Analysis of Research in the Teaching of Mathematics 1957 and 1958

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Contents

"THE AMERICAN EDUCATIONAL SYSTEM — fine as it is in many respects — can be, and as a whole should be, substantially improved . . .

"To attain these ends we conclude that four major areas need specific and urgent attention throughout our educational system: (1) the curriculum and the content of courses, (2) the quality and effectiveness of teachers, (3) the recognition and encouragement of students, and (4) the development of intellectual leadership."

Statement by the President's Science Advisory Committee
EDUCATION FOR THE AGE OF SCIENCE
May 24, 1959

Foreword

THE INCREASED COMPETITION in technical advancement has made the need for high quality scientists, engineers, and technicians stand out in bold relief. Better instruction in mathematics in our public schools can make a contribution to the quality of our scientific personnel. If research findings are implemented in the classroom they likewise can make a contribution. The purpose of this study is to help implement research findings by making available the results of research in the teaching of mathematics that have been reported to the Office of Education during the calendar years 1957 and 1958.

It was for the purpose of disseminating the findings of research on the teaching of mathematics that the Office of Education in cooperation with the National Council of Teachers of Mathematics reported summaries of research in mathematics education in 1952 (Circular No. 377) and in 1954 (Circular No. 377-II). These summaries received many favorable comments and suggestions from readers in mathematics education. As a result of the suggestions, the Office of Education and the Council cooperated further by summarizing research completed in mathematics education during the 2-year period 1955-1956 and also in analyzing the material.

The present bulletin is a summary and analysis of the research in the teaching of mathematics for the calendar years 1957-58. It is hoped that the present study will be helpful both to researchers and classroom teachers in their efforts to improve the teaching of mathematics.

The Office of Education is grateful to the Deans of Graduate Schools and to research workers in mathematics education who supplied the data on which the study is based. Without their cooperation it could not have been prepared. Cooperation in preparing this report was also effected through the Research Committee of the National Council of Teachers of Mathematics. The members of that committee were John J. Kinsella,

School of Education, New York University; Howard Fehr, Teachers College, Columbia University; Sheldon Myers, Educational Testing Service, Princeton, New Jersey; Fred Weaver, School of Education, Boston University; and Kenneth E. Brown, U. S. Office of Education, Department of Health, Education, and Welfare.

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ANALYSIS OF RESEARCH IN THE TEACHING OF MATHEMATICS

Introduction

TO ASSIST in the collection and dissemination of research findings in the teaching of mathematics, the U.S. Office of Education with the aid of the Research Committee of the National Council of Teachers of Mathematics sent an inquiry to 817 colleges that offered graduate courses in mathematics or whose staffs had made previous contributions in this field. The committee received answers to the questionnaire from 399 colleges. Of the 399 colleges, 59 reported research in the teaching of mathematics. The Committee carefully studied the 111 research studies reported by these 59 colleges and selected 73 of them for inclusion in this analysis. Those that were selected are 14 studies by college faculty members, 32 doctoral dissertations, and 27 master's theses. A summary of each is included in the appendix.

If an attempt were made to classify the studies according to the major emphasis there would be considerable duplication. For example, about the same number of studies were concerned with methods as with content. Yet, several studies were on both content and method. Among the other topics considered in one or more of the studies were television in the teaching of mathematics, preparation and competence of teachers, modern mathematics in the high school program, visual aids, activities of mathematics clubs, homogeneous grouping, and programs for the superior student.

Therefore, instead of attempting to classify the findings on the basis of the major emphasis in the research, they are presented according to pertinent questions in mathematics education. Although there is considerable overlapping when the 73 studies are classified according to grade levels, the questions and analyses are reported under three headings: Research in the Teaching of College Mathematics; Research in the Teaching of High School Mathematics; and Research in the Teaching of Elementary School Mathematics. The college level contains 23 studies; the high school level, 28; and the elementary, 22. Within each analysis the names in parentheses denote the investigator whose study is summarized in the appendix.

Research In The Teaching of College Mathematics

ON the college level, research in the teaching of mathematics was concentrated on the preparation and competence of elementary and secondary school teachers of mathematics and the use of television in teaching mathematics. Also included were studies on predicting success in college mathematics. Questions and commentary on these and other problems will now be presented.

1. *What do the studies on the use of televsion indicate?*

The prediction that by the early 1960's the colleges will be flooded with applicants has caused some educators to look to television as a means of helping meet the problem.

One study (Alkire) seemed to indicate that a course in the teaching of arithmetic could be taught more effectively by television than by traditional means, despite the lack of opportunity for questions and discussion. The television medium seemed to engender greater student interest and closer attention than the classroom setting. On the other hand, another investigator (Schied), who taught the calculus by television, ascribed the slightly better learning of the television students to better-prepared lectures. In another investigation (Elliot), college algebra and trigonometry were taught by a combination of television and regularly scheduled help sections. The television students again did decidely better on the examinations. The investigator gave the major credit to the help sections. Too, the fact that there were five different opportunities per day to receive the program was probably an important factor in the results. Finally, another study (Hooten) concluded that the principal criteria for a good mathematics television program are that the mathematics must be "impeccably accurate" and the presentation meaningful to the viewer.

2. *What does research say about the mathematical competence and preparation of the elementary school teacher?*

Previous studies have pointed to the scanty mathematical preparation of elementary school teachers and their lack of arithmetic understanding.

At a large university it was found (O'Donnell) that seniors preparing to be elementary school teachers had a grade mean of 13.14 in arithmetic achievement. A comparison of prospective elementary teachers of India and of an Iowa teachers college (Sahai) showed that the American students were superior in nearly all aspects of arithmetic. A third study (Knight) revealed that a program of mathematical preparation involving concepts of modern mathematics was more effective in producing competence in computation and mathematical reasoning than a traditional course involving arithmetic, algebra, and geometry. Finally, procedures were described (Rudd) for conducting an inservice course for teachers of arithmetic which brought about considerable growth in teacher's understanding of arithmetic.

3. *What does research tell us about the preparation of high school teachers of mathematics?*

In recent years the need for up-grading the preparation of teachers of high school mathematics and science has led to the widespread development of National Science Foundation programs.

A Kansas study (O'Hair) revealed that the mathematics preparation of 126 high school teachers in class A high schools ranged from 12 to 75 semester hours with a mean of 31 hours. Ninety percent of 50 college mathematics instructors and 87 percent of 150 school administrators wanted high school and college mathematics programs revised along the lines of modern mathematics. An Ohio study (Zimmerman) indicated that in a college of education the number of hours completed in mathematics and education by high school teachers of mathematics was practically the same. Although liberal arts students had significantly higher intelligence test scores than education students preparing to teach mathematics, there was no significant difference in the grades in mathematics courses taken by both groups.

While the results of these two studies are encouraging, the situations in some of the other 48 States leave much to be desired.

4. *What are the findings of studies dealing with the pedagogy and organization of mathematics courses?*

In the past such studies have been rare at the college level. The ideas that teaching cannot be stereotyped and that the students are old enough to take charge of their own learning have been quite prevalent among many college instructors.

A method of teaching the calculus emphasizing the use of student-discovery procedures (Cummins) produced better understanding of the subject

and just as much manipulative skill as the traditional approach. At Dartmouth (Kemeny), the special training in small groups within mathematics honors sections increased vastly the number of students entering graduate work in mathematics. In planning a methods course for prospective teachers of mathematics, one investigator (Felder) concluded that the instructor must be prepared both mathematically and professionally, and that the students must be interested in teaching as well as in mathematics.

5. *To what extent do grades and scores on tests predict success in college mathematics?*

Despite the fact that the great number of variables involved in predicting scholastic success practically precludes much accuracy in the endeavor, efforts in this direction continue to be made. It was found in one college (Brown) that grades in mathematics courses could not be predicted from credit-point averages in courses preceding them. The Q-score on the American Council on Education Psychological Test and the P.E.A.T. scores were found to be fairly good predictors of the success of engineering freshmen (Leo).

6. *What should be the program of general mathematics in junior college?*

As in the case of ninth grade general mathematics, the answer to this question varies according to the preparation and ability of the students, and with the social-vocational and geographic factors associated with the given institution. In fact, the content of the course found necessary for terminal students in a California junior college (Rowe) differs little from that found in syllabi of ninth grade general mathematics courses in other parts of the United States.

7. *What are the mathematical needs of semiprofessional engineers?*

In recent years it has been pointed out that the mathematical needs of semitechnical workers have been increasing. One group of the mathematics needs of semiprofessional engineers (Madole) reveals that the courses needed, ranked in terms of importance, are trigonometry, algebra, solid geometry, and analytic geometry. Although calculus is usually required in the preparation of these students, their employers do not even list it as important.

8. *What happens to students who prepare to teach mathematics?*

The problem of holding mathematics teachers in teaching has become a crucial one. One study (Frigo) showed that only 52 percent of the respondents from the classes of 1950-55 in the mathematics department of a New Jersey teachers college were still in the field of education in 1957. Most

of the respondents had taught in grades 7 to 9 during their first year; the subjects taught were usually arithmetic and general mathematics. The salaries of the nonteaching men exceeded that of the male teachers by $1,000. Four-fifths of the men and one-fourth of the women teachers found it necessary to supplement their salaries.

This is not an encouraging picture.

9. *How have teachers of mathematics in liberal arts colleges changed over the years?*

The picture of the mathematics professor was shown (Gavurin) to have changed, since the founding of Harvard, from an individual educated in the classical tradition to a highly-trained specialist in a well-developed field.

Research In The Teaching of High School Mathematics

ON this level research seems to have been influenced to some extent by the recommendation of several organizations of national standing that students of superior ability in mathematics be taught more challenging ideas. Studies of the readiness of students for the earlier introduction of traditional content and plans for revising 12th year mathematics programs are relatively frequent. Discussion of these and other problems follows below.

1. *Can high school students learn some of the concepts of modern mathematics?*

One investigation (Byrkit) involving a small number of high school seniors indicated that the formal study of relations and the learning of certain ideas in number theory were feasible topics for these students.

Another study (Roughead) involving average students at the 10th grade level showed that the basic notions of set theory, including its terminology and its visualization by Venn diagrams, and the graphing of inequalities were interesting and comprehensible to these students. Materials involving the notions of set, variable, relation and function were taught to students in grade 10 and above by methods involving teacher-pupil planning and the discovery of generalizations (Grubb). The 17 students made great gains from September to May in their scores on the Langton First-Year Algebra Test.

2. *What should be the program in 12th year mathematics for the average and above average pupil?*

Four studies (Garb, Kaufmann, Lawton, Clark) dealt with this course, which obviously enrolls students superior in mathematics. The first (Garb) concluded that material from space coordinate geometry, the algebra of sets, logic, statistics, and integral calculus should be included in the course. The second study (Kaufmann) suggested that solid geometry should largely be incorporated into the plane geometry course and that

topics in analytic geometry, statistics, and the calculus replace some of the traditional content.

From the results of a questionnaire to college mathematics instructors and college students in mathematics courses the decision was reached that at the 12th grade level solid geometry as a distinct course should be dropped; that greater emphasis should be placed on graphing, analytic geometry, and the theory parts of trigonometry; and that some work on statistics should be added but no set theory or symbolic logic (Clark).

It is obvious that complete agreement is lacking on this question of the content of 12th grade mathematics.

3. *When can students begin to learn algebra?*

One study (Davis) involving several teachers and classes in upstate New York reports that considerable algebra can be learned as early as the seventh grade.

4. *When can students begin to learn logical proof and demonstrative geometry?*

A unit on deductive proof and an introductory unit in plane geometry were taught in grades from six through ten (Corley). It was found that concepts from demonstrative geometry could be taught as low as the seventh grade with a reasonable expectation of success.

5. *At what grade level should certain measurement concepts be taught?*

One study (Beane) suggested that the concept of "ordering," using such ideas as longer than, nearer than, etc., be taught in the primary grades; that measurement involving the use of different scales be introduced in the middle grades; that the concepts of standard units, the approximate nature of measurement, accuracy, and precision could be learned during the junior high school years; and that a postulational approach to measurement might be tried in a course in demonstrative geometry.

Another study (Boeckmann) emphasized that the development of the notion of approximate data was essential to the teaching of the slide rule at the ninth grade level.

6. *What do the studies on the learning of algebra show?*

In a study of penetration in solving algebra problems (Hammer) the outstanding findings were the uniqueness with which each student arrived at insight and the dependency of successful solving upon the attitude toward solution. Getting an acceptable answer was usually the student's primary concern; understanding the problem and its solution was secondary.

When algebra was organized and taught around certain unifying themes instead of the usual topics, students developed as much manipulative skill, while acquiring a greater understanding of the nature of mathematics. (Kushta).

Using a modification of the Winnetka individualized approach one investigator (Keith) found that "in beginning algebra students made more progress when working independently than when they were being taught as a group."

Two methods of determining the characteristic of a logarithm and the decimal point in the antilogarithm were compared (Clebowicz). What differences there were favored a method involving the counting of the number of places between the decimal point and a reference point immediately to the right of the first nonzero digit.

