

**Abstract Title Page**

**Title:** *Fostering First-Graders' Reasoning Strategies with the Most Basic Sums*

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## Abstract Body

**Background:** Although there is general agreement that *all* children need to achieve fluency with basic sums (single-digit items such as  $7+1$  and  $4+4$ ) and knowledge of basic combinations is one of the most well studied phenomena in educational psychology (Kilpatrick, Swafford, & Findell, 2001), the best method of instruction for these combinations is not entirely clear. Brownell (1935), proponents of the number sense view (Gersten & Chard, 1999; Jordan, 2007), and others (Henry & Brown, 2008) have argued that meaningful memorization of basic sums is more efficient in achieving fluency than their memorization by rote. James (1958) observed that meaningful and secure memorization of new information can be achieved by relating it to what a child already knows (cf. Piaget, 1964). For example, although many first graders can efficiently cite the number after another in the counting sequence, they do not use this knowledge to determine the sum of an  $n+1/1+n$  item and resort to a counting strategy (developmental phase 1). Connecting adding with 1 to existing number-after knowledge yields a **general add-1 rule or reasoning strategy**: *The sum of  $n+1/1+n$  is the number after  $n$  in the counting sequence* (developmental phase 2). With practice, children can use this rule efficiently to deduce the sum of any  $n+1$  combination for which they know the counting sequence, even those not previously practiced. That is, this rule or reasoning strategy can become part of the retrieval network (developmental phase 3; Baroody, 1985; Fayol & Thevenot, in press; National Mathematics Advisory Panel, 2008).

In a meta-analysis of 164 studies, Alfieri, Brooks, Aldrich, and Tenenbaum (2010) found that assisted discovery learning was more effective than explicit instruction or unassisted discovery learning and that explicit instruction resulted in more favorable outcomes than unassisted discovery learning. In other words, “unassisted discovery does not benefit learners, whereas feedback, worked examples, scaffolding, and elicited explanations do” (p. 1).

**Research Question:** The primary aim of the present research was to compare structured discovery, unstructured discovery (haphazard practice), and business-as-usual in fostering fluency with add-1 and doubles combinations.

**Setting:** Interventions were conducted during the school day as pullout, 1-on-1, training sessions at five elementary schools in two school districts serving a medium-size Mid-western city.

**Participants:** Through mental-addition pretesting/screening, 77 children from a sample pool consisting of 156 first graders were identified as eligible for the study. Eligibility was defined as not fluent on more than 50% of the  $n+1/1+n$  items ( $M = 13\%$ ) or the doubles ( $M = 15\%$ ). Descriptive information on participants can be found in Table 1. Participating children ranged in age from 6.0 to 8.1 years old (mean=6.6). Of these children, 58.6% of the children were female. The majority of children were African-American (55.7%; 27.1% Caucasian; 8.6% Hispanic; 8.6% mixed, unknown, or other race). Additionally, 75.7% of participants were eligible for free or reduced-price lunch.

**Intervention:** The preparatory training (Stages I and II) and the primary training by instructional condition (Stages III to V) are detailed in Table 2. The preparatory training was common to all participants and was designed to ensure an adequate base of knowledge for mental addition and the computer-based testing and training. This included remedying knowledge gaps identified by preliminary testing. The primary training was tailored to help children discover and practice a

particular reasoning strategy or provide practice for a particular family of combinations. See Figures 1 to 3 for samples of the primary training.

**Research Design:** All children in the sample pool simultaneously received the 7.5-week long preparatory (Stage I and II) training. During this time, preliminary testing involving a nationally standardized achievement/diagnostic test (TEMA-3) was administered to gauge mathematics achievement and identify gaps in readiness skills for mental addition. After the completion of the preparatory training, participants were individually administered a computer-based mental-addition pretest/screening test that served to gauge fluency with the easiest sums: adding with 0 and 1 and the doubles. Participants fluent on half or fewer than half the adding with 1 and the doubles items were eligible for the present study. Participants were then randomly assigned by class to one of three primary training conditions: (a) structured learning/practice of add-1 reasoning strategy, (b) structured learning/practice of doubles reasoning strategy, or (c) unstructured practice of add-1 and doubles combinations. The computer-assisted experimental interventions were conducted simultaneously, and each lasted 12 weeks. Both preparatory and primary training involved one-to-one, 30-minute sessions twice per week. All project training and testing was conducted at project computer stations in a hallway outside a child's classroom or in a room dedicated to the project. Pullouts occurred in non-literacy time blocks, including mathematics instruction and playtime. All participants were re-tested on the mental arithmetic items two weeks after the training. This delayed mental-addition posttest served to gauge retention of practiced combinations and transfer to unpracticed combinations. Project personnel (University Research Assistants, Research Associates, or Academic Hourlies) implemented all testing and training procedures. Positive assent was obtained for each testing and training sessions. A summary of the research design can be found in Table 3.

