A Meta-Analysis of the Autocorrelation in Single Case Designs

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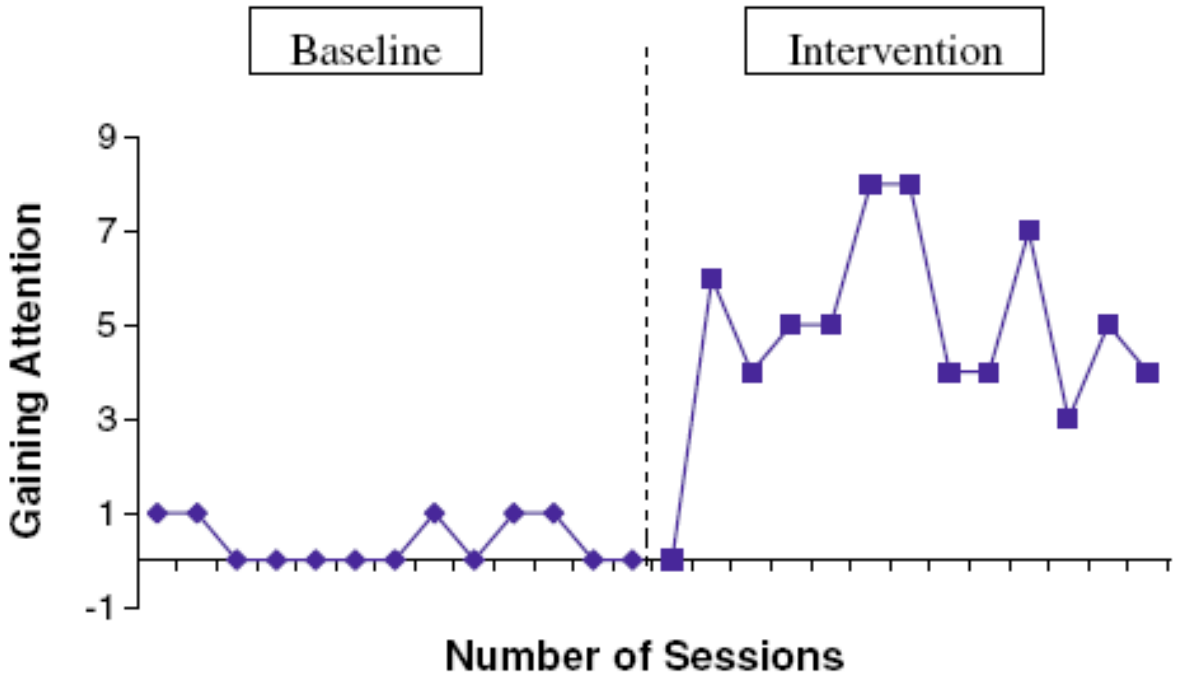
Background – Single Case Designs

- Experimental Design
  - Repeated measurements on the same experimental unit over time



Background – Single Case Designs

- Regression techniques can be used to analyze SCD data in addition to visual analysis

- One problem
  - Observations are nested within the same participant
    - Violates assumption that errors are independently and identically distributed
    - Errors may be serially correlated (i.e. autocorrelated)

Background - Autocorrelation

- Autocorrelation (AC) is typically assessed using a lag-1 AC coefficient
  - correlation between the regression residuals and the same residuals shifted ahead by one unit of time

