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Title: Change in Peer Ability as a Mediator and Moderator of the Effect of the Algebra-For-All Policy on Ninth Graders' Math Outcomes

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Background / Context:
A recent report by the Mathematics Advisory Panel referred to algebra as a "gateway" to later achievement (National Mathematics Advisory Panel, 2008). To address the problem of low academic performance in algebra, an increasing number of states and districts have started to implement a policy of requiring algebra for all students in ninth-grade or earlier. The rationale is that providing all students with an opportunity to learn algebra will likely improve math achievement across the board. However, the well-intended curricular policy may have unintended consequences for some students if the change of curricular offerings leads to a reorganization of all math classrooms in a school.

This policy can be viewed as an effort to eliminate tracking in mathematics instruction. Conventionally, secondary school students are placed in different sequences of course-taking based on their initial academic skills, interests, and future occupational orientations (Powell, Farrar, & Cohen, 1985). The tracking system has been widely criticized for increasing educational inequality especially because low-track classrooms are characteristic of low-level content, low expectations, and often low engagement (Gamoran & Mare, 1989; Lucas, 1999; Oakes, 1985, 2005; Page, 1991). For example, under the tracking system, lower ability 9^{th} graders tend to take remedial math together with students at a similar ability level while their higher ability peers are taking algebra. To eliminate tracking would require enrolling students from mixed ability levels in the same algebra classes.

An earlier study has shown that the algebra-for-all policy adopted by the Chicago Public Schools in 1997 indeed increased algebra enrollment of many lower ability 9th graders who would otherwise have taken remedial math. Researchers did not find a policy effect on lower ability students' math achievement. However, lower ability students' grades and passing rates seemed to have declined after the policy was introduced (Allensworth, Nomi, Montgomery, and Lee, 2010). In the meantime, although the policy did not change the algebra enrollment of higher ability students, it appeared to have an unintended negative impact on the math achievement of those who would have taken algebra regardless of the policy (Nomi, 2010).

We reason that the negative effect on higher ability students' achievement and that on lower ability students' grades and passing rates are at least partly attributable to policy-induced changes in peer ability composition in 9^{th} grade algebra classes. When schools increased algebra enrollment and eliminated remedial math classes, lower ability students who would have taken remedial math were placed in algebra classes together with higher ability students who would have taken algebra regardless of the policy. In general, the heterogeneity of student ability increased while the average ability decreased in algebra classes as a result of the policy.

Average peer ability represents the amount of math knowledge and skills collectively brought by students in a class. In theory, even when the curriculum is given, ability composition of a class may nonetheless influence instructional content, pace, participation structure, peer interactions, and evaluation, which may subsequently influence a student's math learning and relative standing in class. Hence changes in algebra class composition may mediate the effect of the algebra-for-all policy on students' math outcomes. Moreover, the mediation effect may depend on the direction and magnitude of the change in peer ability for each individual student given his or her incoming skills.

Specifically, we hypothesize that experiencing a decline in average peer ability may negatively affect the math learning of higher ability students. This is because teachers tend to gear instruction to the middle of the ability distribution in a heterogeneous class by reducing content coverage or slowing down the pace in de-tracked algebra classrooms (Rosenbaum, 1999). If the policy could be implemented without changing algebra class composition, presumably the policy should have little impact on higher ability students' math learning.

For lower ability students, we hypothesize that experiencing a rise in average peer ability may have mixed effects as it may affect their social-emotional process and learning opportunities in opposite ways. Being placed in the same class with higher ability peers may heighten peer competition, increase anxiety for failure, and lower one's self-esteem due to unfavorable social comparisons. This mechanism is related to the so-called "institutional effect" and "social effect" of instructional organization in the sociology literature (Gamoran, 1986; Pallas, Entwisle, Alexander, and Stluka, 1994). Yet gaining exposure to higher level content in algebra classes is expected to advance lower ability students' math learning unless the changed course content is beyond their reach.

Purpose / Objective / Research Question / Focus of Study:
To explain the negative effects of the policy that required algebra for all 9^{th} grade students in Chicago Public Schools, our causal questions focus on the mediating role of changes in peer ability composition induced by the policy (See Figure 1 for the causal diagram). Specifically, we address the following research questions: (1) Did change in peer ability mediate the policy effects on student outcomes? (2) What would be the direct effects of the policy on student outcomes if schools did not change class composition? (3) Did the mediated effects (i.e., indirect effects) and the direct effects of the policy depend on the direction of change in peer ability for individual students?