7. *How effective are visual-factual devices in teaching mathematics?*

A kit of sixteen visual-factual devices was used in teaching areas, volumes, and the pythagorean relation in the eighth grade (Anderson). The students using the devices scored higher, but not significantly so, on the tests used. There appeared to be no relation between the amount students claimed they used the devices and the scores earned on the tests.

It was determined that teaching aids in plane geometry were not widely used due to inadequate storage facilities, insufficient time for constructing them, lack of money, inconvenience of usage, and difficulty in obtaining such aids (Turney).

8. *What do the studies on the teaching of geometry reveal?*

A textbook procedure for teaching area to seventh grade students was compared with an experimental unit method (Green). The experimental group performed significantly better than the other group. The discovery emphasis seemed especially preferrable for students above the median in intelligence test scores.

Plane geometry was developed by algebraic means (Richards). It was suggested that after three or four semesters of algebra such an approach might be feasible.

9. *Which activities are interesting to members of a mathematics club?*

Tenth grade plane geometry students listened to descriptions of certain topics for a mathematics club and then rated them in terms of interest (Lombardi). The reactions of mathematics teachers were also obtained. The students chose, in order of preference, field trips, construction projects, pure mathematics (topology, fourth dimension, etc.), recreational

mathematics, and history of mathematics; teachers, like the students, placed field trips first.

10. *How does mathematics education in the United States compare with that in other countries?*

In both Norway and Sweden, one study (Wahlstrom) indicates that mathematical ideas are more gradually developed in both the elementary and secondary school grades. Students spend more time studying mathematics in school and at home. Subject matter and instruction are more uniform. Teachers are better prepared but their instruction tends toward the traditional. Much attention is paid to theory. Many applications are made to geometry. Examinations are comprehensive and thorough.

11. *What do groups outside of the school think about the high school mathematics program?*

In Oswego, Illinois, 56 representatives of agriculture, small business, industry, and colleges considered that the same fundamental mathematics training was necessary for all students (Shull). More mathematical training in the processes of arithmetic was requested. Critical thinking was rated as one of the most important products of mathematical training. The colleges and industries desired more advanced mathematical training for high school graduates.

12. *To what extent are the proposals of authorities in mathematics education being put into practice?*

By means of a questionnaire, proposals of authorities in mathematics education were compared with practices in the areas of aims, curriculum, methods, and evaluations (Shetler). The instrument was sent to a 10 percent sample of all secondary schools in the 20 States of the North Central Association.

There was agreement between the authorities and the respondents on aims. Multiple-track programs were well-established. Many teachers still felt their programs were inadequate. The degree of agreement on classroom methods and evaluation was indicisive — high on some items, low on others. The practices recommended by the experts were followed most closely by teachers who spend most of the day teaching mathematics and by those who are receiving professional assistance in their teaching; the larger schools and the city public schools also tended to follow the recommended practices closely.

Research In The Teaching of Elementary School Mathematics

Research in the teaching of arithmetic during 1957-58 dealt mainly with problem solving, the development of meaning and understanding, and the comparison of alternative ways of teaching certain topics. Homogenous grouping, teaching aids, programs for superior pupils, and factors in achievement also received minor attention. The answers given to the key questions are briefly summarized below.

1. *What factors seem to have the greatest influence on success in arithmetic problem-solving?*

High confidence, ability to interpret vocabulary, and computational reasoning were significantly associated with the problem-solving success of sixth grade students (Corle). The type of operations involved, the familiarity of the problem setting, and the presence of superfluous numerical data were found to be the important factors in solving problems in another study (Post). In neither investigation was computational ability found to be a significant factor.

2. *Are lessons in mental arithmetic of much value?*

Sixth grade pupils given a series of prepared lessons in mental arithmetic not only gained significantly in their ability to solve arithmetic problems presented orally but surpassed a control group, taught traditionally, in gains in general arithmetic ability (Damgaard).

3. *Do seventh and eight grade pupils understand the concepts involved in computation?*

On a test of arithmetic meanings more than three-fourths of nearly 400 seventh and eighth grade pupils in a large city scored less than 50 percent (Rappaport). Pupils with high achievement in computation could have low achievement on a meanings test, but there were no pupils with high achievement on meanings and low achievement in computational skill.

4. *What is the most effective way of dividing the time between the development of meanings and computational practice in the intermediate grades?*

Intermediate grade pupils in groups devoting 75 percent or 60 percent of their class time to developmental work achieved significantly higher scores on tests measuring their understanding of arithmetic and computational skill, than pupils in groups devoting a smaller percent of class time to such developmental work (Shipp).

5. *Should the concept of a fraction as a ratio be emphasized at the fifth grade level?*

Of 200 fifth grade pupils, one group was taught a fraction approach in dealing with parts of a collection, while the other group was taught a ratio method (Silvey). When the numbers were 10s or 100s, and the parts were 10ths or 100ths, the ratio approach was easier in examples of the $\frac{a}{b}(c) = d$ type, if either c or d was the unknown. It was suggested that the notion of ratio be introduced before the operations on fractions are taught.

6. *What is the best way to teach the location of the decimal point in division?*

In an experiment involving four sixth grade classes in three communities one group of students was taught to locate the decimal point in the quotient by the usual method of "moving decimal points;" another group was taught to make the number of decimal places in the quotient equal to the excess of those in the dividend over those in the division (Spooner). Tests revealed that neither method was superior in terms of accuracy, but that the second method surpassed the first on the basis of "meaning given to the answers of verbal problems" and the time required to perform the computations.

7. *Is homogeneous grouping in mathematics classes effective at the seventh grade level?*

Six of 17 seventh grade classes were formed into homogeneous groups each by one of six criteria (Rogler). These were low IQ, low achievement, high achievement, low reading ability, low social maturity, and high social maturity. None of these homogeneous groups did any better in achievement than groups of students having similar characteristics but enrolled in one of the 11 heterogeneous classes.

8. *Should a variety of teaching aids be used in arithmetic instruction?*

One investigator reports that using a variety of materials does not produce better results than using only one material, if both procedures are used for the same time (Sole).

9. *What is the best way to teach the concept of area at the fifth grade level?*

One group of children had repeated experiences in finding the areas of the same rectangular figures; the other group found areas of varied irregular figures (Sorenson). Neither method surpassed the other in a test on areas.

10. *What mathematics should be taught to superior pupils in grades 7 and 8?*

One plan is to include the concepts of number, symbolism, measurement and approximation, statistics, operations, and relationships (Bryan). Number systems, history of symbols and numbers, and real and complex numbers are suggested as topics to be explored. By the end of grade 8 a semester of elementary algebra would be completed.

11. *Are 50-minute daily arithmetic periods much better than 40-minute periods?*

In a study involving 600 sixth grade pupils from 15 schools of a large city the group taught for the extra 10 minutes gained significantly more in achievement than the group taught 40 minutes (Daugherty). Another investigator in the same city found that during the first year of a 2-year study the extra 10 minutes made a difference of only .1 grade at the fifth grade level (Denny). However, after the teachers using the 50 minute period were given inservice training, the 50-minute group of pupils did significantly better than the 40-minute group.

12. *How are gains in arithmetic achievement at the eight grade level related to intelligence levels?*

In a well-developed, statewide study in Minnesota it was found that the middle 5 percent group in intelligence at the eighth grade level seemed to gain more mathematics information than either the top or bottom 5 percent in intelligence (McCutcheon).

13. *What changes have taken place in arithmetic as a school subject since 1900?*

Disciples of John Dewey influenced many teachers to base their arithmetic

instruction on child experience rather than abstractions (Tompkins). By 1920 the interpreters of Thorndike had led many educators to believe that the way to teach arithmetic was to analyze it into many independent skills and then give the pupil abstract drill on each of these. Around 1930 a side effect of this philosophy was the social utility notion of teaching only that arithmetic needed in everyday life by nearly everyone. Since 1935 the field psychologists, especially Brownell, have been very influential in making "meaning and understanding" the key words. The latest development is the emphasis on meaning and understanding supplemented by the drill necessary to insure the retention of certain arithmetic skills.

Summary

COLLEGE LEVEL

A STUDY of the research for 1957-58 reveals considerable activity in the area of the preparation and competence of teachers of elementary and secondary school mathematics. However, no investigations of the best ways of developing good teachers of college mathematics were received. If in the 1960's the colleges are inundated with students, the problems of locating and inducting men and women into the teaching of college mathematics are likely to be serious ones.

This is the first 2-year period in which a noticeable effort has been made to appraise the place of television in the teaching of college mathematics. A special issue of the *American Mathematical Monthly* was even devoted to the use of television in teaching. That the television medium does engage the attention and interest of the students, at least, during their early contacts with it, does seem established. Whether the interest is maintained for periods of time greater than a semester needs to be investigated. The achievement of the television group is usually better than that of the usual classroom group. The careful preparation of the television lectures and the stimulation of the professor by the challenge of the new medium are factors that must be considered in experiments evaluating the new medium.

There is still a dearth of studies dealing with the pedagogy of college mathematics. Is it just personality that makes one college teacher strikingly more effective than another with a comparable grasp of his subject? What is there about the way outstanding teachers carry out their instructional activities that makes them so effective? Someone should try to make a clinical study of a sampling of these outstanding men and women in action to see if there are clusters of patterns that might be of aid to others who are less effective. Why shouldn't some of the proposals of Polya and of the mathematicians who tried out and studied the teaching of mathematics to future social scientists be given a test.

Several studies of the power of certain data to predict success in

college mathematics were received. Studies of this type have been legion in the field of high school mathematics and other fields as well. Very seldom is the correlation between the predictors and the criterion greater than 0.70. Perhaps research energy could be used more profitably in other directions.

HIGH SCHOOL LEVEL

During these 2 years many of the studies were probably influenced by organizations like the Commission on Mathematics of the College Entrance Examination Board, the University of Illinois Committee on Secondary Mathematics, the National Science Foundation Institutes, and the National Council of Teachers of Mathematics through its publications, especially *The Mathematics Teacher*.

The studies on the readiness of students to learn algebra and demonstrative geometry earlier were of a tentative, informal sort. Very few students and teachers were involved. The duration of the experiments was short. The subject matter selected was limited. The teaching methodology was not always obvious.

The investigations into what the content of grade 12 mathematics should be were usually done by one individual for one school. Evaluation of the proposals in the classroom was always left for the future. The content of most of the proposals was strikingly similar, in most instances, to the proposals of the groups listed at the beginning of this summary.

There was only one study that dealt with problem-solving. There is a need for more studies like this one, in which the thinking of a small number of students was analyzed in depth in a clinical manner, instead of merely using crude data like test scores on masses of students.

The studies on the learning of algebra and geometry were few and yielded little that was new or revolutionary. There was encouragement in one study, however, in which other means besides performance on techniques were used to compare the relative merits of two ways of organizing the content of algebra.

The study about the degree to which the recommendations of authorities were being put into practice seems important and interesting. A huge responsibility is placed on the "experts;" they must be sure that their judgments are based on evidence rather than on unsupported opinions. The fact that there was high agreement on aims but uneven agreement on classroom methods is not a new anomaly. Does it mean that the verbal statement of aims has different meanings for different individuals? Does it mean that teachers see few relations between aims and methods? Does it mean that aims constitute a creed which is very difficult to put into practice because of the characteristics of students, the inadequacies

in teachers, and the conflicting aims of school administration? These questions should be investigated.

ELEMENTARY SCHOOL LEVEL

The investigators of problem solving in arithmetic agreed that many factors were more important than computational skill. On the other hand, these same investigations uncovered different sets of factors which were important in problem-solving. Would additional "n" investigations find "n" additional discrete sets of factors? Is the number of variables influencing success in problem-solving so large that recipes and instructional strategies are bound to be fruitless?

The research on meanings and understanding usually reveals that pupils taught computations only don't show much understanding. On the other hand, pupils of average IQ and above who receive instruction emphasizing meaning and understanding not only learn to compute just as well as those taught computations but also come to understand arithmetic better. The research of 1957-58 does not dispute these statements. It is a pleasant surprise when students learn what the teacher has not taught them; it is hardly shocking when pupils do not learn what they have not been taught.

The investigations dealing with surveys of arithmetic learning at the city and state level during 1957-58 raise some questions but hardly give answers of broad application. If a 25 percent increase in class time produces a 45 percent increase in grade score, would a 50 percent increase in instructional time result in a 90 percent increase in grade score in arithmetic? Where is the point of diminishing returns?

The study at the State level shows that schools with students of the same IQ and initial mathematical achievement vary in what their students achieve by the end of the year. This is a good beginning, but the "grass roots" type of research requires that the reasons for the variations in achievement be found, and that careful analysis of these reasons lead to a plan for better education in specific schools.

RECOMMENDATIONS FOR FUTURE RESEARCH

The recommendations for research in mathematics education made in 1955-56 apply to 1957-58 as well. Three major points were (1) that crucial problems should be the subjects of investigation; (2) that more research should be of the cooperative, team-like type; and (3) that the results of research should be clearly and adequately reported.

