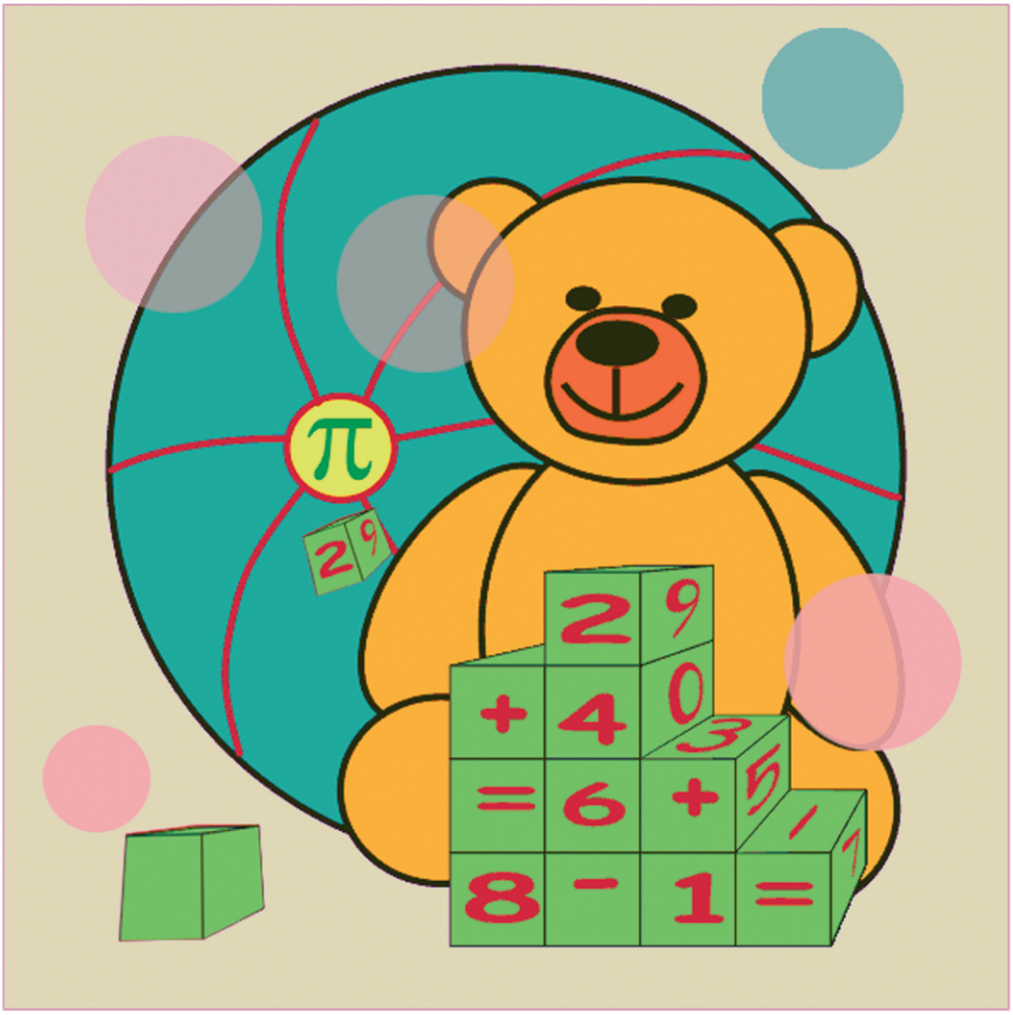
UČITELJSKI FAKULTET
SVEUČILIŠTE JOSIPA JURJA STROSSMAYERA U OSIJEKU

djel za matematiku



International Scientific Colloquium
MATHEMATICS AND CHILDREN
(How to teach and learn mathematics)

Osijek, April 13, 2007

Josip Juraj Strossmayer University of Osijek
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and
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Proceeding of the

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(How to teach and learn mathematics)

Editor: Margita Pavleković

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Editor's Note

The main aim of the Organisational Committee of the international scientific colloqium *Mathematics and Children* is to encourage additional scientific research in the field of mathematics teaching in Croatia.

The development of science and education is a part of a long-term Education Sector Development Plan 2005-2010.

Following the example of Europe and the rest of the world, special attention in the field of education is given to mathematical literacy of children (PISA programme) as well as to mathematics teacher training (quality insurance in higher education).

Mathematics teaching in Croatia faces modified strategic, organizational, social and technical conditions. Introducing one-shift classes in primary schools, including children with special needs (talented ones and those with difficulties) in regular classes, extended day program for all students, two teachers per class, greater mobility of children and teachers in schools and new teaching technologies demand changes in the methodology of mathematical education of both children and future teachers of mathematics. It is important to develop life-long learning programme for teachers of mathematics that includes doctoral studies.

Research in the field of mathematics teaching implies multi- and interdisciplinarity. Therefore a cooperation with scientists outside the field of mathematics (psychologists, special-ed teachers, educators) is an imperative, although we strongly believe that improvements in mathematics teaching should be encouraged within the field of mathematics.

A precondition for developing new approaches and methodologies in mathematics teaching in Croatia is a first-hand experience with the results of international research and standards in mathematics teaching and defining doctoral studies within the same field.

We believe that the lectures, discussions and experience exchange between Croatian and international participants of the *Mathematics and Children* meeting will initiate and intensify scientific cooperation in the field of mathematics teaching on the international level. We would also like for this event to initiate the start of doctoral studies in the field of mathematics teaching in Croatia following the examples from Europe and worldwide.

We are very grateful to numerous Croatian and international scientists who have recognized the importance of this event and managed to find the time to attend this gathering. We would also like to thank the heads and entrepreneurs of the local community who financed this event for the most part.

On behalf of the Organizational Committee, I express my deepest gratitude.

Osijek, April 13, 2007 *Margita Pavleković*

Riječ urednice

Organizacijski odbor međunarodnoga znanstvenoga kolokvija *Matematika i dijete* postavio si je zadatak dodatno potaknuti u Hrvatskoj znanstvena istraživanja u području metodike nastave matematike.

Razvoj znanosti i obrazovanja dio je dugoročnoga prioriteta razvoja Hrvatske (2005 – 2010).

Po ugledu na Europu i svijet u okviru obrazovanja posebna se pozornost pridaje, kako matematičkom opismenjavanju djece (PISA program) tako i izobrazbi učitelja matematike (osiguranje kvalitete u visokom obrazovanju).

Nastava matematike u Hrvatskoj je pred izmijenjenim strateškim, organizacijskim, socijalnim i tehničkim uvjetima. Uvođenje jednosmjenske nastave u osnovne škole, inkluzija djece s posebnim potrebama (talentirane i one s teškoćama) u redovite odjele, produženi boravak za sve učenike, dva učitelja u odjelu, veća pokretljivost djece i nastavnika u školama, nove tehnologije u nastavi iziskuju promjene u metodologiji matematičkoga obrazovanja, kako djece tako i budućih učitelja matematike. Važno je osmisliti cjeloživotnu izobrazbu učitelja matematike koja uključuje i doktorske studije.

Istraživanja u nastavi matematike pretpostavljaju multidisciplinarnost i interdisciplinarnost. Stoga je pri istraživanjima u nastavi matematike neophodna suradnja sa znanstvenicima izvan područja matematike (psiholozima, defektolozima, pedagozima, istraživačima iz područja informacijskih znanosti), iako držimo da razvoj metodike nastave matematike treba njegovati u okvirima matematičke struke.

Pretpostavka je iznalaženju novih pristupa i metodologija u nastavi matematike u Hrvatskoj upoznavanje *iz prve ruke* rezultata inozemnih istraživanja i strane prakse, kako u nastavi matematike tako i definiranju doktorskih studija iz metodike matematike.

Vjerujemo da će izlaganja, rasprava i izmjena iskustava domaćih i stranih izlagača na skupu *Matematika i dijete* potaknuti i intenzivirati znanstvenu suradnju iz područja metodike nastave matematike na međunarodnoj razini. Također želimo da ovaj skup pridonese bržem zaživljavanju doktorskih studija iz metodike nastave matematike u Hrvatskoj po ugledu na već postojeće u Europi i svijetu.

Važnost ovoga skupa prepoznali su brojni domaći i inozemni znanstvenici, a neki od njih uspjeli su odvojiti dio svojega vremena za zajedničko druženje. Zahvaljujemo im, ali i vrijednim čelnicima i poduzetnicima lokalne zajednice koji su u najvećem dijelu sponzorirali održavanje ovoga skupa.

U ime organizacijskoga odbora svima od srca zahvaljujem.

U Osijeku, 13. travnja 2007. *Margita Pavleković*

Invited Lectures

AN OVERVIEW OF THE AUTHORISED CURRICULUM IN *TEACHING MATHEMATICS* HARMONISED WITH THE BOLOGNA DECLARATION AT THE DEPARTMENT OF MATHEMATICS, UNIVERSITY OF SARAJEVO

Šefket Arslanagić[1]

Abstract. *In academic year 2005/2006 University of Sarajevo introduced new curricula harmonised with the Bologna Declaration. At the Department of Mathematics in Sarajevo the scheme 3+2 years of study is followed. Upon completion of a three-year study programme in mathematics, students can continue with their studies by enrolling in one of the 4 branches offered by the curriculum.*

Upon graduating from one of these branches, a candidate is granted an MSc degree in Mathematics Education.

The paper gives an overview of the curriculum referring to the branch Teaching Mathematics.

Key words: *mathematics, teaching mathematics, science.*

After successful completion of the first cycle study programme in mathematics at the University of Sarajevo, a graduate is awarded a BSc degree, and by completing the additional two years of study (the second cycle study programme) a graduate is awarded an MSc degree (with the branch indicated).

There exist four branches:

- Theoretical Mathematics
- Applied Mathematics
- Theoretical Computer Science
- Teaching Mathematics.

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Thus, a Master level programme requires four semesters. After successful completion of the Master level programme in the first three branches, a graduate is awarded an **MSc degree in Mathematics** (with the branch indicated), whereas by completing the study programme in teaching mathematics a graduate is awarded an **MSc degree in Mathematics Education**.

Lectures referring to the branch *Teaching Mathematics* (in the 4th and 5th year of study) will be conducted as follows:

Semester	**Course**	**Lecturer**
Semester I	Algebraic and geometric inequalities	Dr. Šefket Arslanagić, Associate Professor
	Fundamentals of geometry	Dr. Mirjana Malenica, Full Professor
Semester II	Abstraction and generalisation in algebra	Dr. Hasan Jamak, Associate Professor
	Mathematical logic	Dr. Medo Pepić, Associate Professor
Semester III	History and philosophy of mathematics	Dr. Muharem Avdispahić, Full Professor
	Fundamentals of number theory	Dr. Lejla Smajlović, Assistant Professor
Semester IV	Aspects of working with mathematically gifted children	Dr. Šefket Arslanagić, Associate Professor
	History of graduations	Dr. Mirjana Vuković, Full Professor

Out of eight courses that are offered, students take six. Students should pass at least five of the courses they attended. One of the courses might be replaced by some other Master level course offered by some other related faculty or university with prior approval issued by the Doctoral Study Committee.

Dr. Šefket Arslanagić was appointed head of the branch Teaching Mathematics.

Entry requirements include a completed undergraduate study programme in mathematics or any related science with a GPA of min. eight (8) and knowledge of one foreign language.

Applicants who graduated from other related sciences are subject to compulsory testing that would prove their competence for the Master level study programme in Teaching Mathematics.

The aforementioned provision might also be applied to any interested applicant who obtained a BSc degree in mathematics with a GPA less than eight (8).

After passing the required examinations, an applicant defends the Master's thesis done in co-operation with the thesis advisor.

(translated by Ivanka Ferčec)

ROLE OF DIFFERENT REPRESENTATIONS OF MATHEMATICAL CONCEPTS FOR LEARNING WITH UNDERSTANDING

Tatjana Hodnik Čadež[1]

Abstract. *A representation is something that stands for something else. Each representation should consist of the following aspects: (1) what the represented world is; (2) what the representing world is; (3) which aspects of the represented world are being modelled; (4) which aspects of the representing world present the modelling; and (5) what the correspondences are between the two worlds (Palmer, 1978).*

The idea of representation is continuous with mathematics itself. Any mathematical concept if it is to be present in learner's mind must be represented in some way. We distinguish between external representation (environment) and internal representation (mind). External representation refers to all external media, which has as its objective to represent a certain mathematical idea. We mainly use representation with concrete material, graphical representation and mathematical symbols when teaching mathematics to young children. This paper discusses the role of using different external representations in the process of learning and teaching mathematics. The importance of establishing relations between different representations is stressed with a model of representational mappings. Within this theory we have defined two concepts: meaning and understanding. We have considered a child's understanding as his or her ability to translate between different representations of arithmetic operations. A child can give meaning to a particular representation if he or she is able to perform a required transformation within a representation.

***Key words**: mathematics, mathematics teaching, learning with understanding.*

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1. Introduction

Representation of mathematical ideas either with mathematical symbols, graphical representation or representation with concrete material is crucial for communication of mathematical ideas.

We differentiate between internal (mental images) and external (environment) representations. Cognitive development is based on a dynamic process of intertwining mental images and environment. This means that a successful process of learning is an active formation of knowledge in the process of interactions between external and internal representations.

Internal representations, known also as cognitive representations, can be defined as mental images which correspond to our internal definition of 'reality'. Internal representations are defined as mental images or mental presentations (not representations): something that does not have its original, inner world of experiences.

External representations consist of structured symbolic elements whose role is an 'external' presentation of a certain mathematic 'reality'. The term 'symbolic element' signifies elements which are chosen to represent something else. We define the thing that 'represents' another one as a symbol. In mathematics classes pupils are introduced to three different types of symbolic elements or external representations: concrete (didactic) material, graphical illustrations and mathematical symbols. In the following sections, we will be dealing with the role of different representations of mathematical concepts for learning with understanding.

We are going to present each of external representation in mathematics very briefly.

2. External representations

2.1 Concrete representations

The word concrete representation means different things to different people. For someone a concrete representation stands for a particular structured representation which is used only in the process of teaching and learning mathematics, and does not have any special meaning out of that process. We will call such material as structured material, e.g. Dienes blocks. However we also

understand any other concrete material, let us call it unstructured material that a child uses in order to learn a particular mathematical concept, as a concrete representation. Children in Slovene use cubes (Multilink) that can be structured into sticks of 10 and individual cubes to illustrate the place value nature of numbers. For more advanced calculating, Dienes blocks of 'hundreds' (flat blocks), 'tens' (rods) and 'units' (individual cubes) are used to model the procedures, such as 'decomposition' and 'carrying', that are involved in traditional calculatuions.

There is a common view of teachers and parents that children learn mathematics more easily if they have the possibility to manipulate with concrete material. Research in this matter is not unitary. For example, during the 1960s and 1970s Dienes blocks were widely used in the Netherlands, but criticism of their use as being helpful for the representation of abstract number structure, but weak in the representation of number operations when they become more complicated (Beishuizen, 1999) has lead to the use of bead frame and bead strings (Anghileri, 2001). Let us list some other of the authors who researched the role of structured apparatus and unstructured material in the process of teaching and learning mathematics. Fennema (1972) and Fridman (1978) showed positive role that relate more closely to images of a counting strategy of using concrete material at primary level but not in secondary school, while Suydam and Higgins (1977) found manipulating with concrete material useful in whole elementary school. Labinowicz (1985) observed young children using Dienes blocks and came to the conclusion that children had problems establishing relations between these blocks and the place value system of integer numbers. Again, on the other hand, Fuson and Briars (1990) found very positive role of these blocks for learning adding and subtracting integer numbers. Thompson (1992), and Resnick and Omanson (1987) concluded in their research that Dienes blocks had very little influence on children's understanding of arithmetic algorithms in primary school. These contradictory conclusions make us aware that concrete material itself does not ensure successful learning. In other words, the process of teaching and learning mathematics is very complex, and one part of it is also manipulating with concrete material. We believe that manipulating with concrete material without thoughtful reflection on the process of manipulating and without relating concrete representations to other representations in mathematics is not sufficient for successful learning of mathematical concepts. The nature of a mathematical concept, the way of using concrete material, and the material itself define how the learning is going to take place.

2.2 Graphical representations

Graphical representations are mainly used for representing mathematical ideas in teaching and learning primary mathematics. Mathematical textbooks, workbooks, and other material for children are full of graphical representations which differ in originality and correctness. Let us look at the the graphical representation of the concept 'number'. A concrete representation of the number is all countable objects around us. But we do not count everything together. We can only count objects which have a certain characteristic in common and differ in some way at the same time what makes this group of objects countable. This way we can count together balls, colour pencils and pictures, but we cannot count together pictures and balls because in this case it would be hard for young children to end the sentence, "We counted 13 …" 13 of what? Graphical representations of numbers are mostly illustrations of objects, animals and persons which pupils write down with mathematical symbols or numerals. Graphical representations are not used only for illustration of mathematical concepts but for illustration of certain mathematical symbols as well. These representations are there to help pupils remember a certain mathematical symbol easier. Pupils adopt the concept represented by a symbol even before it is introduced as a symbol. Adopting the concept and learning how to write it down with a symbol is going on at the same time. It is not possible to exclude the fact that by adopting a mathematical symbol for a certain concept, pupils learn about that mathematical concept as well.

Graphical representations stand for so-called 'bridge' between concrete representations and representations with mathematical symbols. Let us illustrate this idea we adopted from Heddens (1986) with Figure 1.

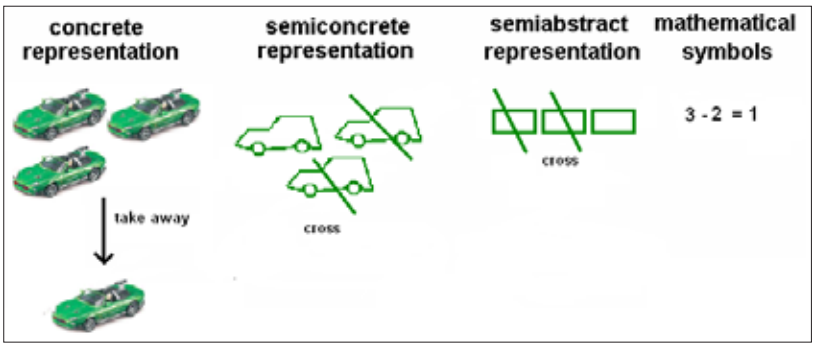


Figure 1: Graphical representations as a bridge between concrete representations and mathematical symbols

Graphical representation, drawn cars (Figure 1), is semiconcrete representation, representation of subtraction with rectangles is an example of semianbstract one (more distant from experience world in our case). Representation with rectangles shown in the picture above (Figure 1) could be a semiconcrete representation in any other situation.

The semiconcrete representation for numbers 1 and 3 could be Roman numbers I and III because they are more 'concrete' as symbols 1 and 3.

As already mentioned, in the teaching and learning process we use a variety of different graphical representations. A graphical representation depends on the nature of a mathematical concept and on a representation with concrete representation. We have to mention a number line as a special case of semiabstract representation in mathematics. A number line causes many problems to children because its interpretation includes both ordinal and cardinal aspects of integer numbers. On one hand, a number is presented with a position on the line, and on the other hand, a number stands for the number of movements on the line. A recent innovation in the Netherlands has been the 'empty number line' (Figure 2) which supports the development of mental strategies. As a response to teachers' complaints about children hanging on too long to the materials, such as Multilink cubes, Dienes blocks, numbered lines, and passively reading off answers from the blocks when doing sums, removing all calibration from the number line has enabled children to use it flexibly for 'jumps' of any size, in either direction, providing imagery to encourage and support mental strategies (Anghileri, 2001).

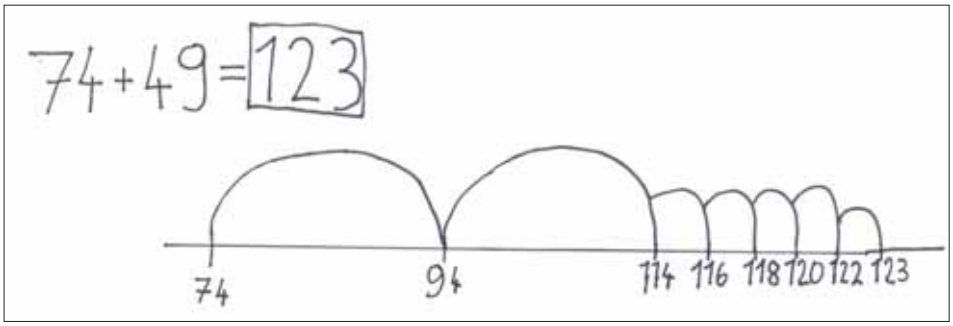


Figure 2: Empty number line

Children need to discuss numbers holistically according to Anghileri (1998), which means that children do not calculate hundredths, tens and units but

'whole' numbers. According to Anghileri (1998), manipulation with concrete material is less important because calculation up to 1000 could be successfully presented to children on a symbolic level. It is hard to believe that these ideas could be accepted in our curriculum in learning arithmetics because teachers and also parents strongly believe that arithmetics without concrete representations and learning the place value system is just not possible. In terms of research these ideas are worth challenging in practice.

Let us briefly mention also manipulation with mathematical symbols.

2.3 Mathematical symbols

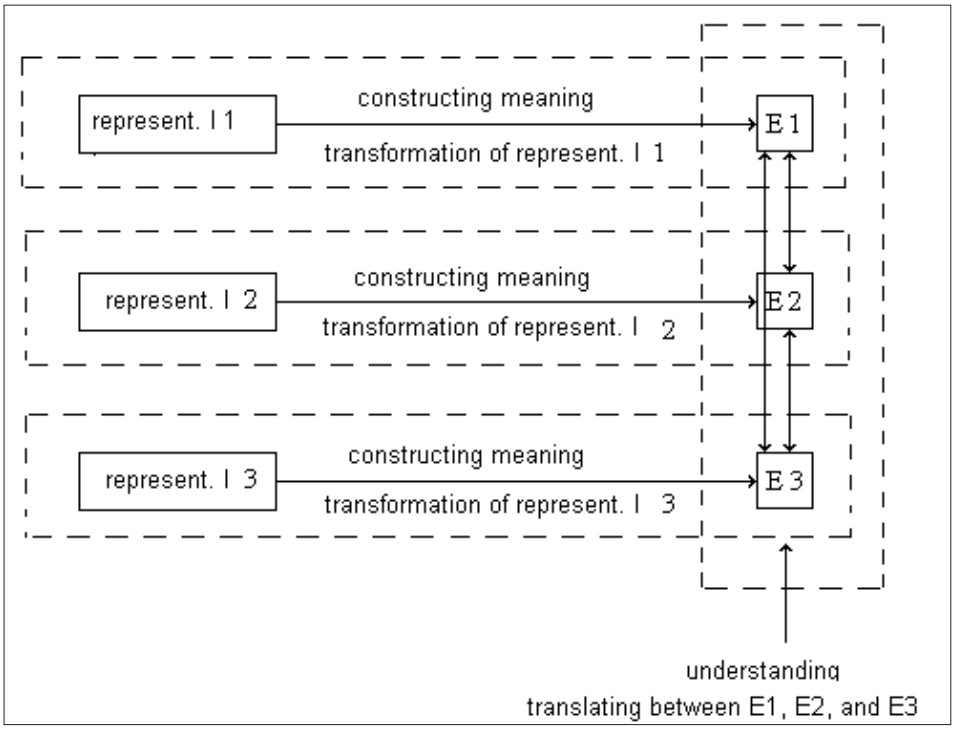
Mathematical symbols which are introduced to young children in the first three years of primary school are numbers from 0 to 9, symbols for operations (+, -, :, ×) and symbols for relations (<, >, =). The number of symbols is small but there are many combinations of these symbols with special rules which hold for particular combinations of symbols. This is what makes handling with mathematical symbols difficult for many children. In many cases children manipulate with mathematical symbols mechanically without understanding. In the primary mathematics, manipulating with concrete material and graphical representation is closely related to mathematical symbols. According to Hiebert (1988), mathematical symbols are a representational system with five stages that children need to proceed in order to be able to manipulate symbols successfully. We are going to mention only the first stage called 'providing relations between symbols and referents' which stresses the importance of providing children with mathematical experience with concrete material as well as graphical representations, and relating these experiences to mathematical symbols. The idea of establishing relations between different representations (not only between mathematical symbols and others) is going to be discussed more in detail in the following sections.

3. Relations between different representations. A model of representational mappings

There are many different explanations of the concepts of meaning and understanding. We are going to define meaning as something closely related to a specific representation, while for understanding this is not true. We define a concept of understanding as child's ability to translate between different repre-

sentations of a mathematical concept. A child can give meaning to a particular representation if he or she is able to perform a required transformation within a particular representation. Let us explain these statements with the following example. If a child is able to perform an operation of division with concrete material, he or she can give meaning to this particular representation. But if a child is able to translate between different representations of division, for example, between concrete representation, graphical representation and a representation with mathematical symbols, then he or she understands an algorithm. In other words, it means that he or she is able to represent his or her manipulation with concrete material both with a picture and with mathematical symbols.

The relations between representations, concepts of meaning and understanding are clearly shown in Figure 3 (Hodnik Čadež, 2001, 2003).



I 1 : concrete representations
I 2 : graphical representations
I 3 : representations with mathematical symbols
E 1, E 2, E 3: representations of I 1, I 2, I 3.

Figure 3: A model of representational mappings

The diagram shown in Figure 3 represents a fundamental framework of our research (Hodnik Čadež, 2001, 2003). We used it for analysing children's understanding of adding and subtracting but we believe that this model could be applied for some other concepts in mathematics as well. We have formulated our fundamental hypothesis that a child who fully translates representations of addition/subtraction within numbers up to 100 from one form of representation to another, and has a very good number concept, can develop his or her method for calculating (adding, subtracting) three-digit numbers. We have confirmed this hypothesis on the basis of a case study research (Hodnik Čadež, 2003).

Let us illustrate the model of representational mappings with the following example. An implicit representation I1 could stand for a representation with structured material. If a child is able to perform an operation, e.g. 28 + 5 with that material, he or she transforms implicit representation I1 into explicit representation E1 or, in other words, he or she gives meaning to that concrete representation. If we put this in other words, we can conclude that no representation represents by itself It always needs an interpreter who transforms implicit representation into explicit. If a child then makes relations between different explicit representations of an arithmetic algorithm or, in other words, recognises the same concept represented with different representations, we might say that a child understands an arithmetic algorithm, which results in transfer from the previous to the following learning. In our case that meant that a child was able to create his or her own algorithms for addition and subtraction within numbers up to 1000, although these numbers and algorithms were not taught in school before the research took place.

4. Conclusions

External representations in mathematics: concrete, graphical and symbolic do not "represent" by themselves, they need an interpreter. There is a variety of external representations in mathematical curriculum; the interpreter is a pupil, who establishes a mental interaction with the proposed representation. The pupils' interpretation of the representation is linked with his or hers understanding of the mathematical concept represented by the representation. The way in which the concept is represented by the external representation also plays an important role. During the process of teaching and studying mathematics we understand the concrete experience, linked to the pupils' life and the translation

of this representation into a graphic representation or symbolic one as something natural. Usually we do not stop to think that every external representation needs an explanation, a dialog, a "dynamic" interpretation and that the pupils' understanding of a representation would be easier to understand, if he or she were given a chance to create these sort of representations by themselves. We must not presume, that the concrete material, pictures in mathematics, which are often very colorful and interesting always serve their purpose. The representations in mathematics do not always form a mental link with the pupils' understanding of mathematical concepts. Next to concrete, graphic and symbolic representations in mathematics, we must mention the speech, which is also a representational system and is treated in a close relation to all previously listed representations.

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THE SCIENTIFIC FRAMEWORKS OF TEACHING MATHEMATICS

Zdravko Kurnik [1]

Abstract. *In the process of cognition and learning of the laws of nature scientists use special techniques – scientific methods of research. Basic methods of scientific thinking and research are: analysis and synthesis, analogy, abstraction and concretization, generalization and specialization, induction and deduction.*

Mathematics as a science and mathematics as a subject are closely related. The connection is, along with other, also established by scientific principle. Consolidation of that connection has by far influenced the changes in mathematics education. Barycenter of modern mathematics education lies in introducing the scholars with scientific work and development of their thinking. Here are some settings:

- *The work of mathematics teacher with the pupils in a class is by far different from the work of mathematics-scientist, but there are also some common features. Pupils in educational process, either alone or with teachers help, also discover and realize new mathematical truths. Most important is a discovery of the path to independent creative pupil's work. Therefore, the before mentioned scientific methods are important for the modern mathematical education as well. A creative teacher, choosing suitable problems and applying these methods, can enable pupils for work that is very close and similar to research work.*
- *Mathematics in development is a concrete and inductive science, and mathematics itself is abstract and deductive science.*
- *Very important procedure is analogy. It pervades our whole thinking, everyday speech, artistic creation and also high scientific researches. Analogy is very useful in mathematic education as a vivid means for connecting and easier understanding of teaching materials, and also as a means of developing creati-*

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vity and creative thinking in a pupil. In solving a problem pupils are directed to consider some similar, close problem and mimic the steps of its solution.

- *In processing the mathematical notions, teacher realizes scientific principle if he correctly conducts the process of notion forming (observation, the idea of a notion, forming of the notion) and abides some basic conditions that a definition of a notion must meet (appropriateness, minimal content, brevity, suitability, applicability, modernity). The process itself is gradual and its success is reinforced by five important scientific methods : analysis, synthesis, concretization, apstraction and generalization. The critical step in processing of a notion is the shift to the level where abstraction begins, because the shift from concrete to abstract is very hard for some pupils.*
- *In processing the theorems teacher realizes scientific principle if he teaches his pupils to correctly and precisely form a theorem, to clearly differentiate the assumption from statement of a theorem, to form an inversion and an opposite of a theorem and accomplishes understanding of the methodics of proving the theorem.*
- *Scientific principle is also realized by clearly distinguishing definitions and theorems.*

Treatment of some mathematical materials in our textbooks is quite often incompatible with the scientific principle.

Key words: *mathematics, math teaching, science.*

Today, mathematics classes are mostly conducted professionally. However, teaching mathematics is a very complex and demanding process. In order to have successful classes, competence represents a necessary, but not a sufficient, condition. Complexity is successfully resolved by establishing a stronger connection of mathematics with other sciences. In that way we obtain a process that should be carried out harmoniously within several frameworks.

The main frameworks are the following: *linguistic frameworks, competence frameworks, methodological frameworks, scientific frameworks, pedagogical frameworks and psychological frameworks.*

Since harmony cannot be achieved easily, there have been failures and weaknesses in mathematics teaching and mathematics classes in general that significantly influence the quality of mathematics education. This in turn reflects badly on achieving goals of modern mathematics classes, the focus of which is

placed on introducing pupils into independent research work, developing capabilities of problem solving as well as developing their own opinion.

In this paper several assumptions and problems emerging within scientific frameworks of mathematics teaching will be described.

- Relationship between mathematics as a subject at school and mathematics as a science is *inter alia* established by the *scientific principle*. The scientific principle referring to mathematics classes consists of necessary harmonisation between teaching contents and methods on the one hand, and requirements and regularities of mathematics as a science on the other. This means that a mathematics teacher should introduce pupils to those facts and form those mathematical notions in their minds which are now scientifically confirmed.

While teaching mathematical notions a teacher realises the scientific principle if he/she carries out the process of forming notions correctly (observing, perceiving, forming a notion) and adheres to fundamental rules a definition of the notion has to satisfy (adequacy, minimum content, conciseness, genuineness, appropriateness, applicability, modernity).

When teaching theorems, teachers realise the scientific principle if they teach their pupils how to formulate a theorem correctly and precisely, distinguish between an assumption and an assertion of a theorem, formulate the converse of a theorem, formulate an opposite assertion, and if they make their pupils understand the methodology of proving theorems.

E 1. A critical point in teaching a notion is the transfer to that particular level on which abstraction starts, since the transfer from *concrete* to *abstract* is for some pupils rather difficult. The same conclusion holds for generalisations through inductive series of concrete cases.

The scientific principle is also realised by distinguishing clearly between definitions and theorems.

E 2. Pupils will have difficulties in getting a clear idea of mathematics if mathematics textbooks contain the following statements:

1) A *parallelogram* is a quadrangle with two pairs of mutually parallel sides of equal lengths.

The definition of a parallelogram contains two of its main characteristics, i.e. "opposite sides are parallel" and "opposite sides are of equal length". However, these characteristics are equivalent, meaning that each of them suffices for a proper definition of the parallelogram. Therefore, the definition should include only the first characteristic, which is quite appropriate for the notion *parallelogram*, whereas the second one should be omitted and proved separately as a theorem.

2) We can prove that $a^0 = 1$.

This formulation implies a conclusion that this is an assertion, a theorem. However, $a^0 = 1$ is a definition reached by consent which is introduced in order to have the rule of dividing powers $a^m : a^n = a^{m-n}$ also hold for the case $m = n$.

3) Straight lines meeting at a right angle are called *perpendicular lines*.

An angle whose legs are perpendicular to each other is called a *right angle*.

This is an example of a circular definition: perpendicular lines are defined by means of a right angle, and a right angle is defined in terms of perpendicular lines. It is actually not clear what is defined.

E 3. Students start with their education in teaching mathematics by attending the first workshop in teaching mathematics called NOTIONS IN MATHEMATICS I. That topic is dealt with at lectures much later, but it was selected for the purpose of checking the *level of students' foreknowledge* on such an important mathematical issue. Mathematical notions students have to define in this workshop are the following:

Ellipse, homothety, complex number, convex set, taking a root, quadratic equation, logarithmic function, zero of a polynomial, orthocentre of a triangle, inversely proportional quantities, polynomial, percentage, right angle, area, relation, sphere, similarity, translation, vector, altitude of a parallelogram.

The results are as expected: *very poor*. They show that students' knowledge referring to mathematical notions is rather confused. Their works rarely offer a correct definition. Without knowing the principles of defining a mathematical notion at that moment, students enter everything they know about that notion into their definition (examples, properties). In this way, instead of a short, precise and complete definition of a notion, there arises a comprehensive text on the basis of which one cannot figure out what it is about!! Such confusion,

which can also be called *ignorance* or *lack of knowledge*, could not be a means for successful classes. Results imply that a serious approach to this topic is required. Later on, in the chapter dealing with ways of thinking, the topic is methodologically worked out in detail, after which there follows a workshop in teaching mathematics called NOTIONS IN MATHEMATICS II. Students are expected to define the following mathematical notions:

Central symmetry, function, hyperbola, isometry, angles, rectangular parallelepiped, linear equation, logarithm, skew lines, perpendicular planes, pyramid, proportional quantities, rectangle, solution to simultaneous equations, perpendicular bisector of the segment, chord, trapezoid, cylinder, volume, closed interval.

Needless to say that the results are much better now, although the knowledge is still not as good as it should be. Some gaps in knowledge are rather difficult to be filled in, and in order to teach mathematics successfully, students must have a complete overview of the topic in question.

♦ In the process of cognition and introduction to the laws of nature researchers apply special means, i.e. scientific methods of research. Fundamental methods of scientific thinking and research are as follows: analysis and synthesis, analogy, abstraction and concretisation, generalisation and specialisation, induction and deduction.

There follows a brief description of these methods:

Analysis is a scientific research method based on breaking down the unit as a whole into parts, studying its parts and drawing conclusions on the unit on the basis of the obtained results. The opposite is *synthesis*.

Analogy is one type of similarity. *Inference by analogy* is a mental procedure whereby from observing that two objects coincide in a certain number of properties or relations, a conclusion is drawn that they also coincide in other properties or relations which were not directly observed on one object.

Abstraction is mental extraction of a general important property of the object or phenomenon under observation from other properties which are not important for a particular study, and rejection of these unimportant properties. The opposite is *concretisation*. It might be characterised as a mental activity which unilaterally focuses on some side of the object under observation excluding thereby relations with its other sides.

Generalisation is a transfer from observation of the given set of objects to a corresponding observation of its superset, i.e. generalisation of methods by which a border between the given set is crossed and more general notions and assertions are made. The opposite is *specialisation*, i.e. a method which strengthens the internal structure of the given set of objects.

Induction is a method of inference through which a new general proposition is obtained from two or more individual propositions. As a method of research, *induction* implies the following: while observing some set of objects, special objects from that particular set are observed and those properties of theirs are determined which are then assigned to the whole set. The opposite is *deduction*.

♦ Working as a mathematics teacher in class differs a lot from working as a mathematician-scientist, but there are some common characteristics. Either individually or supported by their teacher, pupils also discover new mathematical truths. It is especially important to find the way towards independent creative work of pupils. Therefore scientific methods are mentioned which are important for modern mathematics classes. They provide a strong link between mathematics as a subject at school and mathematics as a science. By selecting suitable problems and applying these methods, a creative teacher might educate and train pupils for work close to scientific.

E 4. During classes a mathematics teacher often says: "the analysis shows", "let us look at several concrete examples", "analogously", "this series of facts infers the conclusion", "the result of these observations is a generalisation", "by specialisation we obtain the formula", "mathematical sets are abstract", etc. Do pupils understand these words? How can we check whether they do?

The problem we face is a serious one, since even mathematics students majoring teaching profiles have problems understanding the aforementioned notions. Therefore, at a rather early stage of their mathematics education and in accordance with their age, pupils should get gradually and adequately taught to *analyse, synthesize, concretize, abstract, induce, deduce, generalise, specialise, observe analogies*, regardless of the fact whether they will seriously deal with mathematics later on or not. In contrast to common acquisition of matter, this represents a higher level of education in mathematics, and mathematical inference is invaluable since it might be applicable to many other activities.

E 5. Pupils' failures in mathematics and lack of knowledge are partially a consequence of the fact that classes are primarily conducted at a lower level where matter is to be learnt whereas a higher level is entirely neglected.

E 6. In mathematics classes synthesis is usually not preceded by analysis, which influences significantly clarity of teaching and understanding of the problem, reducing at the same time a cognitive value of the teaching process itself. To a certain degree, analysis is necessary in every research and should not be avoided.

An example illustrating the importance of *analysis* are *textual tasks*. Why do these tasks often cause a lot of troubles both to pupils and teachers, so that some teachers tend to avoid them? The explanation might be found in the nature of the tasks themselves. Every such task actually consists of two tasks: compilation of equations by translating from a common language into the language of mathematics and solving equations. The former is not always easy; it requires certain mental effort and knowledge of the procedure of analysis, assuming sometimes that pupils know without any explanation. This is the point troubles start from, the result of which is most frequently repulsion towards such problems. However, reducing the problem to solving equations is extremely useful, since it enables development of logic thinking, inventiveness, observation and capability of conducting independent minor research. Thus, it is not good to avoid such problems, but to explain them in a methodologically suitable manner, in order to have them fulfil their purpose of education.

♦ Mathematics emerging is a *concrete* and *inductive* science, whereas mathematics itself is an *abstract* and *deductive* science. What is the situation like with a mathematics teacher in that respect? Primary school mathematics classes are also primarily concrete and inductive. A mathematics teacher comes to abstract propositions, generalizations, by observing concrete objects and concrete examples as well as by inductive inference. That way is close and adjusted to pupils of that age. The inductive procedure consists of a series of inductive steps which lead to understanding a general. It starts with concrete objects and special cases, inductive inferences are obtained by analogy, and we try to generalise the facts observed. It means that induction is closely related to *concretisation, specialisation, analogy* and *generalisation*. Advantages of applying induction are as follows: realisation of the principle from easier to more difficult, from simple to more complex, studying new abstract notions and expressions through observation and checking, guiding pupils to learn and use new notions, stating new assertions, etc. There are quite a lot of contents in school mathematics which require an inductive procedure, some of them being various rules, laws, formulae, theorems, especially if they are not strictly derived or proved.

The opposite is *deduction*, which is carried out after induction and on a higher level of mathematics classes and pupil's education.

E 7. Inductive teaching requires an adequate number of individual and special cases. Often a mathematics teacher considers an insufficient number of such cases, so that derived assertions become unconvincing and unclear, a consequence of which is pupils' lack of knowledge. Another frequent teacher's failure is that he/she does not give a chance to a greater number of pupils to participate in the process of building an inductive series.

E 8. Derivation of generalisations is also a critical point of mathematics classes, since some pupils have problems understanding the transfer from concrete and individual to general. Therefore, a mathematics teacher has a responsibility to make this transfer as easy as possible.

♦ *Analogy* represents a very important scientific procedure. It permeates our thinking, everyday speech, creativity, but demanding scientific research as well. Analogy is also very useful in mathematics classes. During a class, a mathematics teacher usually says or asks: "similarly", "analogously we obtain", "in the same way we prove", "triangles coincide", "this is a related task", "which relation are figures observed in?", "here we might repeat the described procedure", "what in space corresponds to a rectangle?", etc. These simple sentences have a deeper meaning and an important goal. By repeating such structures a teacher deliberately points out analogy. In this way analogy becomes a vivid means of linking and easier mastering of mathematics contents, since certain previously mastered contents are revised, encouraging thereby development of pupils' creative thinking and creativity in general. While solving a problem, pupils concentrate on observation of some similar or related problem and imitate the procedure of solving it. What is more important, analogy enables the teacher to constantly alter teaching aspects and methods for the purpose of improving classes and making them more efficient.

E 9. Analogy has not been used enough in mathematics classes.

That is a pity, since there are so many related objects and their properties. Let us mention only a few: triangle and tetrahedron, square and cube, parallelogram and rectangular parallelepiped, circumference and sphere, circle and sphere, rules for numbers, ellipse and hyperbola, analogous formulae, etc.

E 10. If, by describing a certain mathematical content, textbook authors did not describe a possibility of applying analogy, then most frequently it will not be used in the class.

*

On the basis of the aforementioned, it can be easily concluded that scientific methods have their place in mathematics classes. With one remark: a mathematics teacher does not have to be a scientist to be able to apply correctly and appropriately the scientific principle and scientific methods. This offers itself in mathematics classes. Solving a problem includes something discovering and something creative. Therefore, a teacher is only required to encourage curiosity and independent mental work with pupils and to show them ways to new discoveries. If scientific procedures are applied adequately and correctly, with a feeling for complexity of mathematical contents and mathematical ways of thinking, taking into consideration mathematical capabilities of every individual pupil, mathematics classes might be expected to be successful. Otherwise, pupils will have major problems mastering the contents in question, and in time they might get a wrong impression that mathematics is more difficult than it really is. Unfortunately, in mathematics textbooks, and consequently in mathematics classes, not enough attention is paid to the regularity of applying scientific procedures. For elaborations of some mathematical contents it might be even stated that from that point of view they are wrong. And this violates the scientific principle.

(translated by Ivanka Ferčec)

AN EVERGREEN PROBLEM

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***Emil Molnár*[1]**

(To the memory of my Mentor, Professor Ferenc Kárteszi
on 100th Anniversary of his Birth)

Abstract. *The problem of Robinson and Cannibal is seemingly of Russian origin(?).*

As my former colleague László CZÁCH told me (at an entrance exam of mathematician students, when he asked it at a competition winner, who solved the first formulated problem - see later on - in 5 minutes), it can be found in a problem book of SHKLIARSKII-CHENTSOV-YAGLOM (I did not find the source exactly).

The problem is an exiting challenge for children in various ages, from the secondary school up to university studies, as well.

Robinson lives in an island at the centre of a round lake of unit radius. Sometimes he boats to the coast for food, but once he has noticed that Cannibal endangers him, running on the coast with unit velocity.

1) How big has to be the boating velocity of Robinson - at least - so that he can arrive to the

coast sooner than Cannibal will be there? (Robinson's speed in running is enough to escape.)

2) How many time units does Robinson take for this manoeuvre, at least?

3)What is the best strategy for Robinson and for Cannibal?

As idealization we assume, of course, that they see each other. They can change their directions in zero time without changing their maximum speed. The exact

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formulation of the problem needs certain abstraction. More concrete data for the lake- radius, for Cannibal's speed, etc. may help (!?) to solve the problem.

Finally, it turns out that the first /heuristic/ /solution /(by intention of the proposers of the problem) provides /good/ /estimates, only/. The /complete solution /needs numerical methods, "simple" by computer, no more elementary.

So we arrived at a new field as /optimal controll theory,/ very important in contemporary mathematics, with a lot of applications.

Key words: *methodology of problem solving, introductory motivation to optimal controll theory.*

1. Intentions, heuristic solution of problem

Problem solving is the most natural human activity. Everybody finds certain tasks in his work. Mathematicians solve various problems and develope specific methods for newer more difficult problem circles.

Teaching problem solving is the most important aim of teaching mathematics. Georg (György) PÓLYA wrote wounderful books on the methodology of these topics (see e.g. [P48]). Thus he further developed the Hungarian traditions. Living documents for this is the *Mathematical and Physical Journal for Secondary Schools (Középiskolai Matematikai Lapok,* in Hungarian, since 1894, founded by Dániel ARANY). There are many problem collections on Hungarian mathematics competitions (see e.g. [M89] of the present author, where the methodological aspects can be found as well).

My Mentor, Professor Ferenc KÁRTESZI was also a master how to motivate new mathematical fields by an attractive, natural introductory problem. His scientific book [K76] also shows this intention. I learned a lot of nice geometrie topics from his Hungarian book series (see e.g. [K66]).

This paper intends to expose such a problem. Children had already read the story of Robinson Crusoe by Daniel DEFOE in early grammar school age. This can motivate their efforts to understand the problem in the summary above. Some figurative models may help in understanding. Fig.1 here only illustrates some important concepts.

A first one is the *angular velocity*. Let $v = |\mathbf{v}|$ denote the maximal booting velocity of Robinson (briefly R), $V = |\boldsymbol{V}|$ is the maximal running speed of a

cannibal, let his name be also Cannibal (C). Then R can balance C's run by circular boating of radius $\frac{v}{V}r$, where r denotes the radius of the lake. That means, then they have the same angular velocity

$$\omega := \frac{\Delta\varphi}{\Delta t} = \dot{\varphi} = \dot{\psi} = \frac{V}{r} = \frac{v}{\left(\frac{v}{V}r\right)}. \tag{1.1}$$

Thus in a circle of any smaller radius than $\frac{v}{V}r$, R can reach a point opposite to C related to the lake centre, denoted by *O*. Increasing his radius step by step till $\rho = \frac{v}{V}r$, R will be opposite to C, i.e. *O* lies between them, their angular distance is just π. Now R cannot improve his position further with respect to C. Thus he tries the shortest way R*M* $= r\left(1-\frac{v}{V}\right)$ in time

$$\frac{r}{v}\left(1-\frac{v}{V}\right) < \frac{\pi r}{V} \Leftrightarrow \frac{1}{1+\pi} < \frac{v}{V}, \tag{1.2}$$

i.e. he arrives in less time than C reaches the meeting point *M*. By the intention of the source book mentioned in our above summary, this is „the solution", resonable enough at the first glance. The speed $\frac{1}{1+\pi} \approx \frac{1}{4.14}$ of R related to that of C is much more better than the direct „brute force" estimate

$$\frac{r}{V} < \frac{\pi r}{V} \Leftrightarrow \frac{1}{\pi} < \frac{v}{V}. \tag{1.3}$$

Of course, we applied idealization by assuming that C follows the shore, without difficulties. If C changes opposite direction then R also "reflects" his motion in the same moment, if necessary.

We see that these arguments do not provide any time estimate, and do not guarantee the optimality yet.

My chess colleague in the university team BEAC, Ferenc DEÁK called my attention to this circumstance, tanks to him. This will be discussed later on in Section 3.

2. Time estimate by differential equation

In Fig.2 we sketch the idealized situation in a Cartesian coordinate system (O, x, y) where polar coordinates (r, φ) are introduced as well.

Robinson (R) starts boating at O with velocity vector $\mathbf{v}$ of his maximal speed $v = |\mathbf{v}|$, moving away Cannibal (C). He starts from C(r,0) at the x-axis, say in positive (anticlockwise) direction, with velocity vector $\boldsymbol{V}$ of his maximal speed $V = |\boldsymbol{V}|$. R can follow the angular velocity $\dot{\varphi} = \frac{V}{r}$, $\varphi = \frac{V}{r}t$, (t denotes time variable) of C. That means, his angular velocity will be $\dot{\varphi} = \dot{\psi}$, where $\psi = \varphi + \pi$. He guarantees this by his velocity component $\mathbf{v}_\psi$ proportional with C's velocity V as follows:

$$\text{C:}\quad X = r\cos\varphi \qquad \varphi = \frac{V}{r}t \qquad \boldsymbol{V}: \dot{X} = -r\sin\varphi\cdot\dot{\varphi} \qquad \dot{\varphi} = \frac{V}{r}$$

$$Y = r\sin\varphi \qquad \dot{Y} = r\cos\varphi\cdot\dot{\varphi} \tag{2.1}$$

$$\text{R}(x,y):\quad x = \rho\cos\psi \qquad \mathbf{v} = \qquad \dot{x} = \dot{\rho}\cos\psi - \rho\sin\psi\cdot\dot{\psi}$$

$$(\varphi,\psi)\quad y = \rho\sin\psi \qquad = \mathbf{v}_\rho + \mathbf{v}_\psi: \qquad \dot{y} = \dot{\rho}\sin\psi + \rho\cos\psi\cdot\dot{\psi}$$

Now, the angular velocity argument yields

$$\mathbf{v}_\psi = -\frac{\rho}{r}\boldsymbol{V}, \qquad \text{i.e. } \rho\sin\psi\cdot\dot{\psi} = -\frac{\rho}{r}\left(-r\sin\varphi\cdot\dot{\varphi}\right) \tag{2.2}$$

$$\rho\cos\psi\cdot\dot{\psi} = -\frac{\rho}{r}\left(-r\cos\varphi\cdot\dot{\varphi}\right) \tag{2.3}$$

implying $\psi = \varphi + \pi = \frac{V}{r}t + \pi$, indeed. Then

$$v = |\mathbf{v}| = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{\dot{\rho}^2 + \rho^2\dot{\varphi}^2} = \sqrt{\dot{\rho}^2 + \rho^2\frac{V^2}{r^2}} \tag{2.4}$$

provide a separable differential equation for $\rho(t)$ and its solution:

$$v^2 = \dot{\rho}^2 + \rho^2 \frac{V^2}{r^2}, \text{ hence } \rho(t) = \frac{v}{V} r \sin \frac{V}{r} t \tag{2.5}$$

$$\text{for } \frac{V}{r} t \in \left[0, \frac{\pi}{2}\right], \qquad \rho(t) \in \left[0, \frac{v}{V} r\right].$$

We get in (2.2) the boating path of Robinson by substitution of ρ and Ψ:

$$\mathrm{R}(x,y): \; x = -\frac{1}{2}\frac{v}{V} r \sin \frac{2V}{r} t,$$

$$y = -\frac{1}{2}\frac{v}{V} r \left(1 - \cos \frac{2V}{r} t\right), \qquad t \in \left[0, \frac{\pi}{2}\frac{r}{V}\right]. \tag{2.6}$$

This is half circle of radius $\frac{1}{2}\frac{v}{V} r$ which corresponds to the arc C(r,0), C'(r, $\frac{\pi}{2}$), the quarter circle of radius r of Cannibal.

Now it comes the short way $R'M$ (as $R\overline{M}$ in Fig.1) for escaping just by formulas (1.2). Then C runs angle $\frac{\pi}{2} + \pi = \frac{3\pi}{2}$ to M in Fig.2. He makes this way by its speed V under time

$$T_C = \frac{3\pi}{2}\frac{r}{V} \geq T_R \tag{2.7}$$

bigger than R' s time. Equality is allowed here and also in formulas (1.2) as idealization for the problem (since R's run speed is enough to escape).

We have obtained a good estimate for the „optimal time" of R' s manoeuvre, indeed.

3. On the optimality of the problem, the best strategy

It may be surprising that both „solutions" above are not optimal in strict sense, although they are satisfactory for a resonable manoeuvre.

The optimal solutions, however, are very complicated, just hopeless to determine exactly. A numerical computation (e.g. Maple) can help, of course, if we formulate the best strategy correctly. We sketch this in this section by combining Figures 1 and 2.

To Fig.1 it was critical, why R did choose the shortest way RM in the last step (why not a modification $\overline{RM}$ in Fig.1, or *R'M'* in Fig.2)? This is related with a cosine theorem in a triangle $O\overline{R}\overline{M}$ with angle $\overline{R}O\overline{M}\angle = \alpha$, or in *OR'M'* similarly. Or, why does not leave Robinson his optimal half circle (Fig.2) earlier at a time t_0, at $\beta := \frac{V}{r}t_0 < \frac{\pi}{2}$, where his distance from *O* is

$$\rho(t_0) = \frac{v}{V} r \sin\beta = \frac{v}{V} r \sin\frac{V}{r}t_0 \ ? \tag{3.1}$$

Now from the critical triangle $\overline{R}O\overline{M}$ (e.g. in Fig.1) holds

$$(\overline{RM})^2 = (O\overline{R})^2 + (O\overline{M})^2 - 2\cos\alpha(O\overline{R})(O\overline{M}) \text{ (cosine theorem)} \tag{3.2}$$

i.e. we have R' s time for boating the way by velocity *v* first:

$$\overline{RM} = \sqrt{\rho^2 + r^2 - 2\rho r\cos\alpha} = \sqrt{\left(\frac{v}{V}\right)^2 r^2\sin^2\beta + r^2 - 2\left(\frac{v}{V}\right)r^2\sin\beta\cos\alpha},$$

$$\frac{\overline{RM}}{v} \le (\pi+\alpha)\frac{r}{V} \text{ , i.e. for } \frac{v}{V} \text{ we get the unequality} \tag{3.3}$$

$$\left(\frac{v}{V}\right)^2 \left[(\pi+\alpha)^2 - \sin^2\beta\right] + 2\left(\frac{v}{V}\right)\sin\beta\cos\alpha - 1 \ge 0.$$

These express that R arrives at the meeting point $\overline{M}$ not later than C. The manoeuvre time will be

$$T \le \frac{r}{V}(\pi+\beta+\alpha) \text{ of C.} \tag{3.4}$$

R has the optimal speed, smallest as possible for his manoeuvre. This will determine the optimal $\beta = \frac{V}{r} t_0$ and α as well from the second order unequality (3.3)

$$\frac{v}{V} \geq \frac{-\sin\beta\cos\alpha + \sqrt{(\pi+\alpha)^2 - \sin^2\alpha\sin^2\beta}}{(\pi+\alpha)^2 - \sin^2\beta} =: f(\alpha,\beta). \tag{3.5}$$

The minimum of the right-hand-side by $\alpha, \beta \in \left[0, \frac{\pi}{2}\right]$ provides the optimal $\frac{v}{V}$ and T by (3.4) for Robinson. Our estimates in Section 1 and 2 refer to $\alpha = 0$ and $\beta = \frac{\pi}{2}$. Then we obtained $\frac{v}{V} \geq \frac{1}{\pi+1}$ and $T \leq \frac{3\pi}{2}\frac{r}{V}$. We can check that, at $\alpha = 0$, $\beta = \frac{\pi}{2}$,

$$\frac{\partial f\left(0, \frac{\pi}{2}\right)}{\partial\alpha} < 0 \tag{3.6}$$

does not lead to extremum, indeed. Exact computations are left to the Reader, by computer. The system

$$\frac{\partial f(\alpha,\beta)}{\partial\alpha} = \frac{\partial f(\alpha,\beta)}{\partial\beta} = 0 \tag{3.7}$$

yields a cumbersome numerical problem for determining α and β, then t_0, $\frac{v}{V}$.

You see that we have arrived at a problem of the optimal controll theory. Instead of Robinson and Cannibal we may consider aeroplanes or rockets and ballistic missiles under other assumptions. Problems of astronautics arise also in this field of mathematics and related sciences.

I thank my colleague Jenő SZIRMAI for preparing this manuscript.

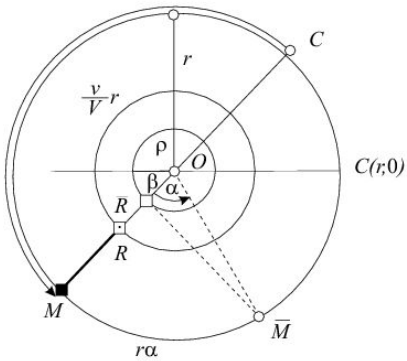

Fig.1.

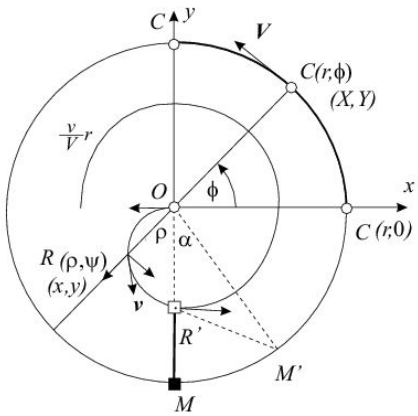

Fig.2

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MATHEMATICALLY GIFTED CHILDREN: WHAT CAN WE TEACH THEM AND WHAT CAN WE LEARN?

Vesna Vlahović-Štetić [1]

Abstract. *Psychologists have first started researching mathematical problem solving and giftedness in the 1920s. Studies made by Thorndike (mathematics) and Terman (giftedness) have opened up these two new areas of psychology, which still share many common features.*

Definitions of giftedness can be classified into four major groups: innate or genetic-oriented definitions, cognitive models, achievement-oriented definitions and systemic definitions. The genetic-oriented approach emphasizes the importance of inborn traits. More recently, it has turned to multiple intelligences, i.e. the notion that giftedness goes beyond high intelligence and is reflected in various domains. Cognitive-based studies and definitions of giftedness claim that cognitive functioning of the gifted is different in particular domains. Thus, gifted mathematicians are more efficient at processing numerical information, but they process verbal information like average subjects. On the other hand, the verbally gifted are more efficient at processing verbal information, while their performance in processing numerical information is like that of average subjects. The achievement-oriented approach stresses the importance of non-intellectual factors (motivation and creativity) for giftedness. The systemic approach emphasizes the role of various social systems for the development of giftedness (the family, the school, the educational system).

At present a great deal is known about the cognitive functioning of mathematically gifted children, their educational achievements and needs. We know that some of their needs can be satisfied through acceleration and enrichment. The

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question that remains unanswered is how familiar teachers are with the specific needs of gifted students and how well they are trained to work with them. Even if teachers are aware of what they should be teaching, this is not enough. Numerous studies have shown that new subject matter alone will not suffice – gifted students also need support in their personal growth, which is particularly true of curricula for the mathematically gifted. It is difficult to answer the question how much teachers are aware of the socio-emotional needs of their mathematically gifted students and how those needs are met in practice. Research has shown that gifted students are willing to express their needs; all we have to do is be willing to learn.

Key words: *gifted children, definitions of giftedness, mathematical giftedness.*

At the beginning of the last century some completely new fields of research opened up in psychology, still a relatively young science at the time. In 1922, E. L. Thorndike published his book *The Psychology of Arithmetic*, today regarded as the beginning of psychologists' research of mathematics. At the same time, L. Terman started his extensive longitudinal research on the gifted, which led to new insights into the development of gifted individuals, and significantly changed the erroneous beliefs of the period (Terman and Oden, 1959).

In the meantime these two fields of psychology, research on the gifted and on mathematical reasoning, have seen significant theoretical and empirical development. The intersection of these two areas of research – mathematical giftedness – may be what mathematicians find most interesting, both with regard to theory and its practical implications.

There are a number of definitions of the notion of giftedness and various approaches to it, which can be, according to Mönks and Mason (2000), classified into four groups:

- innate or genetic-oriented definitions
- cognitive models
- achievement-oriented definitions
- systemic definitions.

These approaches are scientifically-based and have important implications for working with gifted individuals (Vlahović Štetić, 2005), including the mathematically gifted.

The first group of definitions mainly emphasizes the importance of genetic factors for the development of giftedness, which in no way implies denying the importance of environmental factors. According to these theories, gifted individuals are those who have a particular characteristic to the greatest extent in a particular population, which is largely determined by genetics. One of the most famous recent advocates of this approach, Howard Gardner (1983), says: "There are seven different specific abilities, talents or intelligences: logical-mathematical, linguistic, spatial, bodily-kinesthetic, interpersonal and intrapersonal." Society defines giftedness based on its system of values. The western civilizations are more appreciative of some types of giftedness, such as verbal and mathematical-logical giftedness. Thus the mathematically gifted are treated fairly well – their abilities are appreciated in the school environment. Gardner points out that every individual is a specific combination of the mentioned abilities, and a high score in one area, for instance in mathematics, does not mean that the individual is above average in other areas. In other words, a mathematically gifted child is very likely to have mathematically gifted individuals in the family (although this may be latent and not necessarily manifest giftedness), and can be gifted only in the area of mathematics, whereas may be average or even below average in other areas. Gardner emphasizes that it is important for teachers to stimulate various types of talents or intelligences while teaching their subjects – thus teachers of mathematics should think not only how to stimulate mathematical giftedness but also interpersonal or musical giftedness through their subject, while, e.g. history teachers should be thinking how to stimulate mathematical giftedness.

The second group of definitions is directed at cognitive models. Sternberg (2001) considers giftedness to be a path from being a beginner with potential to becoming an expert in a field. This development requires various skills: metacognitive skills, learning skills, thinking skills, declarative and procedural knowledge, and motivation as the main stimulator. Gifted individuals can combine these elements in a superior way; they advance more rapidly and attain a higher level of expertise than average individuals. Researchers within this theoretical framework have studied differences in cognitive functioning between gifted and average individuals. New technologies have made it possible to use complex stimuli, thus allowing new insights in the areas of problem solving, reaction time, short-term and long-term memory. Dark and Benbow (1991) showed that mathematical giftedness is associated with better memory for numbers and spatial locations, while verbal giftedness is associated with better

memory for words. The gifted differ according to the type of information that they can retain in their working memory; they have no general capacity for retention, rather, it depends on the type of information and the type of giftedness. That is to say, we can expect gifted mathematicians to be superior in handling numerical information, be better at storing such pieces of information in their long-term memory and be better at their retrieval, which does not mean that they will remember other types of information more effectively.

The third group of approaches are achievement-oriented approaches. Renzulli (1986), the author of the three-ring theory of giftedness says: "Gifted behavior reflects an interaction among three basic clusters of human traits—these clusters being above average (but not necessarily high) general and/or specific abilities, high levels of task commitment (motivation), and high levels of creativity. Gifted and talented children are those possessing or capable of developing this composite set of traits and applying them to any potentially valuable area of human performance". Abilities, motivation and creativity can be represented as circles of the Venn diagram. Giftedness is their intersection. In other words, intellectual potential, such as the ability of mathematical reasoning, is not sufficient for giftedness – the individual must also have motivation and creativity. Renzulli says that gifted children do not necessarily have to exhibit all three characteristic of gifted behavior, but that they should have a capacity to develop these characteristics later in life. One issue that can be discussed here is the relationship between mathematical giftedness and mathematical creativity. This is a relationship present in various areas: creative individuals are also gifted, but gifted individuals are not necessarily creative. The issue that arises is: are only top professionals mathematically creative, or is there also creativity on lower levels of mathematical knowledge? Sriraman (2005) discusses this issue in detail and concludes that there is mathematical creativity in all age groups, and that teachers can stimulate (teach) their mathematically gifted students to be creative, thus expanding the subset of creative mathematicians in the set of gifted mathematicians.

More recently, systemic approaches to giftedness have emphasized numerous factors in the development of giftedness. Tannenbaum (1983) was the first to offer a definition and a model of giftedness from the perspective of the systemic approach. There are several factors which need to be optimal in order for the potential of the gifted individual to be realized: general ability, special ability, nonintellective facilitators (independence, internal locus of control, motivation, self-esteem, and flexibility), environmental support (from the wider

and immediate environment), and chance. Each of these factors is necessary, but is by itself insufficient to realize the potential. A combination of four factors cannot compensate for a serious deficiency in the fifth, and their relative importance changes with regard to the type of giftedness. Thus, insufficient environmental support or lack of motivation or self-respect will result in the objectively high potential not being manifested as giftedness. Society is responsible for environmental support, and one of the ways in which it must be provided is the school system. Moreover, systemic approaches emphasize the role of social values and society's treatment of the gifted. If we give educational support to gifted mathematicians but at the same time tolerate a climate where they are treated as unusual children or eccentrics, we cannot expect adequate development of the mathematically gifted.

Different approaches to giftedness stress different factors, but according to Sternberg (2004) they also have some common points:

1. giftedness goes beyond the intelligence quotient
2. giftedness consists of cognitive and non-cognitive factors
3. environment is crucial for the realization of giftedness

How do these commonalities apply to mathematically gifted children?

1. Mathematical giftedness undoubtedly transcends the intelligence quotient, which was introduced as a measure of giftedness already by Terman. The intelligence quotient is indeed a measure of general intellectual functioning, but the general quotient alone does not reveal a child's strong points, i.e. the area in which the child may be gifted. Depending on the instrument, the quotient is a measure based on a portion of various abilities (general and specific, such as, e.g., verbal and numerical). The same intelligence quotient may be a result of a higher portion of verbal or a higher portion of nonverbal abilities; these two measures may not necessarily be balanced in an individual. Moreover, studies show that the discrepancy between particular components of the quotient is greater in intellectually superior individuals than in average individuals (Detterman and Daniel, 1989, Wilkinson, 1993). It is also interesting to note that verbally gifted children usually have a more balanced verbal and mathematical ability than the mathematically gifted – their high numerical and mathematical reasoning ability will be more frequently accompanied by average or even below average results in verbal abilities. In other words, the mathematically gifted are less likely to be successfully identified if the intelligence quotient is used as the

sole measure. Similar results are evident in case studies; and Bloom's (1985) retrospective study of twenty mathematically gifted adults showed that none of them have learned to read before school and that six of them had difficulties in learning to read. Although mathematical giftedness should not be equated with general intellectual functioning, it should be noted that studies show that intellectual abilities of the mathematically gifted are above average (Lubinsky and Humphreys, 1990).

2. Mathematical giftedness is not only associated with abilities and knowledge, but also with non-cognitive factors, such as flexibility, openness for the new, tolerance for ambiguity, positive self-concept, curiosity, willingness to take risk and task commitment (Wieczerkowski, Cropley and Prado, 2000). Wieczerkowski (1998, quoted in Wieczerkowski et al., 2000) identified two factors as the basis of motivation to do mathematics in gifted children. The first factor is children's belief about the difficulty of mathematics associated with their belief that they are capable of such achievement. The second is the child's assessment of the value of mathematics, which includes its interestingness, the possibility of using it to fulfill some personal needs such as success, social status or a feeling of self-worth, and the usefulness of mathematics to achieve life goals: academic success or getting a good job. Realization of mathematical giftedness is a result of cognitive as well as non-cognitive factors; therefore, attention should be given not only to the subject matter being taught, but also to children's beliefs about mathematics and their attitudes to it. This is especially true of mathematically-gifted girls. Research on samples of the population tend to indicate a more positive attitude of boys to mathematics (Hyde et al., 1990, Norman, 1977) and a greater math anxiety among girls (Arambašić et al., 2005, Gierl and Bisanz, 1995, Hyde et al., 1990, Ma, 1999).

3. Some gifted children may succeed without the support of their environment, but they are an exception rather than a rule. Like all other gifted children, the mathematically gifted need the support of their immediate environment (the family) and their wider environment (the school). The family can nurture the child's potential from a very early age. The family which shows that it cares about the child's achievement and secures educational support (literature, access to workshops) undoubtedly facilitates the development of a mathematically gifted child. The school is a system responsible for the child's development, which may provide the gifted with different types of support. The most commonly mentioned types are acceleration and enrichment. In Croatia two types of acceleration are possible: early entry to school and grade skipping.

There are no other types, such as, for instance, accelerating a child only in the subject in which he/she is gifted and which is of special interest to him/her – for instance, enabling the gifted sixth grader to take mathematics with seventh graders and all other subject with his/her same-age peers. Enrichment most frequently involves additional classes, and often converts into practicing for competitions, so it is doubtful whether this really caters to gifted students' need for new knowledge and activities. Moreover, the support of the school system is commonly geared towards subject matter and not towards personal development of gifted mathematicians. Along with mathematical knowledge, the curriculum should also develop communication skills, teamwork and positive self-concept, i.e. skills and knowledge that will be profitable to gifted students on a personal level.

Cognitive psychology of education and cognitive developmental psychology deal with abilities of children of various ages. It is certain that gifted mathematicians can learn faster and more than their same-age peers. The question is what they should be taught. The usual subject matter is not challenging enough for the gifted. It should be deepened and expanded, maximizing the applicability and the possibilities of elaboration of what is learned. Experienced mathematics teachers will be able to choose subject matter that is appropriate for children and that children find interesting.

Special attention should be devoted to changing teaching methods. The children's choice of teaching method should be respected, which means that classical teaching should be avoided in favor of group work in small groups, project work or tutoring. If we work with the gifted only on solving more difficult problems (those that appear at competitions), this also soon becomes nothing but familiar routine. They should be taught more complex skills, encouraged to acquire basic knowledge and not only specific facts, the subject matter should be challenging and varied so as to encourage higher thinking processes (synthesis, analysis, generalization, evaluation). Teaching the gifted must include skills such as creative and critical thinking, heuristics and complex methods of problem solving and decision making. They should be taught metacognitive skills – skills of controlling their own learning and monitoring their thought processes and knowledge. When working with the gifted, we should insist on children being producers and not only consumers of knowledge (which is a long-standing deficiency of Croatian schools) and teach them how to clearly and understandably present their work. All this, of course, applies to different types of giftedness, but is particularly pertinent for the mathematically gifted,

because they find it more difficult than for the verbally gifted to communicate their achievements to the wider environment.

The answer to the question of what we can teach them is simple – nearly everything. Of course, it is up to us to find a reasonable measure – to teach them things that they find interesting, challenging, and that will ensure their continued love of mathematics. In doing so, the overall wellbeing of students should always be kept in mind. This means that emphasis should be placed on subject matter that will contribute to the social and emotional development of the child. A gifted mathematician should also be a happy child, well adjusted to his/her environment. This is what adults need to learn – how to recognize children's boundaries and children's needs and not go beyond them.

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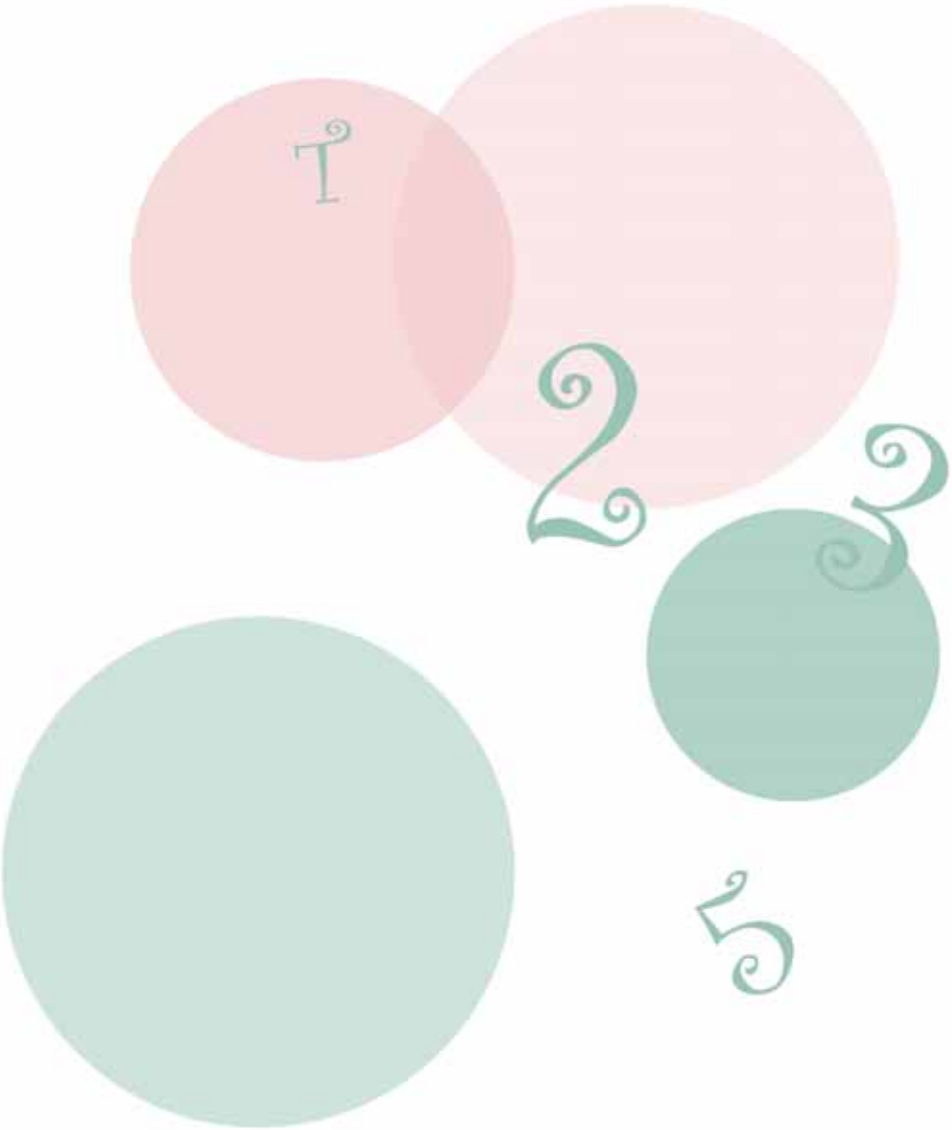
(translated by Mateusz Milan Stanojević)

Short Communications



DIFFICULTIES IN TEACHING MATHEMATICS IN THE SECOND GRADE OF PRIMARY SCHOOL

***Maja Cindrić*[1] *i Josip Cindrić*[2]**

Abstract. *This paper deals with pupils' difficulties concerning adding and subtracting numbers in second grade of elementary school. The paper also gives a statistical overview of pupils' score in first term of the second grade and compares it with scores of other three grades. It also gives an overview of a test taken in second grade, which assesses pupils' knowledge on adding and subtracting. Additionally, the authors suggest possible reasons for this poor score and offer alternative solutions to this problem.*

Key words: *mathematics teaching, difficulties in teaching mathematics.*

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CHILDREN AND SIMPLE COMBINATORIAL SITUATIONS

Mara Cotič [1], Darjo Felda [2]

Abstract. *The present mathematics curriculum for primary schools in Slovenia is for the first time richer in topics pertaining to statistics, combinatorics and probability, gathered under the common term 'Data Processing'– and this in the very beginning of the education cycle (the first triennium).*

The purpose of mathematical education is twofold: to develop mathematical literacy and mathematical reasoning. In primary school mathematical literacy should represent a goal to be achieved by all students. Developing mathematical reasoning, on the other hand, is a very complex activity that is intended particularly, though not exclusively, for students with a special interest in mathematics. While statistics generally belongs to the field of mathematical literacy, combinatorics specifically helps to develop mathematical reasoning. In fact, by learning how to solve combinatorial problems we encourage thinking and inference, develop observation skills and a feeling for the equality-inequality relation; we try to create order from disorder, look for similar or identical patterns, base premises on different laws and perceive the structure of a system. It is not enough, therefore, for teachers to be aware of the goals of teaching combinatorics; they also have to be familiar with the model of developing basic combinatorial terms, which has been adapted and completed after Bruner's model of mathematical notion development. Roughly, the model consists of three levels: the enactive (concrete), iconic (graphic) and symbolic. Each level is further divided into individual sublevels. The enactive level involves the setting and analysis of the starting problem situation, with a subsequent performance of the activity with objects. Then, on

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the iconic level, a graphic representation of the performed activity is created, first by means of a drawing or sketch and then by systematic representations (table, arrow diagram and combinatorial tree). Finally, on the symbolic level, the activity is presented in an even more universal form, the problem being generalised. It should be underlined that at the beginning of the education process children solve simple combinatorial situations primarily through direct experience (play) on the concrete level, while some of them are capable of solving combinatorial problems on the graphic level as well. Given that children have different levels of abilities, it is necessary in the teaching of these topics to apply differentiation as well as individualisation.

The study of these topics in the higher grades of primary school and especially in secondary school will reveal to the students the "heart" of mathematics: combinatorial concepts are expressed through the language of set theory, while the results and methods used in the theory of combinations are very useful and advantageous in other fields of mathematics also, particularly in the theory of probability.

Future teachers should thus in the course of their studies develop the competence ability to bring the understanding of simple combinatorial situations closer to their students.

Key words: *mathematics teaching, statistics, combinatorics and probability.*

Introduction

Until recently pupils in Slovenia started learning combinatorics quite late (in secondary school) and even then only at a formal level. The teaching of maths in secondary school is mostly deductive and on an abstract level and although the notions from combinatorics (permutation, variation, combination, etc.) are totally new to students, teachers do not use appropriate examples and models to explain them. This is certainly one of the reasons why students experience difficulties in understanding them. Mialaret (1969) proved with his study that even in secondary school the success of students' understanding and solving problems depends mostly on abstract and concrete formulation of them. Students in secondary school have problems with understanding combinatorics even if they manage to solve combinatorial problems with appropriate illustrative examples and teacher's help. They seem not to be used to concrete examples when developing basic mathematical concepts and at the same time they have not been introduced even to the most basic combinatorial situations, not even to those, which closely relate to everyday life.

Didactic instructions and aims

With the introduction of a new mathematical curriculum (1998) we wanted to improve this failure by including combinatorics in the very first stages of primary education (first triad), although many other countries do not have it in their maths curriculum for primary school. In the first stages of education combinatorics is not taught in a conventional sense of the word. Pupils are introduced to its basics at a very concrete level through play, which prepares them gradually for abstract thinking.

When introducing combinatorics at a primary school level there are dilemmas about pupil's ability to solve this kind of problems at their age. According to Piaget and Inhelder (1951) a child is capable of solving this kind of problems only on the level of formal operations (11-15). The Piaget's and Inhelder's conclusions that combinatorics is not appropriate on the level of concrete operations of child's development (7-11) are based only on spontaneous children's answers without introducing these issues into lessons. Again, it is necessary to point out that in the first years of education there is no real teaching of this theory because the study of combinatorial situations demands methods to determine the number of elements of a finite set without counting them. Pupils do not use these methods or they are only exposed to simple combinatorial situations in which there are a small number of elements, which can be simply counted.

A number of didactic mathematical researches on appropriateness of introducing combinatorics at the first key stages of education in primary school have disproved Piaget's and Inhelder's statement. Here we point out two researches: the empirical research carried out by Fishbein in Israel already in 1970 and the research done in Slovenia in 1993 as a part of the project Renovation of Primary Schools by M. Cotič and T. Hodnik. The issue was introduced into schools according to a stage of development of children at this age. Pupils start solving combinatorial situations directly through their experiences (games); it means that they manipulate objects (their number should not be large) (Fischbein, 1975). They use objects from their everyday life for activities suitable for their age group. For example:

- they make necklaces (as many as it is possible) using wooden balls of various sizes,
- they put in order chips of various colours or models of geometrical figures, cubes, etc. in every possible way,
- they make all possible numbers using given numerals,

- following instructions they use given letters and syllables to create as many different words as possible … (Fischbein, 1984).

Doing these concrete activities pupils should find out with teacher's help that it is necessary to tackle certain combinatorial situations systematically. At the same time we have to teach pupils to use various graphical presentations (a tree, a diagram, a table, etc.)

In everyday life we are often bombarded with pieces of information that we have to sort out and then be able to use them. Teaching how to solve combinatorial situations is therefore very important because while doing it we:

- develop the ability of observation,
- develop the ability to estimate the relation of equality or inequality,
- try to create an order out of disorder,
- try to discern related or identical patterns and detect principles,
- become aware of a structural system (Felda, 1996).

Levels at solving combinatorial situations

When solving simple combinatorial situations or when learning new notions from combinatorics (permutation, combination, variations, etc.) pupils in the beginning of primary school go through several stages. We adapt these stages from Bruner's or Dörfler's model of developing mathematical ideas (Kokol-Voljč, 1996). We have used the empirical research to check the successfulness of this model; 180 pupils aged 7 to 9 participated in this research (Cotič, 1998). When we were testing them at the end of the project, approximately 80 % of pupils were successful.

I. ENACTIVE (CONCRETE) STAGE

1. Setting up the starting-point for a problem situation
2. Analysis of the situation
3. Realization of activity: - Role-play
   - Presentation using objects

II. ICONIC (GRAPHIC) STAGE

4. Schematizing of the activity (a picture, a sketch)
5. Schematizing of the activity using systematic ways (a table, a combinatorial tree, a diagram)

III. SYMBOLIC STAGE

6. Presentation of the activity in more general form (setting up the calculus for each example)
7. Generalization of the problem

IV. USING THE DEVELOPED NOTION IN NEW SITUATIONS (it acts as an instrument)

All these stages cannot be considered statically for the process of learning these concepts does not follow exclusively the order of proposed stages. All these levels have to be considered flexible and should be variously used in the educational process. They can follow each other in order (for example: concrete activity is represented with a picture that is then described and noted down by symbols), but not necessarily as we can put the concrete activity directly into symbolic level. We can even "perform a concrete activity" using a picture or just describe the "sketched activity" and noted it down by symbols (Tomić, 1984).

It is important to stress here, that the concrete level must not be omitted when forming mathematical ideas at the beginning of education when pupils are at a stage of concrete thinking. It is also important that it should not be too short. The too short stage of concrete level or even its exclusion are usually the reasons for not understanding the basic mathematical notions.

Moving from concrete to abstract level does not happen in the course of one lesson or a day but it is our long-term aim. There are, however, some characteristics of transitions between the stages:

- students perform the activity using objects even on the highest level,
- students first concentrate on processes and intuitive relations and then on answers or symbolic solutions with mathematical expressions,
- after a certain time of activities on a particular stage students can start dealing with similar activities on a higher stage,
- students can anytime play the same game on different levels; a teacher just supervises the moving to a higher stage,
- students have an opportunity to redirect the formation of their own problems and presenting operations (Labinowitz, 1989).

