



**TEDS-M**

# TEACHER EDUCATION AND DEVELOPMENT STUDY IN MATHEMATICS (TEDS-M)

Policy, Practice, and  
Readiness to Teach  
Primary and Secondary  
Mathematics

*Conceptual Framework*

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Glenn Rowley



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**TEDS-M**

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- DPC: Dirk Hastedt (*ex-officio*), Ralph Carstens (*ex-officio*), Falk Brese (*ex-officio*), Sabine Meinck (*ex-officio*)
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### **TEDS-M International Sampling Referee**

- Jean Dumais, Statistics Canada



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The success of any IEA study depends on mobilizing many different sources of support, influence, funding, and expertise in all parts of the world. TEDS-M is no exception. In fact, as a pioneering study unlike any other previous cross-national study, it requires even more willingness to deal with unknowns and to confront new challenges. We are therefore extremely grateful for the extraordinary support and work that has already gone into designing and launching the study described in this conceptual framework document.

At the core of IEA studies are the national research coordinators (NRCs) of each participating country and their national centers. This study has purposefully recruited and actively sought feedback and contributions from the research coordinators and personnel on the conceptual framework and the development of items and reviews of the manuals. Through their participation and insights from the field, the NRCs continue to improve the study daily. The individuals coordinating this effort in the participating countries deserve special mention: Zuwaina Al-Maskari (Oman), Beatrice Avalos (Chile), Horst Biedermann (Switzerland), Sigrid Blömeke (Germany), Tamar Bokuchava (Georgia), Pierre Brochu (Canada), Precharn Dechsri (Thailand), Michal Federowicz (Poland), Evangeline Golla (Philippines), Liv Grønmo (Norway), Feng-Jui Hsieh (Chinese Taipei), Pi-Jen Lin (Chinese Taipei), Maia Miminoshvili (Georgia), Thabo Jeff Mzwinila (Botswana), Ester Ogena (Philippines), Fritz Oser (Switzerland), Supatra Pativisan (Thailand), Luis Rico (Spain), William Schmidt (USA), and Khoon Yoong Wong (Singapore).

The planning for TEDS-M began with a series of international meetings during which this conceptual framework was developed, discussed, and modified in response to feedback. The first meeting was the very influential IEA technical meeting in Belgium in November 2002. Aspects of the TEDS-M framework have been presented annually during the IEA General Assembly meetings in Morocco in 2002, Cyprus in 2003, Taipei in 2004, Helsinki in 2005, Amiens in 2006, and Hong Kong in 2007. The framework has also been discussed in specialized international meetings attended by mathematicians and mathematics educators as well as experts in related areas such as science education, teacher education, international and comparative research, quantitative and qualitative research methodology, and policy analysis (see a list of participants in Section 9, Appendix B). We are very thankful to all these participants for their important role in shaping the TEDS-M conceptual framework.

In addition, the progress of the TEDS-M study was facilitated in part by a feasibility study titled, *Developing Subject Matter Knowledge in Mathematics Middle School Teachers: A Cross-national Study of Teacher Education as a Follow-up of TIMSS*. The final report of the study was released in December 2007 under the name MT21.<sup>1</sup> It was initiated in 2003 by Michigan State University, but limited funding meant only a few countries could be involved. This feasibility study worked with six countries to develop and try out instruments and methods to inform the main TEDS study of secondary mathematics teacher education, and thus it was important in convincing countries and

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<sup>1</sup> See NSF Press Release 07-185 *U.S. middle school math teachers are ill-prepared among international counterparts*. Available online at [http://www.nsf.gov/news/news\\_summ.jsp?cntn\\_id=110845&org=EHR](http://www.nsf.gov/news/news_summ.jsp?cntn_id=110845&org=EHR).

funding agencies of the feasibility of TEDS-M. The feasibility study was carried out with funding from the National Science Foundation (NSF).<sup>2</sup>

The initial concepts and methods for both MT21 and TEDS-M came from the work of Maria Teresa Tatto on teacher education effectiveness and costs as part of the BRIDGES Project, conducted in collaboration with Harvard University (1987–1992) and funded by the United States Agency for International Development; and from the work developed by the Teacher Education and Learning to Teach Study at the National Center for Research on Teacher Education at MSU (1985–1990), which was funded by a grant from the Office of Education Research and Improvement, United States Department of Education.<sup>3</sup>

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2 Schmidt, W., & Tatto, M. T. (2002). *Developing subject matter knowledge in mathematics middle school teachers: A cross-national study of teacher education as a follow-up of TIMSS* (NSF REC 0231886/January 2003). Schmidt, W., Tatto, M. T., Bankov, K., Blomeke, S., Cedillo, T., Cogan, L., Han, S. I., Houang, R., Hsieh, F. J., Paine, L., Santillan, M., & Schwille, J. (December, 2007). *The preparation gap: Teacher education for middle school mathematics in six countries* (MT21 report). East Lansing, MI: Michigan State University (NSF REC 0231886/January 2003).

See also NSF Press release 07-185 *U.S. middle school math teachers are ill-prepared among international counterparts*. Available online at [http://www.nsf.gov/news/news\\_summ.jsp?cntn\\_id=110845&org=EHR](http://www.nsf.gov/news/news_summ.jsp?cntn_id=110845&org=EHR).

3 Tatto, M. T., Nielsen, H. D., Cummings, W. C., Kularatna, N. G. & Dharmadasa, D. H. (1993). Comparing the effectiveness and costs of different approaches for educating primary school teachers in Sri Lanka, *Teaching and Teacher Education: An International Journal of Research and Studies*, 9(1), 41–64. Available online at <http://ed-share.educ.msu.edu/scan/te/mttatto/TATTO010.PDF>.

For NCRTE research, see <http://www.informaworld.com/smpp/content~content=a739614251~db=all>. See also the following:

Tatto, M. T. (1996). Examining values and beliefs about teaching diverse students: Understanding the challenges for teacher education. *Educational Evaluation and Policy Analysis*, 18, 155–180.

Tatto, M. T. (1998). The influence of teacher education on teachers' beliefs about purposes of education, roles, and practice. *Journal of Teacher Education*, 49, 66–77.

Tatto, M. T. (1999b). The socializing influence of normative cohesive teacher education on teachers' beliefs about instructional choice. *Teachers and Teaching*, 5, 111–134.

This framework could not have been completed without our sampling specialists. Pierre Foy, then at the IEA DPC, was the initial consultant. When Jean Dumais of Statistics Canada was appointed sampling referee, he became the author of the sampling section of this document and addressed many more sampling challenges that differed greatly from those of the IEA pre-university studies. We have also benefited from the advice of Marc Joncas, another sampling specialist at Statistics Canada.

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This study builds on the assembled knowledge, of and experience in, many fields and countries. It also takes into account the results of other important comparative international studies in education, such as the Trends in International Mathematics and Science Study (TIMSS), the Organisation for Economic Co-operation and Development's Programme for International Student Assessment (PISA), and the European Union's Eurydice.

The project is highly dependent on the efforts of all the people listed elsewhere on the TEDS-M personnel page. In particular, we would like to acknowledge the key roles played by Inese Berzina-Pitcher, consortium coordinator, and Ann Pitchford, administrative assistant, at MSU.

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TEDS-M owes its existence to its funding agencies. Each of the participating national centers must raise its own funds to cover many of the costs of the study. Our thanks go to these national agencies for their support. We are thankful to the IEA organization for generous funding to get this study underway and for its ongoing financial support throughout the study's duration. The study could not be completed, however, without a very substantial grant to cover remaining international costs. In the case of TEDS-M, this major funding has come from the United States National Science Foundation (NSF).<sup>4</sup> This award was made in 2005. We are thankful to program officer, James Dietz, for his dedication to this project and for his constant support, and to others at NSF for their interest in this new type of cross-national research: Janice Earle, Larry Suter, and Elizabeth Vanderputten.

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The TEDS-M Team  
AUGUST 2008

## Preface

Over the last 50 years, the International Association for the Evaluation of Educational Achievement (IEA) has conducted more than 23 large-scale comparative studies of student achievement. The work associated with teacher preparation as well as experience gained in many of IEA's studies, such as TIMSS, led to a request from members of the organization for an in-depth investigation of teacher preparation and training, particularly in terms of the subject area of mathematics. This document provides the framework and specifications for the first IEA study of teacher preparation and training of primary and lower secondary mathematics teachers.

The framework outlines a comprehensive approach to investigating and understanding teacher preparation around the world and includes an attempt to understand how national policies and institutional practices influence the outcomes of beginning teachers. This focus on what beginning teachers know and can do makes this study the first of its kind.

Development of the framework was a collaborative effort, which involved input not only from individuals who attended a series of expert group meetings but also from study participants. Their contributions were instrumental in helping clarify the many technical issues related to advancing a project of this complexity. IEA is grateful for their work.

In addition to IEA's own resources, critical support for the development of this framework was provided by the United States National Science Foundation (NSF). Without this support, this project would not have been possible.

Although this document represents the work of a considerable number of people, I would like to express my thanks in particular to the study director Maria Teresa Tatto and to the principal investigators Jack Schwille and Sharon Senk from Michigan State University. I also extend sincere thanks to Lawrence Ingvarson, Ray Peck, and Glenn Rowley from the Australian Council for Educational Research for their leadership. Jean Dumais from Statistics Canada and colleagues at the IEA Data Processing and Research Center and the IEA Secretariat also made important contributions to the development of this project and deserve special thanks.

IEA looks forward to the outcomes of this project. The association believes these findings will make a significant contribution to our understanding of how the goals of teacher education policies are realized in practice around the world.

Hans Wagemaker  
Executive Director, IEA  
NOVEMBER 2008



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# 1. Overview of the IEA TEDS-M Study

This document outlines the framework and a comprehensive plan for a cross-national study of primary and secondary mathematics teacher education (TEDS-M) sponsored by the International Association for the Evaluation of Educational Achievement (IEA). TEDS-M 2008, which builds on the results of TIMSS and other earlier studies, focuses on *how teachers are prepared to teach mathematics in primary and lower secondary school*.<sup>5</sup> TEDS-M is a study of the variation in the nature and impact of teacher education programs within and across countries. The study collects and analyzes nationally representative data from participating countries to address contested issues and improve policy and practice in teacher education. This document lays out the goals and justification for this study as well as its design and methodology.

The overall study has three overlapping components:

- COMPONENT I: Studies of teacher education policy, schooling, and social contexts at the national level.
- COMPONENT II: Studies of primary and lower secondary mathematics teacher education routes, institutions, programs, standards, and expectations for teacher learning.
- COMPONENT III: Studies of the mathematics and related teaching knowledge of future primary and lower secondary school mathematics teachers.

The key research questions for the study focus on the relationships between these components, such as relationships between teacher education policies, institutional practices, and future teacher outcomes.

## 1.1 IEA and the Study of Teacher Education

Teacher education has become an area of considerable interest among policymakers in many countries over recent years, a development that underlines the central importance of teacher knowledge to quality learning. IEA's interest in this study reflects the need to produce usable knowledge that will help inform policy to assist in the recruitment and preparation of a new generation of teachers as knowledge demands change and large numbers of teachers reach retirement age.

The TEDS-M 2008 study gathers data at the following three levels of teacher education systems across participating countries:

1. *Outcomes*: What is the level and depth of the mathematics and related teaching knowledge attained by prospective primary and lower secondary teachers? How does this knowledge vary across countries?
2. *Institutions and programs*: What are the main characteristics of teacher education institutions and their programs? In what ways do these vary across countries? What are the learning opportunities available to prospective mathematics teachers (primary and lower secondary)? How are these structured (e.g., what is their level of internal/external coherence)? What content is taught in teacher education programs and how is this instruction organized?

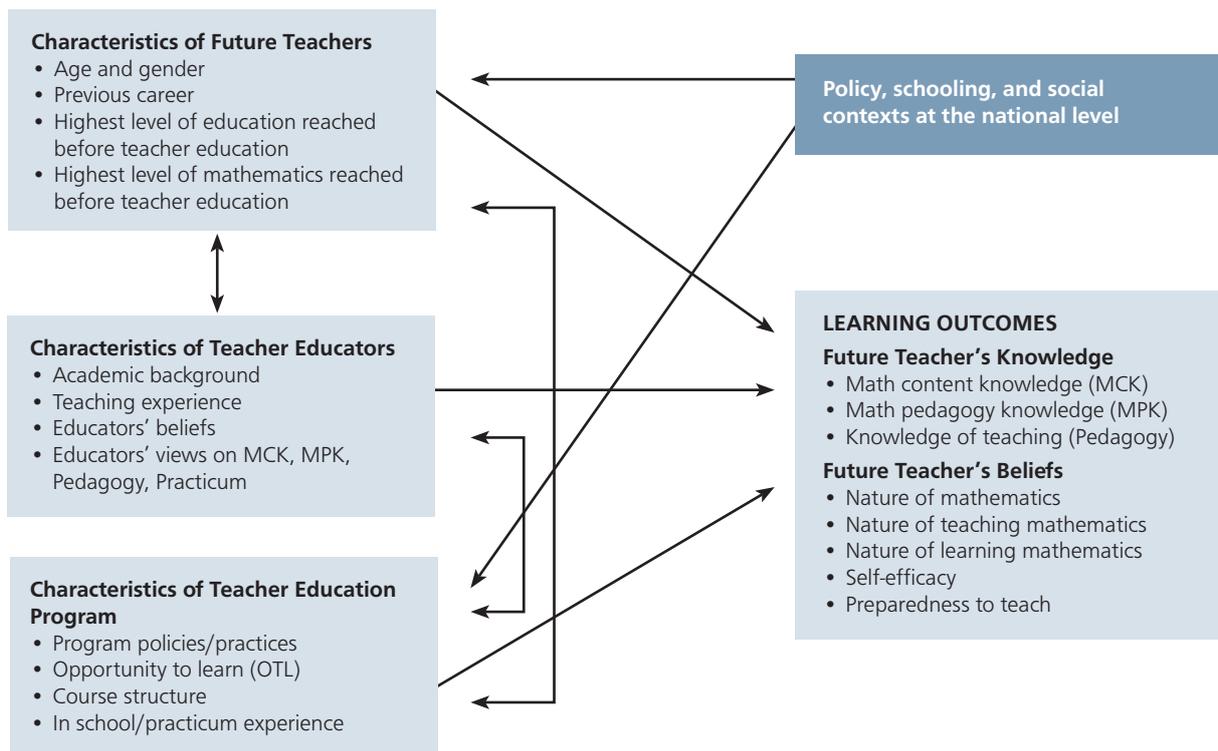
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<sup>5</sup> These teachers include specialist and non-specialist teachers who may end up teaching mathematics.

3. *National policy*: What is the national policy context for teacher education regarding, for example, recruitment, curriculum, quality assurance, and funding? How do these policies vary across countries?

Figure 1 shows the anticipated interrelationships among the key sets of variables examined in TEDS-M and their location in a larger context.

Figure 1 Interrelationships among Variables Examined in TEDS-M



TEDS-M seeks answers to the following research questions:

- What is the impact of mathematics-related policies for quality assurance and the accreditation of teacher education programs on teacher education institutions, programs, and outcomes?
- How do national or program policies influence the recruitment, preparation, graduation, and retention of teachers of mathematics?
- What are the characteristics of teacher education policies, institutions, and programs that lead to high levels of mathematics knowledge and knowledge of mathematics pedagogy in future teachers?
- What relationship is there between the beliefs about mathematics of teacher educators and those of future teachers?
- What kinds of practicum arrangements and school experiences are most effective in preparing future mathematics teachers?
- What are the costs of programs in different settings?

Although the degree to which these questions can be addressed is governed to some extent by the culture of each participating country, the overall goal of this study is to find better ways to help teachers learn what they need to know to teach mathematics well at the primary and lower secondary school levels. In pursuing this goal, this study aims to speak to diverse audiences.

In the case of educational policymakers, the aim is to suggest institutional and program arrangements that are effective in helping teachers become sufficiently knowledgeable. For teacher educators who design, implement, and evaluate teacher education programs, the primary aim is to give them a shared language and a shared database, and then shared benchmarks for examining their programs against what has proved possible and desirable to do in other settings. For mathematics educators, the purpose is to provide a better understanding of what qualified teachers of mathematics are able to learn about the content and the pedagogy of mathematics and the conditions these teachers need to acquire this knowledge. For educators in general and for informed laypersons, the purpose is to provide a better understanding, backed by empirical research, about how and what teachers learn as they prepare to teach.

## 1.2 The Unique Contribution of IEA TEDS-M

To date, no empirical cross-national study based on probability samples has analyzed how education systems prepare teachers of mathematics (or any other subject for that matter) or identified the explicit and implicit expectations for what they should know and be able to do as a result of this preparation. The goal of TEDS-M is to enable countries to compare themselves with other countries in terms of what they expect of their future teachers of mathematics and their ability to help students learn this subject matter. Countries can explore the opportunities other nations give their future teachers in order to meet these expectations. These comparisons should provide valuable insights for policymakers and teacher educators regarding diverse teacher education strategies in specific settings.

As the first cross-national project to attempt this sort of study, IEA's TEDS-M has several signature features (Box 1.1).

### **BOX 1.1: The Unique Features of IEA TEDS-M**

TEDS-M 2008 is the *first*:

- IEA study of higher education
- IEA study of teacher education
- Cross-national study of teacher education based on nationally representative probability samples
- Cross-national study of teacher education to gather data on the knowledge outcomes of teacher education as well as possible determinants of these outcomes
- Cross-national study of teacher education to integrate a specific subject matter (mathematics) with generic issues in teacher education policy and practice
- Cross-national analysis of the curricula of mathematics teacher education
- Large-scale comparative study to address the costs of teacher education.

This study builds on a strong research foundation and explores crucial questions to improve teacher education at a time of rapid global economic and social change. The strong foundations laid by previous studies such as TIMSS, TIMSS-Video, and PISA have allowed the development of a powerful design for this first cross-national study of mathematics teacher education. TEDS-M aims to stimulate fruitful cross-national dialogue among policymakers and educators regarding teacher education policy aimed at improving preparation for and practice in mathematics teaching.

### 1.3 The Need for TEDS-M

The justification for this study and the development of its conceptual framework, design, and methodology are informed by an extensive review of the research literature and rest on the following findings.

**1. *There is significant variation in student achievement levels in mathematics across countries.***

The TIMSS 1999 data showed that, at Grade 8 level, 94% of Singapore students achieved the “top half level” of the international marker of student achievement (i.e., 94% of Singapore students scored above the international mean of 509 points in the mathematics tests). A high percentage of students from Korea (82%), Japan (83%), and Hong Kong SAR (80%) also surpassed the score of 509 points. Students from other countries performed less well. In England, 48% of the tested Grade 8 students achieved the 509 score, a percentage similar to that for students in the United States (45%). In Spain, 36% of students reached the top half level, while in Portugal only 19% of the students did so (Mullis et al., 2000).