### **Data Collection and Analysis:**

*Data Collection:* The test of mental arithmetic fluency included five categories of items: (a) practiced  $n+1/1+n$  items (1+3, 3+1, 1+4, 4+1, 1+7, 7+1, 1+8, 8+1); (b) unpracticed (transfer)  $n+1/1+n$  items (1+5, 5+1, 1+6, 6+1, 1+9, 9+1), (c) practiced doubles items (1+1, 2+2, 3+3, 5+5, 6+6, 7+7), (d) unpracticed (transfer) doubles items (4+4, 8+8, 9+9, 10+10, 12+12); (e) practiced filler items (2+8, 3+4, 3+5, 4+3, 5+3, 7+2). The combinations practiced by condition can be found in Table 4. Note that category *a* items were practiced by the add-1 and the unstructured-practice groups; category *c* items, by the doubles and unstructured-practice groups; and the category *e* items, by all groups. None of the groups practiced category *b* and category *d* items, so category *b* served as transfer items for add-1 and the unstructured-practice groups, and category *d* served as transfer items for the doubles and the unstructured-practice groups. The testing was done in the context of a computer game (see Figure 4).

*Data Analysis:* *Combination fluency* was defined as producing its sum quickly (< 3secs.) and accurately— without counting or evidence of false positives due to a response bias (inflexibly responding with the larger addend or the number after the larger addend on  $\geq 50\%$  of the trials in a testing session where such a response was inappropriate). As the two primary (structured add-1 and doubles) groups targeted different types of skills, each served as a control group for the other. The unstructured-practice group also served as an active instructional comparison group in both sets of analyses to determine if the structured discovery practice resulted in better outcomes than just simply extra practice with the items. Analyses of fluency were done using the proportion correct by a child on a test. ANCOVAs, using pretest mental-

arithmetic fluency, pretest TEMA-3 pretest, and age as the covariates, were used to compare posttest performance of each group on targeted practiced and unpracticed combinations. Effects of treatment were tested using one-tailed significance values given the directional nature of the contrasts (e.g., for the add-1 analyses the structured add-1 group > unstructured practice > control (doubles) group and for the structured doubles analyses the doubles group > unstructured practice group > control (add-1) group. The Benjamini-Hochberg correction was applied to correct for Type I error due to multiple comparisons. Effect size (*Hedge's g*) for all contrasts was also examined due to the limited power of the study and the importance of evaluating effect sizes (Wilkinson & APA Task Force on Statistical Inference, 1999).

**Results:** The mean proportion (and standard deviations) of practiced and unpracticed (transfer) combinations scored as fluent (after scoring false positive due to a response bias) by combination item type or family and condition is detailed in Table 5.

*Adding-with-1.* For practiced  $n+1/1+n$  combinations, planned contrasts revealed that, at the delayed posttest, the structured ( $F[1, 43] = 4.46, p = .021, Hedge's g = .60$ ) and unstructured add-1 groups ( $F[1, 41] = 6.63, p = .007, Hedge's g = .70$ ) outperformed the doubles group, which did not practice the  $n+1/1+n$  combinations. So significant differences were found between the structured and unstructured add-1 groups ( $F[1, 41] = .03, p = .430, Hedge's g = -.05$ ). For unpracticed (transfer)  $n+1/1+n$  items, no significant differences were found between the structured add-1 group and the doubles group ( $F[1, 43] = 1.26, p = .135, Hedge's g = .31$ ) or between the unstructured practice group and the doubles group ( $F[1, 41] = .70, p = .205, Hedge's g = .22$ ). However, effect size differences in both comparisons were in the range of meaningfully significant. No significant differences were found between the structured add-1 group and the unstructured practice groups ( $F[1, 41] = .16, p = .345, Hedge's g = .11$ ). These results indicate that both structured and unstructured add-1 training was more effective in promoting of add-1 rule than the active control group. However, contrary to Alfieri et al.'s (2010) conclusions—participants in the structured add-1 condition did not outperform those in the unstructured add-1/doubles group on practiced and unpracticed (transfer)  $n+1/1+n$  items.

*Doubles.* For practiced double items, planned contrasts revealed that, at the delayed posttest, the structured doubles ( $F[1, 43] = 19.26, p < .001, Hedge's g = 1.00$ ) and unstructured add-1/doubles groups ( $F[1, 41] = 16.48, p < .001, Hedge's g = .87$ ) outperformed the add-1 group, which did not practice the doubles combinations. However, the structured doubles group did not outperform the unstructured add-1/doubles groups ( $F[1, 41] = .06, p = .406, Hedge's g = .06$ ). For unpracticed (transfer) doubles items, the structured doubles group significantly outperformed the unstructured practice group ( $F[1, 41] = 4.95, p = .016, Hedge's g = .51$ ) and marginally significantly outperformed the add-1 control group ( $F[1, 43] = 1.93, p = .086, Hedge's g = .31$ ). The unstructured practice group did not significantly outperform the add-1 comparison group ( $F[1, 41] = .73, p = .199, Hedge's g = -.21$ ). In fact, in the latter comparison, the effect size favored the add-1 comparison group. These results indicate that the structured doubles training was more effective in promoting transfer with these combinations than unstructured practice or the active control group.