The shown model of how a particular notion of combinatorics can be developed is presented by a concrete example. For this purpose we took the concept

of the multiplication rule of combinatorics as it is one of the basic concepts in this field.

I. Enactive (concrete) stage

1. Setting up the starting-point for a problem situation

Students between 6 and 7 years of age are given models of equal (congruent) rectangles and equal (congruent) isosceles triangles, the baseline of the triangles being of the same length as one of the sides of the rectangle. The figures are different colours. The students are left to play with the models first: they use them to compose various structures, sort them by colour or shape … Then they are presented with the following problem: *How many different houses can you make from triangles of three different colours (red, blue, yellow) and rectangles of two different colours (green, orange) if each house is composed of one rectangle and one triangle?*

2. Analysis of the situation

The students analyse how the given problem situation could be solved: how many and which figures the house will be composed of, what has to be taken into consideration, how to recognise the same and the different houses etc.

3. Realization of the activity

The students compose the houses. Given the small number of rectangles and triangles, the majority of the students will be able to find all the different houses (6). Some students compose the houses unsystematically, thus ending up with several "same" houses or missing some of the variants. Such students should be helped by the teacher to understand that it is indispensable to tackle combinatorial situations systematically if we wish to find all the different houses possible. Only a few students will immediately choose a determinate system and follow it. In performing these activities the students should be encouraged to describe in words their work or procedures.

II. Iconic (graphic) stage

4. Schematization of activity (a picture, a sketch)

In this step the students make pictorial presentations of the solutions they arrived at by composing the models of the figures. They first make an unsystematic image:

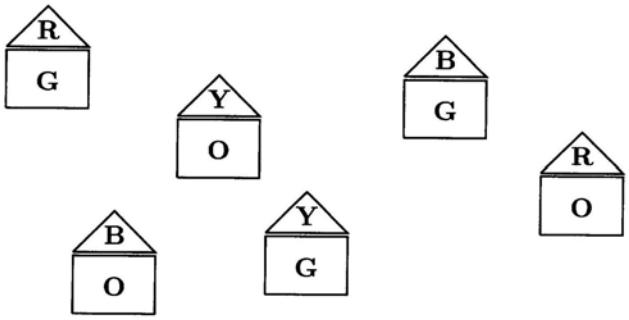

Figure 1. Unsystematic image

Some students discover a system and form, for example, two streets, each with houses with façades of the same colour. Thus the following image is created:

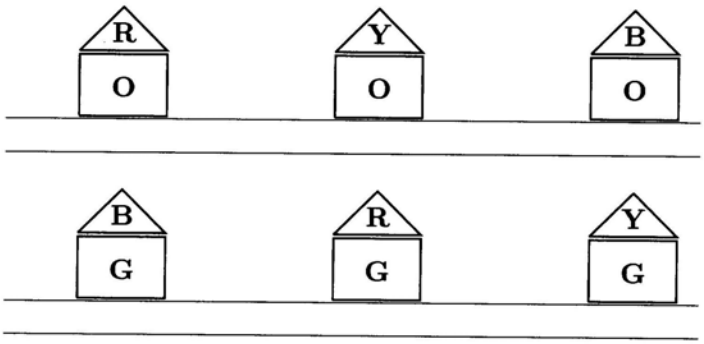

Figure 2. Two streets

5. Schematizing of the activity using systematic ways

The students are gradually taught to present the combinatorial situations with a diagram, a table and a combinatorial tree.

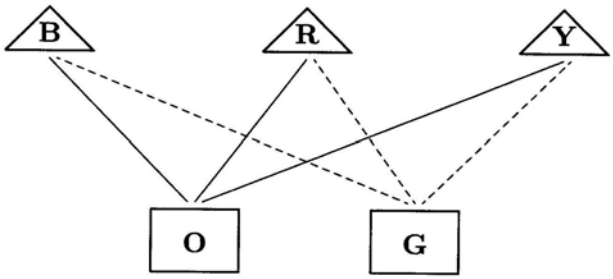

Figure 3. A diagram

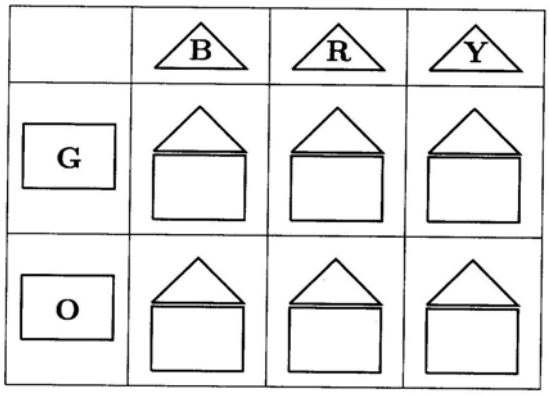

Figure 4. A table

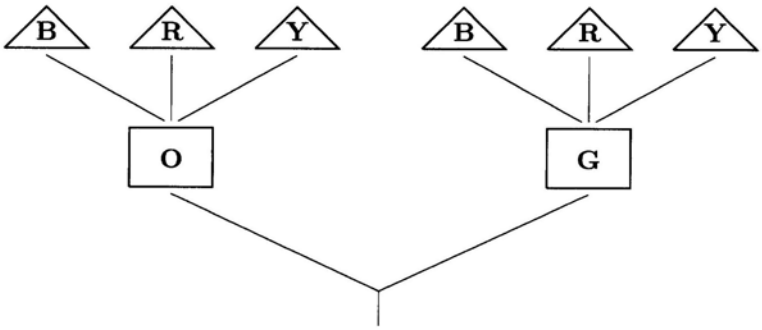

Figure 5. A combinatorial tree

III. Symbolic stage

6. Presentation of the activity in more general form

From the table, diagram and combinatorial tree the students see that they can calculate the number of all the different houses by multiplying the number of the differently coloured rectangles by the number of the differently coloured triangles; in our case: $2 \cdot 3 = 6$.

7. Generalization of the problem (Formal rule)

To the rectangles of two different colours (green, orange) we add a white rectangle. By means of the various pictorial presentations the students find out that now the number of the different houses is: $3 \cdot 3 = 9$.

To the triangles of three different colours (red, blue, yellow) we then add a brown triangle. Based on the pictorial presentation the students write down a calculation with which they calculate the number of all the different houses: $3 \cdot 4 = 12$.

By adding rectangles and triangles of different colours the students arrive at the multiplication rule of combinatorics. If the composed selection is such that we first choose among m possibilities and then, independently of the first selection, among n possibilities, then the number of all possibilities is: $m \cdot n$.

IV. Using the developed idea in new situations

The notion of the multiplication rule of combinatorics is used in new problem situations (for example in problem situations from combinatorics and probability etc.).

Conclusion

The described model how to develop the notion of the multiplication rule of combinatorics was represented by steps for better understanding. But it is obvious that the steps are closely linked with each other and it is difficult to take them separately. They do not always follow each other in a strict order and some of them can sometimes be even omitted (Kokol-Voljč, 1996). That is why we will not separate steps when introducing new notions from combinatorics. We will present the forming of these concepts as a whole process. Pupils at the

beginning of education in primary school usually do not go through all steps. They reach the first five. It depends on pupil's abilities and difficulty of combinatorial situation. As there are mixed ability classes in primary schools, it is necessary to consider the teaching of combinatorics carefully and to stress the individualization and differentiation of the procedure and demands.

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NATIONAL CURRICULUM FRAMEWORK FOR PRIMARY MATHEMATICS EDUCATION - - EUROPEAN EXPERIENCES AND TRENDS

Aleksandra Čižmešija[1]

Abstract. *In this talk we present results of a research study on national curriculum framework documents for primary education in 11 European countries, with a special consideration given to primary mathematics syllabi. We analyzed the national curriculum frameworks of the following countries: Austria, Finland, Ireland, Hungary, the Netherlands, Norway, Germany (Nordrhein - Westfalen), Slovenia, Sweden, England and Scotland. These particular countries were selected to represent European countries with developed and successful educational systems (Scandinavian and Anglosaxon countries), countries whose educational systems had significant influence to the Croatian educational system during past ages (Germany, Austria), as well as neighbouring transition countries with similar educational background to Croatia (Slovenia, Hungary). Since the analyzed documents were published mostly after the year 2000, their comparative analysis provides us to identify actual trends in primary mathematics education in Europe and to compare these trends with the current Croatian subject syllabus and its implementation. The aim of our analysis was to identify similarities and common elements of all studied syllabi, as well as their particularities, and to determine to which extent they match the corresponding current Croatian documents. In our opinion, recent European experiences can contribute developing the new Croatian national curriculum framework for primary education. Moreover, they can foster improvement of mathematics teaching in Croatian schools. The research study was done within the MSES research project 0100500* **Evaluation of syllabi and development of curriculum model for compulsory education, Centre for Educational Research and Development, Institute for Social Research, Zagreb (senior researcher: Branislava Baranović, Ph.D).**

Key words: *national curriculum, mathematics education.*

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DYNAMIC MATHEMATICS CLASS AND THE SMART BOARD
(Poster)

Saša Duka[2], *Damir Tomić*[3]

Abstract. *A few applicable examples of using the smart board in mathematics class with lower primary students will be demonstrated at the conference. Using the smart board in the mathematics class motivates pupils for active participation in the educative process and makes the class more interesting.*

Key words: *mathematics class, the smart board*

(translated by Daniela Dorčak)

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THE DYSCALCULIC CHILD, MATHEMATICS AND TEACHERSTUDY STUDENT'S (Poster)

Lidija Goljevački[1], Aleksandra Krampač – Grljušić[2]

Abstract. *According to a poll which was conducted within the framework of the course Teaching Mathematics among undergraduate ABDs enrolled in teacher study programmes in academic year 2003/2004, prior to their employment students are least confident about detecting children with special needs, designing adjusted programmes, and teaching children with special needs. The paper deals with possibilities and opportunities of gaining confidence and competence with students enrolled in teacher study programmes aimed at successful integration of children with dyscalculia into mathematics classes.*

Key words: *dyscalculia, children with dyscalculia, competence of students enrolled in teacher study programmes, confidence of students enrolled in teacher study programmes.*

1. What is dyscalculia?

Dyscalculia is a set of specific disabilities that affect a child's ability to learn mathematics. They can occur in all or just certain areas of mathematics in children across the whole IQ range. Dyscalculia can affect a child's ability to remember mathematical facts as well as their reasoning of time and money. Dyscalculia cannot be prevented but we can help those children to develop needed skills, which will help them to cope with those problems. Therefore, the most important thing is to detect it as early as possible.

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2. Detected pupils in the primary schools with special needs in osječko-baranjska county in the school year 2006/2007

According to the recent data from January 2007 in Osječko-baranjska county, there are 727 pupils with special needs, who have been integrated into regular classes. A special school model has been developed for those pupils. There are also 95 pupils, who are only partially integrated in special classes. Table 1 shows that in the primary schools of Osječko-baranjska county, 2.799% of the total population are detected with special teaching needs, which is under the world's estimate 10% of the school population.

Table 1. Data about the pupils with special teaching needs in Osječko-baranjska county in the school year 2006/2007.

Area	Number of schools	Number of classes	Number of pupils	School population
Osijek	20	396	9223	224
Osijek – surroundings	9	174	2899	91
Baranja region	11	215	3446	119
Donji Miholjac region	4	94	1753	48
Đakovo region	14	282	5488	150
Našice region	6	161	3250	114
Valpovo region	6	156	3290	76
TOTAL	70	1451	29358	822

The research carried out among the teachers in Osijek primary schools (Pavleković and others, 2007) shows that there are more pupils who need adapted or special programs when learning mathematics than the number of pupils detected. The teachers involved in the process of teaching mathematics have mentioned that one of the main causes is the lack of psychologists and special-education teachers (therapists) in their schools.

The students at the Faculty of Teacher Training cannot achieve the important competence and confidence for such work with pupils with special needs because of the lack of specially trained teachers' support in schools.

3. How to recognize the dyscalculic child

- adopts the idea of a number later than an average child
- understands with difficulty the separation of the whole into the parts
- shows difficulties in making new entities
- has difficulty with the usage of numbers when reading, writing or calculating
- has a mirror image of numbers (reads number 6 instead of number 9 and vice versa)
- shows confusion when reading or writing multi-digit numbers and/or has a mirror image (example the number 43 reads as 34)
- has visual difficulties with calculation signs, confusing the sign "+" with the sign "-" and therefore misunderstands the mathematical operation
- repeats the same number of the same operation several times (if the pupils added in the first exercise, they will do the same operation in the rest on the same page no matter if the mathematical operations have changed in the next exercise)
- has difficulty in remembering and recognizing the numerical data (example: they will not recognize the phone number 580 042; unless it is written in this way: 58 00 42)
- replaces one number with another, even those which do not have the similar shape
- usually writes the spoken number incorrectly
- are slower than their peers
- has difficulties in adding correctly
- omits some steps in solving mathematical exercises

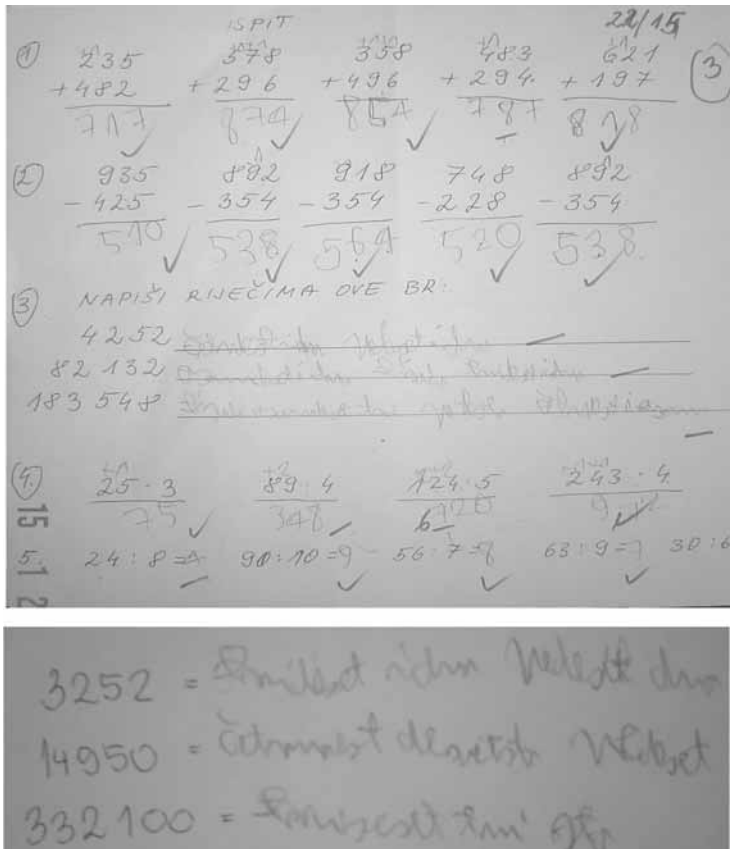
4. How to teach the dyscalculic child

- to teach and examine them orally rather than in a written form
- to use their already known experiences (students' own experiences), exact examples, experiments when teaching
- to use different stimuli – sight, hearing or touch when teaching new lessons
- to avoid pupil's reading aloud

- to use pre-teaching in accordance with their parents (child's learning in advance)
- to check if the pupil has understood the content and definition
- to use the printed text and to avoid the usage of handwritten texts
- to enlarge the space between the numbers and rows in a text
- to bold the letters or digits whenever it is appropriate
- to avoid underlining because it can lead to the visual connection of the letters
- to align the left margin
- to divide the text into smaller paragraphs; to use the hints or numerical orderings in separate rows
- to mark clearly the important parts of the texts in their students' books (definitions, procedures, rules)
- to use the hints and not the whole texts when writing on the blackboard
- to separate clearly the exercises with the same mathematical operation
- to check if the pupil has understood the instructions when being examined in a written form
- the teacher or some pupil-helper can read the exercise to the dyscalculic child when needed
- to write fewer exercises on one page
- to write the exercises from the easiest to the most difficult ones
- to mark the sub-exercises which lead to the solution in more complex exercises
- to limit the time when solving an exercise
- to praise and reward good work and nice behaviour
- to evaluate the motivation and their activities during the lessons; the mark itself should be motivating

5. How to develop the competence and confidence of the students of the teaching studies for active involvement of the dyscalculic child into the teaching process

The experiences show that the students can achieve confidence and competence for successful involvement of the dyscalculic child in the teaching process if they continuously work with at least one child during a longer period (at least one school term) in their classes. During that time it is very important that the student is actively involved in all stages of work with those children with special needs. The students should start with the detection and end up with the evaluation of the child's improvement under the constant monitoring of the student supervisor and the expert team of the primary school. The same work includes the students' cooperation with the child's parents.



The work of the eleven-year-old dyscalculic child with additional difficulties in writing – taken from the diploma of Kristina Đapić 2007)

In order to achieve the students' confidence and competence in organizing the teaching process (mathematics), in which the dyscalculic, dyslexic and disgraphic pupils are integrated, it is suggested to develop the strategy of the partner cooperation of the Faculty of Teacher Education with the primary school (pedagogue, psychologist and therapist); where it is normal to integrate the children into regular classes partially as well as completely.

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(translated by Vlasta Svalina)

Dijete s diskalkulijom,

matematika i studenti učiteljskih studija

A. Krampač - Grljušić
L. Goljevački

Učenici oštećenoga sluha

Učenici s diskalkulijom

Učenici oštećenoga vida

Učenici s motoričkim poremećajima

Učenici s disleksijom

Učenici smanjenih intelektualnih sposobnosti i učenici s autizmom

International Scientific Colloquium
MATHEMATICS AND CHILDREN
(How to teach and learn mathematics)

Osijek, April 13, 2007

IS THE LANGUAGE OF MATHEMATICS DIFFICULT? (The level of technical anguage use among teacher training college students)

Éva Kopasz[1]

Abstract. *In this study we will analyse the problems of language use, both mathematical language and mother tongue, in everyday life, in the scientific world and during comprehension development. We will focus on the level of comprehension and use of some frequent algebraic expressions among training school students. We will also examine whether students are able to find he right expression to describe relationships and logical connections, whether they think or they only rely on previous memories of exercised knowledge.*

Key word: *mathematics teaching, mathematics language.*

1. Introduction

In *Magyar Szemle* [Hungarian Review] Gyula Kodolányi complains that "the language of Mathematics displaces the language of words," and the language of this century will be the language of Mathematics. This statement draws a gloomy picture of the language of Mathematics; on the other hand, mathematicians believe that the lack of mother tongue competence results in difficulties understanding mathematical problems. Our experience is that only those students are capable of comprehending the language of mathematics who have a good command of their own mother tongue.

In mathematics there are some simplifying procedures; for example the use of symbols instead words to assist quicker description. Some simplifying aspirations however can negatively influence problem solving. For example students

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tend to use the collective word "result" instead of "sum", "difference", "product" and "quotient". The lack of knowledge of the abovementioned words might result in faulty solutions. We do not agree with these simplifying trends, partly because they endanger the colourful nature of the mother tongue, and partly because students want to simplify even when it leads to serious misinterpretation of the problem.

Unfortunately we experience in our everyday lives that spoken and written language has diminished, which result in the misinterpretation of words. Short forms are typical of young people who communicate with each other in the form of shortened words in text messages, and we are worried that these short forms will penetrate spoken language, too. Henceforth, we believe we should put more emphasis on the mother tongue and mathematical language use of teachers of the future.

The process of ideation or making concepts in the beginning of primary school studies is very closely linked to language. In his study *Gondolkodás és beszéd* [Thought and Language] Vigotsky discusses the process of making everyday and scientific concepts, their development and their interrelationship in detail. His experiments proved that through adequate own experience and a well structured syllabus *"the development of scientific concepts precede the development of spontaneous concepts"* (p. 206).

Scientific concepts occupy a higher level of awareness than everyday concepts. Moreover, he concludes that "the transmission of conclusions deriving from everyday concepts to scientific concepts is uneven". Transmission is obstructed by the fact that the content of scientific concepts does not overlap or only partly overlaps that of everyday concepts, despite being paronyms. Vigotsky often quoted the Russian author, Tolstoy, whose conviction was "words hardly ever cause problems in understanding… the problem is when the concept the word refers to is missing."(p. 210).

Technical vocabulary is the result of long evolution.„Using terminology and the consistent use of the situation the word refers to is difficult." (Szendrei, p. 399)

The language of mathematics is very exact; definitions only contain the most relevant words and expressions. Mathematical definitions are henceforth difficult to understand, more difficult than a half page long elaboration. Longer texts however are more problematical to memorise precisely, and it is to be feared that important conditions are missed out. The same applies to the wording

of mathematical problems: if they are given briefly then they are difficult to understand, and conjunctions, suffixes and diacritic marks are of greater importance than in any other subjects. However, due to misunderstanding or the lack of understanding the problem cannot be solved or can give more solutions. If students on the other hand were given long problem descriptions, they would run away.

Definitions are only effective if students are already familiar with the meanings of the different words.

"Only by means of definitions unfamiliar concepts cannot be introduced, this is only possible by the abundance of adequate examples." (Skemp, 1971)

This principle is often violated during mathematical education; even textbooks make such mistakes. New concepts are vague without the great quantity of examples, and this will result in comprehension problems. We often face this problem in higher education.

In this paper we will examine what language problems do students of the teacher training college of Baja have in elaborating on mathematical concepts, definitions and logical connections.

2. Hypothesis

Problems of students face in the field of mathematics are of a linguistic origin. They use some words in their everyday meanings, and these often contradict the mathematical meaning. Concepts behind the words are not clear, and imprecision makes the learning of mathematics difficult. Targeted development can improve problem solving skills (deriving from insufficient language competence).

3. Background

We have been monitoring mathematical knowledge, mistakes and comprehension problems of our students. On the basis of previous observations we compiled a list of mathematical problems and the mistakes students often make. We planned a comprehensive research among first year students. As a first step, students were given a test in November 2005, and we wish to publish the results in this paper. The test was written by 49 first year teacher training students. The test consisted of three types of questions:

a) those key-words were to be chosen, which make the statements true or false;

b) multiple choice questions (one solution, three distracters);

c) logical connections were to be filled in to make it a true statement.

Out of the 15 questions of the test, three were picked up for demonstration.

Later student-teacher conversations were taped, and we examined how students can formulate questions relevant to a specific problem, and whether they can correct their insufficient knowledge on the basis of these questions and the answers given to them.

4. Evaluation of the test

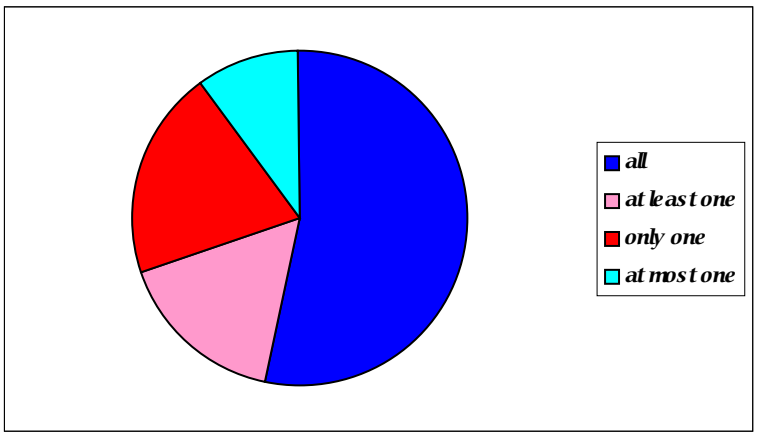
4.1. Mark the answer which makes the statement true.

Products can be divided in a way that .....of its value(s) is/are divided
a. all
b. at least one
c. only one
d. at most one

Our aim was to learn if students,

- are aware of the meanings of at least, only and at most;
- apply wrong analogies during the solution process

Results are reflected in the following pie chart.



We conclude that for most students "at least" and "at most" do not convey meanings. They believe that they are only redundant features, and they only focus on words following them. They do not make an effort to find synonyms for these words since these words are used imprecisely in everyday speech, too. For example a sports commentator, if he considers the match boring, concludes that the only event worth mentioning is that the referee fell. At least the audience had some fun. The word "at least" here means that this was the only event worth mentioning. However, experienced mathematicians would have used the word "at most" to convey the meaning more precisely.

Sums can only be divided if both parts are divided. Results show that students applied this analogy on products, too. This is a typical analogical mistake, which is quite frequent during solving maths problems.

4.2. Addition is associative (sortable). What does the statement mean?

a. Brackets are interchangeable and the sum remains the same.

b. Parts are commutative, brackets are interchangeable, and the sum remains the same.

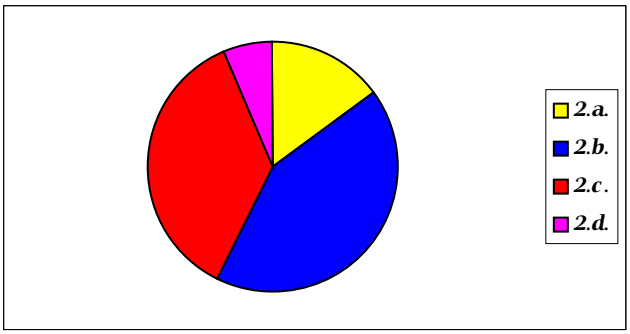
c. Parts can be bracketed provided that the order remains the same.

d. Parts can be added in any order, the sum remains the same.

Our aim was to check if students

- know the exact name of concepts;
- recognise logical connections.

Results are reflected in the following pie chart:



The written test results echo oral test results. Primary school students "learn" by the end of class four that addition is commutative, and the sum remains

the same. We can add first the second value to the first, and we can add the 3rd value to the sum later or we can add the sum of the second and the third values to value one. In the first four classes of primary school it is especially important in oral counting (for example to get full decimals as a partial sum). Later we do not differentiate between these two features of addition so strictly, we apply them together as the problem requires. This way these features will help and not obstruct operations. Grouping alone does not involve changing the order. In mathematics there are operations that are associative but not commutative. For students on the other hand these features are interrelated and do not exist independently of each other.

The word "grouping" is used in another meaning in mathematics and in everyday life, as a synonym for set.

4.3. Complete the following sentence to get a true statement.

a. The quotient does not change if ……..

b. The quotient doubles if ……..

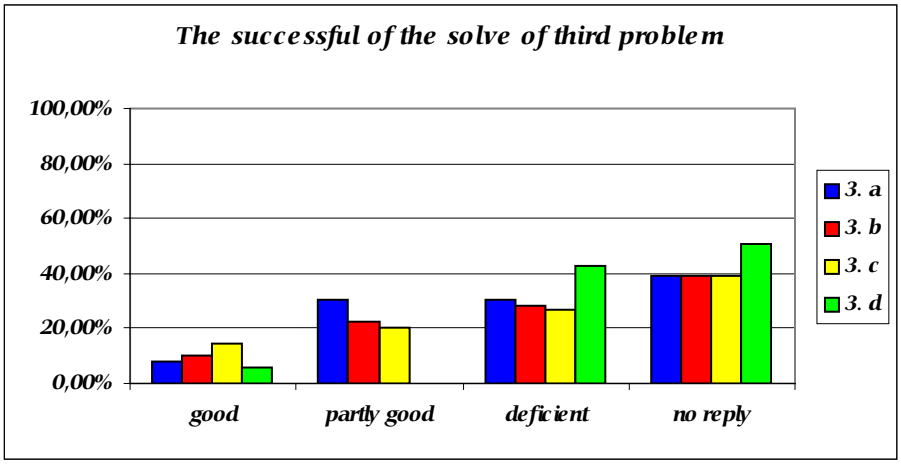
c. The quotient decreases into half if……

d. The quotient increases by 2 if ……..

Our aim with the task was to find out

- if students know these correlations;
- how precisely they can express themselves in the language of mathematics

Results are reflected in the following pie chart.



This task was left out by a lot of students, for different reasons. Students might not have time for that, or they might have left it out deliberately because it was not a multiple choice question or they simply do not like reading.

"Good" answer means precise answer that includes giving abstract correlation. "Partly good" answers are those where students gave only one possible answer for questions *b*, *c*, *d* and gave a unique solution for question *a*. For example, *"we divide and multiply by 2"*.

In case *a* they typically used a preposition [in the Hungarian language a suffix] that should have been used with addition and subtraction. It is a typical mistake, and indicates an erroneous analogy: *"If I increase or decrease the dividend and the divider by the same amount, the quotient does not change"*.

It is obvious from answers that students ignore the notion of zero. This originates from everyday speech, where we do not start with zero when we count. The majority of students made this mistake when they wrote: "I multiply both of them by the same number". Sentences like these are not altogether wrong since they are correct except for one case (dividing by zero).

Unfortunately there was only one student who "managed to" divide by zero: the quotient does not change if *"I divide by zero"*.

Most students wrote that if they increase a number and decrease another one the quotient does not change. This indicates that students mix operational characteristics and the correlations between the four base operations.

In case *b* some answers suggested that some students are not aware of the difference between the dividend and the divider, and the majority of mistakes originated from imprecise use of these concepts. Another typical mistake was that they gave partial solutions, like *"The quotient doubles if I double the dividend."* However this is only true if the divider remains the same.

Typical mistakes in the answers:

I multiply it by 2.
I multiply both of them by 2.
I multiple one value by two.
I square the values.

Questions *b* and *c* were complementary questions, and this was reflected in the answers, too. Those students who managed to complete sentence *b* correctly, gave a good answer for sentence *c*.

Question *d* proved to be most difficult because students had not learnt the formula. Only three correct answers were given, the quotient increases by two if "*I increase the dividend by the doubled divider*".

Although it was not stated that the divider will not change meanwhile, it was a good solution compared to the others. Among incorrect answers there were more references to doubling, and dividing by two. These answers should be appreciated since similar expressions should have been used in previous answers – instead of increasing or decreasing by two. Previous answers (to b or c) suggest that students stored these incorrect notions in their long term memory, and retrieved the information from there. In the new situation however they gave the correlation more precisely, they thought about the answer, and did not only rely on their memory.

5. Conclusions

The hypothesis cannot yet be fully justified since the evaluation of taped discussions is in process.

However, on the basis of the written test, the following conclusions can be made:

• Students memorise too many definitions without understanding their meanings; they do not feel the necessity of giving conditions; henceforth they are only capable of giving partial solutions.

• Mistakes in mathematics are partly of linguistic origin. During discussions most students could correct previous answers, and they realised their mistakes.

• I we develop critical sense in students we might get positive results regarding comprehension and composition skills.

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ASSESSMENT AND EVALUATION IN MATHEMATICS EDUCATION

Željka Milin Šipuš[2]

Abstract. *Present changes in education system in Croatia, beside changes in the national curriculum, involve changes in assessment and evaluation of learning outcomes. The system of external evaluation has been introduced: the system of national exams (for the secondary and the primary level) and the state baccalaureate (matura). In this talk, starting points, aims and experiences of the recent national exams in mathematics will be analyzed. Other types of assessment in mathematics will be discussed as well.*

Key words: *assessment, evaluation, mathematics education.*

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ORIGAMI AND MATHEMATICS

Franka Miriam Brückler[1]

Abstract. *Origami is the traditional Japanese art of paperfolding. Origami models can be used for visualisation of geometric figures and polyhedra, but also to develop the mathematical way of thinking. Origami is easily incorporated in math curricula on all levels of education.*

Children can already at an early age encounter geometry through selfmade origami models instead of the usual already finished, solid ones. As they grow older they are able to fold more complicated models and explore various properties of polyhedra, e.g. symmetry. Even the folding of non-mathematical models can help develop mathematical abilities; for example, after making an origami table the natural question arises: what size should be the paper from which one wants to fold a chair to fit with the table?

Another aspect of origami enhances standard lessons about ruler-and-compass constructions. Origami axioms enable us to make some constructions, like the duplication of a cube, that are impossible to make with ruler and compass. The reason is that ruler-and-compass constructions are geometric equivalents of solutions of quadratic equations, and origami constructions correspond to cubic equations. An additional benefit is that origami constructions are carried out by folding instead of making them in the head or by drawing, This active aspect makes understanding and following the sequence of construction steps easier, and has the didactic benefit of developing deductive reasoning.

Key words: *mathematics teaching, origami, geometrics construction.*

Origami is a well-known japanese art of paperfolding (*ori* = folding, *kami* = paper). The best-known origami objects are various animals folded from one sheet of paper. Even such, apparently nonmathematical, objects are closely related to geometry: after unfolding of the figure one can discover a pattern made

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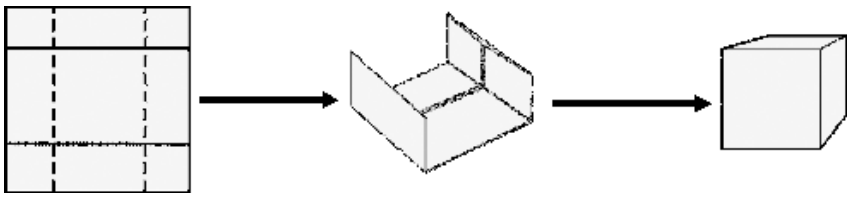
up from polygons bounded by folding lines. Also, the whole object often displays some kind of symmetry. The most important connections between mathematics and origami can be classified as follows:

1. Origami models of polygons and polyhedra;
2. Axiomatic approach analogous to ruler-and-compass constructions, but with some additional possibilities;
3. Analysis of requiered dimensions for folding models o a specific desired size;
4. Connections to higher mathematics, in particular topology and graph theory.

Origami models of polygons and polyhedra

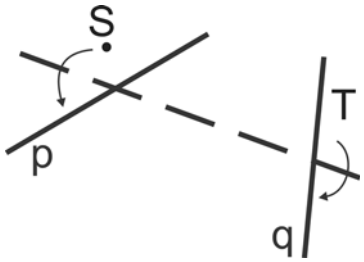
The models range from quite simple ones, which are suitable even for smaller children (thus developing their space sense and motoric abbilities), to very complicated ones requiring much patience and skill. Origami models of polygons and polyhedra can be divided into the ones foldable froma s single sheet of paper (these are more rare and usually they are models of polygons), and models made from several sheets of paper (modular origami).

Example: A cube can be made from six square sheets of paper folded horizontally and vertically to the midlle (picture below, left; full lineas represent mountain folds, and dashed lines represent valley folds). The six pieces („modules") folded to the form shown in the centre picture below and then put together so that the flaps remain inside.



Geometric origami constructions

Traditional education usually involves teaching and learning ruler-and-compass constructions. One of the main objectives of this is to develop deductive reasoning. In the beginning 1990ies origami axioms were introduced (six Huzita axioms), and they have the same mathematical and didactic use as the classic



Euclidean axioms for ruler-and-compass constructions. Additionaly, origami axioms enable us to do some constructions that are impossible to achieve with the ruler and compass. Essentially, ruler-and-compass constructions enable us to solve quadratic equations, and origami constructions can additionally solve cubic equations. korist kao i klasični aksiomi za konstrukcije ravnalom i šestarom, ali imaju i neke dodatne prednosti. In particular, with origami constructions one can solve the cube duplication problem (i.e. the construction of the cube root of 2 if a unit length is given) and the problem of the trisection of a general angle.

Example: The sixth Huzita axiom postulates that „*for two points and two lines there is a folding line such that by folding each point falls on one of the lines*" (picture on the right). Analysis shows that this axiom constructs a tangent on two parabolas with given focuses and directrices, and that corresponds to solving the corresponding analytical cubic equation.

Other connections

Origami can be used for inventive elementary math problems.

Example: If you have already folded a table, how big should the paper be to fold a corresponding chair?

Primjer: If you fold a square paper so that all four vertices fall to the middle, show that you have folded another square and find the relative area to the starting square! If you fold the vertices from the center back to the midpoints of the sides of the square you have folded, check that you'll get another square in the middle and compare its area to the area of the beginning square.

Origami is also connected to higher mathematics, especially with graph theory. The best-known connections consists in the following fact: If a flat origami model is unfolded, the map resulting from the folding pattern is colorable with two colors. This is a consequence of the corresponding graph (vertices are intersections of folding lines, and edges are the folding lines) is an Eulerian graph i.e. all vertices are of an even degree.

Let us finally mention that using origami in the classroom has many didactic advantages: it develops problem solving skills, precise usage of mathe-

matical terminology, usage of fractions, introduction of terms connected to angles, areas, volume, congruence, parallel and perpendicular lines, conics etc., development of deductive reasoning and logic, development of cooperation, foreseeing of outcomes, motoric abilities, aesthetic understanding, visualisation skills ... Another good thing about origami is that open problems in origami mathematics are still frequent, and in contrast to most open problems in mathematics, these are mostly easy to explain, and even nonprofessional could contribute important ideas for their solution. This makes origami an ideal method for introducting the creative aspect of the science of mathematics.

A good web-page for starting to explore the connections between mathematics and origami is Origami & Math, *http://www.paperfolding.com/math/*

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ATTITUDES OF THE STUDENTS OF TEACHING STUDIES TOWARDS MATHEMATICS

Irena Mišurac Zorica[1]

Abstract. *Faced with students' unsatisfactory results in mathematics skills on all levels of education we should constantly reassess the parameters which can influence the process of their acquisition. Teacher in the first four grades is one of the most important components in that process and influences the students to a very large extent. At this level students are presented with the basic mathematics contents which are a foundation necessary for acquiring more advanced concepts in further education and make this period in mathematics education all the more important. Teacher's influence is exercised through ways of teaching and communicating with students, and even more through non verbal communication through which he/she conveys his/her attitudes, associations, as well as fears either voluntarily or involuntarily. Therefore, in this paper we decided to conduct a research into students' – prospective teachers' attitudes towards mathematics. Since they will be teaching mathematics to the youngest segment of the student population it is clear that their attitudes will implicitly or explicitly influence their students' results.*

The starting hypothesis was that the teacher will teach mathematics better if he/she has a more positive attitude towards it. In other words, the person who has a negative attitude towards mathematics or has fears about it will not be successful in teaching it. Students' attitudes were collected through an anonymous survey on a sample of 150 students of the 3rd and the 4th year all taking courses in teaching young learners at the Faculty of Philosophy, University of Split. The results obtained from the survey indicate that on the whole the students have a good, but a not good enough attitude towards mathematics. Still, a large number

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of students show a fear of mathematics, and the associations they have with the word mathematics are either neutral of negative. The respondents also believe that only some rare contents acquired through studying will be of use in their future jobs. We were surprised to find out that a large number of students did not like mathematics in their high school education. Out of all the subjects they will teach in senior classes of elementary school the least number of students has chosen mathematics. The obtained results are unsatisfactory and set a task aimed at changing such attitudes of the prospective teachers.

Key words: *mathematics education, competence of teacher study students.*

PARTNERSHIP AMONG FACULTIES, SCHOOLS AND FAMILIES FOR THE IMPROVEMENT OF MATHEMATICS EDUCATION OF THE GIFTED CHILDREN

(Poster)

Ksenija Moguš[1] i Silvija Mihaljević[2]

Abstract. *In the year 2003 Small School of Mathematics (headed by M. Pavleković) was established within the framework of the project Methodology of Teaching Mathematics approved by the Ministry of Science, Education and Sports. The School was organized to develop self-confidence and competence of teacher education students for mathematically gifted children. The partnership among faculties, schools and families for improvement of mathematics education of the gifted children within the Small School of Mathematics was announced on the Congress of Mathematics Education Teachers in Zagreb, July 2004. (Goljevački, Moguš,2004).*

Key words: *mathematics education, mathematically gifted children, popularization of mathematics, partnership among faculties, schools and families.*

The students of *Small School of Mathematics* are fourth-graders from Osijek primary schools. During the school year teacher education students work with the mentioned school population two periods a week using the method of guided discovery learning. This education has been planned and monitored by university teachers.

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Implementing the Bologna process in Croatian institutions of higher education a programme for the subject *Mathematics and gifted children* was created by M. Pavleković and approved in the year 2004.The programme was designed for teacher education students.

Apart from these students, the university teachers working in Departments of Mathematics, Information Sciences and Psychology have also been included in the programme in the last four years. The partnership among faculties, schools and families implemented in the *Small School of Mathematics* stresses the process of thourough education of teacher education students working with mathematically gifted children.

The special feature of this model is reflected in the closest and constant cooperation between university teachers and their students with the primary school teachers, educators and psychologists that take care of the above mentioned fourth-graders. Students and maths education teachers communicate with the parents of the gifted children in an oral and written form.

The end of the academic year means the end of all educational activities at *Small School of Mathematics* in that year. The end of the year is traditionally accompanied by the final maths quiz made up by the teacher education students and the group of university teachers. The gifted primary students, their teachers and parents (brothers, sisters,sometimes grandparents) take part in the quiz.

At the end of this academic year the fourth final quiz is going to take place at the end of which the fourth-graders will be given certificates on regular attending the *School* and some symbolic prizes as well.

The benefit from the partnership among faculties, schools and families for improvement of mathematics education of the gifted children is manifold and can be noticed as:

1. stimulation of regular identification of mathematically gifted primary school population;
2. stimulation of the gifted students to use their talents outside the school environment (teachers, parents);
3. creation of an appropriate environment where teacher education students can develop their self-confidence and competence to educate mathematically gifted children;

4. setting the required precondition for establishing the expert teams whose task is to identify primary school gifted students and monitor their learning progress;
5. popularization of mathematics;
6. the opportunity for parents to invest into their childrens' benefit and the benefit of our whole society.

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(translated by Jasenka Vincetić)

Partnerstvo

fakulteta, škola i obitelji za napredak matematičke edukacije darovite djece

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UČITELJSKI FAKULTET
SVEUČILIŠTE J.J.STROSSMAYERA U OSIJEKU

EXPERT SYSTEM FOR DETECTING A CHILD'S GIFT IN MATHEMATICS

Margita Pavleković,[1] Marijana Zekić-Sušac[2], Ivana Đurđević[3]

Abstract. *According to Johnson (2000) mathematically gifted children have needs that differ from those of other children. In order to pay special attention to gifted children, teachers usually use mathematical competencies as the only criterion for detecting such pupils. However, it is also important to include other components while deciding about the giftedness in mathematics, such as cognitive components of gift, personal components that contribute the gift development, strategies of learning and exercising, as well as some environmental factors. This paper aims to identify five main components of gift in mathematics based on previous research, and to create an intelligent expert system that will support teachers in detecting gifted children in the fourth year of elementary school. The systems is based on decision rules and forward chaining inference engine used to classify each pupil into one of the four output categories: (1) presumably gifted child in mathematics, (2) child with a special interest in mathematics, (3) child with average mathematical competencies, and (4) child with insufficiently developed mathematical competencies. An empirical research is conducted including 247 pupils, age 10, selected from different elementary schools in Osijek, Croatia. Both, teacher and the expert system assessments, are obtained for each of the pupils. The paper also compares the decisions of the expert system with the teachers' assessments of pupils' gift. The results show a significant difference among teacher and system assessments, and that more pupils are identified as potentially gifted*

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by the system. It implies that an intelligent system leads teacher to consider other components of giftedness in mathematics, and therefore can serve as an efficient methodological tool in detecting gifted children as well as in educating teachers.

Key words: *components of gift, mathematics, intelligent expert system, if-then rules, estimation of gift, t-test*

1. Introduction

The importance of recognizing a child's potential gift for mathematics is emphasized by many authors (Johnson, 2005). The lack of exact definition of the term *gifted child (for mathematics)* causes difficulties in determining gifted children. There are various approaches to giftedness in literature (Vlahović-Štetić, 2002) among which are the approaches oriented to: genetic factors (Terman, Oden, 1959), cognitive models (Sterberg, 2001), achievement (Renzuli, 1986), as well as the system approach (Tannenbaum, 1983). In order to pay special attention to gifted children, teachers usually use only mathematical competencies as criterion for determining a child's gift. However, it is also important to include other components while deciding about giftedness in mathematics. In this paper, we analyze the ways and reasons based on which a teacher estimates a child in the fourth grade of elementary school as a potentially gifted for mathematics and identify five basic components of gift in mathematics. Key variables (attributes) and *if-then* rules are defined for each component as a basis for developing a knowledge base for intelligent expert system that can serve as a decision support system for teachers in determining pupils' gift in mathematics at the fourth grade of elementary school. The system is based on decision rules and forward chaining inference engine, used to classify each pupil into one of the four categories of gift.

An empirical research is conducted at the end of 2006, including 247 pupils, age 10 (fourth grade), at 10 classes of different elementary schools in Osijek.[4] Both, teacher assessment and intelligent system assessment are obtained for each student, and the assessments are compared using statistical tests. The aim of the research was to identify reasons for which a child is categorized as gifted, as well as to determine the differences in teachers' and system's assessments.

[4] research is a part of the project *Little school of mathematics* started at the Faculty of Education at the University of Osijek. Project was announced at the Congress of mathematical teachers in Zagreb, in July 2004 (Goljevački, Moguš, 2004), with the aim of rising the quality of educating future teachers.

The rest of the paper contains a review of previous research in that area, followed by the artificial intelligence (AI) methodology used to develop the intelligent expert system, as well as the model variables for determining the potential gift in mathematics. After that the data about the examinees are described, ending with the conclusion and guidelines for future research.

2. Review of previous research

Research in the area of intelligent systems in education were mostly focused on developing tutoring systems that can serve as a support in learning and teaching a specific topic, with the possibility of including multimedia and personalized approach. For example, Stathacopoulou et al. (2005) propose to use the methodology of neural networks and fuzzy logic for advanced student diagnosis process in an intelligent learning system. Their model enables a system to "imitate" teacher in diagnosing student characteristics and in selecting the learning style that suits those characteristics. The system is tested in the area of learning vector construction in physics and mathematics. Results obtained by the system are compared with the recommendations of a group of experienced teachers, showing that the system is able to manage the diagnostic process, especially for marginal cases, where it is difficult even for teacher to bring accurate evaluation of student. Canales et al. (2006) developed an adaptive and intelligent web-based education system (WBES) which takes into account individual student learning requirements and enables the usage of different techniques, learning styles, learning strategies, and ways of interaction. The architecture of their system follows the standards proposed by the IEEE – LTSA (Learning Technology Systems Architecture), according to which the education systems should be structured into five layers: (1) interaction of learner with the environment, (2) learner-related design features, (3) system components, (4) implementation perspectives and priorities, and (5) operational components and interoperability (code, interfaces, protocols).

However, less research attention is given to the area of intelligent systems for detecting children's gift in particular areas such as mathematics. Johnson (2000) emphasizes the importance and need for accurate detection and further development of mathematical gift, as well as for including criteria other than mathematical competencies.

Saito et al. (2007) investigate the influence of collaboration among schools and universities with the school teachers and university faculty members. Their results show the following: (1) joint lesson planning, observation and reflection contribute to the improvement of teaching methodologies, (2) faculty members and teachers observe that students included in collaboration are more participative, (3) the linkage between students and materials, as well as among students is necessary, (4) collaboration resulted in the development of collegiality within schools and between faculty members and teachers.

Generally, previous research implies that there is a great expansion of AI methodology usage in education, primarily in the area of tutoring tools in the last few years. However, the area of determining the giftedness in mathematics should be more investigated and it is necessary to design an intelligent system that will support detection of gifted children.

3. Methodology

From the first appearance of the term *artificial intelligence* as a scientific discipline until today, a number of techniques have been developed with the aim of creating intelligent machines (Russell, Norwig, 2002). Some of those techniques are expert systems, problem solving, machine learning, natural language understanding, speech recognition, pattern recognition, robotics, neural networks, genetic algorithms, intelligent agents, and others. Although the paper is focused on designing an expert system for detecting children's gift in mathematics, it also gives the guidelines for upgrading the system with other AI techniques, primarily neural networks, in order to classify pupils according to their gift in mathematics.

Expert systems are computer programs able to replace a human expert in decision making process (Mišljenčević, Maršić, 1991). Besides offering an advice for making a decision, such systems are capable to explain decision process by presenting the knowledge that was used by the system while making a decision. For those reasons expert systems belong to so called «white box» methods that are transparent in presenting their way of finding a solution. Expert systems are usually used for problems that have a narrow domain, such as car selection, or stock market trade, or diagnosing heart disease, or similar.

Structure of a standard expert system is presented in Figure 1.

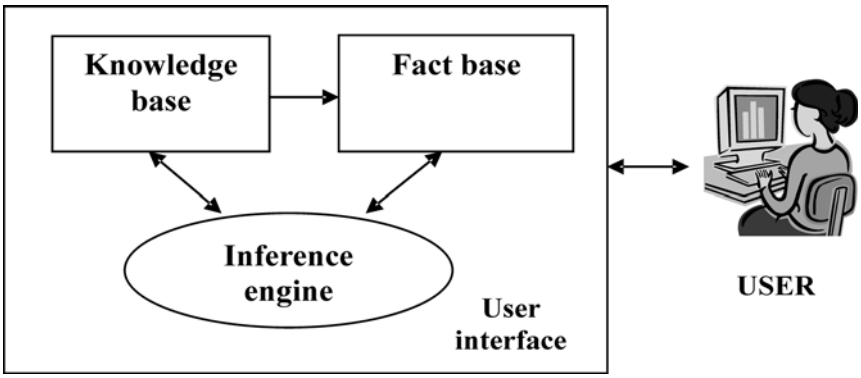


Figure 1. Expert system structure

Knowledge base is the source of knowledge about particular domain acquired from an expert in that area (Čerić et al, 1998). Knowledge can be represented in the form of production rules, semantic networks, predicate logic, etc. Production rules were used in this paper as one of the most frequent form. The base of facts presents a set of facts that describe the problem under consideration (for example, facts can be pupil's grades). Inference engine manages a searching path towards the solution, where the search is conducted by examining facts in the base of facts, as well as knowledge in the knowledge base. User interface enables communication between users and expert system, and it also contains a mechanism for explaining a path used in finding a solution. It is important to build a user-friendly interface in order to enable easy communication of humans with the system.

Knowledge in an expert system represents a set of information "structured to be appropriate for usage in the process of problem solving within a problem domain" (Čerić, Varga, 2004). Among a number of specialized software tools called "expert system shells" that enable knowledge representation and search, we use Exsys Corvid, in which we define variables (i.e. attributes) in the knowledge base, logic blocks and nodes that construct production rules, while forward chaining is used as a search engine (Mišljenčević, Maršić, 1991). Production rules in intelligent systems differ from the rules used in sequential processing in procedural programming, since they consist of (Mišljenčević, Maršić, 1991): data describing the current state of environment, a set of rules in the form: *IF* <condition> *THEN* <action>, and rule interpreters that determine the order of rule execution. Each production rule is defined by a logic relation with possible values of *true (T)* or *false (F)*. For many real problems, in some

occasions it is not possible to determine true or false value of a relation with the certainty of 100%. Therefore, it is possible to introduce a certainty factor or probability whether a condition is satisfied or not.

In order to create the expert system the following steps (phases) were used in the paper:

1. defining the problem to be solved, and possible decision options
2. knowledge base design – defining variables (attributes)
3. defining production rules and evaluating options
4. user interface design
5. expert system usage
6. statistical comparison of assessments made by expert system and teachers

The knowledge base of the expert system is created on the basis of four years of team work and research conducted by the faculty members, students, and teachers at *Little school of mathematics* at Faculty of Education, University of Osijek. The results of that research match with the findings of Saito et al. (2007). During the winter semester 2006/07, an expert in the area of mathematical methodics was working in the collaboration with colleagues, students, teachers, and parents, with a group of pupils from the fourth grade of elementary school (age 10) that had special interest in mathematics. Knowledge acquired from literature, heuristics concerning the methodology of teaching, completed project assignments, as well as pupils' achievements, were the construction threads for creating the expert system knowledge base.

3.1. Defining the problem to be solved by the expert system

Expert system makes the decision about the category of a child's gift (age 10). Possible options of the decision are:

A) **presumably gifted child in mathematics** – the pupil is motivated and supported by environmental factors towards it achievements. Her/his knowledge, skills, and application of mathematics are on the level that overcomes expectations of mathematics curriculum for that age. With the appropriate teaching strategies teacher and mentor enhance and guide the development of pupil's competencies towards the realization of her/his gift. The pupil learns actively, controls its progress and prepares for public assessment of its knowledge and skills, i.e. competition in mathematics.

B) **child with a special interest in mathematics** – the pupil's knowledge, skills and mathematical application is on the level or somewhat above the level of expectations of mathematics curriculum for that age. However, a pupil belonging to this category expresses an extra interest towards mathematics and is also supported by the environment, although she/he is not willing to expose its knowledge and skills to public assessment at competitions in mathematics.

C) **child with average mathematical competencies** – pupil shows no interest for additional practice in mathematics, but her/his achievements are on the level of expectations of mathematics curriculum for that age. Appropriate learning methods are used to systematically enhance the development of pupil's mathematical competencies.

D) **child with insufficiently developed mathematical competencies** – pupil whose knowledge and skills in mathematics show that in order to achieve expected mathematical competencies she/he needs an additional support of parents and environment.

3.2. Knowledge base design – defining variables (attributes)

In the process of defining variables, i.e. attributes that will constitute the expert system knowledge base, it is important to include assessment about mathematical competencies of pupils, cognitive components of gift, personal components that contribute the development of gift, environmental factors, as well as efficiency of active learning and exercising methods that enhance the development of mathematical competencies and possible realization of gift. Each of the five model components is represented by blocks, divided into sub blocks, i.e. groups of different competencies, and finally to variables that constitute production rules. Depending on the importance (i.e. weight) of a particular block for the decision, points are determined for each block. The framework of the knowledge base model (blocks and sub blocks), together with appropriate points, is presented in Figure 2.

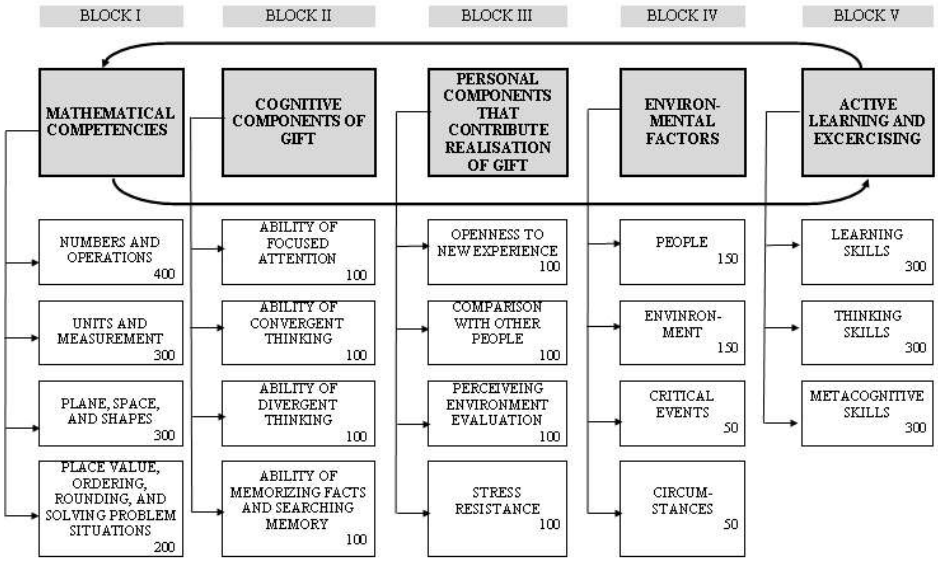


Figure 2. Components of mathematical gift included in the expert system knowledge base together with points representing the weight of a particular component

The block of mathematical competencies (block I) includes four groups of variables in the area of: (a) numbers and counting, (b) units and measurement, (c) plane, space, and shapes, and (d) place value, ordering, rounding, and solving problem situations. In that way, a child's knowledge and skills in counting and measurement, usage of mathematical language and communication, solving problems and modelling, as well as skills of mathematical argumentation are included in the assessment of gift. Each sub block is additionally divided into variables whose values are loaded from the user, i.e. teacher. In the block of cognitive components (block II), the intellectual potential is represented, determined by genetic factors of each pupil. In that block, by adjusting and changing the strategies of active learning and exercising, we examine a pupil's ability of focusing attention, ability of finding a path towards the solution, and ability of fast searching from long-term memory. From personal components that contribute the realization of gift (block III), we observe: openness to new approach of learning, positive image of herself/himself, autonomy (not being afraid to be alone, fulfilled by activities they do, believing that they can influence their success, being persistent in work, taking responsibility and initiative), resistance to stress (perceiving failure as an opportunity for acquiring new experience). In order to determine the giftedness in mathematics it is also important to consider the improvement in active learning and exercising, described in block

V, which includes: learning skills (distinguishing important from unimportant, combining and organizing information in a meaningful structure, selective comparison and connecting new information with existing ones in long-term memory), thinking skills (judgment, comparison, assessment, estimation, evaluation, imagination, discovering and creating new, as well as bringing thoughts into action), and metacognitive skills (planning exercises, "keeping track" of self improvement, regulating its behaviour if it does not give certain results) in the fourth grade of elementary school.

And, last, but equally important, the detection of giftedness in mathematics is also influenced by environmental factors that can affect the development of a potential giftedness towards its realization (block IV). Those environmental factors are: support of teachers (additional courses), support of parents (help in exercising mathematics, financial support), and support of mentor.

3.3. Defining production rules and evaluating options

On the basis of variables mentioned above, logic blocks are created in the form of *if-then* production rules, whose logical values (*true* or *false*) imply appropriate evaluation of options of the expert system decision. Figure 3 presents a part of production rules that form the block *Mathematical competencies,* sub block *Units and measurement*.

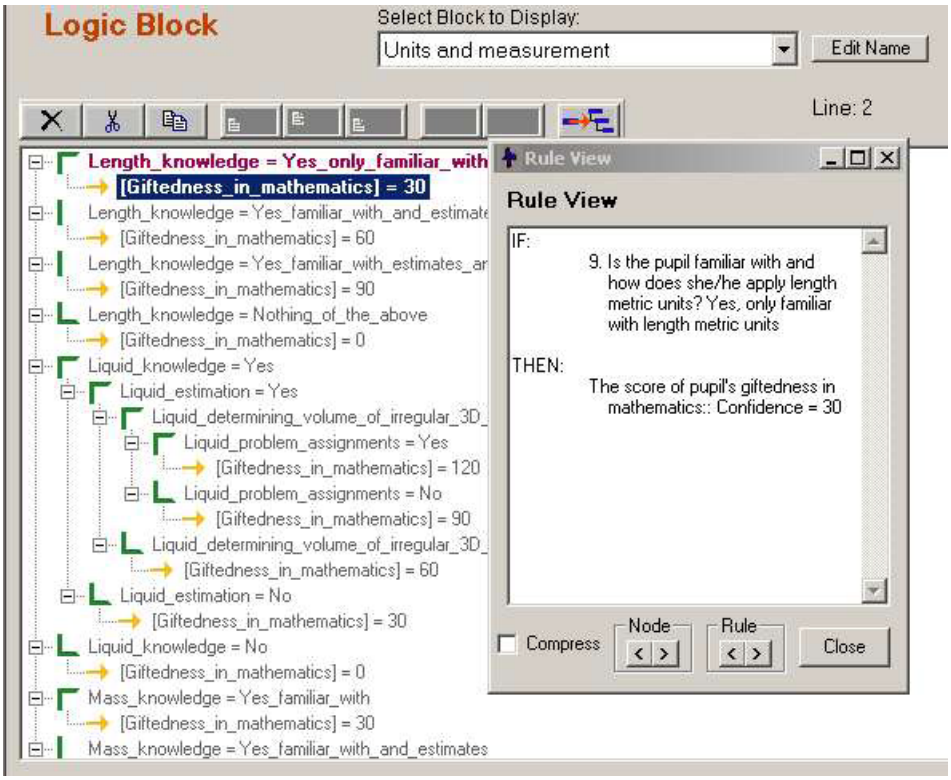


Figure 3. A part of the expert system knowledge base –Mathematical competencies – Units and measurement block

The total knowledge base of the expert systems consists of 250 production rules grouped into five main blocks presented in Figure 2. The process of evaluating options is defined on the basis of heuristics. The method used for searching production rules is the forward chaining, meaning that the search starts from the attribute values at marginal nodes and moves up accumulating points towards the final goal – to make a decision about the category in which the system places a child according to her/his gift in mathematics.

Each examinee x from set of examinees N, $kN = 247$, at i-th node v_i of the program is assigned with exactly w_i points. The final system decision $f(x)$ about the membership of the variable x in one of the categories A, B, C, D that describe a *child's gift in mathematics* explained in details in section 3.1., is generated according to the formula:

$$f(x) = \sum_{i=1}^{61} w_i \tag{1}$$

such that:

$$x \in \begin{cases} A, & 2622 \leq f(x) \leq 3300 \\ B, & 1943 \leq f(x) \leq 2621 \\ C, & 1264 \leq f(x) \leq 1942 \\ D, & 585 \leq f(x) \leq 1263 \end{cases} \tag{2}$$

Categories *A*, *B*, *C*, and *D* are partitive subsets of the set *N* (the union of all four subsets is equal to the set *N*, while the intersection of each two subsets is an empty set).

3.4. User interface design

Using the Exsys Corvid software package, a visual user interface is designed, aimed to conduct communication of the system with a user in two ways: off-line (on a local computer), and on-line through a web interface based on Java runtime technology. Criteria for designing the interface were the following: simple usage, clearness, and availability to the final users through the web. An example of a user interface window is presented in Figure 4. Using the interface, a user enters the values of variables (attributes) that are considered as input values by the system in production rules, and transformed into output values for each decision option.

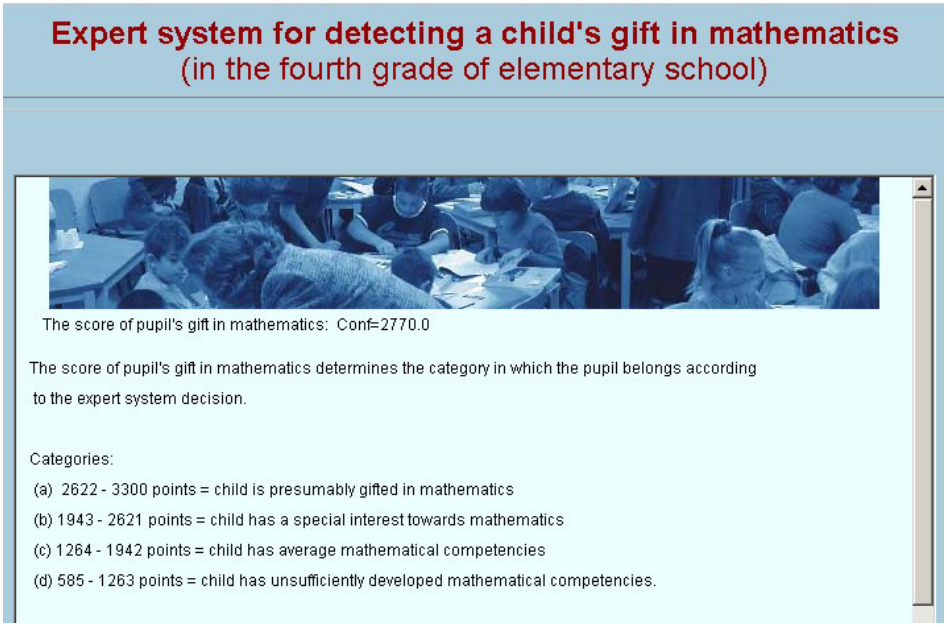


Figure 4. User interface of the expert system – the final screen

3.5. Expert system usage

After testing its formal and logical accuracy, the expert system is used to detect children's gift in mathematics in a survey conducted in ten elementary schools in Osijek.

3.6. Statistical comparison of assessments made by expert system and teachers

On the basis of the conducted survey, a descriptive statistics of the assessments is computed, the correlation coefficients are analyzed, and statistical t-test for dependent samples is used to compare the difference in assessments of teachers and the system.

4. Examinees

The survey included the sample of 247 pupils, age 10, from ten elementary schools in Osijek, Croatia in December 2006. The smallest number of pupils in a class was 17, while the largest number was 30. The sample is stratified since two or three pupils from each of the selected classes with a special interest in

mathematics attended the *Little school of mathematics* at the Faculty of Education in Osijek. Data were collected using assessment lists of the children's mathematical competencies filled out by their teachers. By answering the last survey question a teacher categorizes a child into one of the four categories without knowing the results of the expert system. Therefore, it is possible to examine the differences among teachers' and system's assessments of children's gift in mathematics.

5. Results

5.1. Assessments of gift given by teachers and the expert system

Descriptive statistics of the assessments of gift given by teachers and the expert system is shown in Table 1.

Table 1. Descriptive statistics of assessments by teachers and the expert system

Variable	Mean	Minimum	Maximum	Standard deviation
Gift in mathematics – teacher assessment	2.287	1.000	4.000	0.837
Gift in mathematics – system assessment	2.429	1.000	4.000	1.025

Mean value of the teacher and system assessments denotes that the system in average assigns children's gift into higher category than a teacher, while their standard deviation denotes that there is a larger deviation among the categories of gift estimated by the system, i.e. teachers are more likely to categorize pupils into categories close to each other. The t-test of differences in means indicates that there is a statistically significant difference between mean assessments of teachers and the system ($t= 3.03972$, $p<0.002624$, $df=246$). The Pearson correlation coefficient between the teachers' and system's assessments is 0.3 ($p<0.05$), showing a statistically significant, although not strong, connection among the two assessments.

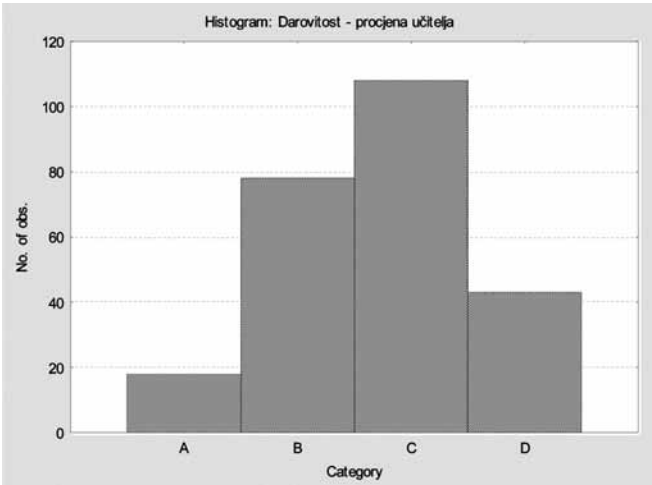
Table 2 shows frequencies of pupils assigned to categories of gift according to assessments of teachers and the system. It is obvious that the system categorize 19.03% of pupils into the highest category A – *presumably gifted child in mathematics*, while teachers categorize a significantly smaller number of pupils into that category (7.29%). The t-test of differences in proportions indicates

that the difference in assessments for that category is statistically significant (p=0.001). Although there are also differences in the number and percentage of pupils categorized into other categories, only the difference of the category C is statistically significant on the 5% level (p=0.0231). The system assigns 34% of pupils into category C – *child with average mathematical competencies*, while teachers assign 43.73% of pupils into that category, implying that teachers tend to categorize most of the pupils into the average group. When category B is observed – *child with a special interest in mathematics* – a larger number of pupils is assigned by teachers than by the system, while the situation is vice versa in the category D – *child with insufficiently developed mathematical competencies*.

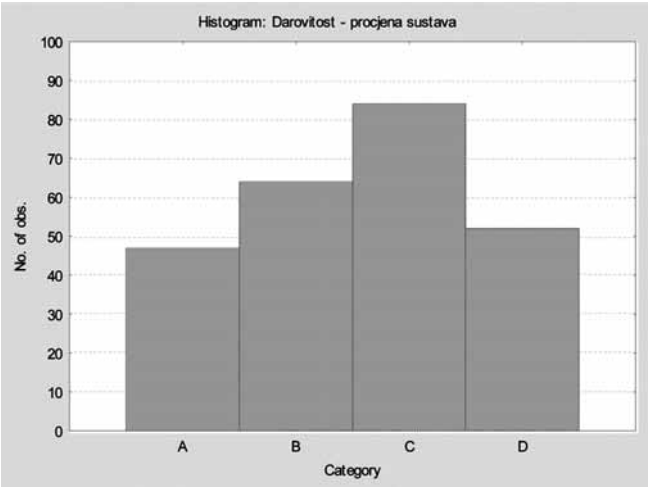
Table 2. Frequencies of pupils assigned to categories of gift according to assessments of teachers and the system

Category	Teacher assessment		System assessment		T-test of differences in proportions
	Number of pupils	%	Number of pupils	%	
a	18	7.287	47	19.028	p=0.001
b	78	31.579	64	25.911	p=0.1382
c	108	43.727	84	34.008	p=0.0231
d	43	17.409	52	21.053	p=0.2577
Total	247	100.000	247	100.000	

Graphical representation of frequencies with the frequency histograms is shown in Figure 5 – a) and b).



b) teacher assessments



b) expert system assessments

Figure 5. Frequency histograms of pupil categories according to a) teacher assessments, and b) expert system assessments

For better insight into the differences among the assessments by teachers and the system, the rate of agreement is computed (see Table 3). Teacher and the expert system agreed in categorizing 131 pupils, yielding the rate of 53.04%.

Table 3. Number and percentage of pupils with a match in assessments by teacher and the system

Description	Number of pupils	%
Existence of match in assessments of teacher and expert system	116	46.964
No match in assessments of teacher and expert system	131	53.036
Total	247	100.000

In order to additionally investigate the agreement in assessments in relation to categories, the confusion matrix presented in Table 4 can be observed.

Table 4. Confusion matrix of the assessments by teachers and the system

	Gift – system assessment				Total number of pupils
Gift – teacher assessment	A	B	B	D	
A	13	3	2	0	18
B	31	31	15	1	78
C	3	29	56	20	108
D	0	1	11	31	43
Total number of pupils	47	64	84	52	247

Values on the diagonal of the confusion matrix present the number of pupils estimated in the same category by a teacher and the system. It can be seen that the largest absolute match is present in category C – *child with average mathematical competencies*, where 56 pupils are assigned into that category by a teacher and the system. The reason for that is also in the largest number of pupils in that category estimated by both estimators. The least number of pupils assigned to the same category is present in category A – *presumably gifted child in mathematics*, where teachers and the system agreed for only 13 pupils. It is interesting to observe the numbers above and below the matrix diagonal, which explain the differences in assessments by category in details. If we look at the data in the first row of Table 4, 3 out of 18 pupils assigned to category A by teachers the system categorized as B, 2 were categorized as C, while none of them was categorized as D. However, 31 out of total number of 78 pupils that is assigned to the category B by teachers, is assigned to the category A by the system, while 15 of them the system assigned to category C, and 1 of them to category D. Similar situation is in the third and the fourth row of the matrix, where it is confirmed that the system assigns a large number of pupils into one category higher than teachers. Data in columns of the confusion matrix show the way in which teachers assessed pupils who are assigned to a certain category by the system.

Table 5. Number and percentage of pupils with a match in assessments by teachers and the system in relation to categories

Category	Number of pupils	%
A	13	9.92%
B	31	23.66%
C	56	42.75%
D	31	23.66%
Total	131	100.00%

Table 5 shows the proportion of each category in the number of pupils that are assigned into the same category by teachers and the system (for 131 pupils that have a match in total). It is obvious that, when teachers and the system agree in their assessments, they assign most of the pupils into the category C (42.75%), while category A – *potentially gifted child in mathematics* consists of 9.92% of the total number of equally categorized pupils.

It can be concluded from the analysis of similarities and differences of gift assessments that there are statistically significant differences of assessments, especially for categories A and C, and that 9.92% of pupils is assigned into the category of children potentially gifted in mathematics both by teachers and the expert system. However, the system assigns more pupils to the category of potentially gifted (19.028%), and also assigns a certain number of pupils into one category higher than teachers do.

6. Conclusion

The paper discusses the reasons for determining a child's gift in mathematics, and differences in assessments of gift given by teachers and the expert system. On the basis of previous research and heuristics, a model for assessing giftedness of children in the fourth grade of elementary schools is suggested, consisting of five main components of gift in mathematics. Besides mathematical competencies, the model includes cognitive components of gift, personal components that contribute realization of gift, environmental factors, as well as the efficiency of active learning and exercising methods that enhance the development of mathematical competencies and possible realization of gift. The key variables are defined for each component of gift, as well as production rules that constitute the knowledge base of the expert system for detecting a child's gift in mathematics. Using the knowledge base and the inference engine, the expert system categorizes a child into one of the four categories of gift. The assessments of children's gift obtained by teachers and the expert system are compared by statistical tests.

The results show that teachers and the expert system agree in their assessments for 53.04% of pupils, and that there are statistically significant differences in assessments, especially for the category of potentially gifted children, and the category of children with average mathematical competencies.

Due to the fact that the expert system, which includes more components of gift in its knowledge base, assigns more children into the category of potentially gifted, it can be concluded that the usage of such system could influence teachers to take other components of gift into consideration when deciding about gift in mathematics. Therefore, the system could be used as an efficient methodological tool in detecting gifted children as well as in educating teachers. Further research in this area could include students of education and psychologists as assessment subjects, and focus on possible methodological improvement of the software tool by including other techniques of artificial intelligence, such as neural networks, genetic algorithms, intelligent agents, and others.

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In Memoriam

Boris Pavković, Full Professor

1931 – 2006

BORIS PAVKOVIĆ
(portrait of a distinguished methodologist and popularizer of mathematics)

Mirko Polonijo[1]

Abstract. *Last year a year-long member of the PMF - Department of Mathematics in Zagreb, university professor dr. sc. Boris Pavković, passed away. He was a great lover of mathematics, geometry in particular, researcher and instructor. Through his scientific, professional, pedagogical and social work he contributed significantly to the development, understanding and popularization of geometry and mathematics in our community.*

Towards the end of the 1970's professor Pavković began lecturing a two-year course in mathematics teaching methodology and thus by means of his knowledge, experience and talent, aided by his teaching and pedagogical instincts, greatly influenced modern structuring and presentation of mathematics teaching methodology at Croatian universities.

As a methodologist and popularizer of mathematics he has left an imprint on the past fourty years of teaching mathematics in our primary and secondary schools. His influence will long remain present through his books and articles, colleagues and coworkers, as well as former students.

Therefore, the entire methodological and popularizing work of professor Pavković calls for and deserves a detailed analysis and overall recognition.

Key words: *mathematics methodology, popularization of mathematics, mathematics education.*

Professor Boris Pavković passed away in Zagreb on June 6, 2006 after a brief and difficult illness. The ceremony took place on June 9 at the Zagreb Crematorium, and the urn was placed on June 13 on Mirogoj. In memory of the dear

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colleague and distinguished professor there was a commemoration on June 28, 2006 at the PMF- Department of Mathematics, University of Zagreb. Life and work of the late professor Pavković, outlined in words of respect and gratitude, was captured in eulogies of professors Pavle Pandžić, Mirko Polonijo, Vladimir Volenec, Sanja Varošanec, Sibe Mardešić and Željka Milin-Šipuš. Additionally anticipated speakers, professors Margita Pavleković and Ivan Kamenarović, due to health problems, sent written eulogies which were read on their behalf.

Professor Boris Pavković was born on November 20, 1931 in Zagreb to father Josip (1904-1977) and mother Hermina, born Petriša (1905-1999). "I come from a functionary family", he wrote in his biography attached to the 1960 job application. He had three brothers, Bruno (born 1935), Branko (1943-1983) and Božidar (1945-1997). After departing he left his beloved behind: wife Marija, daughter Jasna Orešić and granddaughter Sunčana.

He finished elementary school in Čakovec, where he learned Hungarian language, as Čakovec was under Hungarian authority/occupation at the time. In the same town he passed the so-called minor course exam in 1947. Josip Pavković's family returned to Zagreb in the same year and professor Pavković enrolled in V. (boys') grammar school where he also graduated in 1951. In the same year he passed the so-called major course exam. Shortly before being admitted to hospital, on May 25, 2006 with his former schoolmates professor Pavković celebrated his 55th high school graduation anniversary.

In autumn of 1951 professor Boris Pavković enrolled in the Faculty of Science of the University of Zagreb as a mathematics major. "He completed all prescribed studies", in other words graduated in mathematics on January 30, 1957 at the Department of Mathematics and Physics, study course applied mathematics, and was "awarded the title" of a graduated mathematician. His graduation thesis was written under the mentorship of professor Stanko Bilinski, who would later on also supervise his dissertation and act as year-long "boss" of the Institute.

Immediately after graduation professor Boris Pavković was employed at the Secondary School of Wood Technology, where he had already taught mathematics as a graduate student. In the autumn of 1957 he joined the army due to mandatory military service. Upon return he taught mathematics at the Secondary School of Civil Engineering.

In the autumn of 1959 he was appointed assistant at the Department of Mathematics of the Faculty of Mechanical Engineering and Naval Architecture in Zagreb. In the period of two academic years, 1959/60 and 1960/61, he worked at the same faculty as the assistant to the distinguished professor Danilo Blanuša, with whom he also developed a year-long friendship ("It was fantastic working with him!", professor Pavković stated in an interview published in *Matka* vol.51 (2005) by author L.Gusić)

In the autumn of 1961, professor Boris Pavković was appointed assistant at the Institute of Geometry of the Faculty of Science.

He remained at this Institute till his retirement, building his career as a scientist, methodologist, author of textbooks and popularizer of mathematics.

As one of the postgraduates of the first generation of postgraduate studies in mathematics (initiated in the academic year 1960/61), professor Pavković obtained his Master's degree on April 27, 1966 with his work *Focal points of continuous mapping*, under the mentorship of professors Siba Mardešić and Pavle Papić.

In the academic year 1971/72 professor Boris Pavković undertook a study residence at the Moscow State University (MGU) Lomonosov. This specialization in Moscow (completed in 1974), particularly his work and experiences within a seminar of the noted geometer B. A. Rosenfeljd, was the key moment in the future scientific work of professor Boris Pavković.

He defended his dissertation *Application of differential geometry of curves and planes in isotropic spaces*, created under the mentorship of professor S. Bilinski, on May 15, 1974. The evaluation commission consisted of professors Stanko Bilinski, Dominik Palman and Danilo Blanuša.

He was appointed assistant professor on April 1, 1975. He became senior scientific associate and immediately afterwards associate professor in 1980 (the electoral commission consisted of professors Dominik Palman, Sibe Mardešić and Svetozar Kurepa). After the appointment to scientific advisor in 1989, professor Boris Pavković was promoted in the same year into the scientific-lecturing rank of university professor (the members of the electoral commission were professors D. Palman, V. Volenec and M. Prvanović). He retired on October 1, 1994.

Scientific work and contribution of the late professor Boris Pavković belongs to the area of differential geometry of space with projective metrics, particularly differential geometry of isotropic space:

B. Pavković, *Eine Verallgemeinerung der Frenetschen Formeln im isotropen Raum,* Glasnik Mat. **4**(**24**)(1969), 117-122.

B. Pavković und V. Volenec, *Über die Potenzpunkte der halbkonfokalen (n-1)-Rotationsquadriken,* Glasnik Mat **4**(**24**)(1969), 275-282.

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B. Pavković, *Pseudogeodätische und Unionlinien auf Flächen im isotropen Raum $I_3^{(1)}$,* Glasnik Mat. **10**(**30**)(1975), 115-124.

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B. J. Pavković, *The general solution of the Frenet system of differential equations for curves in the Galilean space* G_3, Rad JAZU **450**(1990), 123-128.

B. J. Pavković, *Relative differential geometry of surfaces in isotropic space*, Rad JAZU **450**(1990), 129-137.

The major scientific results of professor Boris Pavković are contained in the complete description of plane differential geometry in certain spaces with projective metrics, and a detailed analysis of Frenet systems in these spaces.

Furthermore, it is extremely significant to note his work on issues of mathematics teaching methodology. He is particularly responsible for our longterm, quality realtionships with Austrian and Hungarian geometers in both areas. He had a special capability of motivating younger colleagues to engage in scientific work with him. His openness and unselfishness enabled him to assist people by offering cooperation and advice, in most cases on his own initiative. Naturally, this did not end after his retirement, caused by his weak health after a difficult operation in 1983.

He helped with joy, particularly younger people and those who needed help most. Being able to connect with people naturally, he shared his broad knowledge, experience and skill gladly and limitlessly with students, graduates, as well as those who under his supervision and constant care and advice completed their theses and dissertations. For this reason all of them remained grateful and attached to him. Under professor Pavković's supervision approximately hundred students graduated, approximately 10 were awarded their Master's degree and seven their Ph.D.

His early inclination towards geometry and methodology and his choice to pursue them as his career was described in an interview (Školske novine, June 23, 1992, conducted by his friend professor B. Dakić):

"Upon my enrollment in mathematics studies I was lucky that geometry was taught by two excellent professors, prof. dr. Rudolf Cesarec and prof. dr. Stanko Bilinski. It was their „fault" that I fell in love with geometry. Their lectures were interesting, not only in their content, but also in the way they lectured,

and were accented by a high sense of structure. Everything I learned at the university about methodology, I learned from them. One characteristic of their lectures was their poetic nature. I will never forget a lecture by prof. Cesarec in Basic geometry. After he had completed a formula, in order to emphasise its fundamental role he said: "This formula represents the key to the safe in which the most beautiful secrets of hyperbolic geometry are stored". After mentioning this I believe it is clear why I chose geometry as my calling and why I became a methodologist as well. Besides, I need to note that it is geometry that is particularly challenging to methodology. Anyway, it is well known that the aforementioned professors created an entire school of good lecturers and that this fact became the main feature of the Institute of Geometry at the time."

Within the undergraduate studies he taught many courses, some of which were *Elementary mathematics*, *Descriptive geometry*, *Differential geometry*, *Linear algebra and mathematics teaching methodology*, and within postgraduate studies *Riemann's geometry*.