**2. *The school mathematics curriculum varies among countries, but we know little of how this variation affects teaching and learning.***

The heterogeneous performance of students in different countries has been explained in terms of important differences in the curricula of mathematics, including substantial variation in the topics included in the curriculum (textbooks, content standards, and teachers’ reports) across countries (McKnight et al., 1987; Valverde, Bianchi, Wolfe, Schmid, & Houang, 2002). For instance, at a macro level, by Grade 8 in Japan all students have studied a good deal of algebra, but in many countries most students are just beginning to study algebra. In TIMSS 1999, only half of the participating countries covered the topic “estimating computations” and only about a fourth of these countries covered “complex numbers and their properties.” This variability is also evident in complex performance expectations in Grade 8 mathematics textbooks. In Iran and in Slovenia, the incidence of such expectations was over 70%. The rate was between 40% and 50% in textbooks from the Czech Republic, Germany, and Spain. In Australia, Colombia, Hungary, and Hong Kong SAR, the incidence of complex performance expectations was less than 10% (Mullis et al., 2000).

**3. *Mathematics teaching varies in quality across countries.***

Similar variability was found in the TIMSS 1999 Video Study (Hiebert et al., 2003), which rated a sub-sample of mathematics lessons in five countries for coherence, presentation, student engagement, and overall mathematics quality. Ratings ranged from 1 for low to 5 for high. The study reported an average rating in each country: Hong Kong SAR (4.0), the Czech Republic (3.4), Switzerland (3.3), Australia (2.9), the Netherlands (2.7), and the United States (2.3). Although these results should be interpreted with caution because of the use of small sub-samples, the differences in

the ratings suggest important variation in the teaching of mathematics that may help explain differences in achievement tests across countries.

The 1999 TIMSS Video Study also pointed to other important differences in opportunities to learn mathematics across these countries, such as the content included in the lessons; the coherence across the mathematical problems presented in lessons; the topics that were introduced; the procedural complexity with which these topics were presented; and the quality of the individual student work and homework in class. Likewise, in the earlier, first TIMSS Video Study, an expert panel rated the overall quality of mathematics lessons in the samples for Germany, Japan, and the United States. The lessons were sorted into three quality categories: low, medium, and high. In Japan, 51% of the lessons were rated as medium quality and 39% of the lessons were rated high. In the United States, 89% of the lessons were rated low quality; no lesson was rated high. In Germany, low-quality lessons made up 34% of the whole sample, and high-quality lessons made up 28% of the entire sample. A similar pattern across these three countries was found regarding mathematical concepts that were “developed” by the teacher with the participation of students (77% in Japan and 22% in the United States) as opposed to simply “stated” by the teacher (23% in Japan and 78% in the United States) (Stigler & Hiebert, 1997, 1999).

#### 1.4 Assumptions Underlying the TEDS-M Design

Since the literature indicates that mathematics is taught in different ways in different countries and that this variation may be related to the ways in which future teachers are prepared, we turn to the issues posed by studies of teacher preparation. This study is constructed to provide a strong empirical base from which to assess the impact of teacher education policy, as mediated through institutional and organizational arrangements, on the quality and depth of teachers’ mathematical knowledge and abilities. Previous work supports our making certain assumptions to serve as the starting points for TEDS-M.

*1. Teacher education is understood and implemented differently across national settings and even between institutions within the same country.*

The recent OECD studies on recruiting, preparing, and retaining effective teachers indicate that teacher education varies in very important ways across the 25 participating countries. The synthesis report *Teachers Matter* (Organisation for Economic Co-operation and Development/OECD, 2005) indicates, for example, that in some countries students start teacher education directly after secondary school. However, in others, prospective teachers already have a university degree or other experience. In a number of countries, universities are not the only providers and others may include teachers’ colleges and/or polytechnics.

The report also indicates that some teacher education programs are “concurrent” in that teacher education is combined with undergraduate preparation in a discipline outside education. Other programs are “consecutive” in that the teacher education program with a focus on pedagogy and teaching is taken after the candidate has finished a first degree in a subject-matter area. According to the report, some countries have a “unitary” system with only one “route” to becoming a teacher. Others are “segmented” and offer more than one route. Some routes are concentrated within university faculties of education while others are spread through a number of faculties in large universities.

According to the report, programs also vary greatly in duration. “The range is from 3 years (e.g. for some primary teachers in Ireland and Spain) up to 6.5 years for some secondary teachers in Germany, 7 years in the Slovak Republic and 8 years in Italy” (p. 21). Duration also varies within countries. The report notes that program content differs in terms of subject-matter knowledge, pedagogic knowledge, educational sciences, educational psychology, and practical experience (OECD, 2005).

The report also indicates that different agencies are responsible for the certification of new teachers. These include institutions of higher education, professional bodies, and state authorities, which impose varying requirements. For example, “a number of countries (e.g., France, Germany, Greece, Italy, Japan, Korea, Mexico, and Spain) use a competitive approach ... Examinations may include observation of the candidate’s teaching, in-depth interviews or consideration of portfolios with records of achievement and work experience” (p. 22).

Differences in organizing and implementing teacher education are expressed in the diverse language used to describe the components of teacher education around the world (Eurydice, 1998, 2002; Stuart & Tatto, 2000; UNESCO, 1998). For example, the word “pedagogy” has a wide array of meanings, ranging from a narrow technical focus on teaching technique (as used in the United States) to a broad concern with everything that happens in the classroom, including its moral and philosophical underpinnings (Hamilton & McWilliam, 2001). The broader view is represented in European discourse on teacher education, where the term “general pedagogy” may be used to designate all non-subject-matter-specific theoretical aspects of teacher education programs. In the United States, these would be considered “educational foundations.”

Terms such as “practicum” and “student teaching” also vary across contexts. For example, in some settings, the practicum experience may occur throughout a student’s university career and be tightly connected to university coursework; in other places, the practicum—or field experience more generally—is a stand-alone experience. Similarly, the chief activity of the practicum may require the future teacher to take the lead in lesson planning and teaching students (as is often the case in a number of systems in the European Union and the United States). In other places, such as China, the practicum experience, or “student teaching,” emphasizes observation and detailed preparation of a very small amount of classroom teaching (perhaps as little as four to six periods in total).

Even when the number of weeks required do not differ dramatically, there is variation in the nature, focus, and substance of the activities in question (Britton, Paine, Pimm, & Raizen, 2003; Paine, 1990). The challenge, in this case, is gaining clarity about the various forms that teacher education takes (in logic, timing, content, and structure) and remaining acutely sensitive to this variation in language in order to deepen understanding of the phenomenon in question. The definitions included in this framework therefore are not meant to be definitive, but rather a beginning point for this process of clarification.

The differences in teacher education are also reflected in its costs. The economic dimension of teacher preparation includes the variable costs of becoming a teacher across countries as well as the expenses institutions incur in preparing teachers across the disciplinary spectrum. The conceptual and empirical work conducted by Tatto and her colleagues during their pioneering international study on the effectiveness and costs of different models of teacher preparation (Tatto, Nielsen, Cummings, Kularatna,

& Dharmadasa, 1990, 1993) served as a prototype for the preparatory study for TEDS-M (i.e., MT21, also known as P-TEDS) and TEDS-M itself.

**2. *Teacher education and teacher learning are complex, contested processes.***

Any research project based on the aims discussed here has to consider the complexities of teacher education and teacher learning. These complexities relate to lack of agreement among experts, policymakers, and reformers about what knowledge is most important to teach; competing views concerning the importance of subject matter, pedagogy, and knowledge of students; the relationship between theory and practice; disagreement over what teachers learn best from experience; the lack of standardization or even shared expectations for many aspects of programs; and variation in the prior knowledge of future teachers (Schwille & Dembélé, 2007; Tatto, 1999a, 2007).

**3. *Knowledge of the content to be taught is a crucial factor influencing the quality of teaching.***

A central motivation behind policy and programs related to the education of mathematics teachers is to increase the quality of teaching and consequently the quality of student learning. But the quality of teaching involves multiple dimensions and therefore cannot be assessed on the basis of only one dimension (Tatto, 2001). Nevertheless, while experts may fail to reach a consensus on the relative importance of these dimensions, all agree that knowledge of some kind is central (Monk, 1994; Porter, Floden, Freeman, Schmidt, & Schwille, 1988). There remain, however, marked differences among stakeholders on what knowledge is important for teachers to acquire, how it is to be acquired, and how important that knowledge is to ensuring a teacher becomes a good teacher (Grossman, 1990). Of particular importance for formal teacher education is the common criticism which asserts that teachers who are well-prepared in mathematics content knowledge can generally learn on the job most of what they need to teach well.

**4. *Teacher education requires understanding of and addressing how teachers should think about mathematics, teaching, and learning.***

There is no universally accepted definition of knowledge for mathematics teaching, but nevertheless the last two decades have seen considerable progress in the development of domain-specific understanding of teacher knowledge. The roots of the concept of knowledge for teaching can be traced to ideas expressed by Lee Shulman in his 1985 presidential address to the American Educational Research Association. Later, Shulman (1987) identified three categories of teachers' knowledge: subject-matter knowledge, pedagogical content knowledge, and curricular knowledge. According to Shulman, *subject-matter or content knowledge* is the set of fundamental assumptions, definitions, concepts, and procedures that constitute the ideas to be learned. *Pedagogical content knowledge* (PCK) includes useful forms of representation of those ideas, powerful analogies, examples, and explanations of a subject, insights into what makes the learning of specific topics easy or difficult, and the conceptions that students of different ages and backgrounds bring with them to the learning of the topic. *Curricular knowledge* involves understanding how the topics are arranged over time across schooling experiences.

Many scholars (among them, Ball & Bass, 2000; Ball & Cohen 1999; Even, 1993; Even & Tirosh, 2002; Fennema & Franke, 1992; Lappan, 2000) have since used Shulman's categories of knowledge to characterize knowledge for teaching mathematics. Fennema and Franke (1992) argue that different contexts demand different knowledge and that teacher knowledge cannot be separated from teacher beliefs (see also DeCorte, Op't Eynde, & Verschaffel, 2002; Grouws, 1992).

Other scholars, such as Fan and Cheong (2002), Kilpatrick, Swafford, & Findell (2001), and Ma (1999), have introduced related terms to describe aspects of knowledge for teaching mathematics. For instance, Fan and Cheong (2002) suggest three aspects of pedagogical knowledge: pedagogical curricular knowledge, consisting of knowledge of teaching materials and resources, including textbooks and technology; pedagogical content knowledge, including knowledge of ways to represent concepts and procedures; and pedagogical instructional knowledge, consisting of knowledge of teaching strategies and classroom organizational models.

Measuring teacher knowledge, let alone the knowledge of *future teachers*, is a complex and challenging task and there is little agreement on what to measure and how to measure it (Ball, Lubienski, & Mewborn, 2001). Hill, Sleep, Lewis, and Ball (2007) give a good overview of attempts in the United States. They cite the Praxis series, INTASC, the NBPTS assessments, and their own work at the University of Michigan with the Learning Mathematics for Teaching (LMT) project.

Measuring teacher knowledge cross-nationally requires paying attention to the differences between what Europeans call *didaktik* or *didactique* and what Americans call knowledge of pedagogy and teaching methods. In their useful review of the didactics of mathematics and the professional knowledge of teachers, Boero, Dapuetto, and Parenti, (1996) concluded that due to differences and difficulties created by local institutions, and by cultural aspects and traditions, ‘tools and results of research in the didactics of mathematics frequently remain outside the core of mathematics teacher education, with a considerable waste of skills and energies.’

Since this review by Boero and colleagues, the work of Baumert, Blum, and colleagues in the COACTIV project in Germany has advanced the study of didactics in secondary mathematics. Influenced by Shulman’s work, they defined three aspects of mathematics teachers’ professional knowledge: knowledge of mathematical tasks, knowledge of student misconceptions and difficulties, and knowledge of mathematics-specific instructional strategies. A key finding of their work was that, ‘when mathematics achievement in grade 9 was kept constant, students taught by teachers with higher pedagogy content knowledge (PCK) scores performed significantly better in mathematics in grade 10.’ This finding confirms many other findings in the United States and elsewhere, namely that the so-called PCK should be considered a core candidate for creating powerful learning environments that continue to support students’ knowledge construction.

The TEDS-M framework defines mathematical knowledge for teaching as comprising two main subsets of knowledge: mathematical content knowledge and mathematics pedagogical content knowledge. We hypothesize that the latter consists of at least three components—mathematical curricular knowledge; knowledge of planning for mathematics teaching and learning (pre-active); and enacted mathematics knowledge for teaching and learning (interactive). These components correspond roughly to Shulman’s (1987) three categories of knowledge for teaching, and to the pedagogical curricular knowledge and pedagogical content knowledge defined by Fan and Cheong (2002).

***5. Knowledge for teaching involves consideration of the situational contexts where teachers will teach***

Teaching involves what has come to be called situated knowledge (Putnam & Borko, 2000). Situating knowledge within preparation for the practice of teaching mathematics refers not only to the varied classroom settings in which teachers ultimately practice, but also to the following: teachers' own prior primary and secondary schooling; the courses in which university-level content knowledge of mathematics is acquired; the courses in which the pedagogy of teaching mathematics is most emphasized; the classroom contexts for acquiring learning about mathematics in teaching during field experiences; and special arrangements for internships.

The knowledge developed or modified in each of these contexts contrasts with what has been called general knowledge (applicable across situations and settings) and contrasts even more with theoretical knowledge (general knowledge explicitly formulated in terms of interrelated concepts and basic ideas). This conceptualization raises a question: is it sufficient to measure general or theoretical knowledge without regard to situated knowledge? (See Hammer & Elby, 2002, in this regard.) Indeed, a major issue in research on teacher education is how to deal with the situated character of this knowledge. Ball et al. (2001) take a strong position on this: "What matters ultimately is not only what courses teachers have taken or even what they know, but also whether and how teachers are able to use mathematical knowledge in the course of their work" (p. 450). Nevertheless, some teacher education programs are based on the view that pre-service teacher education programs should mainly be concerned with theoretical knowledge, leaving practical, situated knowledge to be acquired later and largely on the job.

***6. Teacher education embodies a developmental logic of how teachers acquire professional knowledge for the teaching of mathematics and other subjects.***

Munby, Russel, and Martin (2001) summarize research comparing expert and novice teachers according to a paradigm of cognitive science:

Expert teachers possess richly elaborated knowledge about curriculum, classroom routines, and students that allows them to apply what they know to particular cases. Where novices may focus on surface features or particular objects, experts draw on a store of knowledge that is organized around interpretive concepts or propositions that are tied to the teaching environment ... (p. 889)

Programs of teacher education typically embody this developmental logic of how teachers acquire their professional knowledge. For example, a common justification of such programs is that theoretical knowledge prepares teachers to acquire practical knowledge in their initial years of practice, leading eventually to a state of expert professional knowledge. But evidence is lacking to show that this model predicts how well novices construct their professional knowledge or even that it represents an ideal model to help someone learn to teach mathematics. While the process of how teachers can best acquire expert knowledge continues to be contested, the literature contains important insights about the state of novices' subject-matter knowledge for teaching. (See Even, 1993, for a study of future secondary school teachers' understanding of the function concept and their responses to student solutions and errors; and see Schmidt, 1994, and Van Dooren, Verschaffel, & Onghena, 2002, for Canadian and Belgian studies on the preferences of future teachers for algebraic as opposed to arithmetic solutions to word problems.)

*7. Teacher education is assumed to be linked to student achievement, but this relationship is poorly understood.*

The motivation for TEDS-M derives in part from the findings emerging from TIMSS, TIMSS-Video, PISA, and other studies which show that many students across the globe do not attain desired levels of competence and understanding in mathematics. If we agree that teacher education is a component of schooling, then we must also agree that its relationship to teachers' and students' knowledge, although assumed important and proven influential by some studies (Hill, Rowan, & Ball, 2005; Mullens, Murnane, & Willett, 1996; Sanders & Rivers, 1996; Tatto et al., 1993), is also poorly understood. For instance, the Second International Mathematics and Science Study (McKnight et al., 1987) and the TIMSS 1996 and subsequent publications (Valverde et al., 2002) show that curriculum and content coverage are important for student achievement. TEDS-M explores whether future teachers acquire the knowledge related to the topics that have proved difficult for the students of teachers across many—if not all—countries in these studies. For comparison, and in order to examine levels of consistency in teacher preparation, TEDS-M also considers topics where achievement across countries is generally high.

## 2. TEDS-M DESIGN

This section describes the details of the design adopted to answer the research questions posed in the previous section.

- To provide data on the POLICY AND CONTEXT of mathematics teacher education, TEDS-M studies the policies that influence primary and lower secondary teachers' achieved level and depth of mathematics and related teaching knowledge, and how teacher policies influence the structure of primary and lower secondary mathematics teachers' opportunities to learn. Data on policy and context are collected at the national level through what is described below as *Component I* of TEDS-M.
- To provide data on the PROCESSES of mathematics teacher education, TEDS-M studies the opportunities to learn (OTL) available to future primary and lower secondary mathematics teachers that enable them to attain the knowledge they need to teach mathematics. It also studies the structure of the opportunities, the content taught in teacher education programs, and the organization of instruction. These data are collected at national, institutional, teacher educator, and future teacher levels, as detailed below under *Component II*.
- To provide data on the OUTCOMES of mathematics teacher education, TEDS-M studies the level and depth of the mathematics and related teaching knowledge attained by future primary and lower secondary teachers, and how this knowledge varies across programs, routes, and countries. This is achieved through collection of data from representative samples of future teachers in the last year of their programs, as described below under *Component III*.

Box 2.1 provides definitions of important terminology used in TEDS-M.

### **BOX 2.1: Definitions of Terms Used in TEDS-M**

#### **Opportunity to Learn (OTL)**

OTL is defined as an experience with an anticipated or intended learning outcome. TEDS-M applies this concept to teacher learning. OTL for teachers can occur at any point in the continuum of teacher learning, from the opportunities associated with schooling before entry into a formal teacher preparation program to the opportunities given to experienced teachers throughout their careers. TEDS-M endeavors not only to capture indicators of OTL throughout the teachers' professional life-cycle but also to concentrate on the opportunities that future teachers have to learn mathematics, mathematics pedagogy, and the general pedagogy provided by their pre-service preparation programs.

#### **Route**

This refers to the sequence of opportunities to learn that lead future teachers from the end of their general secondary schooling to being considered fully qualified to teach in primary or lower secondary schools. In other words, a route is a prescribed pathway through which the teacher education programs are made available in a given country. Teacher preparation (TP) programs within a given route share a number of common features that distinguish them from TP programs in different routes. Different countries have different sets of routes.

**Concurrent Routes**

The route is concurrent if its first phase consists of a single program that includes studies in the subjects future teachers will be teaching (academic studies), studies of pedagogy and education (professional studies), and practical experience in the classroom.

**Consecutive Routes**

The route is consecutive if it consists of a first phase for academic studies (leading to a degree or diploma), followed by a second phase of professional studies and practical experience (leading to a separate credential/qualification). Thus, no route can be considered consecutive if the institution or government authorities do not award a degree, diploma, or official certificate at the end of the first phase. Moreover, it may be customary or required for future teachers to do the first and second phases in different institutions.

**Practice (Apprenticeship) Routes**

If the route consists predominantly of school-based experience, with other institutions playing only a minor, marginal, supporting role, the route is primarily a practice or apprenticeship route.

**Teacher Preparation (TP) Institution**

This is a secondary or post-secondary school/college/university that offers structured OTL (i.e., a *program or programs*) on a regular and frequent basis to future teachers within a route of teacher preparation.

