The Sequential Tests of Educational Progress level-4 achievement test will be administered to both the experimental and control pupils at the beginning and at the end of the school year. In addition, an achievement test prepared by SMSG will be administered to all classes at the end of the school year. Also, achievement tests prepared by SMSG will be administered to the experimental classes at approximately 2-month intervals.

Another pedagogical question concerns the selection of a program that will provide the best possible mathematics education for all levels of ability. Thus we must ask the question: "For what level of ability is this curriculum the most suitable?"

Philosophical Criteria

We now come to the crucial basis for evaluation. What is the purpose of mathematics instruction? Where are we going? What are we striving to attain? It is not sufficient to know that we can teach a given concept at the fifth-grade level. We must decide whether we ought to teach it, whether it is more important than other ideas which we could teach.

To determine what we ought to teach we need to spell out our objectives. Although these goals may include general objectives such as responsible citizenship, or ethical character, I will assume that these have equal likelihood of being attained by traditional or new curricula. When you ask teachers to state the specific results they expect of mathematics instruction they usually come up with a list such as the following:

1. The student has a knowledge and understanding of mathematical processes, facts and concepts.
2. The student has skill in computing with understanding, accuracy, and efficiency.
3. The student has the ability to use a general problem-solving technique.
4. The student understands the logical structure of mathematics and the nature of proof.
5. The student associates mathematical understandings and processes with everday situations.
6. The student recognizes and appreciates the role of mathematics in society.
7. The student develops study habits essential for independent progress in mathematics.
8. The student develops reading skill and a vocabulary essential for progress in mathematics.
9. The student is stimulated to participate in mental activities such as creativeness, imagination, curiosity, and visualization.
10. The student develops attitudes leading to appreciation, confidence, respect, initiative, and independence.

Now the problem arises as to what specific facts, processes, or skills you wish to teach. Here you must choose. Those appropriate for a given ability level or for given vocations will certainly vary. Not all mathematicians agree as to which are most suitable for college or for science. Most agree with Mr. Adler that classical as well as modern topics should be included. Most agree that basic computational skills are still needed. One of the things which the various curriculum groups should do is to spell out the specific competencies needed for success in business, government, and citizen activities, as well as in science and mathematics. Then we will have a basis for rendering judgment relative to the competence attained by a given program. Traditional tests are not satisfactory for this purpose.

Here are some specific recommendations of the Joint Commission on the Education of Teachers of Science and Mathematics:

1. SMSG might profit from a careful comparison between the present curricular projects and earlier ones. Such a study may reveal the factors which contribute significantly to a successful program. The following elements may be relevant: adequate support, development of text materials, experimental teaching, a favorable climate for getting relevant information to teachers and supervisors and for getting cooperation between school teachers and mathematicians.

2. In addition to defining the general objectives of curriculum projects, considerable effort must still be made in defining operationally the criteria which reflect the objectives (other than mathematical knowledge and skills) of the mathematics programs. Teachers, pupils, and parents react strongly to evaluative instruments (e.g., college board examinations); hence, SMSG should support efforts to construct tests reflecting all the many objectives of the programs.
3. Long-range follow-up studies, particularly of students who go on to college, should be made to find out whether the training provided by the new programs meets the demands made upon it and whether it produces the hoped-for results concerning attitudes towards mathematics.
4. SMSG should encourage joint research between mathematicians and behavioral scientists concerning the learning process and the formation of attitudes towards mathematics.

Measurement

If we accept these goals we then have a basis for measuring the achievement of our students. Can we do this so that we can render judgment as to the relative effectiveness of a given curriculum? In many ways this will be impossible. For example, in terms of facts and skills, only those common to both can be tested. This is likely to be an inadequate sample of the learning from either curriculum. New tests must be devised, not only on the common topics but on the common goals, such as problem solving, communication skill in reading, writing, and presenting mathematical ideas; attitudes, application, discovery, and creativeness.

Besides these we need long-range evaluations in terms of continued study of mathematics (what happens to registrations?), continued success in mathematics, success in related fields such as science, success in selected vocations, and success in citizenship.

Summary

Finally, then, the evaluation of a school mathematics curriculum is not a single process. It will need the judgment of many specialists: the mathematician, the psychologist, the educator, the scientist, the research worker, and the teacher. It will need a comprehensive testing program which includes tests not as yet devised. It will need research in the form of comparison studies and follow-up studies that will require several years for completion. In the meantime, we must make our choices based on value judgments. In making our choices, however, we do have mathematical, psychological, pedagogical and philosophical criteria.

How the State Supervisor of Mathematics Can Stimulate Local Leadership

JAMES H. ZANT

REAL IMPROVEMENT OF MATHEMATICS in the schools must take place with the teachers in the classrooms. It is not implied here that in this process intelligent administration is not a valuable asset. The point of view of this paper is that actual improvement occurs only when the mathematics teacher is both stimulated and provided with some definite, regular means of acquiring the background necessary to use the new concepts in his teaching.

The State Supervisor of Mathematics must find some means of stimulating the mathematics teachers in his State. Further, he must find the means to provide them with definite instruction concerning the principles of mathematics and to convince them that mathematics must be taught as a structure—as a body of knowledge based on fundamental, understandable, and consistent principles of logic.

The State supervisor's obligation is twofold: to make teachers want to improve the school mathematics program and to make it possible for them to learn the basic mathematics needed to initiate and carry on such a program. Teachers, through no fault of their own, have not been taught the kind of mathematics they need to teach the new and exciting material of the new programs being suggested and widely used. It has been demonstrated in our State (which has no State supervisor) that teachers *can* be stimulated to learn the mathematics needed for successful teaching of modern mathematics courses at both secondary and elementary levels.

The State Supervisor must use any and all devices possible to stimulate teachers so that they will want to improve their classroom activities. This can be done by teachers' meetings of many sorts. Teachers' meetings, both State and district, are held annually in nearly all States. More can be accomplished, however, by holding local town or county meetings to which nearby teachers may be invited. Such meetings have been used to inform and interest them in new and different mathematics programs. The activities at these meetings can be many and varied, and the State supervisor can play a leading role in many of them. He should, however, also rely upon other sources as well. College teachers can be interested in this problem and can serve as a source of information. Their prestige in the State and community is very helpful. Other teachers who have had experience with teaching new programs are most valuable, and their first-hand experience should not be ignored. Demonstration classes should be used whenever possible. Teachers have a hard time believing that some of the suggested topics can be taught to children. Seeing it done can be impressive. All meetings with teachers should include time for discussion and questions and answers.

Individual contacts should not be ignored. Interested teachers can learn enough about the new programs to teach one course in an acceptable manner. Many teachers have attended summer institutes sponsored by the National Science Foundation and others. Although it is true that most of these institutes stress mathematical content, but do not show the participating teachers how this content may be used in the classroom, the teachers can, with some effort, successfully teach a modern course of mathematics the following year. Instances can be cited in our State where this has been done.

The State Supervisor might suggest that the Foundation's Summer Institute for Mathematics in his State provide the participants with an opportunity to plan how they can use some new approach or other in their teaching the next year. The Foundation is committed to the goal of improving the teachers' knowledge of the new content in mathematics. All programs sponsored by the Foundation reflect this goal. That the teachers should improve their knowledge of this new content is indeed necessary, but in addition they should consider various methods of presenting it to their students.

A study of methods may be done effectively by using not more than one-fourth of the participants' time during the summer institute. The ideal way perhaps is to provide a seminar during the entire period in which the teachers can work on this problem. During the NSF Summer Institute for High School Mathematics Teachers held at the Oklahoma State University in 1960 the following procedure was used: Participating teachers enrolled for two specially provided courses in the content of mathematics. Three courses were available, one on algebra, one on geometry, and one on the mathematics of the 12th grade; the latter was called Analysis and Statistics. The entire Institute program was tied closely to the School Mathematics Study Group textbooks, grades 9–12, but, these books were not used in the above-mentioned classes as texts. The professors did base their lectures on the concepts included in the textbooks, but when time was available went beyond the level of the high school books. In addition to the two courses they selected, the participants were required to spend approximately two hours a day in a seminar or working on a seminar problem.

Lectures consumed about one-fourth of the time allotted to the seminar period. These lectures discussed the general idea of modern mathematics programs for the schools and the School Mathematics Study Group program in some detail. Several special lecture s discussed other experimental programs. One of these lecturers was Professor Charles Brumfiel of the University of Michigan, who discussed the books being prepared by Brumfiel, Shanks, and Eichols.

Early in the summer the participants were asked to indicate special interests, so that the group might be divided into smaller study groups for the main part of the seminar activity. In most instances these smaller groups were separated by grade-level interest. The teachers were told to choose a group and level which they felt would be useful to them in their classroom work the following year. There was one subgroup, for example, made up of teachers who expected to teach 9th grade the following year. Many of the subgroup knew already that they would be using the SMSG *First-Year Algebra*. This subgroup examined the book critically in order to familiarize themselves with the content and the way algebra was organized and structured in the book, and it suggested some additions and deletions. Finally, the subgroup submitted a written report on what they had done. This report was made available to each participant after the Institute closed.

Other subgroups carried out similar projects and submitted written reports on their findings and recommendations. The reports were bound in a single volume and copies were made available.

The purpose of the seminar was twofold: to give the participants an overall picture of mathematics developments in the secondary schools, and to have them study the SMSG textbooks thoroughly enough so that they could go into the classroom in September and teach the material with competence and confidence. This purpose was accomplished with nearly all of the participants, and a large percentage of them actually taught mathematics from the SMSG textbooks during 1960–61.

This discussion so far has dealt largely with the stimulation of the individual teacher or of a few teachers in a particular school district. This is important and is a necessary step in the early stages of introducing new content and point-of-view in teaching. We in Oklahoma are convinced that this is an effective way of getting a modern program underway in our schools. A single teacher teaching one or two sections of general mathematics, elementary algebra, or plane geometry can have a profound effect in a community. The fact that the pupils understand and are interested in mathematics, often for the first time, becomes well known in the school and among the parents. Other teachers become interested and other parents want to know why *their* children are not in these classes. From this simple beginning we have evidence in many school systems that the offering is much expanded the following year.

In fact, it is not necessary for teachers to wait until they have new textbooks to start improving their teaching. They should be encouraged to do something immediately. They can begin with one or more units, such as units on exponents, sets, or inequalities. Teaching material for these should be made available to the teachers and, since it must be integrated into the work based on traditional textbooks, it usually cannot be lifted bodily from the SMSG or other textbooks. Teachers can make these adaptations under proper guidance in a summer institute; but working alone, without adequate stimulation and help and with all of the other things a teacher is expected to do, it is not realistic to assume that they can integrate the new mathematics with the present traditional material. The State Supervisor of Mathematics can assume leadership in organizing inservice education institutes, but he may wish to seek the counsel and help of competent teachers in the State and of mathematicians in the colleges.

It is often worthwhile to work with the entire mathematics staff of a school system. Under present conditions this necessitates some program of inservice education for the teachers. It also involves some cooperation with the administration and perhaps a nearby college.

Since Dr. Gundlach's paper will probably deal with preservice and inservice education for teachers in a large school system, brief reference will be made here to a neglected source of personnel useful in working with smaller schools. We refer to the comparatively large number of former participants of the NSF Academic Year Institutes. Before June 1960, 3,344 mathematics and science teachers had received a full academic year of training at the graduate level in these Institutes. Approximately 1,500 per year are being added to this total. Despite the fact that 95 percent of these participants remain in the teaching profession and perhaps 80 percent return to high school, this potential for competence and leadership is not being effectively used.