In class professor Pavković participated and delighted with his lectures at other universities as well (Osijek, Split, Rijeka) by significantly contributing to raising the level and awareness of mathematical culture at these faculties of education.

He was a top lecturer, regardless of his audience, clear and systematic in his expression and explanations, comments and notes, always brilliantly, carefully and methodically prepared. To his listeners each of his lectures was a new, content-filled, learning experience in mathematics and teaching mathematics.

By heading for many years the Entrance Examination Committe, he established great connections and cooperation with many young colleagues, instructing them at the beginning of their teaching careers in various skills of a level-headed examiner.

Towards the end of the 1970's professor Pavković took over lectures within the course *Mathematics teaching methodology*. Due to his wide knowledge and talent, as well as his teaching instincts, this moment caused a significant turn in the state of affairs at our faculty in the area of modern structuring and presentation of this previously neglected discipline.

He was also the head of the scientific project in the area of mathematics teaching methodology.

As for methodology, he used to say that it was his "inner" calling:

"I can't explain it, I love that job. To me it's always a challenge to find ways of explaining something complicated. My favourite weapon is the living word. Unofrtunately, I don't like to write. I must mention here the influence of an acclaimed mathematician and methodologist, a professor at the Stanford University, George Polya, American of Hungarian origin. For many years he gave lectures at that university which were intended for future professors of mathematics and wrote many books in which he deals with these issues. I'd like to take this opportunity to draw attention to two of them, *Mathematics and Plausible Reasoning* and *Mathematical Discovery*. (…) All the topics are richly illustrated by means of concrete mathematical content from the area of elementary mathematics. His views on teaching are in accord with the Reccomendation of American Mathematical Society, whose main idea is constructed according to the principle "Guess, research and prove". By this we mean that on a "small scale" one must imitate the creative activity of mathematicians. The afore-mentioned principle is the foundation of all my methodological endeavours." (qtd. in Školske novine)

Indeed, within the entire methodological work of professor Pavković the implementation of basic ideas of G. Poly (1905-1985) is clearly visible. In his work he implemented the recommendation, i.e. to use all methods used by mathematicians in their research, in mathematics instruction as well. Of all the teaching methods he favoured the heuristic one, attempting to, by means of appropriate tasks, encourage students and pupils to discover the laws individually and try to prove them.

Professor Pavković was also the first one who successfully designed the course *Elementary mathematics* by means of which thirty years ago the gap between the secondary school level of acquired knowledge and mathematics studies at PMF- Department of Mathematics was to be overcome. He is co-author of the university textbook according to which the afore-mentioned, as well as some other courses are being taught:

B. Pavković, D. Veljan, *Elementarna matematika I*, Tehnička knjiga, Zagreb, 1992, 399 stranica

B. Pavković, D. Veljan, *Elementarna matematika II*, Školska knjiga, Zagreb, 1995, 609 stranica

He wrote a number of interesting articles in elementary mathematics:

B. Pavković, "Fotogrametrija", *Matematičko fizički list* 12 (1961/62), 159-160.

S. Kurepa, B. Pavković, "Površina poopćenog kruga", *Matematičko fizički list* 17 (1966/67), 54-59.

B. Pavković, "Dokaz iracionalnosti vrijednosti trigonometrijskih funkcija", *Matematičko fizički list* 29 (1978/79), 5-6.

B. Pavković, "Geometrijski način rješavanja Pellove jednadžbe", *Matematičko fizički list* 33 (1982/83), 75-78.

V. Devčić, B. Pavković, D. Veljan, "Seminar za stručno usavršavanje profesora matematike", *Matematika* 1 (1983), 87-90.

B. J. Pavković, "Lagrangeov zakon i njegove primjene", *Matematičko fizički list* 38 (1987/88), 4-9.

A. Rubčić, J. Rubčić, B. Pavković, "O trokutima pridruženim poligonima", *Matematičko fizički list* 38 (1987/88), 121-126.

B. J. Pavković, "Metoda analogije i primjene u nastavi", *Matematika* 1 (1988), 20-27.

B. Pavković, "Primjena metode afine geometrije", *Matematika* 4 (1990), 17-30.

B. Pavković, B. Dakić, "Funkcionalne jednadžbe", *Matematičko fizički list* 42 (1991/92), 65-72.

B. Pavković, P. Mladinić, "Sferna geometrija i Eulerova formula-još jedan dokaz", *Bilten Seminara iz matematike za nastavnike mentore*-Kraljevica 1996, HMD i Element, Zagreb, 1996, 102-107.

B. Pavković, P. Mladinić, "Polinomska geometrija", *Bilten Seminara iz matematike za nastavnike mentore*-Novi Vinodolski 1997, HMD i Element, Zagreb, 1997, 94-100.

B. Pavković, P. Mladinić, "Gaussova konstrukcija tangenata kružnice", *Matematičko fizički list* 48 (1997/98), 65-67.

B. Pavković, P. Mladinić, "Polinomska geometrija", *Matematičko fizički list* 49 (1998/99), 135-140.

B. Pavković, P. Mladinić, "O nastavi transformacija algebarskih izraza", *Poučak* 2/3 (2000), 60-63.; također u Zbornik radova 1. kongresa, HMD, Zagreb, 2000, 259-262.

B. Pavković, "O djeljivosti brojeva", *Zbornik radova 1. kongresa*, HMD, Zagreb, 2000, 263-271.

B. Pavković, "Metoda posebnih slučajeva", *Zbornik radova 6.susreta nastavnika matematike*, HMD, Zagreb, 2002, 381-387.

B. Pavković, P. Mladinić, "Geometrija polinoma", *Zbornik radova 2. kongresa*, HMD, Zagreb, 2004, 280-281.

Many expert topics were dealt with in his books:

B. Pavković, ¸B. Dakić, *Polinomi*, Školska knjiga, Zagreb, 1987, 179 stranica

B. Pavković, *Diofantske jednadžbe*, Društvo mladih matematičara Pitagora, Beli Manastir, 1988, 14 stranica

B. Pavković, *Kongruencije*, Društvo mladih matematičara Pitagora, Beli Manastir, 1988, 16 stranica

B. Pavković, *Inverzija u ravnini i njene primjene*, Društvo mladih matematičara Pitagora, Beli Manastir, 1990, 22 stranice

B. Pavković, B. Dakić, Ž. Hanjš, P. Mladinić, *Male teme iz matematike*, HMD i Element, Zagreb, 1994, 192 stranice

B. Pavković, B. Dakić, P. Mladinić, *Elementarna teorija brojeva*, HMD i Element, Zagreb, 1994, 202 stranice

B. Pavković, P. Mladinić, *Arhimedova metoda težišta*, HMD i Školska knjiga, Zagreb, 1998, 64 stranice.

Together with colleagues from the Institute of Geometry he wrote a faculty handbook:

Z. Kurnik, D. Palman, B. Pavković, *Zadaci iz nacrtne geometrije, Mongeova projekcija*, Tehnička knjiga, Zagreb, 1973, 236 stranica

In co-authorship professor Pavković wrote three very important secondary school handbooks which went through many repeated, rewritten, corrected, expanded and complemented editions (they were refered to as the so-called

white handbook, green handbook, etc.), and can be found today as part of the latest grammar school textbooks:

B. Pavković, N. Horvatić, *Zbirka zadataka iz matematike 1*, Školska knjiga, Zagreb, 1973, (prvo izdanje)

B. Pavković, D. Svrtan, D. Veljan, *Matematika 3, zbirka zadataka za treći razred srednjeg usmjerenog obrazovanja*, Školska knjiga, Zagreb, 1977 (prvo izdanje)

B. Pavković, D. Veljan, *Zbirka zadataka iz matematike 1 za prvi razred srednjeg usmjerenog obrazovanja*, Školska knjiga, Zagreb, 1984 (prvo izdanje)

Numerous co-authorships of professor Pavković in which he was often the one who contributed most to the common work are further witness to his gift of cooperation, giving and friendliness.

Also significant is his work as a translator due to which we have obtained several valuable foreign mathematical works in our language:

G. Choquet, *Nastava geometrije*, Školska knjiga, Zagreb, 1974, 198 stranica (preveli s francuskog D. Palman i B. Pavković)

A. I. Fetisov, *O euklidskoj i neeuklidskim geometrijama*, Školska knjiga, Zagreb, 1981, 258 stranica (preveli s ruskog D.Palman i B. Pavković)

G. Polya, *Matematičko otkriće*, HMD, Zagreb, 2003, 434 stranice (preveli s engleskog B. Pavković, P.Mladinić i R.Svedrec)

I. N. Bronštejn i suradnici, *Matematički priručnik*, Goldenmarketing-Tehnička knjiga, Zagreb, 2004, XLIV + 1168 stranica (preveli B. Pavković, I. Uremović, D. Veljan i dr.; stručna redakcija B. Pavković i D. Veljan)

Moreover, in connection to different mathematical titles, professor Pavković acted as editor, professional consultant, reviewer, but also as proofreader and draftsman of mathematical pictures.

At the PMF-Department of Mathematics professor Boris Pavković was the head of the Institute of Geometry (1992-1994), head and assistant head of the Seminar of Geometry, as well as Seminar of Differential Geometry, and one of the founders and the first year-long head of the Department of Mathematics Teaching Methodology (1990-1992).

His function of vice dean for instruction was performed in the academic years 1981/82 i 1982/83.

For his year-long and undeniable contribution to popularization of science, in particular mathematics, professor Boris Pavković was awarded the state prize "Fran Tućan" in 1992.

In the aforementioned interview for "Školske novine", to the question of what it means to popularize mathematics, considering the fact that many non-mathematicians, but also mathematicians, are very skeptical of such a concept, professor Pavković replied:

"To popularize mathematics means firstly to get as many people as possible interested in learning about it, and after that find ways to get them acquainted with its value in the most approachable way possible: the first step is relatively simple, one should use the most available and the most interesting media for the age group you want to target. For children those are comics and television. The difficulties arise at the second step and due to those difficulties many people become skeptical. There are indeed many areas of mathematics that are virtually impossible to popularize in the sense in which we speak of here. It needs to be said, though, that lately many new disciplines have been developed, mostly parallel to the development of computer science, such as graph theory, concrete mathematics, ennumerative mathematics etc., in which there are segments that are possible to present in a very approachable manner. The job of a popularizer is to notice those segments and subject them to an appropriate analysis. Therefore, it is possible to talk about mathematics from a popularistic point of view, but it requires great effort. I would like to add that my answer to the same question would be much more complete and content-packed, if I could present it in front of a blackboard with a chalk in my hand. In that case I could support it with numerous concrete examples."

Professor Pavković was a year-long member of the Croatian Mathematical Society, much more active than its most active members. At his 70th birthday celebration at the Institute of Geometry in 2001 it was noted with great admiration that it was none other than professor Boris Pavković who won the greatest number of votes in the election for the new assembly od the HMD. This was not the first time that that happened.

On several occasions he was the member of the Chairmanship of the Society, its Board of directors or the Executive board.

Particularly important was the work of professor Pavković in the teaching section of the Mathematics Society. During his entire service he was the pillar of teachers' evenings by giving numerous lectures, hosting meetings and creating new content. At the Society anniversaries it was expected that professor Pavković would be the one to best describe the work of the teaching section:

B. Pavković, "Djelatnost Društva u proteklih 40 godina - nastava matematike (povodom 40. obljetnice Društva matematičara i fizičara SR Hrvatske)", *Glasnik Matematički* 24(44) (1989), 659-662.

B. Pavković, "O radu nastavne sekcije za matematiku", *Matematika* 1 (1990), 73-77

B. Pavković, "Djelatnost Društva u nastavi u proteklih 50 godina (povodom 50. obljetnice HMD-a)", *Glasnik Matematički* 30(50) (1995), 380-384.

In order to understand the aforementioned 40 and 50 years of the Society it needs to be said that in 1945 the Mathematics and Physics Section of the Croatian Science Society was founded, and in 1949 the independent Society of Mathematicians and Physicists. Within the latter society in 1974 two new sections were founded, one of them being the Mathematics Section. In 1990 it grew into what is known today as the Croatian Mathematical Society. One must mention that after 1995 not a single possible "round" anniversary of the Society, no matter how one calculates it, was celebrated.

Professor Pavković also wrote about the great Ruđer Bošković, as well as his role-models, professors R. Cesarac and S. Bilinski:

B. Pavković,B.A.Rozenfeljd, "Ruđer Bošković", *Voprozi istorii estetstvoznanija i tehniki*, Moskva, 1974

B. Pavković, "Rudolf Cesarec - povodom 100. godišnjice rođenja", *Matematika* 1 (1990), 78-83.

B. Pavković, "Stanko Bilinski (povodom 80-tog rođendana)", *Istorija matematičkih i mehaničkih nauka* 4 (1991), 71-83.

B. Pavković, "Rudolf Cesarec - znanstvenik i pedagog", *Mathematical Communications* 1 (1996), 67-74.

B. Pavković, V. Volenec, "In memoriam: Stanko Bilinski (22.4.1909.-6.4.1998.)", Glasnik Matematički 33(55) (1998), 323-333.

Throughout many years he diligently took part in designing different teaching programmes in mathematics, he was a regular lecturer at seminars for te-

achers, regional and state, at teacher Meetings, and at teacher Conferences. It is precisely due to his undertaking nature and support that these manifestations of teacher meetings have continued with their activities (Meetings since 1992, and Conferences since 2000).

Ever since the beginning of the magazine *Matka* in 1992 until his final departure, professor Boris Pavković was the chief editor of this popular magazine for primary school pupils. He is awarded the most credit for the quality and the duration of the magazine as a means of expanding mathematical knowledge and not of school material, as well as the source for developing creative thinking. In the editorial address to the first volume, as the chief editor professor Boris Pavković revealed "what and why so" should a mathematical magazine for primary school children look like. For this reason we give you the complete editorial address:

"Dear children! Before you is the first volume of the mathematical magazine for primary school pupils. We named it *Matka*, because it is the nickname, hopefully of endearment, that you gave to mathematics. Mathematics is one of your school subjects which many of our students face with problems, moreover, to some it is even a constant nightmare. Yet nowadays you can't do without mathematics. It is present in our everyday life, and directly or indirectly it is applied in areas which only superficially have no connection to it (medicine, psychology, linguistics, different social studies, etc.). For this reason, whether you like it or not, mathematics has to be studied hard if you wish to continue your education on a higher than primary level. Fear of mathematics is the fear of the unknown. By means of studying and better acquaintance with mathematics that fear is gradually overcome. We would like *Matka* to contribute to that as well, which was the main incentive for its initiation by the Croatian Mathematical Society. Our society has been publishing M*atematičko~fizički list* for secondary school students for over 40 years. *Matka* is intended for you – the youngest ones. Mathematics needs to be studied from early childhood. We want to introduce you to ideas and structure of mathematics, ways of thinking and concluding which we encounter on the way to solving problems. We would like to prepare you for creative application of mathematical knowledge in the most diverse situations. We would like to help you in experiencing joy of a mathematical discovery. We believe that with *Matka* you will grow to love "matka". The aforementioned goals have determined the contents of the magazine. Inside it we will publish articles whose content will not be based on monotonous and dry listing of the facts, but will deal with ideas that enable solving of certain types of mathematical problems. The focus should therefore be on the essence of

mathematics. At the end of each article there are exercises by means of which one can test the degree of success in acquiring the described method. In other cases as well the exercises in the magazine will be of particular importance. We invite you to solve them patiently and persistantly. We will notify you regularly about mathematics and computer science competitions for primary school pupils, publish the results of the competitions, as well as the names of the winners. There will be much humour, fun mathematics, mathematical crossword puzzles and the section for our youngest ones. Within the variety of texts you will be introduced to the historical development of mathematics, as well as biographies of noted mathematicians. We won't neglect computer science either. (…) Write to us about what you would like to read about in your magazine. Send us your contributions with anectodes from mathematics classes in your school, activities of mathematical groups, interesting exercises that you found, etc. We would love to publish them. Sincerely yours, Boris Pavković"

Professor Pavković has significantly contributed to the foundation of Childrens' Mathematical Library for Pupils, and as the member of the Board of directors of the Croatian Mathematical Society he initiated the entry of Croatia into the international Kangaroo Mathematics Contest.

Professor Boris Pavković loved mathematics, taught it and popularized it with great skill. In this he was aided by his knowledge in foreign languages and his affinity towards literature, as well as his inborn diligence.

Aside from that he was incredibly funny, often bordering on black humour. He was also creative in telling jokes. His good spirits did not leave him even during the most difficult times.

The basic characteristic of this hard-working man was goodness; professor Pavković was good, but also withdrawn.

As a man, professor Pavković was in more than one respect like the character of boy Nemeček from his favourite book *The Boys from Pavel's Street* by Hungarian writer Ferenz Molnar - withdrawn, unobtrusive, devoted, resolute, faithful, sincere, noble, dedicated to the common cause and prosperity.

Everyone who has ever met professor Boris Pavković received a piece of knowledge and goodness. By knowing him, we became better people. For this reason we will value and respect him, forever.

(translated by Željka Nemet)

MATHEMATICS IN PLAY AND LEISURE ACTIVITIES – LEGO BUILDING BRICKS

Tomislav Rudec[1]

Abstract. *This paper contains some interesting facts as well as two types of exercises referring to LEGO building bricks. Although being of the same type, the exercises are of different complexity, i.e. some of them are going to be easy even for pre-school children, whereas some of the exercises will be difficult even for professional mathematicians. They mostly represent a combination of geometry and combinatorics.*

Key words: *combinatorics, geometry*

Introduction to LEGO world

The main character in the story of creation of LEGO is Danish carpenter Ole Kirk Christiansen. More than construction work Ole liked to make wooden setups, figures and toys. He carved doll houses and builiding bricks and in time he became so good at it that he decided to start making only toys. From Danish words leg and godt (*to play and good*) he constructd the name of his firm – LEGO. Until today LEGO belongs to Christiansen family, and since 1979 the presindent is Ole's grandson Kjedl Kirk Christiansen.

Since 1958 the building bricks are being manufactured in current shape and size, and until today (2006) more than 300 million building bricks were manufactured around the world, or in other words around 10 for each person in the world! Basic and the most common dimensions of the building bricks are 2x2 and 2x4, and beside them LEGO has manufactured tens and thousands of other shapes and dimensions. Figures, wheels and other toys have shared feature: they perfectly fit one into another in first and one hundred and first construction.

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Brick with dimensions 2x2

Brick with dimensions 2x4

Today LEGO is world's fourth toy manufacturer (after firms Mattel, Hasbro and Bandai), and LEGO fun club has around 2 million members. LEGO bricks have recently been pronounced (by Forbes magazine) the best toy of the 20th century. Construction possibilities are really numerous – from two 2x4 bricks of the same colour one can construct as many as 24 different figures, and from the six identical bricks, mathematicians have calculated using computers, one can construct as many as 915 103 756 figures.

Mathematical exercises with LEGO bricks

Exercises with LEGO bricks generally cannot be solved using usual mathematical tools (e.g. equations or geometric formulas). There are mathematical theories on tailing, construction and similar, but reducing the following exercises to their results would be too complicated for non-experts (as well as experts). Therefore, the reader of the following exercises does not need any significant mathematical knowledge – everybody is invited to solve them.

We will assume that all figures are from one part, or in other words, the building bricks are combined in one form.

1. Ground-plan

Exercise 1. Using as many as you like LEGO bricks 2x2 and 2x4 (the colour is of no importance), construct a figure whose ground-plan ("view from above") is a full rectangle with dimensions 4x6.

Solution: see picture

Figure of the ground-plan with dimensions 4x6 with four big bricks and the other with three big and one small brick. The second version will be considered as more successful.

In this exercises we will use only 2x2 and 2x4 bricks, and the aim is to construct a shape, i.e. figure which, viewed from above, we see as a full rectangle of wanted dimensions. (The shape we see when we look at some object from above is called the ground-plan of that object.) The wanted figure should be before all constructed with as few levels (orders, rows) as possible, and then as few bricks as possible. Let's imagine that every big brick 2x4 is worth three points, and every little 2x2 is worth two points. The task is to construct the most economical figure, figure with as few points as possible. (If it is possible, a small brick should be used instead of a big one (that is of course more difficult), and it is better (and more difficult) to use one big brick instead of two small because then the total number of bricks is smaller).

Exercise 2. Construct the most economical figure of the ground-plan whose dimensions are 4x4

Solution: It is sufficient to put two 2x4 bricks one next to the other. But, we are not yet finished because that figure is not from one piece, so one more brick 2x2 should be used to connect these two, e.g. from the top. When we put the constructed figure on the table and look at it from above, we see square whose dimensions are 4x4, i.e. 4x4 ground-plan of the figure. The figure is constructed from two rows, and we used two 2x4 bricks and one 2x2 brick, i.e. altogether 8 points and that is the best version.

4x4 ground-plan figure

Exercise 3. Construct the most economical figure of the ground-plan 3x2

Solution: 2 small bricks are worth 2 points = 4 points

3x2 ground-plan figure

Exercise 4. With as few points as possible construct the figure of the ground-plan 3x5

Solution: It is not possible to construct the figure with dimensions 3x5 from two levels (try). One of the solutions is given in the picture. The number of points is 10.

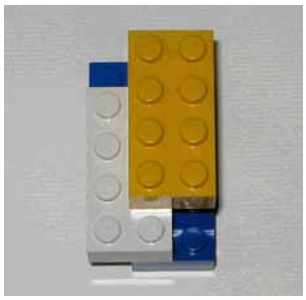


Figure of the ground-plan with dimensions 3x5

Exercise 5. Using as few levels as possible, and with as few points as possible, construct a figure of a given ground-plan.

x	2	3	4	5	6	7	8	9	10
2		2N 4B							
3	X			3N 10B					
4	X	X	2N 8B						
5	X	X	X						
6	X	X	X	X					
7	X	X	X	X	X				
8	X	X	X	X	X	X			
9	X	X	X	X	X	X	X		
10	X	X	X	X	X	X	X	X	

2. Few bricks for a lot of figures

Exercise 6. How many different figures (shapes) can be constructed from two 2x2 LEGO bricks of the same colour?

Solution:

Figure A *Figure B* *Figure C*

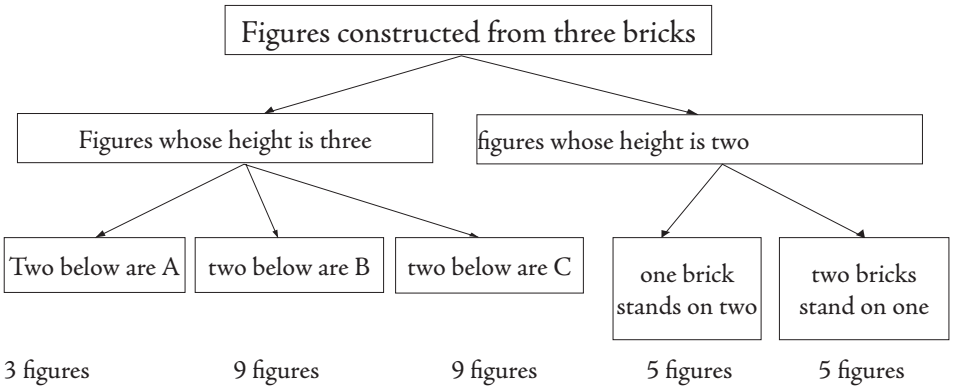
Exercise 7. How many different figures can be constructed from three 2x2 LEGO bricks of the same colour?

Solution: Three bricks can be matched into a figure whose height is three (with three levels) and a figure whose height is two. If we are constructing a figure whose height is three, we must firstly construct first and second level, and we can do that, according to the previous example, in three ways. Therefore, $3 \cdot 3 = 9$ ways.

But, this option is wrong and it nicely shows that in exercises with LEGO bricks we will not be able to use a lot of mathematical tricks.

The error occurs because when we are matching two bricks, the one below is symmetric, so it doesn't matter how we shall turn it before we put the other brick on it, which is not the case in already matched bricks.

Nevertheless, we can take something from mathematics, and that is systematics. All possible figures constructed from three bricks can be divided into different groups according to their appearance. One of the possible divisions, according to the previous exercise and figures A, B and C from the picture, looks like this:



In conclusion, the total number of solutions (3+9+9) + (5+5) = 21+10 = 31.

Exercise 8. How many different figures can be constructed from two 2x2 figures of different colours?

Solution: Let's say those colours are, to be more specific, blue and yellow. Obviously, this figures can be divided into those with blue bottom brick and those with yellow bottom brick, and for each of these cases, according to the exercise 6, there are three possibilities. In total: 2 forms · 3 figures = 6 wanted figures. (Here is some mathematics!)

Exercise 9. Fill in the empty fields in the table. In each field enter how many different figures can be constructed from bricks which determine that field. Numbers that were calculated in the examples are already entered (e.g. number 31 means that from two small blue and one small blue, i.e. three small blue bricks, in total 31 different figures can be constructed). Some of the fields in the table present the same exercise, i.e. fields marked with A. Number inside the field marked with * is difficult to get, in the field with ** very difficult, and in the field with ****, of course, very very difficult!

	1 small blue	1 small yellow	2 small blue	2 small yellow	1 small yellow and 1 small green	1 big yellow
1 small blue	3	6	31			A
1 big blue		A	*	*	*	
2 big blue	**	**	****	****	****	**

(translated by Mirta Kopić)

BASIC KNOWLEDGE OF MATHEMATICS AND TEACHER TRAINING

Sanja Rukavina[1]

Abstract. *The technological development and computer accessibility have led to numerous discussions about the Mathematical contents to be acquired during the course of compulsory education. In other words, the question of basic Mathematical literacy is becoming relevant. The main question being raised is about the actual knowledge and skills a contemporary man should possess in order to successfully participate in modern society.*

Bearing in mind that teachers play an important role in every educational process, when dealing with teaching Mathematics, a few questions need to be answered:

- What are the basic Mathematical skills each student should acquire during the process of compulsory education?,

- Which competencies are necessary for realizing the teaching of Mathematics?,

- Which are the main goals and objectives of Mathematics teacher training?

Considering teaching Mathematics in Croatia starts as early as the first grade of elementary school, the previously mentioned questions affect not only specialized Mathematics teachers and their education but also teachers who start the process of systematic Mathematical instruction at the beginning of elementary school. Taking into account that many students - future elementary teachers - are not particularly interested in Mathematics nor understand its significance for student development, special attention should be given to their Mathematical training, as well as developing a positive attitude towards Mathematics.

Key words: *basic knowledge of mathematics, teacher training.*

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The technological development and computer accessibility have led to numerous discussions about the Mathematical contents to be acquired during the course of compulsory education. Is it necessary to learn as much Mathematics as we used to or did the use of computers make it easier for us to manage even without Mathematics? Do we maybe have to learn more of Mathematics than before? Which Mathematical skills do we need to acquire? In other words, the question of basic Mathematical literacy is becoming relevant. These discussions are accompanied with discussions on scientific literacy and issues dealing with which are the basic contents to be covered during the course of compulsory education in Physics, Chemistry, Biology, and other natural sciences.

The main question being raised is about the actual knowledge and skills a contemporary man should possess in order to successfully participate in modern society. Furthermore, this question is equally relevant for family life, work, and the fact that every contemporary adult citizen, with their vote, has the opportunity to participate in the decision-making process which affects society as a whole.

After determining what will be considered as basic knowledge, teachers will play an important role. When dealing with teaching Mathematics, a few questions need to be answered:

- What are the basic Mathematical skills each student should acquire during the process of compulsory education?,
- Which competencies are necessary for realizing the teaching of Mathematics?,
- Which are the main goals and objectives of Mathematics teacher training?

Considering teaching Mathematics in Croatia starts as early as the first grade of elementary school, the previously mentioned questions affect not only specialized Mathematics teachers and their education but also teachers who start the process of systematic Mathematical instruction at the beginning of elementary school. Taking into account that many students - future elementary teachers - are not particularly interested in Mathematics nor understand its significance for student development, special attention should be given to their Mathematical training, as well as developing a positive attitude towards Mathematics.

Basic Knowledge of Mathematics

It seems as though the current conflict between supporters of Mathematics and those who are trying to reduce its contents in schools is additionally exacerbated by the technological development. "The supporters" believe that the technological development has intensified the need for mathematically skilled personnel, while the latter claim how we do not even have to know the multiplication table anymore because of the existence of various calculating devices. This conflict is a result of identifying minimal and basic knowledge of Mathematics, i.e. minimal and basic Mathematical literacy.

The concepts of minimal and basic literacy greatly differ and we normally, when not dealing with Mathematics, take this into account, maybe even subconsciously. For example, we are ready to call one illiterate when they do not know or do not follow spelling rules even though they evidently possess a minimal level of literacy (being able to sign their own name). The educational system is expected to "produce" a person on a higher level of literacy than what we consider to be basic literacy. Thus, the difference between minimal and basic knowledge of Mathematics should also been taken into consideration and it should be expected from everybody to have acquired basic Mathematical skills upon the completion of compulsory education.

Most would say that knowing how to add, subtract, multiply, and divide correctly are basic Mathematical skills to be acquired during the course of compulsory education. Only a few would add percentages to the equation. Considering the development of information technology has made computation much easier, this point of view can easily lead to the reduction of Mathematical contents in schools. Regrettably, the "reductionist" point of view is found among early primary school teachers.

The basic knowledge question in Mathematics is to be answered by thinking which prerequisites should be fulfilled in order for one to be considered a successful member of today's society and in what way does acquiring certain Mathematical skills help us in fulfilling those prerequisites.

In every day affairs, we are expected to plan, be responsible, use time and resources efficiently, think critically, make decisions, communicate, negotiate, function in a team, and assume leadership in certain situations. Therefore, basic mathematical skills should include those skills whose practical applicability is

not self-evident, like for example, solving word problems which most people are ready to discard as redundant. But with solving word problems, we are learning how to pose questions, analyze, and recognizing relevant facts. Additionally, we learn how to subject our own conclusions to scrutiny, use trial and error and compare results. These are all skills we will need later in life.

Geometry skills are also readily declared irrelevant by "non-Mathematicians". However, except learning basic properties of geometric figures, we also learn how to compare different objects, recognize their similarities and differences and classify them according to their properties. By solving measurement problems we are preparing ourselves for numerous life situations, as well as for mastering other school subjects.

Acquiring different Mathematical skills, we also acquire a logical way of thinking, techniques of approximation and determining when a result is precise enough for a given situation, alertness to reasonableness of certain statements, or how to properly interpret graphics. The demand for those skills is even greater with the rise of technology. Many careers nowadays require having additional Mathematical education, in addition to basic Mathematical skills.

Teacher training

It is not to be expected from all classroom teachers to be persons with explicit Mathematical interests. The same request could rightfully be made by other subjects studied in early primary school. Unfortunately, when Mathematics is concerned, the situation is often quite the opposite. Many classroom teachers have come to their occupation by "running away" from Mathematics and have not developed a liking for Mathematics nor see the importance of Mathematical education.

Which competencies are expected from classroom teachers when it comes to teaching Mathematics and how are they to be attained? We would like classroom teachers to possess certain Mathematical skills and understand the significance of Mathematical education to students. It is not good for students to start their systematic Mathematical education with a teacher who finds minimal Mathematical knowledge sufficient.

All study programs for future classroom teachers provide for acquisition of required Mathematical skills. The demand for knowing Mathematical contents

and understanding Mathematical concepts must surely be satisfied because it represents a foundation without which quality teaching cannot be realized. In addition, special attention should be given to developing a positive attitude towards Mathematics as a school subject, if not for Mathematics as a science. This positive attitude should be a result of understanding the need for learning Mathematical contents; considering teachers have to be aware of the fact that Mathematical skills are not an end in themselves and that through them students acquire other necessary knowledge and skills. This fact should never be forgotten and it should be emphasized whenever it is appropriate.

Students should be encouraged to notice how abstractness, objectivity and permanence of Mathematics cannot be disrupted. That is the reason Mathematics is extremely useful, both when directly applied as well as a tool in developing logical thinking. If future classroom teachers become aware that the acquisition of basic (not minimal) Mathematical skills is something it should be rightfully insisted upon, their work in the classroom will represent an excellent beginning for Mathematics education.

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SOLVING LINEAR EQUATIONS USING COMPUTER'S DRAWING TOOLS

Miljenko Stanić[1]

Abstract. *In this paper I shall present a very simple, but effective teaching method of solving linear equations that is suitable for primary school pupils. I will present the problem and construct the solution using diagrams only and avoiding any standard formalism. We would guide pupils towards the solution using simple common sense operations, such as take out of or put into some geometrical figures enclosed in drawn entities. According to Piaget' s theory of cognitive development of the child, which is not disputed essentially, primary school pupils deal with the concept of the conservation of quantity and prefer real operations. Therefore, this approach is suitable to their ability. The method has to present the equality of quantity in two entities representing different sides of the equation, applying the operations of adding/subtracting of objects to the relevant entities. The purpose is to get the solution of equations defined over the set N by repeating the above mentioned operations. The teacher can use the same method to extend solutions over the sets Z and Q.This method is good as an auxiliary support (Vygotsky) in understanding formal laws in arithmetical structures not only over the set N, but Z and Q, too. For drawing and analysing, we use computer's drawing tools, usually Word ones.*

Key words: *well formed diagrams, conservation of quantity, syntax, semantics.*

Many mathematical concepts and problems can undoubtedly be presented and solved with a diagrammatic approach. This approach is of a great heuristic value in the teaching of mathematics, while in some proofs it is irreplaceable. Here we will present the solving of equations in a sequence of diagrams that we

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will draw with a computer. I would like to comment on two aspects offered by such an approach.Firstly, its logical-semantic status and secondly, its methodological applicability in teaching.

1. Logic of diagrams

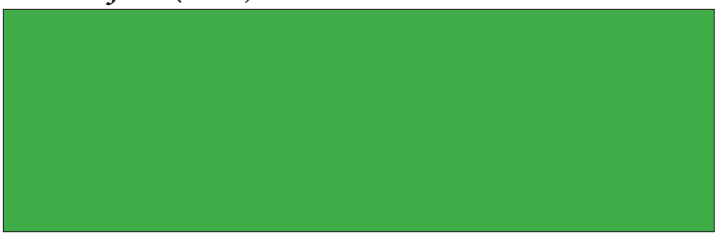
Regarding this topic, I would like to deal with the issue whether a diagrammatic presentation of the procedure of solving linear equations is complete. Namely, is there an equation that cannot be solved by diagrams or, vice versa, is it possible to present an equation in a diagrammatic way without it having an equivalent in the formal-linguistic notation? For example, let us consider Venn's diagrams. With closed curves and one rectangle we can present operations among sets quite effectively in the school practice. Yet, the presentation itself is not complete in the above sense. For example, empty sets cannot be presented with the standard presentation so that we have to enrich the diagram with new elements, such as shading or entering a new sign in order to emphasise that this closed part is empty or that it is not empty, etc. Presentations modified in this way become complete and gain a status that is bigger than heuristic means in the teaching of mathematics, namely, they become a legitimate strategy of presenting the naive theory of sets ([2]).The available space does not allow me to present in its complete the syntax and the semantics of the diagrammatic solving of linear equations.

Following the terminology from ([1]), as *well formed diagrams*, *wfds*, we will call those diagrams that represents a mathematical concept or object (in our case linear equation) in a good, purposeful and efficient form, in the complete sense of the word.

1.1. Syntax

Primitive diagrammatic objects (*icons*)

1. House: Kuc_j =



2. Boxes: $Pak^1_{n,j}$ =  $Pak^2_{n,j}$ =  $Pak^3_{n,j}$ = 

All *Pak* icons are mutually congruent squares which can differ only in colour.

Let us represent with P^1, P^2 and P^3 the set of all colourless, blue or green boxes.

3. Sticks: $Stp^1_{n,j}$ =  $Stp^2_{n,j}$ =  $Stp^3_{n,j}$ = 

All *Stp* icons are mutually congruent rectangles which can differ only in colour.

4. Sticks /n: $Stp^{i/h}_{ng,j}$ is a rectangle with the height that is the h-th part of the appropriate $Stp^i_{n,j}$, $1 \leq g \geq h$

Let us represent with S^1, S^2 and S^3 the set of all colourless, blue or green sticks, and their relevant parts. With index $j \in \{1, 2\}$, we represent that the icon lies within Kuc_1 or Kuc_2. We call index n by the *name* of the icon (n being a natural number), to be determined in the following way. Let Ikn^k_{nj} be a common representation for icons Stp^k_{nj} or Pak^k_{nj}. Let us represent with n_{ij} the cardinal number of icons that lie in Kuc_j, ($i = s(tick)$ or $i = p(box)$), then, to a new icon Ikn^k_{ij} we will assign the *name* $n = n_{ij}+1$.

Diagrams:

Let us represent diagrams with s Δ. Δ consists of icons in the following way:

A) $\Delta = Kuc_1 \cup Kuc_2$ (under the condition that: $Kuc_1 \cap Kuc_2 = \emptyset$) is a *diagram*.

B) If Δ is a diagram, then $\Delta' = \Delta \cup \{Stp^k_{n,j} \mid Stp^k_{n,j}$ lies within $Kuc_j\}$, $k \in \{1,2,1/h,2/h\}$, $h \in N/\{0\}$ and $j \in \{1,2\}$ is also a *diagram*.

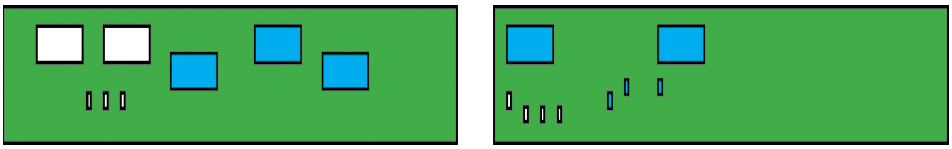
C) If Δ is a diagram, then also $\Delta' = \Delta \cup \{Pak^k_{n,j} \mid Pak^k_{n,j}$ lies within $Kuc_j\}$, $k \in \{1,2,1/h,2/h\}$, $h \in N/\{0\}$ and $j \in \{1,2\}$ is also a diagram.

Let us represent with: $\Pi j = \{Ikn^k_{nj} \mid Ikn^k_{nj}$ lies within $Kuc_j\}$

Let us represent with: $\Phi nj = \{Stp^k_{mj} \mid Stp^k_{mj}$ lies within $Pak^{k'}_{n,j}\}$

It is obvious that: $\Phi_{nj} \subseteq \Pi_j$

Example 1 (*diagram*) **:**



The solvable sequences of diagrams

The sequences of diagrams : $\Delta 0$, $\Delta 1, \ldots, \Delta n$ we will call solvable if the following conditions are satisfied:

a) $\Delta 0$ is a diagram that we will call initial diagram.

b) Δi (**0 < i < n-**1) is a diagram transformed from diagram $\Delta i\text{-}1$ by applying one of the rules

i) ***Izv:*** $\Delta_i = \boldsymbol{Izv}\,(\Delta_{i-1})$

ia) Let for Δ_{i-1} be valid that: $Stp^2_{n,1} \in \Delta_{i-1}$ and $Stp^2_{m,2} \in \Delta_{i-1}$, then is $\Delta_i = \Delta_{i-1} / \{Stp^2_{n,1}, Stp^2_{m,2}\}$

ib) Let for $\Delta i\text{-}1$ be valid that: $Pak^2_{n,1} \in \Delta_{i-1}$ and $Pak^2_{m,2} \in \Delta_{i-1}$, then is $\Delta_i = \Delta_{i-1} / \{Pak^2_{n,1}, Pak^2_{m,2}\}$

ic) Let for Δ_{i-1} be valid that: if from $Stp^k_{n,1} \in \Delta_{i-1}$ or $Stp^k_{m,2} \in \Delta_{i-1}$ it follows that $k \neq 2$ then is

$\Delta_i = \Delta_{i-1} \cup \{Stp^1_{x,1}\} \cup \{Stp^1_{y,2}\}$, $(x = n_{s1}+1, {}_y = n_{s2}+1)$, $Stp^1_{x,1} \in \Pi_1$ and $Stp^1_{y,2} \in \Pi_2$

id) Let for Δ_{i-1} be valid that: if from $Pak^k_{n,1} \in \Delta_{i-1}$ t or $Pak^k_{m,2} \in \Delta_{i-1}$ it follows that $k \neq 2$ then is

$\Delta_i = \Delta_{i-1} \cup \{Pak^1_{x,1}\} \cup \{Pak^1_{x,2}\}$, $(x = n_{p1}+1, y = n_{p2}+1)$, $Pak^1_{x,1} \in \Pi_1$ and $Pak^1_{y,2} \in \Pi_2$t

ii) ***Ubc:*** $\Delta_i = \mathbf{Ubc}\,(\Delta_{i-1})$

iia) Let for $\Delta i\text{-}1$ be valid that: $Stp^1_{n,1} \in \Delta i\text{-}1$ i $Stp^1_{m,2} \in \Delta_{i-1}$, then is
$\Delta_i = \Delta_{i-1} \cup \{ Stp^3_{x,1} \} \cup \{Stp^3_{y,2}\} =$

$= \Delta_{i-1} / \{ Stp^1_{n,1}, Stp^1_{m,2}\}, (x = n_{s1}+1, y = n_{s2}+1), Stp^3_{x,1} \in \Pi_1$ and $Stp^3_{y,2} \in \Pi_2$

iib) Let for Δ_{i-1} be valid that: $Pak^1_{n,1} \in \Delta_{i-1}$ i $Pak^1_{m,2} \in \Delta_{i-1}$ then is
$\Delta_i = \Delta_{i-1} \cup \{ Pak^3_{n,1} \} \cup \{Pak^3_{m,2}\} =$
$= \Delta_{i-1} / \{ Pak^1_{n,1}, Pak^1_{m,2}\}, (x = n_{p1}+1, y = n_{p2}+1), Pak^3_{x,1} \in \Pi_1$ and
$Pak^3_{y,2} \in \Pi_2$

iic) Let for Δ_{i-1} be valid that: if from $Stp^k_{n,1} \in \Delta_{i-1}$ or $Stp^k_{m,2} \in \Delta_{i-1}$ it follows that $k \neq 1$ then is

$\Delta_i = \Delta_{i-1} \cup \{ Stp^2_{x,1}\} \cup \{Stp^2_{y,2}\}, (x = n_{s1}+1, y = n_{s2}+1), Stp^2_{x,1} \in \Pi_1$ and $Stp^2_{x,2} \in \Pi_2$

iid) Let for Δ_{i-1} be valid that: if from $Pak^k_{n,1} \in \Delta_{i-1}$ or $Pak^k_{n,2} \in \Delta_{i-1}$ it follows that $k \neq 1$, then is

$\Delta_i = \Delta_{i-1} \cup \{ Pak^2_{n,,1} \} \cup \{Pak^2_{n,2}\}, (x = n_{s1}+1, y = n_{s2}+1), Pak^2_{x,1} \in \Pi_1$ and $Pak^2_{y,2} \in \Pi_2$

iii) ***Pod:*** $\Delta_i = \mathbf{Pod}\,(\Delta_{i-1})$

Let for Δ_{i-1} be valid that :

1) If $Pak^k_{n,j}\, Pak^{k'}_{n',j'} \in \Delta_{i-}$ are, then is $k=k'=2$ and $j' = j$

2) If $Stp^k_{n,j}\, Stp^{k'}_{n',j'} \in \Delta_{i-1}$ are, then is $k=k'=1$ or is $k=k'=2$ or $k=k'=g/h$ is and $j' = j$

in which is $g \in \{1,\ldots,h\}$, $h \in N/\{0\}$

3) $|(\Delta i\text{-}1 \cap S^k)| \geq |(\Delta^{i-1} \cap P^{k'})|$

Let us order the set : $\Delta_{i-1} \cap S^k = \{ Stp^k_{n1,j}, Stp^k_{n2,j}, \ldots\ldots, Stp^k_{nt,j} \}$

Let us order the set: $\Delta_{i-1} \cap P^{k'} = \{ Pak^{k'}_{m1,j'}, Pak^{k'}_{m2,j}, \ldots\ldots, Pak^{k'}_{mu,j'}\}$, in which it is valid $j \neq j'$, $u \leq t$.

In order to represent Δi with a diagram, we have to define the auxilliary function:

$$f: \{n_1, n_2, \ldots, n_t\} \rightarrow \{m_1, m_2, \ldots, m_u\},\ k_l = t - l + 1$$

If $k_l \geq u$, $\mathrm{rem}'(l) = \begin{cases} rem(l,u) & \mathrm{rem}(l,\mathrm{u}) > 0 \\ u & \mathrm{rem}(l,\mathrm{u}) = 0 \end{cases}$ $\mathrm{f}(n_l) = \mathrm{m}_{\mathrm{rem}'}(\mathrm{l},)$

If $k_l \leq u$, onda je $\mathrm{f}(n_l) = 0$.

$\Delta_i = (\Delta_{i-1}/\{ Stp^k_{n1,j} \mid k_l \geq n_p \}) \cup \{ Stp^k_{f(nl),j'} \mid Stp^k_{f(nl),j'} \in \Phi_{\mathrm{f}(n1),j'}, \mathrm{f}(n_l) > 0\}$

iv*) *Lom: $\Delta_i = \boldsymbol{Lom}\,(\Delta_{i-1})$

$\Delta_i = (\Delta_{i-1} / \{ Stp^k_{n,j} \}) \cup \{ Stp^{i/h}_{\mathrm{ng},j} \mid g \in \{1,2,3,\ldots,\mathrm{h}\} \}$ for a $h \in \mathrm{N}/\{0\}$.

c) Diagram Δ_n is called a terminal diagram that satisfies for a $j \in \{1,2\}$, $\Pi_j = \varnothing$.

d) *The status of solvency* of the solvable transformation of diagrams.

Let us consider $\Pi_{j'} \subset \Delta_n$ under the condition that $\Pi_j = \varnothing$, i $j \neq j'$

We can distinguish four cases regarding the contents of the set $\Pi j'$.

i) if $\Pi_{j'} = \bigcup_{i=1}^{p} \Phi_{n_i j'}$, $n_i \in \{n_1, n_2, \ldots, n_p\}$, $\Phi \mathrm{ni,j'} \neq \varnothing$, $(1 \leq \mathrm{i} \geq \mathrm{p})$ then we will say that the *initial* diagram has been solved in the *unique* way.

ii) if $\Pi_{j'} = \varnothing$, then we will say that the initial diagram has the undeterminate solution.

iii) if $Ikn^k_{n,j'} \in \Pi_{j'}$ then is $Ikn = Stp$, then we will say that the *initial* diagram has no solution.

iv) if $Ikn^k_{n,j'} \in \Pi_{j'}$ then is $Ikn = Pak$ and $\Phi ni,j' = \varnothing$, $(1 \leq \mathrm{i} \geq \mathrm{p})$ then we will say that the initial diagram has the solution 0.

1.2. Semantics – interpretation

Each icon from the system of diagrams is assigned a whole integer or a variable.

Icons: to $Stp^k_{n,j}$ will be assigned a positive integer 1 if k=2 or k=3, i.e. a negative (-1) for k=1.

To $Pak^k_{n,j}$ we will assign the variable x if k=2 , i.e. $(-x)$ if k=1.

$\kappa = |\Pi_j \cap \mathrm{S}^i|$ i.e. quantity, the cardinal number of white sticks (i=1) or blue sticks (i=2) in j-house, which we will interpret as $(-\kappa)$ or (κ).

$\kappa = |\Pi j \cap P i|$ i.e. quantity, the cardinal number of white boxes (i=1) or blue boxes (i=2) in j-house, which we will interpret as (-a) a negative coefficient or (a) a positive coefficient with a variable.

The content of each house is interpreted as one side of the equation, which, once ordered, can be reduced to the form: $\mathbf{a}x + \mathbf{b} = \mathbf{c}x + \mathbf{d}$.

Solving the set is, actually, a procedure of solving equations in which we use a consecutive application of the cancellation rule for the operations of adding.

The solution of equities we can read in Δ_n terminal diagram in the solving sequence, as a number of sticks in the box, or $x = |\Phi n\, j|$.

In this presentation of solving linear equities we reduced to coefficients to the set of integers, namely **a**, **b**, **c**, **d**$\in \mathbb{Z}$, but the solutions could be from the set of rational numbers **Q**.

2. Teaching Practice

I am constructing the basis for the teaching practice, namely for the games with diagrams, on the assumption that it is the task of the teaching practice of mathematics in the early school age to use cognitive competences of children in implementing mathematical contents. Namely, in an ideal situation, the teacher tries to discover in the existing, not necessarily school knowledge of the child, her or his mathematical contents.

Let us begin with the initial diagram.

1. We ask a pupil to give a good look at the diagram. She or he would have to observe that the diagram consists of two parts, the *left* and the *right* one, confined by «houses».

2. Verbal interpretation: We will acquaint the pupil with the fact that we are starting from the assumption that both parts of the diagram contain the same quantity of *Stp* icons, i.e. the same quantity of «sticks». In this quantity we can include at will a great number of "sticks" built into the "house", as if they were "bricks" from which the house is built. Sticks without colour we can consider to be "bricks" that we have taken out of from the wall of the house. Green sticks are building material that we put onto white sticks or "bricks" that we build in the walls of the house. Sticks coloured in blue are additional

material in the house, as if they were its occupants. Within blue rectangles we have a certain quantity of blue sticks hidden. Within colourless rectangles there is an unknown number of sticks taken out of the wall of the house. The part of wall that is removed, we can "mend" with green rectangles.

3. Acquaint pupils with the rules for creating new diagrams from the initial one.

   Rule ***Izv:*** means taking out of, removing from both sides of the diagram a "stick" or a "box" simultaneously, while at the same time you can remove a "stick" or a "box" from the "wall of the house" as well.

   Rule ***Ubc:*** means putting a "stick" or a "box" into both sides of the diagram simultaneously.

   Rule ***Pod:*** means inserting "sticks" from one "house" into "boxes" of another house.

   Rule ***Lom:*** means dividing "sticks" from one "house" into an equal number of "small sticks".

4. Support of psychology: psychologists believe that children in the age of 7 to 12 years have mastered the concept of *conservation of quantities* (Piaget [3]). If we go through the rules of transformations, we can easily notice that if we start from the assumption that the *left* and the *right* side of the *initial* diagram have the same *quantity* of "sticks", by applying the rule of transformations in the *solvable sequence* of diagrams we will preserve the equality of *quantities* of the *left* and *right* side of the transformed diagram.
5. Application in teaching practice: with a good preparation, we can demonstrate to pupils the transformation of diagrams in the form of a game. The winner is the pupil who first opens the terminal diagram. It is also possible to cheat, i.e. the irregular application of syntactic rules.

   Counter player should discover the "cheat" or correct the irregular application in the diagram, taking an extra move as a reward, etc. Mathematical gains represent mental building bridges

   (Vygotsky [3]) towards integer, that is rational numbers.

Example 2: Let us apply the solving of one of the problems that I have taken from the existing school collection of problems for pupils of the 3rd grade ([4]).

Problem: The teacher has prepared a number of mathematical problems for the work in the maths group. It has to be found out how many problems the

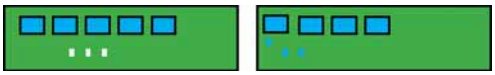
teacher has prepared and how many pupils there are in the group if we know that:

1. If every pupil is to get 5 problems, 3 problems are missing .

2. If every pupil is to get 4 problems, 3 problems will remain .

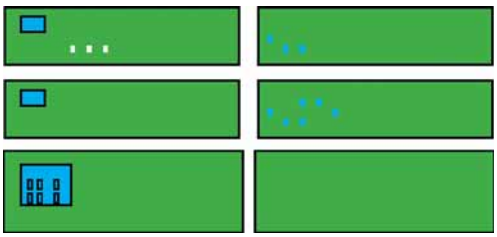
Diagrams-iconic interpretation: *Sticks* are *problems*. *Boxes* determine *the number of pupils in a group*. Left or right side we interpret as the *total number of problems* that the teacher has prepared.

The right side describes claim 1) while the right side claim 2).



Look: *1* problem has been given to all pupils Δ_0 2. problem in the same way…

If we give 5 to every p., then from the «wall» they take 3 more, i.e. if we give 4 to every pupil, then 3 remain non distributed



Δ_1= ***Izv***(Δ_0): app. to boxes 4 times consecutively

Δ_2 = ***Ubc*** (Δ_2): app. To sticks 3 times consecutively

Δ3 = Pod(Δ2) Terminal diagram.

The group has 6 pupils and 27 problems have been distributed.

3. Use of Computer

This problem is actually intended for solving with a computer. Each teacher can easily use the above diagrams using just **Drawing** tools that every Microsoft product is equipped with.

As a helping means, for an easier calculation of solutions, and as an effective heuristic help for teachers, I have made two computer programmes on Excel pages, the file named **Punavreća** ([5]) . On the sheet **jednadžbe** a programme has been inserted, with the help of which every teacher can demonstrate to her or his pupils the rules of transformation in the above diagrams. First, she or he has to write in the linear equation the solution of which we are looking for. By fixing the same equation on the screen, we get the *initial* diagram. By applying

commands **Uzmi** or **Dodaj** we carry out the taking out of/putting into of the icons in the house, doing it *simultaneously*, and thus, in the best of possible ways we demonstrate the rules ***Izv*** or ***Ubc***. Offered commands correspond with other rules and present them in the best way. I can recommend this programme for the beginning, for motivating and warming up pupils for the game. On the sheet **igra** you will find the software that requires a greater interaction of pupils and the computer. Pupils themselves insert icons and create the initial diagram. They have to **fix** the chosen diagram. Then the game can start and it lasts until the *terminal* diagram. The computer checks if the content in the boxes is the right solution.

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(translated by Tatjana Dunatov)

DEVELOPING THE PROBLEM-SOLVING SKILLS OF CHILDREN SUFFERING FROM DYSCALCULIA THROUGH MATHEMATICAL TASKS WITH A TEXT

Straubingerné Kemler Anikó[1]

Abstract. *My research topic is the analysis of the relationship between mathematics and everyday life in case of children suffering from dyscalculia, more precisely I am interested how difficulties of everyday life occur on maths lessons and also what we could do on these lessons so that these children with dyscalculia could cope more easily in everyday life.*

The aim of my lecture is to draw attention to those methods applied in the teaching of Mathematics that are suitable for helping children with learning difficulties catch up and are also suitable for developing these children in the framework of the classroom community.

Apart from the standard therapy of dyscalculia (concept of numbers, basic rules of arithmetic, continuous practice), I would recommend the use of problem-solving activities. Mathematics has to teach not only the calculations, but it also has to aim at teaching children think logically and enable them to solve problems. Mathematical tasks provide numerous opportunities for that. It is important even for children who have difficulties with calculations, since they will be able to use them in their lives later.

I also find it important to show the above mentioned strategies to teacher trainees in the framework of their training programme. On the one hand teacher trainees have to be aware of numerous strategies in order to be able to give an example for children with different ways of thinking; on the other hand they have to be able to understand the way each individual child thinks. In order to enable them to do that, I show and analyse examples that can be used in the classroom.

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Mathematical tasks with a text are particularly suitable for applying problem-solving strategies since they provide an opportunity for applying a wide range of strategies. These tasks can be found in every area of mathematics and they can also be found at each level of education.

In my lecture I will show different ways of solving tasks with a text that can be found in the lower primary classes of primary schools paying special attention to those solutions that can be used with less able children or with children who have dyscalculia.

For those children, who have difficulty in recognising abstract symbols or for whom abstract thinking and model-making are difficult, other methods including trials, parallel changes, demonstrations (repetend-method, heaps or diagrams), reasoning or the use of charts etc. might be more helpful than the algebraic solution (an open sentence). From the arithmetic solutions I will focus on one particular strategy – reverse reasoning. I will also show how thinking methods applied in mathematics can be made more understandable for children with the help of examples and games taken from other areas of life.

Through solving mathematical tasks with a text we can develop children's skills to understand, see correlations and solve problems. The possible solutions are numerous, there is not a single universal method, but numerous ideas can be offered to choose from. My lecture aims at proving that the other methods shown can be as valuable as algebraic solution, or from the point of view of developing logical thinking and problem-solving thinking they are even more valuable than algebraic solution. That is why they are so important for children with dyscalculia who are averse to algebraic solutions.

Key words: *problem-solving, learning disability, dyscalculia, mathematical Tasks with a Text.*

My research topic is the analysis of the relationship between mathematics and everyday life in case of children suffering from dyscalculia, more precisely I am interested how difficulties of everyday life occur on maths lessons and also what we could do on these lessons so that these children with dyscalculia could cope more easily in everyday life.

The aim of my lecture is to draw attention to those methods applied in the teaching of Mathematics that are suitable for helping children with learning difficulties catch up and are also suitable for developing these children in the framework of the classroom community. I believe that mathematical correction should also focus on strengthening and developing of children's personality,

since this can serve as a basis for developing other skills and it can enable children to live a happy and successful life.

Therefore, besides the traditional therapy for dyscalculia (number concept, routinising the four basic arithmetical rules through ongoing practice) I would emphasise the familiarisation with problem-solving methods. This can be important even for those children who have difficulty with counting and do not like using abstract mathematical signs, since this is something they would be able to use in their later lives.

Familiarisation with the strategies shown is important in teacher training, as well. On one hand, future teachers need to be aware of numerous strategies so that they could show examples for children with different thinking skills and also they should be able to transfer knowledge and skills with the help of which students will be able to solve mathematical tasks with a text. On the other hand, teachers have to be able to understand each individual child's way of thinking.

Mathematical tasks with a text provide numerous opportunities for applying problem-solving strategies. Mathematical tasks with a text can be found in every area of mathematics at all levels of primary education.

In my lecture I will show methods for solving mathematical tasks with a text that occur in the lower primary classes of primary school with a special attention to those methods that can be applied by children with dyscalculia and also by less able children, as well.

What is dyscalculia?

„Learning disability" (inability to learn, difficulty, disturbance, disorder), as a category have become common recently, although it was described as long ago as the late 1800s in medical journals. The term was first used by Samuel Kirk in 1962 at a conference, which was organised for experts dealing with children with brain injury or children having difficulty with perception. /S. Kirk, 1962./ Then experts from different research and practical fields unified to get to know the phenomenon more deeply. One of the learning disorders is dyscalculia. According to Hrivnák Ilona, dyscalculia refers to the partial lack or disorder of counting skills and it should not bee mixed with the complete lack of counting skills, which is referred to as acalculia. Children suffering from dyscalculia are

those children whose skills that are necessary for learning mathematics are beaten abnormally compared to other skills that are necessary for learning other subjects. / Hrivnák Ilona, 2003./

One can find a wide range of data about the frequency of occurance. Among the age-group of 11-12 year old children, the number is about 6-7 %, and the rate is the same for boys and girls. This means that there are approximately 60.000 students suffering from dyscalculia in primary and secondary schools in Hungary, who are looking forward to getting support.

/ Dr. Márkus Attila, 1999./

The typical symptoms of serious counting disorder at primary age are the following:

- counting mistakes of the same type that occur fairly often (with the four basic arithmetical rules going over ten, keeping the remainder, taking into account the directions when doing subtractions, with multiplication with multi-digit multiplier finding the place for multiplication-fragments, using symbols and signs, creating lines, writing and reading increasing and decreasing lines.)
- Problems with concept (multiplication, division, conceptualising a fraction, reading and writing decimal fractions, the differences between a plane figures and a solid body, circumference, superficial extent)
- basic problems with the concept of quantity (converting units of time, length).

+ The content lack of different formula and relations used in mathematics, chemistry and physics at school is also related to this question. Therefore, their application is out of question, in vain is there the calculator at their disposal. The formation of analogous and abstract thinking is also problematic for children suffering from dyscalculia. In Hungary children suffering from dyscalculia take part in school lessons, but they have extracurricular lessons with experts (speech therapist, special needs teacher). These lessons are held in Educational Advisory Centres and Committees for Examining Learning Skills and Abilities. Fortunately, the number of these institutions and experts is sufficient and there are special programmes and equipment available. The aim of dyscalculia-therapy is to set the ground for learning mathematics, to form the skills and expertise, to help the process of abstraction in order to use it for obtaining in-

formation independently, and also to develop underdeveloped and wrong psychic functions or to compensate for them./ Dékány Judit, 1995./

The success of these activities depends on the cooperation among experts, parents and school teachers. That is why school teachers have a great responsibility. A supportive and helpful attitude can provide a sense of security for children with dyscalculia. It is essential that we must make children like mathematics. If children take part in the lessons with pleasure, then we can use the numerous opportunities hidden in it for development.

The role of mathematical tasks with a text

There are two main areas of work with mathematical tasks with a text in lower primary classes: interpreting operations and developing problem-solving skills in the field of model-making. Those children who have problems with recognising abstract signs or have difficulty with abstract thinking or model-making, can be helped a lot by getting acquainted with other methods and reasoning instead of using algebraic solutions (open sentences).

Mathematical tasks with a text can help to develop children's skill to understand or their ability to find the essence. Uncovering connections, separating the known and the unknown can be done in the text even without numbers. Mathematical tasks with a text can help to develop perception-cognition, attention, memory, thinking, speech and linguistic skills, as well. They teach children self-discipline, endurance and being able to concentrate for a longer period of time. When interpreting the text and solving the problem, children's logical thinking will develop. They also help interpret the four fundamental arithmetic operations, recognise inversion through tasks with a "reverse" text. They can also help to recognise analogies and they develop abstract thinking.

The method of solving can be found without numbers. That should always be told by the children. However, it might happen that children will not be able to use an open sentence to do that. Using mathematical tasks with a text appropriately in maths lessons can contribute largely to the success of extracurricular dyscalculia-therapy.

Solving strategies

Children often encounter problems that can be solved without using mathematical models. We should exploit their enthusiasm and we should provide a scope for the variety of solutions, but gradually and carefully we should introduce the use of mathematical tools, since in case of complex problems or larger numbers, only the use of models can help. In case of children with dyscalculia this process is much slower. We should be careful to use a relatively simple system of signs. When looking at college students, we often experience the opposite of it – they immediately try to find a mathematical model even if the problem could be solved with other methods in a much simpler way. The problems described in the following can be solved with open sentences as well, but now I would like to show the role of other procedures and methods.

1. Acting out or performing a story

In order to be able to solve a problem, one has to understand that. It can be done by acting the story out or performing it with the help of different objects. If children are able to do that, then it means that not only has he understood the problem, but has also managed to find the answer to the problem. For, example in case of the next task the situation is the following:

„There are ducks and rabbits in a courtyard. We know that they have 8 heads and 22 legs altogether. The question is how many animals are there of each kind?"

The task can be represented with drawings, but moving towards model-making we can also use paper disks (heads) and plastic sticks (legs), as well. First we put the heads on the table, then we put two legs under each one and then we distribute the remaining legs. Ducks will have two legs and rabbits will have four. In case of children with dyscalculia the problem can even be presented without numbers: "They have so many heads and so many legs". The solution can be acted out according to the procedure mentioned above. Manipulation with objects can help to teach concepts, thinking procedures and methods. /Krapf, 1937/. We are also aware that this is an age-related phenomenon, as well but in case of children with dyscalculia it is even more important, since their abstract thinking develops at a later age.

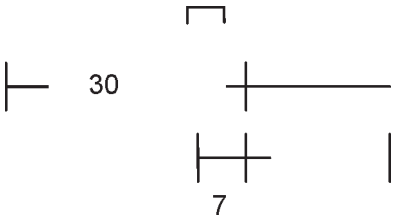
2. Using drawings and signs of relation

Understanding can be helped if children make a drawing about the mathematical task with a text. This drawing is usually reality-based at the beginning then it can become more and more abstract, this is an important moment in the process of abstraction. We often apply a reverse technique – children have to tell what the picture tells. This way children can go along the concrete way of solution and turning into a mathematical model in parallel, when they translate the information that can be seen in the picture into the language of mathematics. (operations, open sentences). / C.Neményi 1999./

3. Using illustrations, diagrams

The simplest abstract illustration is the use of periods. This method works especially well if children learnt how to use the kit if colour plastic sticks as a tool when learning the concept of numbers and operations. (representing the number as an index-number). For example, in case of the next task::

„When Dad was 30 years old, his son was 7 years old.. Now dad is twice as old as his son. How old is the son now?"



4. Rephrasing a text

Rephrasing the text in a different format can be a very important tool for understanding. The story should be told and analysed should first be done aloud in the framework of a dialogue between the teacher and the student. This can provide an example for children how he/she should think and reason alone later, possibly having a "dialogue" with him/herself. A lot of children find the solution this way with the help of a text and reasoning, although they might not be able to write it down.

When solving problems the method most often followed is the "straight-direction" train of thought. However, there are often problems when a reverse

train of thought can make the solution simpler. These are the problems when we know the final state and we are trying to find the starting phase. In these cases we have to rephrase the original text in a way that we start out from the final state. There are numerous areas in mathematics that contain these types of tasks - ranging from lower-primary mathematics to advanced mathematics. Reverse train of thought or reverse reasoning can be applied for this kind of tasks. This principle is applied by the method of „disassembling" used for solving open sentences in lower primary classes.

The thinking methods used in mathematics can often be made more understandable for children with the help of examples, games and procedures taken from totally different areas of life. For example: let's give into a child's hand a paper figure that has been folded by us and let's ask him/her to fold the same figure. The child will disassemble the figure until he/she reaches the starting state, then he/she will fold it repeating the folds in a reverse order. Thus he/she has discovered the strategy: reverse reasoning! or: let's play them a short scene from a film and then let's show the same scene starting from the end. This is humorous this way, but it is even more important that it will be seen how we can get from the beginning to the end through a series of events - and vice versa, as well: from the final state to the beginning. Children can also perform such a series of actions in the right order or in a reverse order. Then we should follow with a text: what they do in one order and what they do in reverse order.

The following solutions all use the reverse train of thought in a different format. For example: „I have thought of a number , I have added 5 to it, then I have divided it by 2, and then I have subtracted 17 from it, thus I had 10 as a result. Which number have I thought of?"

a) we do not use an open sentence, we just reverse the train of thought from for the text. Thus even children with dyscalculia can solve the task (without using any abstract symbols).

b) we illustrate the series of operations with the help of cards (the cards contain the mathematical operations with the numbers related to them), then we will place the operations that have to be done underneath in a reverse order:

$\rightarrow +5 \rightarrow :2 \rightarrow -17 \rightarrow \ = 10$

$\leftarrow -5 \leftarrow \cdot 2 \leftarrow +17 \leftarrow = 10$

Let's do the series operations placed this way!...So you have thought of 49.

c) based on the series of operations, we can use an open sentence for the task. This is much more complex than the placement with cards, we have to be careful about the use of brackets here.

5. *The trial/error method*

This method has a great significance. On one hand, it can provide a sense of achievement for children. On the other hand through the trials he/she will discover connections relationships among quantities.

„There were 2 flowers in the vase. I have added some to them , thus there were 5 flowers in the vase altogether. How many flowers have I added?" The story can be written down in the form of an open sentence: 2 + $\square$ = 5 The solution, however, will not work with the inverse of the incomplete operation at the beginning. Instead, children can try to find the solution with the method of trial and error. They will try what they will get by adding different numbers to 2 and they will choose the one that makes up 5.

Or for example: „Peti and Dani were picking apples, they have picked 80 altogether. Peti has collected 10more than Dani. How many apples has each of them collected ?" Children can disassemble 80, and they can choose the one where the distinction is 10 a between the two numbers.

6. *Using charts*

It is worth recording the results of trials in a chart. For the majority of people, visual representation helps to recognise connections. By recording information in a chart, children will learn to conjugate the quantities related to each other. The chart can also be used well when there is no evident answer to the question due to the nature of the data, for example: „The ducks and the rabbits in the courtyard had 20 legs altogether. How many ducks and rabbits were there?". Or in more complex mathematical tasks with a text (see the ones described in 3 above), we can make a chart using one piece of information, from which we can find out the pair of data that corresponds to the other connection. In this particular case, we can make a chart about the corresponding ages of the father and son, and then we try to find when the father will be twice as old as his son.

7. The method of equal alterations

Children can be made to discover this problem-solving procedure during the procedure described in 1. when children put various objects according to the text. Let's look at the task described there. Let's see what will change if we replace a duck for a rabbit or vice versa! The number of heads will not change, but the number of legs will always decrease or increase by 2. The train of thought can be traced easily in one's head, but the differences among children are indicated by that fact whether they are able to do that in their head, or they need modelling. This method develops function-like thinking, if we observe what alteration a particular alteration results in. As a beginning state, it is worth assuming that all of them are the same and make the alterations afterwards.

Summary

Numerous methods can be used to solve mathematical tasks with a text, we cannot give a general method, but we can provide numerous ideas. In this lecture I have proved that the other methods shown can be equally valuable, moreover, from the point of view of developing logical thinking and problem-solving thinking they can be even more valuable than solving the problem with an equation. That is why they are especially important for children suffering from dyscalculia who try to avoid algebraic

From all the methods described above, I would emphasise reverse reasoning, which is a useful method for problem-solving. Krutetski considers the transfer from one train of thought to reverse train of thought the most important fundamental skill of acquiring mathematics. /Krutetski, 1977./ it can provide great support not only to the acquisition of mathematics, but also to problem-solving in everyday life.

The most important elements of the dyscalculia-therapy are still the number-concept, the routinising of the fundamental operations and their continuous repetition, because these children generally have problems with long-term memory. But it would also be important for these children to acquire knowledge that is useful on the long run, as well, and one of this could be getting to know problem-solving procedures. In the meantime we can also achieve to make them happy adults and live a life without psychic injuries. And apart from experts working for pedagogic services, teachers working in schools can also do a lot to achieve that.

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THE CONCEPT OF THE SQUARE AND THE RECTANGLE AT THE AGES OF 10-11

Szilágyné Szinger, Ibolya[1]

Abstract. *I took part in a developing tecahing experiment for which I had personally designed the material of the lessons and the method of processing it. I participated in the lessons as an observer. I invited a training teacher of mathematics to teach in 4th grade of the Training Primary School of Eötvös József College. In the course of the developing teaching I examined the formation of several geometric concepts but in this essay I deal with the development of the concept of the square and the rectangle in detail.*

My question for research is to see how our teaching of geometrics in lower primary – within this the teaching of the concepts of the square and the rectangle – relates to the Van Hiele geometric levels and to see how effectively the concrete material activities occuring at these levels contribute to the development of the concept of the square and the rectangle.

My hypothesis is that in lower primary (grades 1-4) the first two Van Hiele phases of geometric teaching can be realized. It is not possible to move onto the third level by the end of lower primary. Although concept classes are formed (rectangle, square) but there are hardly any links between them. Children do not yet perceive the inclusive relationship.

P-H. Van Hiele divided the geometric learning process into 5 levels. At the level of global recognition of geometric objects (level 1), children perceive geometric objects as a whole. They easily recognize the different objects based on their shapes, learn the names of objects but do not recognize the relationship between the objects and their parts. They do not recognize the cuboid in the cube, the rectangle in the square because these are completely different things for them. At

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the descreptive level (level 2), children dissect objects into parts then put them together. Observation, measuring, drawing and modelling play an important role at this level. Students observe and list the properties of objects (parallelism or perpendicularity of planes and sides, having right angles, symmetry properties etc.) but they do not make definitions and do not yet recognize the logical relationships between properties.

In the essay I present the developing teaching experiment and its observations which I support with measurement results.

Key words: *mathematics teaching, square, rectangle.*

Through May-June 2006, I took part in a developing tecahing experiment for which I had personally designed the material of the lessons and the method of processing it. I participated in the lessons as an observer. I invited a training teacher of mathematics to teach in 4th grade of the Training Primary School of Eötvös József College. In the course of the developing teaching I examined the formation of several geometric concepts but in this essay I deal with the development of the concept of the square and the rectangle in detail.

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My hypothesis is that in lower primary (grades 1-4) the first two Van Hiele phases of geometric teaching can be realized. It is not possible to move onto the third level by the end of lower primary. Although concept classes are formed (rectangle, square) but there are hardly any links between them. Children do not yet perceive the inclusive relationship.

For small children a directed process of geometric learning already begins in kindergarten. The development of the concepts of geometric objects (geometric bodies, geometric figures) starts by examining the shapes of the environment. Specifying the properties typical for these sets of objects already represents a higher level of learning.

P-H. Van Hiele divided the geometric learning process into 5 levels.

At the level of global object recognition (level 1) children perceive geometric figures as a whole. They easily recognize them by their shape, they learn their names but they do not perceive the relationship between the figure and its parts. They do not recognize the cuboid in the cube, the rectangle in the square because these are completely different things for them.

At the descreptive level (level 2), children dissect objects into parts then put them together. They recognize the planes, edges, points of geometrical bodies. They also recognize geometric figures, which are bordered by curves, sections, points, as the planes of geometric bodies. Observation, measuring, drawing and modelling play an important role at this level. Students observe and list the properties of objects (parallelism or perpendicularity of planes and sides, having right angles, symmetry properties etc.) but they do not make definitions and do not yet recognize the logical relationships between properties. Just because they recognize the common properties of the square and the rectangle, we cannot expect them to draw the conclusion that a square is a rectangle.

At the level of local logical arrangement (level 3), students establish relationships between objects and their properties. The possibility now appears to deduce certain properties of objects from others. They understand the role of specification and definition. But the line of logical deductions is determined by the textbook (and the teacher). The creation of a need to prove deductions begins but is limited to certain objects only. At this level a square is already a rectangle.

Education corresponding to level 4 (attempt at creating a complete logical structure) and level 5 (axiomatic structure) falls under the competence of secondary and higher education.

Each learning period of the Van Hiele model builds on and expands the thinking developed in the previous phase. Moving from one level to the next is continuous and gradual while students acquire the mathematical concepts corresponding to each of the different levels. This process is specifically influenced by teaching, its content and method. In order to develop the right geometrical thinking, none of the levels may be skipped. Each level has its own characteristic language, system of symbols, logical structure. An important educational aspect of Van Hiele's theory is that no students at a lower level should be expected to understand instructions worded adequately for a higher level. According to Van Hiele, this is the main reason of failures in mathematical education.

With regards to the teching of mathematical concepts, R. Skemp, mathematician-psychologist, states the following:

„No concepts of a higher level than the ones he already knows can be communicated to anybody through definitions, it can only be done in a way where a multitude of appropriate examples are provided. Because in mathematics the before mentioned examples are almost all various concepts, first of all we need to make sure that the students have already acquired these concepts. … Choosing the right examples is a lot more difficult as one would think. The examples must have those common properties which form the concept itself but must have no other properties in common." (1975)

When building a concept it has to be placed in the system of already existing concepts (assimilation). However, it might be necessary to modify the already existing system, scheme (accomodation) in order to place the new concept in it. The balance of assimilation and accomodation is essential for proper concept development. If this balance is upset – i.e. assimilation is not followed by proper accomodation –, the student's own explanatory principles will gradually become part of his mathematical concepts which may lead to conceptional confusion. This is where the role of the teacher, whose task is to maintain this balance, becomes important.

The developing educational experiment referred to at the beginning of this essay includes 16 lessons, the aim of which was to implement teaching of geometry according to the Van Hielle model. In the first lesson I had the 26 fourth grade students write a pre-test which helped me to confirm that stepping from level one (global recognition of objects) to level two (analysis of objects) and the further development of geometrical thinking is possible at this stage. When putting together the pre-test, I took into consideration the material of the previous year (grade 3) and the experience of the training teacher. The first lesson of the developing experiment was the first lesson of the 4th grade subject matter at the same time.

The first two exercises of the pre-test had been designed to demonstrate the concept formed about the square and the rectangle. The first exercise is related to the realization (formation) of the concepts of quadrangle, square and rectangle while the second one is related to the identification (recognition) of concepts. In exercise one I asked for the drawing of a quadrangle, a rectangle and a square. In exercise two, from 16 different geometric figures, those ones had to be chosen which were bordered by straight lines only, those which were quadrangles, the rectangles and finally the squares.

The results of exercise 1 (26 students) are summarized in the following chart:

a) Drew a general quadrangle for a quadrangle (people)	9
Drew a square for a quadrangle (people)	11
Drew a rectangle for a quadrangle (people)	5
Was unable to draw a quadrangle (people)	1
b) Drew a rectangle correctly (people)	26
c) Drew a square correctly (people)	25

The solution of the task may be called successful as 1 child was unable to draw a quadrangle and also 1 child was unable to draw a square. The ratio (42%) of those children who drew a square for a quadrangle also is relatively high.

Evaluation of exercise 2:

a) Listed without mistake all those geometric figures that are bordered by straight lines only. (people)	24
b) Listed without mistake quadrangles. (people)	21
c) Defined rectangles correctly by including squares as well. (people)	1
Did not include squares among rectangles but made no other mistake.	5
Did not include squares among rectangles but included general parallelograms. (people)	18
d) Listed squareswithout mistake. (people)	14
Omitted the square standing on its point from the list. (people)	7

The measured data shows that in case of the rectangle and the square more development is required in concept identification. Nearly 20% of the students did not included squares among rectangles but made no other mistakes. A further almost 70% did not include squares among rectangles either but included the general parallelogram. Nearly 90% of the students did not consider the square to be a rectangle. This perfectly correponds to the first two levels of Hiele. It is interesting however, that 27% of the children did not recognize the square in the square standing on its point. The mistakes that occured indicated that going forward I'll have to place great emphasis on providing and discussing proper examples and counter examples for the sake of correct concept development. When using the word „proper" I mean the appropriate quantity

of examples and counter examples on one hand and an appropriate variety of these (e.g. they should encounter rectangles and squares in different positions, too) on the other. Furthermore, they should make the key properties of concepts recognisable for children while enabling them to filter out the unimportant ones.

When planning the lessons I focused on making sure that children discover geometric concepts first through concrete experiences, in real games and material activities, then at a visual level (drawing) and finally at a symbolic level (spoken and written language).

Exrcises for concrete material activities:

E.g. : 1. What is the simpliest way to cut out a square from a rectangle?

2. A rectangle or a square is cut into two triangles along its diagonal and then further geometric figures are created from these triangles.

3. Creating and naming various geometric figures from a stripe of paper with single straight cuts.

4. Creating rectangls of different length but a given height from a stripe of paper. Etc.

During this task a boy called Bence turned to the teacher in desperation: „For one of them all the sides are the same. I've measured it. Each side is 4 cm. This is a square. It won'tbe good."

The teacher reacted in the following way:

„Well, this is how it turned out to be. You have got such a special rectangle. As you can see, the square is a special rectangle. Your solution is good."

Bence gave a sigh of relief. The teacher showed this rectangle of Bence to the class. As it turned out he wasn't the only one who managed to cut a rectangle like that. The teacher repeated to the class she had told Bence previously. We were happy we had the chance to highlight the relationship betwen the square and the rectangle.

Some exercises at visual level:

E.g. 1. Making squares and rectangles on a point grid.

2. Making various quadrangles on a point grid.

3. Making various triangles on a point grid.

4. Making quadrangles with specified properties. Etc.

The discussion of the properties of the various geomtric figures took place at the symbolic level: determining the number of sides and points for polygons, quadrangles, rectangles, squares. Examination of the length, parallelism, perpendicularity of the sides. Determining the number of symmetry axes and the size of angles formed by adjoining sides. The properties of the square and the rectangle were also compared. When the teacher asked whether all the properties of the rectangle are also true for the square, the majority of the answers was no. Two children thought that this statement was true. Although the teacher tried to explain this to the students, several of them told her she was wrong because „the rectangle has sides of two different lengths while the square hasn't". The examination of geometric properties naturally took place together with the visual presentation of the figure in question.

I closed the developing teaching experiment with a test worksheet. The worksheet was completed by 25 students in class 4.c. Upon my request, 23 students in class 4.a. and 24 students in class 4.b. also completed the worksheet. In these classes mathematics was taught by another training teacher. I hereby present only those exercises of the worksheet that are related to the development of the concept of the square and the rectangle.

Exercise one was related to the identification of the concepts of the quadrangle, the rectangle and the square where quadrangles, rectangles and squares had to be chosen from 15 geometric figures.

I summerized the outcome of the exercise in the below chart:

	4.c	4.a	4.b
a) Listed quadrangles without mistake. (people)	25	16	20
b) Defined rectangles correctly, included squares here. (people)	2	2	2
Did not include squares among rectangles but made no other mistake. (people)	13	11	3
Did not include suqares among rectangles but included general parallelograms. (people)	10	10	19
d) Listed squares without mistake. (people)	20	9	14
Omitted the square standing on its point from the list. (people)	5	10	9

The recognition of quadrangles was perfect in the experimental class. 52% of the students did not include squares among rectangles but made no other mistake. This improvement is significant compared to 20% in the pre-test. 40% of students did not include suqares among rectangles but included general parallelograms. Although this is a significant improvement compared to the previous 70% but I still consider this ratio high. 90% of the children still do not consider the square a rectangle. The ratio is nearly the same in the control groups. This data supports my hypothesis that it is not possible to move to level 3 of the Van Hiele model in geometric thinking by the end of lower primary. Only the completion of the firts two levels is realistic.

There was a positive change in the recognition of squares as well. 80% of children (as opposed to the earlier 58%) listed suqares correctly.

Another exercise of the test worksheet was related to the qualities of the square and the rectangle. Children had to underline those statements from the ones given that were true about squares and in the second part of the exercise the ones that were true about rectangles. When evaluating the exercise, I only include perfect performance. 52% of the students in the experimental group found all the true statements regarding squares while this ratio was 35% and 42% in the control groups. The source of mistakes can partly be found in the inappropriate interpretation of the words „opposing" and „adjoining" and partly in the fact that the concept or parallelism and perpendicularity are not stable enough yet.

The last exercise of the test worksheet also supported my hypothesis. In this exercise the logical value of the following statements had to de determined:

The rectangle is a special square.

The square is a special rectangle.

The sides of the square are not equal.

