Abstract Title Page

Title: ***Young Children's Use of a Shortcut to Solve Addition Problems***

Authors: Veena Paliwal, Arthur J. Baroody, Erin E. Reid, and David J. Purpura
(University of Illinois at Urbana-Champaign)

Abstract Body

Background: Typically, achieving fluency with a basic combination or family of combinations involves three phases: counting, reasoning, and retrieval (Baroody, Bajwa, & Eiland, 2009; Kilpatrick, Swafford, & Findell, 2001; Rathmell, 1978; Steinberg, 1985). Initially, children typically need to compute sums and differences by using a counting strategy (developmental phase 1). They then often devise or learn more efficient reasoning strategies that permit them to use known facts to deduce unknown sums and differences (developmental phase 2). With time, children achieve fluency by using a highly efficient retrieval system that entails recall of specific facts or automatic reasoning strategies (developmental phase 3; National Mathematics Advisory panel [NMAP], 2008).

The basic number combinations embody a rich network of patterns and relations (Folsom, 1975; Trivett, 1980). Understanding such arithmetic regularities can facilitate fluency with basic combinations by providing a basis for inventing or understanding various reasoning strategies (developmental phase 2). Reasoning strategies can serve as a shortcut that eliminates or minimizes the need for laborious computation. For example, the sum of the near double 8+7 can be determined by avoiding the less efficient process of counting and using the near-doubles reasoning strategy. If 7+7 = 14 and 8 is one more than 7, then 8+7 must be one more than 7+7 or 14, and so its sum is 15. Using mathematical patterns or relations as a computational shortcut is an example of intelligent problem solving (Wertheimer, 1945). With practice, reasoning strategies can become automatic and become a part of the efficient retrieval system (developmental phase 3; Baroody, 1985; Fayol & Thevenot, in press).

Baroody, Purpura, Eiland, and Reid (2012; Paper 2) investigated the efficacy of a computer-assisted instructional program designed to promote use of the *near-doubles* (if 7+7=14, then 8+7=? is 15 as 8+7=7+(7+1)=(7+7)+1=14+1) and *make-ten* (9+8=9+(1+7)=(9+1)+7=10+7) strategies. The results indicated that the training was no more efficacious than drill-and-practice when fluency with n+8/8+n and n+9/9+n was assessed. Informal observations indicated that some children benefited from the training; however, this was not reflected by performance on the outcome measure, which required quick (> 3s.) and accurate responses. It is possible that the children understood and could apply the relevant reasoning strategy, but the computation demands hindered their performance on the outcome measure. For example, when solving 9+8=?, children may be able to apply the near doubles strategy (e.g., 8+9=8+(8+1)=(8+8)+1) but not be able to efficiently and fluently recall 8+8=?. Might a different (shortcut) task that takes computation out of the equation yield more meaningful information regarding the learning of reasoning strategies?

Research Question: The primary purpose of the study was to determine if computer-based training programs promoted fluent and flexible use of reasoning strategies to solve addition problems using different tasks. Specifically, does participation in strategy training result in the fluent application of the target strategy on a traditional mental arithmetic task? Does participation in strategy training result in the flexible application of the target strategy on an alternative shortcut task?

Setting: The research was conducted in five schools in two districts serving a mid-sized mid-western community.

Participants: A total of 74 first graders (6.1 to 7.6 years old, mean=6.6) participated in the study. Of these children, 46% of the children were female, 61% of the participants were black or Hispanic or multiracial. Additionally, 28% of participants were eligible for free or reduced-price lunch. Descriptive information on participants can be found in Table 1.

Intervention: The training (Stages I and V) is detailed in Table 2 of Paper 1. The preparatory training (Stages I and II in Table 2) was identical for all participants. For Stage III to V, participants were randomly assigned to the make -ten or the near doubles group and received structured training of specific reasoning. For example, participants in the near doubles condition were trained to solve 8 + 9 = ? by relating it to a known fact, 8 + 8 = 16; whereas make-ten participants were trained to answer 8 + 9 = ? by relating the problem to the known fact, 10 + 7 = 17.