Studies sampling these participants show that they earn more money, have a higher status in the schools, do a more effective job of classroom teaching, and engage in more professional activities than do other teachers. They also have favorable attitudes toward this phase of their education experience.[1] Only a small percentage

[1] See *Field Survey Academic Year Institute Participants for 1956–57, 1957–58*, by L. A. Ostlund. Oklahoma State University, Arts and Sciences Research Studies, 1961.

of them, however, have any administrative responsibilities. (Those having such responsibilities are mostly the ones who are also high school principals.) These men and women are scattered over the United States as classroom teachers, many in small schools. They are available, eager, willing, and competent to help their fellow teachers. A project to make adequate use of them in the schools is waiting to be designed and consummated. It can be done and in the field of mathematics perhaps the State supervisors of mathematics may have to do it.

Perhaps it appears that the school administrators have been ignored in this scheme for stimulating local efforts to improve mathematics teaching in the schools. This is not intended, but the theme of this paper is that the effort and leadership should be concentrated in competent, well-trained mathematics classroom teachers. The administrators have been very cooperative and helpful in advancing the program in the schools of our State, but the actual accomplishment has been achieved by the individual teacher in the classroom. The administrator must be willing to provide the teaching material and facilities, time for preparation and inservice education, as well as stimulation and encouragement. As individuals, administrators usually do not have the knowledge of content and point-of-view so urgently needed by the teacher, nor should they be expected to. In a sense they follow the procedure often recommended for handling gifted students; that is, give the teacher what he needs to teach good mathematics and then get out of his way. A good teacher will do the rest!

A word should be said about curriculum materials for a modern mathematics program in the schools. It is not realistic to expect teachers in small schools to write their own materials to any large extent. The State Supervisor must depend on teaching materials already available. Fortunately, such materials have been written and are available. The State Supervisor is familiar with—or should make himself familiar with—these materials and with textbooks that are now beginning to appear.

In Oklahoma we have made extensive use of the SMSG textbooks, grades 4 through 12. Something like 40,000 of them will be used in the State next year. There are several reasons for doing this: (1) Teachers in the State, at both the school and college level, have had a considerable part in writing and trying out these books. (2) We have found them teachable and effective in the classroom. (3) They are readily available. (4) Our administrators have been willing to buy them in large numbers. We freely admit a bias toward these materials and their wide use is not meant as a criticism of other teaching materials.

A word of warning is in place here: we should examine carefully and critically new books that will be published in the next few years for this purpose. The terms "modern mathematics" and "contemporary mathematics" (not really descriptive of what we are trying to do in the schools) imply to many that new or "modern" concepts should be introduced into the textbooks. Hence, books (particularly revisions of older textbooks) will be published introducing material on sets, for example, but will make no further use of this knowledge. Such things have been done before and we should expect them to happen again.

Something more should be said about preservice education for all teachers who teach mathematics in the schools. The content of this training has been discussed, but what about the State Supervisor's role in getting changes made in the college curriculum and in State certification and accreditation procedures?

From the standpoint of college programs for prospective teachers two things may prove useful: There should be a State level committee on mathematics and an attitude or point of view on the part of administrators, especially the superintendents, that it is the obligation of the college to turn out teachers qualified to teach the new materials in mathematics. If you do not have a State committee on mathematics go about getting one appointed. The logical spot for it is in the Curriculum Division of the State Department of Education. In Oklahoma, the State Mathematics Committee was appointed by the Oklahoma Curriculum Improvement Commission, a part of the Curriculum Division. The Commission, operating largely on membership fees from schools in the State, has a relatively small budget. The committee should include representatives from mathematics teachers at all levels, elementary school through college, school administrators, and State Department officials. A committee of about 25 members seems to be about the right size. Above all it should have active, courageous leadership. Perhaps the State Supervisor can serve as chairman, but an interested college mathematician will also be effective. If he is carefully selected, his prestige will add much to the acceptance of the committee's recommendations to the schools as well as to the colleges.

The State committee need not, probably should not, have any power of decision regarding the adoption of a State curriculum in mathematics; but it should make recommendations and have the courage to make them to any group, when it feels that mathematics and the teaching of mathematics can be improved. Recommendations by such a committee with a properly chosen chairman will have an effect on the colleges in the State. New courses, or the same courses taught from a modern point of view, can be suggested for preservice education of teachers. These suggestions are likely to be accepted by the colleges.

After the proper atmosphere has been created, the committee, or a cooperating university, can suggest some thoroughgoing study of the problem of preservice education of mathematics teachers at all levels. Most colleges have it in their power through the standard procedures of the State Department of Education, Certification Division, to change their teacher-training standards when they are ready to do so. From the standpoint of the mathematical organizations definite suggestions are already available. For example, the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America has a Teacher-Training Panel that has made definite suggestions for improved courses in mathematics for teachers. CUPM has the financial backing and the willingness to help any college or group of colleges implement such a plan. Someone in each State should start the colleges working on this project. The State Supervisor through an active State committee can do this. New teachers coming into the schools will then be adequately trained to teach a modern program in mathematics.

By talking with superintendents the State Supervisor can do much to stimulate or create the point of view that is an obligation of the colleges to turn out qualified teachers. If the State committee can foster this attitude, and especially if there is at least one college in the State where such new teachers are available, the point of view will grow rapidly. This is one of the values which can come from getting a single class in a modern program established in a school system. If it is successful, and it usually is, the administration will want it expanded. This poses the problem of retraining the other teachers in the school which we have already discussed. If the superintendent has hired a beginning teacher just out of college, it is easy to raise the question as to why this new graduate is not qualified to teach this sort of mathematics. The schools have a right to expect such training on the part of new teachers and many of them are beginning to demand it. It is within the responsibilities and the obligations of the State Supervisor to suggest that administrators press this point.

At this juncture, we might consider what a program for the preparation of secondary school mathematics teachers should do. Here are some suggestions:

Seven Guidelines for the Preparation of Secondary School Mathematics Teachers

1. Include a thorough, college-level study of secondary school mathematics curriculum.

2. Take into account the sequential nature of the mathematics curriculum and in particular provide the prospective teacher with an understanding of the mathematics which his students will meet in subsequent courses.
3. Include a major in mathematics, with courses chosen for their relevance to the secondary school curriculum.
4. Include sufficient preparation for the later pursuit of graduate work.
5. Take into account the recommendations for curriculum improvement now being made by various national groups.
6. Include work in related areas where mathematics is used.
7. Have its fifth year consist largely of subject-matter courses.

Regarding changes to be made in State certification and accreditation we recognize that the State mathematics supervisor is in a strategic position. He is already a member of the State Department of Education and the directors of teacher education and certification already have a movement under way to improve the qualifications and preservice education of both mathematics and science teachers. Sponsored by the National Association of State Directors of Teachers Education and Certifications (NASDTEC) and the American Association for the Advancement of Science (AAAS), this movement has resulted in a certification study which began under a grant from the Carnegie Corporation of New York on December 1, 1959. Composed of representatives of teacher education, college academic faculties, State Departments of Education, the public schools, national, professional, and academic organizations and accrediting organizations, regional conferences have been held and recommendations have been made. From these recommendations guidelines have been constructed. These were discussed later and constructively criticized by various groups, and then revised.

The points of view of the mathematicians and the various studies under way in modernizing the school mathematics program have been given much attention and have had a marked influence on the final recommendations of the NASDTEC–AAAS study. Further, as observed by Dr. Gail S. Young of the Department of Mathematics, Tulane University, the NASDTEC–AAAS certification study was "the first occasion . . . for mathematicians to influence policy of State Departments of Education and educational administration on a large scale."[2] Also the facts noted above have impressed college and university mathematicians to the extent that they will help to establish an adequate mathematics curriculum for future teachers of mathematics.

This paper has contended that improvement in the school mathematics program at any level can take place only if the classroom

[2] The NASDTEC-AAAS Teacher Preparation and Certification Study. *American Mathematical Monthly*, 67:8:793-97, October 1960.

teachers are both stimulated and given an opportunity to improve their own knowledge of the content of mathematics. It has been demonstrated repeatedly over the country that teachers can learn the necessary content and that they can teach the new programs successfully. In the writer's opinion elaborate schemes worked out at top levels by State Departments of Education or national committees will not get the job done. It is well known that many of the materials now being used in the new programs have been included in recommendations of committees and commissions for the last 50 years. Many of us have served on these bodies, yet little change has occurred. We still have had good students going through the schools without, for example, having any concept of the algebraic structure of the number system.

The implications for the State Supervisor of Mathematics are clear: (1) Work with the individual teachers at the classroom level from the elementary school through college. (2) Stimulate them to want to improve their teaching of mathematics. (3) By any means possible, provide them opportunities to learn the necessary content to teach the new material.

The assistance of the administrators must be obtained, since time and materials cost money. When teachers are enthusiastic and are producing results, administrators are eager to help.

This task will not be easy. It will consume a great deal of time and energy. From results achieved in our State, however, one can say that the effort will be very rewarding. It can probably be assumed generally that if the State Supervisor of Mathematics does not do it, the task will not be done. The Supervisor himself must be a competent professional person; he must know the materials that are available; he must be able to command the respect of teachers, administrators, and college mathematicians in the State. With these attributes, or a willingness to acquire them, the Supervisor of Mathematics can expect that the teaching profession in his State will support him enthusiastically.

How the State Supervisor Can Promote Preservice and Inservice Education for Mathematics Teachers

BERNARD H. GUNDLACH

THE PRESENT STATE of transition in which the school mathematics curriculum finds itself raises three fundamental questions:

1. Why is a revision necessary?
2. What are the essential features of a desirable revision (a) with regard to content; (b) with regard to teaching approaches and methods?
3. What can be done to implement a revised mathematics curriculum as quickly and as painlessly as possible?

In this presentation I am primarily concerned with the third question, more precisely: What can supervisors do to further such an implementation?

I do not wish to go into detail here concerning the results in the various fields, but since aspects of questions 1 and 2 have considerable bearing on question 3, I will summarzie briefly those answers which, in my opinion, can be given to questions 1 and 2.

Why is revision necessary? It is necessary because the society of which we are a part and the particular needs of this society have changed drastically over the past 25 years or so. To fill these ever more urgent needs adequately, we simply must bring the mathematics curriculum up to date.

Content

The essential features of the revised mathematics curriculum fall into two categories: (1) content and (2) teaching approaches or methods. In regard to content, we must recognize the fact that a considerable amount of mathematics now being used in modern business and industry, and in modern life generally, is mathematics which has been created quite recently and which is therefore almost completely absent from our traditional curriculum. To mention only a few, I am referring to such topics as Statistics, Linear Programming, Decision-Making Theories, and Techniques of Machine Computation. Of course, in order to make room for new topics without infringing on other equally important subject matter, we will have to delete or de-emphasize some of the traditional topics. Such changes in content can be brought about by writing new and/or additional text materials and by bringing the teachers up to date. After all, it is they who will have to use the new materials in their classrooms. The teachers must become thoroughly familiar with these materials. This is possible, though not so easily done as said.