**Conclusions and Educational Implications:** Participants in the unstructured add-1 training achieved comparable gains in fluency with  $n+1/1+n$  items as those in the structured add-1, and children in these groups achieved greater fluency with  $n+1/1+n$  items than did peers with active-control group (cf. Alfieri et al., 2011). Although this pattern of results suggests that additional

practice is all that is needed to promote fluency with these basic combinations, several considerations suggest that participants in both the structured *and* unstructured add-1 training probably discovered the add-1 rule rather than memorized  $n+1/1+n$  facts by rote. First, these results were achieved *despite* only 27 repetitions for each of the eight practiced  $n+1/1+n$  items—substantially less practice than thousands of repetitions per item necessary to achieve memorization (by rote) of these facts specified by earlier models and computer simulations of arithmetic learning (e.g., Shrager & Siegler, 1998; Siegler & Jenkins, 1989). Second, transfer to unpracticed  $n+1/1+n$  items is consistent with rule-governed, not rote, learning.

In contrast, the structured doubles condition was more effective than the unstructured practice group in promoting transfer to unpracticed doubles. This difference may have been due to the former group's recognition that the sum of all doubles is an even number and the obvious relation between such sums and skip counting. These contrasting results indicate that the relative efficacy of structured and unstructured instruction/practice is not as straightforward as Alfieri et al. (2011) imply. Specifically, the effectiveness of structured and unstructured discovery varies from combination family to family—depends on the salience of a pattern or relation and fluency with developmental prerequisites (e.g., participants in both the structured and unstructured add-1 groups were fluent with the number-after relations, which is necessary for the fluent application of the number-after rule for adding 1). Whether structured discovery of the add-1 rule might be more effective than unstructured discovery for younger, less developmentally advanced children needs to be examined.

The effectiveness of the add-1 training in promoting fluency with practiced  $n+1/1+n$  items and, more importantly, transfer (fluency with unpracticed  $n+1/1+n$  items) provides additional supporting evidence for what Alfieri et al. (2011) called “the generation effect” with a genuine school (ecologically valid) task. They defined a *generation effect* as the enhancement of learning and retention when learners are permitted to construct their own knowledge in some way, such as generating their own generalization. The results are consistent with the positions outlined by both the NMAP (2008) and the number sense view (Gersten & Chard, 1999; Jordan, 2007) that *both* forming associations via practice and recognizing relations such as the number-after rule for adding 1, are key aspects of achieving fluency and that reasoning processes can be an efficient basis for determining the solutions to basic combinations. Indeed, practice may not be merely a vehicle for strengthening a factual association but an opportunity to enrich memory of a combination by actively creating new connections with it. That is, information stored in long-term memory may not have a permanent form and may be changed each time the memory is recalled (Nader & Hardt, 2009). For example, recalling that the number after “seven” is “eight” while solving (calculating)  $7+1=?$  may help children to construct or strengthen the successor principle—each counting number is exactly one more than its predecessor. Recognizing the connection between adding 1 and known number-after relations and activating the successor principle may allow the generalization that the sum of any  $n+1$  item is the successor of  $n$ . Such a representation allows children to use their (automatic) knowledge of the generative rules for counting to efficiently deduce the sum of any  $n+1/1+n$  item for any known part of the counting sequence. This hypothesis fills an important theoretical gap identified by Siegler and Ramani (2009): “Future models of arithmetic [might] benefit from including retrieval structures or other mechanisms that embody numerical magnitude representations” (p. 556). Specifically, the add-1 rule is the connection between the representations of counting and numerical magnitude that embody the successor principle and retrieval structures. For all of these reasons, learning the add-1 rule should be a focal point or primary goal of first grade instruction.

## Appendices

### Appendix A. References

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Table 1

*Characteristics of the Participants by Condition*

		Condition		
		Structured Add-1	Structured Doubles	Unstructured Practice
Age range		6.1 to 7.3	6.1 to 8.1	6.0 to 7.1
Median age		6.6	6.5	6.7
Number of boys : girls		14:11	6:18	10:12
TEMA-3 range		71 to 109	75 to 114	70 to 112
Median TEMA-3		89	89.5	93
Free/Reduced lunch eligible		21	21	18
Black/Hispanic/Multiracial		13	19	18
Family History	Single-parent	7	7	4
	Parent under 18	0	0	0
	Parents w/out HS	1	0	0
	ESL	2	1	2
Medical/Developmental Condition	Birth complications	0	1	0
	Visual impairment	0	1	1
	Language delay	2	2	1
	Speech services	1	2	2
	Spina bifida	1	0	0
Behavioral Condition	ADHD	2	3	2
	Aggressive	8	4	1
	Passive/withdrawn	1	5	1
Attrition		1 moved	1 moved; 1 refused	2 moved; 1 refused

Table 2

*Five Stages of Computer-Assisted Mental-Arithmetic Training*

A five-stage approach was used to help primary-level children make the transition from concrete arithmetic (Phase 1) to efficient mental arithmetic (Phase 3). A child's solutions in the first three stages below were untimed; solutions in the last two stages were timed. The blue shading indicates preparatory training common to all three conditions. The orange shading indicates the experimental training that differed by condition.