**Program**

Within each of the sampled teacher preparation (TP) institutions, there are likely to be one or more TP programs provided. A program is a specific pathway that exists within an institution and within a route that requires students to undertake a set of subjects and experiences, and leads to the award of a credential on completion. For example, a single TP institution may provide a concurrent program that prepares primary teachers, a concurrent program that prepares secondary teachers, and a consecutive program that accepts graduates from that institution or other tertiary institutions and prepares them to become secondary teachers.

**Level**

The TEDS-M survey is directed at two different groups of future teachers and their corresponding TP institutions and educators: future teachers who are expected to teach mathematics—most likely as generalists—at primary schools, and future teachers who are expected to teach mathematics at lower secondary schools. In this manual, these two groups are referred to as two distinct “levels.”

**Educators of Future Teachers (or “educators” for short)**

Educators of future teachers are persons with regular, repeated responsibility to instruct or mentor future teachers within a given teacher preparation program.

**Future Teachers**

Future teachers are defined for the purposes of TEDS-M as persons enrolled in a teacher preparation program that is explicitly intended to prepare teachers qualified to teach mathematics in any of the grades at primary or lower secondary school level.

## 2.1 TEDS-M Studies of Policy and Context (Component I)

In order to investigate teacher education policy and its context in each country and to prepare for subsequent surveys, TEDS-M includes a number of national studies as follows. Together, these studies form *Component I* of TEDS-M.

1. The study of routes
2. The study of teacher education policy
3. The studies of national and teacher education curricula
4. The study of the costs of becoming and preparing a mathematics teacher.

### 2.1.1 The Study of Teacher Preparation Routes

TEDS-M includes a study of primary and lower secondary mathematics teacher education routes that lead in each participating country from completion of secondary school to full qualification to teach. (Box 2.1 above sets down the *definition* of route used in TEDS-M, while Box 2.2 provides background information on the *concept* of route as it applies to this study.) TEDS-M seeks to clearly identify routes in order to distinguish how they differ in major respects, such as their structure, their curriculum, the capabilities and the backgrounds of their future teachers, and the grade levels and types of schools for which each route prepares its graduates.

Clarity about the routes in each country is essential to allow for comparability of analyses. Clear distinctions are needed, for example, between routes in which formal teacher education follows the completion of a university degree (known as consecutive routes) and routes in which subject preparation and formal teacher education are combined into a single program leading to a degree and eventual teaching certification (*concurrent* routes). The study of routes relies on a series of questionnaires to compile the necessary data.

#### **BOX 2.2: Background Information on Efforts to Define the Concept of Route**

As part of our effort to bring greater precision to the TEDS-M definition of the key concept of *route*, we benefited from the work of the European Eurydice project. The Eurydice Phase One Questionnaire for Initial Training, dated January 2001, offers a basic definition of what Eurydice call models and what this study calls routes. The Eurydice definition is based on two cross-cutting distinctions.

The first is the distinction between general education (defined as general courses and mastery of the subject-matters that future teachers will have to teach when qualified) as opposed to professional training (defined as the “theoretical and practical part of training devoted to teaching as such,” including short periods of assigned field experience) and final required on-the-job training. The second distinction is between concurrent models/routes (where general education and professional training are offered together from the outset of the program) and consecutive models/routes (where there is a first phase of general education and, if considered necessary, a second phase of professional training). Based on these two distinctions, Eurydice posits six models or, in TEDS-M terminology, routes, as follows:

- Two concurrent routes:
  - One combination of general education and professional training without a required final on-the-job training phase; and
  - One such combination with a final on-the-job training phase.

- Four consecutive routes:
  - One with a general education component only (i.e., no professional training);
  - One with a general education component, plus required final on-the-job training phase;
  - One with a general education component followed by a professional training component, but no required final on-the-job training phase; and
  - One with a general education component followed by a professional training component, and then a required final on-the-job training phase.

TEDS-M adds the following factors to this definition when defining routes/approaches:

- Mathematics-specific distinctions, such as mathematics content knowledge versus mathematics pedagogical content knowledge versus general pedagogical knowledge. The basic Eurydice definition sets mathematics content knowledge as part of general education and sets mathematics pedagogical content knowledge and general pedagogy as part of professional training.
- Degree of specialization in subject matter and/or grade levels for which future teachers are being prepared. Diplomas, certificates, and licenses vary widely between and often within countries—particularly in federal systems—with preparation targeted on many different combinations of grades (such as 1–3, 1–4, 1–6, 1–8, 4–6, 6–8, 7–8, 7–9, and so on).
- ISCED level of diplomas/degrees awarded during or at the end of teacher preparation.
- Identity and qualifications of teacher educators (e.g., requirements for experience in primary and secondary schools).
- Selectivity of teacher education routes and the placement/nature of selection points (e.g., use of external examinations).
- Hours devoted to different components of routes (e.g., professional training may be long or short).
- Nature of on-the-job training in the route—formal versus informal.
- Locus of control, including degree of student autonomy, teacher educator autonomy, and institutional autonomy in the construction of the route.
- Residential versus distance education routes.
- Other combinations of Eurydice components not readily represented in the six basic Eurydice routes.

#### **2.1.1.1 TEDS-M Route Questionnaire**

This questionnaire asks the national research coordinators (NCRs) to identify all routes leading to teaching of primary school or lower secondary school mathematics in their system of teacher education. Once the NCRs know the routes to be studied by TEDS-M, they complete the route questionnaire to provide further information on each route. This questionnaire requires drawing on other sources of information, including interviews and focus groups as needed. Topics covered are:

- The legislative/regulatory framework for teacher education.
- Characteristics of the institutions, programs, and sequence that make up the route (including duration and numbers of institutions).
- External examinations required and credentials awarded in each phase.
- Nationally prescribed or recommended curriculum content for the routes.
- Nature and amount of school-based practicum experience in the route.
- Levels at which curriculum decisions are made for the route.
- Qualifications required of teaching staff in the route.

### 2.1.2 The Study of Teacher Education Policy

The TEDS-M study of teacher education policy seeks to explore in more depth the following questions:

- (a) What are the policies that regulate and influence the design and delivery of mathematics teacher education for primary and lower secondary teachers within and across countries?
- (b) How do countries' distinctive political, historical, and cultural contexts shape mathematics teaching and learning? How do these influence policy and practice in mathematics teacher education?
- (c) What are the consequences of policy for the development of standards for degrees, coverage of topics, and certification practices?
- (d) What are the consequences of policies for the recruitment, selection, preparation, and retention of future mathematics teachers? Are these policies coherent across the board or do they compete with one another?

TEDS-M is particularly interested in understanding the national<sup>6</sup> policies and practices in each participating country that concern monitoring and assuring the quality of teacher education programs and institutions and the quality of graduate teachers from those programs. To answer these questions and complete the international report for *Component I*, we prepared guidelines for acquiring additional information on each of the routes summarized in a more standardized form on the route questionnaire. The guidelines ask NRCs to produce a *country report* on national policy and the organization of mathematics teacher education. Such a report serves two purposes:

- (i) As a stand-alone report, which could be submitted as part of an international report for *Component I*
- (ii) As input for cross-national analyses.

The country report is designed to complement the route questionnaire and to provide the contextual narratives needed to interpret the cross-national summary statistics. The cross-national report is designed to produce data that are standardized and therefore directly comparable. Box 2.3 summarizes the intended content for these reports.

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<sup>6</sup> In countries with federal systems, the relevant policies may be formed at the state, provincial, or institutional level.

### **BOX 2.3: Outline for Content of Country Reports on Policy and Context**

#### **Introduction**

##### ***Part One—Context and Organization***

- Historical, cultural, and/or social factors
- Teaching career, teacher labor market, teacher working conditions
- Structure and organization of teacher education

##### ***Part Two—Quality Assurance Arrangements and Program Requirements***

- Entry standards/selection
- Accreditation systems for teacher education
- Curriculum requirements
- Practicum and field experience requirements
- Staffing requirements
- Standards and requirements for entry to the teaching profession

##### ***Part Three—Resources and Reforms***

- Financing of teacher education institutions and programs
- Public debates concerning reform of teacher education

### **2.1.3 The Study of Curriculum**

TEDS-M continues the tradition of examining outcomes of education in relation to the curricula studied.

#### ***2.1.3.1 Primary and Secondary Mathematics Curricula***

TEDS-M assumes that teacher education does not occur in a vacuum and thus is influenced by important forces. One of these—possibly the most relevant to our study—is the relationship between the primary or secondary school curriculum standards and the degree to which teacher education emphasizes these standards. TEDS-M therefore analyzes mathematics curricula from Grades 1 to 8 of the participating countries using an updated method from that used in TIMSS and including only the analysis of programs of study and standards (National Center for Education Statistics/NCES, 1996, 1997; Valverde et al., 2002). The coding schemes used for the primary and secondary mathematics curricula provide the basis for the analysis of the teacher education curricula.

TEDS-M explores the correspondence between the teacher education mathematics curriculum at the national level and the primary and lower secondary mathematics curricula, standards, and examinations. The two types of curriculum analysis provide validation for the domains included in the “future teacher” booklet and in the educator questionnaires.

#### ***2.1.3.2 Curricula of Mathematics Teacher Preparation (First phase—national level)***

Building upon previous work analyzing primary and secondary curricula (Valverde et al., 2002), TEDS-M developed a framework and a methodology for analyzing the curriculum of teacher education. This methodology is required to address the research questions in TEDS-M *Component II* on the practices and content associated with actual routes and programs, and the research questions in the TEDS-M *Component III* on the impact of these practices on the professional knowledge of future teachers.

While there is usually plenty of documentation about school curricula, the same does not apply to curricula for teacher preparation programs. For instance, many countries that have national standards or syllabi for school subjects do not have national standards for teacher education. In teacher education, many courses either do not use detailed written syllabi, or the syllabus used differs from educator to educator and/or from institution to institution. And if there are documents describing the expected or anticipated learning outcomes of the practicum, analysis of these can prove particularly challenging. The initial phase of the study of the teacher preparation curricula therefore examines documents prescribing or describing the national curriculum of teacher education in countries where these exist, or an aggregated analysis of the local or institution curricula when no documents at the national level are available.

### ***2.1.3.3 Curricula of Mathematics Teacher Preparation (Second phase— institutional program level)***

A protocol has been developed to analyze curriculum documents from the teacher education mathematics curricula in the selected routes and programs in relation to mathematics standards for primary and secondary students in each participating country. In addition, at the institutional level, the protocol examines the relationship between the content covered and the performance expectations of courses in the mathematics teacher education curriculum and the local or national examinations for teacher certification or licensing. Such analyses are expected to produce an initial profile of the intended curriculum in mathematics teacher education in terms of the knowledge, pedagogy, dispositions, and other knowledge future teachers are exposed to as they get ready to teach. The data for the second phase of the curriculum studies are collected at the same time that the institutions are surveyed (see Section 2.2, Study of Teacher Preparation Institutions and Programs and Their Outcomes).

### **2.1.4 The Study of the Costs of Becoming and Preparing Mathematics Teachers<sup>7</sup>**

In attempting to understand what it costs to prepare teachers to teach mathematics at the primary and secondary levels in a large number of countries, the costs study brings an economic dimension to TEDS-M. The study involves two phases (outlined immediately below). The conceptual framework for studying costs in teacher education was first developed by Tatto and her colleagues in an empirical study published in 1993. The scope was expanded to include analysis of wage and salary profiles, and it was tested in the preparatory study for TEDS-M so that it could be used for the main TEDS-M survey.

#### ***2.1.4.1 Cost Study (First phase—wage and salary profiles)***

TEDS-M estimates teacher age-wage and age-salary profiles in at least 15 of the participating countries and compares those profiles with similarly educated individuals in mathematics-oriented professions. This set of studies allows TEDS-M (once income per capita in country is controlled for) to test if a relationship exists between the relative pay of teachers and achievement on the TIMSS and PISA tests. The first part of the cost studies was tested in the preparatory study for TEDS-M, and it is also used for TEDS-M.

<sup>7</sup> The sections on the TEDS-M cost studies derive, in part, from the following document: Carnoy, M., Brodziak, I., Loyalka, P., Reininger, M., & Luschei, T. (2006). *How much would it cost to attract individuals with more math knowledge into middle school (lower secondary) teaching: A seven-country comparison*. Palo Alto, CA: School of Education, Stanford University.

The first phase of the cost study assumes that the higher the relative pay of teachers, the easier it is to attract individuals who have higher mathematics skills into teaching, and that it is these people who are most likely to be the better mathematics teachers. While pecuniary returns are certainly not the only reason that individuals choose to enter teaching as a career, we can still assume that removing the pecuniary “barrier” allows more individuals who might become good mathematics teachers to enter teaching.

#### **2.1.4.2 Cost Study (Second phase—the teacher education cost study)**

This second phase of the teacher education cost study explores the costs incurred by teacher education programs. The education of a mathematics-oriented professional obviously begins before university: the high school courses that this person takes are likely to focus primarily on mathematics and science subjects, and the high schools that he or she attends may be more costly per student than the high schools attended by students who tend not to go into mathematics-oriented professions. While the study notes these points, it will not attempt to measure them, even though they may be important. Rather, it will focus on the post-secondary education process for primary and lower secondary mathematics teachers.

A complicating factor is that there is considerable variation in the post-secondary process for mathematics teachers. Any study of educational costs should try to capture that variation by making estimates for different institutional settings and different courses of study, if these are prevalent in a country. We can expect, for example, different costs for primary, lower secondary, and upper secondary school teachers. Another important point is that the teaching force has trained over many years.

The study focus is on the costs associated with formal post-secondary education—university and normal school training. These costs include the cost of university staff, and costs such as living subsidies provided to students.

The second phase of the cost study is attached to the institution survey (see Section 2.2, Study of Teacher Preparation Institutions and Programs and Their Outcomes).

## **2.2 Study of Teacher Preparation (TP) Institutions and Programs and Their Outcomes (Components II & III)**

The TEDS-M study of institutions and programs is designed to answer the following questions:

- (a) What kinds of institutional and field-based opportunities to learn do institutions and programs provide for future primary and lower secondary teachers of mathematics?
- (b) How are program expectations, curriculum, and standards enacted?
- (c) What are the qualifications and prior experiences of the university mathematics lecturers/educators/teachers and teacher educators responsible for implementation of these programs?
- (d) What beliefs about teaching and learning mathematics are promoted by teacher education programs?
- (e) What beliefs do future teachers hold about teaching and learning mathematics at the end of their preparation?
- (f) What knowledge of mathematics do the participating countries expect their future teachers to acquire?
- (g) What depth of understanding are they expected to achieve?

- (h) What is the knowledge for mathematics teaching (e.g., of the content, pedagogy, curriculum, and attitudes) that future primary and lower secondary mathematics teachers actually have at the end of their teacher education and once they are considered “ready to teach”?
- (i) What factors explain how much impact routes, programs, and practices have on the mathematics knowledge and beliefs of future teachers of mathematics?
- (j) What characteristics of future teachers help explain their ability to master this knowledge?

### **2.2.1 TEDS-M Sampling: Basic Principles and Definitions**

One of IEA’s design principles requires cross-national comparisons to be based on national probability samples. During the four decades of IEA history, considerable experience has been gained in planning and executing complex and challenging sample-based studies. Nevertheless, a study of the routes, programs, and outcomes of teacher education poses new challenges that have not been present in IEA studies of student achievement. In fact, TEDS-M is IEA’s first survey study in post-secondary education. The basic IEA approach to the study remains the same: the development of a master international sampling design to fit the purposes and key research questions of the study; adaptation of this master plan to national contexts by the national research centers; and negotiation by an international sampling referee of any differences in judgment about the adequacy of these national adaptations.

TEDS-M conducts three independent surveys: (i) national surveys of the teacher preparation (TP) institutions through use of an institutional program questionnaire; (ii) a national survey of educators and mentors of future teachers in the TP institutions through use of a teacher educator questionnaire; and (iii) a national survey of future teachers in the sampled institutions and routes who are preparing to teach mathematics (either as specialists or generalists).

The samples for the study are based on a multi-stage sampling design with probability proportional to size (PPS). The sample detail, specifications, and procedures are available in the TEDS-M sampling manual and will also be available in the TEDS-M technical report.

### **2.2.2 Study of Teacher Preparation (TP) Institutions and Programs**

The main objective of this study is to explore the intended, as well as the implemented, programs of teacher preparation in the selected institutions in each country.

#### ***2.2.2.1 Institutional Program Questionnaire***

This questionnaire collects information about the characteristics of each program within the selected TP institutions. It contains the following sections: program description, future teacher background, selection policies, program content, field experience, program accountability and standards, staffing, program resources, and reflections on the program. It also includes questions on resources used to operate teacher education programs, and a general profile of teacher educators (e.g., credentials, professional path, courses taught).<sup>8</sup>

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<sup>8</sup> The instruments developed for the study (with the exclusion of the knowledge tests) will be available in the TEDS-M technical report.

### 2.2.3 Study of Educators of Future Teachers

TEDS-M conceptualizes the educators of future mathematics teachers as a particularly important vehicle through which the *intended* teacher education curriculum becomes the *implemented* curriculum. Although our preparatory study and field trial data show that the characteristics of mathematics instructors and teacher educators' vary widely, the full range of this variability and how it influences the institutions in which these people work is not known. At present, there is no cross-national data based on scientific sampling of this very important population that would allow us even to begin to answer these questions. In fact, little survey data of any kind exists on this population. The TEDS-M framework uses the notion that the interaction of teacher educators with the curriculum and with prospective teachers shapes instruction and the OTL available to mathematics teachers.

#### 2.2.3.1 Educator Questionnaire

A single educator questionnaire is used to gather data from the two major groups of educators described above.

For both the primary and lower secondary levels, the *Survey for Mathematics, Mathematics Pedagogy, and General Pedagogy Educators* addresses the following domains: general academic background; teaching background; professional experience; research experience; field-based instruction; OTL in the courses taught; coherence of the teacher preparation program; beliefs about mathematics; and preparedness for teaching mathematics. Parts of this questionnaire are designed to be similar to the future teacher booklet, given that it examines the correspondence and differences between educators' and future teachers' responses (on such things as knowledge, pedagogy, and beliefs) as an indicator of coherence and possible impact.

#### *National option: Questionnaire for field-based educators*

A national option was devised to include the individuals responsible for helping future teachers in the same program/route learn from practice in primary or lower secondary schools. Such persons have roles that vary greatly across countries and that may or may not be highly formalized, and there is no standard international terminology to designate the supervisors of *practicum*. The survey of these individuals as a TEDS-M national option allows further exploration on a country-by-country basis before they are included in a more formal definition of the target population.

The domains addressed by this questionnaire include: general academic background; teaching background; teacher education experiences; OTL for field-based instruction; beliefs about mathematics; and preparedness for teaching mathematics. As with other IEA national options, the plan for surveying this group of educators was negotiated with the lead ISC at MSU and was approved by the IEA sampling referee and the DPC. Countries that choose this option collaborate in a cross-national analysis of these data.

### 2.2.4 Study of Future Teachers

TEDS-M targets future teachers who are in their *final year of training* before they are eligible to become practicing teachers of mathematics in primary and in lower secondary schools (either as generalist teachers or as mathematics specialists). The study provides indicators of the *achieved* curriculum of teacher preparation.