Teaching Approaches

Even more difficult is the task of making parallel changes in the teaching approaches or methods of presentation. A change in approach demands a change in *attitude* toward mathematics, and that is a most difficult feat to accomplish for people who over a period of many years have elaborated for themselves a rather fixed mode of procedure. While it is true that methods of teaching are determined to a large extent by psychological and pedagogical factors, it is also true that success or failure of a certain teaching approach depends on the extent to which the teacher is competent in the subject matter. Both factors *together* make for good teaching and lasting learning.

No matter what is being said, it is still true that for the vast majority of our students mathematics is primarily a tool—a most essential one, to be sure—a tool designed to enable them to make their way in a society rapidly becoming more and more technologically oriented and automated. The rate at which changes take place at the present time is appalling. No signs of a letup of this trend are anywhere in sight.

In the more slowly paced society of the past, for which our traditional approach to mathematics teaching was designed, the chief motivation for learning mathematics was its *social usefulness*. This meant that our teachers performed an adequate job when they handed their students specific tools for specific problem situations and provided plenty of practice in handling these tools. In the society of tomorrow, which will be that of our students, this will not be sufficient. Traditional problem situations disappear from the public scene as rapidly as automated methods take over. We can no longer be sure that the problem situations for which traditional mathematical tools or models were designed will still exist 8 or 10 years from now, when our students will enter productive society and will be looking for satisfactory jobs. They will face entirely new and largely unpredictable problem situations for which the traditional tools will be woefully inadequate. In such a situation there is only one thing we can do: Instead of teaching our students how to *use* mathematical tools, without concern as to where these tools came from and how they came into being, we must now teach them how to *make* their own mathematical tools and models for whatever problem situations the future may hold in store.

At first sight this seems an almost impossible task. A closer look at many experimental results discloses that it is not only possible, but is certainly not more difficult than the traditional task. The reason is this: Mathematics, past, present and future, has a certain basic scheme, something that mathematicians call the *structure of mathematics*, which permeates it in all of its phases and is active in it on all levels—kindergarten through graduate school—and which will continue to be there as long as men exist and make mathematics. This structure, which appears concretely as a rather small, finite set of *structural properties*, can be taught and must be taught from now on as the underlying and meaning-giving pattern of all mathematical instruction. It goes without saying that only a teacher who really knows how a certain piece of mathematics is structured can pass on this information to his students. In brief, instead of handing our students a set of ready-made tools for well-defined problem situations, we must now strive to give them a "kit" of basic mathematical elements, together with a rather simple set of composition rules, which will enable the learner to put together his own mathematical tools and models for whatever problem situation he may have to face. In a nutshell, while the traditional approach was focused primarily on how to *use* mathematics, the new approach is centered around how mathematics is being *made*.

Our discussion of the making of mathematics leads us into the realm of creative activity and creative imagination. As teachers, we know that children have and like to use creative imagination. They like to take things apart and then put them back together again. In the new approach to mathematics we make the most of this ability of theirs. Careful observation followed by individual exploration and self-discovery, creative challenge, and the atmosphere of true adventure are the hallmarks of the new teaching approach to mathematics. We have found that for such an approach to be successful it must be started early—kindergarten or first grade, whichever the case may be. This means we must start it when we begin the teaching of elementary arithmetic.

Algorisms

Arithmetic is and always has been a collection of algorisms or computational schemes, each aimed at finding or writing, with a minimum of expended time and effort, the most convenient numeral for a number sought as the answer to a problem situation. Some of these algorisms are relatively simple; for example, how to find the most suitable numeral for the sum of 3 and 5. Other algorisms, such as written multiplication or long division of whole numbers, not to mention the division algorisms for fractional and decimal numbers, are much more difficult to learn and remember. Moreover, let us face it, most of these algorisms and calculating techniques are far from inspiring and challenging. Rote learning and the drudgery of memorization seem inevitable and lead to frustration and boredom, the latter being the most dreadful enemy of any inspired learning.

In the past, we have tried desperately to motivate the learning of these algorisms by pointing to their undisputed social and technical usefulness. Now that this usefulness has itself become highly problematic, we have to replace it by another and, indeed, stronger motivating agent. This agent makes the learning of mathematics on all levels almost completely self-motivating. Algorisms do not just happen. They are essentially nothing but short, abbreviated, and therefore frequently obscure, arrangements of numerals duplicating the fundamental operations and relations that occur between numbers according to the structural properties of arithmetic. As such, they are concerned with numerals rather than with numbers, with convenient writing arrangements rather than with meaning and understanding. In contrast to this approach, our updated presentation is aimed first at bringing to light all of the underlying structural properties which alone bring meaning and understanding into mathematics, and second at showing how certain mechanical algorisms will produce the desired results without going through the whole long but logical chain of thinking steps every time. Let me give you just a few typical examples from elementary arithmetic.

Example 1. $7 \times 17 = \square$

Algorism:

$$\begin{array}{r} 17 \\ \underline{\times 7} \\ 119 \end{array}$$

By the Structural Properties:

$$\begin{aligned} 7 \times 17 &= 7 \times (10+7) \\ &= (7 \times 10) + (7 \times 7) \\ &= 70 \quad + 49 \\ &= 70 \quad + (30+19) \\ &= (70+30) + 19 \\ &= 100 \quad + 19 \\ &= \quad 119 \end{aligned}$$

Or take this long division of whole numbers:

Example 2. $1491 \div 7 = \square$

Algorism:

$$\begin{array}{r} 213 \\ 7\overline{)1491} \\ \underline{14} \\ 9 \\ \underline{7} \\ 21 \\ \underline{21} \end{array}$$

By the Structural Properties:

$$\begin{array}{r|r} 7\overline{)1491} & \\ \underline{700} & 100 \\ 791 & \\ \underline{700} & 100 \\ 91 & \\ \underline{70} & 10 \\ 21 & \\ \underline{21} & \underline{3} \\ 00 & 213 \end{array}$$

Now let us take a quick look at a division involving two fractional numbers

Example 3. $\frac{3}{4} \div \frac{5}{6} = \square$

Algorism:	By the Structural Properties:
$\frac{3}{4} \div \frac{5}{6}$	$\frac{3}{4} \div \frac{5}{6} = \square$
$\frac{3}{\cancel{4}_{2}} \times \frac{\cancel{6}^{3}}{5} =$	$\frac{9}{12} \div \frac{10}{12} =$
$\frac{9}{10}$	$\frac{9 \div 10}{12 \div 12} =$
	$\frac{9 \div 10}{1} = 9 \div 10 \text{ or } \frac{9}{10}$

Of course, the algorism in each case is faster and more convenient; it was designed to be so. We do wish to teach the algorism, but only *after* we have led our students, using the method of self-discovery to the greatest possible extent, step by step through the background of meaning which makes the algorism possible. No algorism is understandable as such—it can be understood only by deriving it from the structural properties of arithmetic.

You will begin to see that this sort of teaching does not really require the learning of *new* mathematics. Instead, there needs to be a fresh and somewhat deeper look at the traditional algorisms and computing techniques, at many of the so-called "basic facts", to give them real meaning and purpose. Then and only then do they become truly useful and applicable. Only then will our students learn them in a truly meaningful manner.

The New Text Materials

You are aware of the fact that new and modified text materials have been written. Not only this, but they have been tried with very satisfactory results. The children can learn them and learn them considerably faster than the disconnected bits in the traditional approach. What is more, they enjoy working with them.

The children are tremendously enthusiastic. Not so their teachers. The teachers not only have to learn new materials; they have to unlearn and relearn. Relearning is acknowledged as much more difficult than learning from scratch. In most of our 50 States, elementary teachers are required to take 3 hours of "methods of teaching arithmetic" for certification. Those 3 hours are the only ones they receive in college as preparation for teaching mathematics. Some States require an additional 3 to 6 hours of mathematics, but these consist largely of traditional college freshman courses in elementary algebra, trigonometry, or plane geometry. Little if any of this material proves helpful to them in their elementary teaching. As a result, many elementary teachers have developed their own techniques and teaching approaches, frequently based upon incorrect mathematics. Some of these teachers are now uneasy about any change or revision. Frankly, I cannot say that I blame them.

Does this mean that as a Nation we must wait until all teacher-training institutions have produced adequate preparatory programs for elementary and secondary mathematics teachers? Or, to put it differently, that for the next 10 years or so we will simply have to get by with the traditional curriculum and approach? Not only would this be most unfair to the younger generation in our classroom, but it would endanger us as a Nation, endanger us beyond repair. We are engaged in a life-and-death struggle on a worldwide scale. The true battlefields of this struggle are not geographic locations but the minds of our youngsters. From this viewpoint, we are simply facing a basic survival question and may as well make up our minds now that working toward an updated mathematics curriculum no longer admits the easy-way-out attitude—"let's wait a while and see where all this is really going before making the wrong move . . . ." If we take such an attitude, it may develop that there is no time left for making the "right" move. Let us have the courage to experiment, a kind of courage which this Nation has always had in the past when facing a crisis. We do face a crisis today, the like of which we have never before experienced. We simply must live up to it.

There is no reason why local successes cannot be repeated on a nationwide basis. What has worked for the Greater Cleveland Area could work just as well (with proper modifications, of course) for other suburban districts, for cities all over the country, and for our rural school districts.

Useful and mathematically sound materials and texts are now available. Our youngsters are capable and ready to go, but our teachers have yet to master the new materials and techniques. So our precise problem is how to bridge the gap or, rather, how to shorten the time-lag which of necessity exists between knowing what we must do and actually doing it effectively.

Four Problem Areas

From my experiences in the Greater Cleveland Mathematics Program (which involves some 120,000 students and their 4,000 teachers and which in addition to our own text and inservice materials uses SMSG and UICSM materials), I have come to the conclusion that this lag can be shortened very considerably by concentrating one's efforts upon four principal problem areas. Of course, it takes people to do it. As State supervisors you are strategically situated for doing it. The four problem areas on which you must go to work immediately— and I mean now, this summer— are the following:

1. You must set out to create a favorable climate of opinion among your school public, parents, and community; and you must do it with the full cooperation of your school administration.
2. You must make available new texts and teaching materials, including magazines, journals, and research findings, and begin to think about a text-selection committee capable of working in accordance with predominant national trends and local needs.
3. You must lay the groundwork for a well-organized inservice and preservice education program for all of your teachers, K through 12.
4. You must begin to organize a permanent resource structure, based upon certain key personnel and capable of reaching every classroom and every school of your system.

Let me elaborate each of these four points.

A Favorable Climate

Creating a favorable climate of opinion among the school public at large is a fundamental point; it requires the organization of a large-scale public relations program. In the first year of the GCMP we made the almost fatal mistake of not informing our public as to what we were trying to do and why we were doing it. Of course, children did take work home from school and perhaps asked their parents for help. Many of their parents were completely floored when they saw problems and exercises that seemed to have little if any resemblance to what they remembered having done in school long ago. Rather than admitting to their children that times have changed and that youngsters in 1961 must learn about things that simply did not exist 25 or 30 years ago, they took a conservative and even hostile attitude toward the revised program along the lines of the ancient fallacy: that what was good enough for grandpa in his time should certainly be good enough for little Johnny today. In a free and democratic society, where we do not have and do not wish to have revisions dictated by the pen of one man or by a small group in absolute command, our only hope to meet the challenge which we are facing is a completely informed public.