Some crucial problems were attacked in 1957-58: teaching mathematics by television; finding out whether precollege students can learn some

of the concepts of modern mathematics; studying the holding power of the schools with respect to mathematics teachers and the readiness of superior pre-high school students for work in algebra and geometry; and ways of increasing the number of undergraduate students who decide to do graduate work in mathematics.

As in 1955-56 there does not seem to be one reported research study involving the active team-work of two or more investigators. As a result, most of the studies reported are, in general, narrow in scope, limited in duration, restricted to only a few teachers, and carried out in a constricted geographical area. Thus random sampling, which is a necessary condition for the use of statistical tests of significance and for drawing generalizations, was not utilized. Yet such statistical tests were used!

The results of research cannot be clearly and adequately reported in pamphlets like this unless the needed information is supplied. As a minimum, the problem and subproblems should be clearly stated; the number and characteristics of the subjects of the experiment, including the teachers, should be provided; the duration of the experiment should be specified; the methods or treatments used should be clearly described; the method of sampling should be spelled out; the validity and reliability of evaluation instruments should be given; and the findings should be separated from the conclusions and recommendations.

Unanswered Questions In The Teaching of Mathematics

THE FOLLOWING QUESTIONS are those asked or implied by the research workers whose studies were reported during 1957-58.

1. What are the most effective means of changing attitudes toward mathematics?

2. What are the most effective means of increasing interest in mathematics?

3. Would a discovery approach applied to upper undergraduate mathematics courses produce more learning?

4. What are the characteristics of good teachers of mathematics in terms of what they do and how they function?

5. How should the present college mathematics program for prospective secondary school mathematics teachers be revised?

6. How can creative thinking in mathematics be evaluated?

7. To what extent have the mathematics programs of American colleges been revised to include more of "modern mathematics?"

8. What concepts of algebra can be successfully taught in grades 4 to 6?

9. How can understanding of algebra, in contrast with manipulative skill, be evaluated?

10. What "dead wood" should be pruned from present courses in arithmetic? Algebra? Geometry? College mathematics?

11. How can graphing be taught more effectively?

12. What are appropriate materials and exercises for teaching mathematical proof at the ninth grade level through algebra?

13. Are integrated mathematics courses at the college freshman level more effective than separate courses in college algebra and trigonometry?

14. Are integrated mathematics courses at the 10th or 11th grade level more effective than separate courses in plane geometry, advanced algebra, and trigonometry?

15. Are elementary school teachers with strong preparation in mathematics m effective than those with weak preparation?

16. Are secondary school teachers with strong preparation in mathematics more effective than those with weak preparation?

17. To what extent should social applications of arithmetic be taught at the seventh and eighth grade levels?

18. How do elementary school students of differing ability attack arithmetic problems?

19. What is the most effective way of teaching the "three cases" in problems involving percent?

20. What kind of inservice training is most effective in improving teachers of secondary school mathematics?

21. How effective is the teaching of algebra through its physical applications?

22. How effective is the teaching of plane and solid geometry simultaneously?

23. What training in mathematics do our present high school mathematics teachers have?

24. Aside from manipulative skills, what understanding of algebra do high school mathematics teachers possess?

25. What knowledge of logic is possessed by the high school teacher of mathematics?

26. How much knowledge of college mathematics is retained by high school mathematics teachers after 1 to 10 years?

Appendix: Summary of Research Studies

ALEXANDER, MAUDE J. A Development of Syllabi for the Teaching of Algebra and Geometry at Glenn Vocational High School. (M.Ed., 1958, Birmingham-Southern College, Birmingham, Ala.)

Major Faculty Adviser. Ray Black.

Problems. — To determine: (1) a relationship between vocational mathematics and regular high school mathematics, and (2) to develop syllabi for teaching algebra and geometry at Glenn Vocational High School.

Procedure. — High school mathematics and shop theory texts were reviewed to compare the usual high school mathematics problems with the shop problems. Instructors in the school were consulted.

Major Findings and Conclusions. — (1) The basic principles of mathematics were found to be the same in the high school mathematics problems and the shop problems. (2) The shops require more difficult, but more practical problems. (3) Syllabi incorporating the functional material were developed.

ALKIRE, G. DON. Mathematics Instruction via Television. (1958, Fresno State College, Fresno, California.)

Problem. — To determine the effectiveness of instruction via television.

Procedure. — The course, Advanced Problems in the Teaching of Arithmetic, was offered via television by Fresno State College in the fall of 1956-57. Fifteen programs of 1-hour each were televised. Eighty-five students took the course; of the 64 replies from the 71 who were teaching, 49 were in elementary schools, 11 in junior high schools, and 4 in senior high schools.

A questionnaire was devised to evaluate the degree of instructiveness, the quality of instruction, and the adequacy of the technique of televising. About 80 students responded to this instrument both at midterm and at the end of the course.

The results of the midterm and final examination scores of the television students was compared with three other sections of the course not taught by television.

Major Findings and Conclusions. — (1) Compared with regular classroom instruction, the television instruction was more interesting; made understanding a little less difficult; yielded greater retention; produced greater attention; and made learning as efficient despite the lack of class questions and discussions. (2) The quality of the instruction was approved by the students. (3) The television technique, especially the use of visual aids, was very satisfactory to the students. (4) Based on the same examinations given to the four groups, the television class achieved to a significantly higher degre than the other three classes.

ANDERSON, GEORGE R. Visual-Tactual Devices: Their Efficiency in Teaching Area, Volume, and the Pythagorean Relationship to Eighth Grade Children. (Ed.D., 1957, The Pennsylvania State University, University Park.)

Major Faculty Adviser. — Abram Vander Meer.

Problem. — To measure the efficacy of a kit of 16 visual-tactual devices used in an eighth grade arithmetic unit involving areas, volumes, and the Phythagorean relation.

Procedure. — A total of 541 eighth grade students from three junior high schools were divided about equally into an experimental and a control group. After 6 weeks of the same kind of instruction there followed a period of 8 weeks in which the teachers of the experimental group used a kit of 16 visual-tactual devices in explaining new material to the students; the devices were available at all times to the students for self-help and study. The control groups did not use any manipulative devices.

On a progress test given at the end of the six-week period, 204 pairs of students matched on intelligence and scores were formed. The criterion for comparing the two groups of students was the combined score on tests on surfaces, solids, and right triangles given during the period of 8 weeks. Questionnaires were administered to determine student preferences for their four major subjects at the beginning and end of the period of 14 weeks, and to find out the helpfulness and frequency of use of the devices.

Major Findings and Conclusions. — (1) On the criterion test the experimental group consistently scored higher than the control group, but not significantly so. (2) The evidence for judging the efficacy of the devices for students with high mental ages was inconclusive. Students in the experimental group with low mental ages scored lower, but not significantly so, on the criterion test than did students in the control group with low mental ages. (3) There appeared to be no relation between the amount students stated they used the devices and the scores earned on the criterion. (4) The use of the devices appeared to have little effect on student preference for mathematics in contrast with their three other major subjects.

BEANE, DONALD GENE. Concepts of Measurement and their Implications for the High School Curriculum. (M.A., 1958, University of Illinois, Urbana.)

Major Faculty Adviser. — Kenneth B. Henderson.

Problems. — (1) To identify a set of concepts and principles basic to an understanding of measurement; (2) To determine which of these concepts and principles can probably be taught in high school mathematics and/or science classes.

Procedure. — The literature dealing with measurement was read.

Major Findings and Conclusions. — (1) The concept of ordering should be taught in the primary grades. (2) Experience in measuring with different scales can be introduced in the middle grades. (3) The concepts of standard units, significant digits, the approximate nature of a measurement, accuracy, and precision can be taught in the junior high school. (4) Perhaps in the junior high school the types of scales associated with physical magnitudes can be taught. (5) A postulational approach to measurement might be introduced in a plane geometry course. (6) Trigonometry, or the sciences, might be used to teach the difference between fundamental and derived measurement.

BOEKMAN, HERMAN. Teaching Approximate Data and the Slide Rule. (M.S., 1958, Illinois State University, Normal.)

Major Faculty Adviser. — T. E. Rine.

Problem. — To develop a unit on the teaching of the slide rule for a course in secondary school elementary algebra.

Procedure. — A unit for teaching the

slide rule was developed with special attention to developing an understanding of approximate data.

Major Findings and Conclusions. — (1) The unit was effective in teaching the slide rule to 86 percent of the students. (2) Placement of the decimal point was the most frequent error made as a result of the method of teaching the slide rule. (3) The use of a classroom scale model was influential in the success of the unit. (4) Success in using the slide rule was highly associated with success in the unit on approximate data. (5) The unit on the slide rule made the determination of the characteristics of logarithms in a later unit easy.

BROWN, SISTER MARY DENISE, O.S.U. A Study of the Mathematics Program in the Institute of Technology at Saint Louis University. (Ed.M., 1958, Saint Louis University, St. Louis, Missouri.)

Major Faculty Adviser. — Waldo A. Vezeau.

Problem. — to determine the relations between grades earned in mathematics courses in the engineering school of Saint Louis University, St. Louis, Missouri.

Procedure. — Various statistical methods, especially regression and correlation analysis, were used to determine whether credit point averages in mathematics courses taken earlier could be used to predict the corresponding averages in mathematics courses taken later.

Major Findings and Conclusions. — Grades in previous courses did not correlate highly with those in the courses that followed. Hence, such previous grades could not be used for predictive purposes.

BRYAN, EUNICE ROBBINS. A Mathematics Program for Superior Upper Grade Pupils. (M.S. in Ed., 1958, Illinois State University, Normal.)

Major Faculty Adviser. — Francis R. Brown.

Problem. — (1) To learn what provisions are being made for mathematically superior pupils in the seventh and eighth grades, and (2) to suggest a mathematics program for them.

Procedure. — Professional literature was studied. A questionnaire to determine what was being done for mathematically superior students in grades seven and eight was answered by 124 teachers of these grades.

Major Findings and Conclusions. — (1) The survey of the literature indicated a need for special provisions for the gifted child in the two grades but also that little had been done. (2) The results of the questionnaire showed that a small percentage of the superior pupils was given a broader and more accelerated program in mathematics in grades seven and eight. (3) The proposed program was centered around the concepts of number, symbolism, measurement and approximation, statistics, operations, and functions. The program is an accelerated one that will take these superior students into the first half of ninth grade algebra by the end of the eighth grade. Number systems; history of symbols and numbers; real, imaginary, and complex numbers; and modern mathematics are recommended. Three teaching units were developed.

BYRKIT, DONALD RAYMOND. Sets and Number Theory in the High School. (M.S., 1958, Illinois State University, Normal.)

Major Faculty Adviser. — T. E. Rine.

Problem. — To develop a unit in the foundations of modern mathematics teachable at the senior level of the high school.

Procedure. — Units were developed on relations, number theory, sets, transformations, and semigroups and taught to eight students.

Major Findings and Conclusions. — (1) The class seemed to understand the

unit on relations since on a test the average score was over 27 out of 30 points. (2) A similar conclusion was drawn about the unit on number theory since the average score was about 37 out of 40 points. (3) Perhaps due to the short time spent on transformations and semigroups the average score was about 11 out of 15 points. (4) Since the average score was about 87 out of 100 points, the investigator concluded that these high school students could understand some material from the foundations of modern mathematics.

CHANDLER, DONALD GEOFFREY. A Proposed Method of Learning the Multiplication Table on the Junior High School Level. (M.Ed., 1957, College of Education, Ohio State University, Columbus.)

Major Faculty Adviser. — Nathan Lazar.

Problem. — To devise a method for learning the multiplication table on the junior high school level.

Procedure. — Methods of learning skills in fields outside of mathematics were compared.

Major Findings and Conclusions. — The quasi-intentional method was chosen. This involves guessing the answer to a combination when it cannot be recalled. Then the guess is checked by consulting a multiplication table. The requirement to guess focuses the student's attention on his effort to remember. It is recommended that the method be tested with students.

CLARK, DOROTHY T. An Investigation into the Relative Importance for College Preparation of the Topics of Twelfth Grade Mathematics. (1958, Montclair State College, Upper Montclair, N.J.)

Problem. — To determine what should be the content of the 12th grade college preparatory mathematics courses at West Orange High School, West Orange, N.J., in order to provide the students with the best possible foundation for success in college work.

Procedure. — Questionnaires were sent to mathematics instructors in 133 colleges admitting West Orange graduates. As a check, 58 college freshmen were also queried. The questionnaire was constructed from a list of topics found in ten 12th grade textbooks. After the removal of prerequisite material this list contained 74 topics. The degree of importance of each topic was checked by the college instructors. A simpler but similar questionnaire was sent to the 58 college freshmen.

Eighty-nine colleges and 41 freshmen replied. By appropriate weighing of the degree of importance the topics were ranked under three headings: general and technical curricula, general curricula only, and technical curricula only.

Major Findings and Conclusions. — From an analysis of the responses it was decided to drop solid geometry as such, to retain slide rule instruction, to place greater emphasis on theory in trigonometry and algebra, to include some work with statistics, but not to include set theory or symbolic logic, except possibly as optional additions to the course.