### ***National option: Additional second cohort of future teachers***

For purposes of comparison, countries are given the option of surveying the entering future teacher cohort as well as the graduating cohort within the selected institutions. They can do this as a *national option* in consultation with the lead ISC at MSU and the IEA sampling referee, and with the DPC.

#### ***2.2.4.1 Future Teacher Booklets***

These booklets measure the *intended* and *achieved* knowledge of mathematics and of mathematics pedagogy of future teachers in their last year in the sampled teacher education programs. The booklets also ask about the future teachers' general background, program learning opportunities, and beliefs about and attitudes toward teaching and learning mathematics.

Because, in the case of consecutive routes, only future teachers in the last year of the second phase TP institution tend to be surveyed, their OTL for mathematics content is approximated from the study of the national curriculum as well as from their self-report of what they have been exposed to in earlier institutions at secondary and post-secondary levels.

The future teacher questionnaire is ambitious in the domains it attempts to cover. As mentioned earlier, the study of teacher education represents a new practice for IEA. The parameters for application of the questionnaires for future teachers come from our feasibility study, the actual field trial, and other related studies, such as TIMSS. The TEDS-M item pilot and field trial carried out in 2006 and 2007 indicated that future teachers would be unlikely to respond to a questionnaire that exceeded 90 minutes of actual administration time in one session. Table 2.1 shows the overall structure of the future teacher booklets, with allocation of time per section. As shown, the booklet is administered in one 90-minute session.

*Table 2.1 Overall Booklet Structure and Allocated Times for Administration<sup>a</sup>*

	<b>Time (min)</b>
Part A: General Background	5
Part B: Opportunity to Learn	15
Part C: Mathematics for Teaching	60
Part D: Beliefs about Mathematics and Teaching	10

*Note:* <sup>a</sup> The general knowledge for teaching questionnaire is a national option for the main study.

Details about the items in this booklet are contained in Section 3, Measurement Specifications and Booklet Design.



### 3. MEASUREMENT SPECIFICATIONS AND BOOKLET DESIGN

#### 3.1 Development, Trialing, and Review of Questionnaire Items<sup>9</sup>

The process of item development for TEDS-M has been thorough, extensive, and rigorous.

The conceptual challenges of measuring the outcomes of teacher preparation in terms of teacher knowledge and belief are considerable. TEDS-M builds on our development study (i.e., MT21), which produced an earlier and shortened version of a questionnaire for future lower secondary teachers to measure knowledge of (i) mathematics, (ii) mathematics pedagogy, and (iii) general knowledge for teaching. A number of belief scales and preparedness scales based on the literature were also included. These instruments were trialed on a small-scale basis in six countries with promising results and served to inform the instrument development in TEDS-M.

Lessons from the MT21 feasibility study made it necessary to add a substantial number of mathematics and mathematics pedagogy knowledge items to the lower secondary portion of the questionnaire so that we could develop a *true test* of knowledge in these domains and report by sub-domains in important areas of mathematics and mathematics pedagogy. Similarly, and since our development study did not include the study of future primary teachers, TEDS-M developed or adapted a large number of items in order to test knowledge and to report by sub-domains.<sup>10</sup> The TEDS-M research team accordingly solicited items from the Knowing Mathematics for Teaching Algebra (KAT) Project at Michigan State University, the Learning Mathematics for Teaching (LMT) Project at the University of Michigan, from researchers in Australia, and from the participating countries. The TEDS-M MSU team developed items as well.

The item pilot took place in June 2006, and this was followed by a major *field trial* of multiple forms of the assessment instruments and questionnaires for most of the participating countries in March–April 2007. Thus, many of the items used in the main study have been subject to five rounds of international trialing in diverse countries (when combining the efforts from our development study and from TEDS-M).

At each stage of the item development process, expert panels examined the content validity and appropriateness of the items. These reviews took into consideration clarity, correctness, cultural relevance, classification within the framework of domains and sub-domains, relevance to teacher preparation, and curricular level. In preparation for the field trial, detailed manuals were written for all aspects of the study (sampling, survey administration, and procedures, etc). Scoring guides and rubrics were prepared for all constructed response items, and sample responses were collected to provide a basis for training. All these materials were thoroughly reviewed and revised when appropriate in close collaboration with the country teams. Two scoring training sessions were carried out in preparation for the field trial and two more in preparation for the main study. Further details on the design of the main study and timeline can be found below and in Section 9 (Appendix A).

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<sup>9</sup> The items included as examples in the appendix were tested in the field trial and dropped from the final data collection after the data were analyzed to determine their psychometric properties and their fit with the underlying constructs in the study.

<sup>10</sup> Another important lesson from MT21 is the need to provide careful attention to the sample design in a future feasibility study of any kind but most importantly when the study concerns higher education.

## 3.2 Knowledge for Teaching Mathematics: Instrument Development

Knowledge for teaching mathematics consists of two constructs: mathematics content knowledge and mathematics pedagogical content knowledge (see the introduction of the framework for the theoretical origins of these constructs).

### 3.2.1 Mathematics Content Knowledge Framework

With a view to providing useful links to TIMSS data, the TEDS-M lead team proposed to members of the first NRC meeting that the instrumentation for *primary* future teachers should build on the content and cognitive domains from the TIMSS framework as described in the TIMSS 2007 assessment framework (Mullis et al., 2007). The NRCs agreed to this proposal. They also agreed with the notion that the emphasis in the TEDS-M mathematics assessment should be on the content that teachers are *required to teach*. They furthermore agreed that the test should measure mathematical content knowledge of at least two years beyond the level they are expected to teach. This proviso implied that both the Grade 4 and Grade 8 TIMSS frameworks would inform instrument development for the content knowledge of future primary school teachers.

Similarly, the conceptualization and instrumentation of mathematical knowledge for the *secondary level* for TEDS-M builds on the content areas used by the TIMSS 2007 content domain assessment framework, and the TIMSS advanced assessment framework, as well as on the framework developed by the feasibility study for TEDS-M. Hence, the four subject areas assessed at both the primary and the secondary levels are (i) number, (ii) algebra, (iii) geometry, and (iv) data (see Table 3.1). As is the case for items developed for future primary teachers, TEDS-M items for students preparing to teach at lower secondary school concentrate on the mathematics taught in lower secondary grades. However, mathematics topics from upper secondary school and university study are also addressed.

Table 3.1 Mathematics Framework: Content Knowledge Domains

NUMBER	Whole numbers <sup>P S</sup> Fractions and decimals <sup>P S</sup> Number sentences <sup>P S</sup> Patterns and relationships <sup>P S</sup> Integers <sup>P S</sup> Ratios, proportions, and percents <sup>P S</sup> Irrational numbers <sup>P S</sup> Number theory <sup>P S</sup>
GEOMETRY	Geometric shapes <sup>P S</sup> Geometric measurement <sup>P S</sup> Location and movement <sup>P S</sup>
ALGEBRA	Patterns <sup>P S</sup> Algebraic expressions <sup>P S</sup> Equations/formulas and functions <sup>P S</sup> Calculus and analysis <sup>S</sup> Linear algebra and abstract algebra <sup>S</sup>
DATA	Data organization and representation <sup>P S</sup> Data reading and interpretation <sup>P S</sup> Chance <sup>P S</sup>

Note: <sup>P</sup> primary level; <sup>S</sup> secondary level.

Source: TIMSS 2007 content domain assessment framework (Mullis et al., 2007); TIMSS 2008 advanced assessment frameworks (Garden et al., 2006)

The three main components of the TIMMS 2007 framework for the cognitive domains were also adopted for the TEDS-M mathematics items for future teachers (Table 3.2). Using the same conceptual framework for subject matter and cognitive domain for future primary and lower secondary teachers simplifies the analysis and interpretation of TEDS-M data. At the primary level, more emphasis was given to items addressing the application domain, followed by the knowing and reasoning domains. At the secondary level, more emphasis was placed on application, followed by reasoning and knowing domains.

The curricular level of each mathematics content item was also categorized into novice (indicating mathematics content that is typically taught at the grades the future teacher will teach); intermediate (indicating content that is typically taught one or two grades beyond the highest grade the future teacher will teach); or advanced (indicating content that is typically taught three or more years beyond the highest grade the future teacher will teach). For both the primary and secondary levels, the largest number of items is targeted to measure intermediate level content. Section 9 (Appendix A) provides further details of the mathematics content framework.

*Table 3.2 Mathematics Framework: Cognitive Domains*

<b>Knowing</b>	
Recall	Recall definitions; terminology; number properties; geometric properties; notation.
Recognize	Recognize mathematical objects, shapes, numbers and expressions; recognize mathematical entities that are mathematically equivalent.
Compute	Carry out algorithmic procedures for addition, multiplication, division, subtraction with whole numbers, fractions, decimals, and integers; approximate numbers to estimate computations; carry out routine algebraic procedures.
Retrieve	Retrieve information from graphs, tables, or other sources; read simple scales.
Measure	Use measuring instruments; use units of measurement appropriately; estimate measures.
Classify/Order	Classify/group objects, shapes, numbers, and expressions according to common properties; make correct decisions about class membership; order numbers and objects by attributes.
<b>Applying</b>	
Select	Select an efficient/appropriate operation, method, or strategy for solving problems where there is a known algorithm or method of solution.
Represent	Display mathematical information and data in diagrams, tables, charts, or graphs; generate equivalent representations for a given mathematical entity or relationship.
Model	Generate an appropriate model, such as an equation or diagram, for solving a routine problem.
Implement	Follow and execute a set of mathematical instructions; draw figures and shapes according to given specifications.
Solve Routine Problems	Solve routine or familiar types of problems (e.g., use geometric properties to solve problems); compare and match different representations of data; use data from charts, tables, graphs, and maps to solve routine problems.

Table 3.2 Mathematics Framework: Cognitive Domains (contd.)

<b>Reasoning</b>	
Analyze	Determine and describe or use relationships between variables or objects in mathematical situations; use proportional reasoning; decompose geometric figures to simplify solving a problem; draw the net of a given unfamiliar solid; visualize transformations of three-dimensional figures; compare and match different representations of the same data; make valid inferences from given information.
Generalize	Extend the domain to which the result of mathematical thinking and problem-solving is applicable by restating results in more general and more widely applicable terms.
Synthesize/Integrate	Combine (various) mathematical procedures to establish results, and combine results to produce a further result; make connections between different elements of knowledge and related representations, and make linkages between related mathematical ideas.
Justify	Provide a justification for the truth or falsity of a statement by reference to mathematical results or properties.
Solve Non-routine Problems	Solve problems set in mathematical or real-life contexts where future teachers are unlikely to have encountered closely similar items, and apply mathematical procedures in unfamiliar or complex contexts; use geometric properties to solve non-routine problems.

Source: TIMSS 2007 cognitive domain assessment framework (Mullis et al., 2007).

### 3.2.2 Mathematics Pedagogical Content Knowledge Framework

As noted in Section 1.4 of this document, the development of the framework for the mathematics pedagogical content knowledge was informed by the feasibility study for TEDS (Schmidt et al., 2007) and the work of other researchers in the field (Ball & Bass, 2000; Hill, Rowan, & Ball, 2005; Hill, Schilling, & Ball, 2004; Tatto et al., 1993), as well as by feedback from the TEDS-M expert groups and the NRCs. Table 3.3 shows the framework used for both the primary and the secondary item development. For the main study, the two sub-domains “mathematical curricular knowledge” and “knowledge of planning for mathematics teaching and learning” (pre-active) are combined.

### 3.2.3 Booklet Design and Reporting by Sub-scales for Mathematics Content and for Mathematics Pedagogical Content Knowledge

TEDS-M developed scales for mathematics and for mathematics pedagogical content knowledge by using item response theory (IRT) methods.<sup>11</sup> TEDS-M classifications for the cognitive domains and for the curricular levels were used solely for purposes of test development in order to achieve a good balance among these categories of the framework. Scales were not developed for these.

These decisions have implications for the main study instruments:

- To enable reliable reporting of measures for each sub-domain, the number of the items has to be reasonably large. However, because testing time for the mathematics assessment is limited to 60 minutes, no one participant can be expected to answer all items in it. Therefore, more than one test booklet is required, and the items in blocks need to be rotated among the assessment booklets.
- Because some countries have “small” teacher preparation institutions (i.e., institutions that prepare fewer than 10 future lower secondary teachers in the last year of the program), problems arise if many different assessment booklets are used.

<sup>11</sup> Scales will also be developed for other areas such as beliefs, OTL, and preparedness to teach. Whether these will be based on IRT methods remains to be determined.

By using a rotated block design, TEDS-M increases the scope of the mathematics measurements without increasing the administration time. The field trial confirmed the use of this strategy for the main study.

Five blocks are considered appropriate for the TEDS-M main study at the primary level. Table 3.4 presents the proposed five-booklet, five-balanced incomplete blocks (BIB) design for the primary-level knowledge instrument.

*Table 3.3 Mathematics Pedagogical Content Knowledge (MPCK) Framework*

<b>Mathematical Curricular Knowledge<sup>a</sup></b>	<ul style="list-style-type: none"> <li>Establishing appropriate learning goals</li> <li>Knowing different assessment formats</li> <li>Selecting possible pathways and seeing connections within the curriculum</li> <li>Identifying the key ideas in learning programs</li> <li>Knowledge of mathematics curriculum</li> </ul>
<b>Knowledge of Planning for Mathematics Teaching and Learning (pre-active)</b>	<ul style="list-style-type: none"> <li>Planning or selecting appropriate activities</li> <li>Choosing assessment formats</li> <li>Predicting<sup>b</sup> typical students' responses, including misconceptions</li> <li>Planning appropriate methods for representing mathematical ideas</li> <li>Linking the didactical methods and the instructional designs</li> <li>Identifying different approaches for solving mathematical problems</li> <li>Planning mathematical lessons</li> </ul>
<b>Enacting Mathematics for Teaching and Learning (interactive)</b>	<ul style="list-style-type: none"> <li>Analyzing or evaluating students' mathematical solutions or arguments</li> <li>Analyzing the content of students' questions</li> <li>Diagnosing typical students' responses, including misconceptions</li> <li>Explaining or representing mathematical concepts or procedures</li> <li>Generating fruitful questions</li> <li>Responding to unexpected mathematical issues</li> <li>Providing appropriate feedback</li> </ul>

*Notes:*

<sup>a</sup> This framework pays attention to the temporal dimension of teacher knowledge as well as the way in which mathematical categories refer to different types of knowledge.

<sup>b</sup> Attention to choice of verbs may prove useful in distinguishing between pre-active and interactive dimensions of the categories.

The TEDS-M design for the primary booklets for the field trial is shown in Section 9 (Appendix A) of this framework report. Experience gained from the TEDS-M item and field trials indicates that, on average, respondents may be able to answer 30 items in 60 minutes. Each primary block therefore contains a minimum of 12 items, each worth one to three score points, and requires 20 minutes of administration time.

At the secondary level, the small size of target populations within institution, within program, and within country imposes further restrictions. Table 3.5 represents the proposed three-booklet, three-BIB design for the TEDS-M main study at the lower secondary level.

*Table 3.4 TEDS-M BIB Design for the Primary-level Mathematics Knowledge Instrument<sup>a</sup>*

<b>Booklets</b>	<b>Assessment Blocks</b>		
1	B1	B2	B
2	B2	B3	B4
3	B3	B4	B5
4	B4	B5	B1
5	B5	B1	B2

*Note:* <sup>a</sup> B1, B2, B3, B4 and B5 represent the five primary mathematics blocks.

*Table 3.5 TEDS-M BIB Design for the Secondary-level Mathematics Knowledge Instrument<sup>a</sup>*

<b>Booklets</b>	<b>Assessment Blocks</b>	
1	B1	B2
2	B2	B3
3	B3	B1

*Note:* <sup>a</sup> B1, B2 and B3 represent the three secondary mathematics blocks.

Each secondary block contains an average of 15 items, each worth one to three score points, and requires 30 minutes of administration time. Section 9 (Appendix A) shows the booklet design for the main study at the secondary level.

The field trial showed these designs to be empirically viable. The design provided precise scores, thus permitting within-country analysis, with institutions as one of the hierarchical units of analysis. The comparative analysis relied on the use of institutional, route, and country means, as these are more reliable than individual participant scores.

Because of time limitations for data collection in the main study, some sub-domains were collapsed in order to achieve acceptable levels of reliability.

At both the primary and the secondary levels, number and data items are grouped together so that three sub-domains of content knowledge (number and data, algebra, and geometry) can be reported. Similarly, at primary level, only two sub-domains of mathematics pedagogy (curriculum and planning and enacting) are reported. At the lower secondary level, no sub-domains are reported for mathematics pedagogy knowledge.

### 3.3 National Option: Exploring General Knowledge for Teaching

General knowledge for teaching is the most unexplored knowledge area in the research literature. Not surprisingly, this area was slow to develop in our feasibility study. At the time of the field trial, much progress had been achieved, but it was not enough to develop a rigorous measurement instrument. Nevertheless, many of the NRCs in TEDS-M have a strong commitment to this area of knowledge, and the Joint Management Committee (JMC) therefore decided to designate it as a national option. Although the TEDS-M framework for this area initially drew from our feasibility study, it has now been changed to include “holistic scoring” and has been simplified as well for use in TEDS-M. As in the other knowledge areas, the items benefited from input from the TEDS-M expert groups as well from the NRCs’ input. The basic premise is that, in addition to teachers’ knowledge of content, the knowledge that teacher educators in a number of countries call *general knowledge for teaching* is essential to good mathematics teaching. This dimension includes the following knowledge categories: (a) students (i.e., knowledge of students), including the influence of socioeconomic status on teaching and learning; (b) classroom environment; (c) instructional design; and (d) diagnostics and assessment.

The cognitive dimensions for understanding general knowledge for teaching are substantive knowledge of theory, practical knowledge and approach, and reasoning and judgment regarding pedagogy-oriented issues (Table 3.6). Future teachers are asked about their knowledge of methods and practices and their ability to apply pedagogical knowledge to teaching-related situations. Reasoning reflects the professional judgment that is considered another key aspect of the general knowledge of teaching. On a smaller scale, these items also examine the ability of the future teacher to make connections between theory and practice.

Table 3.6 *General Knowledge for Teaching Framework*

Item Description	Key Variable
Purpose of assessment at the end of a unit	Reasoning
Purpose of assessment at the start of a unit	Reasoning
SES: Explanation of phenomenon	Substantive focus
SES: Accommodations in teaching	Approach and reasoning
Planning instruction: Evaluation of a lesson plan	Substantive focus
Student thinking: Cognitive development	Substantive focus
Student thinking: Relevant theory	Substantive focus
Student thinking: Link between the theory and the situation	Reasoning
Student thinking: Teacher’s approach to the situation	Approach and reasoning
Student thinking: Role of teacher in seeing justification for a given answer in test	Reasoning
Role of increased waiting time in instruction	Substantive focus
Motivation: Method to motivate a student	Approach
Assessment: Purposes of assessment as a learning tool	Substantive focus

The items included in this instrument measure general knowledge for teaching in areas that are strongly represented in the teacher education curriculum analyzed for TEDS-M. These items include knowledge of students, knowledge of classroom management, knowledge of instructional design, and knowledge of assessment. Most items measure knowledge of theory and about a third of them measure knowledge of practice.