In our second year of the GCMP we gave general presentations of our program, as well as detailed subject-matter lectures, to over 200 PTA groups and talked to many civic and professional organizations, soliciting in each case their moral support. Once we had put our cards on the table, we always obtained their wholehearted support.

We did succeed in creating a climate of opinion favorable to our new program and in bringing about an atmosphere wherein fathers and mothers worked along with their youngsters in an effort to keep pace.

The PTA in some communities occasionally puts on a mathematics session for interested parents instead of a pot-luck supper. These sessions were always well attended, there was great eagerness to learn, and much excellent homework was done. As a result of this effort in public relations the students received at home the moral support and the encouragement without which the best teacher's finest efforts must remain spotty and incomplete. It developed that three 2-hour sessions were generally sufficient to bring the parent of an elementary child up to date, so that he could not only understand what the child was supposed to do, but also tender a helping hand when needed. Junior high school parents required five sessions. Thus far, we have had no experiences with high school parents, but should have some by the end of next year.

There is yet another important aspect to a well-informed and favorable public opinion. A new program such as ours of necessity creates certain additional expenditures. When our patrons are made to see that a topnotch, up-to-date educational program is not only the greatest thing they can provide for their youngsters, but also one of the key factors in our present worldwide struggle for survival, they will adopt quite a different attitude toward higher taxes for educational purposes than the one that many of them now profess to have. But it is essential to offer them a real chance to see and understand these needs in concrete detail.

Much of the financial support for the GCMP comes voluntarily from Cleveland business and industry, both small and large. The leaders in this important area of our national life know quite well that their own strength rests upon the strength of the Nation, and that our Nation's real strength lies in the minds and potential abilities of our younger generation and in the degree to which teachers can develop them. It speaks well for the farsightedness and dedication of American business and industry that they are willing and eager to support educational programs to the extent that they do. Each State, each district, each city has its own particular business and industry. What has been done in Cleveland can be done elsewhere. Make your problems and needs known to business and industry in detail; convince them of the urgency of up-to-date educational programs. I am sure they will help.

Of course, such a public-relations program is possible only with the full support and active cooperation of State and local school administrations. In their annual conventions the school administrators of this Nation have been given the basic facts and needs as far as known. They do not need to be convinced all over again. They know the time has come when something must be done, and that their active help and support are needed on all levels. The Greater Cleveland Mathematics Program (GCMP) came into being through the earnest concern and interest of 21 suburban superintendents, all of whom had put a revised mathematics program with top priority on a list of 12 urgent school problems.

It is the superintendent who in any given case must decide whether a certain limited amount of money is to be spent on new band uniforms, new physics laboratory equipment, or a new mathematics program. We know that our Nation is wealthy enough for our schools to have all three, but if such a decision has to be made in certain cases, the superintendent is the man who makes it. If he is not convinced already that a topnotch, up-to-date, instructional program is the primary objective of any school system, he must be made to see it now. For as the superintendent goes, so go the principals; and as the principals go, so go the classroom teachers. This, then, is the area in which you must start. The U.S. Office of Education here in Washington has enough printed information available to build a good, solid case for the cause of more and better, but primarily better, school mathematics. I can even think of several cases in which an enlightened public has forced a hesitant school administrator to give full support to a new instructional program.

I have discussed public relations and the creation of a favorable climate of opinion because without such a favorable climate, program revision becomes extremely difficult and controversial. However, I do not mean that one must take a whole year or so to create such a climate of opinion before starting a revised mathematics program. The two must simply go hand in hand.

A Representative Selection of New Materials

I want to emphasize the importance of having a representative selection of new materials available for study. There are many mathematics study groups, so many that I can list here only a few of the ones which strongly indicate an overall national trend. Among these are the following:

SMSG with materials for grades 4 through 12 and some enrichment topics.

UICSA with materials for grades K through 6.

UICSM with materials for grades 9 through 12.

The Madison Project at Syracuse University with materials for upper elementary and junior high school classes.

The former Ball State Experiment in Mathematics with materials mainly for grades 8 through 11.

The GCMP with materials for grades K–6 and special topics for junior high school classes.

Although there are differences among the materials they all agree on a sound structural approach to mathematics.

Two guides will be eminently helpful to you on curriculum materials. The first of these is a small publication entitled *Studies in Mathematics Education, Summer 1960*. Its subtitle is "A Brief Survey of Improvement Programs for School Mathematics." As the titles indicate, you will find a rather complete list of the foremost mathematics study groups and improvement programs, together with detailed grade-level indications and the names of the respective directors. You will also find the addresses of these groups, so that you can write for sample materials or be placed on their permanent mailing lists.

The other guide is the National Council of Teachers of Mathematics with its main office right here in Washington.[1] The Council is a most reliable source of information about the new curriculum, content, and methods, and the rapidly developing national trends. Its two Journals, *The Arithmetic Teacher* and *The Mathematics Teacher*, bring a wealth of pertinent information to all mathematics teachers, enabling them to a large extent to bring themselves up to date and remain up to date. The Council's yearbooks are veritable treasure books of good mathematics and teaching approaches.

Many of the men and women who served on the original writing teams of experimental groups such as SMSG are now released and are under contract to commercial publishers of mathematical textbook series. Many publishers are now engaged in producing a new and updated series containing the basic principles and approaches which have proved most successful in the experimental programs. Some of these new series are available, others will be on the market before fall, and all will be available by fall 1962. Some will go all out in the new direction and others will strike a reasonable compromise between old and new. Still others will hardly be more than revisions and modifications of the previous series because the publishers are waiting to prepare a completely new edition until a more definite trend has become evident. Soon you will have a wide range of texts from which to choose. Search now for good people to be on selection committees.

You should keep track also of Programmed Learning. No matter what you may hear about such programs, get all available materials

[1] 1201 16th St. N.W., Washington 6, D.C.

and make an objective study of them. There are already certain aspects of programmed learning—and I do not necessarily refer to the so-called "teaching machines" but to books—which are sure to stay. For example, documented reports show that slow and poor learners apparently can benefit immensely from such an approach. You must keep informed in this area.

Make sure that enough pertinent information is available to all of your teachers for survey and study. There are still many teachers in this country who ask with regard to SMSG, "What does that mean?"

Inservice Education

I wish to state at the very outset that you cannot expect any of your teachers to be able to handle new or modern text materials, experimental or commercial, without thorough inservice education. All attempts have ended in dismal failure when such materials were simply placed into the hands of some unsuspecting teachers with the benign words: "Here, take these and look them over during the summer. They are the new materials you are going to teach next fall." The more obvious reasons for failure, such as being unprepared and being steeped in old-fashioned attitudes, have already been mentioned.

You know that the great majority of your teachers did not receive an adequate preparation to present mathematics in the light of the new approach. Not only is it unfair to the teachers to demand the impossible from them, but it is outright detrimental to their students who will only be confused by what their teachers do not thoroughly understand themselves. Never make this fatal mistake, no matter how great the pressure for a new program may be. Summer institutes are fine and have already given a tremendous uplift to the national situation. But do not expect too much from one such institute. Twenty years of a certain teaching routine cannot be changed in four to eight weeks. This takes much more time and patience than are available in such concentrated sessions as summer institutes. As a matter of fact, I would recommend that you urge into institutes only those of your teachers who have already had some inservice education within the system.

Our experience in the GCMP tends to indicate that one year (9 months) of inservice education, topped off with a good workshop or a National Science Foundation Institute, is adequate. For this sort of inservice program I would suggest one weekly 90-minute session which should be half lecture-demonstration and half work. No homework should be associated with inservice education, except in very special cases. These weekly sessions should be held in a locality with proper classroom facilities, and if possible projection equipment should be provided. Every third session might consist of movies, film-strips or slide materials, followed by a discussion period. Whenever possible, a special lecturer or demonstrator should come to address the group. No attending teacher should have to drive more than one hour to reach the class. In the GCMP, the average driving time was 19 minutes.

The obvious problem that now arises is how to staff such inservice education programs. The most sensible procedure is to select one of the larger cities in your State having considerable business and industry and at least one college or junior college in the vicinity. From one such city you can then spread out to cities of similar structure and eventually to smaller places. After that, you can tackle the rural areas, since by then you will have personnel available, recruited from previous programs. Some of your main staff ought to come from the State Department of Education. Several States—for example, Connecticut—have a staff of visiting experts operating out of the central office; results appear to be very satisfactory.

The help of nearby colleges and universities should be enlisted. In the mathematics departments of most colleges and universities a great concern exists to improve mathematics teaching in the public schools. These departments, when properly approached, should be an almost ideal source for additional staff. As I said earlier, this will require funds for compensating these people. Without money no inservice plan will ever work. There are also a considerable number of traveling lecturers provided by Mathematics Associations and the various local Councils of Mathematics. In Ohio, for example, our Council provides regular traveling workshops anywhere in the State for nothing more than traveling expenses.

Good outside personnel are essential to get a program started, but once under way, your most important source of manpower comes from the classrooms in your own schools. Almost every building has at least one teacher who is particularly interested and well prepared in mathematics on some given level. You may not know this person now, but you can certainly locate him by consulting local supervisors and principals. You will want to seek such teachers for special training as group and discussion leaders. They must, however, be fully recognized by their respective school administrators and be furnished compensation and released time. This is a most important point. Although it is quite true that most of our teachers are willing and even eager to improve themselves, it is also true that most of them are busy people outside of school, with community and family obligations.

Go out and find a small number of basic key people, keeping in mind that, for a start, enthusiasm and attitude count much more than subject-matter mastery. Without such people you will never get off the ground; with them you can go as far as you wish. Frequently you will find an excellent junior high school mathematics teacher who would make a good discussion leader and/or lecturer for an upper elementary group. Similarly, some really interested senior high school teacher may work with a junior high group. In reality our schools are full of resource teachers. They must be found, tapped, and put to work. They must be recognized by all as leaders in their field. There will be some gripes and envy, but where do we have situations without these human features?

I would begin the first year of the inservice program by deciding on levels of instruction and training. In the GCMP during the first year we grouped the teachers by grades. We could do this since the urban concentration enabled us to have lecture and work sessions of up to 400 teachers at a given hour. In most cases, a more realistic division would consist of four groups: K–3, 4–6, 7–9, and 10–12. There is some evidence that these four groups are adequate. I always want my teachers to be familiar at least with the mathematics that precedes and with that which follows their own grade level. However, the first two or three meetings should be joint sessions addressed by one or two topnotch people in modern mathematics education. A good enthusiastic start is half the enterprise. These people should give encouragement to your teachers, alleviate their fears and dislikes and, in general, do these two things: stimulate a good deal of enthusiasm and outline the program as a whole. Another opening speaker should talk on the *why*, the *what*, and the *how* of the revised program.

You will then have to decide upon one set of materials to be studied during the year. In my considered opinion, it would not make too much difference which set of materials you use, as long as you stay safely with one of the top national programs. The essential structural features of mathematics which, in turn, largely determine the most suitable teaching approach, are the same in all of them.

Once started, you or the local supervisor will recognize certain teachers who will stand out through their attitudes and individual contributions. Keep close track of them; they will become the key people in the spreading organization. If possible, obtain a brief report from each group about once a month or have such reports forwarded to you through the local supervisors. Get all possible help from the local superintendent, curricular experts, counselors, and principals. We noticed soon after we had started that our best coordinators were elementary principals.