CLEBOWICZ, ALEXANDER EMIL. An Extension of the Johnson-Neyman Analysis to Three Matching Variables Applied to a Study of Two Methods of Determining the Characteristic of a Common Logarithm (Ed.D., 1958, Teachers College, Columbia University, New York, N.Y.)

Major Faculty Adviser. — Myron F. Rosskopf.

Problem. — (1) To compare the effectiveness of two methods of determining the characteristic of the common logarithm of a number and of locating the decimal point in the antilogrboard of a given logarithm. (2) To extend the Johnson-Neyman method of statistical analysis to

three matching variables and to apply this to problem (1).

Procedure. — Four classes involving both high school and college students were taught by a method (A) in which the logarithm of a number was determined either by the number of digits to the left of a number greater than 1, or by the number of zeros between the decimal point and the first nonzero digit when the number was less than 1. Four other classes were taught by a method (B) in which use was made of a reference point (C) placed immediately after the first nonzero digit of the number. The number of places between the reference point and the decimal point determined the characteristic. In both methods the decimal point was determined in the antilogarithm by procedures corresponding to those in the inverse process.

Measures for both groups of students were obtained on intelligence, numerical ability, and verbal reasoning, as well as scores on two criterion tests. By means of the Johnson-Neyman technique the ranges of values for each background trait, wherein the difference in achievement between groups A and B was significant, was determined.

Major Findings and Conclusions. — (1) High school students of average or better ability taught by method B scored higher on a criterion test than those taught by method A. However, after both groups had sufficient time to learn their respective methods, there was no appreciable difference between them. (2) College students in method B whose background scores were higher scored better, in a statistically significant sense, than their counterparts in method A.

CORLE, CLYDE G. Thought Processes of Sixth Grade Pupils While Solving Verbal Problems in Arithmetic. (1958, The Pennsylvania State University, University Park.)

Problem — To determine the importance of certain factors in the arithmetic problem-solving of sixth grade pupils.

Procedure — Seventy-four sixth grade pupils were interviewed, one at a time, while they solved eight verbal problems involving arithmetic. Each pupil read a problem orally and then silently before working out the solution on a card. Tape recordings were made of each interview. The interview and solution were evaluated for accuracy, concept evaluation, reasoning methods, degree of confidence, vocabulary rating, and reading efficiency. The relationships between accuracy in problem solving and five problem-solving procedures were tested by means of the coefficient of contingency.

Major Findings and Conclusions. — (1) Pupils tend to be overconfident of their problem-solving ability. (2) High confidence and problem-solving efficiency show a statistically significant relationship. (3) Correct interpretation of vocabularly is related to problem-solving efficiency. (4) Pupils made little reference to life applications in solving or explaining their problems. (5) Computational reasoning is an effective procedure in problem-solving. (6) There is no significant relationship between oral reading fluency and ability to work problems correctly. (7) Computational accuracy is not a major factor in problem-solving.

CORLEY, C. S. An Experiment in Readiness for Logical Thinking and Demonstrative Geometry. (Ph.D., 1959 George Peabody College, Nashville, Tenn.)

Major Faculty Adviser. — J. Houston Banks.

Problem. — To determine the minimum age at which average and superior pupils can master proofs in demonstrative geometry.

Procedure. — Classes from grades 6 through 10 were taught a unit on deductive proof and an introductory unit on plane geometry. Grade 10 was used as a control group. Achievement tests were administered at the conclusion of the unit. The relative achievement of the groups was determined.

Major Findings and Conclusions. — Grade 6 pupils showed insufficient maturity to master the material studied. There was rapid growth in Grade 7. Succeeding grades showed slower, but consistent, improvement.

On the basis of this experiment, demonstrative geometry can be introduced as early as grade 7 with a reasonable expectation of success.

CRUMLEY, RICHARD D. A Study in Predicting Success in Algebra. (M.A., 1957, Iowa State Teachers College, Cedar Falls.)

Major Faculty Adviser. — Irvin H. Brune.

Problem. — To find a reliable method of predicting the marks of students in algebra for guidance purposes.

Procedure. — The multiple correlation was determined between the algebra marks of a set of students and (1) scores on test 4 (Quantitative Thinking) of the Iowa Tests of Educational Development, (2) composite scores of tests 1-8 of the same series, and (3) intelligence quotients as measured by the Otis Quick-Scoring Mental Ability Test.

Major Findings and Conclusions. — The coefficient of multiple correlation was found to be 0.78, which is significant at the one percent level. A predicting equation was obtained.

CUMMINS, KENNETH BUNDETTE. A Student Experience — Discovery Approach to the Teaching of the Calculus. (Ph.D., 1958, The Ohio State University, Columbus.)

Major Faculty Adviser. — Harold P. Fawcett.

Problem. — To determine whether a student experience — discovery approach to the calculus — produces more proficiency and understanding than the usual methods of college mathematics teaching.

Procedure. — Representative calculus textbooks were studied. Study guides, designed to encourage a maximum of student discovery, were prepared; the general plan of these was to encourage the making of generalizations from numerical examples.

The course was given to one calculus section at Kent State University, Kent, Ohio, throughout each of three quarters. Comparisons in terms of tests were made with control classes taught traditionally. Student reactions were obtained via a questionnaire and a student paper on "What I Conceive the Calculus To Be." A modest study of the change in attitude toward mathematics was also made.

Major Findings and Conclusions. — The experimental group performed significantly better on tests on understanding the calculus, and just as well on manipulative problem-solving proficiency.

DAMGAARD, GENEVIEVE TURNER. A Study to Determine the Significance of a Mental Arithmetic Program for Grade Six. (Ed.D., 1958, Colorado State College, Greeley.)

Major Faculty Adviser. — Edward Kelley.

Problem. — To determine the effect of prepared lessons in mental arithmetic on (1) the success of students in solving, without pencil and paper, mental arithmetic problems presented orally, and on (2) growth in arithmetic as measured by standardized tests.

Procedure. — A test in mental arithmetic was constructed and given at the beginning and end of the experimental period of 13 weeks to 351 sixth grade students from 12 sixth grade classrooms from 11 schools. Instructional lessons in mental arithmetic were given twice a week to the experimental group. A standarized test in arithmetic was also given to all students at the beginning and end of the experiment. Matched in terms of their scores on the standarized arithmetic test, 102 pairs were drawn randomly from the matched pairs in the total

group. Analysis of variance by three levels was used.

Major Findings and Conclusions. — (1) The series of lessons in mental arithmetic produced a statistically significant change in the ability to solve without paper and pencil mental arithmetic problems presented orally. (2) All three ability groups benefited from these lessons by about the same amount. (3) The series of lessons in mental arithmetic significantly increased the general arithmetic ability of the experimental group.

DAUGHERTY, JAMES LEWIS. A Study of Achievements in Sixth Grade Arithmetic in Des Moines Public Schools. (Ed.D., 1957. Colorado State College, Greeley.)

Major Faculty Adviser. — Paul McKee.

Problem. — (1) To compare arithmetic achievement of sixth grade pupils of Des Moines in 50 minute classes with those in 40 minute sections; (2) to analyze the fitness of certain standardized tests in arithmetic for Des Moines schools.

Procedure. — From a total of 596 sixth grade pupils experimental and control groups, taught arithmetic 50 and 40 minutes per day respectively, were formed having the same mean on Form M, Advanced Battery, of the Iowa Every-Pupil Test of Basic Arithmetic Skills. Six months later another form of the same test was used. The differences in gains on these tests between the experimental and control groups were used to determine which practice was better.

Forms L and M of the Iowa test were both given to a distinct group of 100 students to test for equal difficulty. A percentage of errors study was used for this purpose.

The validity of the test forms for Des Moines sixth grade students was determined.

Major Findings and Conclusions. — (1). The 50-minute group surpassed the 40-minute group in gains in overall arithmetic achievement by a statistically significant amount. On Part I (Fundamental Knowledge) the former group gained one grade year more than the latter. On part II (computation) the 40-minute group did better. On part III (problems) the 50-minute group did better. (2) It was decided that forms L and M were not of equal difficulty. (3) Forms L and M were found to be invalid tests for the beginning of grade six. (4) It was recommended that 50-minute arithmetic periods be adopted.

DAVIS, ROBERT BENJAMIN. The Madison Project. (1957, Syracuse University, Syracuse, N. Y.)

Problem. — To determine whether algebra can be taught at the seventh grade level.

Procedure. — For the past 2 years algebra has been taught in the seventh grade of certain central New York State schools. Teachers had monthly conferences with the investigator.

Major Findings and Conclusions. — Seventh grade students seem to be able to learn algebra.

DENNY, ROBERT RAY. A Two- Year Study of the Effects of an Increased Time Allotment upon Achievement in Arithmetic in the Intermediate Grades. (Ed.D., 1958, Colorado State College, Greeley.)

Major Faculty Adviser. — Annie McCowen.

Problem. — To determine the effect on the arithmetic achievement of fifth and sixth grade pupils of increasing the daily class time devoted to arithmetic from 40 to 50 minutes.

Procedure. — A standardized arithmetic test was used to compare the gains made in seven schools using 50 minutes per day for instruction in the fifth and sixth grades with those made in seven comparable schools using 40

minutes per day for a period of 2 years. During the second year the teachers of the 50 minute groups participated in an inservice program.

Major Findings and Conclusions. — (1). The gains of the 50-minute groups surpassed those of the 40-minute groups by a statistically significant amount. (2). The inservice program was successful since the second year superiority of the 50-minute over the 40-minute groups was much greater.

ELLIOT, H. MARGARET. Teaching Freshman Mathematics by Television. (1957, Washington University, St. Louis, Missouri).

Problems. — To compare the results obtained by teaching college algebra and trigonometry by televised lectures and accompanying help sections with those achieved by teaching the courses traditionally.

Procedure. — For each of 58 lessons a 45-minute kinescoped lecture supplemented by text and problem assignments for each lesson were administered to 475 students. The students had five opportunities per day to hear the lecture. Three 2-hour examinations and a 3-hour final examination were given. Grades were based on these tests and on homework which was weighted by 0.2.

Extensive help sections with individual attention were carried out by student assistants, supervised to some extent by two professors.

Major Findings and Conclusions. — Of the 475 television students, 69 percent received grades of D or better, in contrast with 60 percent of the 387 students taught by traditional methods. Students in the television group showed far more interest and enthusiasm for the calculus material. The bright students were not forced to listen to repeated explanations; the slower students were given all of the help they needed.

The elaborate machinery, the extensive use of instructional man-hours, and the large initial expenses make this sort of program impractical for courses with small enrollment.

The closed circuit did not work well.

It was felt that the success of the program was due principally to the help sections conducted outside of the television viewing.

FELDER, VIRGINIA. A Proposal for a Methods Course to be Used in the Education of Teachers of Secondary School Mathematics. (Ed.D., 1959, Teachers College, Columbia University, New York, New York.)

Major Faculty Adviser. — Howard F. Fehr.

Problem. — To propose a plan for a methods course to be used in the education of teachers of secondary-school mathematics.

Procedure. — Professional literature concerning the history and development of methods courses was studied. Methods course textbooks were reviewed. Those individuals providing similar methods courses in Mississippi and outside of it were interviewed. Other qualified educators replied to a "letter of opinion."

Criteria relating to the instructor, the students, and the course were established.

Major Findings and Conclusions. — The instructor must have mathematical and professional preparation. The students must be interested in teaching as well as mathematics. The methods course must be correlated with other parts of the student's education, must be adaptable to the needs and resources of particular college programs, and must be concerned with principles of teaching as well as with specific techniques.

A three-semester-hour course was proposed.

FRICO, HANS. The Status in the Fall of 1957 of the Graduates, Classes of 1950 through 1955, of the Mathematics Department of the New

Jersey State Teachers College, Montclair, New Jersey. (1958, Montclair State College, Upper Montclair, New Jersey.)

Problems. — To determine the status in the fall of 1957 of the graduates, classes of 1950 through 1955, Mathematics Department, Montclair State College, Montclair, New Jersey.

Procedure. — A check-list method of inquiry was employed. The files of the Mathematics Department and of the Alumni Secretary were checked against each other. The filled-in checklists were grouped according to sex as well as to teaching and nonteaching activities.

Major Findings and Conclusions. — (1) One hundred two graduates, including 76 percent of the men and 62 percent of the women, responded. (2) Fifty-two percent of the respondents were in the field of education in the fall of 1957. (3) Seventy-three percent of the men and 67 percent of the women were married. (4) About one-third of the nonteaching respondents and four-fifths of the teaching personnel had done graduate work. (5) Salaries of the beginning teachers rose from below $3,000 in 1950 to about $3,400 in 1955. (6) Most of the respondents had taught their first year in grades 7 to 9, mainly general mathematics and arithmetic. (7) For various reasons slightly less than half were no longer teaching. About one-fourth left teaching after one year, while one-tenth never taught. About half of all the nonteaching respondents, mostly married women, planned to teach in the future. (8) The salaries of the nonteaching men exceeded male teachers by almost $1,000. The corresponding salaries for women were about the same. Four-fifths of the men and one-fourth of the women teachers found it necessary to supplement their salaries.