A more detailed description of the overall design for the field trial booklets can be found in Section 9 (Appendix A).

### 3.4 Question Types

#### 3.4.1 Format

Three question formats are used for the TEDS-M knowledge questions: multiple-choice (MC), complex multiple-choice (CMC), and open constructed-response (CR). Each multiple-choice question counts one point toward scores and sub-scores. Complex multiple-choice response items consist of one stem followed by a list of choices, each of which is scored correct or incorrect. Open constructed-response questions are scored from zero up to one, two, or three points, depending on the depth of understanding demonstrated in answering the question. Up to 74% of the questions are multiple-choice and complex multiple-choice response; the rest are open constructed-response. For the main study, open constructed-response items contribute about 27% of the items.<sup>12</sup> Some of these items combine multiple-choice formats with open-ended formats. The weight of the open constructed-response items to the whole assessment is therefore not negligible. The analyses of these items provide rich information on patterns of responses in knowledge items.

In theory, multiple-choice and complex multiple-choice items can be used to measure any of the knowledge domains. However, because of the situated nature of teacher knowledge, this format does not allow respondents to provide detailed interpretation of situations and demonstration of the knowledge that is required to teach mathematics. In contrast, open constructed-response items allow respondents to develop a response to a question and to demonstrate the depth of their thinking on mathematics knowledge and mathematics teaching knowledge.

Examples of item types developed for the knowledge measurements are given in Section 9 (Appendix B). These include samples of the actual items trialed but not selected for the main study.

### 3.5 Beliefs

Similar to the arguments given about the importance of content and general knowledge in teaching, there is wide agreement that beliefs are an important influence on teaching. Nevertheless, there is no conclusive evidence that beliefs can be effectively influenced by teacher preparation or that they are an intrinsic characteristic of those individuals who become teachers (Tatto & Coupland, 2003). In TEDS-M, this measurement area is informed by previous work done by the Teaching and Learning to Teach Study at MSU (Deng, 1995; Tatto, 1996, 1998, 1999b, 2003), and by the work of other international scholars (Grigutsch, Raatz, & Törner, 1998; Ingvarson, Beavis, Danielson, Ellis, & Elliott, 2005; Ingvarson, Beavis, & Kleinhenz, 2007). The TEDS-M beliefs scales include questions in five areas: beliefs about the nature of mathematics; beliefs about learning mathematics; beliefs about mathematics achievement; beliefs about preparedness for teaching mathematics; and beliefs about program effectiveness.

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<sup>12</sup> The proportion of question formats varies by level of education (primary and secondary). Exact percentages are MC and CMC: 72.8% for primary and 73.5% for secondary. For CR, they are 27.1% for primary and 26.5% for secondary.

### **3.5.1 Beliefs about the Nature of Mathematics**

The items included in this area include questions that explore how future teachers perceive mathematics as a subject (e.g., mathematics as formal, structural, procedural, or applied). The items are based on work by Grigutsch et al. (1998) and by Ingvarson et al. (2005, 2007).

### **3.5.2 Beliefs about Learning Mathematics**

This area includes questions about the appropriateness of particular instructional activities, questions about students' cognition processes, and questions about the purposes of mathematics as a school subject.

### **3.5.3 Beliefs about Mathematics Achievement**

This area taps into future teachers' beliefs about various teaching strategies used to facilitate learning of mathematics. Other questions explore beliefs about how mathematics learning may take place, and yet others explore the application of attribution theory to teaching and learning interactions (e.g., innate ability for learning mathematics).

The items used to measure the area described in Sections 3.5.1, 3.5.2, and 3.5.3 come from a number of studies, including those by Deng (1995), the feasibility study for TEDS-M (Schmidt et al., 2007), and several studies by Tatto (1996, 1998, 1999b, 2003).

### **3.5.4 Beliefs about Preparedness for Teaching Mathematics**

The fourth area of belief relevant to TEDS-M concerns the extent to which future teachers perceive their teacher preparation has given them the capacity to carry out the central tasks of teaching and to meet the demands of their first year of practice. The items in these scales are therefore designed to explore different areas of teacher preparation impact. At the end of the questionnaire, a direct question is used to confirm these views.

The preparedness scale used in the TEDS-M study is based on the ACER *Preparedness to Teach* inventory, a robust measure based on extensive research (Ingvarson et al., 2005, 2007). For the TEDS-M study, the items included measure preparedness to teach in areas such as assessment, management of learning environments, and practices for engaging students in effective learning, and the extent to which teachers become active members of their professional community.

### **3.5.5 Beliefs about Program Effectiveness**

In addition to asking a question about the overall effectiveness of teacher preparation to help future teachers learn to teach mathematics, six items probe future teachers' beliefs about the degree to which their instructors modeled good teaching practices, used research, and used evaluation and reflection in their courses. The future teachers are also asked if their instructors valued their (the future teachers') various experiences before and during their teacher preparation program.

Section 9 (Appendix B) shows examples of the items included in the beliefs sub-scales. In all belief areas, many more items were trialed than used—a practice that brings efficiency and precision to the final TEDS-M data collection because the result is shorter instruments based on only the best-performing items.

### 3.6 Opportunities to Learn (OTL)

TEDS-M uses the concept of OTL as central to explaining the impact of teacher preparation on teacher learning. Torstén Husén was the first person to define and use this term in order to explain student learning in IEA's First International Study of Achievement in Mathematics (FIMS):

... [w]hether or not . . . students have had the opportunity to study a particular topic or learn how to solve a particular type of problem presented by the test ... If they have not had such an opportunity, they might in some cases transfer learning from related topics to produce a solution but certainly their chance of responding correctly to the test item would be reduced. (Husén, 1967, cited in Burstein, 1993)

Others, prior to Husén and since, have simply defined this concept as “time allowed for learning” (Carroll, 1963). Scholars have also continued to argue that the concept of OTL “has changed how researchers, educators, and policymakers think about the determinants of student learning” (McDonnell, 1995, p. 317) and, in the case of TEDS-M, about the determinants of teacher learning. Moreover, IEA studies dating back to FIMS have empirically tested teachers' perceptions of students' OTL and found them to correlate highly with student achievement scores for mathematics test items.

TEDS-M explores this concept in teacher education. Inclusion of OTL in the study serves a number of purposes: as an explanation of differences in levels of knowledge; as an indicator of curricular variation among countries; as an aspect of fairness (e.g., appropriateness of language of test items); and as a representation of the diversity of content, both overall and for distinct groups of teachers. (For a full treatment of OTL as applied to teaching and learning, see Floden, 2002.)

The TEDS-M survey includes a number of items and scales to allow exploration of the OTL that future mathematics teachers have across countries. These include opportunities to learn the following: (i) university- or tertiary-level mathematics; (ii) school-level mathematics; (iii) mathematics education/pedagogy; (iv) and education/pedagogy. They also include opportunities to learn (v) how to accommodate classroom diversity and to reflect on practice; (vi) how to learn from school experience and the practicum, and (vii) how to learn in a coherent teacher education program. The items and scales as a whole evaluate coherence of the teacher education program, given that TEDS-M hypothesizes that *program* coherence has an important impact on teacher learning.

Section 9 (Appendix B) contains selected examples of the items included in the OTL scales, as described in the following sections (3.6.1 to 3.6.7).

#### 3.6.1 Opportunity to Learn University- or Tertiary-level Mathematics

The items in this area explore if future teachers have studied key mathematics topics (e.g., geometry-related topics, algebra, number theory, calculus, functions, etc.). Because opportunities to learn in this area often occur before future teachers enter teacher preparation, these questions ask future teachers whether or not they have studied such topics “ever.”

### **3.6.2 Opportunity to Learn School-level Mathematics**

This area includes items exploring the interaction between mathematics pedagogy and mathematics content as it concerns the school curriculum, learning techniques for teaching mathematics topics, and student learning. In addition, this area includes items relating to the emphasis given to learning mathematics (e.g., at the level of the school curriculum or at a deeper level or beyond and with no relation to the school curriculum).

### **3.6.3 Opportunity to Learn Mathematics Education/Pedagogy**

This area includes items exploring the interaction between mathematics content and pedagogy. Additional scales include items concerning the use of learning strategies in mathematics. Future teachers are asked to indicate whether or not they have studied each topic as part of their teacher preparation program. Other questions ask the future teachers how often they engage in a number of activities and learning strategies in their teacher preparation program.

### **3.6.4 Opportunity to Learn General Knowledge for Teaching**

This scale includes items about topics considered relevant for all teachers to know, such as educational theory, general principles of instruction, classroom management, curriculum theory, and so on. As with the items in the previous knowledge areas, these items ask future teachers if they have studied such topics as part of their teacher preparation program.

### **3.6.5 Opportunity to Learn to Teach for Diversity in and Reflection on Practice**

This area includes items about development and use of materials for teaching as well as items about accommodation of diverse levels of student learning. Other items focus on learning how to assess and reflect on one's own teaching, and on developing strategies to improve one's own professional knowledge.

### **3.6.6 Opportunity to Learn in Schools and through the Practicum**

A far more extensive section asks more in-depth questions regarding in-school experience. These questions ask future teachers whether they spent time in the classroom in a primary or secondary school and, if so, for how long. The questions also ask the future teachers if they had a school supervisor assigned to them, the particular activities in which they engaged and at what levels, and whether they found the in-school experience helpful. An additional set of questions asks about diverse characteristics of the practicum (e.g., the role of the mentor, feedback received, standards, methods used, and the level of mathematics knowledge and pedagogy of the classroom teacher or mentor).

### **3.6.7 Opportunity to Learn in a Coherent Teacher Education Program**

The coherence area includes items exploring program consistency across courses and experiences offered to future teachers, and whether there are explicit standards with expectations for what future teachers should learn from their respective programs.

### 3.7 Background and Demographics

As is the case with all questionnaires in TEDS-M, the future teacher questionnaire includes a number of indicators of future teachers' backgrounds and demographics. These indicators are of intrinsic interest and were selected on the basis of what the literature tells us about the types of factors that are likely to help us understand differences between and within countries. The questionnaire includes such variables as age, gender, and parents' education/socioeconomic status; language spoken at home; nature and level of secondary school mathematics knowledge; information on academic/general education; area of specialization; routes into teaching; degrees obtained; teaching/work experience; motivation/plans/intention to become a mathematics teacher; special circumstances/personal costs of becoming a teacher; and how long the future teacher plans to stay in the profession. See Section 9 (Appendix B) for an example.

## 4. DATA ANALYSIS

As described in the introduction to this framework, an important goal for this study is to provide policy-relevant information regarding the mathematics preparation of future teachers. The data are therefore reported comparatively to show variation in the mathematics and other related knowledge of future teachers as it occurs across the different routes to teacher preparation evident among the participating countries. Specifically, the analysis endeavors to accomplish the following:

- Show the level and depth of the mathematics and related teaching knowledge attained by prospective primary and lower secondary teachers across the different routes studied in the participating countries. These findings are examined vis-à-vis expectations apparent in the standards set in the K–12 mathematics curriculum and in the mathematics and related content teacher education curriculum.
- Characterize the OTL provided in terms of the level and depth of knowledge and in terms of the internal and external coherence and other features related to the organization of instruction.
- Characterize the policy environments that support future primary and lower secondary teachers at national and institutional levels attain high levels of mathematics preparation for teaching mathematics.

### 4.1 Preliminary Analysis

For initial reporting purposes, descriptive statistical analysis (e.g., means, standard deviations, and correlations) for selected individual items will be used to provide information about patterns of responses on questionnaire items by programs, across institutions, across routes within each country, and across countries.

### 4.2 Item and Scale Analyses

Within TEDS-M, several sets of questions were designed to create scales or broader measures of important constructs. Item analysis and scale development include three primary steps. First, mathematics and mathematics pedagogy knowledge and belief items are evaluated for functionality (few missing responses, variability in responses). Second, items are combined into sets based on the intended constructs as defined in Section 3 of this document. Third, based on this prior intent for the items, confirmatory factor analysis is used to assess the quality of each scale and, where appropriate, of second-order correlations among sub-scales. Fourth, based on the fit-indices and the item-loadings, decisions regarding selection of items for final assignment to scales are considered (DeVellis, 2003). Next, scaling methods based on item response theory (IRT) are used to establish measurement invariance as a function of item functioning across countries/languages (e.g., differential item functioning) and to establish secure meaningful scale scores measuring each construct. IRT analysis provides information about item fit, item contribution (information) to the scale, and other measurement properties of the scale, including reliability. This process is appropriate for both dichotomously and polytomously scored cognitive items and for the rating scale of attitudinal items. Additional validity-related evidence regarding expected relationships among scales are assessed with inter-scale correlations (Bond & Fox, 2001).

### 4.3 Analysis of Variability within and between Countries

The analysis models multivariate relationships while taking into account the nested structure of the data. For example, via multilevel modeling, the analysis examines relationships between future teachers' mathematical knowledge and contextual information regarding teacher education programs and national teacher education policies. Multilevel models allow for the assessment of relationships within each level of data (within [aggregated] programs, within institutions, and within routes) and of interactions among levels (e.g., where the effect of a particular institutional characteristic depends on the route or on the nature of a national policy). Alternatively, or in addition to, the described first approach to modeling, the analysis is expected to model all three levels (policy context, institutions, and outcomes), with non-significant variables removed one at a time until the model is left with only significant variables. Models using (aggregated) programs as the units of analysis are also explored.

The plan for analysis includes the implementation of statistical tests appropriate for each research question. These analyses typically include descriptive analyses of the variables involved in the particular research question, followed by an evaluation of the distributional properties of variables for selection of appropriate statistical tests and the modeling of outcomes.

Scale score difference is examined (using the general linear model) for background variables such as gender, degree level, etc. The general linear model (GLM) allows for weighted least-squares analyses and for multivariate analyses (simultaneous testing of multiple outcomes) with explanatory variables in multiple forms, including group membership (categorical variables) and covariates (continuous variables).

In the case of categorical variables, non-parametric statistical procedures are proposed (e.g., Spearman's rank correlations, Chi-square analyses, or Kruskal-Wallis tests). Finally, in order to understand the nature of the relationship among the major constructs proposed in the conceptual model, structural equation modeling may be employed to assess the interaction of major constructs and contextual variables. Additional procedures are used to accommodate the sampling design in the estimation of variances and standard errors.

Each analysis involves estimation of a common effect size to facilitate comparisons of multiple effects. These analyses also explore the effects of item-by-country interaction and, as in other IEA studies, use national sub-samples for international calibration.

### 4.4 Curriculum Analyses

The TEDS-M curriculum analyses of syllabi, policy documents, and other curriculum data will permit comparison of curricular differences between programs, institutions, routes, and countries. For example, appropriate indices are used to estimate correspondence between K–12 curriculum standards and the teacher education curriculum at the primary and secondary levels. This correspondence is examined in relation to background variables and other institutional contextual variables. Curricular variables are also used in the multivariate modeling. Displays illustrate correspondences across curricular levels.

#### 4.5 Cost Analyses

Mathematics teacher preparation costs vary among institutions within countries and vary greatly across countries. The data on costs are limited to that obtained from questionnaires and are supplemented with data gathered from correspondents in the participating countries. These data compare the costs per program regarding the annual budget for the particular year of the study as well as those costs contributed by other programs and/or departments. The study also collects data on the budget for instruction (e.g., teaching staff salaries), and on whether or not future teachers are given subsidies for living expenses. Other questionnaire data such as the number of future teachers prepared yearly by level allow us to estimate the total cost of preparing primary, middle school, and high school teachers of mathematics in various institutions in each country. These costs per student are then compared with the results from the main part of the study, which show the opportunities that future teachers are given to learn mathematics. These comparisons bring insights into whether teachers are better prepared in mathematics teaching largely as a function of higher spending on their training, or whether other factors are at play.



## 5. TIMELINES/PRODUCTS (Abridged)

TEDS-M spans four years.<sup>13</sup> The following is a summary of the main activities and products (P) for those years.

### October 2005 to September 2006

- Revision and distribution of the first and second drafts of the conceptual framework document (P)
- Initiation of national studies of policy and curriculum analysis
- Initiation of literature reviews of mathematics teacher education research
- Review, adaptation, and development of new items for future primary-level teachers
- Review, adaptation of feasibility study (MT21) instruments, and additional item development for future lower secondary-level teachers
- Item pilot, revision, and development of additional items
- Development of instruments and procedures for field trial
- Design of national TEDS-M probability samples of TP institutions, future teachers, and educators
- 1st NRC meeting 13 to 16 February 2006, Hamburg  
2nd NRC meeting 3 to 8 September 2006, East Lansing, Michigan, USA

### October 2006 to September 2007

- Conduct field trial of instruments and procedures (P)
- 3rd NRC meeting 25 to 29 June 2007, Chinese Taipei
- Finalization of instruments for the studies of institutions, educators, and future teachers based on the field trial of instruments and procedures
- Execution of sample designs
- Review of survey operation procedures manuals (P)
- Revision and release of field trial edition of the conceptual framework document (P)
- Continuation work on national reports on policy studies and curriculum analysis (P)
- Quality control monitoring seminar, 27 to 28 August 2007, Amsterdam, the Netherlands
- Data management seminar, 27 to 28 September 2007, Hamburg, Germany
- Draft and review of final international report on cost study, Phase 1 (P)

### October 2007 to September 2008

- Scoring training workshop for constructed response items, 29 November to 1 December 2007, Miami, USA
- Quality control monitoring seminar, 21 to 22 January 2008, Amsterdam, the Netherlands
- Scoring training workshop for constructed response items, 7 to 9 April 2008, Fribourg, Switzerland
- K–12 curriculum, national teacher education curriculum and institution syllabi coding workshop, 9 to 13 June 2008, Warsaw, Poland
- 4th NRC meeting, 1 to 5 September 2008, Bergen, Norway

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<sup>13</sup> Additional funding will be sought for a study of beginning teachers that will follow the TEDS-M survey, with the aim of extending the measures of the achieved curriculum of teacher education to the first years of teaching.

- Revision and publication of final version of conceptual framework document (P)
- Publication of final international report on cost study (P)
- Main data collection in Southern Hemisphere and Northern Hemisphere
- Entering, scoring, cleaning, weighting, and scaling of data
- Analysis of data
- Draft and review of international reports Vols. I through V

**October 2008 to December 2009**

- 5th NRC meeting, March 2009, Chicago, Illinois, USA
- 6th NRC meeting, June 2009, Santiago, Chile
- Release of international report on national policy studies (P)
- Final review of draft international report Vols. I through V
- Final review of draft cost study

**January 2010 to September 2010****January 2010**

- Press release January 2010, Washington DC
- Release of international reports Vols. I, II, III (P)
- Release of international report on cost study (P)

**June to September 2010**

- Release of international reports Vols. IV and V (P)
- Release of the TEDS-M technical report (P)
- Release of international database and user guide (P)

## 6. DISSEMINATION

### 6.1 Release of TEDS-M Study Materials to the Public

TEDS-M is scheduled to end its first study cycle in 2009. In line with IEA policy, the study instruments (including items and other measurement strategies) and data are released into the public domain after publication of the international reports. Moreover, because this study may be extended to beginning teachers, a substantial number of items will not be released so that approximate trend data can be obtained in subsequent years. Experience from other IEA surveys indicates that the release of the database for TEDS-M will take place approximately nine months after the release of the international report.