The very first goal of any inservice program is to dispel fears and apprehensions in your teachers. Assure them that they do not really have to relearn all of mathematics or arithmetic. Convince them with concrete examples that what counts most is a change in attitude toward the subject—a new look, a different perspective. Go very slowly at first, and make sure that each group sets its own pace. It is not important how much ground is covered, but how deeply one looks into every bit of material.

A group should take just one topic at a time, always from the integrated viewpoint of subject matter and teaching approach. When possible, have specially gifted or interested teachers give classroom demonstrations using nonselected groups of children. Also urge your guest speakers and visiting lecturers to use live classroom demonstrations. Frequently one 50-minute demonstration makes a certain point better than a 3-hour inspired talk. Throughout, emphasize the underlying structural properties over any computational details, encourage the discovery method as opposed to the lecture-demonstration approach, concentrate on developing thinking ability in students rather than memorization. You can help the various groups with a mimeographed periodic circular describing how to make a certain teaching device or telling about a certain successful enrichment topic suitable for a particular age group of students.

Throughout this first inservice period, no teacher should take any half-digested new topics into the classroom; warn all teachers repeatedly against this tempting action. On the other hand, certain teaching approaches or discovery methods may be applicable immediately to a whole variety of topics, including certain good puzzle problems and arithmetical games.

All elementary teachers, K through 8, should master the structural properties of the arithmetic of whole numbers as listed below:

Addition—

- Is commutative.
- Is associative.
- Has a left and right identity element (0).
- Has an inverse (subtraction).

Multiplication—

- Is commutative.
- Is associative.
- Has a left and right identity element (1).
- Has an inverse (division).
- Is left and right distributive over addition.

The set of whole numbers—

Is closed under addition and multiplication.

Of course, this is not the language to use with children in the lower elementary grades. For those children, it is the *spirit* of the structural properties which must guide the modern teaching approach.

In addition, all teachers must acquire some degree of familiarity with the simple language of sets and set relations. All teachers will have to understand that in arithmetic there exists fundamentally only one operation, addition. Subtraction is taught as the inverse of addition; multiplication is presented as "repeated addition of the same number"; division is made plausible first as "repeated subtraction of the same number" and is then solidified as the inverse operation of multiplication. All algorisms pertaining to these operations are motivated in terms of the structural properties from which they are derived. This is taught by the discovery method.

Continuous Readiness of the Basic Structure

Although it is true that such an inservice program as described in this paper will be needed only until you can hire newly certified young teachers who are thoroughly familiar with all phases of modern elementary mathematics, there are three points which make it desirable to maintain the basic organizational structure of the program in continuous readiness.

First, the same basic structure with certain simple modifications can be used also for a revised program in a different subject matter area, for example, a revised language curriculum with a structural grammar approach (of which there is much talk in well informed circles); or for a revised social studies curriculum; or for other programs in sciences or foreign languages.

The extension of the number sets from the set of the whole numbers to that of the integers and then to the set of the rational numbers is a minor problem once the same basic structural properties have been taught and understood. Further work in arithmetic should relate these 10 structural properties to real life and should use them continuously once they have been introduced. Many movies and filmstrips are now available which feature good presentations of the structural properties. As much as possible use these for your inservice education programs.

As a matter of fact, your inservice program, once started, will soon begin to propagate itself with only an occasional gentle prodding here and there. From the operation of your very first inservice program you will get a list of names of people who will become the real key people in its further spread and development.

Second, many years will pass before we can hire *all* the well-prepared mathematics teachers that our Nation needs. My best guess is that this will *begin* to happen 5 or 6 years from now. Many of my colleagues, however, speak in terms of 8 or even 10 years.

Third, since our society and its needs are changing so fast, to keep ourselves informed and up to date will become more and more a problem all by itself. Even if the original inservice groups graduate to the level of study groups or journal-reading seminars, some definite measure will have to be taken to make certain that in the future we do not again fall as far behind the times as we have now done. Thus, some form of inservice education will almost likely become a permanently established feature of all modern school systems.

At the end of first year's inservice program, you should be able to compile a list of key teachers. If possible, there should be a teacher for every grade level in every school building. For the future, this teacher will be prepared to help any of the other teachers in the building, by answering their questions, by teaching a demonstration class for them, or by making available helpful printed materials and bibliographies. It is desirable that most of the commonly occurring teaching problems be handled within a building or at least within any one school district.

During the program's second year you should provide a monthly workshop session, by grade-level grouping, for all of these resource teachers or coordinators. In these sessions they discuss the problems that occur most frequently. They also are given more advanced training in difficult subject-matter areas and are brought up to date on recent developments in those areas. These people are urged to attend meetings of local councils and those of the State Council of Teachers of Mathematics. Their schools should pay their way. These people will play a decisive role in textbook selections, and their services will be sought for presentations and classroom demonstrations for PTA groups. In many cases they can function as mathematics consultants for their respective principals.

As far as preservice education is concerned, these recognized key people may again be called upon to introduce newcomers and substitute teachers to the program. If your inservice program can be scheduled for evening sessions, prospective teachers and even high school seniors who plan to make a career of teaching should be invited as regular guests.

A very small and highly selected group of your own teachers should be instrumental in organizing and staffing a preservice workshop just before the beginning of each school year. In most cases you can have an effective workshop using the services of just one supervisor and one outside speaker. The important details will be managed by your own experienced mathematics teachers, thus giving much inner strength to the school system.

I know that the organization of such an inservice program as I have discussed is a hard task, fraught with many difficulties that I have not even mentioned. But all of these difficulties can be met successfully when they arise. On the other hand, without such a program you cannot expect to achieve progress toward an updated mathematics curriculum. Do not be discouraged should your first attempt fall below expectations. Of one thing you can be certain: You will have a list of teachers who will be instrumental in providing a much more successful program the second year. This is what really counts. After all, the reputation of our profession depends upon good, capable, dedicated men and women. Once you have located them, your further problems will be minor as compared to those of the first year. Do not forget the many great and powerful organizations on call, ready to help you out of any difficulty you might encounter.

Programs of this type have been conducted and the experiences gathered in such pilot projects are now available. We may thus avoid repeating mistakes. There is really no reason why it cannot be done just as well and even better in many other places, and there are many good and compelling reasons why it should be done—*now*.

The Role of the State Supervisor in Encouraging Research and Implementing Research Findings

JOHN J. KINSELLA

RESEARCH spelled with a small "r" does not mean the same thing to every individual. At one extreme someone may want to find out how old Columbus was when he discovered America. At the other, an investigator in medical research may seek to discover whether one type of vaccine is better than another for treating a certain kind of polio.

Research in mathematics education includes a variety of activities. It may involve sophisticated statistical techniques for testing a hypothesis connected with a theory of wide application. It may consist of determining the evolution of a certain practice or of a set of topics. It may require careful, deductive analysis of some theory of instruction. It may demand close observation and recording of a few students' reactions to certain experiences over a limited period of time. It may call for a survey of present conditions as a basis for next steps.

Research spelled with a capital "R" is not an easy task in mathematics education. The problem of controlling or taking into account certain variables is extremely difficult. All teachers are not the same. The same teacher is not as constant from day to day; neither are the students. Neither is the interaction between teacher and student. A child in a group does not always behave the way he does in a conference with a teacher. It is no wonder that some observers insist that teaching is an art and that research is of little help.

The physical scientists have available extremely accurate and precise instruments of measurement. Our instruments for appraising understanding, attitudes, and interests are crude by comparison. Aside from the problem of scaling these instruments it turns out that the fields of investigation basic to the study of education, such as psychology and sociology, have not developed to the point where they can be of first-class help to us in appraising some of these types of learning.

In these preliminary remarks it has not been my intention to "cast a pall of gloom," to borrow a trite phrase. On the contrary, my purpose is to save you from expecting cure-alls from research in education. The findings of such research must be examined cautiously and tested further in a variety of situations. As an antidote to pessimism, the great improvement in the quality of research in education over the past 40 years is encouraging.

In the time remaining I propose to do four things: (1) Summarize some of the more strongly supported findings in mathematics education, (2) give you some of the tentative results of experiments with the new mathematics (and suggest some implications), (3) indicate what I think needs to be done to make research efforts more productive of more soundly based results, and (4) suggest (with your permission) how you can contribute to the enterprise.

Selected Research Findings

Problem solving, as applied to word statements of quantitative situations in arithmetic and algebra, has been the subject of repeated experimentation for at least the past 40 years. Investigators have tested a variety of methods for developing the ability to solve these "problems". What are the results? The answer is that special methods to teach such problem solving have no advantage over giving the student a variety of problems and telling him to solve them by any method he chooses.[1]

During much of this century preceding 1940, the teaching of arithmetic and algebra was characterized by great emphasis on developing skill in performance by a heavy drill program. After 1940 (and before the period of the new mathematics) increased attention was given to what has been called "meaning and understanding" and less attention to blind manipulation. During those years preceding the new mathematics, teaching was characterized by the following: great dependence on the place value of the decimal system, more time spent on developing concepts through experience inductively, more effort made to rationalize algorisms and the relations between algorisms, and greater provision made for learning through discovery. The results? The meaning and understanding classes did just about as well as others did in computation, retained longer

[1] Burch, R. L. An Evaluation of Analytic Testing in Arithmetic Problem Solving (Doctoral dissertation, Duke University, 1949).

Hanna, Paul. Arithmetic Problem Solving. (Doctoral dissertation, Teachers College, Columbia University, 1929.)

Washburne, C. W. and Osborne, R. Solving Arithmetic Problems. *Elementary School Journal*, 27: 219-28 and 296-304, November and December 1926.

what they learned, and were able to transfer their learning to new problems more successfully.[2]

Does demonstrative plane geometry as traditionally taught lead to a gain in the ability to reason more critically in nonmathematical situations? No. Students of comparable mental ability improved just as much as the geometry students even though they didn't study mathematics or logic. Can geometry be taught to improve such critical thinking without serious loss in geometric knowledge? Yes.[3] Should geometry be somewhat deemphasized so as to achieve this general education aim? Research cannot answer that question. The answer demands a value judgment. The new programs have made such a judgment. Their answer is "no".

Does a course in solid geometry involving proofs lead to a gain in space perception? No. By comparing ability-matched groups (some taking solid geometry and some not), researchers found in two large and carefully done experiments, that the gains in spatial perception made by the nonsolid-geometry groups exceeded the gains of the solid-geometry groups.[4] Is solid geometry a requirement in many colleges? No, except in a pitifully small number.

Do teaching aids, such as models, manipulative materials, films and filmstrips lead to the better learning of mathematics? The answer is unclear. It may be that the results depend on the teacher's skill

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[2] Brownell, W. A. *and* H. Moser. Meaningful vs. Mechanical Learning. Duke University, 1949.

Burkhard, Sarah. A Study of Concept Learning in Differential Calculus. Doctoral dissertation, Teachers College, Columbia University, 1956.

Cummins, Kenneth B. A Student Experience-Discovery Approach to the Teaching of the Calculus. Doctoral dissertation, The Ohio State University, Columbus, 1958.

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Swenson, E. Organization and Generalization as Factors in Learning. Learning Theory in School Situations. University of Minnesota Studies in Education, 1949.

Thiele, C. L. Contributions of Generalization to the Learning of Addition Facts. Doctoral dissertation, Teachers College, Columbia University, 1938.

Van Engen, H. *and* E. Glenadine Gibbs. General Mental Functions Associated with Division. Iowa State Teachers College, 1956.