GARB, REGINA HELEN. A Proposal for a Twelfth-Year Mathematics Program for Selected College Preparatory Students at Bloomfield Senior High School, Bloomfield, N. J. (Ed.D., 1958, Teachers College, Columbia University, New York, N. Y.)

Major Faculty Advisor. — H. F. Fehr.

Problem. — To plan a new program for selected college preparatory students in 12th year mathematics at Bloomfield Senior High School.

Procedure. — A survey of textbooks in use in high schools and colleges was made via questionnaires. Curriculum studies, doctoral studies, curriculum guides, high school and college mathematics textbooks, and periodicals were studied.

Major Findings and Conclusions. — A unified course entitled "Mathematics 12" was proposed. It consists of (1) reorganizing the content of solid geometry, trigonometry, and algebra III; (2) introducing some coordinate geometry of space, algebra of sets, nature of proof, logic, statistics, and integral calculus; and (3) providing for a mathematics seminar period.

GAVURIN, LESTER L. Teachers of Mathematics in Liberal Arts Colleges of the United States, 1888-1941. (Ph.D., 1957, Teachers College, Columbia University, New York, New York.)

Major Faculty Adviser. — Howard F. Fehr.

Problem. — To develop a clear and accurate picture of the teacher of mathematics at liberal arts colleges in the United States from 1888-1941.

Procedure. — The individuals whose careers were studied taught either at one of the 25 oldest colleges in the United States, or at one of 117 sample colleges selected on the basis of size, age, administrative control, and geographic location. Particular emphasis was given to the training and professional work of these teachers.

Major Findings and Conclusions. — The picture of the teacher of mathematics in the 25 oldest colleges changed from an individual trained in the classical tradition to a highly trained specialist in a well-developed field. The careers of those who taught at the 117 colleges followed the same stages as those from the oldest colleges.

GILLESPIE, VERNA GRACE. The Effect of Kindergarten Training on Achievement in Reading and Arithmetic in Grade 2. (M. Ed., 1958, University of Washington, Seattle.)

Major Faculty Adviser. — Henry R. Fea.

Problem. — To determine the relationship of kindergarten attendance to success in reading and arithmetic in grade 2.

Procedure. — Tests appraising mental ability, reading achievement, and arithmetic achievement were given to 279 grade 2 pupils in Vancouver, British Columbia. These were separated into two groups, with and without kindergarten experience, and were comparable in intelligence, chronological age, mental age, sex, and school experience.

Major Findings and Conclusions. — A study of test results showed that there was no significant difference in achievement. The test data did not support the assumption that attendance of kindergarten affects achievement of pupils in reading. However, it was suggested that the results in arithmetic achievement were inconclusive because the test was not valid.

GREEN, MARGARET PATRICIA. A Comparative Study of Two Approaches to the Teaching of Area in the Junior High School. (M.A., 1958, Iowa State Teachers College, Cedar Falls.)

Major Faculty Adviser. — H. C. Trimble.

Problem. — To compare the amount of information learned and retained by seventh grade students of Beloit, Wisconsin, through a textbook and a unit approach to the topic of area.

Procedure. — Analyses of various kinds were used to determine whether the scores on a three-part test on areas indicated the superiority of either the textbook or unit approach.

Major Findings and Conclusions. — The unit method proved superior by several criteria. The discovery aspect of the unit approach seemed especially preferable for students above the 50th percentile in intelligence scores.

GRUBB, BETTY SAYRE. A Proposal for Emphasizing Modern Mathematics in Elementary Algebra. (M.A., 1958, Iowa State Teachers College, Cedar Falls.)

Major Faculty Adviser. — Irvin H. Brune.

Problem. — To prepare, teach, and evaluate materials correlated with present-day textbooks in elementary algebra so as to emphasize modern concepts.

Procedure. — Materials to create, maintain, and extend pupil understandings of set, variable, relation, and function were prepared and used to supplement the standard textbook course in elementary algebra. All of the 21 students were in grade 10 or higher. The method of teaching involved teacher-planned situations which encouraged the students to discover ideas, principles, and relations. The Langton First-Year Algebra Test was given at the beginning and end of the school year.

Major Findings and Conclusions. — The gains in scores on the Langton test were extensive among the 17 students who finished the course. Another outcome was greater pupil sensitivity to reasonable answers and possible answers.

HAMMER, DONALD I. Penetration of Mathematical Problems by Secondary School Students. (Ed.D., 1957,

Teachers College, Columbia University, New York, N. Y.)

Major Faculty Adviser. — Howard F. Fehr.

Problem. — To examine the mathematical problem-solving of secondary school students in order to discover (1) the extent of their penetration, and (2) general characteristics of problems and individuals that affect penetration.

Procedure. — Twelve students from an accelerated 10th-grade class were given five problems of a kind normally encountered in grades 9 and 10. The students thought aloud as they worked each problem. Recordings of the "spoken thoughts" were recorded on tape and transcribed. This material was supplemented by the students diagrams and written work.

The extent of the students' penetration was appraised by judgments of (1) their understanding of the general nature of the problem; (2) their ability to discover explicit conditions and requirements; (3) their ability in discovering implicit conditions and requirements; and (4) their success in perceiving the total structure of the problem. The number of subjects attaining each of these levels was reported, and the manner in which their penetration occurred was discussed.

Major Findings and Conclusions. — Complete penetration occurred in only 15 percent of the protocols. The most noticeable characteristic of the protocols was the uniqueness of each subject's development of understanding.

The subjects' attitude toward solutions seemed to have the greatest bearing on their success. Getting an acceptable answer was the primary concern; understanding the problem and its solution was secondary. Thus, there was a tendency to grasp at superficial clues and to employ learned methods of solution hastily and uncritically.

HEISE, PATRICIA McGINNIS. Enriching the Teaching of High School Algebra by the Integration of Physics (M.S., 1958, Illinois State University, Normal.)

Major Faculty Adviser. — T. E. Rine.

Problem. — To determine (1) why certain physics concepts should be integrated in the teaching of algebra, and (2) if certain concepts of physics can be integrated with the teaching of algebra.

Procedure. — A random sampling of physics teachers in Illinois was made to determine whether physics students who had completed algebra had difficulty with problems of a mathematical nature. A list of algebraic concepts was obtained by analyzing the content of three algebra textbooks widely used in southern Illinois. An analysis of the algebraic concept used in solving physics problems was made.

Major Findings and Conclusions. — The replies of the physics teachers indicated that half of the students who had completed algebra before enrolling in the physics course had trouble with problems of a mathematical nature. A frequency count of algebraic concepts in physics was made. A definite relationship between algebra and physics was established.

HOOTEN, JOSEPH JR. The Production of Televised Mathematics Programs. (Ed.D.; 1958, Teachers College, Columbia University, New York, New York.)

Major Faculty Adviser. — Myron F. Rosskopf.

Problem. — To develop a handbook for the production of televised mathematics programs, and to develop criteria for the effective presentation of mathematics via television.

Procedure. — The criteria were classified under (1) the objectives of the television program, (2) the content of the program, (3) audience identification, (4) audience suitability, (5) the nature

of television and the program, (6) visualization and illustration, (7) the talent, and (8) evaluation.

Major Findings and Conclusions. — The handbook was written. Each criterion was expanded and substantiated. A series of questions was used to summarize the criteria and to make them easy to apply. The primary criterion is that the mathematics must be impeccably accurate and the presentation meaningful to the viewer.

IZZO, JOSEPH ANTHONY. A History of the Use of Certain Types of Graphical Representation in Mathematics Education in the Secondary Schools of the United States. (Ph.D., 1957, Teachers College, Columbia University, New York, N. Y.)

Major Faculty Adviser. — M. F. Rosskopf.

Problem. — To determine the use of certain types of graphical representation in mathematics education in the secondary schools of the United States.

Procedure. — Three hundred sixty-one algebra textbooks, 17 books on integrated mathematics, 138 books on plane, or plane and solid geometry, and 111 trigonometry books were examined for their use of graphs.

Major Findings and Conclusions. — (1) Change in attention to graphical representation since 1900 was more evident in elementary algebra than in any other branch of secondary school mathematics. (2) Algebra textbooks emphasized graphs heavily, but differed as to whether the main emphasis should be in the first or second course in algebra. (3) First year algebra books published since 1923 have placed more importance on the graph as a learning device. (4) In recently published plane geometry books the graph has been used in connection with locus problem and analytic geometry.

KALINOWSKI, WALBERT. The Use of Senior Mathematics Students as Instructors in General College Mathematics. (1959, St. John's University, Collegeville, Minnesota.)

Problem. — To compare the effectiveness of senior mathematics students as teachers of general college mathematics with that of a regular member of the mathematical faculty.

Procedure. — The teaching was done and the results compared.

Major Findings and Conclusions. — It was found that freshman students achieved as well under the student instructors as under the regular member of the mathematical faculty. The study seems to indicate that this is a good way to recruit future teachers of mathematics.

KAUFMANN, JEROME EDWARD. Revised Course for Senior High School Mathematics. (M.S. in Education, 1958, Illinois State Normal University, Normal.)

Major Faculty Adviser. — Francis R. Brown.

Problem. — To propose the basic foundation for a course in advanced mathematics to replace the traditional course in solid geometry for high school seniors in small and medium-sized schools.

Procedure. — Mathematics textbooks were studied. One hundred six high school teachers of mathematics in Illinois were asked about their methods of teaching solid geometry; their opinion of what the content of the advanced course should be; and the continuity pattern of the proposed course. The opinions of authorities in mathematics education were studied.

Major Findings and Conclusions. — (1) There is a need for revision in the high school mathematics program. (2) Solid geometry topics can be integrated into a plane geometry course. (3) Analytic geometry, statistics, and elementary notions of the calculus should be offered in the high school. (4) Teachers and

authorities in mathematics education differ in their judgment of the importance of mathematical instruments and history of mathematics in high school instruction.

KEITH, VIRGIE IRENE. A Comparative Study of the Respective Outcomes Stemming from Two Distinct Patterns of Mathematics Teaching. (M.S., 1958, Virginia Polytechnic Institute, Blacksburg.)

Major Faculty Adviser. -- Homer Howard.

Problem. — To compare the effectiveness of teaching high school mathematics on a pattern based on the Winnetka System, involving a highly individualized approach, with a more traditional plan, emphasizing lectures and recitations.

Procedure. — Three groups of pupils in beginning algebra were taught by the traditional plan during the first semester and by the Winnetka System during the second semester. The Iowa Algebra Aptitude Test was administered at the beginning of the year. The Seattle Algebra Test was given at the end of the first semester and the Lankton First-Year Test at the end of the second semester.

Major Findings and Conclusions. — Pupils made more progress under the Winnetka system than under the traditional method.

KEMENY, JOHN G. Honors Mathematics at Darthmouth. (1956, Dartmouth Collge, Hanover, N. H.)

Problem. — To interest the high aptitude mathematics student during his freshman and sophomore years in a mathematical career.

Procedure. — Students were put into special honor sections, as soon as they entered college, on the basis of College Board scores. These students received special training in small groups, with particular emphasis on understanding, rather than on techniques.

Major Findings and Conclusions. — Over the past 5 years it was found that the procedure described increased vastly the number of mathematics students going on to graduate work. Science departments also profited greatly from this honors program.

KNIGHT, LYMAN COLEMAN. A Study of the Effectiveness of the Subject Matter of Modern Mathematics in the Preparation of Elementary School Teachers. (Ed.D., 1959, University of Pittsburgh, Pittsburgh, Pennsylvania.)

Major Faculty Adviser. — W. D. Sones.

Problem. — To compare the effectiveness of the subject matter of modern mathematics with that of traditional mathematics in the preparation of elementary school teachers.

Procedure. — An experimental group of 30 students was taught a course in elementary mathematics consisting of such modern mathematics topics as mathematical systems, number systems, mathematical proof groups, fields, and sets. The control group of 29 students with the same instructor was taught a traditional college review course, involving the fundamentals of arithmetic and the basic ideas of geometry and algebra. Pretests and posttests of mathematical reasoning, computational skill, and preference of subject areas were administered.

Major Findings and Conclusions. — (1) The control group did not make significant gains in the three variables used. (2) The experimental group made significant gains in mathematical reasoning and computational skill. (3) The experimental group was significantly better than the control group on the midterm test of the following semester.

KNIGHTS, FRANCES ELLURA. The Development of an Instrument to Predict Success in Analytic Geometry of Entering College Freshmen in Engineering and the Indication of Some

possible Improvements Advisable in Their Secondary School Mathematics Courses. (Ed.D., 1957, The Pennsylvania State University, University Park.)

Major Faculty Adviser. — Robert B. Patrick.

Problem. — To develop an instrument to predict a student's probable success in analytic geometry, and, as a result of analyzing the data, to indicate some possible improvements advisable in some of the students' secondary school mathematics courses.