Research Design: Each participant's mathematics achievement and general number sense was gauged using nationally standardized test (TEMA-3). All children then received the same preliminary concrete addition and estimation training to determine if they have required prerequisites to benefit from the training. Next, participants were given mental-addition pretest and randomly assigned to either a near-doubles or training make-ten condition. The near-doubles group did not practice the make-ten strategy or the make-ten items and this served as control group for the make-ten condition and vice versa. Participants were retested on the mental-addition test two weeks after completing the training. The shortcut test was given to the participants an average of 4 days (range = 1 to 14) after the post-test. Project personnel (University Research Assistants, Research Associates, or Academic Hourlies) implemented all testing and training procedures.

Data Collection

Mental Arithmetic. Mental arithmetic testing included both practiced and unpracticed items from the intervention. Testing was conducted within the context of a computer game. Problems included in the mental arithmetic testing are described in Table 2.

Shortcut Task. The shortcut task was designed to measure if the participants have an understanding of the reasoning strategies (near doubles and make-ten) and could use them flexibly. The shortcut task was modeled after that used by Baroody, Ginsburg, and Waxman (1983), Canobi (Canobi, 2004; Canobi, Reeve, & Pattison, 2003), and others. It entails first presenting a the child with a "helper" item such as 7+7=?. The computer provides feedback on the helper item and moves the problem to the left of the screen. Next, the target item (8+7=?) is presented while the helper item is left in view. The tester does not mention that the previous problem sometimes helps in answering the next problem. If the child understands the near-doubles strategy, he/she can avoid computing the answer to 8+7 by, for example, looking at the previous equation (helper item) and adding 1 to its sum. In effect, the task gauges whether a child can use a reasoning strategy to shortcut (eliminate) computational effort. The shortcut task also includes "non-helper" items and "control" problems in which the previous item does not provide a computation shortcut. Such pairs of items serve to check whether a child appropriately applies a reasoning strategy. A list of items used in the shortcut task is found in Table 3. Some of the same near-doubles (5+4, 7+6, 8+9, 9+8, 6+7, 8+7, 4+5), make-ten (9+8, 9+9, 6+9, 9+5, 9+7, 8+9), and other items (6+8) used in the mental-addition testing were also used for the shortcut task. As the mental arithmetic items were presented in semi-random order so that no related items followed in succession (e.g., 5+5 was presented in a different set than 6+5), this task provided baseline data on a child's solution response time (and accuracy) if a helper problem is not present. A more sophisticated scoring system than used previously (Baroody, et al., 1983 Canobi, 2004; Canobi et al., 2003) was developed. The six criteria used to evaluate the use of a reasoning strategy include:

- *Explanation*. For each item, two points are awarded if the child clearly states the use of the applicable reasoning strategy. An explicit explanation (e.g., 5+6=? is 11 because 6 is one more than 5 and 5+5=10 so 5+6 is one more and is 11) shows very strong evidence of understanding and use of shortcut principle. One point is awarded if the child implicitly states (e.g., for 9+5=? after 10+4=14 a child responds that this is the same as previous one or this is make-ten strategy but does not clearly explain the strategy when probed) the use of the applicable strategy. An implicit explanation for a particular problem illustrates that a child may have some understanding of the shortcut principle but does not demonstrate a clear understanding of strategy under consideration.

- *Reduction in response time (RT)*. One point is awarded if the RT for the problem is reduced by 33% when compared with the RT for the same problem on the mental arithmetic post-test. A child who knows and uses the strategy as a shortcut should be able to respond quickly relative to a child who does not know or use the principle. Half of a point is awarded if the RT in the shortcut testing is reduced by 25 % compared with the RT for the same item in the mental arithmetic post-test.
- *Improved accuracy*. One point is awarded if the child's answer is correct on the shortcut task but not on the mental arithmetic post-test. The presence of a computational shortcut could facilitate a child to answer accurately in the shortcut task as opposed to the mental arithmetic posttest where a computational shortcut was not provided. This shows that the child might be using the previous problem and arithmetic relationship in question to answer correctly in the shortcut task.
- *Absence of counting*. Half of a point is awarded if the child used counting to arrive at an answer on the mental arithmetic post-test but did not use counting on the shortcut task. The use of a computational shortcut indicates the child used the underlying mathematical relation to determine the sum instead of the inefficient method of counting.
- *Looked at the previous problem*. One point is awarded if a child looks at the previous problem multiple times. Half of a point is awarded if the child looks at the previous problem once before answering the problem. Looking at the previous problem before answering a new problem reflects that a child is using previous problem to answer the newer one. Multiple looks at the previous problem recertifies the use of previous problem in answering given problem.
- *Overall RT*. Half of a point is awarded if the child takes less than three seconds to answer a problem, there is no evidence of counting, and any of the criteria listed above are met. A RT of less than 3 seconds in the absence of any other criteria listed above, or is accompanied by counting, is not awarded any points as this might be a case of coincidence and it is not clear why child is answering fast in the shortcut task.