[3] Fawcett, H. P. The Nature of Proof. 13th Yearbook of the National Council of Teachers of Mathematics, 1938.

Ulmer, G. Teaching Geometry to Cultivate Reflective Thinking. *Journal of Experimental Education*, 7: 18–25, September 1939.

[4] Brown, Francis R. The Effect of an Experimental Course in Geometry on Ability to Visualize in Three Dimensions. Doctoral dissertation, University of Illinois, 1954.

Ranucci, Ernest R. Effect of the Study of Solid Geometry on Certain Aspects of Space Perception Abilities. Doctoral dissertation, Teachers College, Columbia University, 1952.

in using these aids and his ability to make them an essential part of a learning program rather than an appendage or sometime thing.[5]

How effective is the use of TV in teaching mathematics? Most studies show that about the same amount of knowledge is acquired under TV and non-TV instruction. However, seldom is TV used alone. Help-sections and repeated showings are factors in some of the experiments. Whether the novelty effect is lasting has not yet been answered. The teacher still has an important role to play.[6]

To what extent can success in mathematics be predicted? The answer seems to be that the success of an individual can be predicted only in terms of a probability that increases with his score on the predictive instruments. The correlation coefficient between the predictor variables, such as previous school grades, marks in mathematics, intelligence, and aptitude tests seldom exceeds .75. The standard error of estimate is so large in this case that a regression equation is of little value for individual prediction.

What is the extent of elementary school teachers' preparation in mathematics? About one-quarter of the colleges require high school credit in mathematics for entrance. The amount varies between one and two units of high school mathematics. In college it would be rare for a student preparing to teach mathematics in the elementary school to have more than two semester hours in mathematics and two in methods of teaching mathematics.[7]

Do undergraduates in the curriculum for elementary school teachers fully understand the mathematics of the elementary school? The answer in nearly all research studies is "no."[8]

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[5] Anderson, G. R. Visual Tactual Devices: Their Efficiency in Teaching Area, Volume, and the Pythagorean Relationship to Eighth Grade Children. Doctoral dissertation, The Pennsylvania State University, 1957.

Johnson, D. A. An Experimental Study of the Effectiveness of Films and Filmstrips in Teaching Geometry. *Journal of Experimental Education*, 17: 363-372, March 1949.

Sole, David. The Use of Materials in the Teaching of Arithmetic. Doctoral dissertation, Teachers College, Columbia University, 1957.

[6] Elliott, H. Margaret. Teaching Freshman Mathematics by Television. *American Mathematical Monthly*, 65: 440-43, June-July 1958.

Jacobs, James N. *and* Joan K. Bollenbacher. Teaching Seventh-Grade Mathematics by TV. *The Mathematics Teacher*, 53: 543-47, November 1960.

Wells, David W. The Relative Effectiveness of Teaching First Year Algebra by Television-Correspondence Study and by Conventional Methods. Doctoral dissertation, University of Nebraska, 1960.

[7] Grossnickle, F. E. The Training of Teachers of Arithmetic. *The Teaching of Arithmetic, In* 50th Yearbook, Part II, National Society for the Study of Education. University of Chicago Press, 1951. p. 203-31.

[8] Glennon, V. J. A Study in Needed Redirection in the Preparation of Teachers of Arithmetic. *The Mathematics Teacher*, 42: 389-96, December 1949.

Orleans, J. S. The Understanding of Arithmetic Processes and Concepts Possessed by Teachers of Arithmetic. College of the City of New York, 1952. Summary in *Elementary School Journal*, 53: 501-12, May 1953.

What is the attitude of prospective elementary school teachers toward the teaching of mathematics? The alarming answer is that a large percent of them either dislike it or have a fear of it.[9]

What are the recommendations of the Mathematical Association of America for the mathematical preparation of elementary school teachers? The Association proposes four semesters of mathematics, including number systems, basic concepts of algebra, and informal geometry. Further, it would like to see about one-fifth of these teachers equipped with about six more semesters of mathematics.[10]

What is the state of preparation of secondary school mathematics teachers? The U.S. Office of Education cooperated with the States of Maryland, New Jersey, and Virginia in 1959 in an excellent study of this problem.[11] Covering about 800 teachers, the study found that although 7 percent of them had taken no college mathematics, they usually taught general mathematics. The mean number of semester hours of mathematics was 23. About 60 percent of the 800 had taken calculus or more advanced courses, but only 20 percent of this group had taken such work after 1950. Although two-thirds had taken a course in practice teaching, only one-half of that group had taken a course in methods of teaching mathematics. Of this one-half only 15 percent had taken such a course since 1950. About one-half of the teachers were graduates of liberal arts colleges; the others had done their work in teachers colleges or schools of education.

Similar studies need to be done in other States. It is doubtful that the results would present a more optimistic picture. One conclusion seems inescapable: If we are going to try to improve mathematics teaching in a State or in the Nation, we should first find out what has been the preparation of those who will be primarily responsible for the improvement. If the newest advanced mathematics a teacher has had is 10 years old, the probability is high that what is retained is only a small precent of what was originally learned. Perhaps, then, it is fortunate that only a small percent of our students will be ready for calculus in the 12th grade. An interesting question for a survey type of research is the nature of the population of secondary school teachers who have attended NSF institutes, the number

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[9] Dutton, W. H. Attitudes of Prospective Teachers toward Arithmetic. *Elementary School Journal*, 52: 84-90, October 1951.

O'Donnell, John R. Levels of Arithmetic Achievement, Attitudes toward Arithmetic, and Problem-Solving Behaviors Shown by Prospective Elementary Teachers. Doctoral dissertation, The Pennsylvania State University, 1956.

[10] Mathematical Association of America. Recommendations for the Training of Teachers of Mathematics. *The American Mathematical Monthly*, 68: 1-11, January 1961.

[11] Brown, Kenneth E. and Obourn, Ellsworth S. Qualifications and Teaching Loads of Mathematics and Science Teachers (Circular No. 575). U.S. Department of Health, Education and Welfare. Washington: U.S. Government Printing Office, 1959.

of institutes they have attended, and the nature of the mathematics they have studied.

A ray of hope lies in the possibility of self-teaching. Many of us in secondary schools and colleges have had the experience of getting ready to teach a course we never have taught. Very often a good place to begin is with the textbooks to be used by the students, especially if answers are available. This usually leads to the study of books on the same, or similar subjects, at a more advanced level. The second or third year of teaching a new course is usually the most enjoyable one.

In the case of the U.S. Office of Education study of the teachers in the three States it was found that the teaching load was heavy in terms of number of students, number of different preparations, and number of extra-curricular activities, aside from the usual school-record keeping. In such a situation a teacher would almost have to use his summer to get ready for a new course.

The situation in the elementary school as far as mathematics education is concerned is a critical one. More than one study has shown that after grades 3 or 4 children show a noticeable decrease in their liking for mathematics. It is certainly possible that during this period many children of ability come to detest mathematics. If by some magic the teachers of these grades could suddenly acquire more knowledge of mathematics and a greater liking for it, fewer capable children would decide that mathematizing is a cruel, boring experience. We should all keep in touch with Professor David Page of the University of Illinois and Professor Robert Davis of Syracuse University concerning this problem, as well as with Professor Fred Weaver and his group, who are close to the SMSG experiments at this level.

I have noted previously that attention to meaning and understanding without neglecting performance has led to promising results. Those of us who are acquainted with the text materials of UICSM, SMSG, and UMMaP know that they are characterized by appeals to understanding and learning through discovery. Neither the power to do nor the ability to think is neglected. On the basis of these criteria alone, the teaching results should be promising.

Experiments With the New Mathematics

Information about the results of experiments is of two types. The first consists of informal, one-teacher investigations with one or two classes. The second is descriptive of the preliminary results of teaching SMSG materials as reported by the Mathematics Section of the Minnesota National Laboratory in grades 6 to 12.

In one study, units on relations, number theory, sets, transformations, and semigroups were taught to eight high school seniors. Except for the last two units, the average grade on the home-made tests was close to 90 percent.[12]

Professor Robert Davis of Syracuse University and others working in the central New York State area obtained evidence that children in grades 3 and 7 could learn some of the basic ideas of algebra, such as variable, equation, and directed number.[13]

In a Midwest class, units on set theory, Venn diagrams, and the graphing of inequalities were taught to an average 10th-grade class for 6 weeks. Scores on tests of knowledge and attitude indicated that the experiment was a success.[14]

Another investigator tried to find out the lowest grade at which average and superior students could master proofs in demonstrative geometry. His conclusion was that demonstrative geometry can be introduced as early as grade 7 with a reasonable degree of success.[15]

In the Minnesota experience four average 6th-grade classes used the SMSG 7th-grade materials. Two of the four teachers had never taught the material before. The Sequential Test of Educational Progress in Mathematics was given at the beginning and end of the experiment. Students of all ability levels profited from the course. At the end, almost half of these 6th-grade students scored above the national median for 8th-grade students.[16]

In another Minnesota comparison 13 teachers each taught one 7th-grade class, using SMSG material; and each taught another class, using a conventional text. In terms of gains on the STEP test the SMSG classes surpassed the conventional groups by an amount that was significant at the 1-percent level.[16]

One more illustration may suffice. Six 9th-grade classes used the SMSG 9th-grade booklets. On the STEP test they did about as well as 11th-grade students.[16]

Other Minnesota reports, showing similar results, dealt with the SMSG programs for grades 8, 10, 11, and 12.[16] If it can be assumed that the STEP tests are valid and reliable measures of improvement in mathematical understanding, these results are remarkable. The

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[12] Byrkit, Donald R. Sets and Number Theory in the High School. Master's thesis, Illinois State Normal University, 1958.

[13] Davis, Robert B. Mathematics for Younger Children—The Present Status of the Madison Project. *New York State Mathematics Teachers Journal*, 10: 75–79, April 1960.

[14] Roughead, William G. An Experiment in Tenth-Grade Modern Mathematics. Master's thesis, Illinois State Normal University, 1958.

[15] Corley, G. S. An Experiment in Readiness for Logical Thinking and Demonstrative Geometry: Doctoral dissertation, George Peabody College, 1959.

[16] Minnesota National Laboratory. Preliminary Evaluation of SMSG Texts and Courses. St. Paul; The Laboratory, 1960.

only other possible question would be whether the teachers and students were typical.

It seems likely on the basis of these embryonic reports that some mathematical concepts can be taught much earlier than is customary, and that some of the new mathematics can be learned by a high percentage of the students.

Wise Use of Research Energy

In general, my suggestions for the wise use of research energy are to give attention to important problems rather than trivial ones, to up-grade the quality of research designs and procedures, and to improve the organization and administration of research activities.

The important thing is primarily a matter of value judgments, rather than a revelation from research. Essentially, we want more students to become more interested and more successful in learning more important mathematics, not only for the welfare of the Nation, but also for the benefit of the individual. Teams of mathematicians and classroom teachers have made judgments as to the mathematics that is important and capable of being learned at various chronological and mental age levels. What important research, then, is still needed?

We need much more experimentation in more States, using the new materials with students of different levels of ability and with teachers of diverse characteristics. Should certain topics be dropped? Should more attention be given to some topics and less to others? Should some topics be delayed and others introduced earlier?