Procedure. — Four tests of comparable difficulty were devised to test the students' command of 34 concepts considered by the investigator and a jury of experts to be needed for success in analytic geometry. Each of the four sections of the freshman students of the 1956 class of Pennsylvania State University who were enrolled in engineering, chemistry and physics, mineral industries, and agricultural engineering took one of the four tests; an extensive statistical analysis was carried out.

Major Findings and Conclusions. — Students need (1) more practice in using abstract symbols other than numbers, (2) a better understanding of definitions and relationships, (3) more experience in analyzing problems, (4) more experience in applying information to new situations, and (5) more practice in making associations and generalizing.

KUSHTA, NICHOLAS PAUL. A Comparison of Two Methods of Teaching Algebra in the Ninth Grade. (Ph.D., 1958, the University of Chicago, Chicago, Illinois.)

Major Faculty Adviser. — Maurice L. Hartung.

Problem. — To compare the effectiveness of teaching algebra in the first half of the ninth grade by a method involving organization around certain unifying concepts with the traditional approach.

Procedure. — In each of five schools the same teacher taught two classes comparable in predicted success in first semester ninth grade algebra. Both the concept group and the traditional group consisted of 131 students. At the beginning of the 17-week experiment each teacher collected data from the school records and administered a specially constructed attitude scale and an instrument for measuring interests. At the end of the experimental period the teacher used the Seattle Algebra Test for the End of the First Half, a test to measure the understanding of the nature of mathematics and repeated the use of the attitude scale and interest instrument.

The traditional group used a standard textbook and the concept group used a specially prepared manuscript.

Major Findings and Conclusions. — (1) There was no statistically significant difference between the two groups as far as manipulative skill was concerned. (2) The concept group surpassed the traditional group by a statistically significant amount in its understanding of the nature of mathematics. (3) The scores on the attitude and interest instruments did not differ significantly between the two groups. (4) The more skillful the teacher in the concept method, the greater was the improvement in the areas investigated. (5) A slight modification of the concept approach produced scores in manipulative skill equal to those achieved in the traditional approach.

LAWTON, MILDRED T. A Twelfth Year Mathematics Program for Hunter College High School. (Ed.D., 1958, Teachers College, Columbia University, New York, N. Y.)

Major Faculty Adviser. — Myron Rosskopf.

Problem. — To formulate for Hunter College High School a senior mathematics course for students who are taking a 4-year major-interest sequence and who have completed the 11th year of New

York State's integrated mathematics program (intermediate algebra and plane trigonometry).

Procedure. — The literature on gifted students was studied. Twelfth-year mathematics programs in some other schools were examined. The history of the mathematics curriculum at the school was reviewed. A set of criteria for determining a 12th-year mathematics program at the school was selected. Alternative courses were tested against these criteria. An outline of the proposed course, as well as suggestions for implementation and evaluation, was developed.

Major Findings and Conclusions. — A course in mathematical analysis seemed most desirable. Seminar work and individual projects were incorporated into the course.

LEO, REVEREND BROTHER CYRIL, *F.S.C.* A Study of the 1956 Engineering Class at Manhattan College. (M.A., 1958, Manhattan College, New York, N.Y.)

Major Faculty Adviser. — Reverend Brother C. John, F.S.C.

Problem. — To illustrate the mathematical treatment of a mass of data by determining the value of certain tests for predicting success of engineering freshmen at the end of the first semester.

Procedure. — The distribution of the scores of 280 freshmen on certain predictive tests were tested for normality. Correlations were found between an "Index of Success" at the end of one semester with (a) scores on the Q-test of the A.C.E. for college freshmen; (b) the P.E.A.T. test; and (c) mathematics scores on the tests of the College Entrance Examination Board.

Major Findings and Conclusions. — It was found that the Q-score and P.E.A.T. score were fairly good predictors. The mathematics scores on the C.E.E.B. test had a low correlation with the "Index of Success."

LESSINGER, MICHAEL LEON. An Evaluation of an Enriched Program in Teaching Geometry to Gifted Students. (Ed.D., 1957, University of California, Los Angeles.)

Major Faculty Adviser. — May V. Seagoe.

Problem. — To investigate the design, implementation, and evaluation of an enriched program in geometry for gifted students.

Procedure. — Gifted students of 118 I.Q. or above studied during the second semester six enrichment exercises entitled Generality and Greek Thought; Functions and Algebraic Loci; Models and Mathematics; Observation and Classification; Axioms, Postulates and Algebra; and Frames of Reference. A control group, taught by the same teacher, was taught the same geometric material as the first group but did not study the enrichment exercises.

Major Findings and Conclusions. — The experimental group (1) showed a better grasp of subject matter; (2) demonstrated a greater understanding of mathematics in general; (3) had greater ability to apply geometry to other fields; (4) developed more creativity and originality; (5) did no better than the control group in assimilating new mathematical material and in appreciating mathematics.

LOMBARDI, LOUIS G. An Investigation to Determine Interesting Activities for a Mathematics Club on the Tenth Grade Level at Union High School, Union, N.J. (1958, Montclair State College, Upper Montclair, N.J.)

Problem. — To work out a program of activities for a prospective mathematics club in the senior high school of Union, N.J.

Procedure. — Twenty preselected mathematics topics were presented to 134 students in classes of 10th grade plane geometry at Union High School. The students rated the topics in terms of

interest. Sixty-five teachers in neighboring counties ranked the topics as to their fitness for mathematics clubs.

Major Findings and Conclusions. — The five categories from which the topics were selected were: (1) field trips; (2) industrial arts projects, such as the tomahawk trisector; (3) pure mathematics, such as elementary topology; (4) recreational mathematics; and (5) history of mathematics.

MADOLE, JAMES EDWARD. Junior College Mathematics for the Semi-Professional Engineer. (M.S., 1958, Illinois State Normal University, Normal.)

Major Faculty Adviser. — T. E. Rine.

Problem. — To determine the need for revision of the mathematics program of the junior college with respect to the training of semiprofessional engineers.

Procedure. — Catalogues of the public junior colleges of Illinois were analyzed to determine what mathematics is now being offered. An analysis was made of the ranking by heads of the engineering departments of various industrial firms of the value of certain mathematical topics in the professional training of semiprofessional engineers.

Major Findings and Conclusions. — Trigonometry, college algebra, analytic geometry, and the calculus have been the mathematics courses taken by students preparing to be semiprofessional engineers. Not later than 1956 intermediate algebra had been added to this list. Further, electronics technician, chemical technician, drafting technician, and engineering aide have become important positions. Of the 3,643 persons employed in the engineering departments more than one-third have 4 years of college training in engineering. More than one-fourth are high school graduates with less than 2 years of such training. Nearly one-sixth have less than 4 years of training at the college level.

The persons ranking the importance of mathematics for semiprofessional engineers were evenly divided as to whether the mathematics preparation was adequate. The subjects most frequently checked for importance were trigonometry, algebra, solid geometry, plane geometry, and analytic geometry, in that order. Of the 162 topics listed, nearly half were rated of some value, and more than one-fourth of much value.

MAJOR, HELEN ELIZABETH. Determination of Specific Problem-Solving Techniques and Their Relation to Problem-Solving in Fifth Grade Arithmetic. (M.Ed., 1957, University of Washington, Seattle.)

Major Faculty Adviser. — Alice H. Hayden.

Problem. — (1) To determine the specific techniques for solving written verbal problems in five fifth grade arithmetic textbooks, and (2) to discover the relationship, if any, between these techniques and the content of the books.

Procedure. — The problem-solving techniques were located and the problems classified according to their degree of unity about a topic of social significance.

Major Findings and Conclusions. — There was no marked relationship between the 28 techniques and the quality level, in terms of the social significance of the problem material. The greatest number of applications of the techniques were to problem-solving involving decimals and measurement. "Analyze the problems orally" was the technique suggested most frequently.

MCCUTCHEON, GEORGE J. An Analytical Study of Achievement in Grade 8 General Science and in Grade Eight General Mathematics in Minnesota Public Schools. (Ph.D., 1957, University of Minnesota, Minneapolis.)

Major Faculty Adviser. — P. O. Johnson.

Problem. — To collect and analyze data

about pupils, teachers, and schools through classes in mathematics and science as taught at the eighth grade level in a stratified random sample of Minnesota Public Schools.

Procedure. — The tests in science and mathematics, the teacher questionnaires, and teacher "logs" were especially constructed for the study. The intelligence quotient was measured with the Otis Quick Scoring Test. Professional literature and research investigations were studied. Pretests and posttests were given 6 months apart in the case of mathematics and 4 months apart in the case of science. Eleven thousand sets of student test data were obtained. Eighty schools were in the sample.

Major Findings and Conclusions (for mathematics only). — (1) There was no statistically significant difference between the mean final achievement in mathematics between boys and girls in 72 Minnesota public schools. (2) There was a statistically significant difference in mean final achievement in mathematics between schools when adjusted for inequalities in mean pretest scores and in mean intelligence quotients. (3) There was a statistically significant difference in mean final achievement in mathematics between the top, middle, and low 5 percent of the pupils in terms of intelligence quotients. (4) In the sample, 60 percent of the schools had statistically significant gains in mathematics achievement; 37 percent showed insignificant gains; 3 percent showed a loss. (5) The middle 5 percent seemed to gain more mathematics information than the high or low 5 percent.

McFarland, Sister Mary Ferrer. Revision and Curriculum Development in Mathematics from First Grade through College. (1958, Saint Xavier College, Chicago, Illinois.)

Problem. — To develop a revised mathematics curriculum from first grade through college.

Procedure. — Since 1952 the faculty of St. Xavier College has been working on the problem of curriculum revision in mathematics.

Major Findings and Conclusions. — (1) At the elementary level the treatment of multiplication and division as inverse operations should be given greater emphasis. The cumulative, associative and distributive laws should be taught at appropriate levels. The notion of a fraction as a ratio should be given more attention. (2) At the secondary level a course in algebra and simple number theory should prove more interesting than general mathematics. The treatment of magnitude and proof should be extended more to mathematics less outmoded than Euclid's geometry. Courses for a 4-year high school program should be mathematics of number, mathematics of magnitude, analytic geometry applied to physics, and mathematics coordinated, respectively. (3) At the college level four courses, geared to varying ability and background, are proposed for the freshman year: Elementary mathematics and methods for the future elementary school teacher; systems of mathematics, a survey course for those who have a good background but who will probably not use mathematics directly; college mathematics, a traditional course for those who will make extensive applications of mathematics; and introduction to mathematical analysis for the better prepared student.

Moore, Donald Albert. Current Methods of Introducing and Giving Meaning to the Notation of Decimal Fractions. (M.Ed., 1957, College of Education, The Ohio State University, Columbus.)

Major Faculty Adviser. — Nathan Lazar.

Problem. — To investigate the methods of introducing and giving meaning to the notation of decimal fractions.

Procedure. — The origin and development of the notation for decimal fractions were reviewed. Modern theories on

the methods of introducing the notation were studied. Nine textbook series were analyzed to determine the development of the concepts essential for understanding decimal fractions. An analysis of the development of decimal fraction notation in fifth grade arithmetic books was made.

Major Findings and Conclusions. — Decimal fractions are introduced as another way of writing common fractions, or as an extension of our notation for whole numbers. Textbooks fail to develop concepts of 10ths and 100ths before introducing the decimal notation. They also fail to provide a rationale for the notation.

It is recommended that the abacus be used to develop the rationale. How this can be done is illustrated.

O'DONNELL, JOHN ROBERT. Levels of Arithmetic Achievement, Attitudes toward Arithmetic, and Problem-Solving Behavoir Shown by Prospective Elementary Teachers. (Ed.D., 1958, The Pennsylvania State University, University Park.)

Major Faculty Adviser. — Clyde G. Corle.

Problem — To survey three factors which may influence the teacher's effectiveness in teaching arithmetic to elementary school children: (a) achievement in arithmetic, (b) attitude toward arithmetic, and (c) arithmetical problem-solving behavior.

Procedure. — One hundred nine elementary education seniors at the Pennsylvania State University took the California Achievement Test and Remmers "Attitude Toward Any School Subject." They also worked 10 problems from the California Test while their comments about their procedures and methods in problem-solving were tape-recorded.

Major Findings and Conclusions. — (1) The mean for arithmetic achievement was 13.14; students with 4 years of high school mathematics made the highest scores. (2) Feelings of like and dislike for mathematics were not closely related to the number of years of mathematics taken. (3) There was little sex difference in the attitude toward arithmetic and problem-solving. (4) Attitude toward arithmetic and like or dislike for arithmetic seemed to be different attitudes.

O'HAIR, CARL D. The Pre-Service Preparation of High School Mathematics Teachers in Selected High Schools of Kansas. (M.S., 1958, Fort Hays Kansas State College, Hays.)

Major Faculty Adviser. — W. Clement Wood.

Problem. — To determine (1) the amount of preservice preparation in mathematics completed by mathematics teachers in class A high schools of Kansas, and (2) what the preparation of these teachers should be.

Procedure. — Questionnaires were sent to 126 high school teachers, 150 administrators, and 50 college mathematics instructors. The answers to these inquiries were analyzed.