Stage Name	Rationale
<b>Stage I: Preparatory Concrete Training</b> (7 sessions; ~ 3.5 weeks)	<p><b>Aim:</b> Support phase 1 and ensure recognition and understanding of the formal symbolism for addition and subtraction (e.g., <math>7+1</math> or <math>8-2</math>) by connecting it to concrete or meaningful situations and their own informal solutions. (Items presented as meaningful word problems AND symbolic expressions. Children were encouraged to solve problems in any way they wish, including informal counting-based strategies). To this end, virtual manipulatives, such as 10 frames and number sticks, were presented as an option.)</p> <p><b>Plan:</b> For each of 7 sessions, there were 3 sets. Set 1A served a vehicle for learning how to navigate the program (e.g., use the mouse). Set 1B and 2A to 7A introduced virtual manipulatives (e.g., record a score using a ten frames and dots). Set 2B to 7B involved solving word problems (relating expressions or equations to a concrete model). Set 1C to 7C focused on relating part-whole terminology to equations and composition and decomposition, which underlie a number of reasoning strategies.</p>
<b>Stage II: Preparatory Mental Arithmetic (Estimation) Training</b> (8 sessions; ~ 4 weeks)	<p><b>Aim:</b> Serve as a developmental bridge between using Phase 1 strategies promoted by Stage (informal counting-based strategies with objects such as fingers or a ten frame) and Phases 2 and 3 (using mental-arithmetic strategies involving reasoning or retrieval)—i.e., serve as the transition between exact concrete computation (Stage I) and exact mental arithmetic (Stages III to V). Help children identify and define a good or SMART GUESS.</p> <p><b>Plan:</b> Numerical estimation (approximating the size of a single collection) was introduced first in Sets 8 and 9; arithmetic estimation (approximating the size of sums and differences), in sets 10 to 12. The stage begins visually estimating the <i>number</i> of carrots or frogs. (About how many carrots or frogs did you see)? This provides a basis for estimating the answers to <i>addition and subtraction</i> problems come next.</p>

Table 2 continued

<p><b>Stage III: Strategy Training</b> (8 sessions; ~ 4 weeks)</p>	<p><b>Aim:</b> Except for the control condition, promote phase 2 (help a child discover the relations that underlie a reasoning strategy and thus understand and effectively use a reasoning strategy). <b>Plan:</b> Items presented concretely (bubble lines and ten frames) and symbolically. A child was encouraged to determine an answer in manner of his/her choice. No time limit was set. Re-dos for incorrect responses were done concretely.</p>
<p><b>Stage IV: Strategy Practice</b> (8 sessions; ~ 4 weeks)</p>	<p><b>Aim:</b> Promote phase 3. <b>Plan:</b> Items are presented symbolically. A child is encouraged to make initial response mentally and quickly (“make a smart guess as quickly as you can”). Concrete solutions (bubble lines and ten frames) are now used only as a backup for determining the exact answer (correcting an incorrect response) on second attempts or re-dos. Related items are juxtaposed or immediately follow one another only some of the time. Using concrete means of solving the problems are only for backup; second attempts. In Stage IV we encourage children, through hints in the feedback, to use the relationship they are being trained in to solve problems in a timely fashion.</p>
<p><b>Stage V: Strategy Fluency</b> (8 sessions; ~ 4 weeks)</p>	<p><b>Aim:</b> Cement phase 3. <b>Plan:</b> The child is encouraged to make a good guess (“smart guess”) as accurately and quickly as possible. For an initial incorrect response, the child is given a second chance to revise her answer mentally (a second-chance guess). In Stage V, we are preparing the child for the posttest. Mental arithmetic is emphasized. “Give us your answer right away.” If unsure of an answer, make a SMART GUESS FAST. Our games underscore the importance of a fast and smart guess and the disadvantages of counting. Second-chance guess, no hints, no manipulates. See Table 4 for details.</p>