Setting:
Chicago has the third-largest school system in the United States. The student population is about 50% African-American, 38% Latino, 9% White, and 3% Asian. Approximately 85% of students are eligible for free/reduced priced lunches. Nearly 55% of students scored below the national median on ITBS and a quarter of student failed in mathematics prior to the 1997 policy.

Population / Participants / Subjects:
Our analyses include all the CPS high schools in existence before and after the policy was implemented (n=59 schools). The study population consists of six cohorts of first-time 9th-graders enrolled in CPS high schools between 1994 and 1999 with approximately 20,000 students in each cohort.

Intervention / Program / Practice:
The intervention under study is the curricular policy mandating college-preparatory coursework for all high school students. Prior to 1997, students were required to complete two to three years of mathematics in any subject and many students began high school with remedial coursework (e.g., pre-algebra or general mathematics). In 1997 the Chicago Public Schools (CPS) raised graduation standards requiring Algebra I in the ninth grade, followed by Geometry and Algebra

II in the subsequent years. We restrict our attention to the requirement of algebra enrollment for all 9th graders.

Research Design:
This study uses interrupted time series design to identify the causal effects of the algebra-for-all policy that presented an exogenous shock to the entire school system. The research design can be viewed as a natural experiment as the 1997 policy left little room for self selection of course enrollment. The treatment group includes the three cohorts of students who were enrolled in the 9th grade after 1997; and the control group includes the three cohorts of students enrolled before 1997. Additionally, to minimize threats to internal validity such as concurrent historical events or change in instrumentation, we use a third comparison group—students who attended schools that had already offered algebra for all students prior to 1997. Assuming that these schools were not affected by the system-wide algebra-for-all policy, we intend to remove the confounding of the time-varying factors by comparing the pre-post differences in outcomes between students in schools unaffected by the policy and those in schools that increased algebra enrollment.

Data Collection and Analysis:
Notation. We use T to denote the system-wide policy. Let $t = 1$ denote the algebra-for-all policy and $t = 0$ for the control condition. The mediator of focal interest to us is the ability composition of a math class. We use $C(t)$ to represent average peer ability as a function of the algebra-for-all policy. In assessing the mediated policy effect on math achievement, we compute peer ability as the class average math pretest score relative to the school average score in a given year. This measure of class average ability is independent of between-school differences in student academic composition and of within-school year-to-year variation in student academic composition. In assessing the mediated policy effect on math grades and course failure, we compute peer ability as the class average math pretest score relative to a focal student's pretest score. Because the policy was directly targeted at changing math course-taking of lower ability 9th graders, we use $M(t)$ to represent math course-taking as a function of the policy. Let $M(t) = 1$ if a student takes algebra and 0 otherwise for $t = 0, 1$. Under the algebra-for-all policy, we have that $M(1) = 1$ for all students. The math outcomes of interest include standardized math test score, math grade, and whether a student passes or fails in a math class. A student's potential math outcomes are represented by $Y(t)$ which is equivalent to $Y(t, M(t), C(t))$. We use $\mathbf{X}$ to denote a vector of student and school characteristic measures.

Data sources. This study draws from multiple data sources provided by the Chicago Public Schools. Course transcripts and semester grade files contain information on students' course enrollment. Administrative records contain student demographic information including gender, age, race/ethnicity, special-education status, and residential mobility. Student socio-economic variables were constructed using the 2000 U.S. census block-level data linked to students' home addresses. We use student scores on the Iowa Tests of Basic Skills from third to eighth grade to measure a student's incoming ability. High school achievement test scores come from the Tests of Academic Proficiency (TAP) given at the end of the 9th grade (see Table 1 for details). School-level information includes school type, size, demographics, and academic composition.

Total effect. The total effect of the algebra-for-all policy is defined as the average difference between the potential outcome associated with the algebra-for-all policy and the potential outcome that is not associated with this policy. This is represented by $E[Y(t=1) – Y(t=0)] \equiv E[Y(1, M(1), C(1)) – Y(0, M(0), C(0))]$.