Major Findings and Conclusions. — Fifty-nine of the high school teachers responding indicated that they had majors in mathematics. The range in semester hours of mathematics among all high school teachers responding extended from 12 to 75 semester hours with a mean of 31 hours.

Of the college instructors, 90 percent and of the administrators, 87 percent wanted high school and college mathematics revised along the lines of modern mathematics. Of the mathematics teachers, 66 have taken at least one college mathematics course in the last 10 years.

POST, RICHARD. A Study of Certain Factors Affecting the Understanding of Verbal Problems in Arithmetic. (Ph.D., 1958, Teachers College,

Columbia University, New York, N.Y.)

Major Faculty Adviser. — M. F. Rosskopf.

Problem. — To study factors that affect the understanding of a verbal problem in arithmetic.

Procedure. — The following factors were selected for study: size of numbers, superfluous numerical data, familiarity of setting, number of steps, type of operation, and symbolic terms. Each of these factors was defined at two levels. A total of 128 test items were constructed to include all factor combinations. The problems were administered to 176 fifth and sixth grade students in two schools. Only the choice of numbers and operations was used in scoring the test items.

Major Findings and Conclusions. — The three most important factors were the type of operation, the familiarity of the problem setting, and superfluous numerical data.

RAPPAPORT, DAVID. An Investigation of the Degree of Understanding of Meanings in Arithmetic of Pupils in Selected Elementary Schools. (Ed.D., 1957, Northwestern University, Evanston, Ill.)

Problem. — To determine (1) the degree of children's understanding of basic arithmetic concepts and processes used in computations, and (2) the relationship between computational skill and the understanding of a group of basic arithmetic meanings.

Procedure. — A standarized arithmetic test was administered to 271 seventh grade pupils in nine public elementary Chicago schools and to 110 eighth grade students in three of these nine schools to measure the computation skill attained. Three groups of pupils — high, average, and low — were selected on the basis of their test scores.

A test of 72 items was devised to test the students' understanding of 16 basic concepts and operations derived from the number system itself.

Major Findings and Conclusions. — (1) Seventy-nine percent of the seventh grade and 74 percent of the eighth grade students achieved less than 50 percent on the meanings test. (2) The correlation between the computation test and meanings test was .63 for both the seventh and eighth grades. (3) Pupils with high achievement in computation could have low achievement on the meanings test but there were no pupils with high achievement on meanings and low achievement in computation. (4) The pupils tended to follow mechanical processes rather than meaningful interpretations in doing the exercises. (5) Pupils were more concerned about getting answers than in understanding the meanings of arithmetic.

RICHARDS, JOHN FRANKLIN. An Algebraic Approach to Plane Geometry. (M.A., 1957, Iowa State Teachers College, Cedar Falls.)

Major Faculty Adviser. — Irvin H. Brune.

Problem. — To determine what part analytic geometry and graphing could have in establishing a link between high school algebra and plane geometry.

Procedure. — The history of analytic geometry and graphing was studied. The investigator used an algebraic groundwork to be later combined with synthetic methods in traditional geometry. Sets, equalities, inequalities, and graphing on axes having units were emphasized in constructing the "groundwork."

Major Findings and Conclusions. — The materials developed indicate definite possibilities for an algebraic approach to high school geometry. Perhaps, three or four semesters of algebra should precede the geometry in order to develop the necessary algebraic maturity.

ROGLER, PAUL VINCENT. Some Observations on the Value of Homogeneous

Grouping to Achievement in Seventh Grade Mathematics. (Ed.D., 1957, Teachers College, Columbia University, New York, N. Y.)

Major Faculty Adviser. — M. F. Rosskopf.

Problem. — To determine the effects of different grouping methods on the mathematical achievement of seventh grade students.

Procedure. — In Fairlawn, N. J., six classes in seventh grade mathematics were formed so that each had one of these characteristics: students of low IQ, students of low mathematics achievement, students of high mathematics achievement, students low in reading ability, students considered immature, and students above average in social maturity. A standardized mathematics achievement test was given at the beginning and end of the school year; the results were used to calculate the gains in achievement. It was possible to match only within 10 IQ points a student in a homogeneous group with a student from one of the other groups. Relative difference among gains of paired students were analyzed. These comparisons together with faculty evaluations of the groups constituted the observations of the study.

Major Findings and Conclusions. — The use of homogeneous grouping did not seem to have any direct influence on students' achievement in mathematics.

Teachers recommended that the enrollment in slow groups be limited to 14 per class, and that low-achieving students with emotional problems, or poor attitudes, or work habits be kept out of the slow groups.

ROUCHEAD, WILLIAM GEORGE. An Experiment in Tenth Grade Modern Mathematics. (M.S. in Ed., 1958, Illinois State Normal University, Normal.)

Major Faculty Adviser. — Francis R. Brown.

Problem. — (1) To prepare college level materials on modern mathematics for use by the average tenth grade student. (2) To test the suitability of these for such students.

Procedure. — A unit including an introduction to set theory, Venn diagrams, and the graphing of inequalities and equations was developed and taught for 6 weeks to an unselected, average, 10th grade class. Evaluation consisted of observations by the teacher and appraisals of the changes in scores on tests of attitudes and knowledge given before and after the 6 weeks period.

Major Findings and Conclusions. — (1) Certain college level materials on modern mathematics can be successfully taught to average tenth grade students. (2) These topics were of high interest to these students. (3) Greater understanding of basic mathematical concepts can be achieved through the modern topics than through the traditional topics. (4) The achievement of the class was average as far as the traditional topics were concerned.

ROWE, JACK L. General Mathematics for Terminal Students in California Junior Colleges. (Ed.D., 1957, University of Colorado, Boulder.)

Major Faculty Adviser. — Harold M. Anderson.

Problem. — To develop and evaluate a course in nontransferrable general mathematics for terminal students of California junior colleges.

Procedure. — A study was made of recommendations and findings of relevant investigations of the past few years. A questionnaire organized to select objectives, content, and organization of the course was developed and validated by 16 experts. This instrument was sent to the mathematics instructors of all California junior colleges, 198 terminal college junior college students in two California junior colleges, and 150 recent graduates of a California junior college.

Two experimental classes, two classes in business mathematics, and two control classes not taking mathematics were matched on their scores on the Cooperative School and College Ability Test.

Major Findings and Conclusions. — From the questionnaire responses a syllabus was developed for the course. The main topics were: review of fundamental processes of arithmetic; percentage and its applications; business and consumer concepts; budgeting; graphs; elementary algebra; units of measurement; estimation and approximation; problem solving; and the role of mathematics in the world today.

It was concluded: (1) There was a need for such a general mathematics course; (2) the experimental classes made significant gains in achievement in comparison with the other groups; (3) the course provided practical and useful experience in mathematics; (4) these students were able to learn considerable mathematics, in spite of opinions to the contrary.

RUDD, LONIE R. Growth of Elementary School Teachers in Arithmetical Understandings through Inservice Procedures. (Ph.D., 1957, Ohio State University, Columbus.)

Major Faculty Adviser. — Ruth Streitz.

Problem. — (1) To develop inservice procedures for promoting growth in arithmetical understandings; (2) to determine the extent to which these procedures were effective; (3) to recommend procedures for an inservice program intended to improve arithmetic teaching on the elementary school.

Procedure. — Fourteen groups of teachers in service (116) voluntarily took a test of basic mathematical understandings. The inservice procedures centered around an eight-session course, given within each of four school systems and devoted to the development of 72 arithmetical understandings. A variety of teaching aids were used in the course. The effectiveness of the inservice course was measured by: (1) a test of basic mathematical understandings: (2) teacher conferences; (3) teacher questionnaires; (4) teacher summaries; (5) observations from classroom visitations; and (6) teacher opinion check lists.

Major Findings and Conclusions. — Teacher growth in arithmetical understanding resulted. A variety of means in measuring understanding are necessary; the written summary was very effective. Knowledge of the basic concepts among the teachers increased with the grade level at which they taught. Principals have a better understanding of arithmetic than teachers. Teachers at all levels have a poor understanding of fractions and the rationale of computation. Availability of of teaching aids and experience in their use are needed for the effective use of them.

Recommendations for such an inservice course are: (1) the basic arithmetical understandings should be inventoried as an initial step; (2) participation in the course should be voluntary; (3) inservice credit should be given; (4) discussion and class participation should be utilized; (5) application of what is learned to classroom teaching should be emphasized; (6) the sessions should be no more than an hour and a half in duration; (7) the class size should not exceed 20; (8) the course should cut across grade levels; (9) the course should be given during the early part of the year; (10) conferences and classroom visitations should be used during and at the end of the course to provide additional help to the teachers.

SAHAI, PREM NATH. Arithmetic Competencies Possessed by Prospective Elementary Teachers of India. (M.A., 1958, Iowa State Teachers College, Cedar Falls.)

Major Faculty Adviser. — E. W. Hamilton.

Problem. — To investigate the arithmetic performance of prospective elementary teachers in India, using as a criterian the performance of prospective teachers in training at Iowa State Teachers College.

Procedure. — The similarity of the prospective elementary school teachers of India in many respects to those in the Iowa State Teachers College was shown. A 30-item test dealing with ratio, formulas, place-value notation, fraction concepts and two-step computation was constructed and administered to 90 Iowa students just starting a first course in the teaching of arithmetic. The same test was translated into the mother tongue and given to 127 students at three institutions representing three different training programs for elementary teachers in India.

Major Findings and Conclusions. — One Indian school scored significantly above, and two schools scored significantly below the Iowa State Teachers College.

The Indian schools treat inadequately certain notions, such as the zero concept, negative numbers, irrational numbers, and geometric concepts. The idea of interest on money was taught more effectively in all Indian schools than at the Iowa State Teachers College.

SCHEID, FRANCIS. Calculus Through TV. (1958, Boston University, Boston.)

Problem. — To test the effectiveness of teaching calculus through television.

Procedure. — A one-semester introductory college course in the calculus was presented on television. Twenty lectures of one hour duration were given. Discussion meetings were held in the studio weekly.

Major Findings and Conclusions. — The television group learned the calculus slightly better than the regular classroom group. It is suggested that the reason for this may be better-prepared lectures.

SHETLER, LUTHER LEROY. Practices and Trends in the Teaching of Secondary School Mathematics. (Ed.D., 1958, Indiana University, Bloomington.)

Major Faculty Adviser. — Philip Peak.

Problem — To determine the extent to which current practices in the teaching of secondary-school mathematics agreed with the results of certain research studies and proposals.

Procedure. — Proposals of authorities concerning practices in the teaching of secondary school mathematics were used to construct a questionnaire in the areas of aims, curriculum, methods, and evaluation, and it was validated. This instrument was sent to a 10 percent sample of all secondary schools in the 20 states in the North Central Association. The practices found in use, from the responses to the questionnaire, were tested for their compatibility with the proposals of the authorities.

Major Findings and Conclusions. — (1) The authorities and the schools agreed, in general, on the *aims* of instruction. (2) On methods and evaluation the agreement was spotty — high on some items and low on others. (3) The two groups of teachers who followed most closely the practices recommended by the experts were those who taught mathematics during the major part of the school day and those who were receiving one or more types of professional assistance in their teaching. (4) The larger schools and the city public schools tended to follow the recommended practices more closely than the smaller schools. (5) Many teachers had inadequate classroom equipment and poor library facilities. (6) Multiple-track programs were well-established and increasing. (7) Many teachers felt that their mathematics programs were inadequate.

SHIPP, DONALD E., JR. An Experimental Study of Achievement in Arithmetic and the Time Allotted to the Development of Meanings and

Individual Practice. (Ph.D., 1958, Louisiana State University, Baton Rouge.)

Major Faculty Adviser. — George H. Deer.

Problem. — To determine whether variations in the class time devoted to developmental activities and to individual practice work affects achievement in arithmetic.

Procedure. — Four classes in each of the grades four, five, and six were given a distinct treatment. One of the four treatments involved devoting 75 percent of class time to developmental work, a second 60 percent, a third 40 percent, and a fourth 25 percent. Each of the 12 sections was separated into three levels of mental ability. A standardized achievement test was given before and at the end of 12 weeks to appraise changes in arithmetic understanding, computational skill, problem-solving, and total achievement. Analysis of covariance was used to adjust final test scores in terms of initial test scores.

Major Findings and Conclusions. — (1) Groups devoting 75 percent and 60 percent of class time to developmental work achieved a significantly higher total score than those devoting a lesser percent of class time to such work. These groups showed similar superiority with respect to gains in understanding arithmetic and in computational skill. (2) There were no significant differences in problem-solving. (3) The results described in (1) and (2) were similar at the three ability levels.

SHULL, EVAN DWAIN. Recommendations for Secondary Mathematics in Oswego, Illinois. (M.S. in Ed., 1958, Illinois State Normal University, Normal.)

Major Faculty Adviser. — Francis R. Brown.

Problem. — To determine whether the proper mathematical concepts and skills are being taught in the Oswego Community High School.

Procedure. — An analysis of the literature dealing with major mathematical curricular reorganization since 1900 led to general recommendations for setting up a mathematics curriculum. Of the eighty members of groups associated with agriculture, small business, industry, and colleges contacted, 56 replied to a questionnaire on the mathematical fundamentals that should be emphasized.