The international report will be published as a series with the title: *Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics: The Teacher Education and Development Study in Mathematics International Report*. Volumes in the series are:

- Volume 1: *National Policies and Regulatory Arrangements for the Mathematics Preparation of Future Teachers*;
- Volume 2: *Institutions, Programs, and Opportunities to Learn for the Mathematics Preparation of Future Teachers*;
- Volume 3: *Mathematics and Related Outcomes Achieved among Prospective Primary and Lower Secondary Teachers*;
- Volume 4: *Program Characteristics, Opportunities to Learn, and Outcomes in the Mathematics Preparation of Future Teachers*; and
- Volume 5: *TEDS-M Encyclopedia*.

As in other IEA surveys, the national research centers will be given access to the international data in a pre-released and embargoed form before the release of the international report so that they can prepare their national reports. The international and national reports will probably be released simultaneously.

### 6.2 Format

Because most researchers use SPSS, fully documented and labeled SPSS files are included in the international database. Depending on the requests and needs of countries, SAS or raw data files may also be provided. Codebooks, exhaustively detailing all variables and their corresponding values and characteristics, and a comprehensive user guide are provided alongside the data.



## 7. IEA TEDS-M MANAGEMENT AND ORGANIZATIONAL CHART

### **International Association for the Evaluation of Educational Achievement (IEA)**

The Teacher Education Study in Mathematics (TEDS-M) is sponsored by the International Association for the Evaluation of Educational Achievement (IEA). IEA is incorporated under Belgian law as a consortium of research institutions in 60 countries. Since the 1960s, IEA has carried out many important and influential large-scale, cross-national studies in education, including the very well-known Third International Mathematics and Science Study (TIMSS). During TEDS-M, the IEA Secretariat has managed translation verification and international quality control of the data collection programs.

### **TEDS-M Joint Management Committee**

IEA established the Joint Management Committee (JMC) to act as the decision-making body to oversee the whole TEDS-M study. Maria Teresa Tatto of Michigan State University (MSU) is the study director and chair of this committee. The JMC consists of the study director, the two co-directors from the international study center at MSU (Sharon Senk and John Schwille), and the three co-directors from ACER (Lawrence Ingvarson, Ray Peck, and Glenn Rowley). *Ex officio*, non-voting members include the IEA executive director (Hans Wagemaker), the TEDS-M sampling referee Jean Dumais from Statistics Canada, and other representatives of the IEA Secretariat and the IEA Data Processing and Research Center.

The JMC is responsible for overall project planning and for ensuring sound financial, personnel, and logistical management, as well as for accountability to IEA and the funding agencies. Areas in which the committee acts as coordinator and broker for the study include facilitating relationships with participating IEA national centers as well as with funding agencies; procuring funds; and contracting and subcontracting for the study. It is also responsible for recruiting countries to join the study; for study design, timelines, reports, and archives; and for designating the work of outside experts on the study.

The JMC reports and is accountable to the IEA General Assembly, Standing Committee, and the Technical Executive Group (TEG) on behalf of TEDS-M. The TEG comprises a group of methodologists internationally known for their expertise in the type of research conducted by IEA. Their task is to ensure that the design of each IEA study conforms to IEA technical standards (as outlined in Martin, Rust, & Adams, 1999).

### **The International Study Center at Michigan State University**

MSU is the lead institution for TEDS-M. The major responsibilities of MSU are as follows:

- To provide overall leadership and the administrative direction of the study;
- To develop, incorporate, and oversee all plans for the design and execution of the study within a comprehensive conceptual framework document to be published by IEA;
- To initiate proposals for and seek definition of JMC-assignments for instrument development;

- To coordinate the overall technical/scientific, operational, and analytic aspects for *Components I, II, and III*; and
- To coordinate the publication of the international reports, technical report, and the study database in collaboration with IEA/DPC.

MSU has primary responsibility for the development and authorship of the conceptual framework, the technical report, and the final international reports. In addition, it has primary authorship over the study concerning the mathematics preparation of secondary school teachers, including the development of relevant data-gathering instruments for the item pilot, field test, and main data collection, the conduct of relevant data analyses, and preparation of relevant sections of the final international reports. MSU is responsible for accomplishing other tasks as decided by the JMC and the annual work plan. The MSU international study center is accountable to the United States National Science Foundation (NSF) for its partial funding of TEDS-M, and to IEA.

#### **The Australian Council for Educational Research**

The Australian Council for Educational Research (ACER) team supports the TEDS-M international study center at MSU with technical and research management expertise. ACER has primary responsibility over the study concerning the mathematics preparation of primary school teachers, including the development of relevant data-gathering instruments for the item pilot, field test, and main data collection, the conduct of relevant data analyses, and the preparation of relevant sections of the final international reports. In addition, ACER is responsible for preparing and participating in meetings of the JMC; preparing budgets for ACER's work in the project; accomplishing follow-up actions arising out of national research center meetings as required by the JMC or annual work plan; and accomplishing other such work as decided by the JMC and the annual work plan. The ACER team is accountable to the IEA and to the international study center at MSU.

#### **IEA Data Processing and Research Center (DPC)**

The DPC in Hamburg has been the data-processing department of IEA since 1995. The increasing number of international studies and the increasing number of countries participating in them, along with the ever more complex research designs, have placed higher demands on data processing, analysis, dissemination, and training. The IEA DPC has developed the capability to respond to all such demands. IEA executive director Hans Wagemaker assumes overall responsibility for the IEA DPC, while operative management responsibilities within the DPC rest with Heiko Sibberns and Dirk Hastedt.

IEA DPC responsibilities for TEDS-M include, among others, sampling in close consultation with the TEDS-M sampling referee; support relevant to instrument development, codebooks, operational manuals, within-institution sampling and tracking using the IEA WinW3S software; data entry using the IEA WinDEM software; scoring procedures, documentation, verification and approval of national and cultural adaptations; data cleaning and processing; item almanacs; reliability statistics; creation of an international database to be used with statistical packages; organization of training seminars on the use of these data; and preparation of applicable parts of the TEDS-M technical report.

**TEDS-M International Sampling Referee**

The sampling referee, in consultation with the other partners, is responsible for developing an international master sampling plan for the study. This plan provides guidelines for the national centers to develop national plans that translate the international plan into the specifics of their own contexts and systems. The international sampling referee is then responsible for judging whether each national plan is adequately based on the international plan. This person also negotiates and adjudicates any deviations from the master plan. The referee is also responsible for overseeing the drawing and execution of the sample in collaboration with the DPC. The international sampling referee for TEDS-M is Jean Dumais of Statistics Canada.

**National Research Coordinators (NRCs)**

According to IEA policy and practice, each participating country must have a national center capable of carrying out the study and raising all the necessary funds for the within-country work. Each country designates a national research center and a coordinator (NRC) to direct its part of the study. NRCs play a key role internationally in guaranteeing that their country is adequately and accurately taken into account within the overall study design, analysis, and reporting. NRCs provide input to the project via continuous communication with the project director, the staff in the international study center at MSU, the DPC, other designated TEDS-M staff, and at the NRC meetings.

Under TEDS-M, each NRC is responsible for collecting relevant policy documentation on mathematics teacher recruitment, selection, and certification policies, and teacher policies on mathematics preparation. The NRC also collects information about the mathematics teacher education curricula and primary and secondary school curricula, and the national-level data required for the cost study. The NRC is responsible for filling out protocols and for bringing together focus groups to answer questions left unanswered by the document analysis. In addition, the NRC is responsible for gaining entrance to the teacher education institutions and finding the most appropriate person(s) to answer the institutional questionnaire. NRCs need to have a dedicated investigator per institution to administer surveys to the educators, and to help gain access to the implemented curriculum (e.g., syllabi, texts, and examinations used in the program by the selected educators).

NRCs conduct the teacher education curriculum analysis according to agreed guidelines. They are also responsible for implementing the sample design and for appointing a researcher to institutions to organize the application of the survey of future teachers in their last year in teacher education. Each NRC is encouraged to analyze the TEDS-M national data in the context of data from other countries in order to produce a national report tailored to the issues of teacher education policy and practice most important to the country in question. The NRC should also prepare and execute a dissemination plan to ensure that the findings of the study are brought to the attention of and discussed with target audiences.

The national research centers for TEDS-M represent the following countries: Botswana, Canada (Manitoba, Newfoundland and Labrador, Nova Scotia, Ontario, Quebec), Chile, Chinese Taipei, Georgia, Germany, Malaysia, Norway, Oman, Philippines, Poland, Russian Federation, Singapore, Spain, Switzerland (German-speaking cantons), Thailand, and the United States.

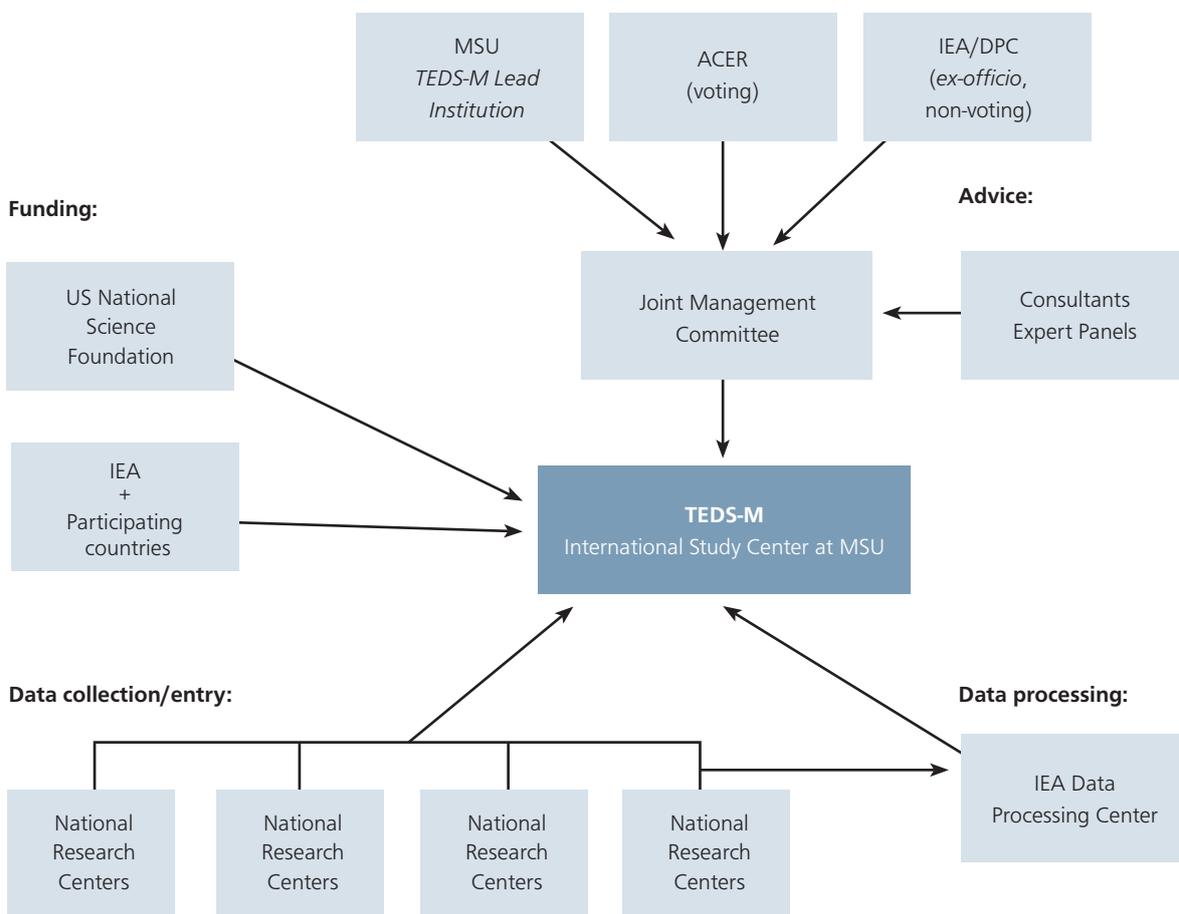
**TEDS-M Project Advisors**

TEDS-M also has a number of project advisors who meet occasionally to provide advice to the JMC. This group of advisors includes experts in the fields relevant to TEDS-M and is chaired by Maria Teresa Tatto of MSU, chair of the JMC for TEDS-M.

**Funding**

In addition to receiving funds from IEA, TEDS-M receives international costs from the United States National Science Foundation (NSF) (REC 0514431 9/15/2005 to 9/15/2009). Maria Teresa Tatto is the principal investigator and project director, and John Schulle and Sharon Senk are co-principal investigators. Each participating country is responsible for funding national project costs and implementing TEDS-M in accordance with international procedures established by the International Study Center at MSU and by IEA.

**TEDS-M Organizational Chart**



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## 9. APPENDICES

### APPENDIX A: STUDY DESIGN AND DATA SOURCES

#### Primary Future Teachers Mathematics Instrument Design

The design for the primary study has the following characteristics:

- A rotated 5-block design (Table 9.1) that permits estimation and analysis of the full covariance matrix, and that includes enough items and score points to generate IRT (item response theory) scales and reports by sub-domain.

Table 9.1 Design for Primary Booklets

Main Primary Booklets: Rotated Blocks for Knowledge Items	
	Mathematics Knowledge for Teaching
Booklet 1	B1PM, B2PM
Booklet 2	B2PM, B3PM
Booklet 3	B3PM, B4PM
Booklet 4	B4PM, B5PM
Booklet 5	B5PM, B1PM

Key: B1PM to B5PM = mathematics blocks primary (1–5).

- Three mathematics content knowledge (MCK) sub-domains (algebra, geometry, and number and data), proportionally represented to reflect the international curricula (as per the TEDS-M analysis of the K–12 mathematics curriculum in the participating countries). Each sub-domain has a minimum of 12 items. The number of score points per MCK sub-domain is at least 17, and likely ranges are shown in the last column of Table 9.2. The two mathematics pedagogy content knowledge (MPCK) sub-domains (curriculum and planning, and enacting) also have at least 12 items each and at least 20 score points. The domains and sub-domains are shown in Table 9.2.

Table 9.2 Primary Items by Sub-domains, Blocks, and Score Points

Sub-domain	Number of Items in Assessment Blocks					Items	Estimated Range of Score Points
	B1	B2	B3	B4	B5		
Algebra	2	3	4	4	2	15	21–27
Geometry	2	2	3	3	2	12	17–22
Number and data	5	4	3	2	4	18	26–39
MathPed1 (curriculum and planning)	2	2	3	2	4	13	22–24
MathPed2 (enacting)	2	3	1	4	2	12	20–24
TOTAL	13	14	14	15	14	70	106–136

- Item assignment to the blocks to ensure sufficient variation of items in terms of cognitive domains, number of score points, and mathematics and/or mathematics pedagogy level (novice, intermediate, and advanced). These distributions are shown in Tables 9.3 and 9.4.

### 9.3 Primary Mathematics Items by Cognitive Domain and Sub-domain

	Knowing	Applying	Reasoning	Total No. of Items
Algebra	10	5	0	15
Geometry	4	5	3	12
Number and data	11	4	3	18

### 9.4 Primary Items by Level of Difficulty and Sub-domain

	Novice	Intermediate	Advanced	Total No. of Items
Algebra	6	5	4	15
Geometry	4	6	2	12
Number and data	8	6	4	18
MathPed1 (curriculum and planning)	1	8	4	13
MathPed2 (enacting)	0	9	3	12

In regard to the levels of difficulty by sub-domain, note that because of the many criteria that had to be met (expert opinions, psychometric data, item types, etc.) and because of the item categorization judgments made about each item, it was not possible to find an “enacting” item in the pool at the novice level. These “levels” are estimates only. It is therefore possible, and even likely, that there will be a need to re-categorize after the initial analysis. This is an empirical aspect of this research study.

### Lower Secondary Future Teachers Mathematics Instrument Design

The design for the secondary booklets has the following characteristics:

- As with the primary booklets, the rotated 3-Block design (Table 9.5) for the secondary booklets allows analysis of the full covariance matrix and provides enough items and score points to generate IRT scales.

Table 9.5 Design for Lower Secondary Blocks

Main Lower Secondary Booklets: Rotated Blocks	
	Mathematics Knowledge for Teaching
Booklet 1	B1SM, B2SM
Booklet 2	B2SM, B3SM
Booklet 3	B3SM, B4SM

Key: B1SM to B3SM = mathematics blocks secondary (1–3).

- Three reporting sub-domains for MCK: algebra, geometry, and number. Data is not a reporting sub-domain. Data is proportionally less represented in the international curricula. This is why the number of the items is not sufficient to build a sub-scale. Data items are included for the completeness of the domains. The MPCK domain will be reported as an intact domain (i.e., it does not include sub-domains). Each of the reporting (sub-) domains has at least 17 score points. The domains and sub-domains are shown in Table 9.6.

*Table 9.6 Lower Secondary Items by Sub-domains, Blocks, and Score Points*

	Number of Items in Assessment Blocks			Items	Score Points
	Block 1	Block 2	Block 3	Total	
Algebra	6	3	3	12	20–25
Geometry	4	5	3	12	21–28
Number	2	2	5	9	17–34
Data*	1	1	2	4	6
MathPed	3	5	4	12	18–29
TOTAL	16	16	17	49	82–122

Note: Although we have included items on data, “data” in itself is not a reporting sub-domain.

- Item assignment to the blocks ensures sufficiently balanced coverage in terms of cognitive domains, number of score points, and mathematics level (novice, intermediate, and advanced). These distributions are shown in Tables 9.7 and 9.8.

*9.7 Secondary Mathematics Items by Cognitive Domain and Sub-domain*

	Knowing	Applying	Reasoning	Total No. of Items
Algebra	2	7	3	12
Geometry	2	5	5	12
Number	4	3	2	9
Data*	1	2	1	4

Note: Although we have included items on data, “data” in itself is not a reporting sub-domain.

*9.8 Secondary Items by Level of Difficulty and Sub-domain*

	Novice	Intermediate	Advanced	Total No. of Questions
Algebra	3	5	4	12
Geometry	3	7	2	12
Number	1	6	2	9
Data*	2	1	1	4
Mathematics Pedagogy	7	5	0	12

Note: Although we have included items on data, “data” in itself is not a reporting sub-domain.

### Overall Design for the Main Study Booklets

The overall structure of the TEDS-M field trial booklets for primary and secondary future teachers is presented in Table 9.9.

To enable coverage of a wide range of content areas and to prevent over-burdening each participant, test items are typically arranged in a number of blocks. These blocks are assigned to booklets so that each participant takes only one form, that is, a subset of the items only.<sup>14</sup>

Table 9.9 Overall Design for Main Study Booklets

Primary Booklet Design					
PM1	PM2	PM3	PM4	PM5	
BACK	BACK	BACK	BACK	BACK	Background
OTL	OTL	OTL	OTL	OTL	Opportunity to Learn
B1PM	B2PM	B3PM	B4PM	B5PM	
B2PM	B3PM	B4PM	B5PM	B1PM	Mathematics Knowledge for Teaching
BEL	BEL	BEL	BEL	BEL	Beliefs about Mathematics and Teaching
Secondary Booklet Design					
SM1	SM2	SM3			
BACK	BACK	BACK			Background
OTL	OTL	OTL			Opportunity to Learn
B1PM	B2PM	B3PM			
B2PM	B3PM	B1PM			Mathematics Knowledge for Teaching
BEL	BEL	BEL			Beliefs about Mathematics and Teaching

<sup>14</sup> The optimal test design is the balanced incomplete (7) blocks (BIB) test design. The IEA-TEG October 2006 and the MSU international study center considered this design classic but too cumbersome.

