

Making Sense of Critical Mathematics Teaching

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This paper highlights a teacher's perspective when changing her accustomed and traditional way of teaching into a pedagogic approach philosophically inspired by critical mathematics education. The focus here is on the practitioner's identities during the teaching process, in a context of change. The research is socio-culturally grounded and involves a methodologically critical ethnography. The teacher's learning was demonstrated through her voice in the end as sensing freedom in her teaching, reflecting on new possibilities and analysing the responsibilities different actors have in the mathematics classroom. Her experiences indicate that a critically mathematics inspired teaching approach has the potential to support teachers' achievement of agency.

Setting the Scene

...it feels as if it is always here (traditional education) – where the students and I end up. We start out with, for example, practical mathematics, but then suddenly I stand there again delivering a whole class explanation and think –wait, what happened now, how did we get here again...” (Elin, conversation)

Learning and Identity

Probably Elin, a mathematics teacher in a Swedish upper secondary school, is not unique in her experience of trying to change her teaching in mathematics education but ending up organising her classroom activities in a “usual” familiar way. Even having the best personal intentions and motivation to try out a new way of organising her teaching, and with an achieved agency (Biesta & Tedder, 2006) to do so, it is still very difficult to succeed. This paper discusses the teacher's perspective when she, in collaboration with the researcher, attempted to change her way of organising her teaching. The research is grounded in socio-cultural theories (Lerman, 2001; Radford, 2008). This is important as we were all, researcher, teacher and students, learning together. Learning is here understood as a process of *knowing* and a process of *becoming* as described by Radford (2008).

In this study the teacher's sense making is illuminated through the analytical tool put forward by Sfard and Prusak (2005) where identity is defined as the “collections of stories about persons or, more specifically, as those narratives about individuals that are reifying, endorsable and significant” (p. 16). Sfard and Prusak explained the notion of identity as a present identity and a designated identity – that is, an identity imagined or expected in the future. They suggested learning to be the act of closing the gap between these two identities, thus a process of *becoming*, resonating with the ideas of Radford (2008).

The purpose of this paper is to explore Elin's identities while she, in collaboration with the researcher, changed her teaching to an approach inspired by critical mathematics education, and by foregrounding societal issues as suggested by Wedege (2010). Her prior way of organising her teaching I would describe as traditional. A traditional mathematics structured teaching is here seen as mainly based on teacher's instruction, textbook exercise work (Andersson & Ravn, in press) and students' individual written assessments. With a teaching approach structured in this way, the authority in the mathematics classroom

resides with the teacher and the mathematics textbook (Bishop, 2008). Consequently, it is hard for students to influence their learning of mathematics. Critical mathematics education (CME) is here understood on two different levels. The first level addresses the microclimate, the relationships between all actors within a particular classroom. Skovsmose (2001, p.123) noted that CME “includes a concern for developing mathematics education in support of democracy, implying that the micro-society of the mathematics classroom must also show aspects of democracy”. In such a teaching situation students’ possibilities of achieving agency and students’ opportunities for negotiating and influencing their learning are taken seriously into account (Biesta & Tedder, 2006).

The second level addresses how mathematics teaching itself is organised (Andersson & Valero, 2009). Our interpretation of the assignment (among others) for mathematics teaching is to give students mathematical knowledge and competence for taking well-grounded decisions in everyday life and to interpret the flow of information and hence be able to understand and participate in political discussions in society. This paper articulates some of the challenges and problematic processes the teacher experienced in her change of teaching approach, before she eventually associated CME with terms and words such as *sensing freedom*, and *inspiration*. But first, I review the literature on teachers’ changing practice. Methodological and analytical concerns will be discussed, and a journey in time through two carefully chosen critical moments will be told.

Changing a Teaching Practice

There is a rich body of literature on teachers challenging and changing their practice (Clarke, 2007). A large part of this research focuses on teachers changing practice in reaction to new curricula and reform implementations. Lasky (2005) took a socio-cultural approach and showed how teachers’ identities are shaped in an American reform context and highlighted the dynamic interplay between political and social context and teacher’s agency and identity. Her results clearly showed that the new reform context constrains teachers’ experiences of agency.