Major Findings and Conclusions. — (1) Among the four groups questioned there was agreement about the mathematical fundamentals. (2) The colleges and industries desire advanced mathematical training. (3) The "modern mathematical approach" was approved for its general education value. (4) Critical thinking is considered one of the most important products of mathematical training. (5) There is a need for basic mathematical training in the fundamental processes.

SILVEY, INA MAE. The Use of Ratio or Fractional Thinking to Explore Collections in Elementary Arithmetic. (M.A., 1958, Iowa State Teachers College, Cedar Falls.)

Major Faculty Adviser. — H. C. Trimble.

Problem. — To test the hypothesis that the ratio approach will provide no different performance in certain elementary arithmetic situations than the fractional approach.

Procedure. — A test was devised to compare the effectiveness of the ratio and fractional approaches in dealing with parts of a collection. This test was given to 203 fifth grade pupils in the schools of Cedar Falls, Iowa.

Major Findings and Conclusions. — The ratio approach was easier than the fractional approach. If $\frac{a}{b}(c) = d$, then the examples involving d or c as the unknown were easier to solve by the ratio approach.

If the numbers involved were small, like thirds or fourths, there was no difference in the difficulty of the two approaches. However, if the numbers involved were 10s or 100s, and the parts were 10ths or 100ths, the ratio approach was easier.

SOLE, DAVID. The Use of Materials in the Teaching of Arithmetic. (Ph.D., 1957, Teachers College, Columbia University, New York, N. Y.)

Major Faculty Adviser. — Howard F. Fehr.

Problem. — To determine whether the use of a variety of materials in the teaching of an arithmetic topic produces better results than the use of only one material.

Procedure. — Twelve classes consisting of 240 children were used in the experiment. A variety of materials was used in some classes and in other classes only one material. The achievement results were compared.

Major Findings and Conclusions. — Using a variety of materials does not produce better results than using only one material if both procedures are used for the same duration. The effectiveness of the learning of arithmetic depended more on the teacher than on the materials used.

SORENSON, DIANE LEE. A Concept of Area: Two Approaches. (M.A., 1957, Iowa State Teachers College, Cedar Falls.)

Major Faculty Adviser. — E. W. Hamilton.

Problem. — To compare the effects of two procedures in teaching area.

Procedure. — Thirty-two fifth grade Iowa children in the 100-110 intelligence text score range and with a minimum grade reading level of 3.5 were separated into two groups of 16 children each. In method I the experiences were repetitions of finding the areas of rectangular figures. In method II the other 16 children were asked to find areas of varied irregular figures. Shortly after the instruction period, each of the 32 children was tested to see if he could (1) recognize area situations in verbal and diagram form; (2) find area; and (3) define area.

Major Findings and Conclusions. — The two methods resulted in no significant differences. Method II seemed to develop more ability in recognizing and finding areas of selected figures, although both groups did about equally well in recognizing area in verbal situations.

SPOONER, GEORGE A. A Comparative Study of Two Methods of Placing the Decimal Point in the Quotient of Two Numbers. (Ed.D., 1958, Teachers College, Columbia University, New York, N. Y.)

Major Faculty Adviser. — H. F. Fehr.

Problem. — To compare, at the sixth grade level, two methods of locating the decimal point in the quotient of division examples involving mixed decimal numbers and decimal fractions.

Procedure. — Method A involves changing the divisor to a whole number by multiplying the divisor and dividend by the same power of 10. The decimal point is placed in the quotient above the decimal point in the new dividend. Method B involves first adding zeros to the dividend until the number of decimal places in it equals or exceeds those in the divisor. The division is performed as if all the numbers were whole numbers. If the number of decimal places in the dividend is greater than the number of decimal places in the divisor, the difference in these numbers represents the number of decimal places in the quotient.

After 12 periods of meaningful instruction in the two method A and the two method B classes, the four sixth grade groups were given a criterion test.

This consisted of 14 examples requiring only the location of the decimal point in the quotient and adding zeros, if necessary, and six verbal problems of a division type. In the latter a complete solution was required including especially the meaningful labeling of answers.

Major Findings and Conclusions. — (1) There were no differences between the results of methods A and B with respect to accuracy in solving the 14 division examples. (2) With respect to the meaning given to the answers to the verbal problems three statistically significant differences favored method B. (3) With respect to the time used in computation three statistically significant differences favored method B.

STEER, DONALD. A Study of Homogeneous Grouping of College Mathematics Classes (M.Ed., 1957, The Ohio State University, Columbus.)

Major Faculty Adviser. — Nathan Lazar.

Problem. — To determine the extent to which the homogeneous grouping of students of mathematics might provide for the needs of students of varying ability.

Procedure. — The literature was reviewed. Representatives of the mathematics departments of Ohio State University, Capital University, Ohio Wesleyan University, and Denison University were interviewed.

Major Findings and Conclusions. — Homogeneous grouping is used in many colleges. Small colleges implement it through separate courses for different levels of ability, and preparation. Large schools are able to provide homogeneous sections within many of their elementary courses.

Scores in mathematics placement tests seem to be the best criterion for placing the students. Except for entering freshman, grades in previous courses in college mathematics are used. Scores of scholastic aptitude tests and high school credits and grades are also used but are often found to be unreliable.

TOMPKINS, SYDNEY WINANS. The Development of Arithmetic as an Elementary School Subject. (Ed.D., 1958, Teachers College, Columbia University, New York, N. Y.)

Major Faculty Adviser. — Howard F. Fehr.

Problem. — To trace the changes in arithmetic education during the first half of the twentieth century.

Procedure. — The investigator reviewed the leading educational theories, the work of the most prominent arithmetic educators, and a sample of courses and textbooks written during the period from the late 19th to the middle of the 20th century.

Major Findings and Conclusions. — Dewey's influence on the development of the activity program and the project method led to an arithmetic program based on physical measurement and children's experiences. In the 1920's interpreter's of E. L. Thorndike's work stressed an arithmetic based on abstract drill and a social utility approach that would limit arithmetic to that needed in everyday life. In the 1930's Brownell's emphasis on meaning and understanding dominated thinking about arithmetic instruction. Since 1940 the accepted theory has become a combination of the "meaning theory" and certain aspects of the drill theory indicated by research and experience to meet the problem of varying abilities. Fallacies in some of the 1900-1950 theories and practices were due to overemphasis on the psychological to the neglect of mathematical considerations. The poor achievement in arithmetic was due to teachers poorly trained in arithmetic and pedagogy. Child psychology has had a beneficial effect on arithmetic learning in its emphasis on designing learning activities and materials in keeping with the maturity of the learner.

TREMBLAY, CLIFFORD WILLIAM. Topology: A One-Group Experiment in High School. (1957, Montclair State College, Upper Montclair, N. J.)

Problem. — To develop, teach, and evaluate a unit on topology for 10th grade students.

Procedure. — A unit on topology consisting of traversable and nontraversable networks, the Koenigsburg Bridge problem, puzzles of a topological nature, and one-sided surfaces, was developed and taught to 28 students in a second year algebra class in Hackensack High School.

About a day and a half was spent in introducing and discussing the material of the four units.

A test over the material was given before the instructional period and at the end of the second day.

Major Findings and Conclusions. — The students showed by their questions and statements that they were interested in the material. The mean gain between test and retest was 16.5; the mean of the scores on the first test was 80, and on the retest 96.

The test did not seem to have a difficulty range large enough to extend the better students.

TURNEY, BILLY L. An Evaluation of Selected Teaching Aids for Plane Geometry. (Ed.D., 1957, College of Education, University of Houston, Houston, Texas.)

Major Faculty Adviser. — William Yost.

Problem. — To determine the value of certain selected teaching aids for plane geometry, and the use of them.

Procedure. — An inquiry form designed to secure illustrations and brief descriptions of devices particularly helpful in the teaching of plane geometry was sent to 1,000 plane geometry teachers in selected schools of Arkansas, Oklahoma, and Texas. The result was 121 teaching aids, many of them original with the senders. An eight-man jury of experts in the field of study then evaluated the aids as a learning aid, an instructional aid, and a practical aid.

Major Findings and Conclusions. — (1) The use of teaching aids in plane geometry is not widespread. (2) Reasons found for not using these aids were: (a) lack of adequate storage facilities; (b) lack of adequate time for constructing the aids; (c) lack of money; (d) inconvenience of usage; and (e) lack of convenient sources of such aids. (3) These aids should not be constructed for the sake of the aid. (4) Many of these aids have multiple uses.

URBANCEK, JOSEPH J. Arithmetic Teaching Techniques. (1957, Chicago Teachers College, Chicago, Ill.)

Problem. — To find successful procedures for overcoming difficulties in the teaching of arithmetic.

Procedure. — A survey was made of the difficulties met by those engaged in teaching arithmetic. These were organized into seven categories. Seven thousand teachers described their most successful procedures in meeting the difficulties. The results of the study were organized into a book of 364 pages.

Major Findings and Conclusions. — The six major categories of difficulties were: vocabulary, mechanics-of-reading, arithmetical, teaching, problem analysis, and reasoning. Under each of these difficulties are related sub-difficulties and descriptions of successful methods of meeting them.

Many teachers have found the book very useful.

WAHLSTROM, LAWRENCE E. Mathematics Education in Sweden and Norway. (1958, Wisconsin State College, Eau Claire.)

Problem. — (1) To determine how mathematics education in Sweden and Norway differs from that in the United States from elementary school through

the first 2 years of college with respect to content, instructional methods, and points of emphasis, and (2) to locate the external factors that affect the teaching of mathematics in Sweden and Norway.

Procedure. — Over 100 classes and classrooms were visited. Teachers, officials, and "experts" were consulted. University classes were attended regularly. Pertinent library publications were read.

Major Findings and Conclusions. — In both Norway and Sweden: (1) a more gradual development of mathematical ideas occurs; (2) students study mathematics more hours in school and have more homework; (3) students study a more uniform curriculum; (4) teachers are better prepared; (5) instruction is very good but tends toward the traditional; (6) freshman university students spend up to 16 hours per week in class compared with 5 hours in the United States; (7) subject matter tends toward much theory and many applications to geometry; (8) examinations are comprehensive and thorough.

ZIMMERMAN, MAUD-ELLEN. The Preparation of Mathematics Teachers at the Ohio State University During the Five-Year Period from 1953 to 1958. (M.A., 1958, The Ohio State University, Columbus.)

Major Faculty Adviser. — Harold P. Fawcett.

Problem. — (1) To study the preparation of mathematics teachers at the College of Education, The Ohio State University, from 1953 to 1958; (2) to compare these students in certain respects with mathematics majors graduating from the University's liberal arts college; (3) to determine the frequency with which other teaching fields accompany the major in mathematics; and (4) to determine whether early expression of preference for teaching as a career had an effect upon academic success or performance in student teaching.

Procedure. — The data needed were obtained from University records. A "t-test" was used to compare the College of Education students and liberal arts majors with respect to scores on Ohio State Psychological Examination and on five mathematics courses taken by a "sufficient" number of students.

Major Findings and Conclusions. — (1) Of the 63 mathematics teachers graduating during 1953-58, the College of Education group achieved greater academic success and had a greater percent doing "excellent" student teaching than those who transferred to the College of Education from another institution or another college of The Ohio State University. (2) The grades in professional courses (mean: 3.30) were higher than those in mathematics (mean: 2.79). The median number of quarter hours completed in mathematics was 41 and in education 39. (3) Early expression of preference for teaching by half of the group generally accompanied higher grades in student teaching but not in academic preparation. (4) Quality of preparation in mathematics, science, and professional courses was highly associated with degree of success in student teaching. (5) The women surpassed the men in almost every area. (6) Seventy-three percent of the teachers chose at least one area of science as an accompaning teaching field. (7) There was no significant difference in grades on the five mathematics courses taken by both the College of Education and the liberal arts groups. However, the mean score of the liberal arts group on the Ohio State Psychological Examination exceeded that of the College of Education group by a statistically significant amount.

ZLOT, WILLIAM LEONARD. The Role of the Axiom of Choice in the Development of the Abstract Theory of Sets. (Ph.D., 1957, Teachers College, Columbia University, New York, N. Y.)

Major Faculty Adviser. — Myron F. Rosskopf.

Problem. — (1) To determine the factors which influenced the axiomatization of Cantor's theory of sets; (2) to discover the origins of the naive theory of sets; (3) to describe the early attempts to organize the theory of sets into an autonomous discipline; and (4) to determine the influence of the axiom of choice in determining the present form of the theory of sets.

Procedure. — The history of the development of sets was studied.

Major Findings and Conclusions. — (1) The axiomatization of the theory of sets was necessary to eliminate certain paradoxes, due principally to Cantor's definition of a set. (2) The theory of sets was motivated by certain researches in the theory of trigonometric series. (3) The axiom of choice is necessary to prove the logical equivalence of several apparently equivalent definitions of finite sets. It is also needed if set theory is to be visualized as a generalized theory of numbers.