Natural direct effect and natural indirect effect. Recent statistics literature has decomposed the total effect into a natural direct effect and a natural indirect effect (Pearl, 2001; Robins & Greenland, 1992). In this study, the algebra-for-all policy was expected to change course offering and course taking and thereby change students' math outcomes even without changing peer ability. The natural direct effect of interest here is the policy effect on student outcomes not to be attributed to policy-induced changes in peer ability. A student's counterfactual math outcome under the policy with peer ability counterfactually remained unchanged by the policy is represented by $Y(1, M(1), C(0))$. Hence, the natural direct effect of the policy with peer ability remaining counterfactually unchanged is represented by $E[Y(1, M(1), C(0)) – Y(0, M(0), C(0))]$. The natural indirect effect is the change in student outcomes that is attributable to the change in peer ability induced by the policy. In other words, it is the policy effect mediated by the change in peer ability and is represented by $E[Y(1, M(1), C(1)) – Y(1, M(1), C(0))]$.

Student subpopulations. As noted earlier, we hypothesize that policy-induced changes in peer ability may affect higher ability and lower ability students differently. Hence we define direct and indirect effects of the policy for subpopulations of students that are relatively homogeneous in responding to the change in policy. Subpopulation memberships are jointly determined by a student's expected change in average peer ability $E[\mathrm{C}(1) – \mathrm{C}(0) \mid \mathbf{X}]$ and the expected change in course-taking $E[\mathrm{M}(1) – \mathrm{M}(0) \mid \mathbf{X}]$ given the student's incoming abilities in reading and math, student demographic characteristics, and school membership. For example, we expect that higher ability students would have a negative value in peer ability change and would have no change in course-taking and that lower ability students would have a positive value in peer ability change as well as a positive value in course-taking change.

Analytic strategy. To estimate the direct and indirect effects moderated by student prior ability, we use the ratio-of-mediator-probability weighting method (Hong, 2010). Conventional methods such as path analysis or structural equation modeling require the assumption of no interaction between treatment and mediator (Holland, 1988). These methods are not applicable in this study because we are particularly interested in examining the interaction between the algebra-for-all policy and the change in peer ability as a mediator. Recent statistical advances relax this assumption typically within the linear or nonlinear regression framework (Pearl, 2010; Petersen, Sinisi, & van der Laan, 2006; VanderWeele, 2009) with few exceptions (Imai, Keele, & Yamamoto, 2010). Among other limitations, none of these strategies provide simultaneous estimation of the confidence intervals for the point estimates of natural direct and indirect effects. Our non-parametric weighting approach not only relaxes the assumption of no treatment-by-mediator interaction but also enhances the robustness of estimation results when there are possible model misspecifications. In essence, we estimate the marginal mean of each counterfactual outcome for each subpopulation of students by weighting every student enrolled in high school after 1997 such that the weighted distribution of their peer ability under the policy approximates their counterfactual distribution prior to 1997. The weight to be applied for this purpose is a ratio of the conditional probability of pre-policy peer ability to the conditional probability of post-policy peer ability. Most importantly, the weighting strategy allows us to estimate the average counterfactual outcome $E[Y(1, M(1), C(0))]$. Recent results have shown that weighting on the basis of propensity score stratification produces causal effect estimates that are robust to misspecifications of the propensity score model (Hong, in press). After dividing the sample into a number of strata on the propensity score for pre-policy peer ability and again stratifying the sample on the propensity score for post-policy peer ability, we estimate the ratio-of-mediator-probability weight for each student enrolled in high school after 1997. The weighted outcome model is a function of the subpopulation-specific direct and indirect effects with minimal model-based assumptions. We use estimated robust standard errors to compute respective confidence intervals.

Findings / Results:

The analysis of pre-policy course-taking has shown that almost all 9th graders whose incoming math skills were higher than 0.5 standard deviations above the overall average took algebra in the absence of the policy. Hence the policy would not change these students' algebra enrollment. However, the results indicated that these same students would have experienced a decline in peer ability on average had they instead entered high school after 1997. Among students whose incoming math skills were no more than 0.5 standard deviations above the overall average, those who were more likely to take remedial math prior to the policy showed a higher likelihood of experiencing an increase in peer ability along with a change in course taking had they instead entered high school after 1997. We are currently conducting mediation analysis.