9.10 Summary of Domains and Sources of Data for TEDS-M National Studies

Dimension/ Constructs	Country	Institution	Educators			Future Teachers
			Maths and maths pedagogy	General pedagogy	Field-based experience	
<i>Sub-domains</i>						
<i>Route Study</i>						
Route to full qualification as primary (mathematics) teacher	x					
Structure	x					
Curriculum	x					
Capabilities and background of future teachers	x					
Types of schools and grade levels graduates are prepared for	x					
Route to full qualification as secondary (mathematics) teacher	x					
Structure	x					
Curriculum	x					
Capabilities and background of future teachers	x					
Types of schools and grade levels for which graduates are prepared	x					
<i>Policy Studies</i>						
Policies on primary mathematics teaching	x	x				
Policies on secondary mathematics teaching	x	x				
Characteristics of teacher education	x	x				
Characteristics of future teachers	x	x				x
Characteristics of educators/staff	x	x	x	x	x	
Requirements	x	x	x	x	x	
Goals (intended and realized)	x	x	x	x	x	
Selectivity	x	x				
Assessment	x	x	x	x	x	
Standards	x	x	x	x	x	
Program accountability	x	x	x	x	x	
Quality assurance	x	x	x	x	x	
<i>Curriculum Studies</i>						
Characteristics of teacher education mathematics curriculum	x	x				
Characteristics of K-12 mathematics curriculum	x	x				
<i>Cost Studies</i>						
Salary	x					
Institution		x				
Private		x				x

9.11 Summary of Domains and Sources of Data for the Study of TP Institutions, Programs, and their Outcomes

Dimension/Constructs	Country	Institution	Educators			Future Teachers
			Maths and maths pedagogy	General pedagogy	Field-based experience	
Sub-domains						
<i>Background and Demographics</i>						
Age, gender, parents' education			x	x	x	x
Nature and level of secondary school mathematics knowledge			x	x	x	x
Academic/general education; area of specialization			x	x	x	x
Routes into teaching; degrees obtained			x	x	x	x
Teaching/work experience/preferences/area of responsibility			x	x	x	x
Motivation/plans/intentions to become a mathematics teacher						x
Special circumstances/personal costs of becoming a teacher						x
How long plans to stay in the profession			x	x	x	x
<i>Opportunity to Learn (preparedness to teach)</i>						
<i>Topics in:</i>						
Mathematics content	x	x	x	x	x	x
Mathematics pedagogy	x	x	x	x	x	x
General pedagogy	x	x	x	x	x	x
Assessment	x	x	x	x	x	x
Planning lessons	x	x	x	x	x	x
School curriculum	x	x	x	x	x	x
<i>Course Structure</i>						
Emphasis (content, pedagogy, practicum, etc.)		x	x	x	x	x
Philosophy (cognition, human development, etc.)			x	x	x	x x
Program type/purpose (e.g. four-year B.Ed.)		x	x	x	x	x
<i>In-school Experience and Practicum</i>						
Quality		x			x	x
Nature		x			x	x
Methods		x			x	x
<i>Academic Teaching and Advising (of future teachers)</i>						
Quality		x				x
Nature		x				x
Methods		x				x

## 9.11 Summary of Domains and Sources of Data for the Study of TP Institutions, Programs, and their Outcomes (contd.)

Dimension/Constructs	Country	Institution	Educators			Future Teachers
			Maths and maths pedagogy	General pedagogy	Field-based experience	
<i>Teaching Methods Used in Program</i>						
Quality		x	x	x	x	x
Nature		x	x	x	x	x
Methods		x	x	x	x	x
<i>Beliefs/Attitudes/Perspectives</i>						
Nature of teacher education			x	x	x	x
Nature of mathematics and mathematics knowledge			x	x	x	x
Nature of teaching and learning mathematics			x	x	x	x
Nature of schools/schooling/general pedagogy			x	x	x	x
<i>Mathematics Knowledge</i>						
Subject-matter domain: number, data, and probability, geometry and measurement, algebra			x		x	x
Cognitive domain: knowing, applying, reasoning.			x		x	x
<i>Mathematics Pedagogy Knowledge</i>						
Mathematical curricular knowledge			x		x	x
Knowledge of planning for mathematics teaching and learning			x		x	x
Knowledge of enacting mathematics for teaching and learning			x		x	x
<i>General Knowledge for Teaching</i>						
Knowledge of students				x		x
Knowledge to manage the classroom environment				x		x
Assessment				x		x
Professional responsibilities				x		x

## APPENDIX B: ITEM SAMPLES

### Sample Items for Mathematics Pedagogy Content Knowledge

#### 1. Primary

Categorization	
Domain:	Mathematics Pedagogy Content Knowledge (MPCK)
Sub-domain:	Enacting—analyzing or evaluating student’s mathematical solutions or arguments; providing appropriate feedback.
Level:	Intermediate
Response type:	Multiple-choice

Mr. Lewis was surprised when one of his students came up with a new procedure for subtraction (pictured below), and he wondered whether it would always work. He showed it to Ms. Braun, next door, and asked her what she thought.

$$\begin{array}{r}
 37 \\
 -19 \\
 \hline
 -2 \\
 20 \\
 \hline
 18
 \end{array}$$

How do you think Ms. Braun should respond?

- Check *one* box.
- A. She should tell Mr. Lewis the procedure works for this problem but would not work for all numbers.
- B. She should tell him this does not make sense mathematically.
- C. She should let Mr. Lewis know that this would work for all numbers.
- D. She should say that this procedure only works in special cases.

Source: Ball and Hill (2002).

Note: Modification Part E, “I’m not sure,” was deleted.

## 2. Primary

<b>Categorization</b>	
Domain:	Mathematics Content Knowledge (MCK)
Sub-domain:	Data
Performance:	Reasoning
Level:	Novice
Response type:	Constructed (short)

A teacher gave the following problem to her class.

The numbers in the sequence 7, 11, 15, 19, 23, ..... increase by 4. The numbers in the sequence 1, 10, 19, 28, 37, ... increase by nine.

The number 19 is in both sequences.

If the two sequences are continued, what is the next number that is in BOTH the first and the second sequence?

(a) What is the correct answer to this problem? \_\_\_\_\_

Source: TIMSS 2003 released item.

### Scoring Guide

Code	Response	Item: TD1P060A
	<b>Correct response</b>	
10	55	
	<b>Incorrect response</b>	
70	Either of the responses "27 and 46" or "19" indicating a misreading of the question.	
71	Any other incorrect value, probably indicating a calculation error.	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off-task).	
	<b>Non-response</b>	
99	Blank	

## 2. Primary (contd)

<b>Categorization</b>	
Domain:	Mathematics Pedagogy Content Knowledge (MPCK)
Sub-domain:	Enacting—analyzing or evaluating students’ mathematical solutions or arguments
Level:	Intermediate
Response type:	Constructed

(b) A student gives the response 27 and 46 to the question above. What is the most likely reason for this response?

Source: ACER, Melbourne, Australia, 2006.

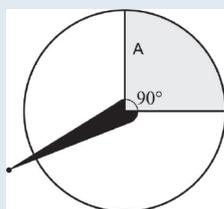
### Scoring Guide

Code	Response	Item: TD1P060B
	<b>Correct response</b>	
20	A response that recognizes that the student has misread, misinterpreted, or misunderstood the question and explains the likely misinterpretation. <i>Examples: The student misinterpreted the question to mean, “What is the next number in each sequence/both sequences?”</i> <i>The student interpreted “BOTH” as meaning give “two” answers.</i>	
	<b>Partially correct response</b>	
10	A limited response that recognizes that the student has misread, misinterpreted, or misunderstood the question but does not explain the likely misinterpretation. <i>Example: The student misread/misunderstood the question.</i>	
11	A response that simply explains that 27 and 46 are the next numbers in each sequence (but does not give a reason <b>why</b> the student has answered this way). <i>Examples: They are the next numbers in each sequence.</i> <i>The student gave the next numbers in each sequence rather than the “same” number in each.</i>	
	<b>Incorrect response</b>	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off-task)	
	<b>Non-response</b>	
99	Blank	
	<b>Non-response</b>	

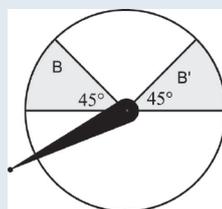
### 3. Primary/Secondary

<b>Categorization</b>	
Domain:	Mathematics Content Knowledge (MCK)
Sub-domain:	Data (chance)
Level:	Primary—advanced; Secondary—intermediate/novice
Response type:	Complex multiple-choice

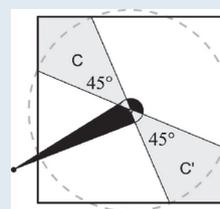
Your students are examining the three spinners shown below. They are discussing the probability that the spinner stops over a shaded region.



Spinner 1



Spinner 2



Spinner 3

Please indicate whether the following statements of four students are Completely True, Partly True or Completely False. **If one sentence is true, but the other is false, check Partly True.**

	Check one box in each row.		
	Completely True	Partly True	Completely False
A. Sherry says, "The probability is twice as large for Spinners 2 and 3 compared to Spinner 1 because they have two regions to stop on and Spinner 1 has only one region."	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
B. George says, "Spinners 1 and 2 have the same probability since the shaded regions have the same area. Spinner 3, however, has a higher probability than Spinner 2 because the shaded region is a larger area."	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
C. Paul says, "Spinners 1, 2 and 3 have the same probability because the angles of the shaded regions are the same size."	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
D. Rainey says, "The probabilities for Spinners 1 and 2 are the same because those areas are the same proportion of the whole circle. For Spinners 2 and 3, however, the probabilities are different because the shaded areas for Spinner 3 are a bigger proportion of the whole square than they are of the circle."	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Source: Prof. W. Whitely, York University, Toronto, Ontario, Canada.

Modification: Degrees added to diagrams. Dotted circle added to third diagram. Some modifications made to wording and also to the rating of the extent to which each statement is correct.

**3. Primary/Secondary** (*contd.*)

*Key:*

- A Completely false
- B Partly true
- C Completely true
- D Partly true

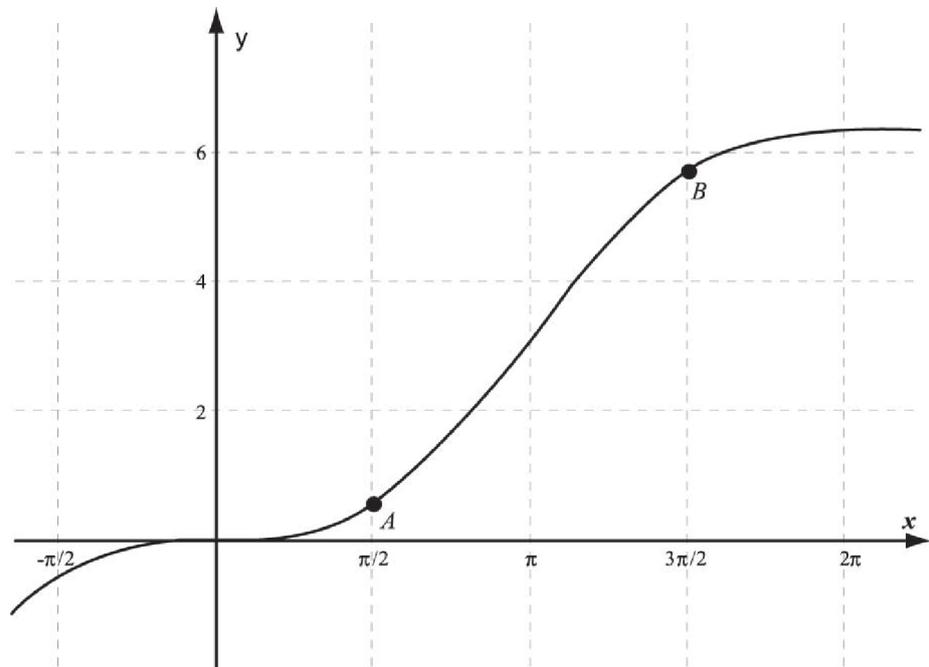
*Notes:* It is unlikely that each part can be considered as dependent. Therefore, this is not suggested as four separate items. However, for measurement purposes, it could be possible to develop a reasonable scoring rule, such as:

- Score 2—all four parts rated correctly.
- Score 1—any three parts rated correctly.

#### 4. Secondary

<b>Categorization</b>	
Domain:	Mathematics Content Knowledge (MCK)
Sub-domain:	Algebra
Performance:	Applying
Level:	Advanced
Response type:	Constructed

The graph of the function  $y=x-\sin x$  is given below.



Determine at which of point A or point B the slope of the graph is greater. Show your work in the box below.

Source: Kiril Bankov.

## Scoring Guide

Code	Response	Item: TD1S120
	<b>Correct response</b>	
20	<p>The response indicates that the slope of the graph at point A and point B is the same, and correct work is shown.</p> <p><i>Example:</i>  <i>If <math>y = x - \sin x</math>, <math>y' = 1 - \cos x</math>. At <math>x = \pi/2</math>, <math>y' = 1</math> and at <math>x = 3\pi/2</math>, <math>y' = 1</math>, so the slopes at points A and B are equal.</i></p>	
	<b>Partially correct response</b>	
10	<p>The response correctly shows the derivative of the given function, but contains error in the value of the derivative at point A (or point B), and therefore concludes that the slopes of the tangents to the given function at point A and point B are not the same.</p>	
11	<p>Response that indicates that the slopes of the graph at points A and B are equal-based on a chain of reasoning that is partly correct and partly incorrect.</p>	
	<b>Incorrect response</b>	
70	<p>Responses that indicate an attempt to take the derivative of the given function, but there is no correct work.</p>	
79	<p>Other incorrect (including crossed out, erased, stray marks, illegible, or off-task).</p>	
	<b>Non-response</b>	
99	<p>Blank</p>	

### Sample Items for Beliefs about Mathematics and Mathematics Teaching

#### 1. Beliefs about the nature of mathematics

To what extent do you agree or disagree with the following beliefs about **the nature of mathematics**?

Check *one* box in each row.

	Strongly disagree	Disagree	Slightly disagree	Slightly agree	Agree	Strongly agree
A. Mathematics is a collection of rules and procedures that prescribe how to solve a problem.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
B. Mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
C. Mathematics involves creativity and new ideas.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
D. In mathematics many things can be discovered and tried out by oneself.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
E. When solving mathematical tasks you need to know the correct procedure or else you would be lost.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
F. If you engage in mathematical tasks, you can discover new things (e.g., connections, rules, concepts).	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
G. Fundamental to mathematics is its logical rigor and preciseness.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
H. Mathematical problems can be solved correctly in many ways.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
I. Many aspects of mathematics have practical relevance.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
J. Mathematics helps solve everyday problems and tasks.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
K. To do mathematics requires much practice, correct application of routines, and problem-solving strategies.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
L. Mathematics means learning, remembering and applying.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>

## 2. Beliefs about learning mathematics

From your perspective, to what extent would you agree or disagree with each of the following statements about **learning mathematics**?

Check *one* box in *each* row.

	Strongly disagree	Disagree	Slightly disagree	Slightly agree	Agree	Strongly agree
A. The best way to do well in mathematics is to memorize all the formulas.	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5	<input type="checkbox"/> 6
B. Pupils need to be taught exact procedures for solving mathematical problems.	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5	<input type="checkbox"/> 6
C. It doesn't really matter if you understand a mathematical problem, if you can get the right answer.	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5	<input type="checkbox"/> 6
D. To be good in mathematics you must be able to solve problems quickly.	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5	<input type="checkbox"/> 6
E. Pupils learn mathematics best by attending to the teacher's explanations.	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5	<input type="checkbox"/> 6
F. When pupils are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process followed.	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5	<input type="checkbox"/> 6
G. In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5	<input type="checkbox"/> 6
H. Teachers should allow pupils to figure out their own ways to solve mathematical problems.	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5	<input type="checkbox"/> 6
I. Non-standard procedures should be discouraged because they can interfere with learning the correct procedure.	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5	<input type="checkbox"/> 6
J. Hands-on mathematics experiences aren't worth the time and expense.	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5	<input type="checkbox"/> 6
K. Time used to investigate why a solution to a mathematical problem works is time well spent.	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5	<input type="checkbox"/> 6
L. Pupils can figure out a way to solve mathematical problems without a teacher's help.	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5	<input type="checkbox"/> 6
M. Teachers should encourage pupils to find their own solutions to mathematical problems even if they are inefficient.	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5	<input type="checkbox"/> 6
N. It is helpful for pupils to discuss different ways to solve particular problems.	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5	<input type="checkbox"/> 6

### 3. Beliefs about mathematics achievement

To what extent do you agree or disagree with each of the following statements about **pupil achievement** in <primary/lower secondary> mathematics?

Check *one* box in *each* row.

	Strongly disagree	Disagree	Slightly disagree	Slightly agree	Agree	Strongly agree
A. Since older pupils can reason abstractly, the use of hands-on models and other visual aids becomes less necessary.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
B. To be good at mathematics you need to have a kind of “mathematical mind”.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
C. Mathematics is a subject in which natural ability matters a lot more than effort.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
D. Only the more able pupils can participate in multi-step problem-solving activities.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
E. In general, boys tend to be naturally better at mathematics than girls.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
F. Mathematical ability is something that remains relatively fixed throughout a person’s life.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
G. Some people are good at mathematics and some aren’t.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
H. Some ethnic groups are better at mathematics than others.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>

#### 4. Beliefs about preparedness for teaching mathematics

Please indicate the extent to which you think your teacher education program has prepared you to do the following **when you start your teaching career**.

Check *one* box in each row.

	Not at all	A minor extent	A moderate extent	A major extent
A. Communicate ideas and information about mathematics clearly to pupils	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
B. Establish appropriate learning goals in mathematics for pupils	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
C. Set up mathematics learning activities to help pupils achieve learning goals	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
D. Use questions to promote higher order thinking in mathematics	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
E. Use computers and ICT to aid in teaching mathematics	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
F. Challenge pupils to engage in critical thinking about mathematics	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
G. Establish a supportive environment for learning mathematics	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
H. Use assessment to give effective feedback to pupils about their mathematics learning	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
I. Provide parents with useful information about your pupils' progress in mathematics	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
J. Develop assessment tasks that promote learning in mathematics	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
K. Incorporate effective classroom management strategies into your teaching of mathematics	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
L. Have a positive influence on difficult or unmotivated pupils	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
M. Work collaboratively with other teachers	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>

### 5. Beliefs about program effectiveness

To what extent do you agree or disagree with the following statements?

The instructors who teach mathematics-related <courses> in your current teacher preparation program:

Check *one* box in each row.