All squares are rectangles as well.

The evaluation of the exercise is the following:

	4.c	4.a	4.b
Determined the logical value of all the statements correctly. (*people*)	6	4	5
Determined the false logical value of the statement „*The rectangle is a special square.*" *correctly. (people)*	16	12	14
Determined the true logical value of the statement „*The square is a special rectangle.*" correctly. (people)	12	9	11
Determined the false logical value of the statement „*The sides of the square are not equal.*" correctly. (*people*)	24	19	23
Determined the true logical value of the statement „*All squares are rectangles as well.*" correctly. (*people*)	12	11	11
Marked with different logical value the statements „*The square is a special rectangle.*" and „*All squares are rectangles as well.*" (*people*)	10	10	11
Determined the logical value of the statements „*The square is a special rectangle.*" and „*All squares are rectangles as well.*" as *true*. (*people*)	7	5	6
Determined the logical value of the statements „*The square is a special rectangle.*" and „*All squares are rectangles as well.*" as *false*. (*people*)	8	8	8

In this exercise the number of perfect answers is low in all student groups. It's no wonder of course as three statements out of four referred to the hierarchy between the square and the rectangle. We cannot be sure either that the children who found the second and fourth statements true were really aware of the subset relationship existing between the square and the rectangle. Those who found these statements to have different logical values (40% of the students in the experimental group, 43% and 46% in the control groups) made mutually contradictory decisions which indicates the uncertainity in determining the relationship between the concept class of the square and that of the rectangle. And those who found both of them false (32%, 34% and 35% of students) really didn't perceive any kind of hierarchy.

I feel that the developing teaching directed by me contributed effectively to deepening the concept of the rectangle and the square. The comparison of the pre-test and the closing worksheet results also proved this. Effectiveness is further proven by the fact that the results, compared to results experienced in the other two parallel classes, were generally better and in some cases much better in the control group.

For a closing thought let me quote György Pólya: „Nothing should be missed that has a chance to bring mathematics closer to students. Mathematics is a very abstract science and that's why it must be presented in a very concrete way." (1977)

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THE USE OF COMPUTERS IN TEACHING MATHEMATICS

Sanja Varošanec [1]

Abstract. *It is only natural that modern mathematics teaching should follow technological developments and that its aim is to introduce new teaching tools into the educational process in order to bring mathematics closer to the pupils, to motivate them for work, and to improve understanding, detecting and learning of mathematical concepts, phenomena and patterns. Just as overhead projectors, slide projectors, epi scopes, tape recorders and similar equipments were introduced into the teaching process in the past decades, so are we now witnessing increasingly frequent use of computers, related electronic devices and software in the process of teaching and learning. As is the case with the use of any other teaching tool, even the use of computers has its advantages as well as its disadvantages. In deciding when, where, why and how to use new technology, the teacher is using the following basic rules as guidance:*

- *The decision about when and how to use or not to use the computer in lessons depends on whether its implementation improves the existing teaching practice.*
- *The decision must be directly conditioned by evaluation whether the use of the computer will bring abut more efficient realisation of the individual teaching unit objectives.*
- *The use of the computer must enable the teacher and the students to achieve something which they wouldn't be able to achieve without the use of the computer, i.e. it must enable more efficient teaching to the teacher and more efficient learning to the student than it would be possible without this technology.*

Considering that in mathematics teaching, in senior grades of the Elementary school, geometry is being taught to the great extent, I shall refer to the use of the dynamic geometry software. It is a tool which enables different understanding of traditional geometrical contents and by means of which research and experimental methods obtain new and more important place in mathematics teaching.

Key words: *teaching tools, computer, dynamic geometry software.*

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FROM ACTIVE EXPERIMENTING TO ABSTRACT NOTION CONCEPT

Amalija Žakelj[1], Aco Cankar[2]

Abstract. *We are trying to present in this article certain didactic aspects of process-didactic approach to teaching and learning which we developed in our research, called »Process-didactic approach and understanding of mathematical notions in primary schools, (Žakelj, 2004). The research took place in 2001/02 and 2002/2003 with pupils aged 12 to 15. The main scientific question was how to research the link between children's thinking (cognitive structures, meta-cognition, and mental strategies) and the approaches in teaching mathematics. We verified it from the aspect of its impact on the quality and type of knowledge.*

We conceived our didactic model on the basis of the theoretical knowledge about children's mental development, including the newly established recognitions about children's thinking as well as our knowledge on social cognition. We have leaned on the theory of development psychology, which studies the development of notions in regard with the development level of children's thinking, and we took note of newly established cognitive-constructivistic recognitions of pedagogical science on learning, which underline children's activity in the learning process.

Key words: *mathematics teaching, active experimenting.*

Experience learning

While establishing the process-didactic approach we took note that the learning process is essentially influenced by the level of thinking development, the structure of existing knowledge and the organisation of children's activities as well a by the encouragements for the environment. At the same time children's

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thinking was interpreted from the point of view of new recognitions on metacognition and from the connection between thinking and language.

The process-didactic method introduces the experience learning in practice, dialogue and all forms of participation (*influence of social interactions*) as an important part of knowledge construction. Experience learning includes: *modelling, active experimenting, independent search for sources, searching for similarities and connections, looking for examples and contra examples, it encourages development of problem knowledge* (solving open problems, understanding of problem situation, putting questions, learning of strategies for solving problems, establishing solutions, presentation of results, clarifying) and introduces forms of the *applicability of mathematics* in other areas and provides *linking/combining of knowledge*. While solving problems the emphasis is on the *processes and strategies of solving problems,* on clarifying, verifying solutions, presenting results and exchange of views. Teachers encourage motivation in the cognitive and social-cognitive conflict. Teachers' role is also in offering children various approaches to learning. What approaches children would use depends on their prior knowledge, on their cognitive maturity, on their orientations and their learning styles.

What is necessary to consider when introducing new notions?

In the assignment, we are describing below, we illustrate, that children's success in solving problems depends on their notion concepts and on their cognitive maturity.

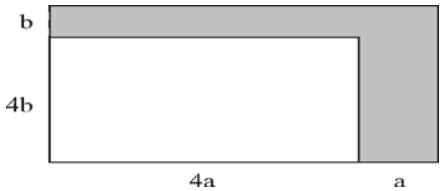
Assignment 1: When do you think two quantities are directly proportional? Give some examples.

Didactic aspect: Demonstration of understanding of the notion direct proportion by giving examples.

Assignment 1 was tested in the research (Žakelj, 2004) in which we among others studied also understanding of the notion direct proportional quantity. Even 77 percents of pupils gave a correct answer, however only 44 percents could give examples of direct proportional quantities. Although the cell of the definitions was full, the cell of notion concepts was by pupils, who could not give any example, empty or it was filled with wrong concepts. The cell of notion concepts fills up by the process of acquiring experiences.

Assignment 2: Calculate what percentage of the figure is coloured.
Didactic aspect: solving problem at a symbolic level.

Picture 1



Also the Assignment 2 was tested in the research (Žakelj, 2004) by which we tested the capability of 12 year old pupils in solving problems at a symbolic level. Only rare pupils could solve the exercise entirely and mathematically totally correct. Most pupils solved the assignment so that *they chose the concrete data by themselves,* some of them *determined them by measuring*, others drew a network and *defined the area unit* or they reached the approximate result by *assessing.* Various pupils' approaches in solving problems show *a link between the cognitive development of children, experiences* children have and the way they solve assignments.

From the way of solving the first and the second assignment we can assume that it is very important that teachers when selecting teaching approaches and when they define assignment difficulty levels make their choice from different presentations of notions, by which they introduce notions gradually and thus make sense in adjusting to the pupils' cognitive maturity and consequently influence their development. In the first example children had too little experience in acquiring notion conceptions, since besides the definition many pupils were not able to give not a single example, which points at their weak conceptions of notions, which are besides the definition a part of a notion structure. The second example shows that pupils have to walk gradually through all the phases: from concrete experiences, and from imaginative and symbolic level to abstract level. Thoughtless and too quick introduction of demanding abstract notions, overtaking their cognitive development, are from the point of view of establishing one's notion conceptions for children extremely difficult and often also ineffective.

From active experimenting to abstract conceptualisation

Development of mental concepts and understanding mathematical notions is for understanding of mathematics essential for the construction of knowled-

ge and it is also a precondition for the transfer of knowledge. It would be impossible to learn a new strategy for each problem. However it happens quite often that many pupils cannot make connection and present information as isolated part. If they listen to the same topic at two different subjects, they cannot connect it, but have two »separate knowledges«. The same thing happens when connecting notions within mathematics. If a pupil understands the relation as dividing two whole numbers, he would not be able to use that notion for learning direct and reverse proportion. It is therefore very important how we learn. A gradual transfer from concrete presentations of notions, through imagery and symbols, to abstract conceptualisation, is indispensable in the construction of notion concepts. It is natural because it in all cases follows the phases of cognitive development. Learning by experience foresees all those phases: active experimenting, researching, finding characteristics on a concrete model, acquisition of concrete experiences, thoughtful observations, which in the last phase can bring to the abstract conceptualisation of a notion.

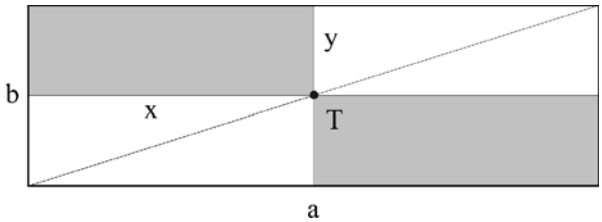
Inductive and deductive approach to solving problems

Pupils for whom it is correct that they first find a theory, the principle, and the rule solve problems deductively. However, if they have difficulties in the transfer from general to concrete, they should solve problems in an inductive way.

At a given problem situation or challenge we always can put several questions, which demand different level and type of knowledge. Therefore the decision on what question and to what depth an individual is going to reach depends on his knowledge. We are giving an example where we can reach out for different levels (from the primary school level to the secondary school level). The approaches to solving problems depend on the level and extent of knowledge. Pupils can solve the challenges of finding areas of shadowed geometrical figures by measuring, collecting partial solutions and put a hypothesis on the basis of the collected data. Of course, when using that approach it is necessary to talk to pupils and tell them that the results and solutions acquired by measuring, can be only an approximate estimation, since measuring only partly provides accurate data or findings. The established findings are always followed by clarifications. It is mostly descriptive at the primary stage possibly with examples. In the given case we can also at the level of primary schools clarify our findings with the help of mathematical facts on congruent of figures and on geometrical

figures, equal in area. In the secondary schools the assignment can be linked with solving of extremal problems by using derivative.

Challenge 2: In an ad hoc position of the point T on the diagonal of a rectangle with sides a and b we draw two parallels to both sides. Calculate the areas of shadowed rectangles on the picture.



Picture 2

Didactic aspect of challenge

We encourage pupils at their work to do the research, to observe, to measure, to compare and to put hypotheses. They make links and use notions of similarity, congruent in figures, equal in areas. This might be one of the chances for pupils to complement and correct possible wrong notion concepts. From the didactic point of view this kind of exercise is useful for building concepts of the notion of areas, of figures, equal in areas, congruent figures and similar figures.

Activity of pupils

The problem requires productive use of mathematical knowledge: areas of rectangle, theorem on congruent and similarities of triangles, linking of knowledge and analysing a given problem situation.

1. Consideration on the challenge and asking question

The insight into the problem enables us to have a careful look at the picture and at finding relationships between geometric elements. For instance, while observing pupils find out: The areas of shadowed rectangles are changing with the movement of the point T along the diagonal. Questions follow:

Formation of question

How do the areas of the shadowed rectangles change if we move the point T along the diagonal?

What is the proportion between the areas of both shadowed rectangles?

What is the proportion between the sum of areas of both shadowed rectangles and the area of the entire rectangle?

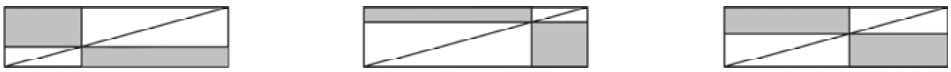
At what position of the point T is the sum of areas of both shadowed rectangles the greatest?

2. Performance

There are several ways of solving problems. Allow us to show two approaches.

a) Inductive approach – by measuring

We can get the necessary data also by measuring and calculating.



Picture 3

We draw several different situations: Point T is located for example close to the vertex, or in the centre of diagonal … For the selected concrete examples of measurements we define the length of the sides, which we need to calculate the areas of shadowed figures. On the basis of measured data we calculate the areas. It is wise to put down the results of measurements and arrange/draw a table. In case of systematically organised data we can see the solution much faster.

The data are written in the table in a general way and pupils are of course going to work with concrete data.

Table 1: Areas of shadowed rectangles when moving point T along the diagonal

Division of a side of the rectangle into n parts	x	y	Area of the first and second shadowed rectangle	The sum of areas of both shadowed rectangles	Proportion between the sum of areas of both shadowed rectangles and the area of the entire rectangle
	0	b	0	0	
8	a/8	7b/8	7ab/64 7ab/64	14ab/64	$14/64 = 2 \cdot 7/8^2$
4	a/4	3b/4	3ab/16 3ab/16	6ab/16	$6/16 = 2 \cdot 3/4^2$
2	a/2	b/2	ab/4 ab/4	ab/2 = 2ab/4	$2/4 = 2 \cdot 1/2^2$
8	7a/8	b/8	7ab/64 7ab/64	14ab/64	$14/64 = 2 \cdot 7/8^2$
		...			
	0	0	0		General: $2(n - 1)/n^2$

3. Conclusion

By observing a concrete sequence children in the primary schools can make conclusions:

- proportion between the sum of areas of the shadowed rectangles and the area of the entire rectangle is $2(n - 1)/n^2$;
- areas of both shadowed rectangles are equal;
- the sum of areas of both shadowed rectangles is the biggest if the point T is in the centre of the diagonal.

4. Clarification

Conclusions can also in the primary schools be clarified with the help of theorems on congruent.

b) Deductive approach – assumption, linking and the use of mathematical notions and rules

By considering similarities we get the result that, form $x_1 = a/4$ the noting of proportion is followed:

$a : b = a/4 : y_1$, from which it follows that $y_1 = b/4$ and $y = 3b/4$. The area of shadowed rectangles is: $a/4 \cdot 3b/4$ and second $3a/4 \cdot b/4$. The areas of shadowed rectangles are equal.

In general: if $x_1 = a/n$, then by using similarity we get the proportion: $a : b = a/n : y_1$, from which we can assume that $y_1 = b/n$ oz. $y = (n - 1)b/n$. Areas of shadowed rectangles are: $a/n \cdot (n-1)b/n$ in $(n-1)a/n \cdot b/n$, from which we can assume that they are equal at each n.

Where is the area the biggest? At the primary school level it can be found out on the basis of observation of the sequence, and at the secondary school level by the help of derivative. Area $a/n \cdot (n-1)b/n$ is the biggest when $n = 2$. This means in cases when the point T is in the centre of the diagonal.

Joint findings

Of we slide the point T along the diagonal, the sum of areas of both shadowed rectangles change from the value 0, when the point T is in the vertex, to the biggest value $ab/2$, when the point T is in the centre of the diagonal.

The proportion of the areas of shadowed rectangles is 1 : 1 or the areas are equal at all positions of point T. Areas are the biggest when the point T is in the centre of the diagonal. The proportion between the sum of areas of shadowed rectangles and the area of the entire rectangle is $2 \cdot (n-1)/n^2$.

CONCLUSIONS

With the challenges shown above we wanted to indicate that for successful teaching and learning also the process is important and not only final aims. In case of such open problems we learn on one hand the strategy of solving problems and on the other the active search for solutions or understanding of notions by various approaches helps pupils to take different views on the contents and eases the construction and understanding of basic notions.

At the transmissive approach to learning and teaching the situation often resembles the one where questions follow each other in fast sequence and there is no time for thinking or checking and clarifying. In the challenge ***Investigate*** *the area of shadowed rectangles* a pupil is confronted with a rather different demand as it were the case if the assignment said: ***Calculate*** *the area of shadowed figure at the given data.* The difference is essential. In the first case pupils inde-

pendently ask questions which they later investigate. For example: are the areas equal, what is the proportion between the areas of shadowed figures, how do the areas change if we slide the point along the diagonal, what is the share of shadowed figure in the rectangle. While solving the problems pupils try to find various ways of solving and different solutions. For example they draw pictures, shape models, measure lengths and calculate areas, analyse pictures and link data, they calculate, compare results, look for congruent of figures, they establish hypotheses, as for example: *»areas are equal«*. They also clarify and demonstrate their findings. In the second case the pupils would on the basis of given data calculate a required area by using a form. They would often perform it as a routine without any thinking. By accumulating various experiences from the first case they gradually fill their notion concepts, which are, as we said in the beginning, the key item for understanding notions.

We can draw a conclusion that learning is more effective if words are given a sense and if the content is not learned by heart, if we repeat with our own words, if we help ourselves by modelling, active experimenting etc. When establishing notion concepts it is not important only that pupils solve as many exercises as possible, but it is also important that they solve complex assignments where they link their knowledge. More complex problems usually include understanding and the use of notions and the mastering of various procedures. Open problems are very good, they initiate discussions among pupils, asking research as well as they open up in the continuation the vast opportunity for further discussions both about the ways of solving problems and about solutions.

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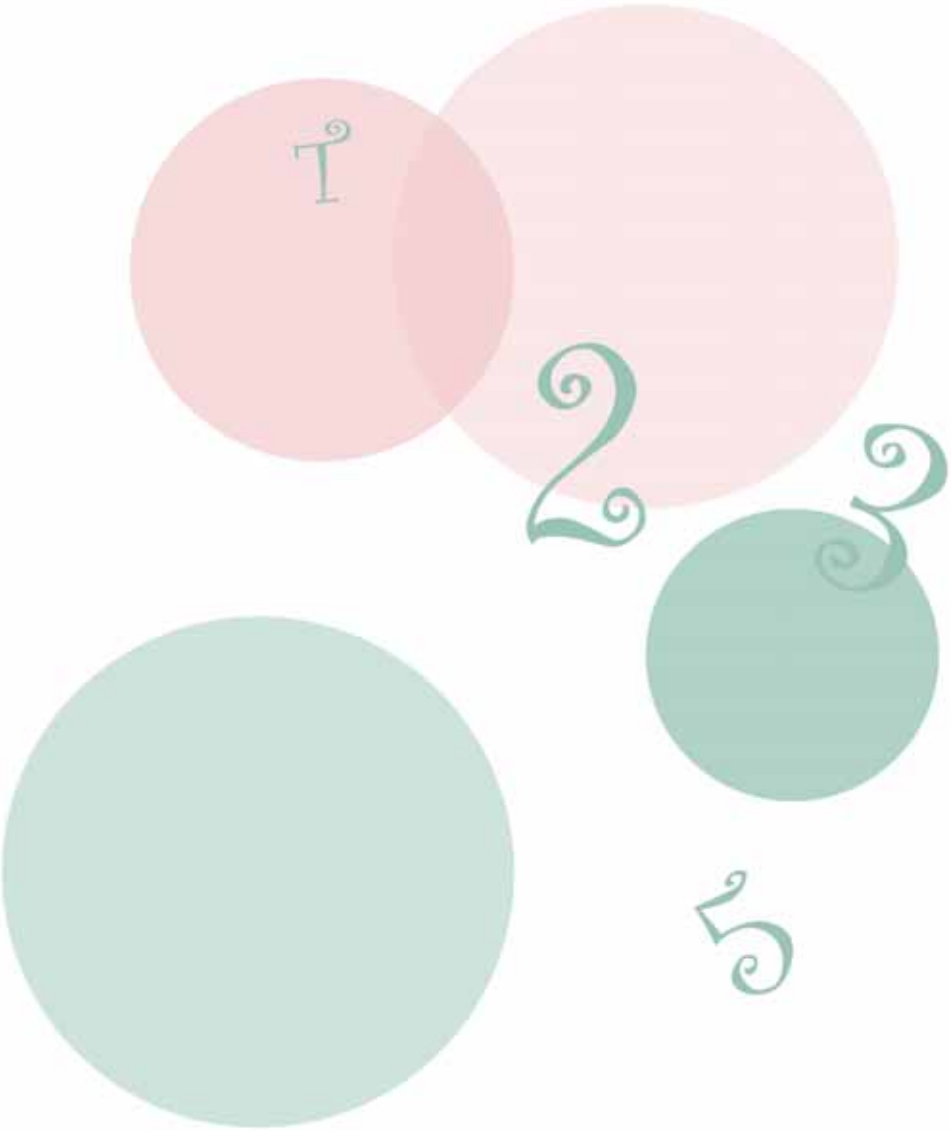
Pozvana predavanja



PRIKAZ ODOBRENOG PROGRAMA IZ METODIKE NASTAVE MATEMATIKE U SKLADU S BOLONJSKOM DEKLARACIJOM NA PRIRODNO-MATEMATIČKOM FAKULTETU U SARAJEVU

Šefket Arslanagić[1]

Sažetak. *Akademske 2005./2006. godine na Univerzitetu u Sarajevu započelo je obrazovanje studenata prema novim programima usklađenim s bolonjskom deklaracijom. Na Odsjeku za matematiku Prirodno-matematičkoga fakulteta u Sarajevu studira se po shemi 3+2 godina. Nakon trogodišnjega studija matematike, student može nastaviti studij na 4 moguća smjera.*

Završetkom jednog od tih smjerova kandidat stiče naučni stepen magistar metodike nastave matematike.

U članku se daje prikaz studija na smjeru Metodika nastave matematike.

Ključne riječi: *matematika, nastava matematike, znanost.*

Nakon trogodišnjeg studija matematike na Odjelu za matematiku PMF u Sarajevu stiče se diploma visoke spreme, a poslije završene još dvije godine zvanje magistra (određenog smjera).

Postoje slijedeća četiri smjera:

- Teorijska matematika
- Primijenjena matematika
- Teorijska kompjuterska nauka
- Metodika nastave matematike.

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Dakle, postdiplomski studij traje četiri semestra. Uspješnim završetkom postdiplomskog studija na prva tri smjera, kandidat stiče naučni stepen **magistra matematičkih nauka** (uz naznaku smjera), a završetkom studija Metodika nastave matematike, kandidat stiče naučni stepen **magistar metodike nastave matematike.**

Predavanja za smjer Metodika nastave matematike (4. i 5. godina studija) odvijaće se prema slijedećem planu (rasporedu):

Semestar	Predmet	Nastavnik
I. semestar	Algebarske i geometrijske nejednakosti	prof. dr. sc. Šefket Arslanagić
	Osnovi geometrije	prof. dr. Mirjana Malenica
II. semestar	Apstrakcija i generalizacija u algebri	prof. dr. Hasan Jamak
	Matematička logika	prof. dr. Medo Pepić
III. semestar	Historija i filozofija matematike	prof. dr. Muharem Avdispahić
	Osnovi teorije brojeva	doc. dr. Lejla Smajlović
IV. semestar	Aspekti rada s matematički nadareni učenicima (MNU)	prof. dr. sc. Šefket Arslanagić
	Historija graduacija	prof. dr. Mirjana Vuković

Od osam ponuđenih predmeta kandidat bira šest koje će slušati. Kandidat je dužan položiti najmanje pet predmeta koje je odslušao. Jedan od odslušanih predmeta kandidat može zamijeniti sa nekim drugim predmetom postdiplomskog studija nekog drugog srodnog fakulteta ili univerziteta uz prethodnu suglasnost Vijeća postdiplomskog studija.

Voditelj smjera Metodika nastave matematike je prof. dr. sc. Šefket Arslanagić.

Uslovi za upis na postdiplomski studij su završen dodiplomski studij matematike ili srodnih nauka s prosječnom ocjenom najmanje osam (8) i poznavanje jednog svjetskog jezika.

Kandidati koji su diplomirali srodne nauke, podliježu provjeri osposobljenosti za postdiplomski studij Metodike nastave matematike.

Odredba iz prethodnog stava može se primijeniti i na zainteresovane kandidate koji su završili studij matematike s prosječnom ocjenom manjom od osam (8).

Nakon položenih ispita kandidat brani magistarski rad koga je izradio u suradnji sa svojim mentorom.

VLOGA RAZLIČNIH REPREZENTACIJ MATEMATIČNIH KONCEPTOV PRI UČENJU Z RAZUMEVANJEM

Tatjana Hodnik Čadež[1]

Povzetek. *Reprezentacija je v prvi vrsti nekaj, kar stoji namesto nečesa drugega. Pri vsaki reprezentaciji moramo opredeliti: (1) reprezentirajoči svet, (2) svet, ki ga reprezentirajoči svet reprezentira (v nadaljevanju svet, ki ga reprezentira), (3) kateri vidiki sveta, ki ga reprezentira, so reprezentirani, (4) kateri vidiki reprezentirajočega sveta reprezentirajo ter (5) povezavo med svetom, ki ga reprezentira, in reprezentirajočim svetom (Palmer, 1978).*

Ideja o reprezentacijah je v matematiki stalno prisotna. Za komunikacijo matematičnih idej je reprezentacija le-teh nujna. Razlikujemo med notranjimi (miselne predstave) in zunanjimi reprezentacijami (okolje). Zunanje reprezentacije so sestavljene iz strukturiranih simbolnih elementov, katerih vloga je 'zunanja' predstavitev določene matematične 'realnosti'. Pri pouku matematike v glavnem uporabljamo konkretne reprezentacije, grafične reprezentacije in reprezentacije z matematičnimi simboli. V prispevku se osredotočamo na pomen uporabe različnih zunanjih reprezentacij v procesu poučevanja in učenja matematike. Povezovanje reprezentacij, kot ključni dejavnik pri učenju o matematičnih pojmih, ponazarjamo z modelom reprezentacijskih preslikav. V okviru tega modela definiramo dva koncepta: razumevanje in pomenjanje. Učenčevo razumevanje razumemo kot njegovo sposobnost prehajanja med različnimi zunanjimi reprezentacijami, pomenjanje pa kot sposobnost rokovanja z določeno zunanjo reprezentacijo.

Ključne besede: matematika, pouk matematike, učenje z razumevanjem.

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1. Uvod

Reprezentacije matematičnih idej, bodisi s konkretne, grafične bodisi z matematičnimi simboli so ključnega pomena pri komunikacij v matematiki.

Razlikujemo med notranjimi (miselne predstave) in zunanjimi reprezentacijami (okolje). Kognitivni razvoj temelji na dinamičnem procesu prepletanja miselnih predstav in okolja (Karmiloff-Smith, 1992). To pomeni, da je uspešno učenje aktivno oblikovanje znanja v procesu interakcij med zunanjimi in notranjimi reprezentacijami.

Notranje reprezentacije, poznamo jih tudi pod izrazom kognitivne reprezentacije (Palmer, 1978), bi lahko opredelili kot miselne predstave, ki ustrezajo našim notranjim formulacijam ‚realnosti'. Notranje reprezentacije opredelimo torej kot miselne predstave oziroma miselne prezentacije (ne re-prezentacije): nekaj, kar nima originala, notranji svet izkušenj.

Zunanje reprezentacije so sestavljene iz strukturiranih simbolnih elementov, katerih vloga je ‚zunanja' predstavitev določene matematične ‚realnosti'. Z izrazom 'simbolni elementi' označujemo elemente, ki jih izberemo za reprezentacijo nečesa drugega. Objekt, ki reprezentira drug objekt (pojem) razumemo kot simbol. Pri pouku matematike ločimo tri vrste simbolnih elementov oziroma tri vrste zunanjih reprezentacij: *konkreten (didaktični) material*, *grafične ponazoritve* in *matematične simbole*. V nadaljevanju vsako od omenjenih zunanjih reprezentacij kratko predstavjamo.

2. Zunanje reprezentacije

2.1 Konkretne reprezentacije

Izraz konkretne reprezentacije lahko različnim ljudem pomeni različne reči. Za nekatere so konkretne reprezentacije v matematiki zgolj strukturirane reprezentacije, ki se uporabljajo izključno pri poučevanju in učenju matematike in nimajo posebnega pomena zunaj tega procesa. Tak material bomo imenovali strukturiran material in eden najbolj pogosto uporabljenih pri pouku matematike so prav gotovo Dienisove plošče, s katerimi ponazarjamo desetiške enote (enice, desetice, stotice, tisočice). Prav tako pa je pri procesu poučevanja in učenja matematike aktualen tudi nestrukturiran konkreten material, s katerim si učenci pomagajo pri usvajanju matematičnih pojmov. Omenimo ‚link' kocke, s katerimi učenci v Sloveniji ponazarjajo števila, z združevanjem kock v stolpce po 10 ponazarjajo desetice… Sicer pa je uporaba nestrukturiranega materiala

pri usvajanju števil in preprostega računanja na začetku šolanja zelo raznovrstna: igrače, žetoni, kamenje, perle… Prav gotovo pa so za računanje v obsegu večjih števil (do 1000) bolj uporabne desetiške enote (Dienesove plošče), saj s svojo obliko ponazarjajo desetiške enote oz. odnose med njimi (proste kocke – enice, paličice – desetice, plošče – stotice, kocke – tisočice). Z omenjenimi ploščami se učenci v Sloveniji učijo tudi računanja s prehodomn, kjer je v ospredju procedura ‚zamenjevanja' (pri odštevanju npr. 1 desetico zamenjamo za 10 enic, pri seštevanju pa npr. 13 desetic zamenjamo za 1 stotico in 3 desetice).

Obstaja splošno prepričanje (učiteljev, vzgojiteljev, staršev), da se učenci laže učijo matematiko, če imajo možnost rokovanja s konkretnim materialom. Raziskave s tega področja v tem niso enotne. Na primer, v letih 1960 do 1970 so na Nizozemskem zelo poudarjali uporabo Dienesovih plošč pri učenju aritmetike, a izkušnje na tem področju so jih pripeljale do ugotovitve, da so plošče po eni strani zelo primerne, uporabne za ponazoritev strukture desetiškega sistema, po drugi strani pa manj uporabne pri reprezentacijah kompleksnih računskih operacij (Beishuizen, 1999), kar jih je vodilo do uporabe drugačnih ponazoril, med drugim do nestrukturiranega materiala (Anghileri, 2001).

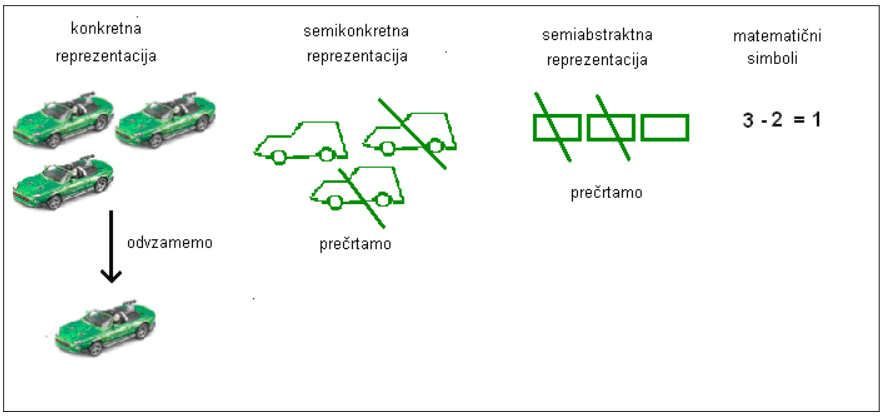
Omenimo še nekatere druge izsledke s področja raziskovanja uporabe konkretnega materiala pri matematiki. Fennema (1972) in Friedman (1978) utemeljujeta njegovo vlogo v nižjih razredih osnovne šole in ne zagovarjata uporabe konkretenga materiala v višjih razredih, Suydam in Higgins (1977) pa poročata o pozitivni vlogi uporabe konkretnega materiala za vse učence. Labinowicz (1985) je opazoval učence razredne stopnje pri rokovanju z Dienesovimi ploščami in ugotovil, da imajo učenci težave s povezovanjem teh plošč z zakonitostmi desetiškega sistema, po drugi strani pa sta Fuson in Briars (1990) ugotovila zelo pozitivno vlogo teh plošč pri učenčevem razumevanju seštevanja in odštevanja naravnih števil. Thompson (1992) ter Resnick in Omanson (1987) pa so ugotovili, da imajo Dienesove plošče zelo malo vpliva na učenčevo razumevanje algoritmov na razredni stopnji. Te nasprotujoče si ugotovitve nas opozarjajo, da konkreten material sam po sebi ne zagotavlja uspešnega učenja oziroma da je učenje kompleksen proces, katerega sestavni del je tudi rokovanje s konkretnim materialom. Prepričani smo, da rokovanje s konkretnim materialom, ki ni osmišljeno z natančno refleksijo procesa rokovanja in ni obravnavano v relaciji z drugimi reprezentacijami v matematiki, ne more voditi k uspešnemu učenju o matematičnih pojmih. Narava matematičnega pojma, način uporabe konkretnega materiala in material sam so dejavniki, ki vplivajo na proces učenja in poučevanja.

2.2. Grafične reprezentacije

Grafične reprezentacije so v matematiki na razredni stopnji najbolj zastopane pri ponazarjanju matematičnih idej. Matematični učbeniki, delovni zvezki ter drugo matematično gradivo so polni grafičnih reprezenatacij, ki se med seboj razlikujejo po domiselnosti, izvirnosti ter korektnosti. Nekatere so celo matematično vprašljive in didaktično neustrezne.

Poglejmo si primer grafičnih reprezentacij, s katerimi ponazarjamo koncept števil. Konkretna reprezentacija za števila so vsi števni predmeti, ki obkrožajo otroka, učenca. Seveda pa ne štejemo vsega po vrsti. Štejemo lahko samo predmete, ki imajo določene skupne lastnosti in jih hkrati lahko razločujemo. Lahko štejemo balone, barvnike, slike, ne moremo pa šteti balonov in barvnikov skupaj iz preprostega razloga: majhen otrok ne zna opredeliti, kaj je preštel oz. nadaljevati stavka,‚Prešteli smo 13 …' (13 česa?). Grafične reprezentacije števil so v glavnem ilustracije predmetov, živali in oseb, ki jih učenci izrazijo tudi s simboli oz. s številkami. Grafičnih reprezentacij pa ne uporablajmo zgolj za matematične pojme, ampak tudi pri ponazarjanju določenih matematičnih simbolov. Učenje o matematičnih pojmih in simbolih zanje poteka v glavnem sočasno (npr. simboli za relacije: <, >, =).

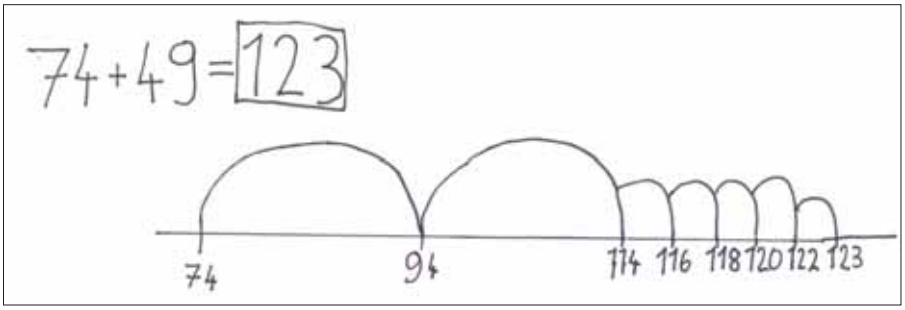
Grafične reprezentacije predstavljajo nekakšen most med konkretnimi reprezentacijami in reprezentacijami z matematičnimi simboli. Heedens (1986) je most, ki vodi od konkretnega proti abstraktnemu, predstavil kot most grafičnih reprezentacij, ki so bodisi semikonkretne bodisi semiabstraktne (slika 1).



Slika 1: Grafične reprezentacije kot most med konkretnimi reprezentacijami in matematičnimi simboli

Grafična reprezentacija na sliki 1, narisani avtomobili, je semikonkretna, pravokotniki pa predstavljajo semiabstraktno reprezentacijo (je bolj oddaljena od učenčevega izkušenjskega sveta). Reprezentacija s pravokotniki (slika 1) bi lahko bila v neki drugi situaciji tudi semikonkretna reprezentacija. Semikonkratni reprezentaciji simbolov 1 in 3 sta lahko zapisa števil z rimskimi številkami I in III, saj sta bolj 'konkretna' od simbolov 1 in 3.

Pri učenju matematike se torej srečujemo z različnimi grafičnimi reprezentacijami. Izbiro grafične reprezentacije določa narava matematičnega koncepta in uporaba konkretnega materiala pri obravnavi tega koncepta. Omenimo še številsko os, kot poseben primer semiabstraktne reprezentacije v matematiki. Številska os povzroča nemalo težav učencem, saj takšna ponazoritev števil vključuje tako ordinalni kot kardinalni vidik števil. Po eni strani je število predstavljeno kot pozicija na osi, po drugi strani pa število predstavlja tudi število premikov po številski osi. Zanimivo različico številske osi so razvili na Nizozemskem, tako imenovano 'prazno številsko os' (slika 2), ki podpira razvijanje učenčevih strategij računanja. Prazno številsko os so razvili kot odgovor na izkušnje učiteljev, ki so pokazale, da učenci predolgo uporabljajo konkreten material: link kocke, Dienesove plošče in reprezentacije na številski osi oz. da so pri računanju na nek način pasivni; zgolj berejo rezultate, ki jih ponujajo ponazorila. Prazna številska os pa omogoča učencem, da poljubno 'skačejo' po osi, si predstavljajo števila na svoj način in razvijajo lastne strategije računanja (Anghileri, 2001).



Slika 2: Prazna številska os

Učenci morajo števila obravnavati celostno (holistično), kar pomeni, da ne operirajo z enicami, deseticami, stoticami…, ampak s števili v 'celoti' (Anghileri, 2001). Anghileri (1998) celo zagovarja, da rokovanje s konkretnim materialom ni tako pomembno in da bi učencem računanje do 100 lahko predstavili zgolj na simbolnem nivoju. Praktično je nemogoče pričakovati, da bi tovrstne ideje

lahko zaživele v slovenskem prostoru, saj učitelji in starši močno verjamejo v učenje s pomočjo konkretnih ponazoril. Je pa ideja o prazni številski osi ter učenju osnovnih računskih operacij zgolj na simbolnem lahko velik izziv za raziskovanje na tem področju.

Omenimo še kratko rokovanje z matematičnimi simboli.

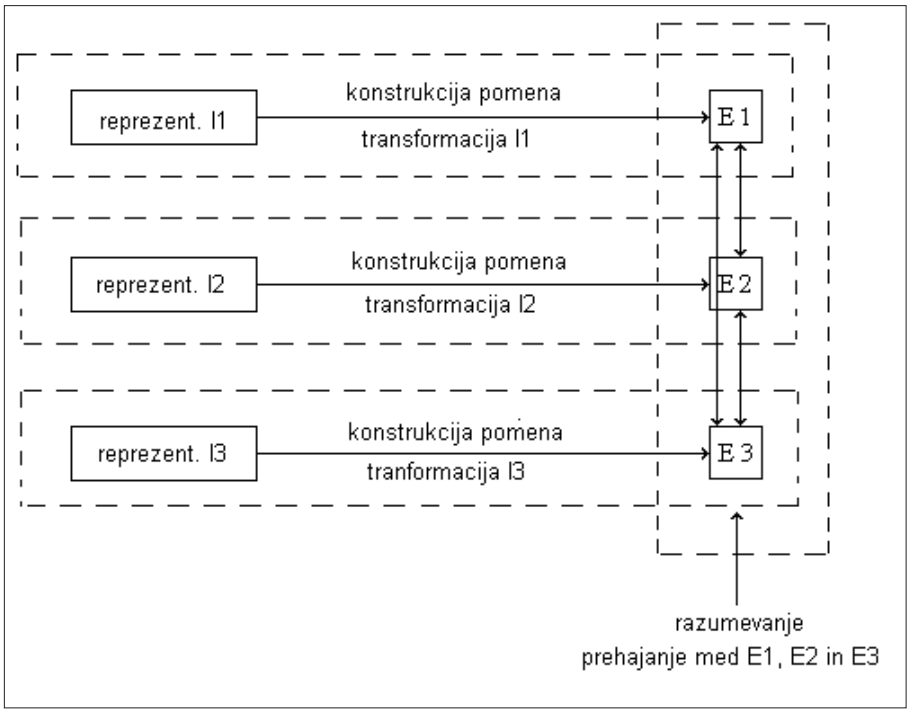
2.3 Matematični simboli

Učenci v prvih letih šolanja spoznajo števke od 0 do 9, znake za operacije (-, +, :, x) ter simbole za relacije (<, >, =). Število znakov je majhno, a je neskočno število kombinacij teh simbolov in pravila, ki veljajo za posamezne kombinacije tisto, kar povzroča učenem nemalo težav pri rokovanju z matematičnimi simboli. Nemalokrat učenci rokujejo s simboli mehanično, brez razumevanja. V procesu zgodnjega učenja matematike je rokovanje s simboli tesno povezano s konkretnimi in grafičnimi reprezentacijami. Hiebert (1988) definira matematične simbole kot reprezentacijski simbol, opredeljen s petimi stopnjami, ki jih mora usvojiti učenec, da lahko s simboli uspešno rokuje. Omenili bomo zgolj prvo stopnjo, to je 'zagotavljanje relacij med simboli in referencami za simbole', kar pomeni, da moramo v procesu učenja in poučevanja omogočiti učencem rokovanje s konkretnim in grafičnim materialom in vzpostavljati relacije med temi reprezentacijami in simboli. Idejo o vzpostavljanju relacij med različnimi reprezentacijami (ne zgolj med simboli in ostalimi) bomo podrobneje predstavili v naslednjem razdelku.

3. Realcije med različnimi reprezentacijami. Model reprezentacijskih preslikav

Poznamo veliko različnih razlag pojmov razumevanje in pomenjanje. Mi bomo definirali pomenjanje kot process, tesno povezan s specifično reprezentacijo, razumevanje pa kot učenčevo sposobnost prehajanja (prevajanja) med različnimi reprezentacijami. S pomenjanjem torej opredelimo učenčevo sposobnost dati določeni reprezentaciji pomen oz. izvesti predvidemo transformacijo v okviru določene reprezentacije. Razložimo oba procesa na primeru operacije deljenja. Če učenec lahko izvede operacijo deljenja s konkretnim materialom, pomeni, da tej reprezentaciji da določen pomen. Učenec, ki reprezentacijo s konkretnim materialom lahko prevede (spremeni) v grafično reprezentacijo in/ali reprezentacijo z matematičnimi simboli, pa operacijo deljenja tudi razume.

Model reprezentacijskih preslikav smo grafično predstavili s sliko 3 (Hodnik Čadež, 2001, 2003).



I 1 : konkretna reprezentacija
I 2 : grafična reprezentacija
I 3 : reprezentacija z matematičnimi simboli
E 1, E 2, E 3: reprezentacije I 1, I 2, I 3.

Slika 3: Model reprezentacijskih preslikav

Slika 3 predstavlja osnovno teorijo naše raziskave (Hodnik Čadež, 2001, 2003). Uporabili smo ga za analiziranje učenčevega razumevanja operacij seštevanja in odštevanja, a verjamemo, da bi ga lahko uporabili tudi pri preučevanju drugih matematičnih konceptov. V naši raziskavi smo potrdili osnovno hipotezo, ki je trdila, da učenec, ki popolnoma prehaja med različnimi reprezentacijami seštevanja in odštevanja do 100, lahko razvije svojo učinkovito strategijo računanja (za seštevanje in odštevanje) v obsegu števil do 1000.

Razložimo zgornji model reprezentacijskih preslikav z naslednjim primerom. Implicitna reprezentacija I1 je lahko reprezentacija s strukturiranim materialom. Če učenec lahko izvede operacijo, npr. 28 + 5 s tem materialom,

pomeni, da je implicitno reprezentacijo spremenil (tranformiral) v eksplicitno reprezentacijo, dal ji je pomen. To z drugimi besedami pomeni, da nobena reprezentacija ne reprezentira sama po sebi, vedno je nujen interpretor, ki implicitno reprezentacijo pretvori v eksplicitno. Če je učenec nato sposoben vzpostavljanja relacij med posameznimi eksplicitnimi reprezentacijami, ali z drugimi besedami, prepozna isti koncept, predstavljen na različne načine, z različnimi reprezentacijami, lahko rečemo, da razume matematični algoritem, v našem primeru prištevanje enic k poljubnemu dvomestnemu številu. Razumevanje pa ponavadi rezultira v transferu že usvojenega na novo učenje. V naši raziskavi je to pomenilo, da je bil učenec sposoben prenesti znanje računskih algoritmov v obsegu do 100, na samostojno oblikovanje računskih algoritmov v obsegu števil do 1000 (učenci se o algoritmih v obsegu do 1000 niso učili na klasičen način, ustvarili so jih samostojno).

4. Zaključek

Poudarimo še enkrat, da reprezentacije v matematiki, konkretne, grafične, s simboli, ne reprezentirajo same po sebi, potrebujejo interpretorja. Obstaja veliko zunanjih reprezentacij, ki obkrožajo učenca pri učenju matematike; učenec je tisti, ki jih interpretira, vzpostavlja miselne interakcije s temi reprezentacijami. Velikega pomena je način predstavitve matematičnega koncepta z zunanjimi reprezentacijami. V procesu poučevanja in učenja matematike pogosto razumemo prehajanje med konkretnimi, grafičnimi in simbolnimi reprezentacijami kot nekaj naravnega, spontanega. Nemalokrat pozabimo, da zunanje reprezentacije potrebujejo razlago, 'dinamično' interpretacijo, v kateri so udeleženi tako učenci kot tudi učitelj. Ne pozabimo, da lahko tudi učenci samostojno ustvarjajo zunanje reprezentacije, jih predstavljajo drugim, o njih diskutirajo. Napačno je predvidavati, da raznovrstne reprezentacije, ki so ponavadi tudi zelo privlačne na pogled, vedno služijo svojemu namenu, to je, ustvarjanju povezav med miselnim procesom in reprezentacijami. V veliko pomoč pri osmišljanju reprezentacij je jezik, prav tako reprezentacijski sistem, ki je v tesni relaciji s konkretnimi, grafičnimi in simbolnimi reprezentacijami.

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ZNANSTVENI OKVIRI NASTAVE MATEMATIKE

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Sažetak. *U procesu spoznaje i upoznavanja zakona prirode istraživači primjenjuju posebna sredstva – znanstvene metode istraživanja. Osnovne metode znanstvenog mišljenja i istraživanja su: analiza i sinteza, analogija, apstrakcija i konkretizacija, generalizacija i specijalizacija, indukcija i dedukcija.*

Matematika kao znanost i matematika kao nastavni predmet usko su povezane. Tu vezu uspostavlja pored ostalog načelo znanstvenosti. Jačanje te veze znatno je utjecalo na promjene u nastavi matematike. Težište suvremene nastave matematike danas leži na uvođenju učenika u istraživački rad i razvoju njihovog mišljenja. Evo nekih postavki:

♦ *Rad nastavnika matematike s učenicima u razredu u mnogo čemu se razlikuje od rada matematičara-znanstvenika, ali postoje i neke zajedničke značajke. Učenici u nastavnom procesu samostalno ili uz pomoć nastavnika također otkrivaju i spoznaju nove matematičke istine. Posebno važno je otkrivanje puta k samostalnom stvaralačkom radu učenika. Zato su navedene znanstvene metode važne i za suvremenu nastavu matematike. Kreativan nastavnik, birajući pogodne probleme i primjenjujući te metode, može učenike osposobiti za rad koji je vrlo blizak istraživačkom radu.*

♦ *Matematika u nastajanju je konkretna i induktivna znanost, a sama matematika je apstraktna i deduktivna znanost.*

♦ *Važan znanstveni postupak je analogija. Ona prožima čitavo naše mišljenje, svakidašnji govor, umjetničko stvaralaštvo, ali i visoka znanstvena istraživanja. Analogija je vrlo korisna i u nastavi matematike kao zorno sredstvo povezivanja i lakšeg svladavanja nastavnog gradiva, te kao sredstvo razvijanja stvaralačkog mišljenja i kreativnosti učenika. Pri rješavanju nekog problema učenici se usmje-*

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ravaju na razmatranje nekog bliskog, srodnog problema i oponašanje postupka njegova rješavanja.

♦ *Pri obradi matematičkih pojmova nastavnik ostvaruje načelo znanstvenosti ako pravilno provodi proces formiranja pojma (opažanje, predodžba o pojmu, formiranje pojma) i pridržava se osnovnih pravila koja mora zadovoljavati definicija pojma (primjerenost, minimalnost sadržaja, sažetost, prirodnost, prikladnost, primjenjivost, suvremenost). Sam proces je postupan i njegovu uspješnost osigurava pet značajnih znanstvenih postupaka: analiza, sinteza, konkretizacija, apstrahiranje i poopćavanje. Kritično mjesto obrade nekog pojma je prijelaz na onaj stupanj u kojem počinje postupak apstrahiranja, jer je prijelaz s konkretnog na apstraktno za neke učenike dosta težak.*

♦ *Pri obradi poučaka nastavnik ostvaruje načelo znanstvenosti ako svoje učenike nauči ispravno i precizno formulirati poučak, jasno razlikovati pretpostavku od tvrdnje poučka, formulirati obrat poučka, formulirati suprotnu tvrdnju, te ako postigne razumijevanje metodike dokazivanja poučaka.*

♦ *Načelo znanstvenosti ostvaruje se također jasnim razlikovanjem definicija i poučaka.*

Obrada nekih matematičkih sadržaja u našim udžbenicima često nije u skladu s načelom znanstvenosti.

Ključne riječi: *matematika, nastava matematike, znanost.*

Nastava matematike danas se pretežno odvija u stručnim okvirima. Međutim, nastava matematike je složen i zahtjevan proces. Za njezinu uspješnost stručnost je nužan preduvjet, ali nije dovoljan. Složenost se uspješno razrješuje jačim povezivanjem matematike s drugim znanostima. Tako dobivamo proces koji se treba skladno odvijati unutar nekoliko okvira.

Glavni okviri su *jezični okviri, stručni okviri, metodički okviri, znanstveni okviri, pedagoški okviri i psihološki okviri.*

Kako sklad nije lako postići, u nastavi matematike dešavaju se propusti i nastaju slabosti koji značajno utječu na kakvoću matematičkog obrazovanja učenika. To se onda loše odražava na ostvarenje ciljeva suvremene nastave matematike koja težište postavlja na uvođenje učenika u samostalan i istraživački rad, razvijanje sposobnosti za rješavanje problema, te na razvoj njihovog mišljenja.

U ovom članku opisat ćemo nekoliko postavki i problema koji nastaju unutar znanstvenih okvira nastave matematike.

♦ Vezu između matematike kao nastavnog predmeta i matematike kao znanosti uspostavlja, pored ostalog, načelo znanstvenosti. Načelo znanstvenosti nastave matematike sastoji se u nužnom skladu nastavnih sadržaja i nastavnih metoda s jedne strane i zahtjeva i zakonitosti matematike kao znanosti s druge strane. To znači da nastavnik matematike treba učenike upoznavati s onim činjenicama i u njihovom mišljenju formirati one matematičke pojmove koji su danas znanstveno potvrđeni.

Pri obradi matematičkih pojmova nastavnik ostvaruje načelo znanstvenosti ako pravilno provodi proces formiranja pojma (opažanje, predodžba o pojmu, formiranje pojma) i pridržava se osnovnih pravila koja mora zadovoljavati definicija pojma (primjerenost, minimalnost sadržaja, sažetost, prirodnost, prikladnost, primjenjivost, suvremenost).

Pri obradi poučaka nastavnik ostvaruje načelo znanstvenosti ako svoje učenike nauči ispravno i precizno formulirati poučak, jasno razlikovati pretpostavku od tvrdnje poučka, formulirati obrat poučka, formulirati suprotnu tvrdnju, te ako postigne razumijevanje metodike dokazivanja poučaka.

P 1. Kritično mjesto obrade nekog pojma je prijelaz na onaj stupanj u kojem počinje postupak apstrahiranja, jer je prijelaz s konkretnog na apstraktno za neke učenike dosta težak. Isti zaključak vrijedi i za izvođenje generalizacija preko induktivnih nizova konkretnih slučajeva.

Načelo znanstvenosti ostvaruje se također jasnim razlikovanjem definicija i poučaka.

P 2. Teško da će učenici dobiti jasnu predodžbu o matematici ako se u udžbenicima mogu naći i ovakve postavke:

1) *Paralelogram* je četverokut koji ima dva para međusobno paralelnih stranica jednakih duljina.

U definiciji pojma paralelogram nalaze se dva njegova bitna obilježja: «nasuprotne stranice su paralelne» i «nasuprotne stranice su jednakih duljina». Međutim, obilježja su ekvivalentna, pa je svako od ovih dovoljno za određenje paralelograma. Zato u definiciju treba ući samo prvo obilježje, što je posve primjereno terminu *paralelogram*, a drugo izostaviti i odvojeno dokazati kao poučak.

2) Možemo dokazati da je $a^0 = 1$.

Formulacija ukazuje na zaključak da se radi o tvrdnji, o poučku. Međutim $a^0 = 1$ je dogovorna definicija koja se uvodi zato da bi pravilo dijeljenja potencija $a^m : a^n = a^{m-n}$ vrijedilo i za slučaj $m = n$.

3) Pravci koji zatvaraju pravi kut nazivaju se *okomiti pravci*.

Kut čiji su kraci međusobno okomiti naziva se *pravi kut*.

Ovo je primjer cirkularne definicije: okomiti pravci definiraju se pomoću pravog kuta, a pravi kut pomoću okomitih pravaca. Zapravo se ne zna što je definirano.

P 3. Početak metodičkog obrazovanja studenata je prva metodička radionica pod nazivom MATEMATIČKI POJMOVI I. Ta se tema na predavanjima obrađuje znatno kasnije, ali je izabrana kao početak zato da se provjeri *razina predznanja* studenata o tako važnom matematičkom sadržaju. Matematički pojmovi koje studenti trebaju definirati u ovoj metodičkoj radionici su:

Elipsa, homotetija, kompleksni broj, konveksan skup, korjenovanje, kvadratna jednadžba, logaritamska funkcija, nultočka polinoma, ortocentar trokuta, obrnuto proporcionalne veličine, polinom, postotak, pravi kut, površina, relacija, sfera, sličnost, translacija, vektor, visina pravokutnika.

Rezultati su onakvi kakve je profesor metodike i očekivao: *vrlo slabi*. Pokazuje se da je znanje studenata o matematičkim pojmovima dosta zbrkano. U njihovim radovima rijetko se može pročitati neka korektna definicija. Ne znajući u tom trenutku načela definiranja matematičkog pojma, studenti u definiciju unose sve što o pojmu znaju (primjere, svojstva). Tako umjesto kratke, precizne i potpune definicije pojma dobiva se opširan tekst iz kojeg se na kraju ipak ne može doznati o čemu se radi! Ovakva zbrka, a može se slobodno reći i *neznanje*, ne bi mogla biti sredstvo uspješne nastave. Rezultati ukazuju na potrebu ozbiljnog pristupa ovoj temi. Kasnije, u poglavlju o oblicima mišljenja tema se metodički detaljno obrađuje, a nakon toga slijedi metodička radionica MATEMATIČKI POJMOVI II. U njoj su studenti trebali definirati sljedeće pojmove školske matematike:

Centralna simetrija, funkcija, hiperbola, izometrija, kut, kvadar, linearna jednadžba, logaritam, mimoilazni pravci, okomite ravnine, piramida, proporcionalne veličine, pravokutnik, rješenje sustava linearnih jednadžbi, simetrala dužine, tetiva kružnice, trapez, valjak, volumen, zatvoreni interval.

Gotovo je suvišno kazati da su sada rezultati bolji, iako to još uvijek nije onakvo znanje kakvo treba biti. Neke se praznine u znanju malo teže popunjavaju, a da bi uspješno izvodili nastavu matematike, studenti moraju potpuno vladati materijom.

♦ U procesu spoznaje i upoznavanja zakona prirode istraživači primjenjuju posebna sredstva – znanstvene metode istraživanja. Osnovne metode znanstvenog mišljenja i istraživanja su: analiza i sinteza, analogija, apstrakcija i konkretizacija, generalizacija i specijalizacija, indukcija i dedukcija.

Evo kratkog opisa tih metoda:

Analiza je znanstvena metoda istraživanja koja se zasniva na raščlanjivanju cjeline na dijelove, proučavanju dijelova i izvođenju zaključaka o cjelini na temelju dobivenih rezultata. Njezina suprotnost je sinteza.

Analogija je jedna vrsta sličnosti. *Zaključivanje po analogiji* je misaoni postupak pri kojem se iz opažanja da se dva objekta podudaraju u određenom broju svojstava ili odnosa izvodi zaključak da se oni podudaraju i u drugim svojstvima ili odnosima koji se kod jednog objekta nisu izravno opažali.

Apstrakcija je misaono odvlačenje općeg bitnog svojstva promatranog objekta ili pojave od ostalih svojstava, nebitnih za određeno proučavanje, i odbacivanje tih nebitnih svojstava. Njezina suprotnost je *konkretizacija*. Možemo je karakterizirati kao misaonu aktivnost kojom se jednostrano fiksira neka strana promatranog objekta izvan veze s njegovim drugim stranama.

Generalizacija ili *poopćavanje* je prijelaz s razmatranja danog skupa objekata na odgovarajuće razmatranje njegova nadskupa, tj. generalizacija metoda kojom se prelazi granica danog skupa objekata i izgrađuju općenitiji pojmovi i općenitije tvrdnje. Njezina suprotnost jest *specijalizacija*, odnosno metoda kojom se učvršćuje unutrašnja struktura danog skupa objekata.

Indukcija je način zaključivanja kojim se iz dvaju ili više pojedinačnih ili posebnih sudova dobiva novi opći sud, a kao metoda *indukcija* je metoda istraživanja kojom se pri proučavanju nekog skupa objekata promatraju posebni objekti iz toga skupa i utvrđuju kod njih ona svojstva koja se zatim pripisuju čitavom skupu. Njezina suprotnost je *dedukcija*.

♦ Rad nastavnika matematike s učenicima u razredu u mnogo čemu se razlikuje od rada matematičara-znanstvenika, ali postoje i neke zajedničke značajke. Učenici u nastavnom procesu samostalno ili uz pomoć nastavnika također otkrivaju i spoznaju nove matematičke istine. Posebno važno je otkrivanje puta k samostalnom stvaralačkom radu učenika. Zato su navedene znanstvene metode važne i za suvremenu nastavu matematike. One čvršće povezuju matematiku kao nastavni predmet i matematiku kao znanost. Kreativan nastavnik, birajući pogodne probleme i primjenjujući te metode, može učenike osposobiti za rad koji je vrlo blizak istraživačkom radu.

P 4. Tijekom nastavnog sata nastavnik matematike često govori: «analiza pokazuje», «pogledajmo nekoliko konkretnih primjera», «analogno se dokazuje», «ovaj niz činjenica inducira zaključak», «rezultat ovih razmatranja je generalizacija», «specijalizacijom dobivamo formulu», «matematički pojmovi su apstraktni» i sl. Razumiju li učenici ove riječi? Kako provjeravamo da oni to razumiju?

Problem je ozbiljan, jer čak i studenti matematike nastavničkih profila imaju problema u vezi s razumijevanjem gornjih pojmova. Zato treba krenuti dosta rano i učenike postupno i primjereno njihovoj dobi naučiti analizirati, sintetizirati, konkretizirati, apstrahirati, inducirati, deducirati, generalizirati, specijalizirati, uočavati analogije, bez obzira hoće li se oni kasnije ozbiljnije baviti matematikom ili ne. Za razliku od običnog usvajanja gradiva, ovo je viša razina matematičkog obrazovanja, a matematički način mišljenja dragocjena je stečevina matematičkog obrazovanja, primjenjiva i u mnogim drugim djelatnostima.

P 5. Neuspjesi učenika u matematici i neznanje koje ispoljavaju nakon završenog školovanja dobrim dijelom su posljedica činjenice da se nastava većinom izvodi na nižoj razini gdje se suviše inzistira samo na usvajanju gradiva, a zapostavljena je navedena viša razina.

P 6. U nastavi matematike sintezi najčešće ne prethodi analiza, a to utječe na jasnoću poučavanja i razumijevanje problema, što znatno umanjuje spoznajnu vrijednost nastave. Analiza je u manjoj ili većoj mjeri nužna u svim istražuivanjima i ne smije se izbjegavati.

Primjer matematičkih sadržaja gdje je analiza važna su školski tekstualni zadaci. Zašto takvi zadaci ipak često zadaju dosta teškoća i učenicima i nastavnicima, pa ih neki nastavnici izbjegavaju? Objašnjenje dobrim dijelom leži u

naravi samih zadataka. Svaki takav zadatak sastoji se zapravo od dva zadatka: sastavljanja jednadžbi prevođenjem s običnog jezika na matematički jezik i rješavanja jednadžbi. Prvi od njih nije uvijek lagan, zahtijeva priličan umni napor i poznavanje postupka raščlanjivanja, analize, što se nerijetko pretpostavlja da učenici znaju i bez objašnjenja. Odatle teškoće, a rezultat je najčešće odbojnost prema takvim problemima. Međutim, svođenje problema na rješavanje jednadžbi višestruko je korisno jer ono omogućuje razvijanje logičkog mišljenja, dosjetljivosti, opažanja i umijeća samostalnog provođenja nevelikih istraživanja. Zato takve probleme nije dobro izbjegavati, već ih treba metodički primjereno objašnjavati, kako bi oni ispunili svoju obrazovnu svrhu.

♦ Matematika u nastajanju je *konkretna* i *induktivna* znanost, a sama matematika je *apstraktna* i *deduktivna* znanost. Kako je s nastavom matematike u tom pogledu? I nastava matematike u osnovnoj školi pretežno je konkretna i induktivna. Učitelj matematike dolazi do apstraktnih postavki, do generalizacija, razmatranjem konkretnih objekata i konkretnih primjera i induktivnim zaključivanjem. Taj način je blizak i primjeren učenicima toga uzrasta. Induktivni postupak sastoji se od niza induktivnih koraka kojima se dolazi do shvaćanja općeg. Počinje se s konkretnim objektima i specijalnim slučajevima, induktivni zaključci nižu se analogijom, a promatrane činjenice nastoje se generalizirati. To znači da je indukcija tijesno povezana s *konkretizacijom, specijalizacijom, analogijom i generalizacijom*. Prednosti primjene indukcije: ostvarenje načela od lakšeg ka težem, od jednostavnog ka složenom, proučavanje novih apstraktnih pojmova i izreka preko promatranja i provjeravanja, navođenje učenika na nove pojmove, iskazivanje novih tvrdnji i dr. Mnogo je sadržaja u školskoj matematici za čiju je obradu potreban i za razvoj učenikova mišljenja važan induktivni postupak. Među takve sadržaje posebno se ubrajaju razna pravila, zakoni, formule i teoremi, pogotovo ako se oni strogo ne izvode ili ne dokazuju.

Obrnuti postupak od indukcije je *dedukcija*. Deduktivni način mišljenja i dokazivanja provodi se poslije indukcije i na višoj razini nastave matematike i obrazovanja učenika.

P 7. U induktivnoj nastavi potreban je primjeren broj pojedinačnih i posebnih slučajeva. Često učitelj matematike razmatra premali broj takvih slučajeva, pa izvedene tvrdnje postaju neuvjerljive i nejasne, a posljedica je manjkavo znanje učenika. Čest je i drugi propust učitelja kad ne daje priliku većem broju učenika da sudjeluju u izgradnji induktivnog niza.

P 8. Izvođenje generalizacija također je kritično mjesto nastave matematike, jer prijelaz s konkretnog i pojedinačnog k općem neki učenici teško svladavaju. Zato je pred učiteljem matematike odgovorna zadaća da svojim metodičkim pristupom i umješnošću učenicima učini taj prijelaz što lakšim.

♦ Važan znanstveni postupak je *analogija*. Ona prožima čitavo naše mišljenje, svakidašnji govor, umjetničko stvaralaštvo, ali i visoka znanstvena istraživanja. Analogija je vrlo korisna i u nastavi matematike. Tijekom nastavnog sata nastavnik često govori ili pita: "slično se izvodi", "analogno se dobiva", " na isti način se dokazuje", "trokuti se podudaraju", "ovo je srodan zadatak", "u kojem su odnosu promatrani likovi?", "ovdje možemo ponoviti opisani postupak", "što u prostoru odgovara pravokutniku?" i sl. Te jednostavne rečenice imaju dubok smisao i važan cilj. Ponavljanjem takvog načina govora u svom izlaganju nastavnik svjesno ukazuje na analogiju. Na taj način analogija postaje zorno sredstvo povezivanja i lakšeg svladavanja nastavnog gradiva, jer se određeno ranije usvojeno gradivo ponovo obnavlja i utvrđuje kao sredstvo razvijanja stvaralačkog mišljenja i kreativnosti učenika. Pri rješavanju nekog problema učenici se usmjeravaju na razmatranje nekog bliskog, srodnog problema i oponašanje postupka njegova rješavanja. Još je važnije što analogija daje nastavniku mogućnost neprestane izmjene nastavnih oblika i metoda u svrhu postizanja učinkovitije nastave.

P 9. U nastavi matematike analogija nije dovoljno iskorištena.

To je prava šteta, pogotovo što postoji toliko mnogo srodnih objekata i njihovih svojstva. Nabrojimo samo neke: trokut i tetraedar, kvadrat i kocka, pravokutnik i kvadar, kružnica i sfera, krug i kugla, pravila za brojeve, elipsa i hiperbola, analogne formule i dr.

P 10. Ako autori udžbenika pri opisu nekog matematičkog sadržaja nisu opisali mogućnost primjene analogije, onda najčešće izostaje njezina primjena i pri obradi toga gradiva u razredu.

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Na temelju svega što je gore rečeno lako je zaključiti da znanstvene metode imaju svoje mjesto u nastavi matematike. Uz jednu napomenu: nastavnik matematike ne mora biti znanstvenik da bi u nastavi pravilno i primjereno primjenjivao načelo znanstvenosti i znanstvene metode. To se u nastavi matematike nameće samo po sebi. Rješavanje svakog problema ima nešto otkrivalačko

i stvaralačko. Zato je potrebno samo da nastavnik u svojih učenicika razvija radoznalost duha, sklonost za samostalan umni rad i da im ukazuje na putove do novih otkrića. Ako se znanstveni postupci primjereno i pravilno primjenjuju, s nužnim osjećajem za težinu matematičkih sadržaja i matematičkog načina mišljenja, uvažavajući matematičke sposobnosti svakog pojedinog učenika, može se očekivati da će nastava matematike biti uspješna. U protivnom, učenici će imati znatnih poteškoća pri svladavanju nastavnog gradiva i oni s vremenom mogu steći pogrešan dojam da je matematika teži predmet nego što to ona uistinu jest. Na žalost, često se u udžbenicima matematike, a onda kao posljedica i u nastavnom procesu, ne poklanja dovoljno pozornosti na pravilnost primjene znanstvenih postupaka. Za obrade nekih matematičkih sadržaja može se čak ustanoviti da su s tog gledišta pogrešne. Time je povrijeđeno načelo znanstvenosti.

MATEMATIČKI DAROVITA DJECA: ŠTO IH MOŽEMO POUČITI I ŠTO BISMO MOGLI NAUČITI OD NJIH?

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Sažetak. *Početak istraživačkog interesa psihologa za rješavanje matematičkih problema i za nadarenost vremenski se poklapaju i sežu u 20-te godine prošlog stoljeća. Istraživanja Thorndikea (matematika) i Termana (nadarenost) otvorila su dva nova područja u psihologiji koja i danas imaju brojne dodirne točke.*

Definicije nadarenosti su vrlo brojne i mogu se svrstati u četiri osnovne skupine: one usmjerene na urođenost odnosno genetske činitelje, one usmjerene na kognitivne modele, one usmjerene na postignuće i sustavski usmjerene. Genetski usmjeren pristup govori o važnosti urođenih predispozicija a u novije vrijeme ukazuje na višestruke inteligencije tj. ideju da darovitost nadilazi ideju visoke inteligencije i da se ogleda u različitim domenama. Kognitivno usmjerena istraživanja i definicije darovitosti jasno govore o različitom kognitivnom funkcioniranju darovitih u pojedinim domenama. Tako će daroviti matematičari puno bolje procesirati numeričke informacije dok će verbalne informacije procesirati kao i prosječni ispitanici. S druge strane verbalno daroviti će efikasnije procesirati verbalne informacije ali će numeričke procesirati kao i prosječni ispitanici. Pristup usmjeren na postignuće ukazao je na važnost neintelektualnih činitelja (motivacija i kreativnost) za darovitost. Sustavski pristup ukazuje na ulogu različitih društvenih sustava u razvoju darovitosti (obitelj, škola, obrazovni sustav).

Danas znanost već puno zna o kognitivnom funkcioniranju matematički darovite djece, o njihovim obrazovnim postignućima i potrebama. Znamo da kroz akceleraciju i obogaćivanje programa možemo zadovoljiti neke njihove potrebe. Otvoreno je pitanje koliko učitelji u praksi znaju o specifičnim potrebama darovitih učenika i koliko su uvježbani za rad s njima. Čak i kad su učitelji svjesni koje

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sadržaje treba poučavati to nije dovoljno. Naime brojna istraživanja govore kako samo novi sadržaji nisu dovoljni – daroviti učenici trebaju i poticaje za osobni razvoj što posebno vrijedi za programe rada s matematički darovitima. Teško je dati odgovor na pitanje koliko su učitelji svjesni socio-emocionalnih potreba matematički darovitih učenika i koliko ih uvažavaju u svom radu. Istraživanja pokazuju da su daroviti učenici spremni iskazati svoje potrebe a mi moramo biti spremni učiti.

Ključne riječi: *darovita djeca, definicije darovitosti, matematička darovitost.*

Početkom prošlog stoljeća psihologija se, kao još relativno mlada znanost, počinje baviti nekim sasvim novim područjima. Tako 1922. godine E.L.Thorndike objavljuje svoju knjigu *The psychology of arithmetics* koja se danas smatra početkom znanstvenog bavljenja psihologa matematikom. Istodobno L.Terman započinje svoje veliko longitudinalno istraživanje darovitih koje je pridonijelo sasvim novim spoznajama o razvoju darovitih pojedinaca i uvelike promijenilo dotadašnje predrasude (Terman i Oden, 1959).

U međuvremenu su se ta dva područja psihologije, istraživanje darovitosti i istraživanje matematičkog rezoniranja, i teorijski i empirijski snažno razvila. Matematičarima je vjerojatno najzanimljiviji presjek ova dva polja istraživanja tj. matematička darovitost, spoznaje u tom području i njihove praktične implikacije.

Kad je riječ o darovitosti onda danas imamo brojne definicije tog pojma, odnosno različite pristupe darovitosti koji se prema Mönksu i Masonu (2000.) mogu razvrstati u četiri skupine:

- definicije usmjerene na urođenost odnosno genetičke činitelje
- definicije usmjerene na kognitivne modele
- definicije usmjerene na postignuće
- definicije sa sustavskim pristupom.

Nabrojeni pristupi imaju svoje znanstveno utemeljenje i važne implikacije za rad s darovitim pojedincima (Vlahović-Štetić, 2005), pa tako i s matematički darovitima.

Prvu skupinu definicija čine pristupi koji prvenstveno ističu važnost genetičkih činitelja za razvoj darovitosti ali to ne znači da negiraju okolinske činitelje. Prema njima daroviti su oni pojedinci koji u najvećoj mjeri u populaciji posjeduju neku osobinu, a to je uvelike određeno genetikom. Jedan od najpoznati-

jih, novijih autora koji pripada ovom pristupu je Howard Gardner (1983.) koji kaže: "Postoji sedam različitih specifičnih sposobnosti, talenata ili inteligencija: logičko-matematička, lingvistička, vizualno–spacijalna, tjelesno-kinestetička, glazbena, interpersonalna i intrapersonalna." Društvo kroz sustav vrijednosti definira što se smatra darovitim. Zapadne civilizacije više cijene neke oblike kao što su verbalna i matematičko-logička darovitost. Tako matematički daroviti imaju relativno dobar tretman – njihove sposobnosti se u školskom okruženju cijene. Gardner ističe kako je svaka individua specifična kombinacija spomenutih sposobnosti i visok rezultat u jednom području, primjerice matematičkom, ne znači da je pojedinac natprosječan i u ostalim područjima. Drugim riječima matematički darovito dijete vrlo vjerojatno ima i u obitelji matematički darovitih pojedinaca (iako to može biti latentna a ne nužno manifestna darovitost) i ono može biti darovito samo u području matematike a da u drugim područjima postiže prosječne ili čak ispodprosječne rezultate. Kao važno za obrazovni sustav Gardner ističe da bi učitelji kroz svoje predmete trebali poticati različite oblike talenata ili inteligencija – tako bi primjerice učiteljica matematike trebala razmišljati kako kroz svoj predmet poticati ne samo matematičku darovitost već i kako poticati interpersonalnu ili glazbenu darovitost. Istodobno bi primjerice učitelj povijesti trebao razmišljati kako kroz svoj predmet poticati matematičku darovitost.

Druga skupina definicija orijentirana je na kognitivne modele. Sternberg (2001.) govori o darovitosti kao putu od početnika koji ima potencijal do eksperta u području. Za taj razvoj, nužne su metakognitivne vještine, vještine učenja, vještine mišljenja, deklarativno i proceduralno znanje te motivacija kao glavni pokretač. Daroviti pojedinci superiorno kombiniraju spomenute elemente, brže napreduju i postižu višu razinu ekspertnosti od prosječnih. Istraživači ovog teorijskog pristupa proučavali su razlike u kognitivnom funkcioniranju između darovitih i prosječnih pojedinaca. Upotreba novih tehnologija omogućila je zadavanje složenih podražaja u istraživanjima i tako omogućila nove spoznaje u području rješavanja problema, vremena reakcije, kratkoročnog i dugoročnog pamćenja. Dark i Benbow (1991.) su pokazale da je matematička darovitost povezana s boljim pamćenjem brojki i spacijalnih lokacija dok je verbalna darovitost povezana s boljim pamćenjem riječi. Nadareni se razlikuju po vrsti informacija koju lako zadržavaju u radnoj memoriji, nema generalnog kapaciteta zapamćivanja kod nadarenih, on je vezan uz vrstu informacije i vrstu nadarenosti. Drugim riječima od darovitih matematičara možemo očekivati superiorno baratanje numeričkim informacijama, bolje pohranjivanje takvih informacija u

dugoročno pamćenje i lakše dozivanje, ali to ne znači da će oni efikasnije pamtiti druge vrste informacija.

Treću skupinu pristupa čine oni usmjereni na postignuće. Renzulli (1986.), autor troprstenaste teorije darovitosti kaže:"Darovito ponašanje pokazuje interakciju triju osnovnih skupina ljudskih osobina: iznadprosječne opće i/ili specifične sposobnosti, visoke usmjerenosti na zadatak i visokog stupnja kreativnosti. Pojedinci koji pokazuju nadareno ponašanje su oni koji imaju ili mogu razviti ovu kombinaciju osobina i primijeniti je u nekom vrijednom području ljudske aktivnosti." Sposobnosti, motivacija i kreativnost mogu se prikazati kao kružnice Vennovog dijagrama. Njihov presjek je darovitost. Drugim riječima intelektualni potencijal kao što je sposobnost za matematičko rezoniranje nije dovoljan za darovitost – kod pojedinca moraju istodobno postojati i motivacija i kreativnost. Renzulli kaže da darovita djeca ne moraju nužno pokazivati sve tri karakteristike darovitog ponašanja ali trebaju imati kapacitet da kasnije u životu razviju te osobine. Ovdje je moguće raspravljati pitanje odnosa matematičke darovitosti i matematičke kreativnosti. Riječ je o odnosu koji se javlja u različitim područjima: kreativni jesu istodobno i daroviti ali daroviti nisu nužno i kreativni. Pitanje koje se može postaviti je i jesu li matematički kreativni samo vrhunski profesionalci ili o kreativnosti možemo govoriti i na nižim razinama matematičkog znanja? Sriraman (2005.) detaljno raspravlja ovo pitanje i zaključuje da o matematičkoj kreativnosti možemo govoriti na svim dobnim uzrastima i da učitelji mogu poticati (poučavati) svoje matematički darovite učenike da budu kreativni te tako širiti podskup kreativnih matematičara u skupu darovitih matematičara.

Noviji pristupi darovitosti tzv. sustavski naglašavaju brojnih činitelja u razvoju darovitosti. Tannenbaum (1983.) je dao prvu definiciju i model darovitosti vezan uz sustavski pristup. Činitelji koji moraju biti optimalni da bi potencijal darovitog pojedinca bio realiziran kao postignuće su: opća intelektualna sposobnost, specifične sposobnosti, ne-intelektualni činitelji (nezavisnost, unutarnja kontrola, motivacija, samopoštovanje, fleksibilnost), okolinska podrška (uže i šire okoline) i slučajnost odnosno sreća. Svaki od navedenih činitelja nužan je ali i sam po sebi nedovoljan za realizaciju potencijala. Kombinacija od četiri spomenuta činitelja ne može nadoknaditi ozbiljan nedostatak na petom, a njihova relativna važnost mijenja se s obzirom na vrstu nadarenosti. Dakle nedostatna podrška okoline ili nedostatak motivacije ili samopoštovanja dovest će do toga da se objektivno visok potencijal neće manifestirati kao darovitost.

Društvo je odgovorno za okolinsku podršku koju mora pružati i kroz školski sustav. Sustavski pristupi ističu i ulogu društvenih vrijednosti i odnosa društva prema darovitima. Ukoliko i pružamo obrazovnu podršku darovitim matematičarima ali istodobno toleriramo klimu da su to zapravo neobična djeca ili čudaci ne možemo očekivati primjeren razvoj matematički darovitih.

Različiti pristupi darovitosti imaju naglasak na različitim faktorima no prema Sternbergu (2004) postoje i neke njihove zajedničke točke:

1. darovitost je više od samo kvocijenta inteligencije
2. darovitost se sastoji od kognitivnih i nekognitivnih činitelja
3. okolina je ključna za realizaciju darovitosti

Kako ove zajedničke točke vrijede za matematički nadarenu djecu?

1. Matematička darovitost je bez sumnje fenomen koji nadilazi ideju kvocijenta inteligencije kojeg je kao mjeru darovitosti uveo još Terman. To jest mjera općeg intelektualnog funkcioniranja ali iz samog općeg kvocijenta ne vidimo koje su djetetove jake strane odnosno u kojem području je darovito. Ovisno o primijenjenom instrumentu kvocijent je mjera izvedena na temelju udjela različitih sposobnosti (generalne ali i specifičnih kao primjerice: verbalnih i numeričkih). Jednak kvocijent inteligencije može se postići većim udjelom verbalnih ili većim udjelom neverbalnih sposobnosti tj. te dvije mjere nisu kod pojedinaca nužno usklađene. Dapače istraživanja pokazuju da je neusklađenost pojedinih komponenti kvocijenta veća kod intelektualno superiornih pojedinaca nego kod prosječnih (Detterman i Daniel, 1989., Wilkinson, 1993.). Podjednako je zanimljiv podatak da su verbalno darovita djeca u pravilu bolje «usklađena» po verbalnim i matematičkim sposobnostima nego matematički daroviti – oni će češće uz visoke numeričke sposobnosti i sposobnosti matematičkog rezoniranja pokazivati prosječne ili čak ispodprosječne rezultate na verbalnim sposobnostima. Drugim riječima matematički daroviti imaju manje šanse da ih se uspješno identificira ukoliko se kao mjera koristi samo kvocijent inteligencije. Slične nalaze daju i studije slučajeva pa je Bloomova (1985.) retrospektivna studija dvadeset matematički nadarenih odraslih pokazala da nijedan nije naučio čitati prije škole, a njih šest je imalo poteškoće u svladavanju čitanja. Iako ne valja matematičku darovitost izjednačavati s općim intelektualnim funkcioniranjem valja reći da studije pokazuju kako su intelektualne sposobnosti matematički darovitih iznadprosječne (Lubinsky i Humphreys, 1990).

2. Matematička darovitost nije vezana samo uz sposobnosti i znanja već i uz nekognitivne činitelje kao što su fleksibilnost, otvorenost za nova iskustva, tolerancija na neizvjesnost, pozitivna slika o sebi, znatiželja, spremnost na rizik i predanost zadatku (Wieczerkowski, Cropley i Prado, 2000.). Wieczerkowski (1988., prema Wieczerkowski i sur., 2000.) je identificirao dva činitelja koja su u podlozi motivacije za bavljenjem matematikom kod darovite djece. Prvi činitelj je dječje uvjerenje o težini matematike povezano s njihovim uvjerenjem da su sposobni za takva postignuća. Drugi činitelj je dječja procjena vrijednosti matematike koja uključuje njezinu zanimljivost, mogućnost da kroz nju ostvare neke osobne potrebe kao što su uspjeh, društveni položaj ili osjećaj vlastite vrijednosti, te korisnost matematike za postizanje životnih ciljeva: akademskog uspjeha ili dobrog posla. Realizirana matematička darovitost bit će rezultat kognitivnih ali i nekognitivnih činitelja pa tako osim poučavanja sadržaja valja voditi računa i o dječjim uvjerenjima i stavovima vezanim uz matematiku. O tome posebice valja voditi računa kad je riječ o matematički darovitim djevojkama. Naime istraživanja provedena na uzorcima iz populacije u pravilu ukazuju na pozitivniji stav mladića prema matematici (Hyde i sur.,1990., Norman, 1977.) te na veći strah od matematike kod djevojaka (Arambašić i sur., 2005., Gierl i Bisanz, 1995., Hyde i sur., 1990., Ma, 1999.).