If change is to be a robust process, and not only an isolated event, Fullan’s (1991) research implies that a stable change involves continuous learning for teachers and possibilities for reflection. For teachers to undertake the intellectual work of engaging in thorough reflections about their own practice and in making decisions for engaging in a process of change, the notion of teachers’ foreground i.e. the potentials she sees in her future practice (Alrö et al, 2008) needs to be acknowledged. Willingness, openness and feelings of trust also facilitate teacher change (Clarke, 2007; Lasky, 2005). Clarke (2007, p.28) suggested ten principles for guiding planning and implementation of teacher development programs. The possibilities for teachers achieving agency are conditional the existence of principles of “identified by teachers”, “degree of choice for participants” and “enable substantial degree of ownership”. Supportive conditions are articulated as “involving groups of teachers”, “solicit teachers’ commitment” and “allow time and opportunities for planning, reflection, and ... to share ‘the wisdom of practice’ (p. 28). Feelings, or teachers’ vulnerability are expressed in words such as “recognize that change is a gradual, difficult and often painful process”(p. 28). At the start of this project it seemed as all these conditions were almost fulfilled.

Methodology

Critical Ethnography

This research is conducted as ethnography (Willis & Trondman, 2000,) with inspiration from critical ethnography (Soyini Madison, 2005; Thomas, 1993). Willis and Trondman (2000, p.5) described ethnography –as a family of methods involving direct and sustained social contact with agents”. The researcher spent five months with two social science classes and their mathematics teacher during lessons, breaks, lunches and administration work. Thomas (1993 p.4) noted, –Conventional ethnographers study culture for the purpose of describing it; critical ethnographers do so to change it”. In the research reported here, things are a bit different as the context in itself is *the change*.

Positioning the Researcher

A researcher needs to consider critically her positioning in the classroom (Patton, 2002). In this particular context the teacher, Elin, chose to position the researcher as an assistant teacher in relation to the students and their parents, though with no assessment obligations. Elin was clearly the responsible class teacher with the authority in the classrooms. Elin and the researcher worked very close together planning the teaching sessions. The labour sharing changed over time as the researcher took more planning responsibility when Elin asked for support due to work pressure. At those times the preparing and planning part of the lessons and students’ projects were done by the researcher, Elin always made the final decisions relating the content to the curriculum and assessment system. The researcher’s role was encouraging and supporting but also, as responsible for pushing the process further, not letting go of the goals of the main research project during tough times.

Data Collection and Data Analysis

The main information was collected through participation in two Swedish social science classes in the students’ first mathematics course in upper secondary school. Field notes were complemented with regular audio-recorded conversations and written material such as personal letters, Internet forum comments and e-mails. The data were coded as suggested by Sfard and Prusak’s (2005, p. 17) analytical framework. An identifying story was represented by the triple ${}_B A_C$ where A was the identified person, B the author of the story and C the recipient. To distinguish between Elin’s different identities, authored by different actors, ${}_A A_C$ is defined as an identifying story told by the identified person by and for herself, ${}_B A_A$ is an identifying story told to the identified person, and ${}_B A_C$ is a story about A told by a third party to a third party. To understand *why* Elin’s identities changed the way they did at those particular times, the framework offered by Sfard and Prusak needed to be complemented. This was done visually with the emerging narratives told by Elin and others arranged over a timeline. Events and incidents at the school and the labour sharing were added on the timeline with the purpose of connecting the narratives with occasions in other parts of the network of the educational practise (Valero, 2009). Out of this exercise, critical moments emerged with stories related to Elin at particular historical times. To reach the endurable quality, Elin have confirmed the writings as reflecting the state of affairs. The next section of the paper introduces Elin and her reasons to engage in the project. The following sections elaborate two chosen critical moments and the last section contains Elin’s reflections on organising her classroom activities inspired by CME.

The Process as Critical Moments

Elin's Decisions on Curriculum and Assessment Issues in CME teaching

For me it's important to clarify what goals they are working towards but also that they themselves know what goals they are working for ... have a continuing dialogue...they are aware of the assessment criteria... and feel they have possibilities to show all the qualities and curriculum criteria or that they get influence on the examination forms ... some of them can't work well on written tests so there must be a variation so everybody feel they have had opportunities to show their knowledge. (Elin, interview).

This particular setting is situated in the social science program's first year at Ericaskolan, in the first compulsory mathematics A course. The grading system in the Swedish upper secondary school is different to most other countries. In upper secondary school the students attend specific subject courses on different levels. Students get graded after each finalised course, adding up to 30-40 different courses during a three-year period, and all grades are equally important when applying for further studies at university. These relatively short courses add extra pressure on teachers who have a limited time to get "everybody through" the courses. Changing teaching organisation may affect students' results negatively. Hargreaves (2001) stressed the importance of teachers engaging in change do so critically considering, for example which students benefit or which will suffer from these initiatives. This critical discussion was conducted on a regular base through the collaboration semester.

I have always spent a lot of time on all the different topics which has made them feel tedious (känns segt), especially when one only work in the book, and have to count all exercises on all the pages, then it takes quite a long time. Maybe one could look closer into what the course curriculum says, slim it and work with them more intensively (Elin, interview).