Table 3

*Practice and Transfer Items for Each Condition*

Structured $n+0/n+1$ Training		Structured Doubles Training		Unstructured $n+0/n+1$ & Doubles Training		All Conditions (Filler Items)	
Practiced Items	Unpracticed Items	Practiced Items	Unpracticed Items	Practiced Items	Unpracticed Items	Practiced Items	Unpracticed Items
0+4 3+0	0+5 6+0	1+1	4+4	0+4 3+0	1+5 5+1	2+8	8+2
0+7 8+0	0+9 7+0	2+2	8+8	0+7 8+0	1+6 6+1	3+4	
1+3 3+1		3+3	9+9	1+3 3+1	1+9 9+1	3+5	
1+4 4+1	1+5 5+1	5+5	10+10	1+4 4+1	17+1 1+18	4+3	
1+7 7+1	1+6 6+1	6+6	12+12	1+7 7+1	4+4	5+3	
1+8 8+1	1+9 9+1	7+7		1+8 8+1	8+8	7+2	2+7
After 2 to 9,	17+1 1+18	Share 2 to 15, 17, & 20		1+1 2+2	9+9		
After 17, & 18				3+3 5+5	10+10		
				6+6 7+7	12+12		

Note. Items on the pretest and delayed posttest were:

Table 4

*Summary of the Research Design the Distinguishing Features of the Experimental Stage II*

Experimental Training Condition	Manual TEMA-3 Pretest (Sep)	Computer-assisted Training Stages I & II: developmental prerequisites for mental addition (Oct-Dec)	Computer-assisted Mental-Addition Pretest <sup>a</sup> (Jan)	R	Computer-assisted Training Stage III to V: Specific to a condition <sup>b</sup> (Feb-Apr)	Computer-assisted Mental-Addition Delayed Posttest <sup>a</sup> (May)
Structured add 1	<b>X</b>	<b>X</b>	<b>X</b>	<b>X</b>	Practiced $n+1/1+n$ immediately after number-after relations	<b>X</b>
Unstructured add 1 & doubles practice	<b>X</b>	<b>X</b>	<b>X</b>	<b>X</b>	Practiced $n+1/1+n$ and <b>doubles</b> in HAPHAZAR D orders	<b>X</b>
Structured doubles	<b>X</b>	<b>X</b>	<b>X</b>	<b>X</b>	Related <b>doubles</b> to skip counting by 2 (sums are always even numbers)	<b>X</b>

*Training*

**Note.** X = activity identical in all conditions; **R** = random assignment.

**Note also.** The structured add-1 condition served as the control for the structured doubles training, and the doubles condition served as the comparison group for the structured add-1 condition. The unstructured add-1 and doubles conditions served as a control for the effects of additional practice.

<sup>a</sup> The pretest and both posttests involved the practiced and unpracticed  $n+0/0+n$ ,  $n+1/1+n$ , near double, and filler sets and the practiced doubles.

Table 5

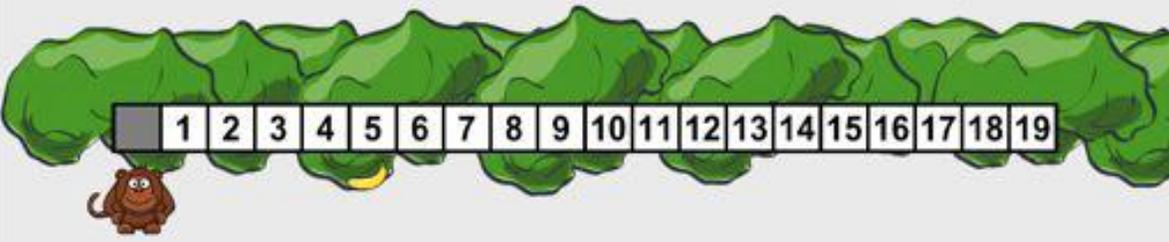
*Pretest and Delayed Posttest Means (and Standard Deviations) by Condition for  $n+1/1+n$  and*

*Doubles Items*

Condition	<i><math>n+1/1+n</math></i>				Doubles			
	<i>Practiced</i>		<i>Unpracticed</i>		<i>Practiced</i>		<i>Unpracticed</i>	
	Pretest t	Posttest t	Pretest t	Posttest t	Pretest t	Posttest t	Pretest t	Posttest t
Structured Add-1 Group	.12 (.14)	.66 (.32)	.17 (.16)	.56 (.33)	.22 (.23)	.40 (.31)	.06 (.11)	.18 (.19)
Unstructured practice	.11 (.13)	.74 (.30)	.17 (.16)	.57 (.38)	.23 (.23)	.71 (.28)	.12 (.17)	.20 (.23)
Structured Doubles Group	.09 (.12)	.48 (.38)	.11 (.14)	.47 (.33)	.24 (.25)	.73 (.27)	.07 (.13)	.28 (.24)

Figure 1: A sample of structured add-1 training (relating add-1 to number after)

The monkey sees a banana and decides to swing out to it and grab it.



1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

Relating number after to  $n+1$ .