3. Neka darovita djeca uspjet će i bez podrške svoje okoline, no oni su prije iznimka nego pravilo. Kao i ostali, tako i matematički daroviti trebaju podršku bliže (obitelj) i šire okoline (škola). Obitelj je ta koja može od vrlo rane dobi njegovati dječje potencijale. Obitelj koja pokazuje da joj je stalo do dječjih postignuća, ona koja omogućava obrazovnu podršku (literatura, pristup radionicama) bez sumnje će olakšati razvoj matematički darovitog djeteta. Škola je sustav koji je odgovoran za dječji razvoj i u kojem je moguće realizirati različite oblike podrške darovitima. Oblici koji se najčešće spominju su akceleracija (acceleration) i obogaćivanje programa (enrichment). Kod nas su moguća dva oblika akceleracije: raniji upis u školu i tzv. preskakanje razreda. Nema drugih oblika kao što bi primjerice bila mogućnost da dijete bude akcelerirano samo u predmetu za koje je darovito i koji ga posebno zanima – da daroviti učenik 6. razreda sluša matematiku s učenicima 7. razreda, a sve ostale predmete sa svojim vršnjacima. Kod obogaćenja programa najčešće je riječ o satima dodatne nastave koji se u brojnim slučajevima pretvaraju u vježbanje za natjecanja pa je pitanje koliko stvarno zadovoljavaju potrebe darovitih učenika za novim znanjima i novim oblicima rada. Drugi je problem što je podrška koja se pruža unutar školskog sustava najčešće orijentirana na sadržaj a nije usmjerena na osobni

razvoj darovitih matematičara. Uz matematička znanja valjalo bi istodobno razvijati komunikacijske vještine, vještine timskog rada ili pak dobru sliku o sebi – vještine i znanja od kojih će daroviti učenici profitirati i na osobnoj razini.

Kognitivna psihologija obrazovanja i kognitivna razvojna psihologija govore o mogućnostima djece različite dobi. Sigurno je da daroviti matematičari mogu naučiti brže i više od svojih vršnjaka. Pitanje je što ih trebamo poučavati. Naime uobičajen sadržaj je darovitima nedovoljno izazovan i valja ga mijenjati tako da se povećaju mogućnosti primjene i elaboracije onoga što se uči kao i da se ide u povećanje dubine i širine ponuđenih sadržaja. Iskusni nastavnici matematike znat će odabrati primjerene i djeci zanimljive sadržaje.

Posebna se pažnja mora obratiti na promjene u načinu poučavanja. Valja poštovati njihov odabir načina poučavanja što najčešće znači da treba izbjegavati klasično poučavanje i rabiti rad u malim grupama, rad na projektima ili rad s mentorom. Ako s darovitima samo rješavamo nešto teže zadatke (one koji se pojavljuju na natjecanjima) uskoro i to postaje samo poznata rutina. Njih valja poučavati složenim vještinama, poticati stjecanje temeljnih znanja a ne samo specifičnih činjenica, organizirati sadržaje koji su dovoljno izazovni i raznoliki da potiču više misaone procese (sintezu, analizu, generalizaciju, evaluaciju). Pouka darovitih mora uključivati i vještine kao što su kreativno i kritičko mišljenje, heuristika, te složene metode rješavanja problema i donošenja odluka. Valja ih poučavati metakognitivne vještine – vještine kontrole vlastitog učenja i praćenja misaonih procesa i znanja. U radu s darovitima treba insistirati na tome da djeca budu proizvođači, a ne samo konzumenti znanja (nešto što je kronični nedostatak naše škole) te ih naučiti da znaju jasno i razumljivo prezentirati svoje uratke. Sve ovo, dakako, vrijedi za različite oblike darovitosti no posebno je relevantno za matematički darovite – oni teže od verbalno darovitih komuniciraju svoja postignuća široj okolini.

Odgovor na pitanje što ih možemo poučiti je jednostavan – gotovo sve. Naravno na nama je da nađemo razumnu mjeru – poučiti ih ono što ih zanima, ono što im je izazov i što će osigurati da i dalje vole matematiku. Pritom valja misliti da je cilj dobrobit učenika u najširem smislu te riječ. To znači da naglasak valja stavljati i na sadržaje koji će doprinijeti socijalnom i emocionalnom razvoju djeteta. Daroviti matematičar treba istodobno biti i sretno, u okolinu dobro uklopljeno dijete. Tu je prostor za učenje odraslih – naučiti kako prepoznati dječje granice i dječje potrebe i kako ne ići preko njih.

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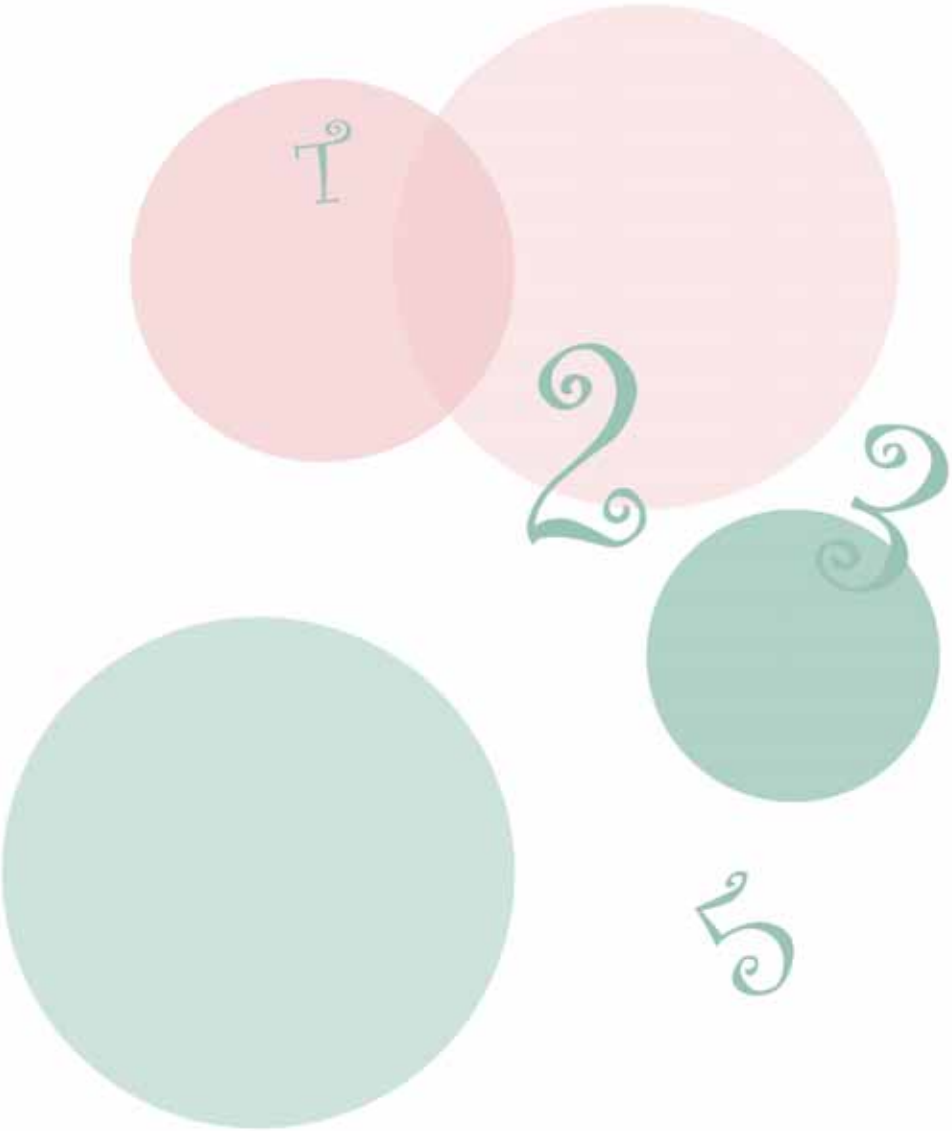
Priopćenja



TEŠKOĆE U NASTAVI MATEMATIKE U DRUGOM RAZREDU OSNOVNE ŠKOLE

Maja Cindrić[1] i Josip Cindrić[2]

Sažetak. *Rad upućuje na učeničke teškoće s zbrajanjem i oduzimanjem prirodnih brojeva do 100 u drugom razredu osnovne škole. Dan je statistički prikaz uspjeha učenika u prvom polugodištu drugog razreda u usporedbi s uspjehom u ostala tri razreda na uzorku od 104 učenika, te rezultate testiranja grupe učenika drugog razreda osnovne škole na temu zbrajanja i oduzimanja prirodnih brojeva do 100. Nadalje se rad osvrće na moguće uzroke lošijeg uspjeha i alternativna rješenja tih problema.*

Ključne riječi: *nastava matematike, teškoće u nastavi matematike.*

Uvod

Prije samog opisa problematike kojom se ovaj rad bavi poželjno je iznijeti ciljeve i obrazovne zadaće za nastavnu cijelinu "Zbrajanje i oduzimanje prirodnih brojeva do 100" istaknute i promovirane od strane autora hrvatskog nacionalnog obrazovnog standarda. Zbrajanje i oduzimanje prirodnih brojeva do 100 sugerira se predočavati i poučavati postepeno sljedećim redosljedom:

- Zbrajanje i oduzimanje desetica
- Zbrajanje dvoznamenkastog i jednoznamenkastog broja (prvotno bez prijelaza desetice, a kada se to usvoji i s prijelazom desetice)
- Oduzimanje jednoznamenkastog broja od dvoznamenkastog (prvotno bez prijelaza desetice, a kada se to usvoji i s prijelazom desetice)

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- Zbrajanje dva dvoznamenkasta broja (prvotno bez prijelaza desetice, a kada se to usvoji i s prijelazom desetice)
- Oduzimanje dva dvoznamenkasta broja (prvotno bez prijelaza desetice, a kada se to usvoji i s prijelazom desetice)
- Zbrajanje i oduzimanje više od dva dvoznamenkasta broja, sa i bez korištenja zagrade

Iako se direktno nigdje ne govori podrazumijeva se usvajanje prikazanih računa "napamet" , a ne pismeno potpisivanjem dva broja jedan ispod drugog , što međutim predlaže i svaki udžbenik odobren od strane Ministarstva znanosti , obrazovanja i športa. Takva tehnika zbrajanja i oduzimanja znači usmeno zbrajanje i oduzimanje prikazano sljedećim primjerima:

- 20 + 40 = 60 – na prikazu konkretnih objekata postavlja se analogija s zbrajanjem jednoznamenkastih brojeva
- 53 + 2 = 55 – konkretnim objektima pokazuje se da broj desetica ostaje nepromijenjen dok se jedinice zbrajaju i pribrajaju desetici
- 36 + 4 = 40 - također preko konkretnih primjera prikazuje sa da jedinice zbrojene daju jednu deseticu koja se pribraja postojećoj desetici
- 84 + 8 = 92 - rastavljajući drugi pribrojnik nadopunjuje se prvi do sljedeće veće desetice i pribrojava se ostatak npr. 84 + 8 = 84 + 6 + 2 = 90 + 2 = 92
- 67 – 5 = 62 - konkretnim objektima pokazuje se da se jednoznamenkasti broj oduzima od jedinica dvoznamenkastog broja , a broj desetica ostaje nepromijenjen
- 41 – 7 = 34 - tu se ponovo poziva na rastavljanje umanjitelja npr.
  41 – 7 = 41 – 1 – 6 = 40 – 6 = 34
- 36 + 57 = 36 + 50 + 7 = 86 + 7 = 95
- 62 – 35 = 62 – 30 – 5 = 32 – 5 = 27

Kao što je moguće uočiti svaki postupak zahtijeva dobro usvojeni prethodni postupak i to usvojen do razine automatizacije, te također automatizaciju računanja u skupu brojeva do 20 koju su morali usvojiti u prvom razredu osnovne škole. Automatizacija računanja podrazumijeva automatsko prizivanje činjeni-

ca iz memorije bez oslanjanja na proceduru računanja, kao što je računanje na prste.

Važno je naglasiti da se za usvajanje navedene tehnike računanja, uz ponavljanje znanja iz prvog razreda osnovne škole, predviđeno otprilike četiri mjeseca.

Da li je takav postupak problematičan za usvajanje ? Kako ga usvajaju sedmogodišnjaci ? Predstavlja li im savladavanje navedene tehnike računanja problem?

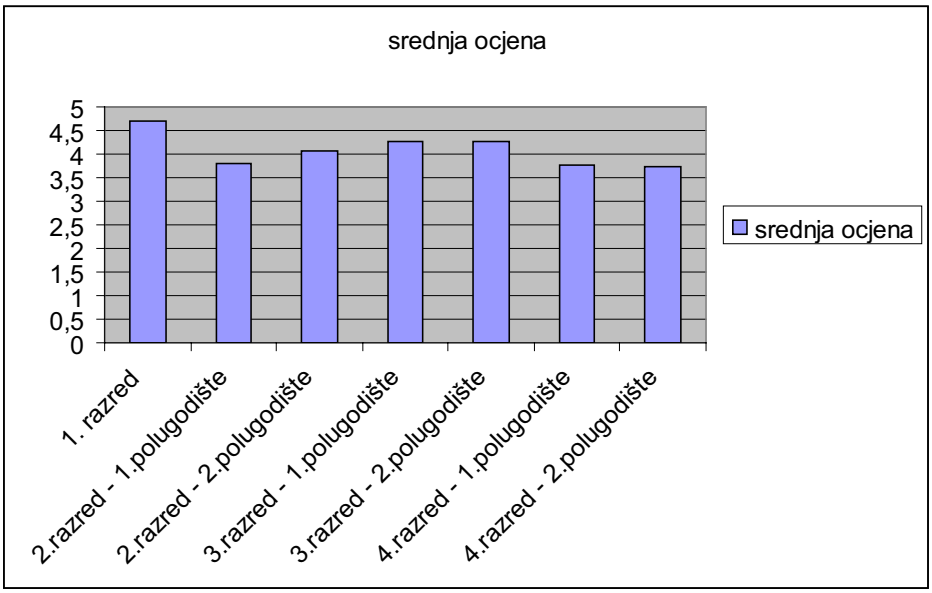
Prilikom individualnog rada s učenicima drugog razreda uočili smo kod pojedinih učenika velike teškoće s učenjem zbrajanja i oduzimanja u skupu prirodnih brojeva do 100 i vođeni tom spoznajom istražili smo kakav uspjeh na prvom polugodištu drugog razreda pokazuje slučajni uzorak od 104 učenika jedne osnovne škole u Zadru.

Uspjeh učenika drugog razreda osnovne škole u zbrajanju i oduzimanju brojeva do 100

Za opisano istraživanje odabran je slučajan uzorak od 104 učenika osnovne škole koji su ove školske 2006./07. godine u petom razredu osnovne škole. Odabrana metoda je kvantitativan opis uspjeha svakog pojedinog učenika u prvom, drugom, trećem i četvrtom razredu na polugodištu i kraju nastavne godine, osim prvog razreda gdje se kvantitativno bilježi uspjeh učenika samo na kraju nastavne godine. Uspjeh 104 učenika u pojedinim razredima prikazuje dana tablica i stupčasti dijagram:

	srednja ocjena
1. razred	4,71
2.razred - 1.polugodište	3,81
2.razred - 2.polugodište	4,07
3.razred - 1.polugodište	4,28
3.razred - 2.polugodište	4,28
4.razred - 1.polugodište	3,78
4.razred - 2.polugodište	3,75

Slika1. : Tablični prikaz prosječne ocjene iz matematike za 104 učenika

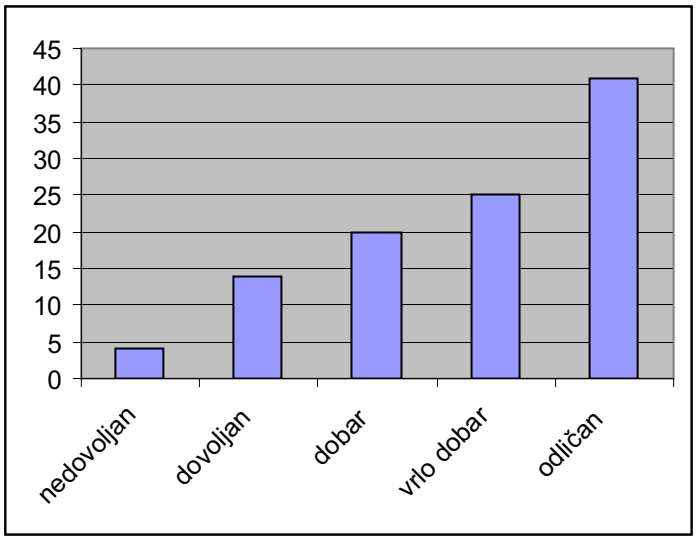


Slika 2. : Histogram prosječne ocjene iz matematike za 104 učenika

Podatci prikazuju nagli pad uspjeha učenika u prvom polugodištu drugog razreda , dok uspjeh u drugom polugodištu raste i tendencija rasta nastavlja se do kraja trećeg razreda,te ponovo pokazuje pad kvantitativnog uspjeha u četvrtom razredu. Uspjeh postignut u prvom razredu osnovne škole ne ponavlja se u prva četiri razreda. Odličan uspjeh u prvom razredu moguće je objasniti tolerantnošću učitelja prema početnicima u učenju, ali i programu prikladnom djeci te dobi. U usporedbi s prvim razredom na prvom polugodištu drugog razreda uočava se znatniji pad uspjeha. Kvantitativno gledajući 3,81 nije loš uspjeh, no pregled ocjena iz matematike učenika u drugom razredu pokazuje čak četiri nedovoljne ocjene na prvom polugodištu drugog razreda.

ocjene	
nedovoljan	4
dovoljan	14
dobar	20
vrlo dobar	25
odličan	41

Slika 3. : Tabličan prikaz ocjena iz matematike na prvom polugodištu drugog razreda za 104 učenika



Slika 4. : Histogram broja ocjena iz matematike na prvom polugodištu drugog razreda za 104 učenika

Taj pokazatelj upućuje na potpuno neusvojeno zbrajanje i oduzimanje brojeva do 100 kod četiri učenika. Da li bi takav uspjeh trebao biti zabrinjavajući? U razgovoru sa osam učiteljica većina učenika s teškoćom usvaja zbrajanje i oduzimanje u potpunosti, a najveći problem je zbrajanje i oduzimanje s prelaskom desetice i to usvajaju samo rijetki učenici. Možemo li dopustiti da savladavanje tehnike računanja nije kod većine učenika svladano?

Vrijeme koje se utroši na svladavanje tehnike računanja je preveliko u odnosu na rezultate koje učenici pokazuju. Da li je potrebno trošiti toliko vremena na usvajanje tehnike računanja koja je učenicima teška, kada se u trećem razredu uči zbrajati pismeno potpisivanjem? Zašto gubiti vrijeme na učenje zbrajanja i oduzimanja napamet, a nakon toga potpisivanjem? Ovaj osvrt ne želi negirati vrijednosti usmenog učenja računanja, već uz njegove prednosti istaknuti i nedostatke.

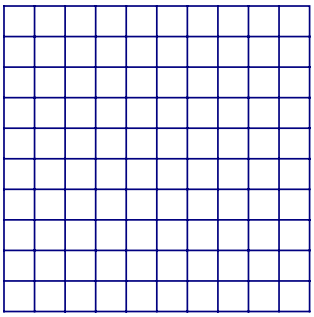
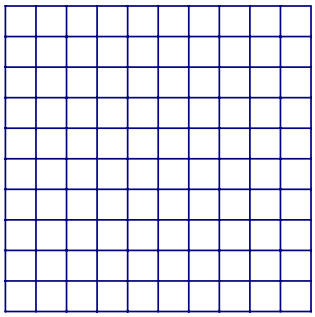
Daljnje istraživanje sastojalo se u testiranju 98 učenika drugog razreda osnovne škole . Naravno ne radi se o istim učenicima, jer su učenici čiji se uspjeh promatrao kroz četiri godine sadašnji učenici petog razreda. Izazov ovom radu bio bi daljnje pračenje ovih 98 učenika u uspjehu iduće dvije godine. Test se sastojao od sljedećih zadataka :

1. Izračunaj :

a) 51 + 27 = b) 48 + 39 = c) 68 - 46 = d) 87 - 59 =

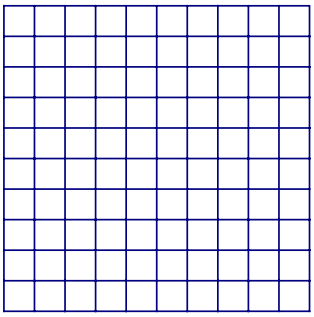
2. Ana i Iva su prijateljice. Ana je za Božić dobila 39 kockica, a Iva je dobila 33 kockice. Da bi sagradile dvorac kakav žele potrebno im je 100 kockica.

a)

	
Oboji crvenom bojicom onoliko kvadratića koliko kockica ima Ana	Oboji plavom bojicom onoliko kvadratića koliko kockica ima Iva

b) Koliko kockica zajedno imaju Ana i Iva.

c) Oboji onoliko kvadratića koliko kockica zajedno imaju Ana i Iva.



d) Da li imaju dovoljno za sagraditi željeni dvorac ?

e) Koliko im kockica nedostaje?

3. Ante , Lucija i Valentina išli su u maškare. Ante je dobio 18 kuna, Lucija 24 kune i Valentina 29. Zajedno žele kupiti loptu koja košta 80 kuna.

a) Koliko kuna su dobili zajedno ?

b) Koliko im kuna nedostaje da bi kupili loptu ?

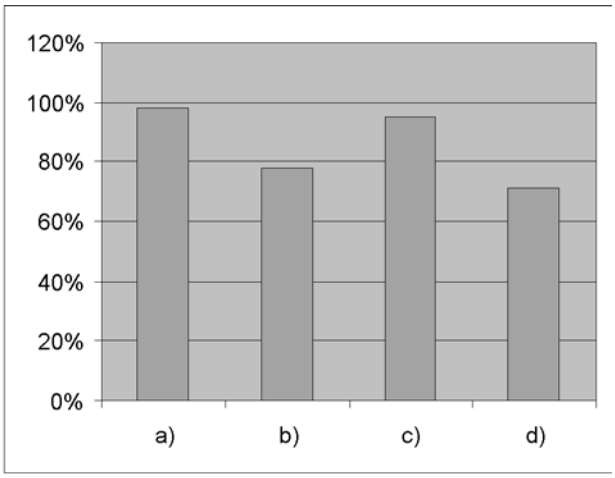
4. U kinu se nalazi 93 ljudi (muškaraca, žena i djece) . Muškaraca je 34, žena je 17 više nego muškaraca.

a) Koliko je u kinu žena ?

b) Koliko je u kinu djece ?

Test je sastavljen da ispita sposobnost rješavanja problemskih zadataka i korištenja tehnike računanja . Prvi zadatak ispituje tehniku računanja, drugi zadatak ispituje sposobnost rješavanja problemskog zadatka uz pomoć crtanja konkretne situacije. Treći zadatak je analogan drugom zadatku samo bez crtanja konkretne situacije.

Statistika prvog zadatka ukazuje da je 71 % učenika u potpunosti savladalo tehniku zbrajanja i oduzimanja do 100.



Kao što je za očekivati zbrajanje i oduzimanje sa prijelazom desetica pokazuje manji postotak uspješnosti. Zanimljivo je istaknuti grešku u 1. d) zadatku gdje se kao odgovor javljaju 32 i 22, što učenici dobivaju oduzimajući veću znamenku jedinica od manje znamenke jedinica , bez obzira koji je broj umanjienik , a koji umanjitelj. Takva greška nije slučajna jer mnogi roditelji da bi pomogli svojoj djeci koja teže savladavaju ovakvu tehniku računanja, poučavaju ih zbrajanju i oduzimanju na principu pismenog zbrajanja i oduzimanja što je u suprotnosti s onim što se radi u školi i to djecu još više zbunjuje. Tada djeca zaboravljaju na prenošenje desetica , jer za to ne vide logično objašnjenje, ili

oduzimaju ono što im je bliže dakle u izrazu 87 – 59 oduzimaju 9 - 7., što im daje rezultat 32 ili 22.

Analiza rješenja drugog zadatka pokazala je da samo 34 % učenika u potpunosti riješilo taj zadatak, dok je još 18% dalo rješenje uz uspješno obojane kvadratiće. Nakon završenog testiranja ispitano je tih 18 % učenika o postupku rješavanja zadatka i načinu na koji su došli do rješenja. Učenici bojajući i prebrojavajući kvadratiće riješili su zadatak. Tih 18 % učenika problem rješavaju koristeći konkretne objekte, a ne oslanjajući se na znanja o zbrajanju i oduzimanju brojeva. Rezultati trećeg zadatka ukazuju na bolji uspjeh, 43 % učenika uspješno je riješilo taj zadatak, dok je četvrti zadatak uspješno riješilo samo 18% učenika. Sagledavajući te rezultate postavlja se pitanje da li se uči zbrajati i oduzimati radi rješavanja problema ili se problemski zadaci rješavaju da bi se ilustriralo zbrajanje i oduzimanje i ono se podiglo na jednu višu razinu. Ciljevi nastave matematike trebali bi biti poticanje i usmjeravanje učenika ka rješavanju problema i korištenje matematičkih znanja u tu svrhu.

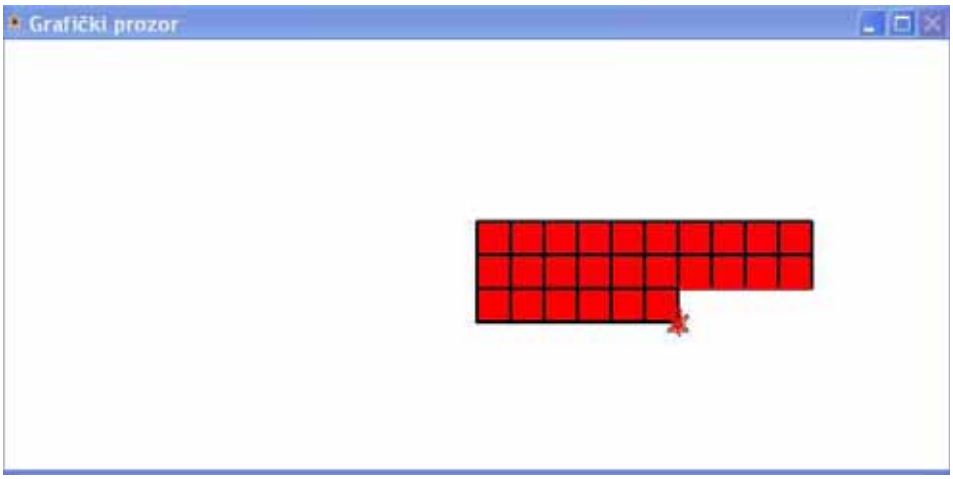
Moguća rješenja teškoća u zbrajanju i oduzimanju brojeva do 100

Prateći težnje reformiranog obrazovnog sustava za smanjenjem nastavnih sati iz matematike, koji je od ove školske godine 2006./07. smanjilo nastavu matematike sa pet sati tjedno na četiri sata tjedno, trebalo bi uvesti i efikasniji način rada. Ako su ciljevi nastave matematike stjecanje temeljnih matematičkih znanja potrebnih za razumijevanje pojava i zakonitosti u prirodi i društvu, stjecanje osnovne matematičke pismenosti i razvijanje sposobnosti i umijeća rješavanja matematičkih problema, tada naglasak treba dati na rješavanje problema kao u testnim zadatcima ,a ne naglasak na tehniku računanja. Hrvatski nacionalni obrazovni standard (HNOS) nadalje ističe odgojnu vrijednost matematike u formiranju ličnosti, razvijanju intelekta, razvijanju logičkog mišljenja i razvijanju stvaralačke sposobnosti pri rješavanju problema. Neuspjeh učenika te dobi neće pridonijeti pozitivnom razvoju ličnosti već strahu od matematike i nesigurnosti u svoje sposobnosti, što djeca često formuliraju : "Nisam ja za matematiku".

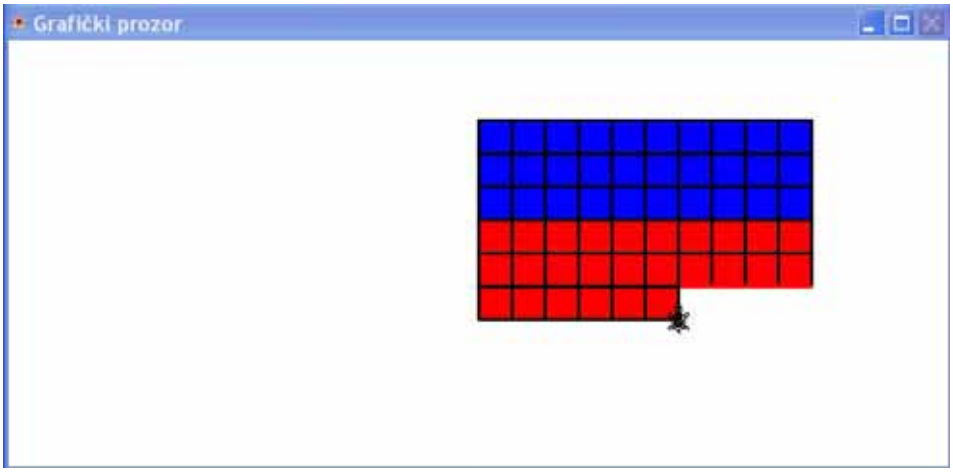
Prema analizi drugog zadatka koja pokazuje da djeca pribjegavaju neformalnim znanjima, trebalo bi im omogućiti učenje uz vizualizaciju brojeva preko konkretnih objekata i to ne samo prilikom uvođenja nastavnog sata , već i prilikom rješavanja problemskih zadataka. U tu svrhu moguće je koristiti pomagala kao što je računaljka (abacus) ili dinamički prikaz zbrajanja što je moguće

pomoću računalne tehnologije. Kao na primjer procedure izrađene u programskom jeziku Logu. Prikazani program upisivanjem zbroja ili razlike dva broja program iscrtava crvene i plave kvadrate kao u primjeru drugog zadatka Aninih i Ivinih kockica, a zbrajanje i oduzimanje prati proceduru zbrajanja i oduzimanja opisanu u uvodu.

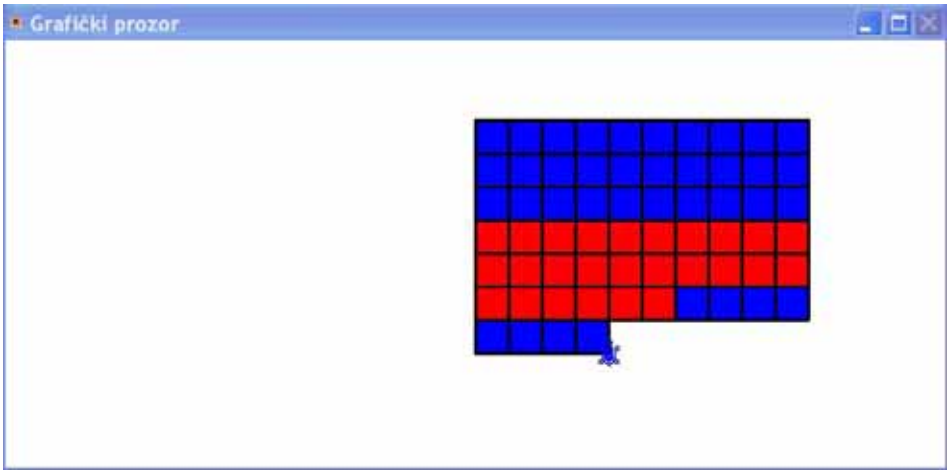
Ako želimo zbrojiti 26 + 38 program crta 26 crvenih kvdrata na sljedeći način:



a potom dodaje 38 plavih kvadrata tako da prvo pribraja desetice ,



a zatim jedinice:



Na analogan način, prateći način poučavanja, moguće je prikazati i oduzimanje. Sadašnje učenje zbrajanja i oduzimanja iziskuje mnogo truda i vremena kojim učitelji ne raspolažu, dakle treba pribjeći novim metodama kao pomoći pri bržem i lakšem svladavanju tehnike računanja. Također bi valjalo ispitati funkcionalnost i mogućnosti uvođenja pismenog zbrajanja i oduzimanja paralelno s usmenim zbrajanjem i oduzimanjem.

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OTROK IN PREPROSTE KOMBINATORIČNE SITUACIJE

Mara Cotič [1], Darjo Felda [2]

Povzetek. *V sedanjem slovenskem učnem načrtu za matematiko za osnovno šolo so prvič vpeljane vsebine iz statistike, kombinatorike in verjetnosti pod skupnim imenom obdelava podatkov, in to že na samem začetku šolanja (v prvem triletju).*

Namen matematičnega izobraževanja je predvsem dvojen: razvijati matematično pismenost in matematično mišljenje. Matematična pismenost naj bi bila v osnovni šoli cilj za vse učence. Razvijanje sposobnosti matematičnega mišljenja pa je zelo kompleksna dejavnost, namenjena predvsem, ne pa izključno, učencem s posebnim interesom za matematiko. Medtem ko statistika v glavnem sodi v področje matematične pismenosti, pa pri kombinatoriki razvijamo predvsem matematično mišljenje. Z učenjem reševanja kombinatoričnih situacij namreč vzpodbujamo razmišljanje in sklepanje, razvijamo sposobnost opazovanja in občutek za relacijo enakosti oziroma neenakosti, poskušamo ustvariti red med neurejenostjo, iščemo sorodne ali enake vzorce, postavljamo predpostavke o zakonitostih in opažamo strukturo sistema. Ni dovolj, da učitelj pozna cilje poučevanja kombinatorike, poznati mora tudi model razvoja osnovnih pojmov iz kombinatorike, ki smo ga priredili in nato dopolnili po Brunerjevem modelu razvoja matematičnih pojmov. Model v grobem sestoji iz treh nivojev: enaktivnega (konkretnega), ikoničnega (grafičnega) in simbolnega. Vsak nivo je razdeljen še na posamezne podnivoje. Tako v prvem enaktivnem nivoju najprej zastavimo izhodiščno problemsko situacijo in jo analiziramo, nato pa izvedemo dejavnost s predmeti. V ikoničnem nivoju grafično prikažemo izvedeno dejavnost najprej

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z risbo ali skico, nato pa še s sistematičnimi prikazi (preglednico, puščičnim diagramom in kombinatoričnim drevesom). Na koncu, v simbolnem nivoju, prikažemo dejavnost v še splošnejši obliki in posplošimo problem. Pomembno je poudariti, da otrok na začetku šolanja rešuje preproste kombinatorične situacije predvsem z neposredno izkušnjo (igro) na konkretnem nivoju, nekateri otroci pa so sposobni reševati kombinatorične probleme tudi na grafičnem nivoju. Zaradi različnih sposobnosti otrok je potrebno pri teh vsebinah izvajati pri pouku tako diferenciacijo kot individualizacijo.

Učenje teh vsebin v višjih razredih osnovne šole in predvsem v srednji šoli učencu razkrije »srce« matematike: kombinatorični koncepti se izražajo z jezikom teorije množic, rezultati in metode iz kombinatorike so zelo uporabni in koristni tudi na drugih matematičnih področjih, prav posebej v teoriji verjetnosti.

Bodoči učitelj bi moral v času študija razviti ustrezne kompetence, da bi znal v okviru svoje profesije otroku približati razumevanje preprostih kombinatoričnih situacij.

Ključne besede: *matematički pouk, statistika, kombinatorika in verjetnost.*

Uvod

Kombinatorika je matematična vsebina, ki so se je doslej učenci v Sloveniji začeli učiti razmeroma pozno (v srednji šoli), in še to le na formalni ravni. Ker je pouk matematike v srednji šoli praviloma deduktiven in na abstraktnem nivoju, učitelji kljub za učence popolnoma novim pojmom iz kombinatorike (permutacije, variacije, kombinacije, osnovni izrek kombinatorike ...) ne uporabljajo ustreznih konkretnih ponazoritev in modelov. To je zagotovo eden od razlogov za težave, ki jih imajo učenci pri razumevanju kombinatorike. Mialaret (1969) je z raziskavo dokazal, kako zelo je od abstraktne ali konkretne formulacije matematičnega problema odvisna uspešnost učenčevega razumevanja in reševanja celo v srednji šoli. Raziskave pa so potrdile tudi, da imajo učenci v srednji šoli težave pri kombinatoriki celo takrat, ko rešujejo kombinatorične probleme z ustreznimi učnimi ponazorili. Skozi svoje dotedanje šolanje so namreč premalo ali skoraj nič uporabljali konkretna ponazorila pri razvijanju osnovnih matematičnih pojmov, hkrati pa se pri matematiki še niso srečali s preprostimi kombinatoričnimi situacijami, niti s takimi, ki so tesno povezane z vsakdanjim življenjem.

Didaktična navodila in cilji

Z novim kurikulumom za matematiko (1998) smo želeli to pomanjkljivost odpraviti, zato smo že na samo začetno stopnjo šolanja v osnovni šoli (1. triletje) vpeljali kombinatoriko, čeprav veliko držav v svojih kurikulumih matematike v osnovni šoli nima te vsebine. Na začetku šolanja še ne gre za pravo učenje kombinatorike, ampak učenec pridobiva prva znanja zgolj na konkretnem nivoju skozi igro. S tem ga postopoma pripravljamo na abstraktno mišljenje.

Pri uvajanju kombinatorike v pouk matematike na začetni stopnji šolanja v osnovni šoli se pojavljajo dileme, ali je učenec na tej stopnji sposoben reševati tovrstne probleme. Po Piagetu in Inhelderjevi (1951) je namreč otrok sposoben reševanja takih vrst problemov šele na stopnji formalnih operacij (11 - 15 let). Zaključki Piageta in Inhelderjeve, da kombinatorika ni primerna na stopnji konkretnih operacij otrokovega razvoja (7 - 11 let), temeljijo samo na spontanih odgovorih otrok, ne da bi te vsebine prej uvajala. Tu je nujno poudariti, da v prvih letih šolanja ne gre za pravo učenje kombinatorike, saj ta kot matematična disciplina zahteva metode, ki nam omogočajo, da v kombinatornih situacijah brez direktnega preštevanja določimo število elementov neke končne množice. Učenci ne uporabljajo teh metod oziroma se srečujejo s tako preprostimi kombinatornimi situacijami, v katerih množica ni "prebogata", zato lahko njene elemente preprosto preštejejo.

Opravljene matematično didaktične raziskave o primernosti uvajanja kombinatorike na začetni stopnji šolanja v osnovni šoli pa so ovrgle trditev Piageja in Inhelderjeve. Tu bi omenili predvsem empirično raziskavo, ki jo je opravil Fischbein v Izraelu že leta 1970, in empirično raziskavo, ki sta jo v Sloveniji leta 1993 v projektu Inoviranje osnovne šole izvedli M. Cotič in T. Hodnik. Vsebina je bila vpeljana na način, ki je primeren razvojni stopnji otroka v tem obdobju. Učenec začne reševati kombinatorične situacije z neposredno izkušnjo (igro); to pomeni, da manipulira s predmeti (seveda mora biti število predmetov majhno) (Fischbein, 1975). Uporablja naj predmete iz svojega vsakdana in z njimi naj izvaja primerne aktivnosti, na primer:

- iz lesenih kroglic različnih velikosti ali barv narediti čim več različnih verižic,
- razporediti raznobarvne žetone (ali modele geometrijskih likov oziroma teles) na vse možne načine,
- ob upoštevanju določenih navodil sestaviti iz danih dveh, treh, štirih črk ali zlogov čim več različnih besed,

- iz danih števk sestaviti vsa možna števila itd. (Fischbein, 1984).

Ob teh konkretnih aktivnostih naj bi učenec z učiteljevo pomočjo uvidel, da se je določenih kombinatornih situacij nujno potrebno lotiti predvsem sistematično. Pri tem moramo otroke naučiti uporabljati različne grafične prikaze (drevo, črtni prikaz, preglednica).

V življenju se velikokrat srečamo s poplavo podatkov, ki jih moramo znati urejevati in uporabljati. Učenje reševanja kombinatornih situacij je zato zelo pomembno, saj z njim:

- razvijamo sposobnost opazovanja,
- razvijamo občutek za relacijo enakosti oziroma neenakosti,
- poskušamo ustvariti red med neurejenostjo,
- iščemo sorodne ali enake vzorce in postavljamo predpostavke o zakonitostih, opažamo strukturo sistema (Felda, 1996).

Učenčevi nivoji pri reševanju kombinatoričnih situacij

Učenec na začetni stopnji šolanja v osnovni šoli pri reševanju preprostih kombinatoričnih situacij oziroma pri usvajanju novih pojmov iz kombinatorike (permutacije, kombinacije, variacije, osnovni izrek kombinatorike ...) "prehodi" nivoje, ki smo jih priredili in nato dopolnili po Brunerjevem oziroma Dörflerjevem modelu razvoja matematičnih pojmov (Kokol-Voljč, 1996). Uspešnost tega modela izgradnje osnovnih pojmov iz kombinatorike smo empirično preverili z raziskavo, v kateri je sodelovalo 180 učencev, starih od 7 do 9 let (Cotič, 1998). Učenci so na preizkusu, ki smo ga opravili po končanem projektu, dosegli v povprečju 80 % možnih točk.

I. KONKRETNI NIVO

1. Zastavitev izhodiščne problemske situacije
2. Analiza izhodiščne problemske situacije
3. Izvedba aktivnosti: - igranje situacij
   - predstavitev s predmeti

II. GRAFIČNI NIVO

4. Shematizacija dejavnosti (risba, skica)
5. Shematizacija dejavnosti s sistematičnimi prikazi (preglednica, kombinatorno drevo, črtni prikaz)

III. SIMBOLNI NIVO

6. Prikaz dejavnosti v še splošnejši obliki (nastavitev računa za posamezen primer)
7. Posplošitev problema

IV. UPORABA RAZVITEGA POJMA V NOVI SITUACIJI
(razviti pojem deluje kot instrument)

Enaktivnega, ikoničnega in simbolnega nivoja ne smemo gledati statično, kot da bi proces učenja pojmov potekal najprej samo enaktivno, nato ikonično in slednjič simbolično. Te različne predstavitvene nivoje moramo pojmovati zelo fleksibilno in jih različno vključevati v pouk; ti trije nivoji si lahko sledijo zaporedno, npr. konkretno enaktivnost prenesemo v sliko in nato sliko opišemo in zapišemo s simboli, in tudi tako, da konkretno enaktivnost prenesemo takoj na simbolno raven. Nadalje lahko po sliki konkretno "operiramo" in obratno, da po sliki opišemo potek in ga zapišemo s simboli (Tomić, 1984).

Pomembno je poudariti, da na začetni stopnji šolanja v osnovni šoli, ko je učenec na stopnji konkretnih operacij, pri oblikovanju matematičnih pojmov ne smemo nikdar izpustiti konkretnega nivoja, hkrati pa ta tudi ne sme biti prekratek. Razlog za nerazumevanje osnovnih matematičnih pojmov je največkrat ravno v tem, da je konkretni nivo izpuščen ali pa je prekratek.

Prehod iz konkretnega nivoja na abstraktni nivo tudi ni cilj ene ure ali dneva, ampak je to dolgoročni cilj. Nujno je še omeniti nekatere značilnosti tega zaporedja nivojev:

- učenci celo na najvišjem nivoju izvajajo aktivnosti z materiali;
- učenci se najprej osredotočijo na procese in intuitivne odnose in šele potem na odgovore ali simbolizacije rešitev z matematičnimi izrazi;
- šele po precejšnjih aktivnostih na določenem nivoju se učenec lahko začne ukvarjati s podobnimi aktivnostmi na višjem nivoju predstavitve;
- v razredu se lahko različni učenci kadarkoli igrajo isto igro na različnih nivojih, učitelji "nadzorujejo" učenca pri prehodu na višji nivo;
- učenci imajo priložnost, da sami usmerjajo nastajanje lastnih problemov in predstavljenih operacij (Labinowitz, 1989).

Prikazano ogrodje modela razvoja pojma bomo konkretizirali s primerom. Za primer smo izbrali pojem *osnovni izrek kombinatorike*, ker je eden najelementarnejših pojmov iz kombinatorike.

I. Enaktivni (konkretni) nivo

1. Zastavitev izhodiščne problemske situacije

Učenci, stari od 6 do 7 let naj dobijo modele enakih (skladnih) pravokotnikov in enakih (skladnih) enakokrakih trikotnikov, ki imajo osnovnico enako dolgo, kot je dolga ena izmed stranic pravokotnika. Liki naj bodo različnih barv. Učenci naj se z modeli najprej igrajo: sestavljajo iz njih različne figure, jih razvrščajo po barvi oziroma obliki … Nato jim zastavimo naslednji problem: *Koliko različnih hišk lahko naredite iz trikotnikov treh različnih barv (rdeče, modre in rumene) in pravokotnikov dveh različnih barv (zelene, oranžne), če je vsaka hiška sestavljena iz enega pravokotnika in enega trikotnika?*

2. Analiza izhodiščne problemske situacije

Učenci analizirajo, na kakšen način bodo rešili dano problemsko situacijo: iz koliko in katerih likov bo sestavljena hiška, na kaj bodo morali vse paziti, kako bodo prepoznali enake oziroma različne hiške ...

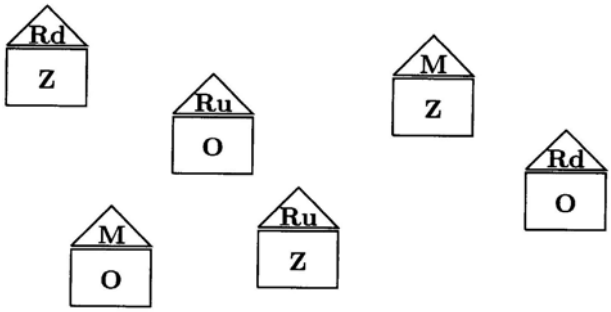
3. Izvedba aktivnosti

Učenci sestavljajo hiške. Ker je število pravokotnikov in trikotnikov majhno, bo pretežna večina učencev našla vse različne hiške (6 hišk). Nekateri učenci hiške sestavljajo nesistematično, zato se jim zgodi, da dobijo tudi "enake" hiške ali pa kakšno od možnosti izpustijo. Ti naj bi z učiteljevo pomočjo ugotovili, da se je kombinatoričnih situacij nujno potrebno lotiti sistematično, če želimo dobiti vse različne hiške. Redki učenci pa se že takoj odločijo za določen sistem. Ob izvajanju aktivnosti naj učenci ubesedijo svoje delo oziroma postopke.

II. Ikonični (slikovni) nivo

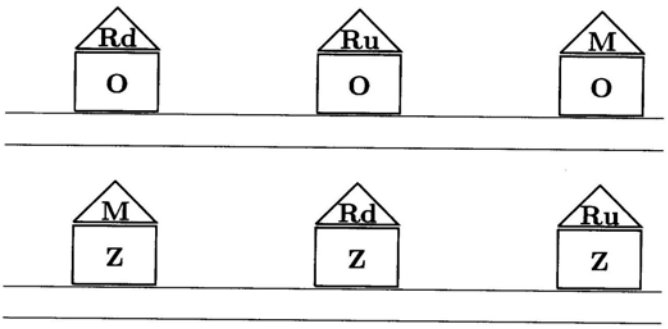
4. Shematizacija dejavnosti (risba, skica)

V tem koraku učenec grafično prikaže svoje rešitve, ki jih je sestavil z modeli likov. Najprej nastane nesistematična slika:



Slika 1. Nesistematična slika

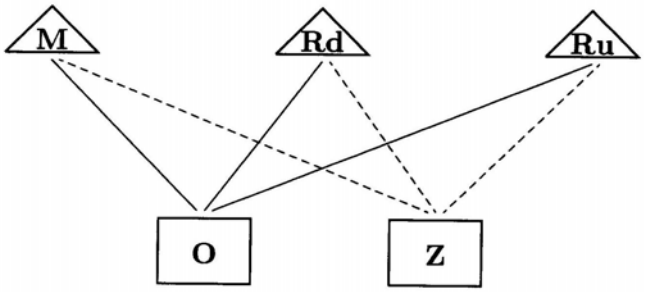
Nekateri učenci odkrijejo sistem, oblikujejo na primer dve ulici, v vsaki ulici so pročelja hiš enake barve. Tako nastane naslednja slika:



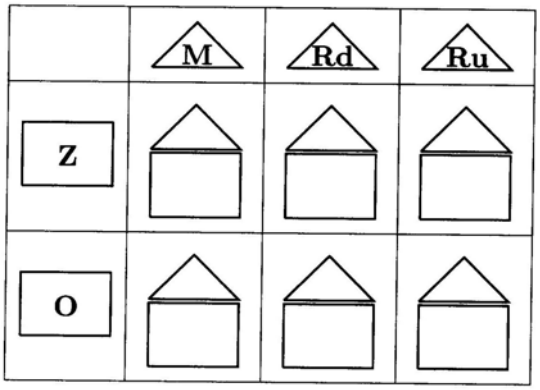
Slika 2. Dve ulici

5. *Shematizacija dejavnosti s sistematičnimi prikazi*

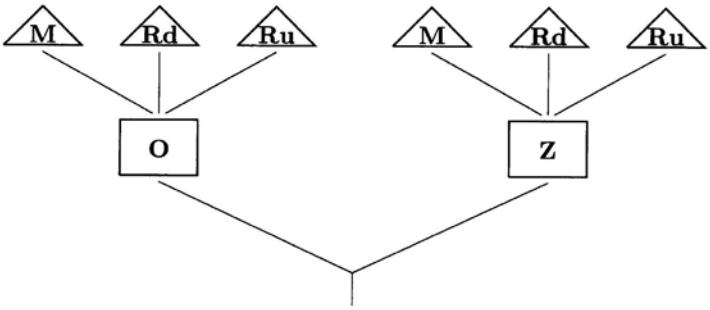
Učence počasi navajamo, da kombinatorične situacije prikažejo s črtnim prikazom, preglednico in kombinatoričnim drevesom.



Slika 3. Črtni prikaz



Slika 4. Preglednica



Slika 5. Kombinatorično drevo

III. Simbolni nivo

6. Prikaz dejavnosti v splošnejši obliki (nastavitev računa za posamezen primer)

Iz preglednice, puščičnega prikaza in kombinatoričnega drevesa učenec uvidi, da izračuna število vseh različnih hišk z množenjem števila pravokotnikov različnih barv in števila trikotnikov različnih barv; v našem primeru: $2 \cdot 3 = 6$.

7. Posplošitev problema (formalno pravilo)

K pravokotnikoma dveh različnih barv (zelen, oranžen) dodamo še bel pravokotnik. Z uporabo različnih grafičnih prikazov učenci ugotovijo, da je število različnih hišk sedaj enako: $3 \cdot 3 = 9$.

Nato dodamo k trikotnikom treh različnih barv (rdeče, modre, rumene) še en trikotnik rjave barve. Učenci ob grafičnem prikazu zapišejo račun, s katerim izračunajo število vseh različnih hišk: $3 \cdot 4 = 12$.

Z dodajanjem pravokotnikov in trikotnikov različnih barv učenci pridejo do osnovnega izreka kombinatorike. Če je sestavljen izbor tak, da zbiramo najprej med m možnostmi, nato pa neodvisno od prvega izbora med n možnostmi, potem je vseh možnosti:

$$m \cdot n .$$

IV. Uporaba razvitega pojma v novi situaciji

Pojem osnovnega izreka kombinatorike uporabimo v novih problemskih situacijah (npr. pri izgradnji drugih pojmov iz kombinatorike in verjetnosti itd.).

SKLEP

Opisani model izgradnje pojma osnovnega izreka kombinatorike smo zaradi nazornosti predstavili v med seboj ločenih korakih. Vendar je že iz vsebine razvidno, da se ti "koraki" dejansko med seboj prepletajo in da je težko med njimi potegniti ločnico. Vedno si tudi ne sledijo v tako strogo določenem vrstnem redu oziroma je včasih kakšen tudi izpuščen (Kokol-Voljč, 1996). Zato pri uvajanju novih pojmov iz kombinatorike ne bomo korake med seboj tako strogo ločevali, ampak bomo oblikovanje posameznih pojmov prikazali kot celovit proces. Poleg tega učenci na začetni stopnji šolanja v osnovni šoli ne "prehodijo" vseh korakov, ampak največkrat dosežejo peti korak, odvisno od narave in zahtevnosti kombinatorične situacije ter učenčevih sposobnosti. Zaradi tako velikih razlik med sposobnostmi učencev je nujno, da se pri poučevanju kombinatorike na začetni stopnji šolanja v osnovni šoli posebno poudari individualizacijo in diferenciacijo postopkov in zahtev.

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NACIONALNI MATEMATIČKI KURIKUL ZA PRIMARNO OBRAZOVANJE - EUROPSKA ISKUSTVA I TRENDOVI

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Sažetak. *U izlaganju prezentiramo rezultate istraživanja nacionalnih kurikulskih dokumentata za obvezno obrazovanje u 11 europskih država i pokrajina, s posebnim naglaskom na njihove dijelove posvećene primarnom matematičkom obrazovanju. Analizirani su nacionalni matematički kurikuli sljedećih zemalja: Austrija, Finska, Irska, Mađarska, Nizozemska, Norveška, Njemačka (Nordrhein - Westfalen), Slovenija, Švedska i Velika Britanija (Engleska, Škotska). Pri tome je reprezentativni uzorak zemalja odabran tako da obuhvaća europske zemlje razvijenih i dokazano uspješnih obrazovnih sustava (skandinavske i anglosaksonske zemlje), zemlje čiji su obrazovni sustavi do sada značajno utjecali na hrvatski sustav odgoja i obrazovanja (Njemačka, Austrija), kao i Hrvatskoj susjedne tranzicijske zemlje slične obrazovne tradicije (Slovenija, Mađarska). Budući da se radi uglavnom o dokumentima novijeg datuma, nastalima mahom nakon 2000. godine, njihova komparativna analiza omogućava identifikaciju aktualnih europskih trendova u primarnom matematičkom obrazovanju te njihovu usporedbu s dijelom novog Nastavnog plana i programa za osnovnu školu u Republici Hrvatskoj koji se odnosi na nastavu matematike. Cilj istraživanja bio je uočiti sličnosti i zajedničke elemente matematičkih kurikula svih navedenih zemalja, kao i njihove specifičnosti, te utvrditi u kojoj mjeri korespondiraju trenutnoj hrvatskoj praksi ili, eventualno, od nje odudaraju. Mišljenja smo da recentna europska iskustva svakako mogu pomoći u izgradnji hrvatskog nacionalnog kurikula, kao i doprinijeti uočavanju potencijalnih slabosti u organizaciji nastave matematike u našim osnovnim školama te njihovom otklanjanju. Istraživanje je provedeno u sklopu znanstvenoistraživačkog projekta MZOŠ 0100500 Evaluacija nastavnih programa i razvoj modela kurikuluma za obvezno obrazovanje, Centar za istraživanje i razvoj obrazovanja, Institut za društvena istraživanja, Zagreb (voditeljica projekta: dr. sc. Branislava Baranović).*

Ključne riječi: *nacionalni matematički kurikul, nastava matematike*

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DINAMIČNA NASTAVA MATEMATIKE I PAMETNA PLOČA (Poster)

Saša Duka[1], Damir Tomić[2]

Sažetak. *Na skupu će se demonstrirati nekoliko prikladnih primjera primjene pametne ploče u nastavi matematike s učenicima mlađe školske dobi. Korištenje pametne ploče na nastavi matematike motivira učenike mlađe školske dobi na aktivno sudjelovanje u nastavnom procesu te čini nastavu zanimljivijom.*

Ključne riječi: *nastava matematike, pametna ploča.*

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DIJETE S DISKALKULIJOM, MATEMATIKA I STUDENTI UČITELJSKIH STUDIJA

Lidija Goljevački[3], *Aleksandra Krampač - Grljušić*[4]

Sažetak. *Prema anketi provedenoj u okviru predmeta Metodika matematike (Teaching Mathematics) među apsolventima učiteljskih studija u Osijeku akademske 2003./04. godine, najmanje samopouzdanja uoči zapošljavanja studenti imaju u području detekcije učenika s posebnim potrebama, izradi prilagođenih programa i poučavanju djece s teškoćama u učenju. U članku se razmatraju mogućnosti i prilike stjecanja samopouzdanja i kompetencija studenata učiteljskih studija za uspješno uključivanje djece s diskalkulijom u nastavu matematike.*

Ključne riječi: *diskalkulija, diskalkulično dijete, kompetencije studenata učiteljskih studija, samopouzdanje studenata učiteljskih studija.*

1. Što je diskalkulija

Diskalkulija je skup specifičnih teškoća koje dijete ima pri učenju matematike. Mogu se pojaviti u svim ili samo nekim područjima matematike, bez obzira na prosječno intelektualno funkcioniranje djeteta. Diskalkulija može utjecati na sposobnost pamćenja matematičkih podataka te na pojam vremena i novca. Diskalkuliju ne možemo otkloniti kod djeteta, ali mu možemo pomoći da razvije vještine koje će mu omogućiti nošenje s tim problemima. Stoga je najvažnije rano otkrivanje diskalkuličnoga djeteta.

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2. Detektirani učenici osnovne škole s posebnim potrebama u osječko-baranjskoj županiji u školskoj 2006./07. godini

Prema najnovijim podacima od siječnja 2007., u Osječko-baranjskoj županiji osnovnoškolskim obrazovanjem obuhvaćeno je 727 učenika s posebnim obrazovnim potrebama koji su integrirani u redovite razredne odjele. Za te učenike izrađen je primjeren model školovanja. Također je 95 učenika djelomično integrirano u posebne razredne odjele. Iz tablice 1. iščitava se da je u ukupnoj populaciji učenika osnovne škole u osječko-baranjskoj županiji njih 2,799% detektirano s posebnim obrazovnim potrebama, što je značajno ispod svjetskih pokazatelja koji govore o 10% takvih učenika u populaciji.

Tablica 1. Podaci o učenicima s posebnim obrazovnim potrebama u Osječko-baranjskoj županiji u školskoj 2006./07. godini

Područje	Br. škola	Br. raz. odjela	Br. učenika	Uč. POP
Osijek	20	396	9223	224
Osijek – okružje	9	174	2899	91
Baranjsko područje	11	215	3446	119
Doljnjo-miholjačko područje	4	94	1753	48
Đakovačko područje	14	282	5488	150
Našičko područje	6	161	3250	114
Valpovačko područje	6	156	3290	76
UKUPNO	70	1451	29358	822

Istraživanje provedeno među učiteljima razredne nastave u osječkim osnovnim školama (Pavleković i dr., 2007.) govori o uvjerenju učitelja iz prakse da je broj učenika s teškoćama u učenju matematike za koje je potreban prilagođen ili poseban program značajno veći od detektiranih učenika s posebnim obrazovnim potrebama. Glavnim uzrokom ovakvoga stanja je nedostatak psihologa i defektologa školi, što potvrđuju učitelji i iskustva u praksi.

Izostanak stručne podrške učiteljima u praksi ima za posljedicu izostanak prilika studentima učiteljskih studija za postizanje nužnih kompetencija i samopouzdanja za rad s učenicama posebnih potreba.

3. Kako prepoznati učenika s diskalkulijom

- pojam broja usvaja znatno kasnije od prosječnoga djeteta
- teško razumije razdvajanje cjeline na dijelove
- pokazuje poteškoće pri izgradnji novih cjelina
- sklon je neispravnoj uporabi brojeva pri čitanju, pisanju i računanju
- zrcalno okreće znamenke te 6 čita kao 9 i obratno
- prilikom čitanja i pisanja višeznamenkastih brojeva narušava i/ili zrcalno okreće redoslijed (npr. 43 čita kao 34)
- vizualno griješe kod prepoznavanja znakova računskih operacija, pa "+" prepoznaju kao "-" i zbog toga obavljaju pogrješnu operaciju
- ponavljaju isti broj ili radnju više puta (ako se u prvom zadatku zbrajalo učenici će vršiti računsku operaciju zbrajanja u svim zadatcima na jednoj stranici, bez obzira na promjenu računskih operacija u sljedećem zadatku)
- imaju poteškoća u pamćenju i prepoznavanju brojevnog niza (npr. svoj telefonski broj 580 042 neće prepoznati ako je napisan na način 58 00 42)
- zamjenjuju jednu znamenku drugom znamenkom, a pri tome te znamenke nisu sličnog oblika
- najčešće pogrešno zapisuju izgovoreni broj
- sporiji su od svojih vršnjaka
- imaju poteškoće u pravilnom potpisivanju pribrojnika
- izostavljaju korake u rješavanju zadatka

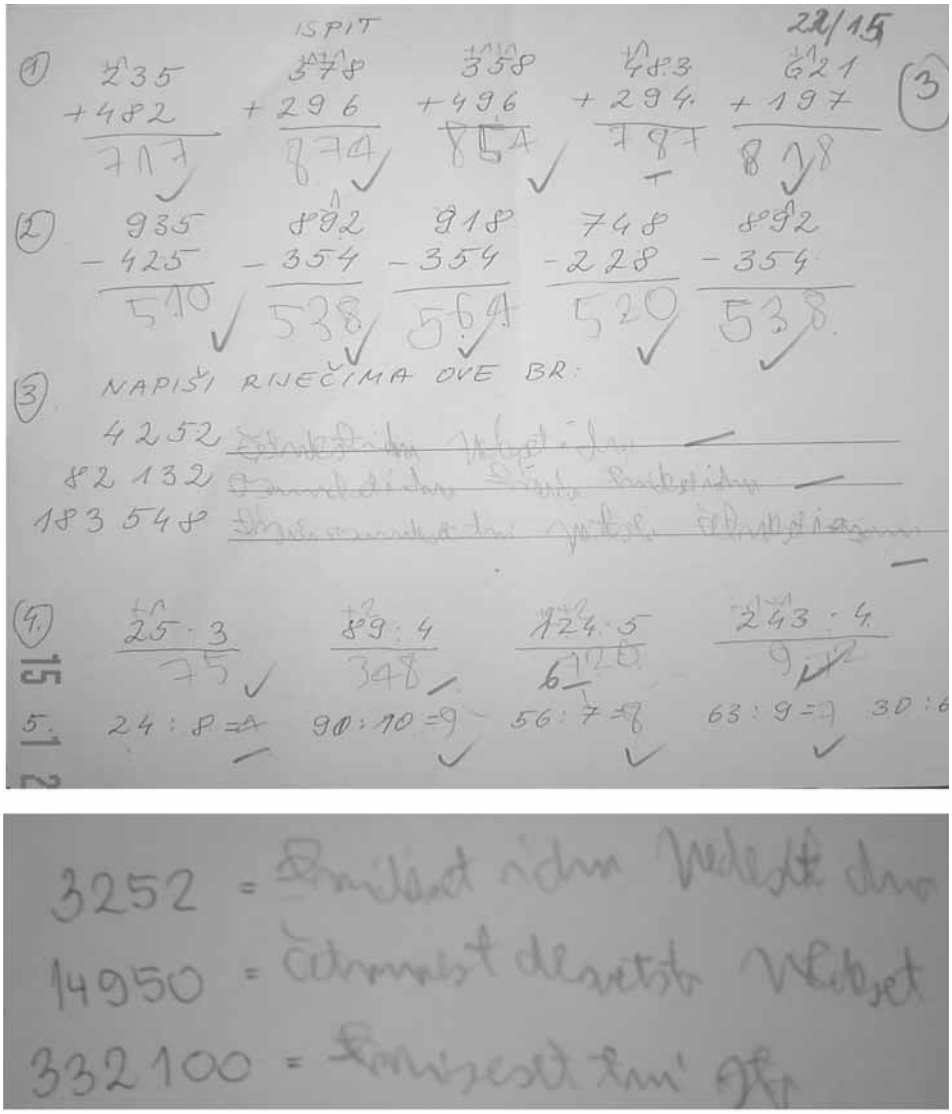
4. Kako poučavati dijete s diskalkulijom

- dati prednost usmenim oblicima poučavanja i provjeravanja pred pismenim oblicima
- pri obradi novih sadržaja koristiti poznato (učenikova iskustva), konkretne primjere, pokuse
- koristiti istovremeno različite vrste podražaja – vidne, slušne, dodirne kod obrade novih nastavnih sadržaja
- izbjegavati učenikovo glasno čitanje
- koristiti učenje unaprijed u dogovoru s roditeljima
- provjeriti je li učenik razumio sadržaj, definiciju
- koristiti tiskani tekst, a izbjegavati tekst pisan rukom

- povećati razmak između slova i redova u tekstu
- podebljati slova, znamenke kad god je to prikladno
- tekst ne treba podcrtavati jer to može dovesti do vizualnog spajanja riječi
- poravnati tekst na lijevoj strani
- tekst podijeliti na manje cjeline, organizirati ga natuknicama ili pomoću numeričkog nabrajanja u odvojenim redovima
- u udžbenicima jasno označiti bitne dijelove teksta (definicije, postupke, pravila)
- pri pisanju na ploču koristiti natuknice umjesto cjelovitoga teksta
- jasno odvojiti skupine zadataka u kojima se koristi ista računska operacija
- pri pismenim zadaćama provjeriti je li učenik razumio uputu
- po potrebi učitelj ili drugi učenik-pomagač može pročitati zadatak učeniku s diskalkulijom
- na jednoj stranici treba biti manji broj zadataka
- zadatke poredati od jednostavnijih k složenijima, od lakših k težima
- u složenijim zadatcima naznačiti podzadatke koji vode ka rješenju
- ograničiti vrijeme rješavanja zadataka
- dobar uradak i ponašanje pohvaliti i nagraditi
- prilikom ocjenjivanja vrjednovati motivaciju i zalaganje na satu, a ocjena znanja treba biti motivirajuća

5. Kako razviti kompetencije i samopouzdanje studenata učiteljskih studija za aktivno uključivanje diskalkuličnoga djeteta u nastavni proces.

Iskustva pokazuju da studenti mogu steći samopouzdanje i kompetencije za uspješno uključivanje diskalkuličnoga učenika u nastavu matematike neposrednim radom s bar jednim djetetom kroz dulje vremensko razdoblje (najmanje jedan semestar) u njegovom razrednom okruženju. Kroz to vrijeme važno je da student bude aktivno uključen u sve etape rada s djetetom posebnih potreba (od detekcije do vrednovanja napretka djetetovih postignuća) pod neposrednim nadzorom mentora i stručnoga tima odgovarajuće osnovne škole. Ovakav rad podrazumijeva i suradnju studenata s roditeljem djeteta.



Uradak 11-godišnjega diskalkuličnoga djeteta s dodatnim teškoćama u pisanju – iz diplomskoga rada studentice Kristine Đapić (2007)

Postizanje samopouzdanja i kompetencija studenata učiteljskih studija za organizaciju nastave (matematike) u koju su integrirana djeca s diskalkulijom, disleksijom i disgrafijom pretpostavlja razrađenu strategiju partnerskoga odnosa učiteljskoga fakulteta s osnovnom školom cjelovitoga tima stručnih službi

(pedagog, psiholog, defektolog) u kojoj, pored integracije djece s poteškoćama u redovne razrede, postoji praksa djelomične integracije djece u redovne razrede.

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NEHÉZ-E A MATEMATIKA NYELVE? (A matematika nyelv használatának szintje a tanítóképzős hallgatóknál)

Éva Kopasz[1]

Összefoglaló. *Kodolányi Gyula a Magyar Szemlében azt nehezményezi, hogy „a matematika nyelve kiszorítja a szavak nyelvét", és hogy századunk nyelve a matematika lesz. Ha ez egy kissé sötét kép is, azt mindenképpen tapasztaljuk, hogy a beszélt nyelv rövidíti, ferdíti a szavakat. Különösen a fiatalok körében lehetünk ennek tanúi. Ezért úgy gondoljuk, oda kell figyelnünk arra, hogy a leendő pedagógusok mennyire értő szinten használják a matematika nyelvét. A fogalomalkotás kezdetben – az iskoláskor elején is – a beszédhez kötött. Egyetértünk Vigotszkijjal abban, hogy a tanítás folyamatában fejlődnek a tudományos fogalmak is, és hogy a „köznapi fogalmakból nyert következtetések átvitele a tudományos fogalmakra nem egyenletes". Az átvitelt tovább nehezíti az, hogy a tudományos fogalmak tartalma – habár a szó azonos – nem fedi vagy nem teljesen fedi a köznapi fogalmakét.*

Előadásomban az kívánom vizsgálni, hogy a bajai tanítóképzős hallgatóknak milyen nyelvi nehézségei vannak a matematika tanulása során.

Az a hipotézisem, hogy a tanítóképzős hallgatók matematikával kapcsolatos problémáinak egy jelentős része nyelvi gyökerű. Bizonyos szavakat köznyelvi értelemben használnak, amelyek részben ellentmondanak a matematikai tartalmaknak. A szavak rejtette fogalmak nem teljes körű értése nehezíti a matematikatanulást.

2005. novemberében vettem fel egy pretesztet az aritmetika témaköréből. Ebben az első évfolyamos bajai tanítóképzős hallgatók közül 49-en vettek részt. A preteszt három fő részből áll:

1. kulcsszavak közül kiválasztani azt, amelyik igazzá/hamissá teszi az állítást

2. négy lehetőség közül kiválasztani az igaz állítást

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3. definíciók átfogalmazása.

A 15 tesztkérdés közül hármat választottam bemutatásra.

Ez a vizsgálódás egy hosszabb munka része. A feltett hipotézisre nem tudok most még egyértelmű választ adni. Csak az írásbeli munkák alapján nem is tudtam minden esetben kiszűrni, hogy a hibák melyike nyelvi eredetű, melyike tükrözi matematikai ismeret hiányát. Feldolgozásra várnak még a hallgatók egy részével készített interjúk, amelyek valószínű, segítenek a problémák besorolásában.

Kulcsszavak: *matematikai oktatás, a matematika nyelv.*

BEVEZETÉS

Kodolányi Gyula a Magyar Szemlében nehezményezi, hogy „a matematika nyelve kiszorítja a szavak nyelvét", és hogy századunk nyelve a matematika lesz. Ez a megállapítás kissé sötét képet fest a matematika nyelvéről, a matematikusok véleménye szerint viszont részben az anyanyelvi ismeretek hiánya okozza a matematikai problémák megértésének nehézségeit. Tapasztalataink szerint csak azok a tanulók képesek pontosan megérteni a matematika nyelvét, akiknek jó szintű az anyanyelvi ismerete.

A matematikában vannak bizonyos egyszerűsítési eljárások, pl. szimbólumokat használnak szavak helyett, amelyek segítik a könnyebb, rövidebb lejegyzést. Előfordulnak azonban olyan egyszerűsítési törekvések, amelyek negatívan befolyásolhatják a feladatok megoldását, pl. az „összeg", a „különbség", a „szorzat" és a„hányados" szavak helyett többnyire az„eredmény" szót használják a tanulók. Az említett szavak ismeretének hiányából téves megoldások születhetnek. Ezekkel az egyszerűsítésekkel azért sem értünk egyet, mivel ez az anyanyelv színességének elvesztéséhez vezet, továbbá megengedésük azt vonja maga után, hogy akkor is egyszerűsíteni próbálják a nyelvet, amikor azok komolyabb félreértelmezéshez vezethetnek.

Sajnos naponta tapasztalhatjuk, hogy a beszélt és az írott nyelv nagyon lerövidült, ez a szavak félreértéséhez, ferdítéséhez vezethet. A rövidítések használata különösen a fiatalokra jellemző, akik rövid sms-ekben, rövidített szavakkal kommunikálnak egymással, s félő, hogy ezek a rövidítések begyűrűznek a beszélt nyelvbe is. Ezért úgy gondoljuk, fokozottabban kell figyelnünk arra, hogy a leendő pedagógusok mennyire értő szinten használják anyanyelvüket és a matematika nyelvét.

A fogalomalkotás kezdetben – az iskoláskor elején is – a beszédhez, nyelvhez kötött. Vigotszkij a Gondolkodás és beszéd című munkájában részletesen foglalkozik a hétköznapi és tudományos fogalmak alakulásával, azok fejlődésének folyamatával és kapcsolatával. Kísérletei azt bizonyították, hogy kellő egyéni tapasztalat valamint jól strukturált tananyag esetén „a tudományos fogalmak fejlődése megelőzi a spontán fogalmak fejlődését" (206.o.).

A tudományos fogalmak a tudatosodás magasabb szintjén állnak, mint a hétköznapi fogalmak. Megállapítja tovább azt is, hogy a köznapi fogalmakból nyert következtetések átvitele a tudományos fogalmakra nem egyenletes. Az átvitelt az is nehezíti, hogy a tudományos fogalmak tartalma – habár a szó azonos – csak részlegesen vagy egyáltalán nem fedik a köznapi fogalmakét. Vigotszkij munkájában többször idézi az orosz szépíró Tolsztojt, akinek az volt a meggyőződése, hogy „majdnem sohasem a szó az, ami érthetetlen, hanem a tanulónál … hiányzik az a fogalom, amelyet a szó kifejez. A szó majdnem mindig készen van, amikor kész a fogalom."(210.p.).

A szakszavak kialakulása lassú folyamat. Nem magának a szónak a megtanulása, hanem annak tartalmi vonatkozása. „Nehéz a szóhasználat és annak a helyzetnek a következetes együttes használata, amire az adott szó vonatkozik." (Szendrei, 399.p.)

A matematika nyelve tudvalevő, hogy nagyon tömör, mivel a definíciók csak a fogalom szempontjából legfontosabb szavakat, kifejezéseket tartalmazhatják. Így a matematikai definíciók a rövidségre való törekvés miatt sokkal nehezebben érthetők, mintha fél oldalon keresztül fejtenénk ki azokat. A hosszú szöveget viszont nehezebb pontosan megtanulni, és nagyobb annak a veszélye, hogy valamely lényeges feltételt kihagynak belőle a tanulók, mint a rövid tömör definíciók esetén. Ugyanez a helyzet a matematikai feladatok megfogalmazásánál is, ha röviden fogalmazzuk meg, akkor nehéz értelmezni, mivel itt a kötőszavaknak, a ragoknak, képzőknek, ékezeteknek sokkal nagyobb jelentősége van, mint más tantárgyaknál, ezek helytelen értelmezése vagy hiánya miatt viszont egy probléma megoldhatatlanná vagy több megoldásúvá válhat. Ha viszont egy szöveges feladatot fél oldalban próbálnánk megfogalmazni, a legtöbb diák elfutna annak megoldása elől.

A közlés szempontjából egy definíció akkor célravezető, ha a befogadók a meghatározásban szereplő kifejezések jelentését már ismerik.

„Definíció segítségével senkinek nem közvetíthetünk az általa ismerteknél magasabb rendű fogalmakat, hanem csakis oly módon, hogy megfelelő példák sokaságát nyújtjuk." (Skemp, 38.p.)

Ezt az elvet a matematika tanítás során elég gyakran megsértjük, ilyen hiba még a tankönyvekben is előfordul. A fogalmak megfelelő mennyiségű példák sokasága nélkül bizonytalanokká válnak, s a matematika nem értéséhez, nem tudásához vezetnek. Gyakran még a felsőfokú oktatásban is találkozunk ezzel a problémával.

Ebben a tanulmányban azt kívánjuk vizsgálni, hogy a bajai tanítóképzős hallgatóknak milyen nyelvi nehézségei vannak a matematika fogalmak, definíciók, összefüggések kifejtése során.

1. HIPOTÉZIS

A tanítóképzős hallgatók matematikával kapcsolatos problémáinak egy jelentős része nyelvi gyökerű. Bizonyos szavakat köznyelvi értelemben használnak, amelyek részben ellentmondanak a matematikai tartalmaknak. A szavak mögötti fogalmak nem tisztázottak, s a pontatlanságok nehezítik a matematikatanulást. Célirányos fejlesztéssel a hallgatók nyelvi hiányosságaiból eredő matematikai problémák megoldását javítani lehet.

2. A KUTATÁS HÁTTERE

Évek óta figyelemmel kísérjük a tanítóképzős hallgatók matematikai ismereteinek alakulását, hibáit, szövegértési gondjait. Korábban készített feljegyzéseink alapján egy feladatsort állítottunk össze azokból a problémákból, amelyek megfigyeléseink szerint folyamatosan előfordultak. Az első éves hallgatók körében egy átfogó vizsgálatot terveztünk. Ennek első lépéseként 2005. novemberében egy tesztet töltöttek ki a hallgatók, amelynek eredményéről ebben a tanulmányban kívánunk beszámolni. A tesztlapot a bajai első évfolyamos tanítóképzős hallgatók közül 49-en oldották meg.

A kérdések három fő részből álltak:

a) kulcsszavak közül kellett kiválasztani azokat, amelyek igazzá vagy hamissá teszik az egyes állításokat;

b) négy lehetőség közül kellett kiválasztani az igaz állítást;

c) megkezdett összefüggéseket kellett kiegészíteni igaz állítássá.

A tesztben szereplő 15 kérdés közül hármat választottunk bemutatásra.

A későbbiekben a hallgatókkal való beszélgetésről magnófelvételt készítettünk, ahol azt vizsgáltuk, vizsgáljuk, hogy mennyire tudnak a hallgatók kérdéseket feltenni egy adott problémához és a feltett kérdések és a válaszok értékelése alapján tudják-e korrigálni a helytelen matematikai ismereteiket.

3. *A TESZT FELADATAI ÉS ÉRTÉKELÉSE*

4.1. Jelölje azt a választ, amelyik igazzá teszi az állítást!

Szorzatot egy számmal úgy is oszthatunk, hogy.....tényezőjét osztjuk.

e. mindegyik

f. legalább az egyik

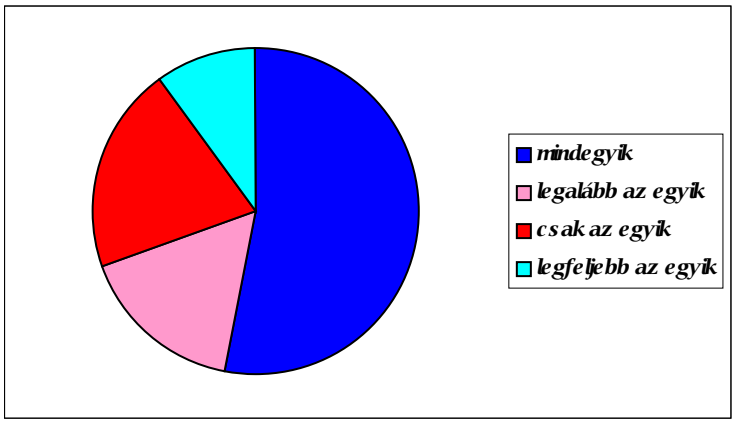
g. csak az egyik

h. legfeljebb az egyik

A feladattal az volt a célunk, hogy megtudjuk,

- mennyire értik a hallgatók a *legalább egy, legfeljebb egy, pontosan egy* szavak tartalmát;
- megoldásaik során alkalmaznak-e hibás analógiát?

A megoldás eredményességét az alábbi grafikon mutatja.



Megállapítottuk, hogy a diákok többsége számára nincs információtartalma a legalább, legfeljebb szavaknak. Egy részük csak felesleges bővítménynek érzi, és csak az utánuk álló szavaknak tulajdonítanak jelentőséget. Nem törekszenek arra sem, hogy megkeressék ezeknek a kifejezéseknek a szinonimáit. A köznyelv sem használja mindig precízen a legalább, legfeljebb szavakat. Például egy riporter – az általa unalmasnak tartott futballmérkőzésről – megállapítja, hogy egyetlen említésre méltó esemény történt: a játékvezető hasra esett. A közönség legalább ezen szórakozott. Ebben a szövegkörnyezetben arra utal a legalább szó, hogy más említésre méltó esemény nem történt, a matematikában járatosak ebben a helyzetben is a legfeljebb szót használták volna, azaz pontosabban fejeznék ki a mondanivalójukat.

Mivel összeget úgy osztok egy számmal, hogy az összeg mindkét tagját elosztom, a fenti megoldásból az tűnt ki, hogy a diákok ezt az összefüggést átvitték arra az esetre is, amikor az összeadás helyett szorzás szerepel. Ez egy tipikus analógiás hiba, ami elég gyakori a matematikai feladatok megoldása során.

4.2. Mit jelent az, hogy az összeadás <u>asszociatív</u> (csoportosítható)?

e. A zárójel felcserélhető, az összeg nem változik.

f. A tagok felcserélhetők és átzárójelezhetők az eredmény nem változik.

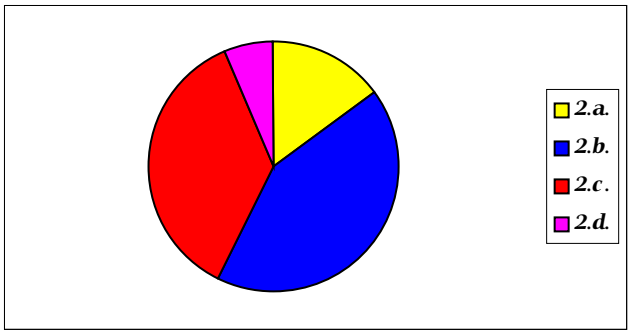
g. A tagok sorrendjének meghagyása mellett az összeg tagjai tetszőlegesen zárójelezhetők.

*h. Mindegy milyen sorrend*ben adom össze a tagokat, az összeg nem változik.