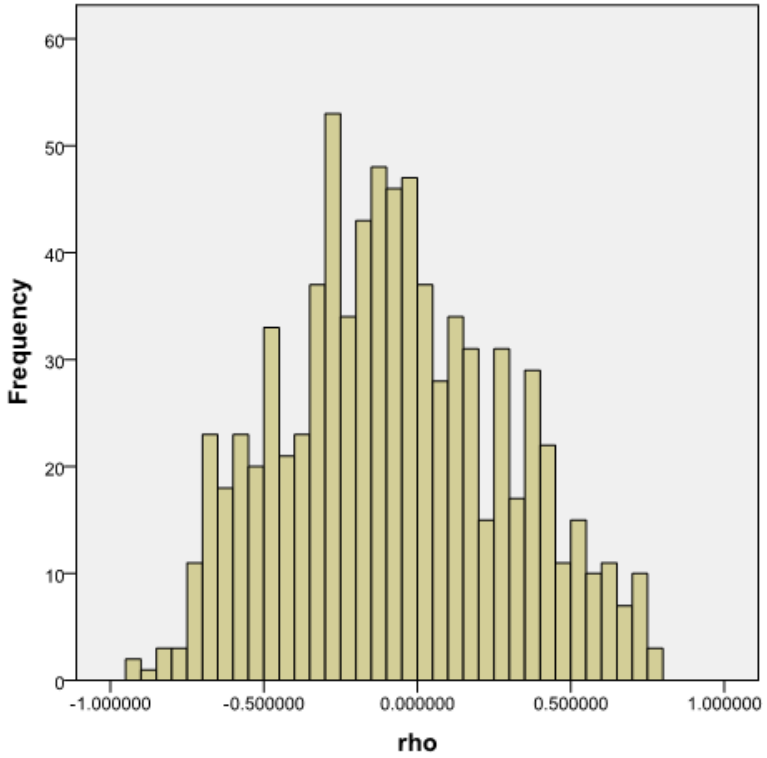
$$r_1 = \frac{\sum_{t=2}^{N} (e_t)(e_{t-1})}{\sum_{t=1}^{N} e_t^2}$$

- Where
  - e_t = the residual at time t
  - N = number of observations in time series

Huitema & McKean (1991)

Background - Autocorrelation

- How prevalent is AC?
  - Some debate
- Shadish and Sullivan (in press)
  - Analyzed 799 SCDs
  - Average AC = -.044, $p < .01$
  - Range = -.931 – .786
  - Significant Heterogeneity
    - (Q = 4306.18, $p < .001$, $I^2 = 81\%$)



Study Purpose

- To get a more accurate estimate of the AC by modeling the hierarchical nature of the data
  - Cases nested within participants nested within studies nested within journals
- To find factors that explain the between-study variability found in observed autocorrelation estimates

Methods

- 799 SCDs from Shadish and Sullivan (in press)
  - SCD data extracted using UnGraph
  - 19 moderator variables coded for each case
  - Time between observations estimated
- Data fit using regression models
  - Treatment, Trend, and Interaction terms
- AC estimates computed with residuals.
  - r_1 estimator
- AC estimates analyzed using multi-level meta-analytic models
  - Random Effects
  - Mixed Effects

Methods

- Simple regression model
  - $y = \beta_0 + \beta_1 time + \beta_2 treatment + \beta_3 time*treatment$
- For a subset of 352 SCDs
  - Longest SCDs (22 observations or more) to avoid perfect fit
  - Higher order trend and interaction terms added to regression model
  - Take into account potential non-linearity
  - For example:
    - Model 2
    - $y = \beta_0 + \beta_1 time + \beta_2 treatment + \beta_3 time*treatment + \beta_4 time^2 + \beta_5 time^2*treatment$

Methods

- Full multi-level model:
  - $\rho_{ijklm} = \gamma_{oo} + \zeta_{oj} + \zeta_{ok} + \zeta_{ol} + \zeta_{om} + \varepsilon_{ijklm}$
  - Where
    - γ_{oo} = grand mean
    - $\zeta \sim N(o,\tau^2)$, j = case, k = participant, l = study, m = journal
    - ε = residual error

Methods

- SAS code – 2 level model

```
Proc mixed  method=ml  covtest  data=temp;
class  ids pid sid jid;
model rho= / ddfm=satterth s cl;
random int / subject=ids;
repeated /group=ids;
parms / parmsdata=betvar
  eqcons = 2 to 800;
run;
```