Data Analysis: As the two groups (near-doubles and make-ten) targeted different strategies and practiced different items, each was used as a control group for the other. Flexible use of a reasoning strategy was indicated by a score $\geq$1.5 on more than half of the corresponding shortcut task items. Analyses were done using the proportion correct on the mental arithmetic test and the total score on the shortcut test. An ANCOVA, with pretest mental arithmetic scores as a covariate, was used to compare the performance of each group on the mental arithmetic post-test. An ANOVA was used to compare the performance of each group on the shortcut task. Fisher's Exact tests were used to determine if group membership was associated with flexible use of a target reasoning strategy.

Results

The impact of structured near doubles training. Performance on the mental arithmetic post-test is reported in Table 4. On the mental arithmetic post-test, participants in the near-doubles condition did not achieve greater fluency on near-doubles items. On the shortcut task, the near-doubles group (M=0.64, SD=0.2) significantly outperformed the control group (M=0.34, SD=0.13) on near-doubles items, $F(1, 72) = 10.13$, $p < 0.002$. Flexible application of the near-doubles strategy was significantly associated with participation in the near-doubles training (p = .015, Fisher's exact test).

The impact of structured make-ten training. On the mental arithmetic posttest, participants in the make-ten condition were significantly more fluent than their near-doubles counterparts. On the shortcut task, the make-ten group (M=0.55, SD=0.23) did not significantly outperform the control group (M=0.43, SD=0.16) on make-ten items $F(1, 72) = 1.3$, $p = 0.258$. Flexible application of the make-ten strategy was not significantly associated with participation in the make-ten training (p = .23, Fisher's exact test).

Conclusions and Educational Implications: The use of the shortcut task can provide a different perspective on the impact of an intervention than a fluency test. For example, a child may have learned a reasoning strategy, but cannot readily recall the knowledge to implement it (e.g., a child may have learned the near-doubles strategy but may not quickly recall that 8+8=16 in order to answer 9+8 fluently. With the shortcut task, the information 8+8=16 is provided; a child need only recognize that it is useful in solving 9+8. Indeed, in contrast to considering only the results of the mental-addition task, the shortcut task results indicated that the near-doubles intervention was at least partially successful: That they learned the near-doubles reasoning strategy and could flexibly apply it, if not fluently (< 3 seconds), as gauged by mental-addition task.

The opposite results for the make-ten group help put the results of the mental-addition task in perspective. Why did this group not perform significantly better on the shortcut task than its comparison group when they exhibited significantly greater fluency on make ten items of the delayed mental-addition posttest? The reason may be that participants in both conditions received make-ten training in their classrooms, and many understood the make-ten strategy and used it to shortcut computational effort on the shortcut task. However, the make-ten intervention was partially successful in that this training significantly improved participants' speed in implementing the make-ten strategy—enabling them to achieve significantly greater fluency on the mental-addition task. Another reason could be the fact that the participants in the make-ten group had achieved fluency with make-ten number combination (as revealed by mental arithmetic post-test), a result that shows that they have moved to developmental phase three (retrieval) of achieving combination fluency and could use automatic recall without using a reasoning strategy (developmental phase two).

The findings of this study reveal the importance of near-doubles and make-ten strategies in achieving fluency with addition of single-digit number combinations. The intervention was effective in helping participants to move from developmental phase 1 (counting) to developmental phase 2 (reasoning) with near-doubles number combinations and developmental phase 3 (recall of specific facts or automatic reasoning strategies) with make-ten number-combinations. In order to understand what constitutes the use of a shortcut strategy, there is a need to understand the foundation of these strategies. Some researchers have revealed that the use of shortcut strategy is related with counting knowledge (Baroody, 1987; Thompson, 1997). There is little research done on early predictors of shortcut strategies. Such type of research could help in improving the task to monitor use of a shortcut strategy.