A feladattal az volt a célunk, hogy ellenőrizzük

- a fogalmak pontos megnevezését tudják-e a hallgatók;
- ismerik-e az összefüggéseket.

feladat megoldásának eredményességét az alábbi ábra mutatja:



Ezek az írásbeli válaszok egybecsengenek a kollokviumokon adott szóbeli válaszokkal. Negyedik osztály végére„megtanulják" a gyerekek - az összeadásnál maradva -, hogy fel lehet cserélni a tagokat, vagy hogy az összeg nem változik attól, ha pl. az 1. és 2. tag összegéhez adjuk a 3. tagot vagy az első taghoz adjuk a 2. és 3. tag összegét. Alsó tagozaton ennek különösen a szóbeli számolásnál van szerepe (pl. hogy kerek tízeseket kapjunk részösszegként). Később már nem különítjük el az összeadás e két tulajdonságát ilyen mereven, hanem - ahogyan a feladat kívánja – együttesen alkalmazzuk. Pontosan azért, hogy később ezek a tulajdonságok ne gátolják, hanem segítsék a műveletvégzést. Azaz önmagában a csoportosítás nem foglalja magában a tagok sorrendjének megváltoztatását, hiszen a matematikában vannak olyan műveletek, amelyek asszociatívak, de nem kommutatívak. A hallgatók megoldásai azt mutatják, hogy számukra ez a két tulajdonság összekapcsolódik, nem független egymástól.

A csoportosítás szót a matematikában és a hétköznapi életben használjuk más vonatkozásban is, például a halmaz szinonimájaként.

4.3. Fejezze be úgy az alábbi mondatokat, hogy igaz legyen!

e. A hányados nem változik, ha……..

f. A hányados 2-szeresre nő, ha……..

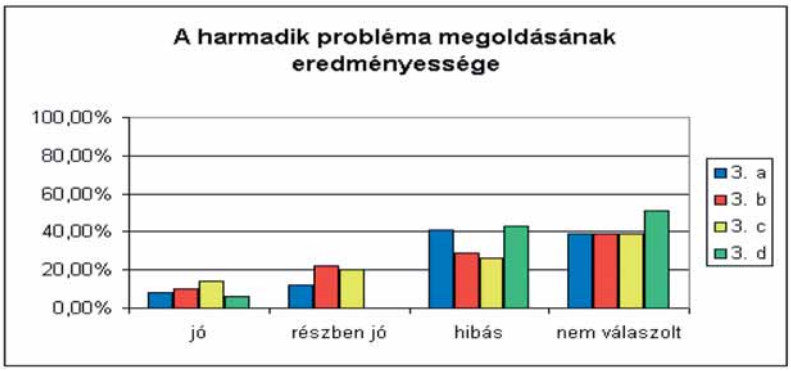
g. A hányados felére csökken, ha……

h. A hányados 2-vel nő, ha……..

A feladattal arra kerestük a választ, hogy

- ismerik-e a diákok ezeket az összefüggéseket;
- milyen pontosan tudják kifejezni magukat a matematika nyelvén?

A feladat megoldásának eredményességét az alábbi ábra mutatja.



Ezt a feladatot sok hallgató nem oldotta meg vagy azért, mert nem volt rá ideje vagy, mert ez nem feleletválasztós volt, s nem tudták a választ vagy, mert egyszerűen nem szeretnek írni.

Jónak csak a pontos, absztrakt összefüggés megadását fogadtuk el. A „részben jó" –nak azokat a megoldásokat tekintettük, ahol a hallgatók a *b, c, d* esetekben csak az egyik lehetséges változatot adták meg , az *a* esetben pedig egyedi konkrét megoldást írtak. Pl. *„osztunk és szorzunk is kettővel."*

Az *a* esetben többen a „-val",„-vel" ragokat használták, amelyek az összeadással és kivonással való növelésre illetve csökkentésre utalnak. Ez tipikus hiba, szintén téves analógiára utal:*„Ha az osztandót és az osztót is ugyanannyival növelem vagy csökkentem, a hányados nem változik".*

A válaszokból egyértelműen az is kiderült, hogy a hallgatók nem foglalkoztak a nullával. Ez a hétköznapi nyelv hatása, mivel a mindennapi számolásokat nem a nullával kezdjük, ezért ott a nullát nem tekintjük számnak, hiszen az a „semmi". Ezt a hibát a hallgatók nagy része elkövette, pl. „mindkettőt ugyanazzal a számmal szorzom.", de ez nem tekinthető teljesen hibásnak, mert hiszen egy eset, a 0-val való szorzás kivételével mindig teljesül.

Szerencsére csak egy hallgató volt, aki tud 0-val osztani, mivel azt írta, hogy nem változik a hányados, ha *„0-val osztok".*

A hallgatók elég nagy része írta azt, hogy ha az egyik számot növeli, a másikat csökkenti, akkor a hányados nem változik. Ez arra utal, hogy a műveleti tulajdonságok nagy-nagy összevisszaságban vannak a tanulók fejében, keverik a négy alapművelettel kapcsolatos összefüggéseket.

A *b* esetben a helytelen válaszok között voltak olyanok is, amelyekből az derült ki, hogy a hallgató nincs tisztában, melyik az osztandó illetve az osztó, a hibák a matematikai szakkifejezések pontatlan használatából eredtek. A másik tipikus hiba az volt, hogy csak részmegoldásokat adtak, pl. *„A hányados kétszeresére nő, ha az osztandót kétszeresére növelem."* Ez csak akkor igaz, ha az osztó változatlan marad.

A hibás válaszok között tipikusak az alábbiak:

2-vel szorzom.
Mindkettőt kétszerezem.
Az egyik tényezőt kétszeresre növelem.
A tagokat négyzetre emelem.

Mivel *b* és *c* kérdések egymás kiegészítései voltak, a válaszokban is ugyanazok a megoldások tükröződtek. Azok a hallgatók, akik helyesen be tudták fejezni a *b* mondatot, azok most is jól dolgoztak.

Legnehezebbnek a *d* rész bizonyult, hiszen erre a diákok nem tanultak formulát. Mindössze 3 helyes válasz született, a hányados 2-vel nő, ha *„az osztandót növelem az osztó kétszeresével."*

Bár elmaradt az a feltétel, hogy közben az osztó nem változik, de ez is nagyon jó eredménynek minősül a többihez képest. A helytelen válaszok között most több volt a 2-szerezésre, felezésre utalás, aminek örülni kell, hiszen ezeknek kellett volna megjelenni az előző válaszokban a 2-vel növelés, illetve csökkentés helyett is. Ez viszont azt mutatja, hogy a korábbi válaszokat az emlékezetükből hívták elő, s valószínűleg azokat úgy tárolták, ilyen módon kerültek be a hosszú távú memóriába. Az új szituációban pontosabban fejezik ki az összefüggést, jobban fogalmaznak, mert gondolkodnak a megoldáson, nem csak emlékezetből próbálnak egy szabályt felidézni.

4. KONKLÚZIÓ

A hipotézisre még nem tudok egyértelmű választ adni, mivel a magnófelvételek kiértékelése folyamatban van.

Az írásbeli munkák alapján az alábbi megállapításokat tehetem:

- Túl sok definíciót, összefüggést tanultak matematika órán a diákok megértés nélkül, így nem érzik a feltételek szükségességét, ezért csak részmegoldásokat képesek produkálni.
- A matematikai hibák egy része nyelvi eredetű, mert a beszélgetések során, a visszakérdezéseknél a többség tudta korrigálni a korábbi megállapításait, felfedezték azokban a hiányosságokat.
- A tanulók kritikai érzékének fejlesztésével pozitív irányba léphetünk a helyes szövegértés és a fogalmazás terén is.

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PROVJERAVANJE I VREDNOVANJE ZNANJA MATEMATIKE

Željka Milin Šipuš[1]

Sažetak. *Aktualne promjene u obrazovanju u Hrvatskoj, osim promjena u nacionalnom kurikulu, donose i novosti u provjeravanju i vrjednovanju znanja. Uvodi se sustav vanjskog vrjednovanja obrazovnih postignuća: sustav nacionalnih ispita (kako za srednješkolsko, tako i za osnovnoškolstvo obrazovanje) i državna matura. U izlaganju ćemo se osvrnuti na polazišta i ciljeve, te iskustva provedenih nacionalnih ispita iz matematike, kao i na ostale vidove provjeravanja znanja matematike.*

Ključne riječi: *provjeravanje znanja, vrjednovanje znanja, edukacija matematike.*

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ORIGAMI I MATEMATIKA

Franka Miriam Brückler[1]

Sažetak. *Origami je tradicionalna japanska umjetnost savijanja papira. Među inim, origami modeli mogu poslužiti vizualizaciji geometrijskih likova i tijela, ali i razvijanju matematičkog načina razmišljanja. Origami je lako uklopiti u nastavu matematike na svim razinama školovanja.*

Već mala djeca susret s geometrijom mogu imati umjesto uobičajenih gotovih, krutih modela tijela putem vlastoručno napravljenih modela, a što su stariji moći će izrađivati kompliciranije modele i na njima proučavati razna svojstva poput simetrije. Čak i savijanje nematematičkih modela može doprinijeti razvoju matematičkog razumijevanja; tako se npr. pri savijanju modela stola postavlja prirodno matematičko pitanje koliki treba biti papir čijim savijanjem bismo dobili sjedalicu koja odgovara stolu.

Drugi aspekt origamija je mogućnost da ga se koristi kao dopunu standardnom upoznavanju sa geometrijskim konstrukcijama ravnalom i šestarom. Aksiomi origamija naime omogućuju konstrukcije poput duplikacije kocke koje nisu izvedive ravnalom i šestarom (jer konstrukcije ravnalom i šestarom geometrijski ekvivalenti rješavanju kvadratnih, a origamijem kubnih jednadžbi). Dodatna korist je što se konstrukcije origamijem ne provode u glavi i crtanjem već savijanjem tj. aktivno, što olakšava razumijevanje i praćenje slijeda koraka, a ima jednaku didaktičku korist podučavanja u matematičkom deduktivnom načinu razmišljanja.

Ključne riječi: *nastava matematike, origami, geometrijske konstrukcije.*

Origami je poznata japanska tehnika savijanja papira (*ori* = savijanje, *kami* = papir). Najpoznatiji su origami objekti razne životinje savijene iz jednog komada papira. Čak i takvi, naizgled nematematički objekti, usko su vezani za geometriju: razvijanjem papira vidjet će se uzorak sastavljen od poligona omeđenih

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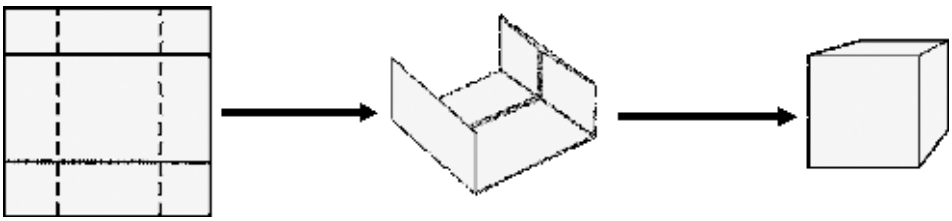
linijama savijanja, a čitavi objekti pokazuju određene vrste simetrija. Ipak, glavne veze između matematike i origamija mogle bi se podijeliti u četiri skupine:

1. Izrada origami modela poliedara i poligona;
2. Aksiomatski pristup analogan konstrukcijama ravnalom i šestarom, ali s nekim dodatnim mogućnostima;
3. Razvoj računskih i analitičkih sposobnosti analizom potrebnih odnosa dimenzija za dobivanje željenih modela;
4. Veze s višom matematikom, osobito topologijom i teorijom grafova.

Izrada origami modela poliedara i poligona

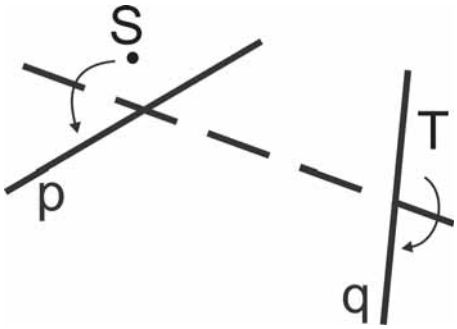
Modeli se kreću od vrlo jednostavnih, pogodnih već za manju djecu (čime se razvija njohov osjećaj za prostor i motorika) do onih vrlo kompliciranih za koje treba puno strpljenja i poprilična spretnost. Origami modeli poligona i poliedara dijele se na one koji se mogu izraditi od jednog komada papira (takvi matematički modeli su rjeđi i većinom se radi o modelima poligona) i modularni origami u kojem se konačni model sastavlja od više dijelova (bez lijepljenja).

Primjer: Kocka se može izraditi npr. od šest kvadratnih komada papira koje savijemo horizontalno i vertikalno prema sredini (slika dolje lijevo; pune linije označavaju linije savijanja prema gore, a iscrtane prema dolje). Tih šest komada („modula") se onda saviju tako da dobijemo oblik kao na slici dolje u sredini. Ti se komadi onda zataknu jedan u drugog tako da izvana ostanu glatki kvadrati, čime se dobije kocka.



Geometrijske konstrukcije origamijem

Kao što je poznato, tradicionalno obrazovanje uključuje upoznavanje s geometrijskim konstrukcijama ravnalom i šestarom, čime se među inim razvija deduktivni način razmišljanja. Početkom 1990ih uvedeni su i aksiomi origami konstrukcija (šest Huzitinih aksioma), koji imaju analognu matematičku (i di-



daktičku) korist kao i klasični aksiomi za konstrukcije ravnalom i šestarom, ali imaju i neke dodatne prednosti. U biti, radi se o tome da klasične konstrukcije ravnalom i šestarom odgovaraju geometrijskom rješavanju kvadratnih jednadžbi, a konstrukcije origami omogućuju i rješavanje kubnih jednadžbi. Specijalno, pomoću origamija (primjenom Huzitinih aksioma) mogu se riješiti problemi duplikacije kocke (tj. konstrukcija trećeg korijena iz 2 ako je zadana jedinična duljina) i trisekcije proizvoljnog zadanog kuta.

Primjer: Šesti Huzitin aksiom postulira *„za dane dvije točke i dva pravca moguće je naći liniju savijanja kojom jedna točka pada na jedan, a druga na drugi pravac“* (slika desno). Analizom značenja tog aksioma vidi se da se radi o konstrukciji zajedničke tangente na dvije parabole kojima su zadani fokusi i ravnalice, što odgovara rješavanju kubne jednadžbe.

Ostalo

Origami može poslužiti i kao izvor elementarnomatematičkih odnosno geometrijskih zadataka.

Primjer: Ako je dijete savilo origami stol iz komada papira, koliko velik treba biti papir čijim savijanjem bi se dobila sjedalica prikladna za taj stol?

Primjer: Ako se kvadratni papir savije tako da se sva četiri vrha nađu u sredini polaznog kvadrata, dokažite da je dobiven kvadrat i odredite omjer njegove površine s polaznim! Ukoliko se sad vrhovi iz sredine presavinu natrag prema van tako da padnu na polovišta strenica kvadrata, provjerite da je u sredini nastao novi kvadrat i usporedite njegovu površinu s polaznim kvadratom!

S druge strane, origami je povezan i s višom matematikom, osobito teorijom grafova. Najpoznatija veza sastoji se u idućem: Ako promotrimo mrežu pregiba koji nastaju kad iz papira izradimo neki origami oblik koji je plosnat, pa papir ponovno izravnamo, onda će se tako dobivena „zemljopisna karta“ moći obojati sa samo dvije boje tako da nikoja dva područja koja graniče po nekoj liniji savijanja nemaju istu boju. Ovaj teorem je posljedica činjenice da kod takvog

modela svaki vrh ima paran stupanj (vrh je sjecište dvije ili više linija pregiba) pa je graf kojemu su vrhovi sjecišta linija savijanja, a bridovi dijelovi tih linija između po dva vrha, Eulerov graf.

Napomenimo na kraju i da korištenje origamija u nastavi rezultira nizom didaktičkih i metodičkih koristi: razvijaju se sposobnosti rješavanja problema, preciznog korištenja matematičke terminologije, upotreba razlomaka i omjera, upoznavanje pojmova vezanih za kuteve, površinu, volumen, kongruenciju, paralelnost i okomitost, konike i dr., razvoj deduktivno-logičkog načina razmišljanja, a omogućuje i razvoj sposobnosti za suradnju, predviđanje ishoda, motoričkih sposobnosti, razumijevanja estetike i sposobnosti vizualizacije ... Posebno zgodno je i to što postoji niz otvorenih problema u matematici origamija čije razumijevanje je velikim dijelom dostupno djeci različitih uzrasta i pritom imaju i mogućnost doprinosa vlastitih ideja, čime se razvija razumijevanje matematike kao kreativne znanosti.

Zgodna web-stranica za početak upoznavanja s vezama između matematike i origamija je Origami & Math, *http://www.paperfolding.com/math/*

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STAVOVI STUDENATA UČITELJSKIH STUDIJA O MATEMATICI

Irena Mišurac Zorica[1]

Sažetak. Suočeni s nezadovoljavajućim rezultatima učenika u matematici na svim razinama obrazovanja, trebamo stalno preispitivati parametre koji mogu utjecati na proces njenog učenja. Učitelj razredne nastave jedna je od važnijih komponenti u tom procesu i ima velik utjecaj na učenike. U razrednoj nastavi obrađuju se najelementarniji matematički sadržaji koji postaju temelj za nadogradnju u predmetnoj nastavi, pa je time značaj ovog perioda u matematičkom obrazovanju itekako velik. Svoj utjecaj učitelj ostvaruje kroz način na koji podučava i komunicira s učenicima, ali još više kroz neverbalnu komunikaciju u kojoj kroz četiri godine učenicima svjesno ili nesvjesno prenosi vlastite stavove, asocijacije i strahove. Iz tog smo se razloga u ovom radu odlučili istražiti kakve stavove o matematici imaju studenti - budući učitelji. Kako će upravo oni u budućnosti matematiku predavati najmlađoj učeničkoj populaciji, jasno je da će njihovi unutarnji stavovi implicitno ili eksplicitno utjecati na učeničke rezultate.

Polazna pretpostavka u istraživanju bila je da će učitelji uspješnije podučavati matematiku ukoliko o njoj imaju pozitivne stavove. Drugim riječima, osoba koja o matematici ima negativne stavove ili osjeća strah prema njoj, neće biti uspješna u njenom podučavanju. Stavove studenata ispitali smo anonimnom anketom na uzorku od 150 studenata 3. i 4. godine učiteljskog studija Filozofskog fakulteta u Splitu. Rezultati dobiveni anketom pokazali su da studenti smatraju da imaju dobar, ali ne izrazito dobar stav prema matematici. Ipak veliki broj studenata iskazuje strah od matematike, a asocijacije na riječ matematika su im neutralne ili negativne. Ispitanici također misle da će im samo veoma rijetki sadržaji naučeni na fakultetu trebati u budućem zvanju. Iznenadilo nas je da najveći broj studenata nije volio matematiku ni u predfakultetskom školovanju. Od svih nastavnih predmeta koje će u budućnosti predavati u razrednoj nastavi, najmanje studenata opredijelilo se za matematiku. Ovakvi rezultati nikako nas ne mogu

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zadovoljiti i stavljaju pred sve nas zadatak da ovakve stavove budućih učitelja što prije mijenjamo.

Ključne riječi: *nastava matematike, stavovi budućih učitelja o matematici.*

1. UVOD

Matematika je bila i ostala jedan od najvažnijih predmeta na svim razinama obrazovanja. To je nastavni predmet koji prije svega razvija logičko mišljenje učenika i podloga je za svako znanstveno, tehničko, informatičko, ali i mnoga druga područja ljudskog djelovanja. Nažalost, svjedoci smo sve slabijih **učeničkih rezultata u** matematici koji se prije svega očituju u velikom broju loših ocjena, u porastu privatnih instrukcija iz matematike, ali i u odabiru škola i fakulteta prema kriteriju „tamo nema mnogo matematike".

Zbog svih navedenih razloga, izuzetno je važan način na koji se podučava matematika, kao i kvalitetan kadar koji će taj složeni posao uspješno odraditi. Obrazovanje učitelja, nastavnika i profesora matematike prvi je i osnovni temelj na kojem se gradi uspješna nastava matematike. Ta činjenica bila je motiv i polazna pretpostavka u ovom radu.

Osnovnoškolsko obrazovanje od samog početka pretpostavlja i podučavanje nastavnog predmeta matematka. U razrednoj nastavi učenici bi trebali upoznati prirodne brojeve i nulu, usvojiti četiri osnovne računske radnje i usvojiti osnovne geometrijske pojmove. U tom periodu stvaraju se temelji odnosa prema matematici i počinje se razvijati formalno matematičko razmišljanje. Upravo stoga, ovaj je period od neprocjenjive važnosti za učenike i u spoznajnom i u emotivnom smislu.

Da bi učitelj bio uspješan u složenom poslu koji obavlja mora prije svega voljeti raditi s učenicima, mora razumjeti njihov način spoznavanja i učenja, ali sigurno mora voljeti i sadržaje koje djeci približava. „Danas je postalo jasno da je učenje cjelovit proces i ne svodi se samo na pamćenje i mišljenje. To je i emocionalni doživljaj, i psihomotorna aktivnost, i socijalni odnos, i proces samoaktualizacije" (Bognar, 1998., 349). Kako učitelj razredne nastave matematiku predaje četiri sata tjedno u okviru redovne nastave, te još po sat dopunske i dodatne nastave, jasno je da on sam mora razumjeti i voljeti matematiku da bi bio uspješan u njenom podučavanju. Ukoliko učitelj osjeća strah ili tjeskobu prema

matematici, sigurno je da „inicira razvoj tjeskobe svojim učenicima, umjesto da u njima stvara pozitivne stavove prema matematici" (Sharma, 2001., 120).

Učitelj, kao ključna figura na početku institucionaliziranog odgoja i obrazovanja, ima snažan utjecaj na učenike. Taj utjecaj ostvaruje kroz način na koji podučava i komunicira s učenicima, ali još više kroz neverbalnu komunikaciju u kojoj kroz četiri godine razredne nastave učenicima svjesno ili nesvjesno prenosi vlastita uvjerenja, asocijacije i strahove. U tom važnom razvojnom periodu učitelj će osim matematičkih znanja i umijeća propisanih nastavnim programom, učenicima eksplicitno i implicitno prenjeti i vlastite stavove o matematici, budući da su „novija istraživanja jasno pokazala kako se stavovi i shvaćanja o ciljevima poučavanja i prirodi učenja odražavaju u ponašanju učiteljica tijekom podučavanja" (Vizek Vidović i dr., 2003., 329). Zbog toga je jasno da će njihovi unutarnji stavovi implicitno ili eksplicitno utjecati na učeničke rezultate.

Polazna pretpostavka u ovom istraživanju bila je da će učitelji kvalitetnije i uspješnije podučavati matematiku ukoliko sami o njoj imaju pozitivne stavove. Drugim riječima, osoba koja o matematici ima negativne stavove ili osjeća strah prema njoj, neće biti uspješna u njenom podučavanju. „Istraživanja u ovom području polaze od pretpostavke kako djelotvornost podučavanja ovisi o stavovima učitelja i učiteljica prema podučavanju i određenim znanjima i vještinama" (Vizek Vidović i dr., 2003., 329).

Odlučili smo istražiti kakve stavove o matematici imaju studenti koji su odabrali učiteljski posao za svoj budući poziv. U istraživanju smo krenuli od nekoliko polaznih pretpostavki. Prva je bila da su stavovi studenata-budućih učitelja o matematici većinom pozitivni, te da učiteljski studij biraju osobe koje su voljele matematiku i koje se u vlastitom školovanju nje nisu bojali. Obzirom da na stav o matematici sigurno najviše utječe najsvježije iskustvo osobe, a to je u ovom slučaju iskustvo s fakulteta, pretpostavili smo i da će studenti koji su na fakultetu imali problema s nekim ispitom iz matematičkih kolegija imati negativnije stavove prema njoj. Treća je pretpostavka bila da će se studenti nakon više od 12 godina svog učenja matematike osjećati kompetentnima za podučavati matematiku od 1. do 4. razreda osnovne škole. Kako su studenti najviše sadržaja matematike naučili na višim stupnjevima obrazovanja, posebno na fakultetu, pretpostavili smo da će sadržaje naučene na fakultetu studenti smatrati korisnim za svoj budući poziv.

2. METODOLOGIJA ISTRAŽIVANJA

Cilj istraživanja bio je saznati kakve stavove o matematici imaju studenti učiteljskog studija, a time indirektno možemo zaključivati i o njihovoj budućoj uspješnosti u podučavanju matematike.

Istraživanje stavova budućih učitelja proveli smo anonimnim anketiranjem 150 studenata učiteljskog studija Filozofskog fakulteta u Splitu, u prosincu 2006. godine. Za uzorak su izabrani studenti 3. i 4. godine studija, dakle studenti koji su već odslušali matematičke kolegije (Matematika 1, Matematika 2, Matematika 3), a i dio programa kolegija „Metodike početne nastave matematike". Smatrali smo da će ti studenti imati potpuno izgrađen stav prema matematici, kao i viziju svog budućeg učiteljskog posla koji pretpostavlja i podučavanje nastave matematike.

Iako se radi o (relativno) velikom uzorku, treba prihvatiti ograničenje da se radi o studentima jednog fakulteta kojima su matematičke kolegije predavali isti nastavnici. To ograničenje u našem istraživanju djelomično umanjuje vrijednost dobivenih rezultata, budući da faktor nastavnika koji je utjecao na studentske stavove nije konzistentna kategorija koja bi općenito i jednako utjecala na cijelu populaciju. Ipak, rezultati koji su dobiveni veoma su indikativni i sigurno nam mogu biti dobar pokazatelj trenutnog stanja u obrazovanju učitelja. Anketni listić kojim je anketa provedena nalazi se u Prilogu ovog rada.

3. ANALIZA REZULTATA

Anketom je ispitano 150 studenata, od čega 91 student treće i 59 studenata četvrte godine studija učitelja Filozofskoga fakulteta u Splitu.

Na samom se početku pokušalo saznati kakve asocijacije studenti imaju na riječ matematika, obzirom da asocijacije mogu pokazati unutrašnje stavove osobe prema zadanom pojmu. Pokazalo se da najveći broj studenata ima neutralne asocijacije, ali zabrinjava činjenica da 37% studenata ima negativne asocijacije na riječ matematika, a svega 21% pozitivne. Usporedbom odabrane asocijacije i godine studiranja ispitanika, uočeno je da studenti treće godine imaju većinom negativne asocijacije, dok studenti četvrte godine imaju neutralne, odnosno pozitivne asocijacije. Takav odnos rezultata smo i očekivali, a uvjetovan je vjerojatno činjenicom da su studenti četvrte godine već položili ispite iz kolegija

„Matematika", te da su odslušali veći dio kolegija „Metodika nastave matematike" u kojem je pristup matematici bitno drugačiji.

Na pitanje boje li se matematike najveći broj ispitanika je odgovorio da se „uglavnom ne boji" (43%), ali veći je broj onih koji se boje matematike (26%), nego onih koji je se uopće ne boje (23%). Postoji 9% studenata koji je se užasavaju, što zasigurno dovodi u pitanje i uspješnost njihovog budućeg poučavanja. Usporednom analizom utvrđeno je da se znatno više boje studenti treće nego četvrte godine studija.

Da bi provjerili polaznu hipotezu po kojoj su studenti učiteljskog studija voljeli matematiku, budući da su za svoj poziv odabrali podučavanje djece i tom predmetu, postavili smo im pitanje jesu li u školi voljeli matematiku. Najveći broj studenata, čak 59% nije previše volio matematiku, dok je broj onih koji su je voljeli (25%) veći od broja onih koji je uopće nisu voljeli (16%). Iz ovih rezultata vidljivo je da je svega četvrtina studenata voljelo matematiku, a 75% njih ne.

Sadržaje naučene na fakultetu studenti uglavnom ne smatraju pretjerano korisnima, pa najveći broj njih smatra da će im samo veoma rijetki biti korisni u njihovu učiteljskom poslu. Zanimljivo je da ispitanici smatraju da su matematiku najbolje naučili u višim razredima osnovne škole i u srednjoj školi, dok je najmanji broj onih koji smatraju da su matematiku najbolje naučili na fakultetu (svega 10%).

U anketnom listiću bile su dane četiri tvrdnje o matematici, a od studenata se tražilo da se opredijele za onu koja je najbliža njihovim stavovima. Najveći broj ispitanika, 53%, odredio se za tvrdnju da matematiku treba razumjeti, što ukazuje na ispravno poimanje uloge matematike u odgojno obrazovnom procesu. Velik je i broj studenata koji smatraju da matematiku treba puno vježbati, 33%, što ukazuje na poimanje matematike kao zahtjevnog predmeta. Svega jedan ispitanik odredio se za tvrdnju da za matematiku treba biti jako pametan, što ukazuje na dvije stvari. S jedne strane to je dobro, jer pokazuje da studenti matematiku ne doživljavaju kao predmet koji selekcionira učenike na „pametne i glupe", a što je česta negativna nuspojava u nastavi matematike. S druge strane takav rezultat može biti posljedica činjenice da studenti za loše rezultate i asocijacije na matematiku ne „krive" sebe, već neke vanjske faktore. Također smo uočili da studenti s pozitivnim asocijacijama o matematici odabiru tvrdnje da

matematiku treba razumjeti i vježbati, dok se studenti s negativnim asocijacijama opredjeljuju i za tvrdnju da iz matematike treba „izvući“ pozitivnu ocjenu.

Kada su trebali izraziti svoj stav prema matematici, većina ispitanika, njih 59%, smatra da imaju dobar stav prema matematici. Ipak velik je postotak onih koji imaju loš stav (26%), odnosno izrazito loš stav (6%), dok svega njih 9% ima izrazito dobar stav. Takav stav zasigurno utječe i na njihova razmišljanja o matematici, što se vidi iz Tablice 1.

Tablica 1.

		stav o matematici			
		izrazito dobar	dobar	loš	izrazito loš
tvrdnje o matematici	matematiku treba puno vježbati	3	30	16	0
	matematiku treba razumjeti	9	52	17	2
	za matematiku treba biti jako pametan	0	0	0	1
	iz matematike treba “izvuci” pozitivnu ocjenu	0	7	5	6

Uspoređujući rezultate ankete, uvidjeli smo da se studenti s lošim i izrazito lošim stavom o matematici mnogo više boje matematike, od studenata s dobrim stavom o matematici, što je vidljivo iz Tablice 2. Ovakav je rezultat očekivan i logičan.

Tablica 2.

		stav o matematici			
		izrazito dobar	dobar	loš	izrazito loš
bojiš li se matematike	uopće se ne bojim	9	24	1	0
	uglavnom se ne bojim	3	51	9	1
	bojim se	1	14	23	1
	užasavam je se	0	0	6	7

Prema rezultatima iz tablice 2. izračunata je empirijska vrijednost hi-kvadrat testa 116,235, pa se uz empirijsku signifikantnost manju od 1% može prihvatiti pretpostavka da postoji statistički značajna povezanost između straha od matematike i stava ispitanika.

Uspoređujući asocijacije na riječ matematika i stav ispitanika o njoj, dobivena je Tablica 3.

Tablica 3.

		stav o matematici			
		izrazito dobar	dobar	loš	izrazito loš
asocijacije na riječ matematika	pozitivne	11	20	0	0
	neutralne	2	53	9	0
	negativne	0	16	30	9

Anketom se pokušalo utvrditi tko je utjecao na stav ispitanika prema matematici, pa smo saznali da je u najvećem broju na njihov stav utjecao predmetni nastavnik u osnovnoj školi, zatim profesor u srednjoj školi, profesor na fakultetu, odnosno učitelj razredne nastave. Uočljivo je i da je studente treće godine najveći utjecaj na stav imao profesor matematike na fakultetu, dok je taj utjecaj znatno manji za studente četvrte godine. Usporedbom stavova o strahu od matematike i utjecaju na stav, dobili smo Tablicu 4.

Tablica 4.

		stav o matematici			
		učitelj od 1. do 4. razreda	nastavnik u osnovnoj školi	profesor u srednjoj školi	profesor na fakultetu
bojiš li se matematike	uopće se ne bojim	6	14	11	2
	uglavnom se ne bojim	7	18	20	18
	bojim se	1	11	8	17
	užasavam je se	1	2	5	5

Prema rezultatima iz tablice 4. izračunata je empirijska vrijednost hi-kvadrat testa 18,256, pa se uz empirijsku signifikantnost od 3,2% može prihvatiti pretpostavka da postoji statistički značajna povezanost između straha od matematike i osobe koja je utjecaju na stav ispitanika. Iz ove je tablice razvidno da studenti na čiji je stav najviše utjecao učitelj razredne nastave imaju najmanji strah od matematike. Studenti na čiji je stav o matematici najviše utjecao profesor na fakultetu imaju znatno veći osjećaj straha prema matematici. Usporedbom prethodnih rezultata uvidjeli smo da studenti na koje je najveći utjecaj na stav imao profesor matematike na fakultetu nisu ni u ranijem školovanju pretjerano voljeli matematiku, što se vidi iz Tablice 5.

Tablica 5.

		stav o matematici			
		učitelj od 1. do 4. razreda	nastavnik u osnovnoj školi	profesor u srednjoj školi	profesor na fakultetu
u školi sam volio matematiku	da, jako	3	12	17	5
	ne previše	9	27	20	29
	uopće ne	3	6	7	8

Sadržaje matematike iz razredne nastave koje su upoznali ispitanici uglavnom razumiju, ali postoji njih 17% koji razumiju samo manji dio naučenih sadržaja. Ta nas činjenica zabrinjava, jer se pitamo kako će ti budući učitelji sutra podučavati djecu sadržajima koje ni sami dobro ne razumiju. Uočljivo je i da ispitanici s pozitivnim stavom o matematici iskazuju znatno veće razumijevanje matematičkih sadržaja od onih s negativnim stavom.

Usprkos ovakvim rezultatima, 43% studenata smatra se kompetentnima predavati učenicima matematiku, dok najveći broj ispitanika (51%) smatra se samo djelomično kompetentnima za podučavanje učenika matematici. U izražavanju stava o kompetentnosti, uočili smo da se kompetentnijima smatraju studenti četvrte godine studija, što je i očekivano.

Zabrinjava nas činjenica da ispitanici ne pokazuju kritičnost prema vlastitoj kompetenciji u odnosu na osobni stav o matematici, što se vidi iz Tablice 6.

Tablica 6.

		kompetentnost		
		da	djelomično	ne
stav o matematici	izrazito dobar	12	1	0
	dobar	41	43	5
	loš	10	26	3
	izrazito loš	1	6	2
asocijacije na matematiku	pozitivne	23	8	0
	neutralne	24	36	4
	negativne	17	32	6

Na kraju se studente - buduće učitelje pitalo koji će predmet najradije podučavati kada budu radili u školi, a dobivena je sljedeća Tablica 7.

Tablica 7.

	broj studenata	postotak
hrvatski jezik	42	28,0
matematiku	10	6,7
prirodu i društvo	55	36,7
likovni odgoj	11	7,3
tjelesni odgoj	16	10,7
glazbeni odgoj	13	8,7

Iz tablice vidimo da je najmanji postotak studenata koji će najradije predavati matematiku, što je rezultat s kojim nikako ne možemo biti zadovoljni.

4. ZAKLJUČAK

Rezultati koji su dobiveni u ovom istraživanju potvrdili su naše bojazni, a time i povećali zabrinutost za buduću uspješnost rezultata početne nastave matematike. Činjenica da više od trećine budućih učitelja ima negativne asocijacije na riječ matematika dovodi nas do zaključka da će svaki treći učitelj razredne nastave u budućnosti djeci predavati predmet o kojem sam ima negativnu sliku.

Indikativna je i činjenica da se veliki broj budućih učitelja (svaki četvrti) boji matematike, a čak 9% njih je se užasava. Obzirom da strah može značajno smanjiti učinkovitost u matematici, jasno je da će utjecati i na uspješnost podučava-

nja tih učitelja. Zabrinjavajuća je i činjenica da tri četvrtine studenata nisu voljeli matematiku ni u školi, pa zaključujemo da učiteljski posao nisu odabrali da bi podučavali matematiku. To potvrđuju i rezultati posljednjeg anketnog pitanja u kojemu se buduće učitelje pitalo koji će predmet najradije podučavati, a najmanji je postotak studenata koji su izabrali matematiku. Iz svega navedenog možemo pretpostaviti da je za buduće učitelje podučavanje matematike samo segment učiteljskog posla kojega prihvaćaju kao nužno zlo, a ne kao vlastiti izbor.

Anketom se pokušalo saznati što sami ispitanici misle o vlastitom stavu prema matematici, a pokazalo se da više od polovine smatra da imaju dobar stav prema matematici, dok trećina ima loš stav. Uspoređujući rezultate ankete, uvidjeli smo da se studenti s lošim stavom o matematici mnogo više boje matematike, od studenata s dobrim stavom. Anketom se pokušalo utvrditi tko je utjecao na stav ispitanika prema matematici, pa smo saznali da je u najvećem broju na njihov stav utjecao predmetni nastavnik u osnovnoj školi, a najmanji učitelj razredne nastave. Rezultati su pokazali da studenti na čiji je stav najviše utjecao učitelj razredne nastave imaju najmanji strah od matematike, dok studenti na čiji je stav najviše utjecao profesor na fakultetu imaju znatno veći osjećaj straha prema matematici. Taj je rezultat u skladu s našom polaznom pretpostavkom da je strah studenata povezan s iskustvima iz ispitne situacije na matematičkim kolegijima. Ispitanici smatraju da sadržaje matematike iz razredne nastave koje su upoznali na fakultetu uglavnom razumiju, ali postoji njih 17% koji razumiju samo manji dio naučenih sadržaja. Iako se taj postotak ne čini velik, postavlja se opravdana bojazan kako će takvi učitelji podučavati djecu sadržajima koje ni sami dobro ne razumiju. Uočljivo je i da ispitanici s pozitivnim stavom o matematici iskazuju znatno veće razumijevanje matematičkih sadržaja od onih s negativnim stavom.

Zbog svega navedenog očekivalo se da će studenti biti veoma kritični prema vlastitoj kompetenciji u podučavanju matematike i da će se pojačano truditi da pojačaju svoja matematička znanja, kako bi na taj način osigurali kvalitetu u vlastitoj nastavi. Rezultati su pokazali da sadržaje naučene na fakultetu studenti uglavnom ne smatraju pretjerano korisnima, a najveći broj ispitanika smatra da su matematiku najbolje naučili u višim razredima osnovne škole. Usprkos tome, gotovo polovina studenata smatra se kompetentnima predavati učenicima matematiku, dok se najveći broj ispitanika smatra djelomično kompetentnima. Zaključujemo da budući učitelji ne pokazuju kritičnost prema vlastitim matematičkim znanjima i stavovima, te da usprkos činjenici da ne razumiju sve

sadržaje, da se boje matematike i da je ni sami u školi nisu voljeli, smatraju da će je kvalitetno i uspješno podučavati.

Možemo zaključiti da je na učiteljskim fakultetima potrebno uložiti dodatni napor kako bi se buduće učitelje senzibiliziralo da osvijeste i mijenjaju unutrašnje stavove prema matematici, a što bi trebalo rezultirati boljim rezultatima nastave matematike.

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PRILOG

ANKETA

Godina studiranja _____

Molim Vas da pažljivo pročitate postavljena pitanja ili tvrdnje i zaokružite samo jedan od ponuđenih odgovora, i to onaj koji najbolje odražava Vaš stav.

1. Kakve asocijacije u tebi budi riječ matematika?
   a) pozitivne
   b) neutralne
   c) negativne

2. U školi sam volio/la matematiku.
   a) da, jako
   b) ne previše
   c) uopće ne

3. Bojiš li se matematike?
   a) Uopće se ne bojim.
   b) Uglavnom se ne bojim.
   c) Bojim se.
   d) Užasavam je se.

4. Sadržaji matematike koje sam učio/la na fakultetu bit će mi korisni za moj budući posao.
   a) da
   b) veoma rijetki
   c) ne

5. Koja ti je od sljedećih tvrdnji najbliža?
   a) Matematiku treba puno vježbati.
   b) Matematiku treba razumjeti.
   c) Za matematiku treba biti jako pametan.
   d) Iz matematike treba "izvući" pozitivnu ocjenu.

6. Tvoj stav o matematici je:
   a) izrazito dobar
   b) dobar
   c) loš
   d) izrazito loš

7. Matematičke sadržaje iz razredne nastave koje sam do sada susreo/la potpuno razumijem?
   a) Da, sve.
   b) Uglavnom sve.
   c) Samo manji dio.

8. Na tvoj stav o matematici najviše je utjecao:
   a) učitelj od 1. do 4. razreda
   b) nastavnik u osnovnoj školi
   c) profesor u srednjoj školi
   d) profesor na fakultetu

9. Osjećaš li se kompetentnim za podučavanje učenika matematici?
   a) da
   b) djelomično
   c) ne

10. Matematiku sam najviše i najbolje naučio/la:
    a) u razrednoj nastavi
    b) u višim razredima osnovne škole
    c) u srednjoj školi
    d) na fakultetu

11. Mislim da ću najradije podučavati:
    a) hrvatski jezik
    b) matematiku
    c) prirodu i društvo
    d) likovni odgoj
    e) tjelesni odgoj
    f) glazbeni odgoj

PARTNERSTVO FAKULTETA, ŠKOLA I OBITELJI ZANAPREDAK MATEMATIČKE EDUKACIJE DAROVITE DJECE (Poster)

Ksenija Moguš[1], Silvija Mihaljević[2]

Sažetak. *U okviru projekta Metodika matematike odobrenog od Ministarstva znanosti prosvjete i športa (voditelj: M. Pavleković) usustavljena je 2003. godine Mala matematička škola s ciljem postizanja samopouzdanja i kompetencija studenata učiteljskih studija za matematičku izobrazbu darovite djece. Partnerstvo fakulteta, škole i obitelji za napredak matematičke edukacije darovite djece u okviru Male matematičke škole najavljeno je na Kongresu nastavnika matematike u Zagrebu, u srpnju 2004. godine (Goljevački, Moguš, 2004).*

Ključne riječi: *matematička edukacija, matematički darovita djeca, popularizacija matematike, partnerstvo fakulteta, škole i obitelji,*

Polaznici *Male matematičke škole* su učenici četvrtih razreda osječkih osnovnih škola. Oni tijekom akademske godine dva sata tjedno rade sa studentima metodom *vođenoga učenja otkrivanjem* (Guided discoveri learning). Nastava je planirana i nadgledana od sveučilišnih nastavnika.

U svjetlu *bolonjskoga procesa* izrađen je i odobren 2004. godine program za predmet *Matematika i nadareni učenici* namijenjen studentima učiteljskih studija (autor programa: M. Pavleković).

U rad su tijekom protekle četiri godine, pored studenata učiteljskih studija, uključeni sveučilišni nastavnici iz područja matematike, informatike, informa-

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cijskih znanosti i psiohologije. Partnerskim odnosom fakulteta, škole i obitelji kroz model *Mala matematička škola* temeljito se pristupilo edukaciji studenata učiteljskih studija u području rada s darovitim učenicima za matematiku.

Posebnost ovoga modela ogleda se u najtješnijoj i kontinuiranoj suradnji sveučilišnih nastavnika i studenata s učiteljima i stručnim službama (pedagozi i psiholozi) osnovnih škola iz kojih dolaze polaznici. Studenti i metodičari matematike komuniciraju također usmeno i pismeno s roditeljima darovite djece.

Završetak akademske godine znači i završetak *Male matematičke škole* u toj godini. Tradicionalno je popraćen završnim matematičkim kvizom kojega osmišljavaju studenti i tim sveučilišnih nastavnika. U kvizu učestvuju daroviti matematičari, njihovi učitelji i roditelji (braća, ponekad bake i djedovi).

Ove godine osmišljava se četvrti takav kviz na kraju kojega se polaznicima dodjeljuju uvjerenja o redovitom pohađanju *Male matematičke škole*, a dijele se i simbolične nagrade.

Korist od partnerskoga odnosa fakulteta, škole i obitelji za napredak matematičke edukacije darovite djece višestruka je a vidljiva je iz:

1. poticanja redovitoga provođenja stručne detekcije darovitih za matematiku u školama (psiholozi)
2. poticanja darovitih učenika na realizaciju darovitosti izvan škole (učitelji, roditelji)
3. stvaranja primjerenoga okruženja za postizanje samopouzdanja i kompetencija studenata učiteljskih studija u području darovitosti
4. uspostavljanje nužnih preduvjeta za konstruiranje ekspertnih sustava za detekciju darovitosti, osobnog nadzora nad napredovanjem u učenju itd.
5. popularizacije matematike
6. prilike da roditelji ulažu u dobrobit svoje djece koja će se sutra reflektiratu u napretku društva u cjelini.

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EKSPERTNI SUSTAV ZA ODREĐIVANJE DJETETOVE DAROVITOSTI ZA MATEMATIKU

Margita Pavleković,[1] Marijana Zekić-Sušac[2], Ivana Đurđević[3]

Sažetak. *Prema Johnsonu (2000.) i drugim autorima, s djecom nadarenom za matematiku potrebno je raditi na drugačiji način od ostale djece. U cilju posvećivanja posebne pozornosti nadarenoj djeci, učitelji obično uvažavaju samo matematičku kompetenciju kao jedini kriterij za određivanje djetetove darovitosti. Međutim, također je važno uključiti ostale komponente kod odluke o darovitosti za matematiku, kao što su: kognitivne komponente darovitosti, osobni činitelji koji pridonose razvoju darovitosti, strategije učenja i vježbanja, kao i neke okolinske činitelje. Cilj rada je na temelju prethodnih istraživanja identificirati pet osnovnih komponenti darovitosti za matematiku, te kreirati inteligentni ekspertni sustav koji će biti potpora učiteljima u određivanju darovitosti učenika u četvrtom razredu osnovne škole. Sustav se temelji na pravilima odlučivanja i mehanizmu zaključivanja ulančavanjem unaprijed, koji se rabi za klasifikaciju svakoga učenika u jednu od četiriju kategorija: (1) dijete za koje se pretpostavlja da je darovito za matematiku, (2) dijete s posebnim interesom za matematiku, (3) dijete prosječnih matematičkih kompetencija i (4) dijete s nedovoljnom razvijenim matematičkim kompetencijama.*

Provedeno je empirijsko istraživanje koje je uključilo 247 učenika iz različitih osnovnih škola u Osijeku (Hrvatska). Za svakoga učenika dobivena je procjena učitelja i ekspertnog sustava. U radu se također uspoređuju procjene sustava i procjene učitelja. Rezultati pokazuju da među tim procjenama postoji statistički značajna razlika, a također je više učenika određeno kao darovito prema odluci

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sustava, što upućuje na zaključak da ekspertni sustav navodi učitelja na promišljanja o ostalim komponentama darovitosti učenika za matematiku, te se stoga njime može koristiti kao učinkovitim metodološkim alatom, kako u određivanju dječje darovitosti za matematiku, tako i u izobrazbi učitelja.

Ključne riječi: *komponente darovitosti, matematika, inteligentni ekspertni sustav, if-then pravila, procjena darovitosti, t-test*

1. Uvod

Važnost prepoznavanja i razlikovanja djece za koje se pretpostavlja da su darovita za matematiku od one koja to nisu naglasilo je više autora (Johnson, 2005). Termin *darovito dijete (za matematiku)* nema dosljednu definiciju, što uzrokuje poteškoću oko određenja darovite djece. U literaturi (Vlahović-Štetić, 2002.) se spominju različiti teorijski pristupi darovitosti, među kojima pristup usmjeren na genetske činitelje (Terman, Oden, 1959.), na kognitivne modele (Sterberg, 2001.), na postignuće (Renzuli, 1986.), kao i na sustavski pristup (Tannenbaum, 1983.). U cilju posvećivanja posebne pozornosti nadarenoj djeci, učitelji[4] obično samo matematičku kompetenciju uzimaju kao jedini kriterij za određivanje djetetove darovitosti. Međutim, također je važno uključiti ostale komponente kod odluke o darovitosti djeteta za matematiku. U ovom radu, u analizi načina i razloga na temelju kojih učitelj vidi neko dijete u četvrtom razredu osnovne škole potencijalno darovitim za matematiku, identificirano je pet osnovnih komponenti darovitosti iz matematike. Za svaku komponentu definirane su ključne varijable (atributi) i *if-then* pravila, na temelju kojih je izgrađena baza znanja za inteligentni ekspertni sustav koji može biti potpora učiteljima u određivanju učenikove[5] darovitosti u četvrtom razredu osnovne škole. Sustav se temelji na pravilima odlučivanja i mehanizmu zaključivanja ulančavanjem unaprijed, kojim se koristi u klasifikaciji svakoga učenika u jednu od četiriju kategorija darovitosti.

Krajem 2006. godine provedeno je empirijsko istraživanje u vezi darovitosti za matematiku kod 247 učenika u 10 odjela četvrtoga razreda osječkih osnovnih škola.[6] Za svakog učenika dobivena je procjena učitelja i inteligentnog su-

[4] termin "učitelj" u tekstu predstavlja učitelja odnosno učiteljicu

[5] termin "učenik" u tekstu predstavlja učenika odnosno učenicu.

[6] istraživanje je provedeno u okviru projekta Mala matematička škola pokrenutoga na Učiteljskom fakultetu u Osijeku. Projekt je najavljen na Kongresu nastavnika matematike u Zagrebu, u srpnju 2004. godine (Goljevački, Moguš, 2004), a cilj mu je bio podići kvalitetu obrazovanja studenata učiteljskih studija.

stava, koje su uspoređene statističkim testovima. Cilj istraživanja bio je utvrditi razloge zbog kojih dijete proglašavamo darovitim za matematiku, te razlike u nalazima učitelja i inteligentnoga sustava o darovitosti istoga djeteta.

U nastavku rada dan je pregled prethodnih istraživanja u ovom području, nakon čega je opisana metodologija umjetne inteligencije korištena za izgradnju inteligentnog sustava, te varijable korištene u modelu za određivanje darovitosti. Zatim su dani podatci o ispitanicima, te na kraju rezultati i zaključak sa smjernicama za daljnja istraživanja.

2. Pregled prethodnih istraživanja

Istraživanja o razvoju inteligentnih sustava u obrazovanju do sada su češće bila usmjerena na tutorske sustave koji mogu biti potpora učenju u svladavanju određenog gradiva, s mogućnošću uključivanja multimedije i personaliziranog pristupa učeniku (studentu).

Stathacopoulou i dr. (2005.), na primjer, predlažu upotrebu metodologije neuronskih mreža i neizrazite (fuzzy) logike za napredno dijagnosticiranje studenta u inteligentnom sustavu učenja. Metode umjetne inteligencije omogućuju na neki način "imitaciju" učitelja u prepoznavanju osobina studenta, te u izboru stila učenja koji pogoduje studentovim osobinama. Sustav je testiran na gradivu konstruiranja vektora iz fizike i matematike. Rezultati dobiveni sustavom uspoređeni su s procjenama grupe učitelja s iskustvom, što je pokazalo da sustav uspijeva upravljati određenom neizvjesnošću u procesu dijagnosticiranja, posebno kod graničnih slučajeva kada je čak i učitelju teško donijeti točnu ocjenu studenta. Canales i dr. (2006) razvili su adaptivni i inteligentni obrazovni sustav temeljen na web-u (WBES), koji uvažava pojedinačne zahtjeve studenata za načinom učenja i omogućuje upotrebu različitih tehnika, stilova učenja, strategija poučavanja, te načina interakcije. Arhitektura njihovog sustava slijedi norme LTSA (Learning Technology Systems Architecture) koje je postavilo društvo IEEE, a kojima se preporuča obrazovne sustave strukturirati u pet slojeva: (1) sloj interakcije učenika s okolinom, (2) sloj dizajnerskih karakteristika koje utječu na učenika, (3) sloj komponenti sustava, (4) sloj perspektiva i prioriteta onih koji podržavaju sustav te (5) sloj operacijskih komponenti i interoperabilnosti (kodiranje programa, sučelja i protokoli).

Međutim, manje je pozornosti istraživača usmjereno na područje razvoja inteligentnih sustava za praćenje darovitosti djece u pojedinim područjima, kao na primjer, matematike. Johnson (2000.) ukazuje na važnost i potrebu pravilnog detektiranja i daljnjeg razvoja darovitosti djece za matematiku te na uključivanje drugih kriterija osim matematičkih kompentencija.

Saito i dr. (2007.) istražuju utjecaj suradnje između škola i sveučilišta na nastavnike u školama i fakultetima. Njihovi rezultati pokazuju sljedeće: (1) zajedničko planiranje nastavnih lekcija, promatranja i refleksije pridonose napretku metodologija poučavanja, (2) nastavnici u školama i na fakultetima primjećuju da takvi učenici/studenti aktivnije sudjeluju u nastavi, (3) potrebno je osigurati povezanost učenika/studenata s nastavnim materijalima, kao i učenika/studenata međusobno te (4) suradnja je potaknula kolegijalnost među školama i fakultetima i njihovim nastavnicima.

Općenito, prethodna istraživanja upućuju na zaključak da u posljednjih nekoliko godina postoji snažna ekspanzija upotrebe metoda umjetne inteligencije ponajprije tutorskih sustava za potporu obrazovanju, ali da područje određivanja darovitosti djece za matematiku treba dodatno istražiti i ponuditi dizajn inteligentnog sustava koji će biti potpora u određivanju dječje darovitosti.

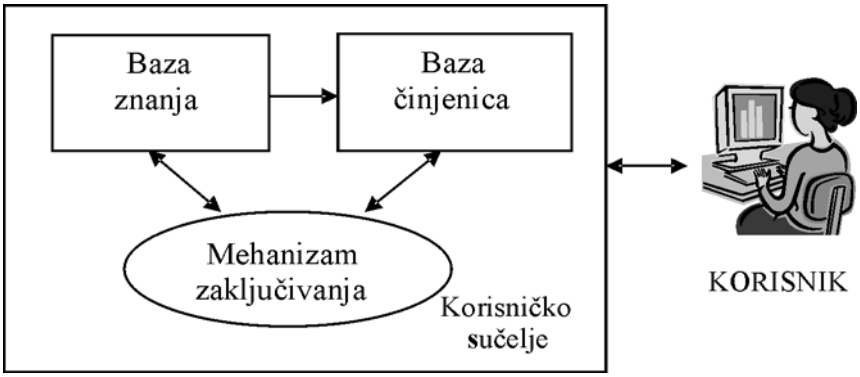
3. Metodologija

Od začetka pojma umjetne inteligencije kao znanstvene discipline do danas razvijene su brojne tehnike kojima je cilj stvaranje inteligentnih strojeva (Russell, Norwig, 2002). Među njima su ekspertni (ili stručni) sustavi, rješavanje problema, strojno učenje, razumijevanje prirodnih jezika, prepoznavanje govora, vidni sustavi (prepoznavanje vidnih uzoraka), robotika, neuronske mreže, genetski algoritmi, inteligentni agenti, i druge tehnike. Iako je u radu naglasak na dizajniranju ekspertnog sustava u području određivanja darovitosti iz matematike, dane su smjernice za nadogradnju sustava s drugim tehnikama umjetne inteligencije, prvenstveno neuronskim mrežama u svrhu klasifikacije učenika s obzirom na darovitost iz matematike.

Ekspertni sustavi računalni su programi koji mogu zamijeniti ljudskog eksperta u donošenju neke odluke (Mišljenčević, Maršić, 1991.). Osim što nude savjet za donošenje odluke, takvi sustavi mogu korisniku i objasniti postupak kojim su došli do rješenja prikazom svojeg znanja kojim su se koristili. Zbog

toga pripadaju u metode tzv. «bijele kutije» (eng. «white box») koje su transparentne, tj. kod njih je vidljiv način dolaska do rješenja. Ekspertni se sustavi rabe kod problema za koje koje postoji usko područje definiranja (tj. uska domena), npr. izbor automobila kod kupovine, ili prijedlog kupovine dionica, ili dijagnoza bolesti srca, i slično.

Dijelovi standardnog ekspertnog sustava prikazani su na slici 1.



Sl.1. Struktura ekspertnog sustava

Baza znanja izvor je znanja o nekom području prikupljen od eksperta za to područje (Čerić et al, 1998.). Znanje se može predstaviti u obliku produkcijskih pravila, semantičkih mreža, predikatne logike i dr., a u radu se koristilo produkcijskim pravilima kao najčešćim načinom prikaza znanja. Baza činjenica predstavlja skup činjenica o stanju nekoga problema koji se rješava (npr. ocjene učenika i sl.). Mehanizam zaključivanja upravlja traženjem puta do rješenja problema, pri čemu se traženje zbiva tako da se ispituju i činjenice u bazi činjenica i znanje u bazi znanja. Korisničko sučelje omogućuje komunikaciju između korisnika i ekspertnog sustava, sadrži i mehanizam objašnjavanja načina na koji je sustav došao do rješenja. Važno je da sučelje bude takvo da korisniku komunikaciju sa sustavom učini što lakšom i pristupačnijom.

Znanjem se u ekspertnom sustavu smatra skup informacija koje su "strukturirane da budu prikladne za upotrebu pri rješavanju problema u nekom području" (Čerić, Varga, 2004.). Među brojnim programskim alatima koji postoje za predstavljanje znanja i njegovo pretraživanje, nazvanih "ljuske ekspertnih sustava", u radu je korišten programski alat Exsys Corvid, u kojem su definirane varijable (odnosno atributi) u bazi znanja, logički blokovi i čvorovi koji čine

produkcijska pravila, dok je kao način pretraživanja znanja korišteno pretraživanje unaprijed (eng. forward chaining) (Mišljenčević, Maršić, 1991.). Produkcijska pravila u inteligentnim sustavima razlikuju se od pravila koja se koriste u sekvencijalnoj obradi kod proceduralnog programiranja, budući da se sastoje od (Mišljenčević, Maršić, 1991.): podataka koji opisuju trenutno stanje vanjskog svijeta, skupa pravila oblika *IF* <uvjet> *THEN* <akcija>, te interpretera pravila koji određuju redoslijed izvođenja pravila. Svako pravilo produkcije definirano je logičkom relacijom koja može imati vrijednost *istina (T)* ili *laž (F)*. Budući da kod mnogih stvarnih problema u nekim prilikama nije moguće sa 100% sigurnošću tvrditi da je nešto istina ili laž (točno ili netočno), u pravila se može uvesti i faktor sigurnosti ili vjerojatnosti da je neki uvjet zadovoljen.

Za kreiranje ekspertnog sustava poduzeti su sljedeći koraci (faze):

(1) definiranje problema koji se rješava ekspertnim sustavom s brojem mogućih opcija
(2) dizajniranje baze znanja – definiranje varijabli (atributa)
(3) definiranje produkcijskih pravila i bodovanje opcija
(4) dizajniranje korisničkog sučelja
(5) testiranje i upotreba ekspertnog sustava
(6) statistička usporedba procjena ekspertnog sustava s procjenama učitelja

Baza znanja ekspertnog sustava kreirana je na temelju četverogodišnjega, neposrednoga rada i istraživanja tima sveučilišnih nastavnika, studenata i učitelja u *Maloj matematičkoj školi* na Učiteljskom fakultetu Sveučilišta u Osijeku. Rezultati naših istraživanja o takvoj suradnji poklapaju se s nalazima do kojih je došao Saito i dr. (2007.). Tijekom zimskoga semestra 2006./07. godine ekspert u području Metodike matematike sa svojim suradnicima i studentima radio je, uz suradnju učitelja i roditelja, sa skupinom učenika četvrtoga razreda s posebnim interesom za matematiku pristiglih s deset osječkih osnovnih škola. Spoznaje iz literature, heuristika u vezi s primijenjenom metodologijom rada, odrađeni projektni zadatci te postignuća učenika, potka su kreirane baze znanja ekspertnoga sustava.

3.1. Definiranje problema koji se rješava ekspertnim sustavom

Ekspertni sustav donosi odluku o kategoriji darovitosti učenika (za dob – četvrti razred osnovne škole). Moguće opcije krajnje odluke jesu:

A) **dijete za koje se pretpostavlja da je darovito za matematiku** – učenik je motiviran i podržavan od vanjskih činitelja, prema postignućima. U znanjima, vještinama i primjeni matematike na razini je koja premašuje očekivanja kurikulskoga pristupa matematici kod vršnjačke dobi. Prikladnim strategijama rada učitelj i mentor, potiču i usmjeravaju razvoj učenikovih kompetencija prema realiziranju darovitosti. Učenik aktivno uči, kontrolira svoj napredak i priprema se za javnu procjenu svojih znanja i sposobnosti, tj. matematičko natjecanje.

B) **dijete s posebnim interesom za matematiku** – učenik je svojim znanjima, vještinama i primjeni matematike u skladu ili nešto iznad očekivanja kurikulskoga pristupa matematici namijenjenoj vršnjačkoj dobi. No, učenik iz ove kategorije pokazuje dodatni interes za matematiku, okolina ga u tome podržava, iako se u pravilu ne želi izlagati javnoj prosudbi svojih znanja i sposobnosti na matematičkim natjecanjima.

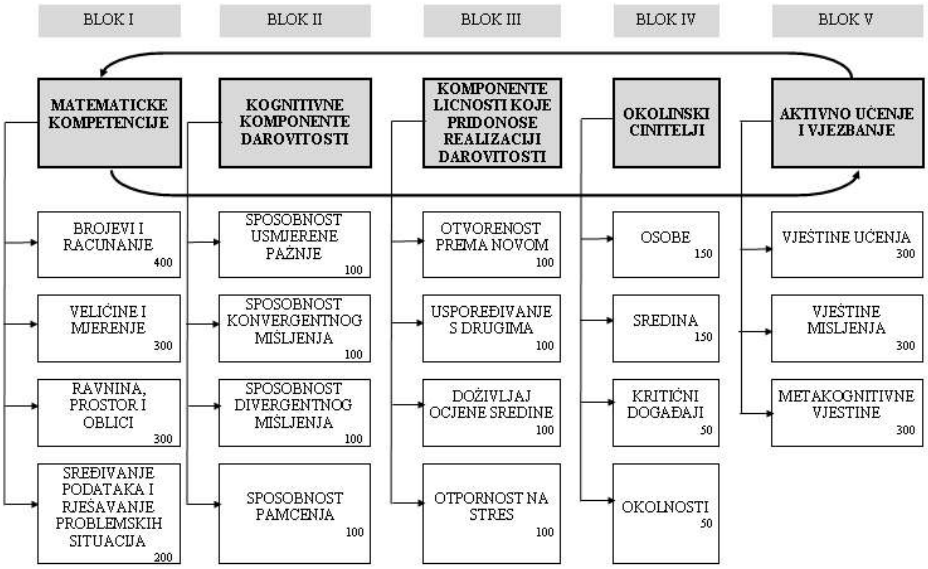
C) **dijete prosječnih sposobnosti za matematiku** – učenik nema interesa za dodatni rad iz matematike, ali su njegova postignuća u okvirima očekivanja kurikulskih pristupa matematici namijenjenih vršnjačkoj dobi. Prikladnim metodama učenja u okviru redovite nastave sustavno se potiče odgovarajući razvoj matematičkih kompetencija.

D) **dijete s nedovoljno razvijenim sposobnostima za matematiku** – učenik čija zatečena znanja, vještine i sposobnosti iz matematike ukazuju da je za postizanje očekivanih osnovnih matematičkih kompetencija potrebna dodatna podrška učitelja i okolinskih činitelja.

3.2. *Dizajniranje baze znanja – definiranje varijabli (atributa)*

Kod definiranja varijabli, odnosno atributa ekspertnog sustava koje će činiti bazu znanja važno je uključiti prosudbu o matematičkim kompetencijama učenika, kognitivnim komponentama darovitosti, komponentama ličnosti koje pridonose razvoju darovitosti, okolinskim činiteljima kao i učinkovitih metoda učenja i vježbanja kojima se potiče razvoj matematičkih kompetencija te moguća realizacija darovitosti. Svaka od tih pet skupina različitih kompentencija u modelu je predstavljena blokovima, koji su raščlanjeni na podblokove, odnosno podskupine kompetencija, te konačno na same varijable koje čine produkcijska pravila. Ovisno o značaju (težini, odnosno utjecaju) pojedinog bloka na odluku

o izboru opcija, definirani su i bodovi za svaki blok. Okvirni dizajn baze znanja (blokovi i podblokovi) zajedno s pripadajućim bodovima prikazan je na slici 2.



Sl. 2. Komponente darovitosti za matematiku uključene u bazu znanja ekspertnog sustava s bodovima koji određuju značaj pojedine komponente

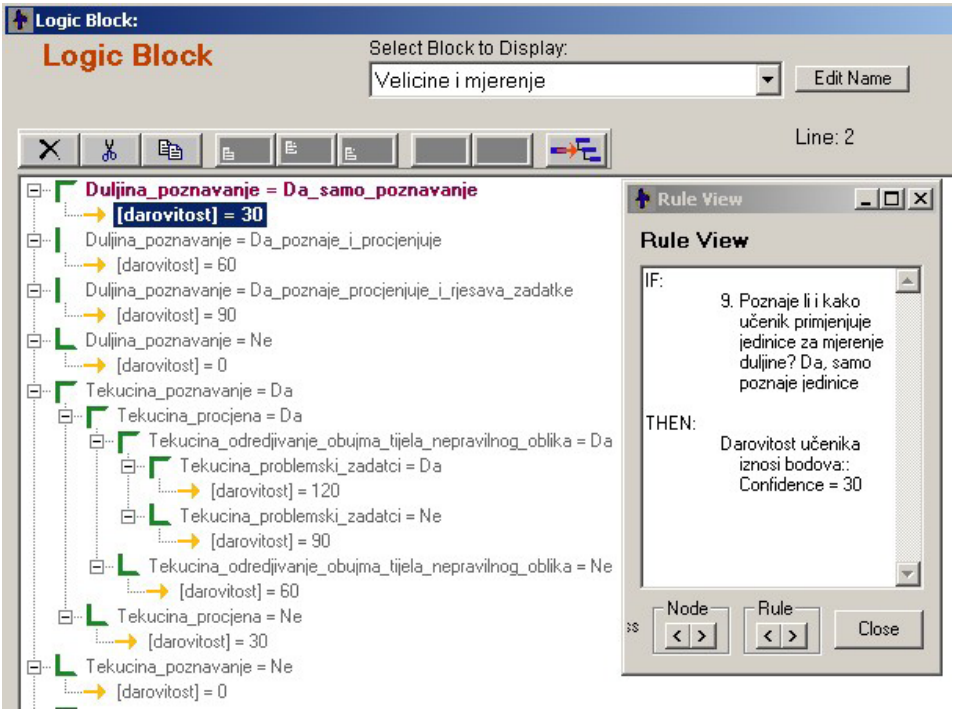
Blok matematičkih kompetencija (blok I) uključuje četiri skupine varijabli iz područja: (a) brojeva i računanja, (b) veličina i mjerenja, (c) snalaženja u ravnini i prostoru, i (d) sređivanja podataka i rješavanja problemskih situacija. Na taj način u procjenu darovitosti uključuju se djetetova znanja i vještine računanja i mjerenja, sposobnosti korištenja matematičkoga jezika i komunikacije, sposobnosti rješavanja problema i modeliranja, kao i sposobnosti matematičke argumentacije. Svaki ovaj podblok podijeljen je dodatno na varijable čije vrijednosti učitava u sustav korisnik, odnosno učitelj. U bloku kognitivnih osobina (blok II) koje predstavljaju intelektualni potencijal određen genetskim činiteljima svakoga učenika, prilagođavanjem i izmjenom strategija aktivnoga učenja i vježbanja, zapravo preispitujemo povećava li se kod učenika sposobnost usmjerene pažnje, sposobnost iznalaženja puta rješavanja te sposobnost brzoga pretraživanja podataka iz dugoročnoga pamćenja. Od komponenata ličnosti koje pridonose realizaciji darovitosti (blok III), kod učenika se zamjećuje (ili ne zamjećuje): sklonost otvorenom i aktivnom pristupu novom, pozitivna obilježja slike o sebi, autonomija (ne zaziru od samoće, ispunjavaju ih aktivnosti kojima

se bave, vjeruju da mogu utjecati na svoj uspjeh, ustrajni su u radu, preuzimaju odgovornost i inicijativu), otpornost na stres (neuspjehe doživljavaju kao priliku za stjecanje novoga iskustva). Također je važno za određenje darovitih za matematiku promišljati o tome poboljšava li se kod učenika aktivno učenje i vježbanje, koje čini blok V, a uključuje: vještine učenja (razlučivanja bitnoga od nebitnoga, kombiniranje i organiziranje informacija u smislenu strukturu, selektivne usporedbe i povezivanja novih informacija s već postojećima u dugoročnom pamćenju), vještine mišljenja (prosudbe, uspoređivanja, procjenjivanja, vrednovanja, zamišljanja, otkrivanja i stvaranja novoga te provedba zamišljenoga u djelo) te metakognitivne vještine (planiranje rješavanja zadatka, praćenje vlastitoga napredovanja, spremnost na promjene pristupa i metoda kod rješavanja problema ukoliko prvotno izabrani put ne rezultira pronalaskom rješenja) učenika četvrtoga razreda.

I na kraju, ali jednako važno, na sigurnije određenje darovitih utječe poznavanje vanjskih činitelja koji mogu djelovati na razvoj potencijalne darovitosti prema realiziranoj darovitosti (blok IV). U darovitih za matematiku, to su: učiteljeva podrška (dopunska nastava), roditeljeva podrška (zajedničko vježbanje matematike, materijalna podrška) te podrška mentora.

4. Definiranje produkcijskih pravila i bodovanje opcija

Na temelju gore navedenih varijabli definirani su logički blokovi u obliku *if-then* produkcijskih pravila, čije logičke vrijednosti (*istina* ili *laž*) povlače odgovarajuće vrjednovanje opcija u odluci ekspertnog sustava. Slika 3. prikazuje dio produkcijskih pravila za blok *Matematičke kompentencije*, podblok *Jedinice i mjerenje*.



Sl. 3. Dio baze znanja ekspertnog sustava – blok Matematičke kompetencije – Veličine i mjerenje

Ukupna baza znanja kreiranog ekspertnog sustava sastoji se od 250 produkcijskih pravila grupiranih u pet glavnih blokova prikazanih na slici 2. Bodovanje opcija definirano je na temelju heuristike. Produkcijska pravila se u sustavu pretražuju metodom ulančavanja unaprijed (eng. forward chaining), pri čemu se polazi od vrijednosti unesenih za krajnje atribute, i slijedom pravila akumulacijom bodova dolazi do cilja – odluke o kategoriji u koju sustav svrstava dijete s obzirom na darovitost za matematiku.

Svakom ispitaniku x iz skupa svih ispitanika N, $kN = 247$, na i-tom čvorištu programa v_i pripada točno w_i bodova. Krajnja odluka sustava $f(x)$ o pripadnosti varijable x jednoj od kategorija A, B, C, D u vezi s *darovitosti djeteta za matematiku* opisane u odjeljku 3.1., dobiva se prema formuli:

$$f(x) = \sum_{i=1}^{61} w_i \tag{1}$$

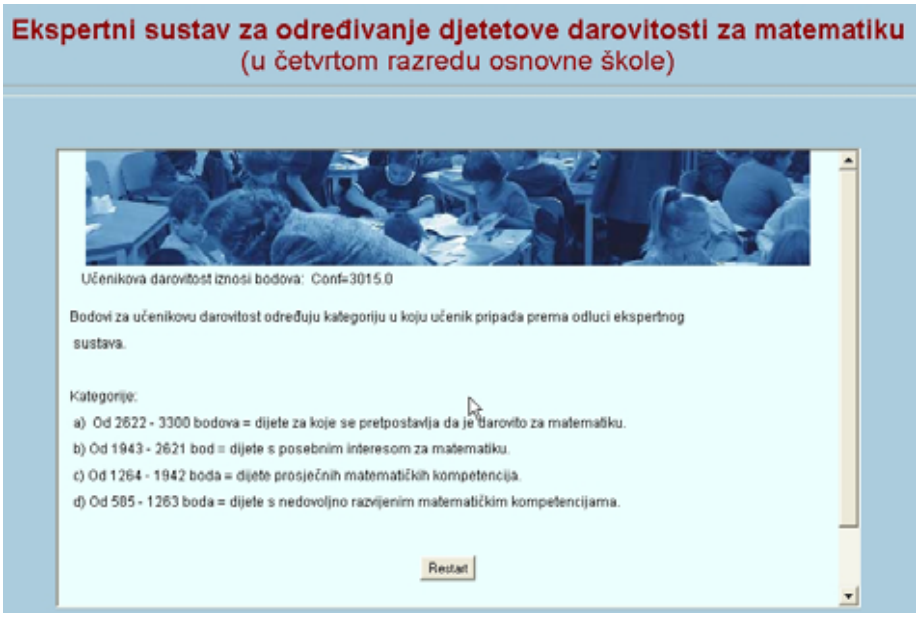
na način:

$$x \in \begin{cases} A, & 2622 \leq f(x) \leq 3300 \\ B, & 1943 \leq f(x) \leq 2621 \\ C, & 1264 \leq f(x) \leq 1942 \\ D, & 585 \leq f(x) \leq 1263 \end{cases} \tag{2}$$

Pri tom su kategorij povi skupa *N* (unija sva četiri podskupa jednaka je skupu *N*, a presjek svaka dva je prazan skup).

5. Dizajniranje korisničkog sučelja

S pomoću programskog paketa Exsys Corvid dizajnirano je i vizualno korisničko sučelje s pomoću kojeg se komunicira s korisnikom i to kroz dva moguća oblika: off-line (na lokalnom računalu) i on-line putem web sučelja korištenjem Java runtime tehnologije. Kriteriji za dizajniranje sučelja bili su: lakoća korištenja, preglednost te raspoloživost krajnjim korisnicima putem web-a. Primjer jednog od prozora korisničkog sučelja prikazano je na slici 4. Putem sučelja korisnici unose vrijednosti pojedinih varijabli (atributa), koje sustav koristi kao ulazne vrijednosti u produkcijskim pravilima te ih transformira u izlazne vrijednosti bodova za svaku opciju odluke o darovitosti.



Sl. 4. Izgled korisničkog sučelja ekspertnog sustava

6. Upotreba ekspertnog sustava

Nakon testiranja formalne i logičke ispravnosti, ekspertni sustav je upotrijebljen u empirijskom istraživanju provedenom u deset osječkih osnovnih škola.

7. Statistička usporedba procjena ekspertnog sustava s procjenama učitelja

Na temelju provedenog istraživanja napravljena je deskriptivna statistika procjena, analizirane su korelacije, a statističkim t-testom za zavisne uzorke uspoređene su razlike u procjenama između učitelja i ekspertnog sustava.

8. Ispitanici

Ispitivanje je provedeno na uzorku od 247 učenika iz deset osječkih osnovnih škola na kraju prvoga polugodišta četvrtoga razreda školske 2006./07. godine. Najmanji broj učenika u jednom odjelu bio je 17, a najveći 30. Škole i odjeli izabrani su namjerno. Naime, po dva odnosno tri učenika s posebnim interesom za matematiku iz tih odjela polaznici su *Male matematičke škole* na Učiteljskom fakultetu u Osijeku. Podatci su prikupljeni listama procjena dječjih matematičkih kompetencija koje je ispunilo deset učitelja o svakom učeniku iz njihova razreda. Svojim odgovorom na posljednje pitanje iz ankete učitelj svrstava dijete u jednu od četiri kategorije bez spoznaje o rezultatima odluke ekspertnoga sustava. Tako je omogućeno ispitati razlike između mišljenja nastavnika i odluke ekspertnoga sustava o svrstavanju djeteta u jednu od četiri ponuđene kategorije.

9. Rezultati

10. Procjene darovitosti od strane učitelja i ekspertnog sustava

Deskriptivna statistika procjene darovitosti djece dobivena od strane učitelja i od strane ekspertnog sustava prikazana je u tablici 1.

Tablica 1. Deskriptivna statistika procjena učitelja i ekspertnog sustava

Varijabla	Srednja vrijednost	Minimum	Maksimum	Standardna devijacija
Darovitost – procjena učitelja	2.287	1.000	4.000	0.837
Darovitost – procjena sustava	2.429	1.000	4.000	1.025

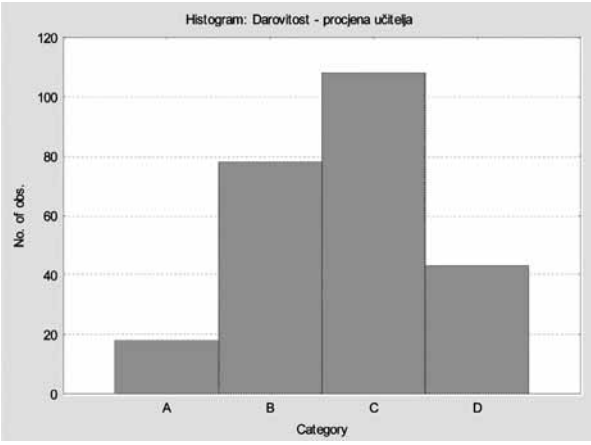
Srednja vrijednost procjena učitelja i sustava ukazuje da sustav u prosjeku ocjenjuje darovitost djece višom ocjenom od učitelja, dok njihova standardna devijacija govori da postoji veće odstupanje u kategorijama darovitosti kod procjene od strane sustava, tj. da su učitelji više skloni svrstati učenike u susjedne kategorije. T-test razlika srednjih vrijednosti ukazuje da postoji statistički značajna razlika između srednjih vrijednosti procjena učitelja i procjena sustava ($t= 3.03972$, $p<0.002624$, $df=246$). Pearsonov koeficijent korelacije između procjena sustava i učitelja iznosi 0.3 ($p<0.05$), što pokazuje statistički značajnu, iako ne jaku, vezu između ovih dviju procjena.

Tablicom 2. prikazane su frekvencije učenika po kategorijama darovitosti s obzirom na procjene učitelja i sustava. Vidljivo je da sustav 19.03% učenika svrstava u najvišu -kategoriju A - *dijete za koje se pretpostavlja da je darovito za matematiku*, dok učitelji svrstavaju znatno manji broj učenika u tu kategoriju (7.29%). T-test razlika u proporcijama upućuje da je razlika u procjenama za tu kategoriju i statistički značajna ($p=0.001$). Iako postoje razlike u broju i postotku učenika svrstanih i u ostale kategorije, na razini 5% signifikantnosti statistički je još značajna samo razlika u procjenama za kategoriju C ($p=0.0231$). Pri tome sustav svrstava 34% učenika u kategoriju C – *dijete prosječnih sposobnosti za matematiku*, dok učitelji svrstavaju 43.73% u tu kategoriju, što upućuje na zaključak o sklonosti učitelja prema svrstavanju najvećeg broja učenika u one s prosječnim sposobnostima. U kategoriju B – *dijete s posebnim interesom za matematiku* – veći broj učenika svrstavaju učitelji nego sustav, dok je za kategoriju D – *dijete s nerazvijenim sposobnostima za matematiku* situacija obrnuta.

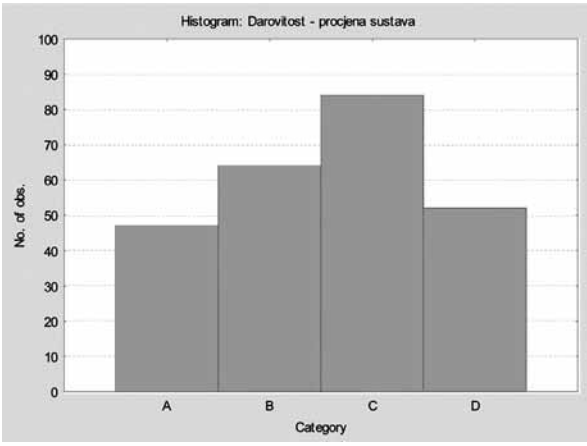
Tablica 2. Frekvencije učenika svrstanih u kategorije darovitosti prema procjenama učitelja i ekspertnog sustava

Kategorija	Procjena učitelja		Procjena sustava		T-test razlika dviju proporcija
	Broj učenika	%	Broj učenika	%	
a	18	7.287	47	19.028	p=0.001
b	78	31.579	64	25.911	p=0.1382
c	108	43.727	84	34.008	p=0.0231
d	43	17.409	52	21.053	p=0.2577
Ukupno	247	100.000	247	100.000	

Grafički prikaz frekvencija putem histograma prikazan je na slici 5 - a) i b).



a) procjena učitelja



b) procjena ekspertnog sustava

Sl. 5. Histogrami frekvencija učenika po kategorijama prema a) procjenama učitelja i b) procjenama ekspertnog sustava

Zbog boljeg uvida u razlike između procjena sustava i procjena učitelja izračunata je stopa podudaranja (vidi *Tablicu 3*). Učitelji i ekspertni sustav su jednako kategorizirali 131 učenika, što čini stopu podudaranja od 53.04%.

Tablica 3. Broj i postotak učenika kod kojih postoji ili ne postoji podudaranje u procjenama učitelja i sustava

Opis	Broj učenika	%
Učenici kod kojih se procjene učitelja i sustava ne podudaraju	116	46.964
Učenici kod kojih se procjene učitelja i sustava podudaraju	131	53.036
Ukupno	247	100.000

Ako se podudaranje procjena od strane učitelja i sustava pogleda detaljnije po kategorijama, dobiva se matrica konfuzije prikazana tablicom 4.