The CME projects had to be well prepared and designed to give students opportunities to reach all curriculum stated goals, on different grading levels, within the given time space of the course. The effect was that special sheets were created titled "Objectives possible to reach within this project" and, "What is needed to show for different grade levels". The outcome of this was threefold. First, it was clear to all participants that care was taken for these issues and by that support was given for the project from school organisation level. Second, from a CME point of view, this made it easier for students to achieve personal agency on assessment and grading issues. They had the opportunity to decide individually what levels they wanted to work on and what goals they wanted to reach within each project. Third, it supported Elin to "get everybody through" as assessment and grading issues became transparent for all actors in the network.

The First Critical moment: Mixing Traditional and Critical Mathematics Education

If I should do the %-counting project again? Not in that way... a project... where they didn't have to hand in exercises from the book as well. But we had that discussion before and then I chose to bring in the book part to feel sure that they did something... it was a control point. (Elin, interview, reflecting back at the very end of the semester)

During the planning stage of the first student project Elin's concern was with the students and their feeling of security and recognition in mathematics education as this was their first year at Ericaskolan. She decided to start the first two weeks with traditional mathematics teaching and assessing with a written test as she saw this as the didactical contract (Brousseau, 1997) they probably were used to from prior mathematics classes. After the initial weeks the first students' project started on the mathematical topics

percentages and decimal fractions. A *‘mathematical frame’* was provided as the *task context* (Wedegé, 1999 p. 206) and the students, working in groups of three (in order to facilitate discussions with peers both about the mathematical content and the critical reflections) decided on a *situation context* (ibid) that made sense for them. Elin wanted the students and herself to feel certainty about students reaching curriculum goals and so, in relation to the objectives and openness of the project they had to solve and hand in book exercises in addition to the presentation as illuminated by her reflection above.

It was an interesting period for Elin and me as a researcher, when we reflected on how we positioned ourselves in the classroom in relation to each other and the students. We reflected on the way we talked with and answered the students and how power was distributed in the classrooms. Reflecting back, it became much easier to change the power relations during the critical projects as it came more naturally there. During the traditional mathematics teaching sequences a change was almost impossible; positioning and power relations seemed to be *‘stuck in the walls’* for all actors in the classroom. As Hargreaves (2000) pointed out, structures of schooling have become so institutionalised over years that they define the essence of schooling itself for the teachers and the students.

Discipline Issues. A critical moment from this period is from the class where some discipline problems occurred with students arriving late and some never bringing their books or calculators. Elin took a respectful and clear conversation with them about these issues. She wanted them to reflect on different possible outcomes of different choices, and what she expected from them if they wanted to pass the course and go on with their studies. Some students changed their behaviour but some did not. A critical moment was when Elin silently wrote on the whiteboard at the end of a busy lesson, at 17.00 on a Tuesday afternoon, *“Those of you who don’t attend, present and pass next week will have to do a written math test”*. Changes of didactical contracts can be hard for *all* participating actors

The Curling Teacher. A change in Elin’s way of describing herself was commencing:

I am fighting with the feeling that this in some way can contribute to that they don’t do the exercises, don’t reach the goals, and won’t pass the course. Maybe this is yet a *“curling behaviour”*. Sometimes I feel like a curling-teacher. I bring extra calculators, extra books and extra papers and pencils. In what way does that support the students becoming independent and taking responsibility? (Elin, e-mail)

At this time Elin used a *‘sweeping’* metaphor when describing her teaching. The picture of a curling teacher refers to a *‘curling parent’*, in Sweden a generalising label for parents who sweep the way in front of their children and by that solve possible problems and tensions and making their children’s life as smooth and easy as possible. Summing up the first story, Elin’s present identities during these weeks were slightly modified. She struggled with acting in a (new) way she wanted and intended, for giving students opportunities to take decisions in their own way and at their own pace. At the same time her *‘curling identity’* for keeping control, interfering in the students’ work and leading the class became more obvious. Her designated identity at this stage can be interpreted as becoming a *“non-curling teacher”*—She started to reflect on whether this was a fruitful way of teaching or not.

The Second Critical Moment: The Statistical Project.

At a later stage of the semester a statistical project in collaboration with the environmental teacher on the theme *“Ecological footprints”*, a project with very high possibilities for students taking their own decisions, commenced. The project ran intensely

for three weeks during mathematics and environmental lessons, with a whole day of displaying results with PowerPoints, papers, posters, presentations, discussions and interactions in the fourth week.