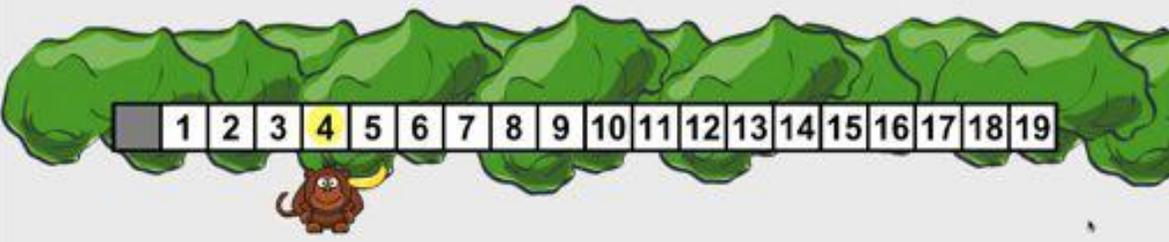
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"Oh no!"

"I landed on 4, but the banana is the number after 4!"

What is the number after 4?

On the counting line below click on your answer.



1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

Figure 1 continued

**Correct!**  
The number After 4 is 5.

A horizontal number line with boxes for numbers 1 through 19. The number 4 is highlighted in yellow, and the number 5 is highlighted in green. A cartoon animal is positioned below the number 4.

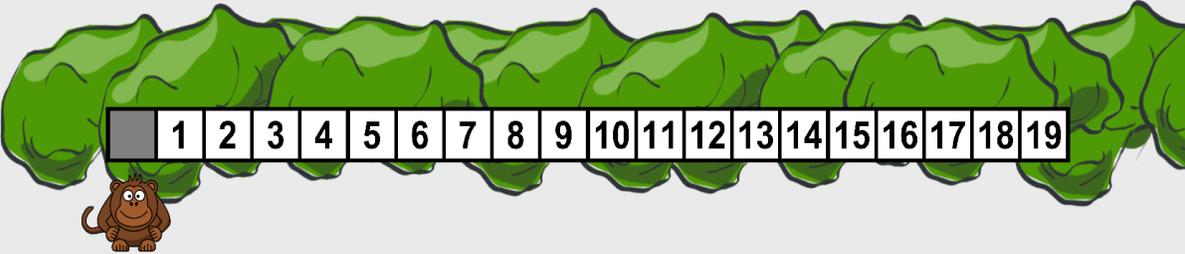
Does knowing the number after 4 is 5 help you figure out  
how much 4 and 1 more is?  
 $4 + 1 = ?$

Click on the [Yes] button if the number After 4 help you answer 4 and 1 more.  
Click on the [No] button if the number After 4 will not help you answer 4 and 1 more.

A horizontal number line with boxes for numbers 1 through 19. The number 4 is highlighted in yellow, and the number 5 is highlighted in green. A cartoon animal is positioned below the number 4. Above the number line, there is a box containing "+1" and another box containing "4".

Figure 2: A sample of doubles training (relating doubles to skip counting)

**The count by 2 monkey swings by 2.**

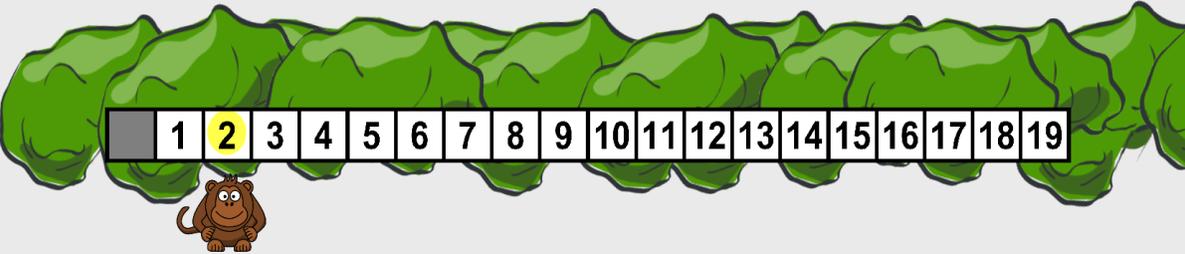


1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

Relating adding doubles to an even sum.

**If the count by 2 monkey swings again where will he land?**

**Click the number on the counting line where you think he will land next.**



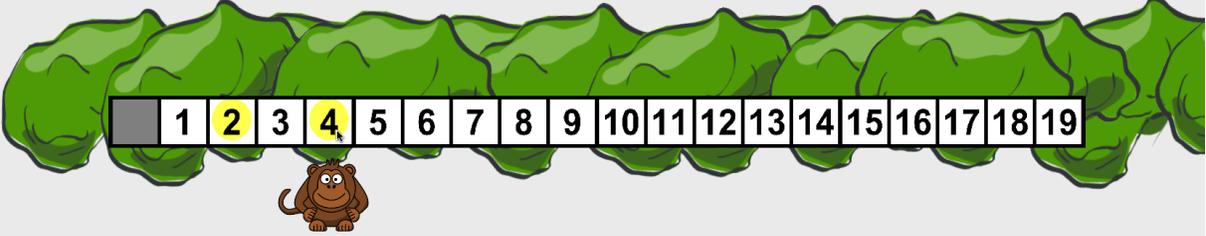
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