Tablica 4. Matrica konfuzije za procjene darovitosti od strane učitelja i sustava

	Darovitost – procjena sustava				Ukupan broj učenika
Darovitost - procjena učitelja	a	b	c	d	
a	13	3	2	0	18
b	31	31	15	1	78
c	3	29	56	20	108
d	0	1	11	31	43
Ukupan broj učenika	47	64	84	52	247

Vrijednosti na dijagonali matrice konfuzije predstavljaju broj učenika koje su učitelji i sustav svrstali u istu kategoriju. Vidljivo je da je najveće apsolutno podudaranje prisutno kod kategorije C – *dijete s prosječnim sposobnostima za matematiku*, gdje je čak 56 učenika svrstano u istu kategoriju i od strane učitelja i sustava, no tome je uzrok i ukupno najveći broj učenika u toj kategoriji od strane oba procjenitelja. Najmanje apsolutno podudaranje prisutno je za kategoriju A – *dijete za koje se pretpostavlja da je darovito za matematiku*, gdje samo kod 13 učenika postoji podudaranje i sustava i učitelja. Zanimljivo je promotriti i brojeve ispod i iznad dijagonale matrice koji detaljnije objašnjavaju razlike u procjenama po kategorijama. Ako pogledamo podatke u prvom retku, od 18 učenika koje su učitelji svrstali u kategoriju A, sustav je 3 učenika svrstao u kategoriju B, 2 u kategoriju C, a niti jednog u kategoriju D. Međutim, od ukupno 78 učenika koje su učitelji svrstali u kategoriju B, sustav za njih 31 smatra da pripadaju kategoriji A, dok je 15 svrstao u kategoriju C i jednoga učenika u kategoriju D. Slična je situacija i u trećem i četvrtom retku matrice, gdje se potvr-

đuje činjenica da velik broj učenika sustav svrstava u jednu višu kategoriju nego što to procjenjuju učitelji. Podaci u stupcima pokazuju način na koji su učitelji procijenili učenike koje je sustav svrstao u neku u četiri kategorija.

Tablica 5. Broj i postotak učenika kod kojih postoji podudaranje u procjenama učitelja i sustava po kategorijama

Kategorija	Broj učenika	%
a	13	9.92%
b	31	23.66%
c	56	42.75%
d	31	23.66%
Ukupno	131	100.00%

U tablici 5 promatra se udio pojedine kategorije kod onih učenika kod kojih postoji podudaranje u procjenama od strane učitelja i sustava (za ukupno 131 učenika). Pri tome je vidljivo da, kada se učitelji i sustav slažu u procjenama, oni svrstavaju najveći udio učenika također u kategoriju C (42.75%), dok kategorija A – *dijete za koje se pretpostavlja da je darovito za matematiku* čini 9.92% od ukupno jednako svrstanih učenika.

Iz analize sličnosti i razlika u procjenama darovitosti učenika od strane učitelja i ekspertnog sustava može se zaključiti da postoje statistički značajne razlike u procjenama posebno za kategoriju A i C, te da je 9.92% učenika svrstano u kategoriju darovitih i od strane učitelja i sustava, ali da sustav više učenika svrstava u kategoriju darovitih (čak 19.028%), a također i određeni broj učenika procjenjuje za jednu kategoriju više od učitelja.

11. Zaključak

Rad se bavi utvrđivanjem razloga zbog kojih dijete proglašavamo darovitim za matematiku te razlika u nalazima o darovitosti istoga djeteta od strane učitelja i od strane ekspertnog sustava. Na temelju prethodnih istraživanja i heuristike kreiran je model procjene darovitosti učenika četvrtih razreda osnovnih škola koji se sastoji od pet osnovnih komponenti darovitosti. Osim matematičkih kompentencija, model uključuje i kognitivne komponente darovitosti, komponente ličnosti koje pridonose razvoju darovitosti, okolinske činitelje, kao i učinkovitost metoda učenja i vježbanja kojima se potiče razvoj matematičkih kompetencija te eventualna realizacija darovitosti za matematiku. Za svaku komponentu definirane su ključne varijable te produkcijska pravila koje čine

bazu znanja ekspertnog sustava za određivanje djetetove darovitosti za matematiku. Na temelju baze znanja i mehanizma zaključivanja ekspertni sustav svrstava učenika u jednu od četiri kategorije darovitosti. Za svakog učenika dobivena je procjena učitelja i inteligentnog sustava, koje su uspoređene statističkim testovima.

Rezultati pokazuju da učitelji i ekspertni sustav procjenjuju jednako samo 53.04% učenika, te da postoje statistički značajne razlike u procjenama, posebno za kategoriju darovite djece i djece s prosječnim sposobnostima za matematiku.

Zbog činjenice da ekspertni sustav, koji u svojoj bazi znanja uključuje više komponenti darovitosti, svrstava i više učenika u kategoriju potencijalno darovitih, može se zaključiti da bi upotreba takvog sustava mogla navesti učitelja na promišljanja o ostalim komponentama darovitosti učenika za matematiku, te se stoga može koristiti kao učinkovit metodološki alat kako u određivanju darovitosti djece, tako i u izobrazbi učitelja. Daljnja istraživanja u ovom području mogu se kretati u smjeru uključivanja studenata učiteljskih fakulteta i psihologa kao subjekata u procjeni o darovitosti istih učenika, te prema metodološkom unapređivanju alata uključivanjem drugih tehnika umjetne inteligencije, kao što su neuronske mreže, genetički algoritmi, inteligentni agenti i druge.

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Zahvale:

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BORIS PAVKOVIĆ
(skica za portret značajnog metodičara i popularizatora matematike)

Mirko Polonijo[1]

Sažetak. *U Zagrebu je prošle godine preminuo dugogodišnji član PMF-Matematičkog odjela, sveučilišni profesor u miru dr. sc. Boris Pavković (20.11.1931.-06.06.2006.). Bio je veliki zaljubljenik matematike, osobito geometrije, njezin istraživatelj i poučavatelj. Svojim znanstvenim, stručnim, pedagoškim i društvenim radom značajno je pridonio razvoju, razumijevanju i popularizaciji geometrije i matematike u našoj sredini.*

Preuzevši krajem sedamdesetih godina prošlog stoljeća predavanja iz dvogodišnjeg kolegija Metodika matematike, profesor Pavković je svojim znanjem, iskustvom i talentom, te predavačkim i pedagoškim instinktom značajno utjecao u modernom koncipiranju i izlaganju metodike matematike na hrvatskim sveučilištima.

Kao metodičar i popularizator matematike bitno je obilježio zadnjih četrdeset godina predavanja matematike u našim osnovnim i srednjim školama. Njegov će utjecaj još dugo posredno trajati kroz njegove knjige i članke, kolege i suradnike, te bivše studente.

Stoga, cjelokupni metodički i popularizatorski rad profesora Pavkovića zahtjeva i zaslužuje detaljnu analizu i sveobuhvatno vrednovanje.

Ključne riječi: *metodika matematike, popularizacija matematike, matematička edukacija.*

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Profesor Boris Pavković preminuo je u Zagrebu 6. lipnja 2006. godine, nakon kratke i teške bolesti. Ispraćaj je bio 9. lipnja na zagrebačkom Krematoriju, a polaganje urne 13. lipnja na Mirogoju. U spomen na dragoga kolegu i cijenjenoga profesora održana je komemoracija 28. lipnja 2006. na PMF-Matematičkom odjelu Sveučilišta u Zagrebu. O životu i radu pokojnoga profesora Pavkovića, izražavajući mu poštovanje i zahvalnost, govorili su profesori Pavle Pandžić, Mirko Polonijo, Vladimir Volenec, Sanja Varošanec, Sibe Mardešić i Željka Milin-Šipuš. Predviđeni govornici, profesori Margita Pavleković i Ivan Kamenarović, spriječeni zdravstvenim tegobama, poslali su pisane poruke koje su pročitane.

Profesor Boris Pavković rođen je 20. studenoga 1931. u Zagrebu, od oca Josipa (1904.-1977.) i majke Hermine rođ. Petriša (1905.-1999.). "Potječem iz službeničke obitelji" napisat će u svojoj biografiji priloženoj 1960. molbi za zaposlenje. Imao je tri brata, Bruna (r.1935.) Branka (1943.-1983.) i Božidara (1945.-1997.). Svojim odlaskom ostavio je voljene: suprugu Mariju, kćer Jasnu udanu Orešić i unuku Sunčanu.

Osnovnu je školu završio u Čakovcu, u kojem je naučio mađarski, jer je od 1941. do 1945. Čakovec bio pod mađarskom upravom/okupacijom. U tom gradu je položio i tzv. niži tečajni ispit 1947. godine. U Zagreb se obitelj Josipa Pavkovića vraća te 1947. pa profesor Pavković upisuje zagrebačku V. (mušku) gimnaziju i maturira 1951. Iste godine položio je i tzv. viši tečajni ispit. Neposredno, dva dana prije odlaska u bolnicu, 25. svibnja 2006., sa svojim školskim drugovima, profesor Pavković je obilježio 55-godišnjicu mature.

U jesen 1951. godine profesor Boris Pavković se upisuje na Prirodoslovno-matematički fakultet Sveučilišta u Zagrebu, grupa matematika. "Završio je propisane nauke", tj diplomirao 30. 1. 1957. na Matematičko-fizičkom odsjeku iz struke matematika, smjer primjenjena matematika, te mu je "podjeljen naslov" diplomiranog matematičara. Diplomski je rad napisao pod mentorstvom profesora Stanka Bilinskog, koji će mu kasnije također biti mentor i doktorske disertacije, te dugogodišnji "šef" Zavoda.

Odmah poslije diplomiranja zapošljava se profesor Boris Pavković u Srednjoj tehničkoj školi drvne struke, gdje je već ranije, kao apsolvent, predavao matematiku. U jesen 1957. odlazi u vojsku, na "odsluženje vojnog roka". Po povratku predaje matematiku u Srednjoj tehničkoj građevinskoj školi.

U jesen 1959. izabran je za tzv. redovitog asistenta na Katedri za matematiku Strojarsko-brodograđevnog fakulteta Sveučilišta u Zagrebu. Dvije akademske godine, 1959./60. i 1960./61. provodi na tom fakultetu kao asistent kod čuvenog profesora Danila Blanuše, s kojim će ostvariti trajno prijateljstvo ("Kod njega mi je bilo fantastično!", reći će profesor Pavković u razgovoru zabilježenom u *Matki* br.51 (2005) koji je vodila L.Gusić).

U jesen 1961., profesor Boris Pavković biva izabran za asistenta u Geometrijskom zavodu Prirodoslovno-matematičkog fakulteta.

U ovom Zavodu ostati će do svojeg umirovljenja, izgrađujući karijeru znanstvenika, metodičara, autora udžbenika i popularizatora matematike.

Kao jedan od postdiplomanada prve generacije poslijediplomskog studija matematike (otvorenog akademske godine 1960./61.), professor Pavković je magistrirao 27. 4. 1966. s radom *Fiksne točke neprekidnih preslikavanja*, pod vodstvom profesora Sibe Mardešića i Pavla Papića.

Akademske godine 1971./72. profesor Boris Pavković na studijskom je boravku na Moskovskom državnom sveučilištu (MGU) Lomonosov. Ova specijalizacija u Moskvi (a koju će nadopuniti 1974.), osobito rad i iskustva u okviru seminara čuvenoga geometričara B. A. Rosenfeljda, bit će ključni korak u budućem znanstvenom radu profesora Borisa Pavkovića.

Doktorsku disertaciju *Prilog diferencijalnoj geometriji krivulja i ploha u izotropnim prostorima*, izrađenu pod mentorstvom profesora S. Bilinskog obranio je 15. 5. 1974. U komisiji za ocjenu i obranu doktorske disertacije bili su profesori Stanko Bilinski, Dominik Palman i Danilo Blanuša.

U zvanje docenta je izabran 1. 4. 1975. godine. Viši znanstveni suradnik, pa odmah potom izvanredni profesor postao je 1980. godine (u izbornom povjerenstvu bili su profesori Dominik Palman, Sibe Mardešić i Svetozar Kurepa). Nakon što je 1989. proveden izbor profesora Borisa Pavkovića za znanstvenog savjetnika, iste godine biva promaknut u znanstveno-nastavno zvanje redovitog profesora (članovi izbornog povjerenstva bili su profesori D. Palman, V. Volenec i M. Prvanović). U mirovinu je otišao 1. 10. 1994. godine.

Znanstveni rad i doprinos pokojnog profesora Borisa Pavkovića pripada području diferencijalne geometrije prostora s projektivnim metrikama, osobito diferencijalnoj geometriji izotropnih prostora:

B. Pavković, *Eine Verallgemeinerung der Frenetschen Formeln im isotropen Raum,* Glasnik Mat. **4**(**24**)(1969), 117-122.

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B. J. Pavković, *Äquiform-metrische Kurven isotroper Räume,* Berichte der Math.-Stat.Sekt., Forschungszentrum Graz, Ber.Nr. 242(1985), 1-14.

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B. J. Pavković and I. Kamenarović, *The general solution of the Frenet system in the doubly isotropic space $I_3^{(2)}$,* Rad JAZU **428**(1987), 17-24.

B. J. Pavković, *The general solution of the Frenet system of differential equations for curves in the Galilean space G_3,* Rad JAZU **450**(1990), 123-128.

B. J. Pavković, *Relative differential geometry of surfaces in isotropic space,* Rad JAZU **450**(1990), 129-137.

Glavni su znanstveni rezultati profesora Borisa Pavkovića sadržani u potpunom opisu ekviformne diferencijalne geometrije u nekim prostorima s projektivnim metrikama i detaljna analiza Frenetovih sustava u tim prostorima.

Osim toga, izrazito je značajno njegovo bavljenje problemima metodike matematike. Posebno je zaslužan za naše dugogodišnje i dobre veze s austrijskim i mađarskim geometričarima na oba područja. Imao je osobitu sposobnost pridobijanja mlađih kolega za znanstveni rad. Otvorenošću i nesebičnošću uvijek je bio spreman pomoći nudeći suradnju i savjete, najčešće samoinicijativno. Dakako, to nije prestalo odlaskom profesora Pavkovića u mirovinu, uzrokovanim krhkim zdravljem nakon teške operacije 1983. godine.

Pomagao je, osobito mlađima, onima kojima je pomoć bila najpotrebnija. Lako stvarajući prisan odnos, svoje je široko znanje, iskustvo i vještinu rado i neograničeno dijelio. Studentima, diplomandima, onima koji su pod njegovim vodstvom i uz njegovu stalnu pažnju, naputke i nadzor izrađivali magistarske radove i doktorske disertacije. Stoga su mu svi ostali neskriveno zahvalni i privrženi. Kod profesora Pavkovića diplomiralo je stotinjak studenata, magistriralo njih desetak, a doktoriralo sedmero.

Svoju ranu sklonost geometriji i metodici te njihov odabir za svoje djelovanje opisao je u jednom zabilježenom razgovoru (intervju u Školskim novinama od 23. 6. 1992. koji je vodio njegov prijatelj profesor B. Dakić):

"Došavši na studij matematike imao sam sreću da su geometriju predavala dva izvrsna profesora, to su bili prof. dr. Rudolf Cesarec i prof. dr. Stanko Bilinski. Oni su glavni "krivci" što sam zavolio geometriju. Njihova su predavanja bila zanimljiva, ne samo svojim sadržajima, već i načinom izlaganja, a isticala su se visokim stupnjem sustavnosti. Ako sam na fakultetu išta naučio o metodici, to je bilo od njih. Jedna značajka tih predavanja bila je i njihova poetičnost. Neću nikada zaboraviti jedno predavanje prof. Cesarca iz Osnova geometrije. Kada je izveo jednu formulu, kako bi nam naglasio njezinu fundamentalnu ulogu rekao je: "Ova formula predstavlja ključić od sefa u kojem se kriju najljepše tajne hiperboličke geometrije". Nakon toga je jasno da je geometrija postala moje opredjeljenje i da sam se preko nje formirao kao metodičar. Osim toga moram istaknuti da je baš geometrija pravi izazov za metodiku. Uostalom, poznato je kako su spomenuti profesori stvorili čitavu jednu školu dobrih predavača i da je to postala značajka ondašnje Katedre za geometriju."

Na dodiplomskom studiju predavao je mnoge kolegije, između ostalih *Elementarnu matematiku, Nacrtnu geometriju, Diferencijalnu geometriju, Linearnu algebru* i *Metodiku nastave matematike,* a na poslijediplomskom studiju *Riemannovu geometriju.*

U nastavi je profesor Pavković sudjelovao i oduševljavao svojim predavanjima i na drugim sveučilištima (osječkom, splitskom i riječkom) značajno pridonoseći podizanju razine matematičke kulture na tamošnjim pedagoškim fakultetima.

Bio je vrhunski predavač, bez obzira kojim se slušateljima obraćao, jasan i sustavan u izričaju i objašnjenjima, komentarima i napomenama. Uvijek izvrsno, pomno i promišljeno pripremljen. Svako njegovo predavanje bilo je slušatelju novo, sadržajno i poučno iskustvo iz matematike i poučavanja matematike.

Vodeći dugi niz godina Povjerenstvo za prijemne ispite ostvario je odličnu vezu i suradnju s mnogim mladim kolegama, poučivši ih na početku njihove nastavničke karijere raznim vještinama promišljena ispitivača.

Krajem sedamdesetih godina prošlog stoljeća profesor Pavković preuzima predavanja kolegija *Metodika matematike*. Zbog njegova široka znanja i talenta te predavačkog instinkta, to je urodilo značajnim zaokretom na našem fakultetu u modernom koncipiranju i izlaganju ove do tada zanemarene discipline.

Bio je također voditelj znanstvenoga projekta iz područja metodike nastave matematike.

Za metodiku je govorio da je njegovo "unutarnje" određenje:

" Ne mogu objasniti, volim taj posao. Za mene je uvijek izazov, kako objasniti nešto zakučasto. Najmilije mi je oružje živa riječ. Nažalost, ne volim pisati. Moram spomenuti utjecaj jednog vrsnog matematičara i metodičara, profesora Sveučilišta u Stanfordu, to je George Polya, Amerikanac madžarskoga podrijetla. On je godinama držao predavanja na tom sveučilištu namijenjena budućim profesorima matematike, a napisao je i mnogo knjiga u kojima tretira tu problematiku. Koristim prigodu kako bih skrenuo pozornost na dvije od njih, to su *Mathematics and Plausible Reasoning* te *Mathematical Discovery*. (…) Sve su teme bogato ilustrirane konkretnim matematičkim sadržajima iz područja elementarne matematike. Njegovi pogledi na nastavu u skladu su s Preporukom Američkog matematičkog društva, čija je glavna ideja sadržana u

principu "Pogađajte, ispitujte i dokazujte". Tu se misli da "u malom" treba imitirati stvaralačku aktivnost matematičara. Navedeni je princip u osnovi svih mojih metodičkih nastojanja." (op. cit. Školske novine)

Doista, u cjelokupnom metodičkom djelovanju profesora Pavkovića jasno je vidljivo provođenje temeljnih ideja G. Polya (1905.-1985.). Djelatno je provodio preporuku da se u nastavi matematike rabe sve metode kojima se služe matematičari u svojim istraživanjima. A od svih nastavnih metoda najviše je cijenio heurističku, nastojeći preko prikladnih zadataka studente i učenike navesti na samostalno otkrivanje zakonitosti kako bi ih onda pokušali i dokazati.

Profesor Pavković bio je također onaj koji je prvi, i to uspješno, realizirao kolegij Elementarna matematika kojim je prije tridesetak godina valjalo premostiti jaz između srednjoškolske razine stečenih znanja i studija matematike na PMF-Matematičkom odjelu. Koautor je sveučilišnog udžbenika po kojem se i danas predaje taj i neki drugi kolegiji:

B. Pavković, D. Veljan, *Elementarna matematika I*, Tehnička knjiga, Zagreb, 1992, 399 stranica

B. Pavković, D. Veljan, *Elementarna matematika II*, Školska knjiga, Zagreb, 1995, 609 stranica

Iz elementarne matematike je napisao niz zanimljivih stručnih članaka:

B. Pavković, "Fotogrametrija", *Matematičko fizički list* 12 (1961/62), 159-160.

S. Kurepa, B. Pavković, "Površina poopćenog kruga", *Matematičko fizički list* 17 (1966/67), 54-59.

B. Pavković, "Dokaz iracionalnosti vrijednosti trigonometrijskih funkcija", *Matematičko fizički list* 29 (1978/79), 5-6.

B. Pavković, "Geometrijski način rješavanja Pellove jednadžbe", *Matematičko fizički list* 33 (1982/83), 75-78.

V. Devčić, B. Pavković, D. Veljan, "Seminar za stručno usavršavanje profesora matematike", *Matematika* 1 (1983), 87-90.

B. J. Pavković, "Lagrangeov zakon i njegove primjene", *Matematičko fizički list* 38 (1987/88), 4-9.

A. Rubčić, J. Rubčić, B. Pavković, "O trokutima pridruženim poligonima", *Matematičko fizički list* 38 (1987/88), 121-126.

B. J. Pavković, "Metoda analogije i primjene u nastavi", *Matematika* 1 (1988), 20-27.

B. Pavković, "Primjena metode afine geometrije", *Matematika* 4 (1990), 17-30.

B. Pavković, B. Dakić, "Funkcionalne jednadžbe", *Matematičko fizički list* 42 (1991/92), 65-72.

B. Pavković, P. Mladinić, "Sferna geometrija i Eulerova formula-još jedan dokaz", *Bilten Seminara iz matematike za nastavnike mentore*-Kraljevica 1996, HMD i Element, Zagreb, 1996, 102-107.

B. Pavković, P.Mladinić, "Polinomska geometrija", *Bilten Seminara iz matematike za nastavnike mentore*-Novi Vinodolski 1997, HMD i Element, Zagreb, 1997, 94-100.

B. Pavković, P. Mladinić, "Gaussova konstrukcija tangenata kružnice", *Matematičko fizički list* 48 (1997/98), 65-67.

B. Pavković, P. Mladinić, "Polinomska geometrija", *Matematičko fizički list* 49 (1998/99), 135-140.

B. Pavković, P. Mladinić, "O nastavi transformacija algebarskih izraza", *Poučak* 2/3 (2000), 60-63.; također u Zbornik radova 1. kongresa, HMD, Zagreb, 2000, 259-262.

B. Pavković, "O djeljivosti brojeva", *Zbornik radova 1. kongresa*, HMD, Zagreb, 2000, 263-271.

B. Pavković, "Metoda posebnih slučajeva", *Zbornik radova 6.susreta nastavnika matematike*, HMD, Zagreb, 2002, 381-387.

B. Pavković, P. Mladinić, "Geometrija polinoma", *Zbornik radova 2. kongresa*, HMD, Zagreb, 2004, 280-281.

Mnoge stručne teme obradio je u knjigama:

B. Pavković, ¸B. Dakić, *Polinomi*, Školska knjiga, Zagreb, 1987, 179 stranica

B. Pavković, *Diofantske jednadžbe*, Društvo mladih matematičara Pitagora, Beli Manastir, 1988, 14 stranica

B. Pavković, *Kongruencije*, Društvo mladih matematičara Pitagora, Beli Manastir, 1988, 16 stranica

B. Pavković, *Inverzija u ravnini i njene primjene*, Društvo mladih matematičara Pitagora, Beli Manastir, 1990, 22 stranice

B. Pavković, B. Dakić, Ž. Hanjš, P.Mladinić, *Male teme iz matematike*, HMD i Element, Zagreb, 1994, 192 stranice

B. Pavković, B. Dakić, P.Mladinić, *Elementarna teorija brojeva*, HMD i Element, Zagreb, 1994, 202 stranice

B. Pavković, P. Mladinić, *Arhimedova metoda težišta*, HMD i Školska knjiga, Zagreb, 1998, 64 stranice.

Zajedno s kolegama iz Geometrijskog zavoda napisao je jednu fakultetsku zbirku zadataka:

Z.Kurnik, D.Palman, B. Pavković, *Zadaci iz nacrtne geometrije, Mongeova projekcija*, Tehnička knjiga, Zagreb, 1973, 236 stranica

U koautorstvu je profesor Pavković napisao tri vrlo značajne srednjoškolske zbirke koje su doživjele mnogobrojna ponovljena, izmjenjena, prepravljena, dopunjena, proširena izdanja (o njima se govorilo kao tzv. bijeloj zbirci, zelenoj zbirci, ...), da bi se dio njih danas našao uklopljen u najnovije, gimnazijske udžbenike:

B. Pavković, N. Horvatić, *Zbirka zadataka iz matematike 1*, Školska knjiga, Zagreb, 1973, (prvo izdanje)

B. Pavković, D. Svrtan, D.Veljan, *Matematika 3, zbirka zadataka za treći razred srednjeg usmjerenog obrazovanja*, Školska knjiga, Zagreb, 1977 (prvo izdanje)

B. Pavković, D.Veljan, *Zbirka zadataka iz matematike 1 za prvi razred srednjeg usmjerenog obrazovanja*, Školska knjiga, Zagreb, 1984 (prvo izdanje)

Brojna koautorstva profesora Pavkovića u kojima je često on bio upravo onaj koji je najviše prinosio zajedničkom uratku, također svjedoče o njegovoj jednostavnosti u suradnji, davanju i kolegijalnosti.

Značajan je i njegov prevoditeljski rad zahvaljući kojemu smo dobili nekoliko vrijednih stranih matematičkih djela na našem jeziku:

G. Choquet, *Nastava geometrije*, Školska knjiga, Zagreb, 1974, 198 stranica (preveli s francuskog D.Palman i B. Pavković)

A. I. Fetisov, *O euklidskoj i neeuklidskim geometrijama*, Školska knjiga, Zagreb, 1981, 258 stranica (preveli s ruskog D.Palman i B. Pavković)

G. Polya, *Matematičko otkriće*, HMD, Zagreb, 2003, 434 stranice (preveli s engleskog B. Pavković, P.Mladinić i R.Svedrec)

I. N. Bronštejn i suradnici, *Matematički priručnik*, Goldenmarketing-Tehnička knjiga, Zagreb, 2004, XLIV + 1168 stranica (preveli B. Pavković, I.Uremović, D.Veljan i dr.; stručna redakcija B. Pavković i D.Veljan)

Osim toga, profesor Pavković se kod raznih matematičkih naslova javljao i kao urednik, stručni redaktor, stručni konzultant, recenzent, ali i kao korektor i crtač matematičkih slika.

Na PMF-Matematičkom odjelu profesor Boris Pavković bio je predstojnik Geometrijskog zavoda (1992.-1994.), voditelj i suvoditelj Geometrijskog seminara, također Seminara za diferencijalnu geometriju te jedan od osnivača i prvi višegodišnji predstojnik Katedre za metodiku nastave matematike (1990.-1992.).

Prodekan za nastavu bio je akademskih godina 1981./82. i 1982./83..

Za svoj dugogodišnji i nezaobilazan doprinos popularizaciji znanosti, odnosno matematike, profesor Boris Pavković postao je dobitnik državne nagrade "Fran Tućan" 1992. godine.

U spomenutom razgovoru za "Školske novine", na pitanje o tome što znači popularizirati matematiku, s obzirom da su mnogi nematematičari, ali i matematičari u tome smislu vrlo sumnjičavi, profesor Pavković je rekao:

"Popularizirati matematiku znači prije svega zainteresirati i pobuditi želju što šireg kruga ljudi da je upoznaju, a nakon toga iznaći načine da ih se na što dostupniji način upozna s njezinim dostignućima: prvi je korak relativno jednostavan, treba se koristiti onim medijima koji su najpristupačniji i najinteresantniji za dobnu skupinu kojoj se želite obratiti. Za djecu su to prije svega strip i televizija. Poteškoće nastaju na drugom koraku i zbog njih se kod mnogih javlja skepsa. Zaista postoje mnoga područja matematike koje je gotovo nemoguće popularizirati u smislu u kojem ovdje govorimo. Valja međutim reći kako su se u novije vrijeme razvile mnoge nove discipline, uglavnom usporedno s razvitkom računarskih znanosti, kao što su teorija grafova, konkretna matematika, enumerativna matematika itd, u kojima postoje segmenti koje je moguće izložiti na vrlo dostupan način. Posao popularizatora jest da te segmente uoči i podvrgne primjerenoj obradi. Prema tome, popularno o matematici moguće je

govoriti, ali to iziskuje velik trud. Dodao bih kako bi moj odgovor na isto pitanje bio znatno potpuniji i sadržajniji, kad bih ga izložio pred pločom s kredom u ruci. Tada bih ga mogao potkrijepiti brojnim konkretnim primjerima."

Profesor Pavković je bio dugogodišnji član Hrvatskog matematičkog društva, aktivniji od mnogih i onda kada su mnogi bili aktivni. Obilježavajući 2001. godine u Geometrijskom zavodu i seminaru njegov sedamdeseti rođendan s neskrivenim zadovoljstom se isticalo kako je upravo profesor Boris Pavković te godine pri izboru za novi saziv Skupštine HMD dobio najviše glasova. Nije to bilo prvi puta.

U nekoliko navrata bio je član Predsjedništva Društva, Upravnog ili Izvršnog odbora.

Osobito je bio značajan rad profesora Pavkovića u nastavnoj sekciji Društva matematičara. Cijeli svoj radni vijek bio je nosivi stup stručno-pedagoških večeri održavši nebrojena predavanja, vodeći te sastanke, osmišljavajući njihove sadržaje. U povodu obljetnica Društva, znalo se da je profesor Pavković taj koji će najbolje opisati rad nastavne sekcije:

B. Pavković, "Djelatnost Društva u proteklih 40 godina - nastava matematike (povodom 40. obljetnice Društva matematičara i fizičara SR Hrvatske)", *Glasnik Matematički* 24(44) (1989), 659-662.

B. Pavković, "O radu nastavne sekcije za matematiku", *Matematika* 1 (1990), 73-77

B. Pavković, "Djelatnost Društva u nastavi u proteklih 50 godina (povodom 50. obljetnice HMD-a)", *Glasnik Matematički* 30(50) (1995), 380-384.

Da bi se razumjelo navedenih 40 i 50 godina Društva valja reći da je 1945. osnovana Matematičko-fizička sekcije Hrvatskog prirodoslovnog društva, a 1949. samostalno Društvo matematičara i fizičara. Unutar ovoga potonjega društva nastaju 1974. dvije sekcije, jedna je Sekcija za matematiku. Ona će 1990. prerasti u današnje Hrvatsko matematičko društvo. Spomenimo da nakon 1995. godine ni jedna moguća "okrugla" godišnjica Društva, ma kako računali, nije obilježena.

Pisao je profesor Pavković i o velikom Ruđeru Boškoviću te svojim uzorima profesorima R. Cesarcu i S. Bilinskom:

B. Pavković,B.A.Rozenfeljd, "Ruđer Bošković", *Voprozi istorii estetstvoznanija i tehniki*, Moskva, 1974

B. Pavković, "Rudolf Cesarec - povodom 100. godišnjice rođenja", *Matematika* 1 (1990), 78-83.

B. Pavković, "Stanko Bilinski (povodom 80-tog rođendana)", *Istorija matematičkih i mehaničkih nauka* 4 (1991), 71-83.

B. Pavković, "Rudolf Cesarec - znanstvenik i pedagog", *Mathematical Communications* 1 (1996), 67-74.

B. Pavković, V. Volenec, "In memoriam: Stanko Bilinski (22.4.1909.-6.4.1998.)", Glasnik Matematički 33(55) (1998), 323-333.

Kroz dugi niz godina, marljivo je sudjelovao u izradi raznih nastavnih programa matematike, bio je stalni predavač na seminarima za nastavnike, regionalnim i državnim, na Susretima nastavnika, na Kongresima nastavnika. Upravo su zahvaljujući njegovom angažmanu i podršci te manifestacije okupljanja nastavnika matematike zaživjele i održali se (Susreti od 1992., a Kongresi od 2000. godine).

Od pokretanja časopisa *Matka* 1992. godine pa do svoga konačnoga odlaska, profesor Boris Pavković je bio glavni i odgovorni urednik tog popularnog lista za učenike osnovne škole. Najzaslužniji je za kvalitetu i trajanje časopisa, promišljajući ga kao mjesto produbljivanja matematičkog znanja, a ne proširivanja školskog gradiva, te kao izvor razvijanja kreativnoga mišljenja . U uvodniku prvoga broja, kao glavni urednik, profesor Boris Pavković otkriva "kako i zašto tako" treba izgledati matematički časopis za osnovnoškolce. Stoga taj uvodnik prenosimo u cjelosti:

"Draga djeco! Pred vama je prvi broj matematičkog časopisa za osnovnoškolce. Nazvali smo ga *Matka*, jer je to vaš naziv, valjda odmila, za matematiku. Matematika je jedan od školskih predmeta s kojim mnogi naši učenici imaju problema, štoviše, nekima je čak stalna mora. Ali danas se bez matematike ne može. Ona se uvukla u sve pore svakodnevnog života, a posredno ili neposredno primjenjuje se i u područjima koja samo naoko s njome nemaju nikakve veze (medicini, psihologiji, lingvistici, raznim društvenim znanostima itd.). Zbog toga, željeli vi to ili ne, matematiku morate vrijedno učiti kanite li se školovati na razini višoj od osnovne škole. Strah od matematike strah je od nepoznatog. Učenjem i boljim upoznavanjem matematike taj se strah postupno svladava. Že-

ljeli bismo da tome doprinese i *Matka*, i to je Hrvatskom matematičkom društvu bio glavni poticaj za njezino pokretanje. Naše društvo već više od 40 godina izdaje *Matematičko~fizički list* za učenike srednjih škola. Matka je namijenjena vama - najmlađima. Matematikom se valja baviti odmalena. Želimo vas upoznati s idejama i strukturom matematike, s načinima razmišljanja i zaključivanja što ih susrećemo pri rješavanju problema. Htjeli bismo vas pripremiti za kreativnu primjenu matematičkih znanja u najraznovrsnijim situacijama. Željeli bismo vam pomoći pri dosezanju radosti matematičkoga otkrića. Vjerujemo kako ćete uz *Matku* zavoljeti "matku". Navedeni ciljevi odredili su i sadržaj lista. U njemu će biti objavljivani članci čiji sadržaj neće biti šturo i suhoparno nabrajanje činjenica već će se u njima obrađivati ideje što će omogućiti rješavanje određenih tipova matematičkih problema. Težište je dakle na biti matematike. Na kraju svakoga članka navode se zadaci pomoću kojih se provjerava stupanj uspješnosti usvajanja opisane metode. I inače, zadaci u listu bit će od osobite važnosti. Pozivamo vas da ih strpljivo i uporno rješavate. Obavještavat ćemo vas redovito o natjecanjima u matematici i informatici učenika osnovnih škola, objavljivati zadatke s tih natjecanja, kao i imena pobjednika. Bit će humora, zabavne matematike, matematičkih križaljki i kutak za najmlađe. U nizu napisa upoznat ćete se i s povijesnim razvitkom matematike, te životopisima čuvenih matematičara. Nećemo zaboraviti ni informatiku. (...) Pišite nam o tome što biste htjeli čitati u svojem listu. Dostavite nam i svoje priloge o zgodama na satovima matematike u vašoj školi, o radu matematičkih grupa, o zanimljivim zadacima na koje ste naišli, itd. Vaše ćemo priloge rado objaviti. Srdačno vaš Boris Pavković"

Profesor Pavković znatno je pridonio pokretanju Male matematičke biblioteke za učenike, a kao član Upravnog odbora Hrvatskog matematičkog društva inicirao je pristupanje Hrvatske međunarodnom natjecanju Klokan bez granica.

Profesor Boris Pavković volio je matematiku, s velikom vještinom ju je poučavao i popularizirao. U tome mu je pomagalo njegovo poznavanje stranih jezika i sklonost literaturi, ali i urođena radišnost.

Osim toga bio je nenadmašno duhovit, često na granici crnog humora. I u pričanju šala je bio kreativan. Duhovitost ga nije napuštala ni u najtežim trenucima.

Temeljna osobina ovoga vrijednog čovjeka je bila dobrota; profesor Pavković bio je dobar, ali samozatajan.

Kao čovjek, profesor Pavković u mnogim je svojim aspektima bio poput lika dječaka Nemečeka iz njegove omiljene knjige *Junaci Pavlove ulice* mađarskoga pisca Ferenza Molnara. Samozatajan, nenametljiv, požrtvovan, nepokolebljiv, vjeran, odan, iskren, plemenit, posvećen zajedničkoj stvari i dobrobiti.

Svatko tko je upoznao profesora Borisa Pavkovića primio je od njega djelić znanja i dobrote. Poznavajući ga postali smo bolji. Stoga ga cijenimo i poštujemo, trajno.

MATEMATIKA U IGRI I RAZONODI – LEGO KOCKICE

Tomislav Rudec[1]

Sažetak. *Ovaj članak donosi nekoliko zanimljivosti te dvije vrste zadataka o LEGO kockicama. Zadaci su, iako istog tipa, vrlo različitih težina, tj. neki će biti laki i predškolskoj djeci dok će se za neke od zadataka morati pomučiti i profesionalni matematičari. Zadaci su uglavnom kombinacija geometrije i kombinatorike.*

Ključne riječi: *kombinatorika, geometrija.*

Glavni lik priče o nastanku LEGA danski je stolar Ole Kirk Christiansen. Više od građevinskih radova Ole je volio izrađivati drvene makete, figurice i igračke. Rezbario je kućice za lutke i kockice za gradnju, što mu je s vremenom krenulo tako dobro da se odlučio baviti isključivo izradom igračaka. Od danskih riječi *leg* i *godt* (*igrati se* i *dobro*) sastavio je ime svoje tvrtke – *LEGO*. LEGO sve do danas pripada obitelji Christiansen, a od 1979. g. upravitelj je Oleov unuk Kjeld Kirk Christiansen.

Od 1958. godine proizvode kockice u današnjem obliku i veličini, a do sada je (2006. g.) u svijetu proizvedeno više od tristo milijardi kockica, ili otprilike, za svakog stanovnika svijeta po pedeset! Osnovne, najčešće dimenzije kockica su 2×2 i 2×4, a osim njih LEGO je proizveo na desetke i stotine drugih oblika i dimenzija. Figurice, točkovi i druge igračke imaju zajedničko svojstvo: odlično pristaju jedna u drugu pri prvom i stotinu i prvom slaganju.

Kockica dimenzija 2 x 2

Kockica dimenzija 2 x 4

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LEGO je danas po veličini četvrti proizvođač igračaka na svijetu (poslije tvrtki Mattel, Hasbro i Bandai), a Klub ljubitelja LEGA broji oko dva milijuna članova. LEGO kockice nedavno su proglašene (časopis Forbes) za najbolju igračku 20. stoljeća. Mogućnosti slaganja su zaista brojne - od dvije 2×4 kockice iste boje možemo sastaviti čak 24 različite figurice, a od šest takvih kockica, matematičari su uz pomoć računala izračunali, čak 915 103 756 figurica!

Matematički zadaci s LEGO kockicama

Zadatke o LEGO kockicama najčešće ne možemo rješavati uobičajenim matematičkim alatima (npr. jednadžbama ili geometrijskim formulama). Postoje matematičke teorije koje se bave popločavanjima, slaganjima i slično, no svođenje zadataka koji slijede na njihove rezultate nestručnjacima bi (a i stručnjacima) bilo prekomplicirano. Čitatelju u zadacima koji slijede neće dakle trebati veća matematička znanja – na rješavanje su pozvani svi .

Za sve ćemo figurice pretpostavljati da su iz jednog dijela, tj. kockice su spojene u jedan oblik.

1. Tlocrt

Zadatak 1. Koristeći po želji puno LEGO kockica 2×2 i 2×4 (ovdje boja nije važna), složi figuricu čiji je tlocrt ("pogled odozgo") puni pravokutnik dimenzija 4×6.

Rješenje: Vidi sliku.

Figurica tlocrta 4×6 sa četiri velike kockice i druga s tri velike i jednom malom kockicom. Drugu ćemo varijantu smatrati uspješnijom

U ovim ćemo zadacima koristiti samo kockice veličine 2×2 i 2×4, a cilj je složiti oblik, tj. figuru koju gledajući odozgor vidimo kao puni pravokutnik tra-

ženih dimenzija. (Lik koji vidimo kad neki objekt gledamo odozgor zovemo tlocrt tog objekta). Traženu bi figuru trebali izvesti prije svega sa što manje nivoa (redova), a onda i sa što manje kockica. Zamisli da svaka velika kockica 2×4 košta tri boda, a svaka mala 2×2 dva boda. Zadatak je načiniti najisplativiju figuricu, figuricu sa što manje bodova. (Treba upotrijebiti malu kockicu umjesto velike, ako je moguće (naravno da je to teže), a isto je tako bolje (i teže) uzeti jednu veliku kockicu umjesto dvije male jer je ukupan broj kockica tada manji).

Zadatak 2. Konstruiraj najisplativiju figuricu tlocrta 4×4.

Rješenje: Dovoljno je staviti dvije kockice 2×4 jednu pokraj druge. No ipak još nismo gotovi jer ta figurica nije složena, nije iz jednog dijela, pa treba uzeti još jednu kockicu 2×2 i spojiti ove dvije npr. odozgor. Kad dobivenu figuricu stavimo na stol i pogledamo ju odozgor, vidimo kvadrat 4×4, tj figuru tlocrta 4×4. Figurica je načinjena u dva reda, a potrošili smo dvije 2×4 i jednu 2×2 kockicu, tj. ukupno 8 bodova i to je najbolja varijanta.

Figurica tlocrta 4×4

Zadatak 3. Konstruiraj najisplativiju figuricu tlocrta 3×2.

Rješenje: 2 male kockice po 2 boda = 4 boda.

Figura tlocrta 3x2

Zadatak 4. Sa što manje potrošenih bodova složi figuricu tlocrta 3×5.

Rješenje: Oblik 3×5 nije moguće izvesti iz dva nivoa (pokušaj). Jedno od rješenja dano je na slici. Broj bodova je 10.

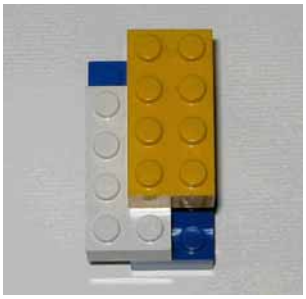


Figura tlocrta 3x5

Zadatak 5. Uz što manje nivoa, a onda i sa što manje bodova složi figuricu zadanog tlocrta.

Broj nivoa (N) i broj bodova (B) upiši u tablicu.

×	2	3	4	5	6	7	8	9	10
2		2N 4B							
3	X			3N 10B					
4	X	X	2N 8B						
5	X	X	X						
6	X	X	X	X					
7	X	X	X	X	X				
8	X	X	X	X	X	X			
9	X	X	X	X	X	X	X		
10	X	X	X	X	X	X	X	X	

2. Malo kockica za mnoštvo figurica

Zadatak 6. Koliko se različitih figurica (oblika) može sastaviti od dvije 2×2 LEGO kockice iste boje?

Rješenje:

Figura A *Figura B* *Figura C*

Osim ove tri nema više figurica, ostale oblike dobivamo drugačijim okretanjem ili jednostavno premještanjem ova tri oblika.

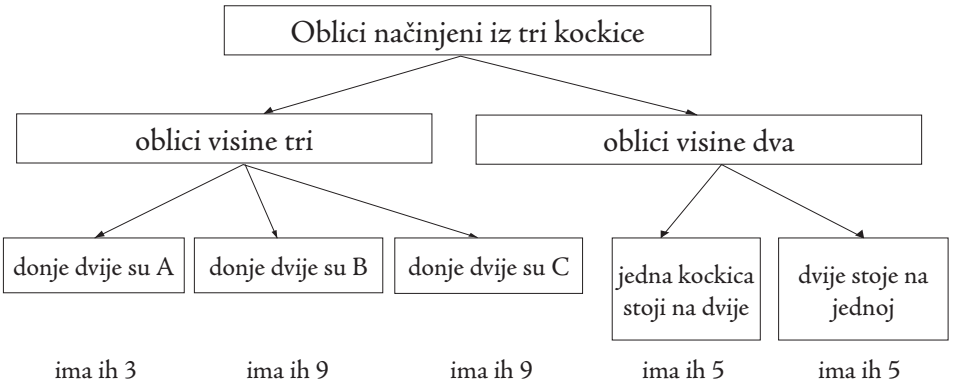
Zadatak 7. Koliko se različitih figurica može sastaviti od tri 2×2 LEGO kockice iste boje?

Rješenje: Tri kockice možemo složiti u figuricu visine tri (uz tri nivoa) i figuricu visine dva. Ako slažemo figuricu visine tri prvo moramo složiti prvi i drugi kat, a to možemo, po prethodnom primjeru, učiniti na tri različita načina. Sada još samo treba staviti treću kockicu na drugu, a to opet po prethodnom primjeru možemo učiniti na tri načina. Dakle, $3 \cdot 3 = 9$ načina.

No, ovo je razmišljanje pogrešno i lijepo pokazuje da u zadacima o LEGO kockicama nećemo moći koristiti puno matematičkih trikova.

Greška je u tome što kada slažemo dvije kockice, donja je kockica simetrična pa uopće nije bitno kako ćemo ju okrenuti prije nego na nju stavimo drugu kockicu, što s dvije već složene kockice nije slučaj.

No od matematike nešto možemo ipak uzeti, a to je sistematičnost. Sve moguće figurice načinjene od tri kockice možemo razdijeliti na različite grupe u odnosu na njihov izgled. Jedno od mogućih raščlanjenja, obzirom na prethodni zadatak i figurice A, B i C sa slike izgleda ovako:



Ukupan broj rješenja je, zaključujemo, (3+9+9) + (5+5) = 21+10 = 31.

Zadatak 8. Koliko različitih figurica možemo složiti od dvije 2×2 figurice različitih boja?

Rješenje: Neka su te boje, radi određenosti, plava i žuta. Očito ove figure možemo podijeliti na one u kojima je donja plava i one u kojima je donja žuta, a za svaki od tih slučajeva, po zadatku 6, imamo tri mogućnosti. Ukupno: 2 vrste · 3 figurice = 6 traženih figurica. (Evo i malo matematike!)

Zadatak 9. Popuni prazna polja u tablici. U svako polje upiši koliko se različitih oblika može sastaviti od kockica koje određuju to polje. Brojevi koje smo izračunali u primjerima već su upisani (npr. broj 31 znači da se od 2 male plave i 1 male plave kockice, dakle tri male plave kockice, može složiti ukupno 31 različita figurica). Neka od polja u tablici predstavljaju isti zadatak, npr polja označena s A. Broj u polju označenom sa * teško je dobiti, u polju s ** vrlo teško, a u polju s ****, naravno, vrlo vrlo teško!

	1 mala plava	1 mala žuta	2 male plave	2 male žute	1 mala žuta i 1 mala zelena	1 velika žuta
1 mala plava	3	6	31			A
1 velika plava		A	*	*	*	
2 velike plave	**	**	****	****	****	**

OSNOVNA MATEMATIČKA ZNANJA I OBRAZOVANJE UČITELJA

Sanja Rukavina[1]

Sažetak. *Razvoj tehnologije te dostupnost priručnih računala potaknuli su brojne rasprave o matematičkim sadržajima koje bi morao usvojiti svaki učenik u okviru obveznoga obrazovanja. Drugim riječima, aktualizira se pitanje temeljne matematičke pismenosti. Osnovno pitanje koje se pri tome postavlja jest što svaki čovjek mora znati i koje vještine mora posjedovati kako bi mogao uspješno participirati u suvremenom društvu.*

Budući da su nastavnici bitan čimbenik svakoga obrazovnog procesa, kada govorimo o nastavi matematike, treba uzeti u obzir i sljedeća pitanja:

- koja su osnovna matematička znanja/vještine što ih mora usvojiti svaki učenik u okviru obveznoga obrazovanja,

- koje su kompetencije neophodne za realizaciju nastave matematike,

- koji su osnovni ciljevi i zadaci obrazovanja nastavnika matematike.

S obzirom da nastava matematike započinje već u prvom razredu osnovne škole, postavljena se pitanja ne odnose samo na predmetne nastavnike matematike i njihovo obrazovanje, već i na učitelje koji u okviru razredne nastave započinju proces sustavnoga matematičkog obrazovanja učenika. Kako mnogi studenti, budući učitelji razredne nastave, ne iskazuju osobit interes prema matematici, a često niti razumijevanje njezina značaja za razvoj učenika i njihova mišljenja, posebnu pozornost treba posvetiti upravo njihovu matematičkom obrazovanju, a od osobitoga je značaja i razvoj pozitivnoga stava prema matematici.

Ključne riječi: *osnovna matematička znanja, obrazovanje učitelja.*

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Razvoj tehnologije te dostupnost priručnih računala potaknuli su brojne rasprave o matematičkim sadržajima koje bi morao svladati svaki učenik u okviru obveznoga obrazovanja. Je li potrebno učiti jednako puno matematike kao i ranije ili nam uporaba računala omogućava uspješno snalaženje i bez poznavanja matematike? Moramo li možda učiti više matematike nego prije? Koja matematička znanja/vještine moramo usvojiti? Drugim riječima, aktualizira se pitanje osnovne matematičke pismenosti. Ove se rasprave provode istovremeno s raspravama o znanstvenoj pismenosti i temeljnim sadržajima koje bi tijekom obveznoga školovanja trebalo obraditi iz područja fizike, kemije, biologije i drugih prirodnih znanosti.

Osnovno pitanje koje se postavlja jest pitanje o tome što svaki čovjek današnjice mora znati i koje vještine mora posjedovati kako bi mogao uspješno participirati u suvremenom društvu. Pri tome ne treba zaboraviti kako se ovo pitanje u jednakoj mjeri odnosi na obiteljski život, radnu sposobnost i na činjenicu da je svaki suvremeni punoljetni građanin često u prilici da svojim glasom za ili protiv participira u donošenju odluka koje se odnose na čitavo društvo.

Sasvim je jasno da će, nakon što odredimo koja ćemo znanja smatrati osnovnim, značajnu ulogu odigrati nastavnici pa, kada govorimo o nastavi matematike, treba obratiti posebnu pozornost i na sljedeća pitanja:

- koja su osnovna matematička znanja/vještine što ih mora usvojiti svaki učenik u okviru obveznoga obrazovanja,
- koje su kompetencije neophodne za realizaciju nastave matematike,
- koji su osnovni ciljevi i zadaci obrazovanja nastavnika matematike.

Budući da nastava matematike započinje već u prvom razredu osnovne škole, postavljena se pitanja ne odnose samo na predmetne nastavnike matematike i njihovo obrazovanje, već i na učitelje koji u okviru razredne nastave započinju proces sustavnoga matematičkog obrazovanja učenika. S obzirom na činjenicu da mnogi studenti, budući učitelji razredne nastave, ne iskazuju osobit interes prema matematici, a često niti razumijevanje njezina značaja za razvoj učenika i njihova mišljenja, posebnu pozornost treba posvetiti upravo njihovu matematičkom obrazovanju, a od osobitoga je značaja i razvoj pozitivnoga stava prema matematici.

Osnovna matematička znanja

Čini se da je u današnjem društvu sukob pobornika matematike i onih koji smatraju da bi matematičke sadržaje u školama trebalo smanjiti dodatno potaknut razvojem tehnologije. Dok prvi ističu kako je razvoj tehnologije povećao potrebu za matematički obrazovanim kadrovima, drugi tvrde kako ni tablicu množenja više nije neophodno poznavati zbog činjenice da nam je gotovo uvijek pri ruci nekakva naprava kojom na brzinu možemo izračunati sve što nam je potrebno. Navedeni je sukob posljedica poistovjećivanja minimalnih i osnovnih matematičkih znanja, odnosno minimalne i osnovne matematičke pismenosti

Minimalna i osnovna pismenost znatno se razlikuju i to, kada ne govorimo o matematici, često i nesvjesno uvažavamo. Na primjer, spremni smo čovjeka koji ne poznaje ili ne poštuje osnove pravopisa nazvati nepismenim iako očito posjeduje minimalnu pismenost (zna se potpisati). Od obrazovnoga sustava očekujemo „produkciju" osoba čija će pismenost biti na razini koja je znatno viša od one što smatramo minimalnom pismenošću. Isto bismo tako trebali uvažiti činjenicu da postoji razlika između minimalnih i osnovnih matematičkih znanja/vještina te očekivati da, po završetku obveznoga obrazovanja, svatko posjeduje osnovna matematička znanja/vještine.

Na pitanje koja su to osnovna matematička znanja/vještine što ih moramo usvojiti u okviru obveznoga obrazovanja mnogi će reći da treba naučiti ispravno izvoditi četiri osnovne računske operacije, a tek poneki među njima navest će kako je uz to potrebno znati i postotke. Kako je razvoj informacijske tehnologije najviše olakšao upravo izvođenje osnovnih računskih operacija, od ovakvoga stava do stava kako je potrebno smanjiti matematičke sadržaje u školama mali je korak. Nažalost, i među učiteljima razredne nastave postoje oni koji će se prikloniti ovakvom načinu razmišljanja.

Odgovor na navedeno pitanje o osnivnim matematičkim znanjima treba potražiti razmišljajući o tome koji su preduvjeti što ih moramo zadovoljiti u današnjem društvu da bismo se mogli smatrati njegovim uspješnim članom te na koji nam način usvajanje određenih matematičkih znanja/vještina pomaže u ostvarivanju tih preduvjeta.

Svakodnevica od nas zahtijeva planiranje, odgovornost, efikasno gospodarenje vremenom i resursima što ih posjedujemo, kritičko razmišljanje, donošenje odluka, sposobnost komunikacije, pregovaranja i rada u grupi te preuzimanje

vodstva u određenim situacijama. Upravo stoga, u osnovna matematička znanja/vještine potrebno je, uz ona matematička znanja čija je primjena svima očita, uvrstiti i one sadržaje čija pragmatičnost nije odmah vidljiva. Tu se posebice ističu zadaci zadani riječima koje su mnogi spremni izbaciti kao suvišne. Rješavajući te zadatke učimo se postavljati pitanja, analizirati, prepoznavati bitne podatke za zadani problem i razlikovati ih od onih koji to nisu. Također se učimo provjeravati vlastite zaključke, procjenjivati vjerojatnost pogreške i uspoređivati dobiveno s očekivanim rezultatom. Sve su to vještine i znanja koja će nam kasnije biti potrebna.

Sadržaji iz geometrije također se ubrajaju među one koje će rijetko koji «nematematičar» proglasiti bitnima. Uočimo, međutim, da je to dio nastavnih sadržaja u kojemu se, osim osnovnih svojstava geometrijskih objekata, učimo uspoređivati različite objekte, uočavati njihove sličnosti i različitosti te ih razvrstavati s obzirom na određena svojstva. Rješavajući probleme mjerenja i zadatke u vezi s njima pripremamo se za brojne životne situacije, kao i za svladavanje sadržaja drugih predmeta.

Usvajanjem raznih matematičkih sadržaja usvajamo i logički način razmišljanja, sposobnost aproksimacije i uočavanja kada je neki podatak dovoljno precizan za uporabu u konkretnoj situaciji, naviku provjeravanja smislenosti određenih tvrdnji, znanje o pravilnoj interpretaciji grafičkih prikaza s kojima se svakodnevno srećemo. Zahtjevi za tim sposobnostima to su veći što je veći razvoj tehnologije, a mnoga zanimanja, uz osnovna matematička znanja, zahtijevaju i dodatno matematičko obrazovanje.

Obrazovanje učitelja

Ne treba očekivati od svih učitelja razredne nastave da budu osobe s izrazitom sklonošću prema matematici. Isti bi zahtjev s punim pravom mogla postaviti i druga područja s kojima se učenici sreću u nižim razredima osnovne škole. U slučaju matematike, situacija je, nažalost, često upravo suprotna i mnogi su učitelji do svoga zanimanja došli «bježeći» od matematike pa tijekom svoga školovanja nisu razvili sklonost prema matematici niti uočili važnost matematičkoga obrazovanja.

Koje kompetencije očekujemo od učitelja razredne nastave kada je u pitanju nastava matematike i na koji način to želimo ostvariti? U svakom bismo slučaju

htjeli da učitelji posjeduju određena matematička znanja te da razumiju značaj matematičkoga obrazovanja učenika. Nije dobro da počeci sustavnoga matematičkog obrazovanja učenika budu vođeni od strane osobe koja minimalna matematička znanja smatra i dovoljnima.

Svi studijski programi u okviru kojih se obrazuju budući učitelji pisani su na način da njihovo svladavanje osigurava usvajanje potrebnih matematičkih znanja. Zahtjevi za poznavanjem matematičkih sadržaja i razumijevanjem matematičkih koncepata svakako moraju biti ispunjeni jer je dobro poznavanje sadržaja preduvjet bez kojega nije moguće ostvariti kvalitetnu nastavu. Uz to, ono čemu bi svakako trebalo posvetiti dužnu pozornost jest razvoj pozitivnoga stava prema matematici kao nastavnom predmetu u školi, ako već ne postoji pozitivan stav prema matematici kao znanosti. Taj bi pozitivan stav trebao proizaći iz razumijevanja potrebe za usvajanjem matematičkih sadržaja; budući bi učitelji morali biti svjesni da matematička znanja/vještine nisu sami sebi svrha i da se kroz njih usvajaju znanja i vještine koja su učenicima potrebna čak i onda kada to na prvi pogled nije očito. Ovu činjenicu nikako ne smijemo zaboraviti i potrebno ju je isticati kad god je to prikladno.

Studente bi trebalo potaknuti da uoče kako apstraktnost, objektivnost i postojanost matematičkih sadržaja ne mogu biti narušeni. Upravo je stoga matematika izuzetno korisna, kako u direktnoj primjeni, tako i u vidu pomoći u razvoju logičkoga razmišljanja. Uoče li budući učitelji da je usvajanje osnovnih (a ne minimalnih) matematičkih znanja/vještina ono na čemu opravdano treba inzistirati, njihov će rad u učionici predstavljati kvalitetan početak matematičkoga obrazovanja učenika.

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INNOVATIVER ANSATZ «MATHEMATIK UND SPRACHE»

Herbert Schwetz[1]

Kurzfassung. *Die Ergebnisse der PISA-Studie 2003 (Schreiner 2006, 104) haben für österreichische Schülerinnen und Schüler wiederum gezeigt, dass sie im Mittelfeld liegen. In einem von IMST (Innovations for Mathematics, Science and Informatics; Initiative zur Veränderung des Mathematikunterrichts des Bildungsministeriums) wird seit Herbst 2006 auf den 3. bis 6. Schulstufen (Grund- und Sekundarstufe I) ein mittelfristig angelegtes Innovationsprogramm durchgeführt. Dieses Programm fokussiert auf den (1) Zusammenhang von Mathematik und Sprache und (2) das Erproben viabler*

Lösungswege (Schwetz 2003, 139). An diesem Programm nehmen 30 Grund- und

Sekundarstufenlehrer teil. In regelmäßigen Treffen werden die Lehrerinnen und Lehrer ermutigt, offene Lernumgebungen zu erproben und Erfahrungen auszutauschen.

Der Innovationsschwerpunkt "Mathematik und Sprache" basiert auf einem vierstufigen mathematikspezifischen Spracherwerbsmodell, nämlich (1) Sprachrezeption, (2) Sprachreproduktion, (3) Sprachproduktion und (4) Sprachreflexion. Den Schülerinnen werden Sachaufgaben mit unterschiedlichem Komplexitätsniveau dargeboten. Besonderes Augenmerk wird aber auf die Sprachproduktion (Verfassen von Sachaufgaben durch Schülerinnen und Schüler) und Sprachreflexion (z.B. Kapitänsaufgaben und überladene Textaufgaben) gelegt. In besondere Weise werden Aufgaben eingesetzt werden, die mehrere Lösungswege (Viabilität) zulassen. In einem Pre-Posttestdesign mit Versuch- und Kontrollklassen werden die Wirkungen der Intervention überprüft.

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Schlüsselwörter: *Innovationsprogramm, Zusammenhang von Mathematik und Sprache, viable Lösungswege, mathematikspezifisches Spracherwerbsmodell.*

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RJEŠAVANJE LINEARNIH JEDNADŽBI POMOĆU SLIKE I RAČUNALA

Miljenko Stanić[1]

Sažetak. *U ovom radu, predstavit ću jednostavan, ali učinkovit način ,pomoću kojeg bismo mogli podučavati učenike u razrednoj nastavi, a to je: kako riješiti linearnu jednadžbu. Potpuno ću se koristiti slikom kao sredstvom rješavanja, izbjegavajući bilo koji standardni formalizam. Prisutni su samo pojmovi iz njihove kompetencije. Na primjer, uzimanje i dodavanje crtaćih likova koji predstavljaju brojeve, odnosno nepoznanice, u nekoj jednadžbi. Po Piagetovoj teoriji kognitivnog razvoja, koja nije suštinski osporena, djeca u razrednoj nastavi ovladala su konzervacijom količine i preferiraju konkretne operacije. Stoga ovaj pristup, slijedi njihove sposobnosti . Metoda mora samo održavati jednakost količine u dvije cjeline, koje predstavljaju različite strane jednadžbe, koristeći operacije dodavanja / uzimanja predmeta istim cjelinama. Cilj je da učenik u nekoliko koraka, ponavljajući spomenute operacije, dolazi do rješenja jednadžbi, definiranih nad skupom N. Pažljivom promjenom centracije pažnje (Piaget), pod vodstvom učitelja, istom se metodom mogu riješiti jednadžbe i nad skupovima Z i Q. Ova metoda dobra je mentalna skela (Vigotsky) za pomoć u shvaćanju formalnih zakona u aritmetičkim strukturama N, ali i u Z i Q. Za crtanje i analizu koristimo se sredstvima koje nudi elektronsko računalo, s uobičajenim crtaćim alatima , kojim je naprimjer, obogaćen Word softverski paket.*

Ključne riječi: *dobro formirana konfiguracija, konzervacija količine,sintaksa, semantika.*

Nedvojbeno je da mnoge matematičke pojmove i probleme možemo prikazati ili rješavati slikovnim pristupom. Slikovni pristup ima veliku heurističku vrijednost u nastavi matematike, a u nekim dokazima je nezamjenjiv. Ovdje ćemo predstaviti rješavanje jednadžbi u nizu slika koje ćemo iscrtati pomoću

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elektronskog računala. U prvom dijelu, predstavljam logičko-semantički status slike i u drugom dijelu, reći ću nešto o metodičkoj primjenljivosti za nastavu matematike .

1. Logika slike

U ovoj temi želimo se pozabaviti pitanjem je li slikovno prikazivanje postupka rješavanja linearnih jednadžbi kompletno? Odnosno, postoji li jednadžba koju ne možemo slikovno riješiti, ili obrnuto da slikovnim putem predstavljamo jednadžbu koja nema ekvivalenta u formalno-lingvističkom zapisu?

Na primjer: podsjetimo se na Vennove diagrame . Pomoću zatvorenih krivulja i jednog pravokutnika vrlo efikasno u školskoj praksi prikazujemo operacije među skupovima. Sam prikaz nije kompletan u gornjem smislu. Na primjer, prazan skup je neprikaziv, u standardnoj reprezentaciji, pa stoga moramo sliku obogatiti novim elementima, kao zasjenjivanjem ili upisivanjem novog znaka da bi naglasili da je taj zatvoreni dio prazan ili da nije parazan, itd. Ovako modificiran prikaz postaje kompletan u gornjem smislu. ([2]).Ovdje mi nažalost raspoloživi prostor ne dopušta da prikažem u potpunosti sintaksu i semantiku slikovnog rješavanja linearnih jednadžbi.

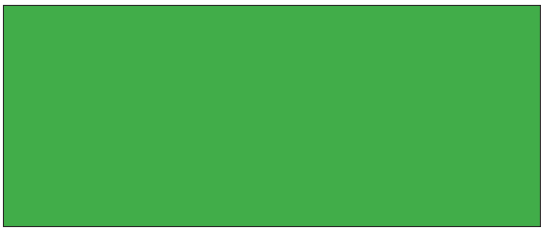
Sljedeći terminologiju iz ([1]), nazvat ću sliku koja prikazuje neki matematički pojam ili objekt (u našem slučaju linearnu jednadžbu) *dobro formiranom slikom (ili konfiguracijom)*(*dfk*).

Prvo, ću predstaviti slikovni abecedarij , a zatim opisati gramatiku *dfk*.

1.1. Gramatika slikovnog prikaza-sintaksa

Osnovni slikovni objekti (ikone)

1. Kućica: Kuc_j =



2. Paketići: $Pak^1_{n,j}$ =  $Pak^2_{n,j}$ =  $Pak^3_{n,j}$ = 

Sve *Pak* ikone su međusobno sukladni kvadrati, koji se mogu razlikovati samo po boji.

Označimo s P^1, P^2 i P^3 skup svih, bezbojnih, plavih ili zelenih paketića.

3. Štapići: $Stp^1_{n,j}$ = ▯ $Stp^2_{n,j}$ = ▮ $Stp^3_{n,j}$ = ▮

Sve *Stp* ikone su međusobno sukladni pravokutnici, koji se mogu razlikovati samo po boji.

4. Štapići /n: $Stp^{i/h}_{n/g,j}$ je pravokutnik s visinom koja je h-ti dio odgovarajućeg $Stp^i_{n,j,}$ $1 \leq g \geq h$.

Označimo s S^1, S^2 i S^3 skup svih, bezbojnih, plavih ili zelenih štapića, i njihovih odgovarajućih dijelova. Indeksom $j \in \{1, 2\}$, označavamo da ikona leži unutar Kuc_1 ili $Kuc_{2.}$

Indeks *n* nazivamo *imenom* ikone (*n* je neki prirodni broj), kojeg ćemo odrediti na sljedeći način.

Neka je Ikn^k_{nj} zajednička oznaka za ikone Stp^k_{nj} ili Pak^k_{nj}. Označimo sa n_{ij} kardinalni broj ikona koje leže u Kuc_j, ($i = s(tapić)$ ili $i = p(paketić)$) , tada, novoj ikoni $Ikn^k_{\cdot j}$ pridružit ćemo *ime*

$n = n_{ij}+1$.

Konfiguracija:

Označimo s Δ sliku koju ćemo nazvati *konfiguracija*. Δ je sastavljena od ikona na sljedeći način:

A) $\Delta = Kuc_1 \cup Kuc_2$ (uz uvjet: $Kuc_1 \cap Kuc_2 = \varnothing$) je *konfiguracija* .

B) Ako je Δ *konfiguracija*, onda je $\Delta' = \Delta \cup \{Stp^k_{n,j} \mid Stp^k_{n,j}$ leži unutar $Kuc_j\}$, $k \in\{ 1,2,1/h,2/h \}$, $h \in\{1,2,3,4,\ldots..\}$ i $j\in\{1,2\}$ također *konfiguracija*.

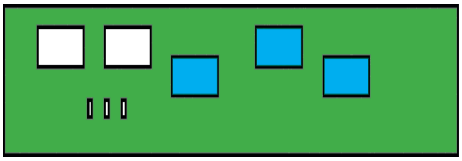
C) Ako je Δ *konfiguracija*, onda je i $\Delta' = \Delta \cup \{Pak^k_{n,j} \mid Pak^k_{n,j}$ leži unutar $Kuc_j \}$, $k \in\{ 1,2,1/h,2/h \}$, $h \in\{1,2,3,4,\ldots..\}$ i $j\in\{1,2\}$ također *konfiguracija*.

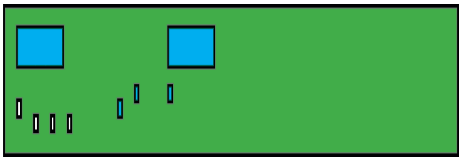
Označimo sa : $\Pi_j =\{ Ikn^k_{nj} \mid Ikn^k_{nj}$ leži unutar $Kuc_j\}$

Označimo sa : $\Phi_{nj} =\{ Stp^k_{mj} \mid Stp^k_{mj}$ leži unutar $Pak^{k'}_{n,j} \}$

Očigledno je: $\Phi_{nj} \subseteq \Pi_j$

Primjer 1. (*konfiguracija*) :





Rješavajući izvod konfiguracija

Konačni niz konfiguracija: $\Delta_0, \Delta_1, \ldots, \Delta_n$ zvat ćemo *rješavajući* ako je zadovoljeno:

a) Δ_0 je neka konfiguracija koju ćemo zvati *početnom*.

b) Δ_i $(0 < i < n-1)$ je konfiguracija izvedena iz konfiguracije Δ_{i-1} primjenom jednim od pravila

i) ***Izv:*** $\Delta_i = \mathbf{Izv}\,(\Delta_{i-1})$

ia) Neka za Δ_{i-1} vrijedi: $Stp^2_{n,1} \in \Delta_{i-1}$ i $Stp^2_{m,2} \in \Delta_{i-1}$, tada je $\Delta_i = \Delta_{i-1} / \{ Stp^2_{n,1}, Stp^2_{m,2}\}$

ib) Neka za Δ_{i-1} vrijedi: $Pak^2_{n,1} \in \Delta_{i-1}$ i $Pak^2_{m,2} \in \Delta_{i-1}$, tada je $\Delta_i = \Delta_{i-1} / \{ Pak^2_{n,1}, Pak^2_{m,2}\}$

ic) Neka za Δ_{i-1} vrijedi: ako iz $Stp^k_{n,1} \in \Delta_{i-1}$ ili $Stp^k_{m,2} \in \Delta_{i-1}$ slijedi da je $k \neq 2$ tada je

$\Delta_i = \Delta_{i-1} \cup \{ Stp^1_{x,1}\} \cup \{Stp^1_{y,2}\} (x = n_{s1}+1, y = n_{s2}+1)$, $Stp^1_{x,1} \in \Pi_1$ i $Stp^1_{y,2} \in \Pi_2$

id) Neka za Δ_{i-1} vrijedi: ako iz $Pak^k_{n,1} \in \Delta_{i-1}$ ili $Pak^k_{m,2} \in \Delta_{i-1}$ slijedi da je $k \neq 2$ tada je

$\Delta_i = \Delta_{i-1} \cup \{ Pak^1_{x,,1}\} \cup \{Pak^1_{x,2}\} (x = n_{p1}+1, y = n_{p2}+1)$, $Pak^1_{x,1} \in \Pi_1$ i $Pak^1_{y,2} \in \Pi_2$

ii) ***Ubc:*** $\Delta_i = \mathbf{Ubc}\,(\Delta_{i-1})$

iia) Neka za Δ_{i-1} vrijedi: $Stp^1_{n,1} \in \Delta_{i-1}$ i $Stp^1_{m,2} \in \Delta_{i-1}$, tada je $\Delta_i = \Delta_{i-1} \cup \{ Stp^3_{x,1}\} \cup \{Stp^3_{y,2}\} =$

$= \Delta_{i-1} / \{ Stp^1_{n,1}, Stp^1_{m,2}\}$ $(x = n_{s1}+1, y = n_{s2}+1)$, $Stp^3_{x,1} \in \Pi_1$ i $Stp^3_{y,2} \in \Pi_2$

iib) Neka za Δ_{i-1} vrijedi: $Pak^1_{n,1} \in \Delta_{i-1}$ i $Pak^1_{m,2} \in \Delta_{i-1}$ tada je $\Delta_i = \Delta_{i-1} \cup \{Pak^3_{n,1}\} \cup \{Pak^3_{m,2}\} =$

$= \Delta_{i-1} / \{ Pak^1_{n,1}, Pak^1_{m,2}\}$ $(x = n_{p1}+1, y = n_{p2}+1)$, $Pak^3_{x,1} \in \Pi_1$ i $Pak^3_{y,2} \in \Pi_2$

iic) Neka za Δ_{i-1} vrijedi: ako iz $Stp^k_{n,1} \in \Delta_{i-1}$ ili $Stp^k_{m,2} \in \Delta_{i-1}$ slijedi da je $k \neq 1$ tada je

$\Delta_i = \Delta_{i-1} \cup \{ Stp^2_{x,1}\} \cup \{Stp^2_{y,2}\}$ $(x = n_{s1}+1, y = n_{s2}+1)$, $Stp^2_{x,1} \in \Pi_1$ i $Stp^2_{y,2} \in \Pi_2$

iid) Neka za Δ_{i-1} vrijedi: ako iz $Pak^k_{n,1} \in \Delta_{i-1}$ ili $Pak^k_{n,2} \in \Delta_{i-1}$ slijedi da je $k \neq 1$, tada je

$\Delta_i = \Delta_{i-1} \cup \{ Pak^2_{n,,1} \} \cup \{Pak^2_{n,2}\}(x = n_{s1}+1, y = n_{s2}+1)$, $Pak^2_{x,1} \in \Pi_1$ i $Pak^2_{y,2} \in \Pi_2$

iii) ***Pod:*** $\Delta_i = \mathbf{Pod}\,(\Delta_{i-1})$

Neka za Δ_{i-1} vrijedi:

1) Ako su $Pak^k_{n,j}$ $Pak^{k'}_{n',j'} \in \Delta_{i-1}$ tada je $k=k'=2$ i $j' = j$.

2) Ako su $Stp^k_{n,j}$ $Stp^{k'}_{n',j'} \in \Delta_{i-1}$ tada je: ($k=k'=1$ ili $k=k'=2$ ili $k=k'=g/h$) i $j' = j$ pri čemu je $g \in \{1,\ldots,h)\}$, $h \in N/\{0\}$.

3) $|(\Delta_{i-1} \cap S^k)| \geq |(\Delta_{i-1} \cap P^{k'})|$

Uredimo skup: $\Delta_{i-1} \cap S^k = \{ Stp^k_{n1,j}, Stp^k_{n2,j}, \ldots\ldots, Stp^k_{nt,j} \}$

Uredimo skup: $\Delta_{i-1} \cap P^{k'} = \{ Pak^{k'}_{m1,j'}, Pak^{k'}_{m2,j'}, \ldots\ldots, Pak^{k'}_{mu,j'}\}$, pritom vrijedi $j \neq j'$, $u \leq t$.

Definirajmo pomoćnu funkciju : $f:\{n_1,n_2,\ldots, n_t\} \rightarrow \{m_1,m_2,\ldots, m_u\}$ $k_l = t - l + 1$

Ako je $k_l \geq u$, $\mathrm{rem}'(l) = \begin{cases} rem(l,u) & \mathrm{rem}(l,u) > 0 \\ u & \mathrm{rem}(l,u) = 0 \end{cases}$, $f(n_l) = m_{rem'(l)}$

Ako je $k_l \leq n_p$, onda je $f(n_l) = 0$.

$\Delta_i = (\Delta_{i-1} / \{ Stp^k_{n1,j} \mid k_l \geq n_p \}) \cup \{ Stp^k_{f(nl),j'} \mid Stp^k_{f(nl),j'} \in \Phi_{f(n1),j'}, f(n_l) > 0\}$

iv) ***Lom:*** $\Delta_i = \mathbf{Lom}\,(\Delta_{i-1})$

$\Delta_i = (\Delta_{i-1} / \{ Stp^k_{n,j} \}) \cup \{ Stp^{i/h}_{n/g,j} \mid g \in \{1,2,3,\ldots,h\} \}$ za neki $h \in N/\{0\}$.

c) Δ_n je *završna* konfiguracija, koja zadovoljava da je za neki $j \in \{1,2\}$: $\Pi_j = \emptyset$.

d) *Status rješenja* rješavajućeg izvoda konfiguracija.

Promotrimo, $\Pi_{j'} \subset \Delta_n$ uz uvijet da je $\Pi_j = \emptyset$, i $j \neq j'$

Razlikujemo , četiri slučaja s obzirom na sadržaj skupa $\Pi_{j'}$.

v) ako je $\Pi_{j'} = \bigcup_{i=1}^{p} \Phi_{n_i j'}$ $n_i \in \{n_1, n_2, \ldots, n_p\}$,$\Phi_{ni,j'} \neq \emptyset$, $(1 \leq i \geq p)$ onda ćemo reći da je *početna* konfiguracija *jednoznačno* rješena.

vi) ako je $\Pi_{j'} = \emptyset$, onda ćemo reći da *početna* konfiguracija ima *neodređeno* rješenje.

vii) ako iz $Ikn^k_{n,j'} \in \Pi_{j'}$ slijedi da je *Ikn=Stp*, tada ćemo reći da *početna* konfiguracija *nema rješenja*.

viii) ako iz $Ikn^k_{n,j'} \in \Pi_{j'}$ slijedi da je *Ikn=Pak* i $\Phi_{ni,j'} = \emptyset$,$(1 \leq i \geq p)$ tada ćemo reći da *početna* konfiguracija ima rješenje 0.

1.2. Semantika – interpretacija

Svakoj ikoni iz sustava slika *pridružujemo* neki cijeli broj ili nepoznanicu.

Ikoni : $Stp^k_{n,j}$ pridružit ćemo pozitivan cijeli broj 1 ako je k=2 ili k=3,odnosno negativan (-1) za k=1.

$Pak^k_{n,j}$ pridružit ćemo nepoznanicu *x* ako je k=2 , odnosno (-*x*) ako je k=1.

$\kappa = |\Pi_j \cap S^i|$ odnosno količina, kardinalni broj bijelih štapića (i=1) ili plavih štapića (i=2) u *j*-kućici koje ćemo interpretirati kao (-κ) ili (κ).

$\kappa = |\Pi_j \cap P^i|$ odnosno količina, kardinalni broj bijelih paketića (i=1) ili plavih paketića (i=2) u *j*-kućici koje ćemo interpretirati kao (-κ) ili (κ) koeficijent uz nepoznanicu.

Sadržaj svake kućice inerpretira se kao strana jednadžbe, koja se poslije sređivanja može svesti na oblik: $\mathbf{a}x + \mathbf{b} = \mathbf{c}x + \mathbf{d}$.

Rješavajući niz je, zapravo, postupak rješavanja jednadžbi gdje se koristimo uzastopnom primjenom zakona kancelacije za operaciju zbrajanja .

Rješenje jednadžbi isčitavamo u Δ_n završnoj konfiguraciji u *rješavajućem* nizu, kao broj štapića u paketiću, ili $x = |\Phi_{n\,j}|$.

U ovom prikazu rješavanja linearnih jednažbi ograničili smo se na koeficijente iz skupa cijelih brojeva, odnosno **a, b, c, d**$\in$**Z**, ali rješenja mogu biti iz skupa racionalnih brojeva **Q**.

3. Metodička svrhovitost

Osnovu za metodičku svrhovitost, igre sa slikama gradim na pretpostavci da je zadatak metodike matematike u ranoj školskoj dobi iskoristiti kognitivne kompetencije djece u implementiranju matematičkih sadržaja. Odnosno, u idealnoj situaciji, učitelj pokušava otkriti u postojećem, pa i ne nužno školskom, znanju djeteta, matematičke sadržaje.

Krenimo od početne konfiguracije.

1. Pozovimo učenika da dobro pogleda sliku.

   Trebao bi uočiti da se slika sastoji od dva dijela, *lijevog* i *desnog*, omeđenih «kućicama».

2. Verbalno tumačenje: Upoznat ćemo učenika da polazimo od pretpostavke da oba dijela slike sadrže istu količinu «štapića». U količinu, uključujemo, i po volji velik broj «štapića» koji grade «kućicu», kao da su to «cigle» iz kojih je izgrađena zgrada. Po boji razlikujemo: štapiće bez boje, štapiće plave boje i štapiće zelene boje.Štapiće bez boje možemo smatrati kao «cigle» koje smo izvadili iz zida kućice. Zeleni štapići su gradivni materijal kojeg stavljamo na bijele štapiće ili «cigle» koje ugrađujemo u zid kućice.Plavo obojeni štapići su dodani materijal u kućici, kao da su njeni stanari.U kvadratićima plave boje skrivena je neka količina plavih štapića. U neobojenim kvadratićima nalazi se nepoznati broj štapića koji su uzeti iz zida kućice. Zelenim kvadratićima «krpamo « dio zida kućice koji je izvađen.

3. Uputiti ih na pravila stvaranja novih slika iz početne.

   Pravilo ***Izv:*** znači izvaditi iz oba dijela konfiguracije «štapić» ili «paketić», istovremeno, pri tom možemo izvaditi «štapić» ili «paketić» i iz «zida kućice».

   Pravilo ***Ubc:*** znači dodati u oba dijela konfiguracije «štapić» ili «paketić», istovremeno.

   Pravilo ***Pod:*** «štapiće» iz jedne «kućice» umetati u «paketiće» druge kućice.

   Pravilo ***Lom:*** «štapiće» iz jedne «kućice» podijeliti na jednak broj «malih štapića».

4. Podrška psihologije: psiholozi smatraju da su djeca u dobi od 7. do 12. godine ovladala pojmom *konzervacije količina* (Pieget[3]). Pregledom pravila izvođenja lako je uočiti da ako pođemo od pretpostavke da *lijevi* i *desni* dijelovi *početne* konfiguracije imaju jednaku *količinu* «štapića», da će se onda primjenom pravila izvođenja u *rješavajućem izvodu* konfiguracija, sačuvati jednakost *količina ljevog* i *desnog* dijela izvedene konfiguracije.

5. Primjena u praksi-nastavi: dobrom pripremom, možemo transformaciju konfiguracija prikazati učenicima kao igru. Igru dobiva učenik koji prvi otvori *završnu* konfiguraciju.

   Dopušteno je i lažiranje, ili *nepravilna* primjena *sintaktičnih pravila*. Kontra igrač mora otkriti «podvalu» ili neispravnu primjenu popraviti u konfiguraciji i za nagradu uzeti potez više, itd. Matematički dobici su postavljanje *mentalnih skela* (Vigotski[3]) prema cijelim odnosno racionalnim brojevima.

Primjer 2. Primijenimo igru na rješavanje jednog zadatka kojeg uzeo sam iz postojeće školske zbirke zadataka za učenike III. razreda ([4]).

Zadatak: Za rad u matematičkoj grupi učiteljica je pripremila niz matematikih zadataka. Treba otkriti koliko je zadataka učiteljica pripremila i koliko ima učenika u grupi, ako se zna da :

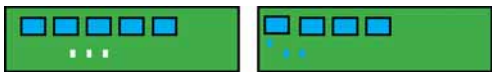
1. Ako bi svaki učenik dobio po 5 zadataka, nedostaju 3 zadatka .