Appendices

Appendix A. References

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Appendix B. Tables and Figures

Table 1

Participant Characteristics by Condition

		Condition	
		Make-ten	Near doubles
Age range		6.0 to 7.5	6.6 to 7.0
Median age		6.6	6.6
Number of boys / girls		17:19	23:15
TEMA-3 range		78 to 127	75 to 125
Median TEMA-3		99	99
Free/Reduced lunch eligible		15	13
Black/Hispanic/Multiracial/ESL		16	19
Family History	Single-parent	9	4
	Parent under 18	1	0
	Parents w/o HS	0	0
Medical/ Develop-mental Condition	birth complications	0	1
	low birth weight	0	0
	fetal alcohol/drug	0	0
	visual impairment	0	0
	speech services	4	0
Behavioral Condition	ADHD	1	2
	Aggressive	2	3
	Passive/withdrawn	4	0

Table 2

Tested and Trained Combinations for Mental Arithmetic testing by Condition

Make-Ten Training		**Both Training Conditions**		**Doubles + 1 Training**	
Practiced Items	Unpracticed	Practiced Items	Unpracticed	Practiced Items	Unpracticed
3+9 9+3	4+9 9+4	9+8	8+9	3+4 4+3	4+5 5+4
6+9 9+6	5+9 9+5			5+6 6+5	6+7 7+6
7+9 9+8	9+7			7+8	8+7
2+10 to 7+10	9+9	4+7		1+1 to 8+8	
10+2 to 10+8				5+10 5+3	
3+5				7+5, 8+5, 10+3	

Table 3

Tested near doubles and make-ten problems for the shortcut task. The order represents helper items followed by tested item.

Make-a ten problems	Near Doubles problems
10+7 9+8	4+4 5+4
4+10 6+8	6+6 7+6
10+8 9+9	8+8 8+9
5+10 6+9	8+8 9+8
10+4 9+5	6+6 6+7
10+6 9+7	7+7 8+7
7+10 8+9	4+4 4+5

Table 4

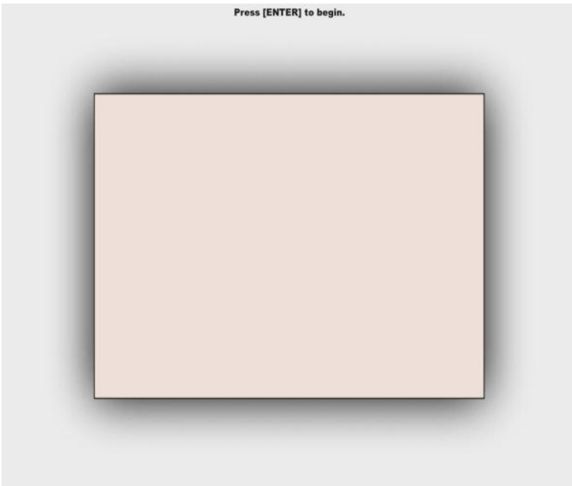
Mean proportion of items mastered in Mental Arithmetic test by condition

	Make-ten items				Near doubles items			
	Practiced		Transfer		Practice		Transfer	
	Pretest	Post-test	Pretest	Post-test	Pretest	Post-test	Pretest	Post-test
Make-ten	.08	.34*	.04	.18*	.17	.21	.08	.17
Near doubles	.08	.12	.04	.11	.16	.27	.09	.16

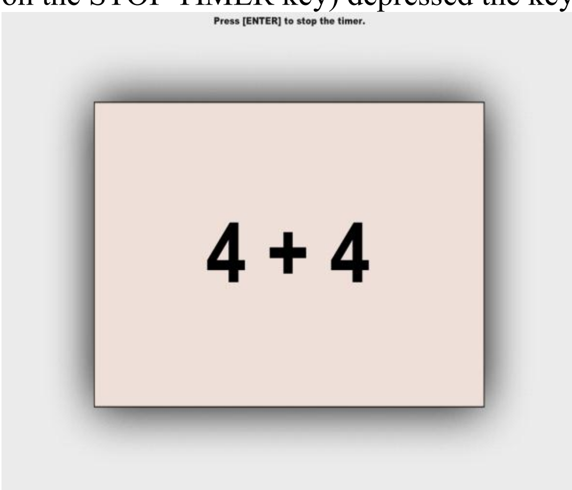
Note. * = The condition with a significantly higher mean.