What are the qualifications and teaching loads of mathematics teachers in a given State and in the individual school systems within a State? I have mentioned this one before. The listing for each teacher of the names of courses taken x years ago will not answer this question. What is the mathematical readiness of the teacher for guiding the learning of the new mathematics right now, in 1961? What are the State and local needs for teachers to carry out this function?

What is the most effective multi-attack program for developing in teachers readiness to teach the new mathematics? What combinations of inservice programs, consultants, study groups, correspondence courses, summer institutes, and television programs are most effective?

Moderate acceleration, homogeneous grouping accompanied by variations in the programs from one group to the other, and various kinds of enrichment have usually been effective with superior students, especially in large schools. What are the most effective combinations of procedures for superior students in small schools?

For too long, perhaps because of the nature of mathematics, we have paid insufficient attention to how students feel about mathematizing, what their attitudes are, and what factors bring about an enduring interest in mathematics. A teacher may be extremely competent in mathematics and technically superior in pedagogies but still leave his students cold about mathematical activity. What are the characteristics of those teachers who seem effective in changing students' attitudes and interests? To what extent can less gifted teachers be guided to greater effectiveness in these respects?

These are a few of the problems I think important enough to be given more attention through research. Of course, there are others, but these seem closer than others to our needs in the sixties and seventies.

I do not believe this is the time and place to say much about improving the quality of research designs and procedures as a means for making wise use of research energy. This matter has been treated extensively by others in various places. From my experience in examining hundreds of research studies in mathematics education, I will make a few brief suggestions.

The methods and treatments used should be clearly defined. For example, just what is meant by the "traditional method," in contrast with some experimental method, is open to wide interpretation.

If statistical tests of significance are to be used, some form of randomness has to be applied, since these tests are based on probability theory. This statement applies to experiments involving nonparametric as well as parametric statistics. It is much too common in educational experiments to meet the assumption that "if there is no reason for believing that the sample is not random, we will assume that it is random." Another gem is the statement that "the results of this experiment apply not only to the school in which it was performed but to all similar schools." Still another is the assumption that representativeness implies randomness of the sample.

Two other practices need to be changed. One is the use of sensitive high-powered statistical methods on measures of very low reliability. This is akin to reading newspaper headlines with a microscope. A second is the assumption that if a difference is statistically significant, then it is important or substantial. Statistical significance is confused with educational significance.

The organization and administration of research are factors in the wise use of research energy.

Can you imagine the medical profession accepting a treatment for a certain disease because it produced promising results in one experiment with 50 patients conveniently located in one medical center? Such experiments must be repeated, or replicated, many times before much credence is given to the results. On the other hand, repetition of encouraging experiments is a rare event in many areas of educational research, including mathematical education. We must do more of it.

Repetition serves purposes other than increasing confidence in the findings. It may make possible their broader application. Just because an experiment involving one teacher in one school in one town with a special group of students reveals certain significant changes in mathematical learning, it does not follow that experiments involving different teachers in different schools in different communities with different groups of students will produce similar results. Such repetition, however, may lead to a more precise description of the conditions under which certain results are likely to be obtained. This is an important conclusion. A good research program provides for repetition of experiments.

Repetition of experiments which are conducted for too short a time is wasteful. In the administration of many experiments more attention must be given to the duration. I know of experiments conducted in terms of hours and days, in contrast with others requiring weeks, months, and years. It is obvious that many kinds of human learning develop slowly, analogous to the growth of some trees. Very often one method or treatment seems to have some advantage over another (but not a significant one) during the time given to the experiment. My guess is that many of these experiments might have revealed significant results had the period of the experiment been longer. Can schools in some States be found which are interested and able to conduct such experiments for longer periods?

Do you recall the Eight-Year Study of the Thirty Schools? Wouldn't it have been absurd to attempt to draw conclusions at the end of one year? It would be just as unfair to demand that SMSG teaching prove itself in one year. Of course, I realize that delaying evaluation is a tactic that has been used by some vested interests at some time in order to protect a prejudgment. Research is rarely welcomed by those who want to preserve the status quo of a program rather than improve it.

Repeating experiments and extending the duration of some of them are necessary but not sufficient for well-organized research. The fruits might be isolated and the results unrelated. We need more cooperative research projects. By this term I mean more than coordination of the work of several individuals on one isolated problem. I mean a research attack on a problem area with responsibility for sub-problems taken over by sub-sets of the entire set of workers, followed by coordination and interrelating of the findings. This operation is comparable to that of a military campaign, in contrast with a limited assault on a small, specific objective. At present, most research in mathematics education is of this narrow sort. Is it possible to use cooperative research in testing the new curriculum proposals? In seeking the most effective inservice education for teachers? In investigating ways of improving the programs for superior students?

Responsibilities of the State Supervisor

Do you know where to find reports of the latest research on mathematics education? Do you have ways of communicating the gist of these to school administrators and mathematics teachers? Have you considered putting into State bulletins and journals of State mathematics teachers associations a section entitled "Did You Know?" or "Have You Tried This?"

Do you urge administrators and teachers to write to you about research they are conducting? Do you publicize these efforts? Are you in a position to provide some consultant help in planning and conducting research below the State level?

When visiting schools, do you try to find instances where new methods are being tried? Do you encourage regional and State conferences to use reports on research? Do you encourage teachers to teach the new content, using films and television? Can the State in any way help local schools free a teacher enough so that he can conduct some promising bit of research?

These are merely samples of the questions that bear on the problem of communicating research findings and encouraging research. No doubt you can add to them as well as subtract from them. My principal hope is that the sum of the directed numbers will be positive and greater than what it was before you paid this visit to Washington.

Appendixes

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Appendix A. Recommended Leadership Activities for State Supervisors of Mathematics

THE LEADERSHIP ACTIVITIES agreed upon by the conferees to achieve the four stated objectives of the conference (see foreward) appear in this section as a list of over 50 items. These items have been classified under public relations, information for educational personnel, research in mathematics education, curriculum planning and development, preservice and inservice education of teachers, new media of instruction, and evaluation.

Although the recommended activities are for the most part generally applicable, supervisors of mathematics in some States, because of local conditions and legislative requirements, may find it necessary to omit or modify some of them.

The recommendations follow:

Public Relations

The State Supervisor of Mathematics should—

- Inform the lay public of present trends in mathematics education from kindergarten to grade 12.
- Spell out, if necessary, the implications of new programs for any given school community.
- Encourage local administrators and teachers to inform parents and the lay public of the changing curriculum.
- Establish rapport with mathematicians in the State, region and/or Nation.
- Work with State and local mathematical associations to provide information regarding current developments in mathematics.
- Make use of such media as newspapers, radio and television programs, addresses to local groups, newsletters, local and State professional organizations, workshops, meetings, and conferences to help disseminate information.

Information for Educational Personnel

The State Supervisor of Mathematics should—

- Inform administrators and classroom teachers periodically of present trends in mathematics education, K-12.
- Inform colleagues in other fields of the merits of current programs in mathematics and solicit their active cooperation.

- Advise school administrators concerning the importance of updating and improving the mathematics curriculum.
- Disseminate information to the schools regarding teacher participation in mathematics institutes.
- Provide information on the use of new instructional media.
- Make available for teachers new texts, teaching materials, journals, and research findings.
- Provide bibliographies; urge that every school establish a mathematics library and foster its use by teachers and students.
- Encourage local schools to purchase, and teachers to use, appropriate current publications, including professional mathematics journals for teachers.
- Seek the aid of the U.S. Office of Education as a clearinghouse for the exchange of current publications which might aid the supervisor in strengthening instruction in his own State.

Research in Mathematics Education

The State Supervisor of Mathematics should

- Collect, analyze, interpret, and disseminate past and present research findings in mathematics education.
- Encourage research within the State and provide help to initiate, plan, and conduct it.
- Share pertinent research information with his colleagues, other State Departments of Education, and the U.S. Office of Education.
- Encourage carefully controlled research on the effect of new instructional media on the teaching of mathematics.

Curriculum Planning and Development

The State Supervisor of Mathematics should

- Encourage the establishment of a continuing, widely representative advisory committee on mathematics.
- Assist actively in curriculum revision by participating, for example, as a consultant or member on a steering and/or production committee.
- Help administrators establish new mathematics programs on a sound basis.
- Encourage local school units to construct their own mathematics curriculum guides.
- Encourage curriculum committees to choose realistic, workable and consistent objectives; and assist in evaluating results in terms of the objectives.
- Initiate, assist, coordinate and cooperate in the production of publications relative to mathematics curriculum planning.
- Stimulate interest in better curricular programs by arranging classroom teaching demonstrations.

- Seek and compile for distribution constructive comments and suggestions from teachers who are working with experimental programs in their classrooms.
- Examine the new programs in mathematics to find common areas of agreement and/or basic differences in philosophy, mathematical structure, and teaching procedures.
- Help teachers become acquainted with the mathematical content of the evolving new programs.
- Bring in specialists, including research mathematicians, to aid in curriculum planning, development of curriculum guides, and inservice education. (State and/or local school districts should supply sufficient funds for this purpose.)
- Assume responsibility for evaluating current instructional materials on the basis of: (1) mathematical structure, (2) philosophical foundations, (3) appropriate psychological principles, (4) pedagogical techniques, and (5) evaluative procedures. (This evaluative information should be made readily available to educators and interested laymen.)
- Act as a coordinator of pertinent local and State agencies in developing an up-to-date mathematics program.
- Cooperate with his colleagues in other States in curriculum development, inservice education, and research.

Preservice and Inservice Education of Teachers

The State Supervisor of Mathematics should

- Cooperate with college personnel and certification agencies concerned with preservice education of teachers.
- Cooperate with State certification authorities to raise standards of teacher certification by recommending appropriate requirements for teaching an up-to-date mathematics program.
- Encourage key teachers to attend mathematics institutes, to gain the confidence of their administrators, and to stimulate enthusiasm and interest among other teachers.
- Encourage, when appropriate, the use of persons from business and industry to assist with inservice programs.
- Encourage local systems to include student-teachers in their inservice programs.
- Encourage the use of live, taped, and/or filmed demonstration teaching situations as part of inservice programs.
- Encourage teachers who have participated in an inservice program to continue their study at a university or at a summer institute.
- Work with directors and staffs of mathematics institutes and assist them in making the course offerings more valuable to teachers.
- Disseminate information to administrators regarding mathematics institutes available to teachers.
- Participate in professional activities directed toward increased teacher competence in subject matter.
- Utilize television and other media to promote inservice programs.

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- Seek and utilize resourceful and knowledgeable teachers, especially institute participants, to assist in stimulating, planning, organizing, and conducting inservice programs.
- Encourage more than one teacher from the same school system to apply for participation in the same mathematics institute and encourage the Institute directors to accept such groups of applicants.
- Encourage teachers to participate actively in professional mathematics organizations.

New Media of Instruction

The State Supervisor of Mathematics should—

- Provide information as to the availability and use of new instructional media.
- Keep an open mind concerning the possibilities of new instructional media, pending more experimental evidence.
- Encourage carefully controlled research as to the effect of new instructional media on mathematics teaching.
- Stress the importance of preserving the recognized objectives of mathematics education regardless of the media used.
- Encourage administrators to solicit the assistance of teachers in the selection and purchase of classroom equipment.