2. Ako bi svaki učenik dobio 4 zadatka, tada će preostat 3 zadatka .

i) Slikovno-ikonička interpretacija:

Štapići su *zadaci*. *Paketići* određuju *broj učenika u grupi*. Lijevi ili desni dio, unutar kućica, interpretiramo kao *ukupan broj zadataka* koje je učiteljica pripremila.

Lijevi dio opisuje zahtjev **1**) a *desni* dio zahtjev **2**).



Pogledaj: *1* zadatak je podijeljen svim učenicima Δ0

2. zada. isto tako…

3. zada. isto tako…

Ako damo 5 svakom uč., onda iz «zida» uzima još 3, odnosno ako ih damo 4 svakom uče. onda još 3 ostaju nepodijeljeni .

Δ_1=***Izv***(Δ_0)**:** prim. na paketiće 4 puta za redom

Δ_2 = ***Ubc*** (Δ_2):prim. na štapiće 3 puta za redom

Δ_3 =***Pod***(Δ_2) *Završna* konfiguracija.Grupa ima 6 učen. i podijeljeno je 27 zadataka.

3. Primjena računala

Zapravo, praktično rješavanje jednadžbi pomoću slika je moguće samo pomoću računala. Svaki učitelj može lako primijeniti gornje crteže, koristeći se samo **Drawing** alatima kojima je oskrbljen svaki Microsoftov proizvod. Kao pomoćno sredstvo, za lakše izračunavanje, napravio sam dva računalna programa na Excel stranicama file **Punavreća** ([5]). Na stranici **jednadžbe** umetnut je program kojim učitelj može učenicima demostrirati pravila izvođenja među konfiguracijama. Prvo, mora upisati linearnu jednadžbu čije rješenje tražimo. Fiksiranjem iste jednažbe na ekranu, dobiva se *početna* konfiguracija. Primjenom komandi **Uzmi** ili **Dodaj** obavljamo vađenje/ubacivanje ikona u kućice, i to *istovremeno*, te na najbolji način, demostriramo pravila ***Izv*** ili ***Ubc***. Ovaj program preporučujem za početak, za motiviranje i zagrijavanje učenika za igru .

Na stranici **igra** naći ćete program koji traži veću interakciju učenika i stroja. Učenici sami umeću ikone i grade početnu konfiguraciju. Izabranu konfiguraciju trebaju **učvrstiti.** Zatim igra počinje i traje do *završne* konfiguracije. Računalo provjerava je li sadržaj u paketićima točno rješenje.

Prijedlog za daljnje istraživanje: prvo, korištenjem slikovnog tumačenja u matematici, primijenjenog na linearnim jednadžbama možemo proširiti na rješavanje jednadžbi s dvije nepoznanice, uvođenjem novih pravila-principa-transformacija koje *će konzervirati količinu*. Mogli bi zamisliti novu tvorevinu koja će pomoću slikovnih demostracija, koristeći se računalom kompletirati, nazovimo to, *račun konzervacije količine*.

Drugo, ne bi bilo tako teško rješavanje linearnih jednadžbi na ovaj način proširiti na izraze s koeficijentima iz **Q**. Dakle, imamo prostora za postavljanje novih Vigotskijevih *mentalnih skela*.

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DISZKALKULIÁS GYEREKEK PROBLÉMAMEGOLDÓ KÉPESSÉGÉNEK FEJLESZTÉSE SZÖVEGES FELADATOK ÁLTAL

Straubingerné Kemler Anikó[1]

Összegzés. *A szöveges feladatoknak sokféle megoldási módszere van, általános módszert nem lehet adni, sok ötletet viszont igen. Ebben az előadásban azt bizonyítottam, hogy a bemutatott egyéb módszerek ugyanolyan értékesek lehetnek, sőt a logikus gondolkodás és a problémamegoldó gondolkodás fejlesztése szempontjából még értékesebbek is, mint az egyenlettel történő megoldás. Éppen ezért nagyon nagy a jelentésük az algebrai megoldástól idegenkedő diszkalkuliás gyerekek esetében is.*

Kiemelném közülük a fordított okoskodást, amely a problémamegoldás egyik igen hasznos módszere. Krutetski a matematika elsajátítása egyik legfontosabb alapképességének tartja egy gondolatmenetről az ellentétes irányú gondolatmenetre való átkapcsolást. /Krutetski, 1977./ De nem csak a matematika elsajátításához, hanem a mindennapi életben való problémamegoldáshoz is nagy segítséget adhat.

A diszkalkuliaterápia lényege ma még a számfogalom, az alapműveletek rögzítése, folyamatos ismétlése, mert ezeknek a gyerekeknek a hosszabb távú memóriájával is általában gond van. De a gyerekek számára nem csak ez lenne fontos, hanem az is, hogy olyan ismeretekhez jussanak, ami hosszabb távon hasznos, és ezek közül az egyik a problémamegoldási eljárások megismerése lehet. Mindeközben azt a cél is elérhetjük, hogy ne érje őket pszichés sérülés, boldog felnőtté váljanak. Ebben pedig nem csak a pedagógiai szakszolgálat szakemberei segíthetnek sokat, hanem az iskolában dolgozó tanítók is.

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Kulcsszavak: *problémamegoldás, tanulási nehézségek, diszkalkulia, matematikai szöveges feladatok*

Kutatási témám a matematika és a hétköznapi élet kölcsönhatásának vizsgálata a diszkalkuliás gyerekek esetében, vagyis, hogy a mindennapi élet nehézségei hogyan jelentkeznek a matematika órán, illetve, hogy mit tehetünk a matematika órán azzal a céllal, hogy ők könnyebben boldoguljanak a hétköznapokban. Ebben az előadásban szeretném ráirányítani a figyelmet a matematikatanítás olyan módszereire, amelyek az osztályközösségen belül is alkalmasak a képességzavaros gyermekek felzárkóztatására és fejlesztésére. Azt vallom, hogy a matematikai korrekció során is a gyerekek személyiségének megerősítésén, fejlesztésén van a hangsúly, hiszen ez jelentheti azt az alapot, amelyen egyéb irányú képességeiket kibontakoztathatják, és később boldog, sikeres életet élhetnek. Ebből adódóan a diszkalkulia-terápia szokásos programja (számfogalom, az alapműveletek rögzítése, folyamatos gyakorlása) mellett a problémamegoldási eljárások megismerését hangsúlyozom.

Ez még azon gyerekek számára is nagyon fontos, akiknek számolási nehézségeik vannak, idegenkednek az absztrakt matematikai jelek használatától, hiszen ez egy olyan dolog, amit későbbi életük során is alkalmazni tudnak.

A bemutatott stratégiák megismertetése a tanítójelöltek képzése során is nagyon fontos. Egyrészt a tanítónak sokféle megoldási eljárást kell ismernie ahhoz, hogy a különböző gondolkodásmódú gyerekek számára mintát tudjon mutatni, ki tudjon alakítani a tanulókban olyan ismereteket, jártasságokat, amelyek birtokában képesek lesznek szöveges feladatokat megoldani a tanult módszerekkel. Másrészt a tanítónak képesnek kell lennie arra, hogy megértse a gyerekek egyéni gondolatmenetét.

Problémamegoldási stratégiák alkalmazására nagyon sok lehetőséget teremtenek a szöveges feladatok. Szöveges feladatok a matematika minden területén és az iskolai oktatás minden szintjén előfordulnak. Előadásomban bemutatom az alsó tagozatban előforduló szöveges feladatok megoldási módszereit, különös tekintettel azokra a megoldási eljárásokra, amelyeket a gyengébb képességű vagy diszkalkuliás gyerekek is alkalmazhatnak.

De mi is az a diszkalkulia?

A „learning disability" (tanulási képtelenség, nehézség, zavar, rendellenesség), mint kategória nem nagyon régóta vált általánosan elfogadottá, bár a jelenség leírásával már az 1800-as évek végétől találkozhatunk orvosi folyóiratokban. A kifejezést 1962-ben használta először egy konferencián Samuel Kirk, ahol agysérülésekkel és a percepció terén problémákkal küzdő gyerekekkel foglalkozó szakemberek találkoztak. /S. Kirk, 1962./ Ezután a különböző kutatási és gyakorlati területek szakemberei egyesítették erőiket a jelenség megismerésére. A tanulási nehézség egyik fajtája a diszkalkulia. Hrivnák Ilona meghatározása szerint: „A diszkalkulia (dyscalculia) a számolási képesség részbeni hiányát, zavarát jelenti, nem tévesztve össze a számolási képesség teljes hiányával, a számolási képtelenséggel, az akalkuliával (acalculia). Diszkalkuliás az a gyerek, aki a matematika megtanulásához célzottan szükséges részképességei fejlődésében kórosan alulmarad a többi tantárgy megtanulását szolgáló részképességek fejlődéséhez képest." / Hrivnák Ilona, 2003./

Az előfordulási gyakoriságról igen különböző adatok látnak napvilágot. A 11-12 éves korosztályban gyakorisága körülbelül 6-7 %, a fiúk és a lányok között ugyanolyan arányban találunk diszkalkuliásokat. Tehát Magyarországon hozzávetőleg 60 000 számolási zavarral küszködő általános és középiskolás várja a segítséget./ Dr. Márkus Attila, 1999./

A súlyos számolási zavar iskoláskori tipikus tünetei:

- Gyakran visszatérő, azonos jellegű számolási hibák (az alapműveleteknél a tíz átlépése, a maradék megtartása, az irányok figyelembevétele kivonáskor, többjegyű szorzóval való szorzáskor a részletszorzatok helyének megállapítása, szimbólumok, jelek használata, soralkotások, növekvő és csökkenő sorok írása és olvasása)

- Fogalmi hiányosságok (szorzás, osztás, törtszám értelmezése, tizedes törtek írása, olvasása, síkidom és test közötti különbség, kerület, terület)

- Alapvető mennyiségfogalmi hiányosságok (idő, hosszúság, űrtartalom, tömeg mértékegységeinek tudása, átváltása terén).

• Ehhez kapcsolódik a különböző iskolai matematikai, kémiai, fizikai képletek, összefüggések tartalmi hiánya. Következésképpen ezek alkalmazása sem le-

hetséges, hiába áll rendelkezésre a számológép. Diszkalkuliás gyerekeknél az analógiás és az absztrakt gondolkodás kialakulása is nehezített.

Magyarországon a diszkalkuliás gyerekek részt vesznek az iskolai tanórákon, de a szakemberek (fejlesztő pedagógus, logopédus) külön is foglalkoznak velük fejlesztő foglalkozások keretében (Nevelési Tanácsadókban, Tanulási Képességet Vizsgáló Bizottságokban). Szerencsére már van kellő számú intézmény és szakember, rendelkezésre állnak fejlesztő programok, eszközök. A diszkalkuliaterápia célja a matematika megtanulásához szükséges biztos alapok megteremtése, a jártasságok és készségek kialakítása, az elvonatkoztatás folyamatának segítése az ismeretek önálló alkalmazásához, valamint a fejletlen vagy hibás pszichikus funkciók fejlesztése, illetve kompenzálása./ Dékány Judit, 1995./ A foglalkozás eredményességének feltétele azonban a szakemberek, a szülők, az iskolai nevelők együttműködése.

Ezért nagyon nagy a felelőssége a tanítóknak. Az elfogadó és segítőkész szemlélet biztonságérzetet jelenthet a diszkalkuliás tanulónak. Alapvető fontosságú, hogy meg kell szerettetni a gyerekekkel a matematikát! Mert ha a gyerek örömmel vesz részt a matematika órán, akkor felhasználhatjuk az abban rejlő sok lehetőséget a fejlesztésre.

A szöveges feladatok szerepe

A szöveges feladatokkal való munkának az alsó tagozatban alapvetően két fő területen van szerepe: a műveletek értelmezése és a problémamegoldó gondolkodás fejlesztése, modellalkotás területén. Azoknak, akiknek az absztrakt jelek felismerése gondot okoz, vagy nehezen megy az elvont gondolkodás, a modellalkotás, sokat segíthet az algebrai megoldás (nyitott mondat) helyett az egyéb módszerekkel, okoskodással történő eljárások megismerése.

Szöveges feladatok megoldásával segíthetjük a gyerekek szövegértő, lényegkiemelő képességének fejlesztését. Az összefüggések feltárása, az ismert és ismeretlen dolgok elkülönítése a szövegből számok nélkül megtehető. A szöveges feladatok által megvalósul az észlelés-érzékelés, a figyelem, az emlékezet, a gondolkodás és a beszéd, a nyelvi fejlettségi szint fejlesztése. A gyerekeket önfegyelemre, hosszabb idejű gondolkodásra, kitartásra neveli. A szöveg értelmezése, és a probléma megoldása során fejlődik a gyerekek logikus gondolkodása. Segíti a szóbeli és írásbeli alapműveletek értelmezését, inverzitás felismertetését a fordított szövegezésű feladatok által. Segíti az analógiák felismerését, valamint

az absztrakciós gondolkodás fejlesztését. A megoldási eljárás is megtalálható számok nélkül. Ezt mondassuk is el mindig a gyerekekkel! Az viszont nem biztos, hogy egy diszkalkuliás gyerek ezt nyitott mondattal meg tudja jeleníteni.

A szöveges feladatok megfelelő matematika órai feldolgozása messzemenően hozzájárulhat az iskolán kívüli diszkalkulia-terápia sikeréhez.

Megoldási stratégiák

A kisgyermek még sokszor találkozik olyan problémával, amit matematikai modell nélkül is meg tud oldani. Ki kell használnunk a lelkesedését, teret kell adni a problémamegoldás sokféleségének, de okos mértékkel, fokozatosan mégis be kell vezetni őket a matematikai eszközök használatába, mert összetett probléma, vagy nagyobb számok körében sokszor csak a modell használata segít. A diszkalkuliás gyerek esetén ez a folyamat sokkal lassúbb. Arra is ügyelnünk kell, hogy viszonylag egyszerű jelrendszert használjunk. A főiskolás hallgatóknál egyébként ennek éppen az ellenkezőjét tapasztalhatjuk, ők a problémák megoldásához rögtön matematikai modellt keresnek, még akkor is, ha egyéb módszerekkel az egyszerűbben megoldható. A következőkben említett problémák nyitott mondatokkal is megoldhatók, most viszont az egyéb eljárások szerepét szeretném bemutatni.

1. A történet eljátszása, megjelenítése

A problémamegoldáshoz először is meg kell érteni a problémát. Ennek eszköze lehet a történet eljátszása, vagy valamilyen tárgyakkal való megjelenítése. Ha ezt a gyerek meg tudja tenni, akkor legtöbbször már nem csak megértette a problémát, hanem közben a kérdésre is megtalálta a választ. Például a következő feladat esetében:

„Egy baromfiudvarban kacsák és nyulak vannak. Tudjuk, hogy 8 fejük és 22 lábuk van összesen. Kérdés, hogy melyik állatból mennyi van?"

Megjeleníthető a feladat rajzokkal, de a modellalkotás irányába haladva például korongokkal (fejek) és pálcikákkal (lábak) is. Kirakjuk először a fejeket, majd mindegyik alá 2-2 lábat, azután a maradék lábakat is kiosztjuk. A 2 lábúak lettek a kacsák, a 4 lábúak a nyulak. Diszkalkuliás gyerekek esetében a probléma akár számok nélkül is feladható: „Ennyi fejük és ennyi lábuk van". A megoldás ekkor is az előbbiek szerint eljátszható.

A tárgyi tevékenység segíthet fogalmak, gondolatmenetek, eljárások tanításában. /Krapf, 1937/. Tudjuk persze, hogy ez életkori sajátosság is, de a diszkalkuliás gyerekeknél ez még fontosabb, mert náluk az elvont gondolkodás később alakul ki.

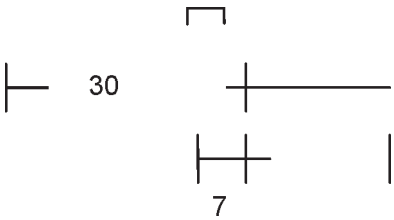
2. Rajzokkal, relációs jelekkel

A megértést segíti, ha a szöveges feladatról a gyerek rajzot készít. Ez kezdetben valósághű, majd egyre elvontabb lehet, ez az absztrakciós folyamat lényeges mozzanata. Gyakran alkalmazzuk a megfordítást is: a gyereknek a képről kell leolvasnia,„Mit mond a kép?" Ezzel a gyerekek párhuzamosan járhatják végig a megoldás konkrét útját, és a matematikai modellbe való átfordítást, amennyiben a képen látható dolgokat „számtannyelven" (műveletekkel, nyitott mondatokkal) is megfogalmazzák.

3. Elvontabb ábrákkal, diagramokkal

A legegyszerűbb elvont ábra a szakaszos ábrázolás. Különösen jól használható módszer, ha a gyerekek a szám- és műveletfogalom kialakítása során használták eszközként a színesrúd készletet (a szám megjelenítése mérőszám formában). Például a következő feladat esetében:

„Amikor apa 30 éves volt, akkor a gyereke 7 éves volt. Az apa most kétszer annyi idős, mint a gyereke. Hány éves most a gyerek?"



4. A szöveg átfogalmazása

A megértésnek rendkívül fontos eszköze a szöveg más formában való elmondása. A történet elmesélése, elemzése kezdetben hangosan, a tanító és a tanuló közti dialógussal történjen! Ez mintát ad a gyereknek arra, hogy a későbbiekben hogyan gondolkodjon egyedül, akár önmagával „párbeszédet" folytatva.

Sok gyerek ilyen formában, szöveggel, okoskodva megtalálja a megoldást, pedig leírni azt nem tudja.

Problémák megoldásánál a legtöbbször követett gondolatmenet az egyenes irányú gondolatmenet. Azonban gyakran olyan problémák is előfordulnak, amikor pontosan egy fordított gondolatmenet egyszerűsíti a megoldást. Ezek azok a típusú problémák, amikor ismerjük a végállapotot, és keressük a kiindulóhelyzetet. Ilyenkor az eredeti szöveget át kell fogalmaznunk úgy, hogy a végállapotból induljunk ki. A matematikában sok területen előfordul ez a fajta feladattípus, a kisiskolás matematikában éppúgy, mint a felsőbb matematikában. Ilyen problémák megoldására alkalmazhatjuk a fordított okoskodást.

Ezt az elvet használja például az alsó tagozatban a nyitott mondatok megoldásának „lebontogatás" módszere.

A matematikában alkalmazott gondolkodási eljárások sokszor az élet egész más területéről vett példákkal, játékokkal, eljárásokkal a gyerek számára jól érthetővé tehetők. Például: Adjunk a gyerek kezébe egy általunk, papírból hajtogatott formát, és kérjük meg, hogy ő is hajtogasson ilyet. Ki fogja bontani, míg el nem jut a kiinduló helyzetbe, aztán összehajtogatja, elismételve fordított sorrendben a hajtásokat. Így önmaga fedezte fel a stratégiát: a fordított okoskodást! Vagy: Vetítsünk le egy rövid filmjelenetet, majd vetítsük le fordítva! Ez így humoros, de ennél sokkal fontosabb, hogy láthatóvá válik, amint a kiindulóhelyzetből cselekvések sorozatán keresztül eljutunk a végállapotba, és fordítva is, a végállapotból a cselekvéssorozat megfordításával eljutunk a kiindulóhelyzetbe. A gyerekek maguk is előadhatnak ilyen cselekvéssort előre, és megfordítva. Ezután kövessük szöveggel: mit tesznek egyik és másik irányban.

A következő megoldási lehetőségek mindegyike a fordított gondolatmenetet használja, különböző megjelenési formában

Például: „Gondoltam egy számot, hozzáadtam 5-öt, aztán elosztottam 2-vel, majd kivontam belőle 17-et, így 10-et kaptam eredményül. Melyik számra gondoltam?"

a) Nem írunk fel nyitott mondatot, csak szövegesen fordítjuk meg a gondolatmenetet. Így a diszkalkuliás gyerekek is megoldhatják a problémát (mindenféle absztrakt jel nélkül).

b) Kártyákkal (rajtuk a matematikai művelet a hozzá tartozó számmal) szemléltetjük műveletsort, majd alatta fordított sorrendben kirakjuk, milyen műveleteket kell elvégezni.

→ +5 → :2 → -17→ = 10

← -5 ← •2 ← +17 ← = 10

Végezzük el a kijelölt műveletsort!...Tehát a 49-re gondolt.

c) Az előbbi műveletsor alapján már nyitott mondatot is felírhatunk a feladatra. Ez már jóval bonyolultabb, mint a kártyás kirakás, itt a zárójelek használatára is tekintettel kell lenni.

5. Próbálgatás

Ennek a módszernek is nagy jelentősége van. Egyrészt sikerélményt adhat a gyereknek, másrészt a próbálgatások során ő maga fedez fel kapcsolatokat, összefüggéseket a mennyiségek között.

„A vázában 2 virág volt. Tettem még hozzá valamennyit, így összesen 5 virág lett benne. Hány virágot tettem hozzá?"

A történet leírható a következő nyitott mondattal: 2 + = 5

A megoldás azonban a hiányos művelet inverzével még nem működik kezdetben. Helyette a gyerekek próbálgatással találják ki a megoldást. Kipróbálják, hogy a 2-höz különböző értékeket hozzáadva mennyit kapnak, és kiválasztják azt, amelyiknek az eredménye 5.

Vagy például: „Peti és Dani almát szedtek, összesen 80 darabot. Peti 10-zel többet szedett, mint Dani. Mennyit szedtek külön-külön?"

A gyerekek felbontják a 80-t, és kiválasztják, hogy melyik az a felbontás, ahol 10 a különbség a két szám között.

6. Táblázatokkal

A próbálkozások eredményeit célszerű táblázatban rögzíteni A többség számára a vizuális megjelenítés segít a kapcsolatok felismerésében. A táblázatos elrendezéssel a gyerek megtanulja párosítani az egymással kapcsolatban

álló mennyiségeket. Akkor is jól használható a táblázat, ha például olyanok az adtok, hogy nem lehet egyértelmű választ adni a kérdésre, például: „A baromfiudvarban levő kacsáknak és nyulaknak összesen 20 lábuk volt. Melyik állatból mennyi volt?". Vagy az összetettebb szöveges feladatokban (mint a 3. pontban említett), az egyik információt felhasználva készíthetünk táblázatot, amelyből kikeressük a másik összefüggést is teljesítő adatpárt. Az adott esetben például táblázatot készíthetünk az apa és a gyerek összetartozó életkorairól, és keressük, hogy mikor lesz az apa életkora a gyerek életkorának kétszerese.

7. Egyenlő változtatások módszere

Ez a problémamegoldási eljárás az 1. pontban említett tárgyakkal való kirakás során is felfedeztethető. Tekintsük az ott ismertetett feladatot! Nézzük meg, mi változik, ha egy nyulat kicserélünk egy kacsára, vagy fordítva! A fejek száma nem változik, a lábak száma viszont mindig 2-vel csökken, illetve nő. A gondolatmenet egyszerűen fejben is végigkövethető, viszont a gyerekek közti különbség éppen abban mutatkozik meg, hogy képes-e ezt pusztán fejben végiggondolni, vagy szükség van modellezésre. A módszer a függvényszerű gondolkodást fejleszti, amennyiben megfigyeljük, hogy egy változtatás milyen változtatást von maga után. Kiinduló helyzetként érdemes feltételezni, hogy mind egyformák, és ezután végrehajtani a cserét.

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A NÉGYZET ÉS A TÉGLALAP FOGALMA 10-11 ÉVES KORBAN

Szilágyiné Szinger Ibolya[1]

Összefoglaló. *Egy fejlesztő oktatási kísérletben vettem részt, amelyhez kapcsolódó tanórák anyagát és feldolgozási módját én terveztem. A tanórákon harmadik megfigyelőként vettem részt. A tanításra az Eötvös József Főiskola Gyakorló Általános Iskola 4. osztályában egy matematika szakos tanító-tanár szakvezetőt kértem fel. A fejlesztő tanítás során több geometriai fogalom alakulását vizsgáltam, de ebben a dolgozatban a négyzet és a téglalap fogalmának alakulásával foglalkozom részletesen.*

Kutatási kérdésem az, hogy hogyan viszonyul az alsó tagozatos geometria oktatásunk, ezen belül a négyzet és a téglalap fogalmának tanítása a Van Hiele-féle geometriai szintekhez, továbbá hogy ezen szinteken előforduló konkrét tárgyi tevékenységek mennyire hatékonyan járulnak hozzá a négyzet és a téglalap fogalomalakulásához.

Hipotézisem, hogy az alsó tagozatos (1-4.osztály) geometriaoktatásban a geometriai gondolkodás Van Hiele-féle szintjeinek első két fázisa reális. A harmadik szintre nem lehet átlépni az alsó tagozat végére. Kialakulnak ugyan fogalomosztályok (téglalap, négyzet), de nincs nagyon kapcsolat köztük. A tartalmazási relációt még nem érzékelik a gyerekek.

P-H. Van Hiele a geometriai ismeretszerzés folyamatát 5 szintre tagolta. Az alakzatok globális megismerésének szintjén (1.szint) a gyerek a tárgyak formáját egészében fogja fel. Megtanulja az alakzatok nevét, nem fogja fel azonban az alakzatok és részeinek kapcsolatát. Nem ismeri fel a kockában a téglatestet, a négyzetben a téglalapot, mert számára ezek egészen különböző dolgok. Az alakzatok elemzésének szintjén (2. szint) a gyermek az alakzatot részeire bontja, majd összerakja. Fontos szerepet kap ezen a szinten a megfigyelés, a rajzolás, a modellezés. A tanuló megállapítja, felsorolja az alakzatok tulajdonságait (lapok,

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oldalak párhuzamossága, merőlegessége, szimmetriatulajdonságok, van derékszöge stb.), de nem definiál és a tulajdonságok közötti logikai kapcsolatokat még nem ismeri fel.

A dolgozatban bemutatom a fejlesztő tanítási kísérletet valamint annak tapasztalatait, amelyet mérési eredményekkel is alátámasztok.

Kulcsszavak: *matematikai oktatás, négyzet, téglalap*

2006 május-júniusában egy fejlesztő oktatási kísérletben vettem részt, amelyhez kapcsolódó tanórák anyagát és feldolgozási módját magam terveztem. A tanórákon megfigyelőként vettem részt. A tanításra az Eötvös József Főiskola Gyakorló Általános Iskolájának 4.c osztályában egy matematika szakos tanító-tanár szakvezetőt kértem fel. A fejlesztő tanítás során több geometriai fogalom alakulását vizsgáltam, de ebben a dolgozatban a négyzet és a téglalap fogalmának alakulásával foglalkozom részletesen.

Kutatási kérdésem az, hogy miként viszonyul az alsó tagozatos geometriaoktatásunk, ezen belül a négyzet és a téglalap fogalmának tanítása a Van Hiele-féle geometriai szintekhez, továbbá, hogy ezen szinteken előforduló konkrét tárgyi tevékenységek mennyire hatékonyan járulnak hozzá a négyzet és a téglalap fogalomalakulásához.

Hipotézisem az, hogy az alsó tagozatos (1-4. osztály) geometriaoktatásban a geometriai gondolkodás Van Hiele-féle szintjeinek első két fázisa valósítható meg. A harmadik szintre nem lehet átlépni az alsó tagozat végére. Kialakulnak ugyan fogalomosztályok (téglalap, négyzet), de nincs különösebb kapcsolat köztük. A tartalmazási relációt még nem érzékelik a gyerekek.

A kisgyermek irányított geometriai ismeretszerzési folyamata már az óvodában elkezdődik. A környezet tárgyainak formáját vizsgálva indul a geometriai objektumok (a mértani test, a síkidomok stb.) fogalmának kialakítása. Ezen objektumok halmazára jellemző tulajdonságok megállapítása már az ismeretszerzés magasabb fokát jelenti.

P-H. Van Hiele a geometriai ismeretszerzés folyamatát 5 szintre tagolta.

Az *alakzatok globális megismerésének szintjén* (1.szint) a gyerek a geometriai alakzatokat mint egységes egészet fogja fel. Könnyen felismeri a különböző alakzatokat a formájuk alapján, megtanulja az alakzatok nevét, nem fogja fel azonban az alakzatnak és részeinek kapcsolatát. Nem ismeri fel a kockában a

téglatestet, a négyzetben a téglalapot, mert számára ezek egészen különböző dolgok.

Az *alakzatok elemzésének szintjén* (2. szint) a gyermek az alakzatokat részeire bontja, majd összerakja. Felismeri a mértani testek lapjait, éleit, csúcsait. A mértani testek lapjaiként a síkidomokat, amelyeket görbék, szakaszok, pontok határolnak. Fontos szerepet kap ezen a szinten a megfigyelés, a mérés, a rajzolás, a modellezés. A tanuló megállapítja, felsorolja az alakzat tulajdonságait (lapok, illetve oldalak párhuzamossága, merőlegessége, szimmetriatulajdonságok, van derékszöge stb.), de nem definiál és a tulajdonságok közötti logikai kapcsolatokat még nem ismeri fel. Attól, hogy a négyzet és a téglalap közös tulajdonságait észreveszi, még nem várhatjuk el, hogy levonja azt a következtetést, hogy a négyzet téglalap.

A *lokális logikai rendezés szintjén* (3. szint) a tanuló már összefüggéseket állapít meg az alakzatok és tulajdonságaik között. Megjelenik a következtetés lehetősége az alakzatok egyik tulajdonságáról a másikra. Megérti a meghatározás, a definíció szerepét. A logikai következtetések menetét azonban a tankönyv (illetve a tanár) határozza meg. Megkezdődik a bizonyítási igény kialakítása, de ez csak az alakzatokra terjed ki. Ezen a szinten a négyzet már téglalap.

A negyedik (*törekvés a teljes logikai felépítésre*) és ötödik (*axiomatikus felépítés*) szinteknek megfelelő oktatás a középiskola és a felsőoktatás feladata.

A Van Hiele-féle modell mindegyik tanulási szakasza az előző által kialakított gondolkodást építi és bővíti tovább. Egyik szintről a másikra való átlépés folyamatosan és fokozatosan megy végbe, miközben elsajátítják az egyes szinteknek megfelelő matematikai fogalmakat. Ezt a folyamatot sajátosan befolyásolja a tanítás, annak tartalma és módszere. A megfelelő geometriai gondolkodás kialakításához egyik szintet sem ugorhatjuk át. Minden szintnek megvan a maga sajátos nyelve, jelölésrendszere, logikai felépítése. Fontos oktatási vonatkozása Van Hiele elméletének az, miszerint az alacsonyabb szintű szakaszban lévő tanulóktól nem várható el, hogy a magasabb szintnek megfelelően megfogalmazott instrukciókat megértsék. Van Hiele szerint ez a legfőbb oka a matematikaoktatás kudarcainak.

A matematikai fogalmak tanításával kapcsolatban R. Skemp matematikuspszichológus a következő megállapításokat teszi:

„Definíció segítségével senkinek nem közvetíthetünk az általa ismerteknél magasabb rendű fogalmakat, hanem csakis oly módon, hogy megfelelő példák sokaságát nyújtjuk. Minthogy a matematikában az előbb említett példák majdnem mind különböző fogalmak, ezért mindenekelőtt meg kell győződnünk arról, hogy a tanuló már rendelkezik ezekkel a fogalmakkal. … A megfelelő példák kiválasztása sokkal nehezebb, mintsem gondolnánk. A példáknak rendelkezniük kell azokkal a közös tulajdonságokkal, amelyek a fogalmat alkotják, de nem szabad rendelkezniük semmiféle más közös tulajdonsággal." (1975.)

Egy fogalom kialakítása során a fogalmakat be kell illeszteni a meglévő fogalmak rendszerébe (asszimiláció), de előfordul, hogy az új fogalom beillesztéséhez szükséges a már meglévő rendszer, séma módosítása (akkomodáció). Az asszimiláció és akkomodáció egyensúlya nélkülözhetetlen a megfelelő fogalomalakuláshoz. Ha ez az egyensúly felbomlik, azaz az asszimilációt nem követi megfelelő akkomodáció, akkor a tanuló saját, egyéni magyarázóelvei fokozatosan beépülnek a matematikai fogalmaiba, ami fogalmi zavarhoz vezethet. Itt válik fontossá a pedagógus szerepe, akinek a feladata ennek az egyensúlynak a fenntartása.

A dolgozat elején említett fejlesztő oktatási kísérlet 16 tanítási órát foglalt magába, amelynek célja a Van Hiele-modell szerinti geometriaoktatás megvalósítása. Az első órán a 26 negyedik osztályos tanulóval egy pretesztet írattam, amelynek segítségével megállapítottam, hogy az első szintről (az alakzatok globális megismerése) a másodikra (az alakzatok elemzése) történő átlépés és a geometriai gondolkodás továbbfejlesztése ezen a szinten lehetséges. A preteszt összeállításánál az előző tanév (3. osztály) tananyagát és az osztályt tanító szakvezető tapasztalatait vettem figyelembe. A fejlesztő tanítás első órája egyben a negyedik osztályos témakör első órája is volt.

A preteszt első két feladata hivatott bemutatni a négyzetről és a téglalapról alkotott helyzetképet. Az első feladat a négyszög, téglalap, négyzet fogalmak realizálásával (alkotásával), a második a fogalmak azonosításával (felismerésével) kapcsolatos. Az első feladatban egy-egy négyszög (a), téglalap (b), illetve négyzet (c) rajzolását kértem, a másodikban pedig 16 síkidom közül kellett kiválasztani a megfelelőket, amelyeket csak egyenes vonal határol (a), amelyek négyszögek (b), amelyek téglalapok (c) és végül, amelyek négyzetek (d).

Az 1. feladat megoldásának eredményességét (26 tanulóét) a következő táblázatban foglalom össze:

a) Négyszögként általános négyszöget rajzolt. (fő)	9
Négyszögként négyzetet rajzolt. (fő)	11
Négyszögként téglalapot rajzolt. (fő)	5
Nem tudott négyszöget rajzolni. (fő)	1
b) Jól rajzolt téglalapot. (fő)	26
c) Jól rajzolt négyzetet. (fő)	25

A feladat megoldása eredményesnek mondható, hiszen 1 gyerek nem tudott négyszöget, illetve ugyancsak 1 nem tudott négyzetet rajzolni. Viszonylag magas azoknak a tanulóknak az aránya (42%), akik négyzetet rajzoltak négyszögként is.

A 2. feladat megoldásának értékelése:

a) Hibátlanul sorolta fel azokat a síkidomokat, amelyeket csak egyenes vonal határol. (fő)	24
b) Hibátlanul sorolta fel a négyszögeket. (fő)	21
c) A téglalapokat helyesen határozta meg, tehát a négyzeteket is ide sorolta. (fő)	1
Nem sorolta a téglalapokhoz a négyzeteket, de más hibája nem volt. (fő)	5
Nem sorolta a téglalapokhoz a négyzeteket, az általános paralelogrammát viszont igen. (fő)	18
d) Hibátlanul sorolta fel a négyzeteket. (fő)	14
A csúcsára állított négyzetet kihagyta a felsorolásból. (fő)	7

A mért adatokból kiderült, hogy a téglalap és a négyzet esetén a fogalomazonosítás további fejlesztést igényel. A tanulók közel 20%-a nem sorolta a négyzeteket a téglalapokhoz, de más hibája nem volt. További közel 70%-uk szintén nem sorolta a négyzeteket a téglalapokhoz, az általános paralelogrammát viszont igen. A gyerekek majdnem 90%-a a négyzetet nem tartotta téglalapnak. Ez a Hiele-féle első két szintnek teljesen megfelel. Érdekes viszont, hogy a csúcsára állított négyzetben nem ismerte fel a négyzetet a gyerekek 27%-a. Az előforduló hibák azt jelezték, hogy a továbbiakban a helyes fogalomalakulás érdekében nagy hangsúlyt kell fektetnem a megfelelő példák, ellenpéldák bemutatására, megbeszélésére. Megfelelőn egyrészt a példák, ellenpéldák kellő mennyiségét, másrészt a változatosságukat értem (pl.: téglalappal, négyzettel különböző helyzetekben is találkozzanak). Továbbá, hogy általuk a fogalomhoz tartozó lényeges jegyek felismerhetők, a lényegtelenek pedig kiszűrhetők legyenek a gyerekek számára.

Az órák tervezése során azt tartottam szem előtt, hogy a gyerekek előbb konkrét tapasztalatok alapján, valóságos játékok keretében, tárgyi tevékenykedés közben, majd vizuális síkon (rajzolás), végül szimbolikus síkon (beszélt illetve írott nyelv) fedezzék fel az elsajátítandó geometriai fogalmakat.

Feladatok konkrét tárgyi tevékenységre:

Pl.: 1. Hogyan lehet a legegyszerűbben téglalapból négyzetet kivágni?

2. Téglalapot, illetve négyzetet egyik átlója mentén szétvágjuk 2 háromszögre, majd ezekből újabb síkidomok alkotása.

3. Papírcsíkból egy-egy egyenes vágással különböző síkidomok előállítása és megnevezése.

4. Papírcsíkból különböző hosszúságú, de adott magasságú téglalapok létrehozása. Stb.

Ez utóbbi feladat végrehajtása közben egy Bence nevű fiú kétségbeesetten mondta, hogy:

„Az egyiknél mindegyik oldal ugyanakkora lett. Megmértem, 4 cm mindegyik oldal, ez egy négyzet. Ez így nem lesz jó."

A tanítónő a következőképpen reagált:

„Hát ez így sikerült. Ilyen különleges téglalapot kaptál. A négyzet látod egy különleges téglalap. Jó a megoldásod."

Bence nagy sóhajtással megkönnyebbült. A tanítónő Bencének ezt a téglalapját az osztálynak is megmutatta. Ekkor kiderült, hogy nem ő volt az egyetlen, akinek így sikerült levágni egy téglalapot. Az előbbieket az osztálynak is elismételte a tanítónő. Örültünk, hogy lehetőségünk nyílt ezáltal rávilágítani a négyzet és a téglalap „rokonságára".

Néhány feladat vizuális síkon:

Pl.: 1. Pontrácson négyzetek, téglalapok alkotása.
2. Pontrácson különböző négyszögek alkotása.
3. Pontrácson különböző háromszögek alkotása.
4. Adott tulajdonságú négyszögek rajzolása. Stb.

A különböző geometriai alakzatok tulajdonságainak megbeszélése már szimbolikus síkon zajlott: sokszög, négyszög, téglalap, négyzet esetén az ol-

dalak, csúcsok számának megállapítása; oldalak hosszúságának, párhuzamosságának, merőlegességének vizsgálata; szimmetriatengelyek számának, a szomszédos oldalak által bezárt szögek nagyságának meghatározása. A négyzet és a téglalap tulajdonságainak összehasonlítása is megtörtént. A tanítónőnek arra a kérdésére, hogy a téglalap minden tulajdonsága igaz-e a négyzetre, többségében nemleges volt a válasz. Két tanuló gondolta úgy, hogy ez igaz. A tanítónő ugyan igyekezett a gyerekeknek ezt megmagyarázni, de többen közölték a szakvezetővel, hogy nincs igaza, mert „a téglalapnak két különböző hosszúságú oldala is van, a négyzetnek pedig nincs". Természetesen a geometriai tulajdonságok vizsgálata mindig az adott alakzat képi megjelenítésével együtt történt.

A fejlesztő tanítási kísérletet egy felmérő feladatlappal zártam. A feladatlapot a 4.c osztályban 25 fő írta meg. Kérésemre a 4.a osztályban 23 tanuló és a 4.b osztályban is 24 tanuló kitöltötte a feladatlapot. Ebben a két osztályban másik szakvezető oktatta a matematikát.

A feladatlapnak csak a négyzet és a téglalap fogalomalakulásával kapcsolatos feladatait ismertetem.

A négyszög, téglalap, négyzet fogalmak azonosításával kapcsolatos feladatban 15 síkidom közül kellett kiválasztani a négyszögeket (a), a téglalapokat (b), illetve a négyzeteket (c).

A feladat megoldásának eredményességét a következő táblázatban foglalom össze:

	4.c	4.a	4.b
a) A négyszögeket hibátlanul sorolta fel. (fő)	25	16	20
b) A téglalapokat helyesen határozta meg, tehát a négyzeteket is ide sorolta. (fő)	2	2	2
A négyzeteket nem sorolta a téglalapokhoz, de más hibája nem volt. (fő)	13	11	3
Nem sorolta a téglalapokhoz a négyzeteket, az általános paralelogrammákat viszont igen. (fő)	10	10	19
c) A négyzeteket hibátlanul sorolta fel. (fő)	20	9	14
A csúcsára állított négyzet(ek)et kihagyta a felsorolásból. (fő)	5	10	9

A négyszögek felismerése a kísérleti osztályban hibátlan volt. A tanulók 52%-a nem sorolta a négyzeteket a téglalapokhoz, de más hibája nem volt. A preteszt 20%-ához képest jelentős ez a javulás. A tanulók 40%-a nem sorolta a négyzetet a téglalapokhoz, az általános paralelogrammákat viszont igen. A korábbi 70%-hoz képest ugyan jelentős ez a javulás, de magasnak tartom ezt az arányt is. A gyerekek 90%-a változatlanul nem tartja téglalapnak a négyzetet. Szinte ugyanez az arány a kontroll csoportokban is. Ezek az adatok igazolják a hipotézisemet, miszerint a geometriai gondolkodás Van Hiele-féle 3. szintjére nem lehet átlépni az alsó tagozat végére, csak az első két szint megvalósítása reális.

A négyzetek felismerésében is pozitív irányban történt a változás. A gyerekek 80%-a (a korábbi 58%-kal szemben) helyesen sorolta fel a négyzeteket.

A felmérő feladatlapnak a négyzet és a téglalap tulajdonságaira vonatkozó feladatában a megadott állítások közül a gyerekeknek azokat kellett aláhúzniuk, amelyek igazak a négyzetre, illetve a feladat második részében a téglalapra. A feladat megoldásának értékelésénél csak a hibátlan teljesítményeket emelem ki. A kísérleti csoportban a négyzet tulajdonságaira vonatkozó valamennyi igaz állítást helyesen állapította meg a tanulók 52%-a, míg a kontroll csoportokban 35%-a, illetve 42%-a. A hibák forrását egyrészt a szemközti, illetve szomszédos szavak nem megfelelő értelmezésében kell keresnünk, másrészt abban, hogy a párhuzamosság és a merőlegesség fogalma még nem elég stabil.

A felmérő feladatlap utolsó feladata is a hipotézisemet támasztotta alá. Ebben a feladatban a következő állítások logikai értékét kellett eldönteni:

A téglalap egy különleges négyzet.

A négyzet egy különleges téglalap.

A négyzet oldalai nem egyenlők.

Minden négyzet téglalap is.

A feladat megoldásának értékelése a következő:

	4.c	4.a	4.b
Mindegyik állítás logikai értékét hibátlanul határozta meg. (fő)	6	4	5
„A téglalap egy különleges négyzet." állítás hamis logikai értékét helyesen állapította meg. (fő)	16	12	14
„A négyzet egy különleges téglalap." állítás igaz logikai értékét jól határozta meg. (fő)	12	9	11
„ A négyzet oldalai nem egyenlők." állítás hamis logikai értékét helyesen jelölte. (fő)	24	19	23
„Minden négyzet téglalap is." állítás igaz logikai értékét helyesen állapította meg. (fő)	12	11	11
„A négyzet egy különleges téglalap." és a „Minden négyzet téglalap is." állításokat különböző logikai értékkel jelölte. (fő)	10	10	11
„A négyzet egy különleges téglalap." és a „Minden négyzet téglalap is." állítások logikai értékét igaznak vette. (fő)	7	5	6
„A négyzet egy különleges téglalap." és a „Minden négyzet téglalap is." állítások logikai értékét hamisnak tartotta. (fő)	8	8	8

Ebben a feladatban a hibátlan megoldások száma mindegyik tanuló csoport esetén alacsony. Ezen persze nem lehet csodálkozni, hiszen a négy állítás közül három a négyzet és a téglalap közötti hierarchiára vonatkozott. Abban sem lehetünk biztosak, hogy azok a gyerekek, akik a második és a negyedik állítást is igaznak tartották, valóban tisztában vannak a négyzet és a téglalap között fennálló részhalmaz viszonnyal. Akik különböző logikai értékűnek tartották ezeket az állításokat (a kísérleti csoportban a tanulók 40%-a, a kontroll csoportokban 43%-a, illetve 46%-a), egymásnak ellentmondó döntéseket hoztak, ami a négyzet és téglalap fogalomosztályok közötti kapcsolat bizonytalan megítélésére utal. Azok pedig, akik mindkettőt hamisnak vélték (a tanulók 32%-a, 34%-a, 35%-a), valóban nem látnak semmilyen hierarchiát.

Úgy gondolom, hogy az általam irányított fejlesztő tanítás hatékonyan járult hozzá a téglalap és a négyzet fogalmának elmélyüléséhez. A preteszt és a záró feladatlap eredményeinek összehasonlítása is ezt igazolta. A hatékonyságot támasztja alá az a tény is, hogy a másik két párhuzamos osztályban tapasztalt eredményekhez képest általában jobb, esetenként lényegesen jobb eredmények születtek a kísérleti csoportban.

Befejezésként Pólya György gondolatát idézem: „Nem szabad semmi olyat elmulasztani, aminek esélye van arra, hogy a diákokhoz közelebb hozza a matematikát. A matematika nagyon absztrakt tudomány – éppen ezért nagyon konkrétan kell előadni." (1977.)

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PRIMJENA RAČUNALA U NASTAVI MATEMATIKE

Sanja Varošanec [1]

Sažetak. *Prirodna je situacija da suvremena nastava matematike prati razvoj tehnologi-je, te nastoji u obrazovni proces uvesti nova nastavna sredstva kako bi se učenicima približila matematika, motiviralo ih se na rad, poboljšalo razumijevanje, otkrivanje i usvajanje matematičkih pojmova, pojava i zakonitosti. Kao što su tijekom prošlih go-dina u nastavni proces kao pomagala ušli grafoskopi, dijaskopi, episkopi, magnetofo-ni i dr., tako smo danas svjedoci sve češćeg poučavanja i učenja uz pomoć računala, pripadnih vanjskih jedinica i programske podrške. Kao i kod svakog drugog nastav-nog sredstva tako i uporaba računala ima svoje prednosti, ali i nedostatke. Pri dono-šenju odluke kada, gdje, kako i zašto koristiti novu tehnologiju, nastavnik se rukovodi ovim osnovnim načelima:*

- *odluka o tome kada i kako uporabiti ili ne uporabiti računalo ovisi o tome unapređuje li ta uporaba postojeću nastavnu praksu,*
- *odluka mora biti direktno uvjetovana procjenom omogućava li uporaba računala efikasnije ostvarivanje ciljeva pojedine nastavne jedinice,*
- *uporaba računala mora omogućiti i učitelju i učenicima da postignu nešto što ne bi mogli postići bez uporabe računala, odnosno učiteljima mora omogućiti poučavanje, a učenicima učenje efikasnije nego bez ove tehnologije.*

Budući da se u nastavi matematike u višim razredima osnovne škole u velikoj mjeri obrađuju geometrijski sadržaji osvrnut ću se na uporabu programa dinamičke geome-trije. Radi se o alatu koji učitelju i učenicima otvara novi pogled na tradicionalne geometrijske sadržaje, te pomoću kojeg metoda istraživanja i eksperimenta dobiva novo, značajnije mjesto u nastavi matematike.

Ključne riječi: *nastavno sredstvo, računalo, program dinamičke geometrije.*

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Prirodna je situacija da suvremena nastava matematike prati razvoj tehnologije, te nastoji u obrazovni proces uvesti nova nastavna sredstva kako bi se učenicima približila matematika, motiviralo ih se na rad, poboljšalo razumijevanje, otkrivanje i usvajanje matematičkih pojmova, pojava i zakonitosti. Kao što su tijekom prošlih godina u nastavni proces kao pomagala ušli grafoskopi, dijaskopi, episkopi, magnetofoni i dr., tako smo danas svjedoci sve češćeg poučavanja i učenja uz pomoć računala, pripadnih vanjskih jedinica i programske podrške. Škole u Hrvatskoj se ubrzano opremaju osobnim računalima, laptopima, raznim vanjskim jedinicama kao što su printeri, projektori, skeneri. Učenici posjeduju džepne kalkulatore, a sve je češća uporaba grafičkih kalkulatora. Sve je to podržano različitim vrstama programa, od koji mnogi imaju edukacijsku komponentu. Pri odabiru pojedine programske podrške, nastavnik mora razmotriti nekoliko pitanja:

- može li konkretni software pomoći podučavanju matematike, povećanju razine znanja, razvoju određenih vještina i poboljšanju razumijevanja matematičkih ideja,
- može li nam konkretni software pomoći pri radu s matematičkim sadržajima, tj. može li nama ili učenicima pomoći pri računanju, crtanju grafova, kreiranju tablica, rješavanju problema, transformiranju izraza i sličnim radnjama,
- pomaže li uporaba konkretnog software-a u izradi nastavnih materijala, čuvanju podataka, pronalaženju već postojećih nastavnih materijala itd.

Opći programski alati s kojima se susrećemo u nastavi su: programi za obradu teksta, programi za rad s tablicama, prezentacijski programi, programski jezici itd. Neki od specijaliziranih programskih alata namijenjenih upravo matematičkoj edukaciji su:

- alati dinamičke geometrije poput The Geometer's Sketchpad, Geogebra, Cinderella, Cabri Geometry,
- grafički alati (napr. Winplot, Dplot, Visio),
- profesionalni matematički programski sustavi (napr. Mathematica, Maple, Derive).

Računala, ili bolje rečeno informacijsko-komunikacijska tehnologija (ICT) u nastavnom se procesu koriste u nekoliko situacija: nastavnik ih koristi pri planiranju i pripremanju za nastavu i rad u školi; učenik kao pojedinac ih koristi van vremena provedenog u školi; nastavnik ih koristi pri radu sa cijelim razredom; grupa učenika ih koristi tijekom rada na školskom satu.

Pri planiranju i pripremanju za nastavni sat nastavnik treba iskoristiti mogućnost pristupa raznim vrstama nastavnog i popratnog materijala. Naime, na internetu i lokalnim elektronskim medijima postoji niz informacija i materijala koje će obogatiti dio sata namijenjen motiviranju učenika, obradi novog gradiva i uvježbavanju obrađenog gradiva. Nastavnik će preuzeti one elektronske sadržaje koji će mu omogućiti efikasnije podučavanje i uvježbavanje nastavnog gradiva. Ne treba zanemariti ni mogućnost bržeg i racionalnijeg stvaranja i čuvanja dokumentacije vezane uz nastavni proces (planovi i programi – opći i za učenike s posebnim potrebama, pisane pripreme, kontrolne zadaće i ispiti znanja, statistike vezane uz vođenje razrednog odjela i drugih školskih i vanškolskih aktivnosti i sl). Također, pomoću ICT nastavnik lako dolazi u kontakt sa sustručnjacima matematičarima, s kolegama nastavnicima, s članovima pedagoško – psihološkog tima i ostalim osobama koje nastavniku mogu prenijeti korisna iskustva i pružiti savjet vezan uz izvođenje nastave matematike ili rad s učenicima. Da bi se ovakav pristup ostvario nastavnik mora ili imati vlastito računalo ili mora imati pristup računalu van školskog vremena. U Hrvatskoj već neko vrijeme traje akcija informatičkog opismenjavanja nastavnika u sklopu koje svaki nastavnik dobiva mogućnost pristupa internetu.

S druge strane poželjno je da i učenik ima pristup računalu van školskog vremena i to bilo kod kuće bilo u školi van nastave ili na nekom drugom pogodnom mjestu (klubovi, knjižnice itd). Pri tome, učenik koristi računala za izradu domaćih zadataka, seminarskih i projektnih radova, za zabavu itd.

Osvrnimo se i na situaciju kad nastavnik koristi računalo pri radu s cijelim razredom. Svjedoci smo opremanja škola sve većim brojem računala, tako da je realno očekivati da će nastavnik biti u mogućnosti organizirati nastavni sat iz matematike u specijaliziranoj učionici opremljenoj računalima, tj. u informatičkoj učionici. U takvoj će učionici nastavnik organizirati obradu i uvježbavanje gradiva tako da svaki učenik ili par učenika radi samostalno za računalom. No, nije ovakav oblik rada rezerviran samo za rad u informatičkoj učionici. Danas je uporaba džepnih kalkulatora postala uobičajena stvar u srednjoškolskoj matematici, a prema, prošle godine uvedenim promjenama, tako će biti i u osnovnoj školi. U bliskoj budućnosti očekuje nas situacija kad ćemo, koristeći džepnu tehnologiju, rad na računalima moći implementirati u svaki nastavni sat koji to dopušta po svom karakteru neovisno o tome imamo li mogućnost rada u specijaliziranoj učionici ili ne. Osim ovakvog individualnog rada na računalima, treba istaknuti i mogućnosti računala kao demonstracijskog nastavnog sred-

stva. Koristeći PC s projektorom, interaktivnu ploču i slični alat, nastavnik je u mogućnosti cijelom razredu prezentirati neki matematički sadržaj, demonstrirati izvjesnu pojavu i/ili zakonitost.

Ponekad je na nastavnom satu moguće organizirati upotrebu nekoliko računala, ali u nedovoljnom broju za sve učenike. Česta je situacija da u razredu gdje se odvija nastava postoji par računala. Tada nastavnik može pripremiti materijal koji će omogućiti grupi učenika da odradi dio sata na računalu. To mogu biti učenici s posebnim potrebama, daroviti učenici ili, općenito, grupa učenika za koje smo pripremili poseban nastavni materijal .

Kao i svako drugo nastavno sredstvo tako i uporaba računala ima svoje prednosti, ali i nedostatke. Pri donošenju odluke kada, gdje, kako i zašto koristiti novu tehnologiju, nastavnik se rukovodi ovim osnovnim načelima:

- odluka o tome kada i kako uporabiti ili ne uporabiti računalo ovisi o tome unapređuje li ta uporaba postojeću nastavnu praksu,
- odluka mora biti direktno uvjetovana procjenom omogućava li uporaba računala efikasnije ostvarivanje ciljeva pojedine nastavne jedinice,
- uporaba računala mora omogućiti i učitelju i učenicima da postignu nešto što ne bi mogli postići bez uporabe računala, odnosno učiteljima mora omogućiti poučavanje, a učenicima učenje efikasnije nego bez ove tehnologije.

Tako nećemo upotrebljavati kalkulator dok se ne svlada tablica zbrajanja i množenja, nećemo upotrebljavati software dinamičke geometrije ako učenici još nisu svladali upotrebu uobičajenog geometrijskog pribora.

Opišimo jednu situaciju gdje je uporaba računala opravdana. U 7. razredu osnovne škole obrađuje se proporcionalnost i ciljevi te nastavne teme su usvajanje koncepta proporcionalnosti i primjena matematičkog postupka u zadacima iz svakidašnjice. I dok se prvi cilj može ostvariti proučavanjem različitih primjera proporcionalnih veličina gdje se obično ograničavamo na rad s prirodnim odnosno cijelim brojevima, drugi cilj će se efikasnije ostvariti dozvolimo li na tim satima uporabu kalkulatora. Naime, nakon što usvojimo ideju proporcionalnosti, primjenjujemo je na zadacima koje crpimo iz realnog svijeta. A u njima su i podaci realistični, dakle, vrlo često ne radi se o prirodnim i cijelim brojevima. No, dijelu učenika račun s razlomcima i decimalnim brojevima još

uvijek nije postao automatizirana procedura te ako pri rješavanju zadataka zabranimo upotrebu kalkulatora ti učenici se suočavaju s nemogućnošću izvedbe točnog računa (prvo pri izračunu faktora proporcionalnosti, a zatim i pri izračunu nepoznate veličine). Drugim riječima, takav učenik ne može uspješno izraditi zadatak u kojem mi, u biti, ispitujemo dvije stvari: je li usvojio ideju proporcionalnosti i zna li izvesti račun.

Dakle, u temama gdje je fokus na analizi problema, a ne na računu potrebnom pri rješavanju tog problema koristit ćemo računala. U takvim situacijama primjena računala omogućava slabijim učenicima preskakanje nekih (za tu temu) manje važnih postupaka i koncentriranje na usvajanje određenog matematičkog koncepta.

Slična se situacija pojavljuje pri ispitivanju nekog geometrijskog svojstva. Umjesto da vrijeme trošimo na crtanje par posebnih slučajeva na temelju kojih ćemo pokušati učenike dovesti do zaključka, uporabom programa dinamičke geometrije fokus sata se prebacuje na analizu i izvođenje željenih zaključaka.

Upotreba računala značajno doprinosi učenju matematike pomažući učenicima pri

- uvježbavanju računanja,
- eksperimentiranju, stvaranju hipoteza koje se odnose na svojstva geometrijskih likova, funkcija i brojeva,
- radu s realističnim podacima i s većim skupovima podataka,
- razvijanju logičkog mišljenja, stvaranju i modificiranju strategija rješavanja omogućenim brzom povratnom informacijom,
- učenju pomoću slika (princip zornosti),
- razvijanju vještina i sposobnosti matematičkog modeliranja na temelju danih podataka.

Kratko rečeno, uporaba računala omogućava učenicima da se koncentriraju na promišljanje o matematičkim idejama, na rješavanje problema na način koji je lakši i efikasniji nego bez tih alata. Tehnologija obogaćuje učenje matematike dozvoljavajući učeniku istraživanje i otkrivanje, a proširuje i vrste problema koji se mogu proučavati.

U nastavi matematike u osnovnoj školi u velikoj se mjeri obrađuju geometrijski sadržaji. Zato ću se osvrnuti na uporabu programa dinamičke geome-

trije. To su računalni programi koji su prvenstveno namijenjeni proučavanju i rješavanju planimetrijskih i stereometrijskih problema. Radi se o alatu koji nastavniku i učenicima otvara novi pogled na tradicionalne geometrijske sadržaje, te pomoću kojeg metoda istraživanja i eksperimenta dobiva novo, značajnije mjesto u nastavi matematike. U ovom trenutku postoje dva programa dinamičke geometrije koji su lokalizirani, tj. prevedeni na hrvatski jezik i koji okupljaju širu matematičku zajednicu oko izrade nastavnih materijala. To su Geogebra i Sketchpad. Oba programa karakterizira mogućnost lakog mijenjanja položaja ucrtanih objekata dok odnosi među njima ostaju nepromijenjeni. Programi animiraju statičnu geometrijsku konstrukciju u pomičnu, dinamičnu sliku koja otkriva nove odnose među geometrijskim objektima koje je možda teško otkriti na klasičnim, statičnim crtežima. Pokazalo se da učenicima viših razreda pružaju izvrsnu motivaciju za učenje matematike i razvijanje interesa za predmet. Sljedećim je primjerom opisana primjena Sketchpada na jednom nastavnom satu matematike u 6. razredu osnovne škole provedenom u informatičkoj učionici.

Primjer. ***Zbroj kutova u trokutu***

Radi se o nastavnoj jedinici 6. razreda osnovne škole. Cilj nastavne jedinice je dokazati i usvojiti tvrdnju da je zbroj veličina kutova u trokutu 180°. Učenici znaju pojam kuta, trokuta, vršnih kutova, kutova uz presječnicu, znaju svojstva vršnih kutova i kutova uz presječnicu. Kroz uvodno ponavljanje provedeno na početku sata učenici se podsjete tih pojmova i svojstava.

Središnji dio sata (20 min) posvećen je eksperimentiranju pomoću Sketchpada. Učenici provode na računalu konstrukciju opisanu na nastavnom listiću na koji zapisuju i svoje zaključke.

NASTAVNI LISTIĆ

1. Nacrtaj trokut ABC.
2. Izmjeri kutove ABC, BCA i CAB.
3. Što primjećuješ vezano uz kutove?_____

_____

4. Zbroji sve veličine kutove tog trokuta. Upiši rezultat na crtu. _____
5. Pomakni točku A.
6. Što se dešava s veličinom kutova trokuta ABC?_____
7. Što se dešava sa zbrojem veličina kutova?

_____

8. Pomakni točke B i C.
9. Što se dešava s veličinom kutova trokuta ABC?_____
10. Što se dešava sa zbrojem veličina kutova? _____
11. Pokušaj zapisati tvrdnju._____

_____

_____

12. Točkom C povuci paralelu sa stranicom BC.
13. Povuci pravce AC i BC.
14. Na svakom od polupravaca s vrhom u C koji ne sadrže druge točke trokuta označi po jednu točku – nazovi ih E, F, G, H.
15. Izmjeri kutove ECF, FCG i GCH.
16. Što primjećuješ? Pojavljuju li se na slici sukladni kutovi?

_____

_____

17. Objasni zašto se to dešava?

_____

_____

18. Promijeni položaj točke C. Što primjećuješ?

_____

_____

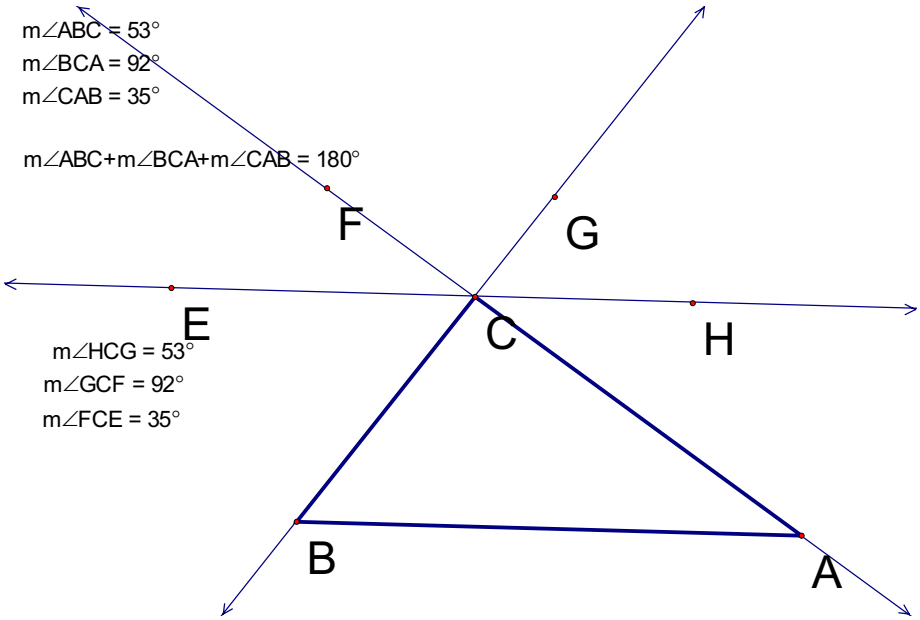
19. Bez izvođenja operacije zbrajanja izračunaj koliki je zbroj veličina kutova ECF, FCG i GCH? Objasni zašto.

_____

_____

Nakon izvođenja eksperimenta slijedi prezentacija i analiza rješenja. Na kraju se izvodi i zapisuje zaključak o zbroju kutova u trokutu i dokaz tog poučka. Novootkriveno znanje se primjenjuje na primjeru pravokutnog i jednakokračnog trokuta.

Slika 1. Izgled ekrana računala po završetku eksperimenta.



OD DEJAVNEGA EKSPERIMENTIRANJA DO ABSTRAKTNIH POJMOVNIH PREDSTAV

Dr. Amalija Žakelj[1], Dr. Aco Cankar[2]

Povzetek. *V članku predstavljamo nekatere didaktične vidike procesno-didaktičnega pristopa učenja in poučevanja, ki smo ga razvili v raziskavi Procesno-didaktični pristop in razumevanje matematičnih pojmov v osnovni šoli (Žakelj, 2004), ki je potekala v letih 2001/02 in 2002/2003 z učenci starimi od 12 do 15 let. Glavno znanstveno vprašanje je bilo, raziskati povezavo med otrokovim mišljenjem (kognitivnimi strukturami, metakognicijo, miselnimi strategijami) in pristopi v poučevanju matematike. Na osnovi tega smo izdelali in preizkusili procesno-didaktični pristop za poučevanje matematike. Preverili smo ga z vidika učinka kakovosti in vrste znanja.*

Didaktični model smo zasnovali na osnovi teoretskega poznavanja miselnega razvoja otrok, vključno z novejšimi spoznanji o otrokovem mišljenju ter poznavanja socialne kognicije. Pri tem smo se oprli na teorijo razvojne psihologije, ki preučuje razvoj pojmov glede na razvojno stopnjo otrokovega mišljenja, ter upoštevali novejša kognitivno-konstruktivistična spoznanja pedagoške stroke o učenju, ki poudarjajo dejavnost učenca v procesu učenja.

Ključne besede: *pouk matematike, dejavni eksperiment.*

IZKUŠENJSKO UČENJE

Pri postavitvi procesno-didaktičnega pristopa smo upoštevali, da na proces učenja bistveno vplivajo razvojna stopnja mišljenja, struktura obstoječega znanja ter organizacija dejavnosti učenca oz. spodbude iz okolja. Hkrati pa smo

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otrokovo mišljenje interpretirali s stališča novejših spoznanj o metakogniciji ter v povezavi mišljenja in jezika.

Model v praksi uvaja izkustveno učenje, dialog ter različne oblike sodelovanja (*vpliv socialnih interakcij*) pomembno vlogo pri konstrukciji znanja. Izkustveno učenje vključuje: *modeliranje, dejavno eksperimentiranje, samostojno iskanje virov, iskanje podobnosti in povezav, iskanje primerov in nasprotnih primerov, spodbuja razvoj problemskih znanj* (reševanje odprtih problemov, razumevanje problemske situacije, postavljanje vprašanj, učenje strategij pri reševanju problemov, postavljanje ugotovitev, predstavitev rezultatov, utemeljevanje) ter uvaja oblike *uporabnosti matematike* na drugih področjih in *povezovanje znanja. Pri reševanju problemov je poudarek na procesih oz. strategijah reševanja*, na utemeljevanju, preverjanju rešitev, predstavitvi rezultatov, izmenjavi mnenj. Učitelj v kognitivnem in socialno-kognitivnem konfliktu spodbuja motivacijo. Učiteljeva vloga je odločilna tudi v tem, da učencem ponudi različne pristope učenja. Katere bo uporabil, je odvisno od predznanja učencev, njihove kognitivne zrelosti, naravnanosti učencev, njihovih učnih stilov.

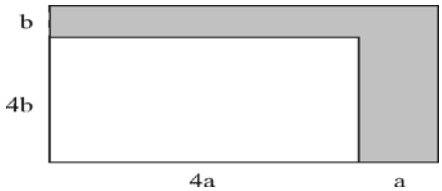
Kaj je potrebno upoštevati pri uvajanje novih pojmov?

Nalogi, ki ju navajamo v nadaljevanju ilustrirata, da je uspešnost učencev pri reševanju problemov odvisna od učenčevih pojmovnih predstav ter od njegove kognitivne zrelosti.

Naloga 1: Kdaj sta dve količini premo sorazmerni? Naštej nekaj primerov.
Didaktični vidik:Izkazovanje razumevanja pojma premo sorazmerje z navajanjem primerov.

Nalogo 1 smo testirali v raziskavi (Žakelj, 2004), v kateri smo med drugim raziskovali tudi razumevanje pojma premo sorazmerni količini. Kar 77 odstotkov učencev je na vprašanje odgovorilo pravilno, vendar pa je le 44 odstotkov učencev znalo navesti primere premo sorazmernih količin. Čeprav je bila celica definicije polna, pa je bila celica pojmovne predstave pri učencih, ki niso znali navesti nobenega primera prazna ali pa napolnjena z napačnimi predstavami. Celica pojmovne predstave pa se napolni v procesu pridobivanja izkušenj.

Naloga 2: Izračunaj koliko odstotkov lika je pobarvanega.
Didaktični vidik: reševanje problema na simbolni ravni.



Slika 1

Tudi nalogo 2 smo testirali v raziskavi (Žakelj, 2004), s katero smo pri dvanajstletnikih testirali sposobnost reševanje problema na simbolni ravni. Redki učenci so nalogo rešili v celoti oz. matematično povsem korektno. Večina učencev je nalogo rešilo tako, da so si *sami izbrali konkretne podatke*, nekateri so jih *določili z merjenjem*, drugi so narisali mrežo ter *določili ploščinsko enoto* ali pa so do približnega rezultata prišli *tudi z ocenjevanjem*. Različni pristopi učencev pri reševanju kažejo *povezavo med kognitivnim razvojem učenca, izkušnjami*, ki jih ima učenec, ter načinom reševanja nalog.

Iz načinov reševanja prve in druge naloge lahko med drugim sklepamo, da je zelo pomembno, da učitelj pri izbiri pristopov poučevanja ter ravni zahtevnosti nalog izbira različne reprezentacije pojmov, s čimer uvaja pojme postopoma in se tako smiselno prilagaja kognitivni zrelosti učencev in posledično vliva na njihov razvoj. V prvem primeru so učenci imeli premalo izkušenj s pridobivanjem pojmovnih predstav, saj poleg definicije veliko učencev ni znalo navesti nobenega primera, kar kaže, na njihove šibke pojmovne predstave, ki so poleg definicije del pojmovne zgradbe. Drugi primer pa kaže, da morajo učenci prehoditi postopoma vse faze: od konkretnih izkušenj, slikovnega in simbolnega nivoja, do abstraktne ravni. Nepremišljeno in prehitro uvajanje zahtevnih abstraktnih pojmov, ki prehiteva njihov kognitivni razvoj, je z vidika izgrajevanja pojmovnih predstav zanje izredno težko in pogosto tudi neučinkovito.

Od dejavnega eksperimentiranja do abstraktne konceptualizacije

Razvoj miselnih predstav in razumevanje matematičnih pojmov je pri razumevanju matematike poglavitnega pomena za konstrukcijo znanja in je tudi pogoj za transfer znanja. Nemogoče bi bilo, učiti se nove strategije za vsak problem. Vendar se žal pogosto pokaže, da veliko učencev ne more delati povezav oz. reprezentirajo informacije kot izolirane dele. Če isto snov slišijo pri dveh različnih predmetih, je ne znajo povezati, ampak imajo dve "ločeni znanji". Enako se dogaja pri povezovanju pojmov znotraj matematike. Če učenec

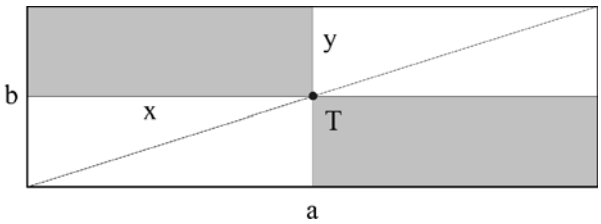
razume razmerje kot deljenje dveh celih števil, ne bo znal tega pojma uporabiti pri učenju premega ali obratnega sorazmerja. Zato je zelo pomembno, kako se učimo. Postopen prehod od konkretnih reprezentacij pojmov, prek slikovnih in simbolnih, do abstraktne konceptualizacije, je nujen pri graditvi pojmovnih predstav. Je naraven, ker tudi sicer sledi fazam kognitivnega razvoja. Učenje z izkušnjo vse te faze predvideva: dejavno eksperimentiranje, raziskovanje, ugotavljanje lastnosti ob konkretnem modelu, pridobivanje konkretnih izkušenj, razmišljajoče opazovanje, ki v zadnji fazi lahko pripeljejo do abstraktne konceptualizacije pojma.

Induktivni in deduktivni pristop reševanja problemov

Učenci, za katere je prav, da najprej poiščejo teorijo, princip, pravilo, rešujejo probleme deduktivno. Če pa težko prehajajo od splošnega h konkretnemu, probleme rešujejo uspešneje po induktivni poti.

Navajamo primer, pri katerem lahko sežemo do različnih ravni (od osnovnošolske ravni do ravni srednje šole). Od ravni in obsega znanja so odvisni tudi pristopi pri reševanju. Izziv o raziskovanju ploščine osenčenih likov lahko učenci v osnovni šoli rešijo z merjenem, zbiranjem delnih rešitev ter na podlagi zbranih podatkov postavijo hipotezo. Seveda se moramo pri takem pristopu z učenci pogovoriti, da so rešitve oz. ugotovitve, pridobljene z merjenjem, lahko le približna ocena, saj smo z merjenjem lahko prišli do le deloma točnih podatkov oz. ugotovitev. Postavljenim ugotovitvam vedno sledi utemeljevanje. Utemeljevanje je na osnovnošolskem nivoju pogosto le opisno, lahko s pomočjo primerov. V danem primeru pa tudi že na ravni osnovne šole, lahko ugotovitve utemeljimo s pomočjo matematičnih dejstev o skladnosti likov oz. ploščinsko enakih likih. V srednji šoli lahko to nalogo povežemo z reševanjem ekstremalnih problemov, z uporabo odvoda.

Izziv 1: V poljubni točki T na diagonali pravokotnika s stranicama a in b narišemo vzporednici k obema stranicama. Razišči ploščini osenčenih pravokotnikov na sliki.



Slika 2

Didaktični vidik izziva

Pri delu učence spodbujamo, naj raziskujejo, opazujejo, merijo, primerjajo, postavljajo hipoteze. Pri tem povezujejo ter uporabljajo pojme o podobnosti, skladnosti in ploščinsko enakih likih. Za učence je to lahko ena od priložnosti, da si dopolnijo ali popravijo morebitne napačne pojmovne predstave. Z didaktičnega stališča je taka naloga koristna pri graditvi predstav o pojmu ploščina, ploščinsko enakih likih, skladnih likih, podobnih likih.

Dejavnosti učencev

Problem zahteva produktivno uporabo matematičnega znanja: ploščine pravokotnika, izrekov o skladnosti in podobnosti trikotnikov, povezovanje znanja in analiziranja dane problemske situacije.

1. Razmislek o izzivu in postavitev vprašanja

Uvid v problem lahko omogoči pozorno opazovanje slike in ugotavljanje odnosov med geometrijskimi elementi. Npr. ob opazovanju učenci ugotovijo: Ploščini osenčenih pravokotnikov se z gibanjem točke T po diagonali spreminjata. Sledijo vprašanja:

Postavitev vprašanja

Kako se spreminjata ploščini osenčenih pravokotnikov, če drsimo s točko T po diagonali?

Kolikšno je razmerje med ploščinama obeh osenčenih pravokotnikov?

Kolikšno je razmerje med vsoto ploščin obeh osenčenih pravokotnikov in ploščino celotnega pravokotnika?

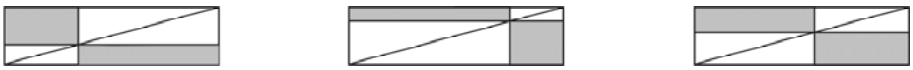
Pri kateri legi točke T je vsota ploščin obeh osenčenih pravokotnikov največja?

2. Izvedba

Poti reševanja je lahko več. Nakažimo dva pristopa.

a) Induktivni pristop – z merjenjem

Do potrebnih podatkov lahko pridemo tudi z merjenjem in računanjem.



Slika 3

Narišemo nekaj različnih situacij: točka T se nahaja npr. blizu oglišča, na sredini diagonale … Za izbrane konkretne primere z merjenjem določimo dolžine stranic, ki jih potrebujemo za izračun ploščin osenčenih likov. Na podlagi izmerjenih podatkov izračunamo ploščine. Rezultate merjenja je smiselno zapisati in urediti v tabelo. Tudi zaradi sistematično urejenih podatkov hitreje vidimo rešitev.

V tabeli so podatki splošno zapisani, učenec pa bo seveda delal s konkretnimi podatki.

Tabela 1: Ploščini osenčenih pravokotnikov pri gibanju točke T po diagonali

Delitev stranice pravokotnika na n delov	x	y	Ploščina prvega in drugega osenčenega pravokotnika	Vsota ploščin obeh osenčenih pravokotnikov	Razmerje med vsoto ploščin obeh osenčenih pravokotnikov in ploščino celotnega pravokotnika
	0	b	0	0	
8	a/8	7b/8	7ab/64 7ab/64	14ab/64	$14/64 = 2 \cdot 7/8^2$
4	a/4	3b/4	3ab/16 3ab/16	6ab/16	$6/16 = 2 \cdot 3/4^2$
2	a/2	b/2	ab/4 ab/4	ab/2 = 2ab/4	$2/4 = 2 \cdot 1/2^2$
8	7a/8	b/8	7ab/64 7ab/64	14ab/64	$14/64 = 2 \cdot 7/8^2$
		…			
	0	0	0		Splošno: $2(n - 1)/n^2$

3. Ugotovitev

V osnovni šoli lahko učenci z opazovanjem konkretnega zaporedja sklepajo:

- razmerje med vsoto ploščin osenčenih pravokotnikov in ploščino celotnega pravokotnika je $2(n - 1)/n^2$;
- ploščini obeh osenčenih pravokotnikov sta enaki;
- vsota ploščin obeh osenčenih pravokotnikov je največja, če je točka T na sredini diagonale.

4. Utemeljitev

Ugotovitve tudi v osnovni šoli lahko utemeljimo s pomočjo izrekov o skladnosti.

b) Deduktivni pristop - sklepanje, povezovanje in uporaba matematičnih pojmov in zakonitosti Z upoštevanjem podobnosti dobimo, da iz $x_1 = a/4$ sledi zapis sorazmerja:

$a : b = a/4 : y_1$, iz katerega sledi $y_1 = b/4$ oz. $y = 3b/4$. Ploščina osenčenih pravokotnikov je: $a/4 \cdot 3b/4$ in druga $3a/4 \cdot b/4$. Ploščini osenčenih pravokotnikov sta enaki.

Splošno: če je $x_1 = a/n$, potem z upoštevanjem podobnosti dobimo sorazmerje $a : b = a/n : y_1$, iz katerega sledi $y_1 = b/n$ oz. $y = (n - 1)b/n$. Ploščini osenčenih pravokotnikov sta: $a/n \cdot (n-1)b/n$ in $(n-1)a/n \cdot b/n$, iz tega sledi, da sta enaki pri vsakem n.

Kje je ploščina največja? Na osnovnošolski ravni to lahko ugotovimo na podlagi opazovanja zaporedja, na srednješolski ravni pa s pomočjo odvoda. Ploščina $a/n \cdot (n-1)b/n$ je največja takrat, ko je $n = 2$. To pomeni takrat, ko je točka na sredini diagonale.

Združimo ugotovitve

Če drsimo s točko T po diagonali, se vsota ploščin obeh osenčenih pravokotnikov spreminja od vrednost 0, ko je točka T v oglišču, do največje vrednosti $ab/2$, ko je točka T v središču diagonale.

Razmerje med ploščinama osenčenih pravokotnikov je 1 : 1 oz. ploščini sta enaki pri vsaki legi točke T. Ploščini sta največji, ko je točka T v središču diagonale. Razmerje med vsoto ploščin osenčenih pravokotnikov in ploščino celotnega pravokotnika je $2 \cdot (n-1)/n^2$.

ZAKJUČEK

S predstavljenima izzivoma smo želeli pokazati, da so pri učenju in poučevanju pomembni tudi procesi in ne le končni cilji. Pri tako oblikovanih odprtih problemih se na eni strani učimo strategij reševanja problemov, na drugi pa dejavno iskanje rešitev ali usvajanje pojmov z različnimi pristopi, učencem

omogoča različne vpoglede v vsebino in mu s tem olajša konstrukcijo in razumevanje osnovnih pojmov.

Pri transmisijskem pristopu učenja in poučevanja je situacija pogosto podobna situaciji, ko si vprašanja sledijo v hitrem tempu in ni časa za razmisleke ter preverjanje in pojasnjevanje. V izzivu ***Raziščite*** *ploščini osenčenih pravokotnikov*, je pred učenca postavljena precej drugačna zahteva, kot bi bila, če bi se naloga glasila: ***Izračunaj*** *ploščino osenčenega lika pri danih podatkih*. Razlika je bistvena. V prvem primeru učenci samostojno postavljajo vprašanja, ki jih nato raziskujejo. Npr.: ali sta ploščini enaki, kolikšno je razmerje med ploščinama osenčenih likov, kako se spreminjata ploščini, če drsimo s točko po diagonali, kolikšen je delež osenčenega lika v pravokotniku. Pri reševanju učenci iščejo različne poti reševanja in različne rešitve. Npr.: rišejo slike, oblikujejo modele, merijo dolžine in računajo ploščine, analizirajo sliko in povezuje podatke, preračunavajo, primerjajo rezultate, iščejo skladne like, postavljajo hipoteze, npr.: »*ploščini sta enaki*«. Svoje ugotovitve tudi utemeljijo in jih predstavijo. V drugem primeru bi učenci na podlagi danih podatkov izračunali zahtevano ploščino z uporabo obrazca. Pogosto bi jo izvedli rutinsko, brez pravega poglobljenega razmišljanja. S pridobivanjem različnih izkušenj iz prvega primera pa si postopoma polnijo pojmovno predstavo, ki je, kot smo omenili ob nalogah na začetku, ključna pri razumevanju pojmov.

Lahko zaključimo, da je učenje učinkovitejše, če besedam damo smisel in se snovi ne učimo na pamet, ponovimo s svojimi besedami, si pomagamo z modeliranjem, dejavnim eksperimentiranjem itd. Pri izgrajevanju pojmovnih predstav ni pomembno le, da učenci rešijo čim več nalog, pomembno je tudi, da rešijo kompleksno nalogo, kjer znanje povezujejo. Kompleksnejši problemi navadno vključujejo razumevanje in uporabo pojmov ter obvladovanje različnih postopkov. Zelo dobri so odprti problemi, ki sprožijo diskusijo med učenci, postavljanje raziskovalnega vprašanja ter v nadaljevanju omogočajo veliko priložnosti za diskusijo tako o poteh reševanja kot tudi o rešitvah.

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