Figure 2 continued

**Correct!**

**The count by 2 monkey swings by 2 every time.**

**So first he swings to 2 and next he swings to 4.**



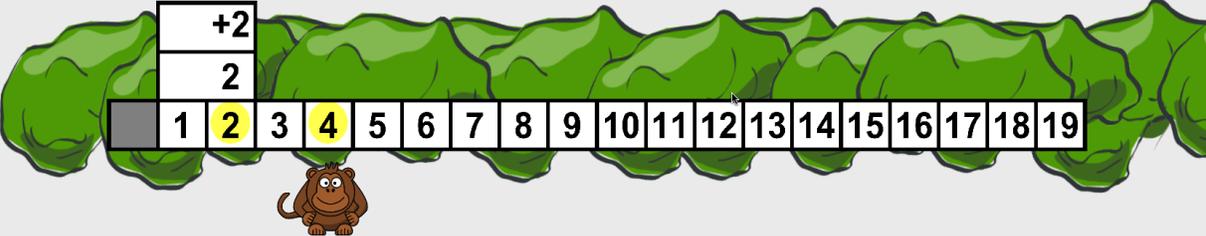
A horizontal number line is shown, starting with a grey box at 0 and ending at 19. The numbers 1 through 19 are in white boxes. The number 2 is highlighted in yellow, and the number 4 is also highlighted in yellow. A small cartoon monkey is positioned below the number 4. The number line is set against a background of green, leafy bushes.

**The counting by 2 monkey counts 2 then 4.**

**Does this help you to figure out how much 2 and 2 more is?**

**Click on the [Yes] button if the count by 2 helps you answer 2 and 2 more.**

**Click on the [No] button if the count by 2 will not help you answer 2 and 2 more.**



A horizontal number line is shown, starting with a grey box at 0 and ending at 19. The numbers 1 through 19 are in white boxes. The number 2 is highlighted in yellow, and the number 4 is also highlighted in yellow. Above the number line, there are two stacked boxes: the top one contains "+2" and the bottom one contains "2". A small cartoon monkey is positioned below the number 4. The number line is set against a background of green, leafy bushes.

Figure 3: A sample of doubles training (two games that relate the sums of doubles to even numbers)

0  
Timer

$2 + 2$       Puppy choice game/ doubles condition.



even 4      even 6      odd 3      odd 5

Correct!!  
 $2 + 2$  is 4

2  
Timer

$2 + 2$



even 4

Figure 3 continued

**The Train master must assemble a train quickly so that it starts on time.**

**For each car that appears decide whether it should go to the odd train track or the even train track.**

**Even** 

---

**Odd** 

Child clicks whether a number or sum of an addition problem is even or odd.

---

**What train does this car belong to?**

**Use the mouse to select whether it is even or odd.**

**Even** 



**Odd** 

Figure 4: Example of mental-addition testing game (Race Car)