	Strongly disagree	Disagree	Slightly disagree	Slightly agree	Agree	Strongly agree
A. Model good teaching practices in their teaching	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
B. Draw on and use research relevant to the content of their <courses>	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
C. Model evaluation and reflection on their own teaching	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
D. Value the learning and experiences you had prior to starting the program	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
E. Value the learning and experiences you had in your field experience and or practicum	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>
F. Value the learning and experiences you had in your teacher preparation program	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>	<input type="checkbox"/> <sub>5</sub>	<input type="checkbox"/> <sub>6</sub>

## Sample Items for Opportunities to Learn

### 1. University- or tertiary-level mathematics

Consider the following topics in university-level mathematics. Please indicate whether you have ever studied each topic.

Check *one* box in each row.

	Studied	Not studied
A. Foundations of Geometry or Axiomatic Geometry (e.g., Euclidean axioms)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
B. Analytic/Coordinate Geometry (e.g., equations of lines, curves, conic sections, rigid transformations or isometrics)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
C. Non-Euclidean Geometry (e.g., geometry on a sphere)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
D. Differential Geometry (e.g., sets that are manifolds, curvature of curves, and surfaces)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
E. Topology	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
F. Linear Algebra (e.g., vector spaces, matrices, dimensions, eigenvalues, eigenvectors)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
G. Set Theory	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
H. Abstract Algebra (e.g., group theory, field theory, ring theory, ideals)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
I. Number Theory (e.g., divisibility, prime numbers, structuring integers)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
J. Beginning Calculus Topics (e.g., limits, series, sequences)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
K. Calculus (e.g., derivatives and integrals)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
L. Multivariate Calculus (e.g., partial derivatives, multiple integrals)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
M. Advanced Calculus or Real Analysis or Measure Theory	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
N. Differential Equations (e.g., ordinary differential equations and partial differential equations)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
O. Theory of Real Functions, Theory of Complex Functions or Functional Analysis	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
P. Discrete Mathematics, Graph theory, Game theory, Combinatorics or Boolean Algebra	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
Q. Probability	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
R. Theoretical or Applied Statistics	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
S. Mathematical Logic (e.g., truth tables, symbolic logic, propositional logic, set theory, binary operations)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>

## 2. School-level mathematics

Consider the following list of mathematics topics that are often taught at the <primary> or <secondary> school level. Please indicate whether you have studied each topic as part of your current teacher preparation program.

Check *one* box in *each* row.

- |   | Studied<br>□ <sub>1</sub> | Not studied<br>□ <sub>2</sub> |
|---|---------------------------|-------------------------------|
| A. Numbers (e.g., whole numbers, fractions, decimals, integer, rational, and real numbers; number concepts; number theory; estimation; ratio and proportionality)   | □ <sub>1</sub>            | □ <sub>2</sub>                |
| B. Measurement (e.g., measurement units; computations and properties of length, perimeter, area, and volume; estimation and error)  | □ <sub>1</sub>            | □ <sub>2</sub>                |
| C. Geometry (e.g., 1-D and 2-D coordinate geometry, Euclidean geometry, transformational geometry, congruence and similarity, constructions with straightedge and compass, 3-D geometry, vector geometry) | □ <sub>1</sub>            | □ <sub>2</sub>                |
| D. Functions, Relations, and Equations (e.g., algebra, trigonometry, analytic geometry)   | □ <sub>1</sub>            | □ <sub>2</sub>                |
| E. Data Representation, Probability, and Statistics   | □ <sub>1</sub>            | □ <sub>2</sub>                |
| F. Calculus (e.g., infinite processes, change, differentiation, integration)  | □ <sub>1</sub>            | □ <sub>2</sub>                |
| G. Validation, Structuring, and Abstracting (e.g., Boolean algebra, mathematical induction, logical connectives, sets, groups, fields, linear space, isomorphism, homomorphism)                           | □ <sub>1</sub>            | □ <sub>2</sub>                |

### 3. Mathematics education/pedagogy

Consider the following list of mathematics education/<pedagogy> topics. Please indicate whether you have studied each topic as part of your current teacher preparation program.

Check *one* box in each row.

	Studied	Not studied
A. Foundations of Mathematics (e.g., mathematics and philosophy, mathematics epistemology, history of mathematics)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
B. Context of Mathematics Education (e.g., role of mathematics in society, gender/ethnic aspects of mathematics achievement)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
C. Development of Mathematics Ability and Thinking (e.g., theories of mathematics ability and thinking; developing mathematical concepts; reasoning, argumentation, and proving; abstracting and generalizing; carrying out procedures and algorithms; application; modeling)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
D. Mathematics Instruction (e.g., representation of mathematics content and concepts, teaching methods, analysis of mathematical problems and solutions, problem-posing strategies, teacher–pupil interaction)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
E. Developing Teaching Plans (e.g., selection and sequencing the mathematics content, studying and selecting textbooks and instructional materials)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
F. Mathematics Teaching: Observation, Analysis and Reflection	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
G. Mathematics Standards and Curriculum	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>
H. Affective Issues in Mathematics (e.g., beliefs, attitudes, mathematics anxiety)	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>

#### 4. Education and pedagogy

Consider the following topics in education and <pedagogy>. Please indicate whether you have studied each topic as part of your current teacher preparation program.

Check *one* box in each row.

	Studied	Not studied
	□ <sub>1</sub>	□ <sub>2</sub>
A. History of Education and Educational Systems (e.g., historical development of the national system, development of international systems)	□ <sub>1</sub>	□ <sub>2</sub>
B. Philosophy of Education (e.g., ethics, values, theory of knowledge, legal issues)	□ <sub>1</sub>	□ <sub>2</sub>
C. Sociology of Education (e.g., purpose and function of education in society, organization of current educational systems, education and social conditions, diversity, educational reform)	□ <sub>1</sub>	□ <sub>2</sub>
D. Educational Psychology (e.g., motivational theory, child development, learning theory)	□ <sub>1</sub>	□ <sub>2</sub>
E. Theories of Schooling (e.g., goals of schooling, teacher's role, curriculum theory and development, didactic/teaching models, teacher-pupil relations, school administration and leadership)	□ <sub>1</sub>	□ <sub>2</sub>
F. Methods of Educational Research (e.g., read, interpret and use education research; theory and practice of action research)	□ <sub>1</sub>	□ <sub>2</sub>
G. Assessment and Measurement: Theory and Practice	□ <sub>1</sub>	□ <sub>2</sub>
H. Knowledge of Teaching (e.g., knowing how to teach pupils of different backgrounds, use resources to support instruction, manage classrooms, communicate with parents)	□ <sub>1</sub>	□ <sub>2</sub>

### 5. Teaching for diversity and reflection on practice

In your teacher preparation program, how often did you have the opportunity to learn to do the following?

Check *one* box in each row.

	Never	Rarely	Occasionally	Often
A. Develop specific strategies for teaching students with behavioral and emotional problems	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
B. Develop specific strategies and curriculum for teaching pupils with learning disabilities	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
C. Develop specific strategies and curriculum for teaching gifted pupils	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
D. Develop specific strategies and curriculum for teaching pupils from diverse cultural backgrounds	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
E. Accommodate the needs of pupils with physical disabilities in your classroom	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
F. Work with children from poor or disadvantaged backgrounds	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
G. Use teaching standards and codes of conduct to reflect on your teaching	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
H. Develop strategies to reflect upon the effectiveness of your teaching	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
I. Develop strategies to reflect upon your professional knowledge	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
J. Develop strategies to identify your learning needs	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>

## 6. School experience and the practicum

During the school experience part of your program, how often were you required to do each of the following?

Check *one* box in each row.

	Never	Rarely	Occasionally	Often
A. Observe models of the teaching strategies you were learning in your <courses>	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
B. Practice theories for teaching mathematics that you were learning in your <courses>	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
C. Complete assessment tasks that asked you to show how you were applying ideas you were learning in your <courses>	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
D. Receive feedback about how well you had implemented teaching strategies you were learning in your <courses>	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
E. Collect and analyze evidence about pupil learning as a result of your teaching methods	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
F. Test out findings from educational research about difficulties pupils have in learning in your <courses>	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
G. Develop strategies to reflect upon your professional knowledge	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
H. Demonstrate that you could apply the teaching methods you were learning in your <courses>	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>

## 7. Coherence of the teacher education program

Consider all of the <courses> in the program including subject matter <courses> (e.g., mathematics), mathematics <pedagogy> <courses>, and general education <pedagogy> <courses>. Please indicate the extent to which you agree or disagree with the following statements.

Check *one* box in each row.

	Disagree	Slightly disagree	Slightly agree	Agree
A. Each stage of the program seemed to be planned to meet the main needs I had at that stage of my preparation.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
B. Later <courses> in the program built on what was taught in earlier <courses> in the program.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
C. The program was organized in a way that covered what I needed to learn to become an effective teacher.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
D. The <courses> seemed to follow a logical sequence of development in terms of content and topics.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
E. Each of my <courses> was clearly designed to prepare me to meet a common set of explicit standard expectations for beginning teachers.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
F. There were clear links between most of the <courses> in my teacher education program.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>

## Sample Items for Background and Demographics

### Motivation/plans/intention to become a mathematics teacher

To what extent does each of the following identify your reasons for becoming a teacher?

Check *one* box in *each* row.

	Not a reason	A minor reason	A significant reason	A major reason
A. I was always a good student in school.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
B. I am attracted by the availability of teaching positions.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
C. I love mathematics.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
D. I believe that I have a talent for teaching.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
E. I like working with young people.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
F. I am attracted by teacher salaries.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
G. I want to have an influence on the next generation.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
H. I see teaching as a challenging job.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
I. I seek the long-term security associated with being a teacher.	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>

## APPENDIX C: EXPERT PANEL MEETINGS

Table 9.12 Specialized Advisory/Expert Panel Meetings for TEDS-M, November 2002

Meeting	Participants	Country/Affiliation
<i>Special IEA advisory meeting on approval of TEDS-M Brussels, Belgium November 4–5, 2002</i>	Fernand Rochette	Belgium (Fl)
	Liselotte Van De Perre	Belgium (Fl)
	Ann Van Driessche	Belgium (Fl)
	Marcel Crahay	Belgium (Fr)
	Julien Nicaise	Belgium (Fr)
	Per Fibaek	Denmark
	Bjarne Wahlgren	Denmark
	Gerard Bonnet	France
	Catharine Regneir	France
	Ranier Lehmann	Germany
	Georgia K. Polydores	Greece
	Bruno Losito	Italy
	Ryo Watanabe	Japan
	Andris Kangro	Latvia
	Jean-Claude Fandel	Luxembourg
	Jean-Paul Reef	Luxembourg
	Seamus Hegarty	UK
Arlette Delhaxe	Eurydice	
Barbara Malak-Minkiewicz	IEA Secretariat	
Maria Teresa Tatto	TEDS-M-MSU	

Table 9.13 Specialized Advisory/Expert Panel Meetings for TEDS-M, June 2003

Meeting	Participants	Country/Affiliation
<i>IEA TEDS expert panel meeting Amsterdam, Netherlands June 16–21, 2003</i>	Peter Fensham	Australia
	Kiril Bankov	Bulgaria
	Martial Dembele	Burkina Faso and Quebec
	Beatrice Avalos	Chile
	Per Fibaek	Denmark
	Sigrid Blomeke	Germany
	Frederick Leung	Hong Kong SAR
	Losito Bruno	Italy
	Ciaran Sugrue	Ireland
	Lee Chong-Jae	Korea
	Loyiso Jita	South Africa
	Marilyn Leask	UK
	Christopher Day	UK
	Michael Eraut	UK
	Drew Gitomer	USA
	Susanna Loeb	USA
	Lynn Paine	USA
	David Plank	USA
	Paul Sally	USA
	William Schmidt	Pre-TEDS-MSU
Adrian Beavis	IEA-TEDS-M ACER	
Lawrence Ingvarson	IEA-TEDS-M ACER	
Jack Schwillie	IEA-TEDS-M MSU	
Maria Teresa Tatto	Pre-TEDS and IEA-TEDS-M MSU	

Table 9.14 Specialized Advisory/Expert Panel Meeting for TEDS-M, December 2003

Meeting	Participants	Country/Affiliation
IEA TEDS expert panel meeting Hamburg, Germany December 1–5, 2003	Peter Fensham Kiril Bankov Beatrice Avalos Per Fibaek Laursent Sigrid Blomeke Frederick Leung Ciaran Sugrue Bruno Losito Tenoch Cedillo Avalos Marcela Santillan-Nieto Loyiso C. Jita Marilyn Leask Angelo Collins Lynn Paine Hans Wagemaker Pierre Foy  Dirk Hastedt Lawrence Ingvarson Jack Schwille Maria Teresa Tatto	Australia Bulgaria Chile Denmark Germany Hong Kong SAR Ireland Italy Mexico Mexico South Africa UK USA USA IEA Boston College, previously at IEA DPC IEA DPC IEA-TEDS-M ACER IEA-TEDS-M MSU IEA-TEDS-M MSU

Table 9.15 Specialized Advisory/Expert Panel Meetings for TEDS-M, 18 September 2006

Meeting	Participants	Country/Affiliation
Expert panel for review of primary TEDS-M items (Mathematics Content Knowledge and Mathematics Pedagogy Content Knowledge) Melbourne, Australia September 18, 2006	Prof Doug Clarke Prof Peter Sullivan Prof Kaye Stacey Dr Gaye Williams Prof Barb Clarke Ann Roche Ray Peck Lawrence Ingvarson	Australian Catholic University Monash University Melbourne University Deakin University Monash University Australian Catholic University ACER ACER

Table 9.16 Specialized Advisory/Expert Panel Meetings for TEDS-M,  
29–30 September 2006

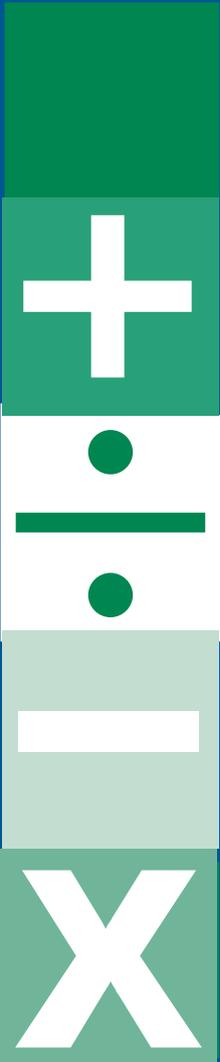
Meeting	Participants	Country/Affiliation
Expert panel for review of TEDS-M items Grand Rapids, Michigan, USA September 29–30, 2006	Kiril Bankov Jarmila Novotna Paul Conway Ruhama Even Kyungmee Park Maarten Dolk Ingrid Munck Hyacinth Evans Lynn Paine Sharon Senk Jack Schwille Maria Teresa Tatto	Bulgaria Czech Republic Ireland Israel Korea Netherlands Sweden West Indies IEA-TEDS-M MSU IEA-TEDS-M MSU IEA-TEDS-M MSU IEA-TEDS-M MSU

*Table 9.17 Specialized Advisory/Expert Panel Meetings for TEDS-M, June 2006*

<b>Meeting</b>	<b>Participants</b>	<b>University</b>
<i>Expert panel for review of TEDS-M items and data from field trial East Lansing, Michigan, USA June, 2006</i>	Edward Aboufadel Sandra Crespo Glenda Lappan Vince Melfi Jeanne Wald Rebecca Walker	Grand Valley State University Michigan State University Michigan State University Michigan State University Michigan State University Grand Valley State University

**APPENDIX D: LIST OF NATIONAL RESEARCH CENTERS PARTICIPATING IN TEDS-M**

<b>Country</b>	<b>Name</b>	<b>Affiliation</b>
Botswana	Thabo Jeff Mzwinila	Tlokweng College of Education
Canada	Pierre Brochu	Pan-Canadian Assessment Program, Council of Ministers of Education, Canada
Chile	Beatrice Avalos	Ministry of Education, Chile, Unidad de Curriculum y Evaluación
Chinese Taipei	Feng-Jui Hsieh Pi-Jen Lin	National Taiwan Normal University, Department of Mathematics; National Hsinchu University of Education, Department of Applied Mathematics
Georgia	Maia Miminoshvili Tamar Bokuchava	NAEC—National Assessment and Examination Center
Germany	Sigrid Blömeke	Humboldt University of Berlin, Faculty of Arts IV
Malaysia	Mohd Mustamam Abd. Karim Rajendran Nagappan	Universiti Pendidikan Sultan Idris
Norway	Liv Grønmo	ILS, University of Oslo
Oman	Zuwaina Al-maskari	Math Curriculum Department, Ministry of Education
Philippines	Ester Ogena Evangelina Golla	Science Education Institute, Department of Science and Technology
Poland	Michał Federowicz	Institute of Philosophy and Sociology, Polish Academy of Sciences
Russian Federation	Galina Kovaleva	Center for Evaluating the Quality of Education, Institute for Content of Methods of Learning, Russian Academy of Education
Singapore	Khoo Yoong Wong	National Institute of Education, Nanyang Technological University
Spain	Luis Rico	University of Granada
Switzerland	Fritz Oser Horst Biedermann	University of Fribourg
Thailand	Precharn Dechsri Supatra Pativisan	The Institute for the Promotion of Teaching Science and Technology
United States	William Schmidt	Michigan State University



This document outlines the framework and a comprehensive plan for a cross-national study of primary and secondary mathematics teacher education (TEDS-M) sponsored by the International Association for the Evaluation of Educational Achievement (IEA).

TEDS-M 2008, which builds on the results of IEA student achievement studies, focuses on how teachers are prepared to teach mathematics in primary and lower secondary schools.

TEDS-M is a study of the variation in the nature and impact of teacher education programs within and across countries. The study collects and analyzes nationally representative data from participating countries to address contested issues and improve policy and practice in teacher education. This document lays out the goals and justification for this study as well as its design and methodology.

The overall study has three overlapping components:

- COMPONENT I: Studies of teacher education policy, schooling, and social contexts at the national level.
- COMPONENT II: Studies of primary and lower secondary mathematics teacher education routes, institutions, programs, standards, and expectations for teacher learning.
- COMPONENT III: Studies of the mathematics and related teaching knowledge of future primary and lower secondary school mathematics teachers.

The key research questions for the study focus on the relationships between these components, such as relationships between teacher education policies, institutional practices, and future teacher outcomes.

## Errata

Page 73, Appendix B

The following Primary item has been incorrectly categorized as Data. It should be Algebra.

### 2. Primary

<b>Categorization</b>	
Domain:	Mathematics Content Knowledge (MCK)
Sub-domain:	Data
Performance:	Reasoning
Level:	Novice
Response type:	Constructed (short)

A teacher gave the following problem to her class.

The numbers in the sequence 7, 11, 15, 19, 23, ..... increase by 4. The numbers in the sequence 1, 10, 19, 28, 37, ... increase by nine.

The number 19 is in both sequences.

If the two sequences are continued, what is the next number that is in BOTH the first and the second sequence?

(a) What is the correct answer to this problem? \_\_\_\_\_

Page 95, Appendix D

Correction: Poland's NRC is Michal Sitek.

Correction: Spain's NRC is Ministerio de Educación: Instituto de Evaluación, Instituto Superior de Formación y Recursos en Red para el Profesorado, y Secretaría del Consejo de Coordinación Universitaria and Universidad de Granada