Figure 1: A sample of shortcut task testing

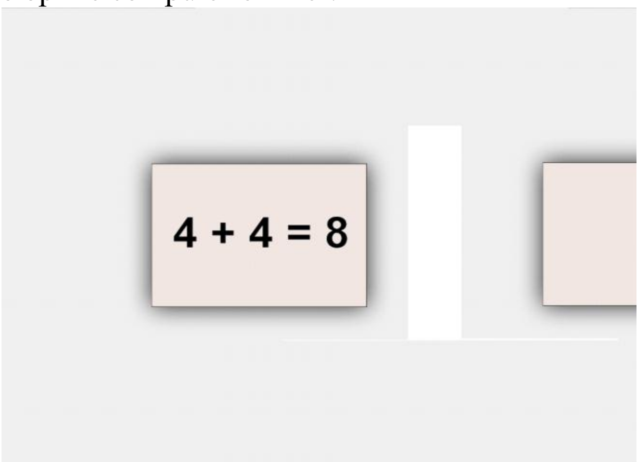
A blank block appeared to provide a focus for the child's attention. When the tester saw that the child is ready, s/he selected the REVEAL PROBLEM on the touch screen to initiate a trial and to start the computer's internal timer.



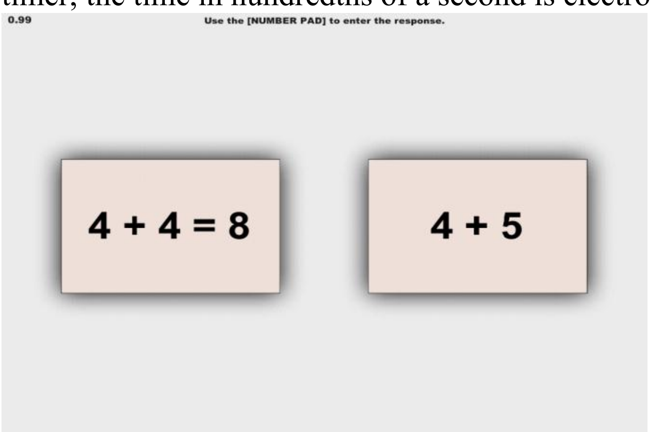
The trial was presented. As soon as the child responded, the tester (who has his/her finger posed on the STOP TIMER key) depressed the key.



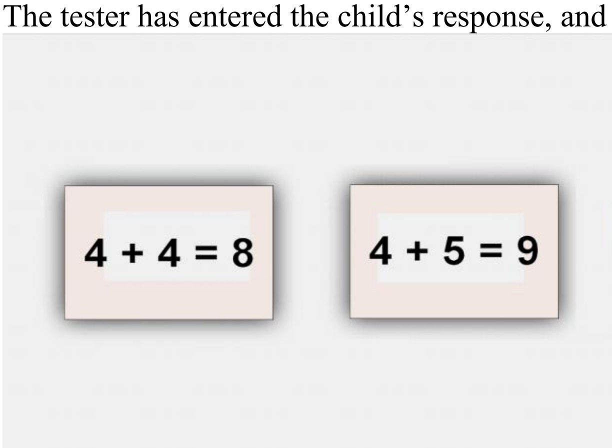
After completing a trial, the correct sum is displayed. The equation then migrates to the left as the new trial moves in from the right. The timer internal to the computer automatically starts when the new item is displayed. The tester hits STOP TIMER as soon as the child responds to stop the computer's timer.



As the trials are related, the issue is whether a child uses the answer of the previous trial to shortcut computational effort. The tester hits the STOP TIMER key on a touch screen to stop the timer; the time in hundredths of a second is electronically recorded.



The tester has entered the child's response, and the correct answer is displayed.



The cycle for a new trial begins.