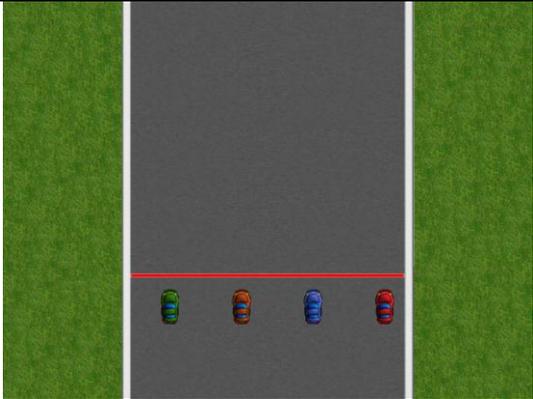
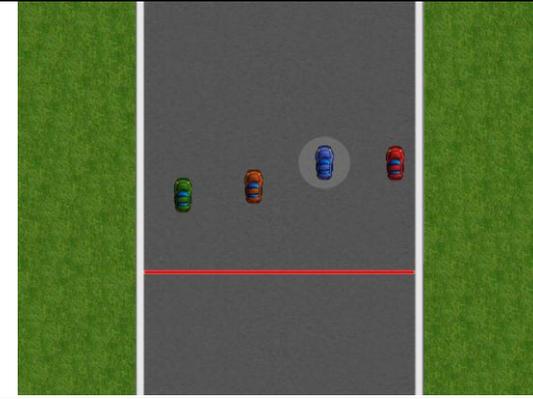
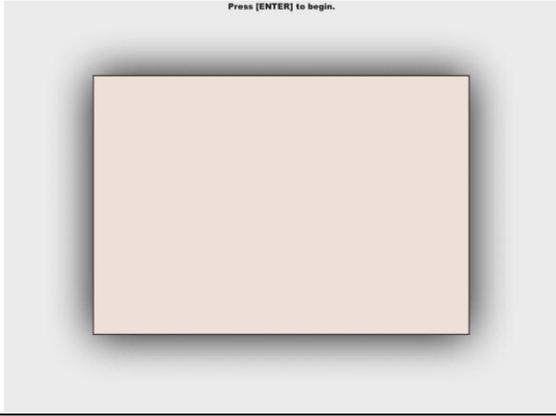
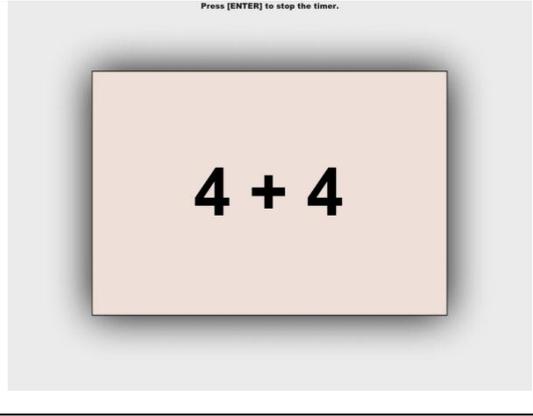
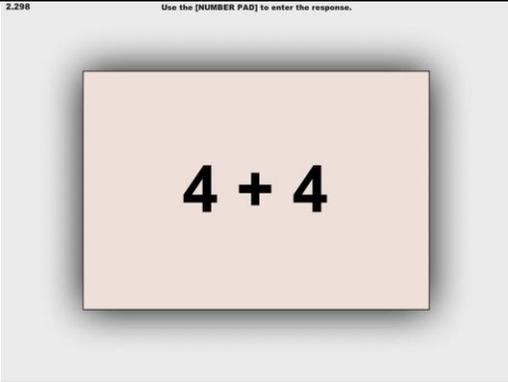
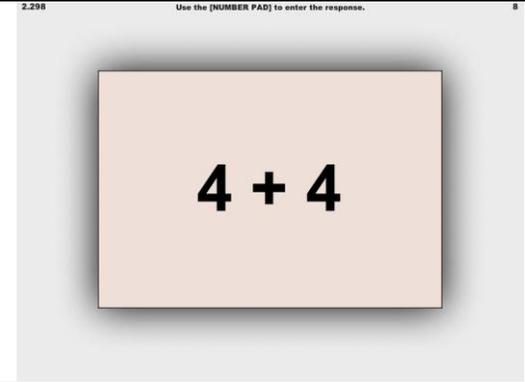
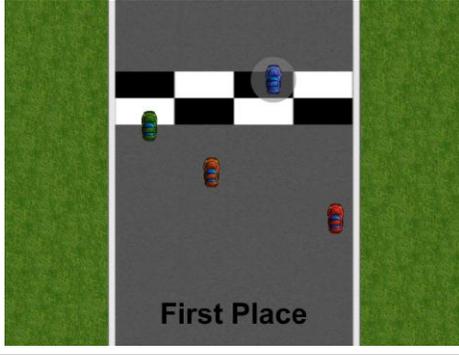
	
<p>a. <b>Prior to testing</b>, a child was given the opportunity to choose a racecar. The tester also chose a car. The game provided a pretext for holding on to a steering wheel, which was intended to eliminate—or at least suppress—finger counting.</p>	<p>b. The computer highlighted the child's racecar. After the testing instructions were given, the tester pressed the <b>ENTER</b> key to start the race. The racecars sped across the starting line, and—after a moment—the screen faded to black.</p>
	
<p>c. <b>During the testing</b>, a blank block appeared to provide a focus for the child's attention. When the child was ready, the tester keyed in <b>ENTER</b> on the number pad to initiate a trial and to start the computer's internal stopwatch.</p>	<p>d. The trial was presented. As soon as the child responded, the tester (who had his/her finger posed on the <b>ENTER</b> key) depressed the key to stop the internal stopwatch.</p>

Figure 4 continued

	
<p>e. The child's RT was displayed in the upper left-hand corner, and the screen prompted: <i>Use the NUMBER PAD to enter the response.</i> The RT is recorded and filed electronically by the computer and, as a backup, manually by the tester.</p>	<p>f. Using the number pad, the tester then entered the child's response and hit <b>ENTER</b> to record the answer electronically. Note that the entered response appeared on the screen in the upper right-hand corner. The tester also recorded the response manually, as a backup, and any relevant information (e.g., the child responded before seeing the trial displayed on the screen, counted objects, finger counting, verbal counting, reasoning strategy, such the number after 7 is 8 or 4 and 4 is 8 and 1 more 9). The tester hit the <b>ENTER</b> key to initiate the next trial.</p>
	
<p>g. <b>After the testing</b>, the race picked up where it left off and continued for about 15 s.</p>	<p>h. Where the child's car finished was determined by the quality of his/her responses.</p>