Conclusions:

Student achievement in algebra is one of the key predictors for academic success. Algebra typically has the highest failure rate among all 9^{th}-grade courses and is thus directly tied to high school graduation rate (Allensworth and Easton, 2007). This study shows that a well-intended policy of universalizing algebra education through changing not only the curriculum but also instructional organization can have mixed effects on these important outcomes. We apply a new analytic strategy to population data that enables us to answer the questions about causal mechanisms that cannot be answered by standard regression-based methods such as path analysis or structural equation modeling. By showing how changes in peer ability mediate the policy effects on student outcomes, this study contributes to the extant research on tracking and de-tracking. While tracking has been widely criticized for increasing educational inequality and de-tracking often seen as a preferable alternative (Boaler & Staples, 2008; Oakes, 2005; Rubin, 2008), some researchers have been concerned about the practical difficulties with de-tracking (Rosenbaum, 1999). Results from this study empirically inform our understanding of how students are influenced differently by changes in peer ability composition as a result of de-tracking. In particular, simply eliminating tracking would have a negative impact on higher ability students as a decline in peer ability composition might lead to a reduction in academic demands within a class. For lower ability students whose course taking were to be changed by the policy, attending algebra classes together with higher ability students tended to increase their failure rate as an increase in peer ability would be accompanied by a decrease in a lower ability student's relative standing in class. Moreover, due to these students' relatively weak math background, learning challenging math content without additional support might also contribute to their failure in algebra classes (also see, Nomi & Allensworth, 2009), which could be an explanation for the direct effects of the policy on lower ability students' grades and failure rate unaccounted for by the change in peer ability. Revealing these subtleties in the causal mechanisms brings us important new insights in evaluating systemic policy initiatives.

Appendices

Appendix A. References

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Appendix B. Tables and Figures

Figure 1.
Causal Relationships among the Algebra-for-All Policy, Peer Ability, and Math Outcomes

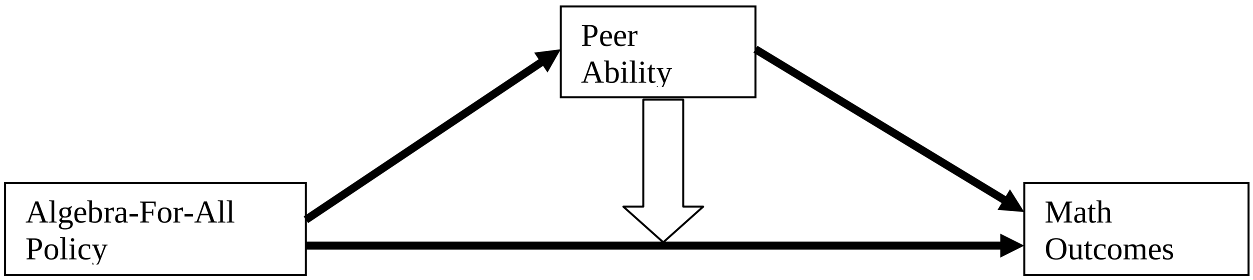

Table 1.
Variable descriptions

Students variables	
9th grade outcomes	
GPA	Average math GPA on a 4 point scale
Course failure	A dummy variable indicating whether students failed math courses
Test scores	Math test scores measured in 9th grade spring semester (TAP)
Other covariates	
Student ability	Latent ability scores in math and English were created for each student using his/her ITBS history from 3rd grade through 8th grade (standardized across all cohorts with a mean of 0 and SD of 1)
Algebra enrollment	A dummy variable with 1=enrolling in algebra and 0=otherwise
Special education	A dummy variable with 1= special education students and 0=otherwise
Gender	A dummy variable with 1=male and 0=female
Race/Ethnicity	A set of dummy variables indicating African-American (ref. group), Asian, Hispanic, and White.
SES	Two variables constructed from the U.S. census data on students' residential block groups: 1) Concentration of poverty (a composite of male unemployment rate and % families under the poverty line. 2) Social status (a composite of the median family income and the average educational attainment. Both were standardized to have a mean of 0 and SD of 1.
Mobility	A set of dummy variables: no moves, one move, two or more moves in the 3 yrs before high school
Age at HS entry	1) Number of months old for high school 2) a dummy variable indicating if students are slightly old 3) a dummy variable indicating if students are young for starting high school
Classroom covariates	
Peer ability level	Classroom average latent math scores
Cohort covariates	
Cohort	A set of dummy variable distinguishing cohorts with the 1994 (i.e., base-year) cohort as a omitted group
Cohort average ability	Cohort average latent math scores