Methods

- SAS code – 5 level model

```
Proc mixed  method=ml  covtest  data=temp;
class  ids pid sid jid;
model rho= / ddfm=satterth s cl;
random int / subject=jid;
random int / subject=sid(jid);
random int / subject=pid(sid jid);
random int / subject=ids(pid sid jid);
repeated /group=ids;
parms / parmsdata=betvar
  eqcons = 5 to 803;
run;
```

Results

- Unconditional model results

	2-Level Model	3-Level Model	4-Level Model	5-Level Model
Fixed Effects				
Intercept	-.028*	-.041*	-.036	-.045
Random Effects				
Cases	.101***	.031***	.031***	.031***
Participants	-	.074***	2e-4	2e-4
Studies	-	-	.073***	.071***
Journals	-	-	-	.004
-2Loglikelihood	723.0	598.1	415.3	415.2

Note: N = 799. *$p < .05$. ***$p < .001$.

Results

- Conditional model results

5-Level Model	
Random Effects	
Cases	.026***
Participants	.002
Studies	.047***
Journals	.002

Note: Satterthwaite degrees of freedom used; rounded to nearest interger.
†$p < .10$. *$p < .05$. **$p < .01$. ***$p < .001$.

5-Level Model	
Fixed Effects	
DV ID	$F(1,732) = 0.43$
SCD Design	$F(4,88) = 1.13$
DV Direction	$F(1,578) = 1.32$
DV Metric	$F(7,336) = 1.33$
Participant has Autism	$F(1,45) = 0.07$
Educational Study	$F(1,92) = 0.00$
Ceiling/Floor Effects	$F(2,751) = 0.07$
Ease of Coding Time	$F(4,86) = 0.55$
Who Coded DV	$F(3,109) = 0.57$
DV Content	$F(3,173) = 0.23$
Number of Sessions	$F(1,309) = 44.44$***
Average Time between Sessions	$F(1,149) = 1.09$
DV Changeability	$F(1,742) = 3.01$†
Participant Age	$F(3,193) = 3.45$*
Participant is a Student	$F(1,157) = 2.62$
Participant has Develop. Disord	$F(1,172) = 10.23$**
Participant has Clin. Diagnosis	$F(1,174) = 0.14$
Location of Study	$F(6,105) = 1.47$
Acceptable Level of IRR	$F(4,429) = 2.35$†

Results

- Results from investigating non-linearity

Model	Parameters	τ_{0j}^2	-2Loglikelihood
Model 1	3	.110	286.8
Model 3	7	.081	196.7
Model 10	21	.060	105.9

Note: N = 352.

Discussion

- Estimating the AC
  - Average AC not significantly different from zero after modeling the full data structure.

Discussion

- Reducing heterogeneity
  - Modeling the full data structure reduces between-case heterogeneity substantially (from .101 to .031)
  - Adding moderator variables further reduces between-case heterogeneity (from .031 to .026)
  - Adding moderator variables also reduces between-study heterogeneity (from .071 to .047)
  - Modeling non-linearity reduces between-case heterogeneity substantially (from .110 to .060)

Discussion

- Significant moderators
  - Number of sessions
    - Positive relationship with AC
    - Short time series are negatively biased
  - Participant age
    - Adults associated with lowest levels of AC
    - Teens associated with highest levels of AC
  - Developmental disorder
    - Those with developmental disorders associated with lower levels of AC

Conclusions

- The nesting of meta-analytic data should be modeled when possible
- Autocorrelation may only be an issue in SCDs with specific characteristics
- A non-trivial amount of non-linearity is likely present in SCD data

Limitations and Future Directions

- Non-linearity findings may not be generalizable to short time series
- No correction used for multiple covariate significance tests
- Additional case- and study-level moderators should be investigated to help explain remaining heterogeneity

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References

- Huitema, B. E., & McKean, J. W. (1991). Autocorrelation estimation and inference with small samples. *Pyschological Bulletin* , *110* (2), 291-304.

- Shadish, W.R., & Sullivan, K.J. (in press). Characteristics of single case designs used to assess Intervention effects in 2008. *Behavior Research Methods*.