I think we could have collaborated with any subject... because it was - the critical discussion – about how we live here, how we can influence and have an impact on how we live, and to react on it. It has been extra up-to-date now with the climate debate and Copenhagen meeting... but we could collaborate with the social science subject as well. It has definitely enriched both the subjects... I believe that mathematics enriches the other subject and it is good to show that mathematics doesn't stand on its own, because it never does; everything –hangs together” (Elin, interview)

Elin commented after the initial lessons: “It was hard for me to feel that I could stand back and not interfere with their work, especially when they were formulating their investigation questions I wanted to start explaining”. Then, Elin became ill and stayed at home for two days. During these days the statistical project continued with me participating as researcher with the environmental science teacher in the classroom. When Elin came back from her sick leave the students had questions on assessment issues and these had to be discussed with Elin. When she met the students, something critical happened.

- Elin, tell me how you experienced the project when you came back to the science class.

- Just then, I was only there to clear some questions... but... everybody was so into what they were doing, everybody sat working, *independent*, it was awesome to see the difference from the introduction. I had only seen one session with that group, then you (environmental science teacher and researcher) had two, but it was anyway a very big difference between the first introduction when they through themselves into ... it felt as if they had been over a peak and worked down slope now, and they were *proud* of what they were doing... (Elin, interview. Authors italics indicating emphasis)

There is an Australian saying that “Teachers won't change their teaching practise unless they believe it is in the best interest for the kids” (P. Clarkson, personal communication). This could be described as a turning point for Elin and her reflections on her teaching. Her present identity after this experience was clearer and more confident; I sensed a relief in her way of being and engaging with the students. An e-mail arriving a month after the researcher left the school illuminates this point:

Reflecting on my personal learning process is difficult. I think I have been influenced in a way that I feel more *freedom* and more *inspiration* in working in different ways in relation to the curriculum goals. Projects, group work, oral assessment. I have also been thinking quite a lot about my teacher roll. I am very caring that the students feel they have enough support from me; I have done extra –*matteugor*”, giving them special exercises. ... One can say I have worked quite hard with those things, but during the (autumn) semester I have been thinking if all this really supported the students. How does their independence and autonomy develop if they always get everything served? Obviously I, as a teacher, am there for the students, but in what way?

During this project (statistical) I have been *fighting a bit with letting go of the responsibility* to the students. Letting them seek information on their own, ask for help when needed – plainly being *responsible for their own learning*. Looking back, now when the new spring semester has started I think the students, even when we do book exercises seems to have learnt a lot by working in the way we have done. With that I mean they use the book and work more together (not always, and not everybody – but many!) (Elin, e-mail, author's emphasis)

Concluding Remarks on Teaching CME

The number of different mathematical topics to be covered during the course put time boundaries on the projects for both students and teacher, who wanted to push the projects mathematically and critically further. In Elin's words:

I think these projects have been meaningful for them. I believe it would have been good to push the projects one step further, to see that what we discussed, e.g. with Swedish language. There are things needing taking care of, e.g. hardly anybody in the class knew where the paper recycle bin was. We ought to do something about that, take the last step so something actually happens. I think that would have been experienced as meaningful so this project won't become yet another school task one does. About influence, I think on different levels, I think of their own personal influence but also that they can feel they can influence outward, that they can see through, and feel confident with that...

Elin's point of view indicates that the students did not express their usual feelings of meaninglessness during the projects. The question "Why do we have to do this?" was not raised during the projects. It rather indicates that the projects could have been pushed further both mathematically and critically. Thus, these projects have developed a sense of meaningfulness for the students. Elin commented on her teaching inspired by CME points towards an increasing awareness of the mathematical impact on the students in society:

I think that has been good, I have noticed that but it is about *them gaining the power*, and then they can in some way *reflect*; wait, what happened, what would happen instead, and if somebody does this to me or how does the stuff I am reading impact on me? That is what I think has been critical in many of these topics. That's what I think has been good. They maybe don't say much but I believe they think about it and that (mathematical knowledge) is a lot about power; I believe they are starting to see that. (Elin, interview, author's emphasis)

This also leads to a belief that every student could benefit from CME. I want to argue, in line with Elin's comment below, that CME is for everybody, not only for social science students in a Swedish context. For an understanding of the world I argue that all students, at several times during their mathematics education should be exposed to CME:

I have been thinking, I don't think this is only for the social science students. Sure, it's possible to relate to the other social science subjects ... but it is not certain that because you are social science students you are more interested in those subjects. What I feel more and more is that *it's about them feeling they have opportunities to influence* [...] they might grow of this kind of mathematics... they feel *responsibility*... and that I don't think is possible when only counting in the math book. (Elin, interview, author's emphasis).

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