Evaluation

The State Supervisor of Mathematics should—

Evaluate the State program of mathematics, and also the local programs, in terms of answers to such questions as the following:

- How effectively am I carrying out the activities recommended above?
- Are teachers' and pupils' attitudes, understanding, and skills improving?
- Does the program provide for individual pupil differences?
- Are mathematics materials and equipment purchased under the NDEA program being utilized effectively?
- Do budgets provide for implementing an improved program?

Appendix B. Panel Discussions

Discussion 1. Introduction of the New Mathematics Into the Curriculum

CHAIRMAN AND PANEL MEMBERS

John Wagner

Isabelle Rucker Arnold Chandler Frank Hawthorne

THE CHAIRMAN noted the great differences among States in their use of nationally known experimental materials in mathematics and attributed the widespread use of such materials in certain States to alert leadership at the State level.

The topics discussed by the panel were the following: (1) the relative merits of new curricular projects, (2) projects that helped introduce the new mathematics into the Virginia curriculum, and (3) the role of the supervisor in energizing and catalyzing teacher re-education.

Relative Merits of New Curricular Projects

Secondary Level

School Mathematics Study Group (*SMSG*).—Prepared by a group of well-qualified mathematicians and teachers. Units are suitable for separate use. Some of the materials are now available for the upper elementary grades.

University of Illinois Committee on School Mathematics (*UICSM*).—Courses produced are strong in pedagogical aspects. Because of their definite sequential nature, units could not easily be adopted for separate use.

Ball State Teachers College Mathematics Program.—Algebra and geometry courses produced, emphasizing axiomatic structure. Materials available in regular textbook form.

University of Maryland Mathematics Project (*UMMaP*).—Program developed for seventh- and eighth-grade mathematics.

Boston College Series.—Emphasize the structure of mathematics, using a historical perspective.

Elementary Level

Madison Project at Syracuse, N.Y.—Seeks to determine the elementary-grade level at which certain mathematical concepts can be taught successfully.

Geometry for Primary Grades.—An experiment (at the Stanford elementary school) exploring the teaching of geometry in grades 1–3.

Projects Helping Introduce the New Mathematics into the Virginia Curriculum

Mathematics teachers were invited to regular regional principals' meetings for a special session on mathematics.

Fifteen inservice mathematics programs were arranged and directed by members of the State Department of Education in areas remote from colleges or universities.

The State Department financed and arranged summer institutes for high school mathematics teachers.

Invited by several school principals, the State Supervisor of Mathematics discussed the new mathematics with the entire school faculty.

A mathematics curriculum study group, K–12, was created to work on materials for three types of students: elementary, slow learners, and college-capable.

The Role of the Supervisor in Energizing and Catalyzing Teacher Reeducation

Mathematics teachers for the most part are reasonably well motivated for inservice education, as evidenced by their widespread participation in various institutes. The major task of the supervisor, in cooperation with mathematicians and school administrators, is to provide adequate inservice programs. Such programs can be implemented by educational films or TV kinescopes, visiting lecturers or consultants in mathematics, regional conferences within reasonable driving distance, State meetings of professional mathematics organizations, TV programs at hours other than 6:30 a.m., and programmed learning.

Discussion II. Curriculum Changes in Science and Their Implication for Mathematics Teaching

CHAIRMAN AND PANEL MEMBERS

John Mayor

Margaret Maury Madeline T. Skirven

James DeRose Robert Henze Paul Hurd Robert Stephenson

The chairman noted that science and mathematics educators need to be mutually aware of the latest developments in both areas. Topics discussed by the panel were the following:

Biological Sciences Curriculum Study (BSCS) of the American

Institute of Biological Sciences.—(1) Three high school courses all covered the same material, but each used a different approach: ecological and evolutionary, genetic and developmental, or biochemical and physiological. (2) A new approach provides continuous laboratory work on a broad subject area for 6 weeks. (3) Charts and graphs, probability, and statistics have been used to a considerable extent in the courses under discussion.

The Chemical Bond Approach (CBA).—A chemistry course was built around two major concepts—chemical bonds and chemical reactions—and the laboratory work was so planned that it would encourage real problem solving. The mathematics used included graphical analysis, quadratic equations, and the concepts of precision and error in measurement.

The Chemical Education Materials Study.—Materials emphasize the theoretical, rather than the traditional, aspects of chemistry. Students are expected to write out experiments in full—not just fill in the blanks on a workbook page.

Developments in Teaching Earth Sciences.—Changes and innovations are taking place primarily at the local level. At the national level a source book of about 20 units at the Duluth Writing Conference arranged by the American Geological Institute under a grant from the National Science Foundation.

Physics Course Prepared by the Physical Sciences Study Committee.—The text does not treat the subject descriptively, but rather in such a way that students must follow a line of reasoning. In the teaching of this course, new topics incorporated in mathematics courses have little to contribute. Student competence in traditional mathematics, however, is most desirable for this course.

Discussion III. Criteria for Evaluating the Mathematics Program of a State Department of Education

CHAIRMAN AND PANEL MEMBERS

George Cunningham

Sylvia (3) Silva Carl Hellman George Roehr

California.—No statewide curriculum guides have been issued for secondary school mathematics, but the curriculum of a given subject comes up for review every 5 years. In 1960 representatives from 78 school districts met to consider the mathematics curriculum. They had the benefit of a report from a group of eminent mathematicians who had evaluated a proposed State Department program. At the meeting, workshop groups evaluated new mathematics programs for several grade levels.

The following questions are useful in evaluating a State program: (1) Does the State Department of Education espouse a good comprehensive mathematics program? (2) Is the State program sensitive to the attitudes, beliefs, and opinions of mathematics personnel in the schools? (3) Is the program so organized that current status information is readily available to the State Supervisor?

Pennsylvania.—(1) The State Department of Education does not have final figures as to the quantities of material and equipment obtained with NDEA funds. The Department feels a concern for the extent of the effective use—or even the possible misuse—of such materials in the classroom. (2) Inservice education programs have resulted in considerable use of new mathematics materials throughout the State. No organized effort has been made, however, to evaluate results. (3) In June 1960 the State Department of Education conducted, as part of its 3-year curriculum revision study, an achievement survey of 17,000 seniors in 118 high schools, representing a sampling of the 100,000 expected graduates. The average mathematics achievement level of the 17,000 was somewhat above the national average.

Puerto Rico.—(1) Education is highly centralized; curriculum development and textbook selection are responsibilities of the central office of the Department of Education. (2) Three evaluative studies have been made of Puerto Rican education: two by Teachers College, Columbia University (in 1926 and 1948); one by the University of Puerto Rico and the School of Education, New York University, jointly (in 1958). (3) The Department of Education makes a study every two years of the preparation of mathematics teachers, and on the basis of the findings plans appropriate inservice education. (4) Since Latin American mathematics textbooks are less adequate and less attractive than the North American counterparts, several of the latter have been translated into Spanish for use by the Puerto Rican schools.

Appendix C. Informal Talks

I. National Science Foundation Programs of Interest to Mathematics and Science Supervisors

Keith R. Kelson

THE NATIONAL SCIENCE FOUNDATION, although conscious of the pressures created by demands for scientific and technological personnel, is committed to a long-range program having continuity and stability rather than to a crash program meeting only current demands of the tense international situation.

The purposes of the NSF as defined by law are to promote the progress of science, to advance national health, prosperity, and welfare, to secure the national defense, and to accomplish other purposes. The Foundation's domain includes the mathematical, physical, engineering, medical, biological, and other sciences.

The NSF engages in four main types of activities to accomplish its purposes: (1) It develops and encourages basic research and education in the sciences. (2) It initiates research programs in the sciences and makes grants to support those programs. (3) It awards scholarships and graduate fellowships in the sciences. (4) It maintains a roster of scientific personnel.

In furthering education in the sciences, including mathematics, the NSF provides fellowships for teachers and research personnel, undertakes projects to improve course content,[1] and sponsors institutes for science and mathematics teachers.[2]

The National Science Foundation encourages and supports experimental curriculum projects such as SMSG, but it feels that it is inappropriate for a Government agency to give a stamp of approval to completed projects. Through institutes it does offer help to get programs started, but as a matter of policy avoids using its influence to encourage the adoption of experimental curriculum projects, even though encouragement might expedite progress. The State Supervisors may effectively bridge the gap between the experimental use of the new materials and their more general use in the schools.

The NSF is interested in elementary school science and mathematics. The Foundation is not certain, however, as to how much it ought to

[1] For example, NSF grants totaling $5 million supported the Physical Sciences Study Committee, which produced materials for a new high school physics course.

[2] Institutes for high school teachers involve an annual expenditure of $20 million.

become involved at the elementary school level. Even assuming that it ought to become involved, the Foundation is even less certain as to what methods would be appropriate for it.

The National Science Foundation is unique in being a Federal Government agency trying to function as a foundation.

II. Research in Mathematics Supported by the U.S. Office of Education

Edwin Hindsman

The Office of Education provides support for research of significance to education through its Cooperative Research Program. The purpose of this program is to develop new knowledge about major problems in education or to devise new applications of existing knowledge in solving such problems.

The program is operated under the terms of Public Law 531, 83d Congress, which authorizes the Commissioner of Education to "enter into contracts or jointly financed cooperative arrangements with universities and colleges and State educational agencies for the conduct of research, surveys, and demonstrations in the field of education."

Since the beginning of the program, the following projects have been supported in the field of mathematics through the regular contract research program:

Title	*Investigator*	*Institution*
ELEMENTARY		
The Development of Mathematical Concepts in Children.	Patrick Suppes	Stanford University.
Abilities of First-Grade Pupils to Learn Mathematics in Terms of Algebraic Structures by Means of Teaching Machines.	Evan R. Keislar	University of California.
An Analysis of Learning Efficiency in Arithmetic of Mentally Retarded Children in Comparison With Children of Average and High Intelligence.	Herbert J. Klausmeier.	University of Wisconsin.
An Evaluation of the Madison Project Method of Teaching Arithmetic (grades 4, 5, and 6).	William F. Bowin	New York State Education Department.

Title	*Investigator*	*Institution*
SECONDARY		
Evaluation and Follow-up Study of Thayer Academy's Summer Advance Study Program in Science and Mathematics.	William F. Cooley	Harvard University.
The Individualization of Junior High School Mathematics.	Joseph T. Sutton	Do.
Systematic Observation of Verbal Interaction as a Method of Comparing Mathematics Lessons.	E. Muriel Wright	Washington University.
GENERAL		
An Evaluative Study of Psychological Research on the Teaching of Mathematics.	Philip H. DuBois	Do.
Discovery and Evaluation of the Structure-of-Intellect Abilities Necessary for Algebraic Thinking.	J. P. Guilford	University of Southern California.
Characteristics of Teachers Which Affect Students' Learning.	Paul C. Rosenbloom.	University of Minnesota.

The following demonstration-research projects in the field of mathematics have recently been signed into contract:

Title	*Investigator*	*Institution*
Experimental Teaching of Mathematics Logic in the Elementary School.	Patrick Suppes	Stanford University.
Enriched Mathematics for Academically Talented Students.	Harry Passow and Miriam Goldberg.	Teachers College, Columbia University.
Implementation, Analysis, and Evaluation of the Madison Project Mathematics Materials.	Robert B. Davis	Syracuse University.
The Effectiveness of an Algebraic Structural Approach to Mathematics in Regular Primary Classrooms as Compared With a Conventional Program of Arithmetic Instruction.	Evan R. Keislar	University of California.

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