

# Comparing the Common Core State Standards in Mathematics and NCTM's *Curriculum Focal Points*

## Introduction

Through the Common Core State Standards (CCSS) Initiative, states and territories have collaborated in the development of a common core of standards in English Language Arts and mathematics for grades kindergarten through twelve that are now being adopted by states. Designed not only for the purpose of providing strong, shared expectations, the Common Core State Standards will also allow adopting states to collectively create and share high-quality tools such as assessments, curricula, instructional materials (such as textbooks and software), and professional development programs.

As educators and policymakers review the CCSS in mathematics, they will want to consider the way these new standards compare to, and build on, existing standards in mathematics. This brief describes the comparison between the CCSS and the National Council of Teachers of Mathematics (NCTM) *Curriculum Focal Points*.

## Common Core State Standards in Mathematics

The K-5 standards provide students with a solid foundation in whole numbers, addition, subtraction, multiplication, division, fractions and decimals—which help young students build the foundation to apply more demanding math concepts and procedures successfully, and move into applications. They also provide detailed guidance to teachers on how to navigate their way through knotty topics such as fractions, negative numbers, and geometry, and do so by maintaining a continuous progression from grade to grade. Having built a strong foundation in K-5, students can move to more complex work in geometry, algebra and probability and statistics in the middle grades to gain a rich preparation for high school mathematics. Students who have completed 7<sup>th</sup> grade and mastered the content and skills through the 7<sup>th</sup> grade will be well-prepared for algebra in grade 8. The high school standards call on students to practice applying mathematical ways of thinking to real world issues and challenges; they prepare students to think and reason mathematically across the major strands of mathematics, including number, algebra, geometry, probability and statistics. Note that the CCSS promote rigor not simply by including advanced mathematical content, but by requiring a deep understanding of the content at each grade level, and providing sufficient focus to make that possible.

The CCSS in mathematics lay out a vision for what all students need to master to be ready for credit-bearing college mathematics courses without remediation. Some of the high school standards are designated by a (+), indicating that they are above the college- and career requirement but necessary for students to take advanced mathematics courses in high school such as calculus, advanced statistics, or discrete mathematics, and to be prepared for Science, Technology, Engineering, and Mathematics (STEM) coursework in college.

## The National Council of Teachers of Mathematics *Focal Points*

The NCTM published its *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence* (2006) as a companion to its comprehensive and influential *Principles and Standards* (2000). The *Focal Points* describes the most important mathematical topics for each grade level and, since the document's release, has been widely used by state mathematics content developers in designing their own standards and curricula. When published in 2006, the *Focal Points* provided fresh guidance on what students should learn each year, and the ways in which the strands of mathematical learning should connect with one another across the grades.

## Achieve's Analysis

Achieve has analyzed the CCSS and *Focal Points* to determine how they compare in terms of **rigor, coherence, and focus** mathematical content. **Rigor** refers to the degree that sets of standards address key content that prepares students for success beyond high school. **Coherence** refers to whether the standards reflect a meaningful structure, revealing significant relationships among topics and suggest a logical progression of content and skills over the years. **Focus** refers to whether the standards suggest an appropriate balance in conceptual understanding, procedural skill, and problem solving with an emphasis on application and modeling; the standards should be teachable within a school year (or across four years of high school), and key ideas in a given grade or topic area should be clear. Standards that are rigorous, coherent, and focused provide better guidance to educators, students, and parents about desired learning outcomes than those that are not. Expert mathematics content analysts conducted a side-by-side comparison of the CCSS and *Focal Points*, looking particularly at the inclusion and treatment of mathematics topics at each grade level. This brief describes their findings.

## Major Findings

- ✓ The CCSS are similarly rigorous to NCTM's *Focal Points*. While some content occurs earlier in the CCSS, the two documents generally describe the same content.
- ✓ While the CCSS and the *Focal Points* are comparable in their coherence and focus, different purposes for each document lead to differences in their levels of specificity.

## Detailed Findings

### Rigor

Because the influential *Focal Points* was an important resource for the developers of the CCSS, the two documents have much in common and generally describe the same content. However, there are some differences in the later elementary and middle grades regarding when topics are included.

- ✓ **Elementary grades:** Through the end of grade 4, the CCSS and the *Focal Points* address much of the same content, although the CCSS introduce some content earlier than the *Focal Points*. By the end of grade 4, both documents expect students to understand the four basic operations with whole numbers, place value, and the meaning and uses of fractions. Both documents introduce statistical topics primarily as reinforcements for work with numbers and measurement in the early grades. The strong foundation in number sense allows students to progress quickly in middle and high school through data, probability and statistics, culminating in content that is generally more rigorous than that found in many state standards. However, specific differences in grade placement are present in a number of instances. For example, the CCSS expect student to establish equivalence of simple fractions with unlike denominators (e.g.,  $1/2 = 2/4$  or  $4/6 = 2/3$ ), and express whole numbers as fractions earlier than *Focal Points*. The CCSS require fluency in the addition, subtraction, and multiplication of fractions by the end of grade 5; the *Focal Points* requires students to do this by the end of grade 6. Despite these differences, they do not lead one document to be more rigorous than the other since by the end of Grade 6, the two documents cumulatively describe substantially similar bodies of knowledge.
- ✓ **Middle grades:** With only a few minor exceptions in geometry and probability, the CCSS and the *Focal Points* describe similar content in the middle grades. Areas of overlap include the major hallmarks of algebra—proportionality, linear expressions and inequalities, and using equations and inequalities to solve real-life and mathematical problems—which prepare students well for the more advanced mathematics they will face in high school. There are some differences between the two documents. For example, the CCSS require students to understand the role of transformations of geometric shapes on a coordinate plane, and in the area of probability, the CCSS expect students to extend their knowledge of probability to compound events in grade 7, where the *Focal Points* does not.

## Coherence and Focus

Achieve's analysis indicates that the Common Core State Standards and the *Focal Points* share some key traits of coherence and focus. For example, both documents develop an understanding of adding, subtracting, multiplying and dividing whole numbers over an average of about three years. Mathematical reasoning is addressed in both documents and both also highlight the importance of explaining and justifying solutions to problems.

Both sets of standards expect students to learn similar amounts of content in each grade level, and the *Focal Points* builds on the pioneering standards first developed by the NCTM in 1989 by articulating clear and realistic priorities for student learning in kindergarten through grade 8. The CCSS were designed to be similarly focused, although they extend through the end of high school. Both documents make clear that curricula should be focused on a tightly defined set of content and skills each year in order for students to build a strong foundational understanding of mathematics before they begin high school coursework. In particular, both documents provide an in-depth treatment of number and operations, with reinforcement from other content areas. This emphasis allows students to progress more quickly through later content, as the groundwork for more advanced mathematics has been laid.

Despite these similarities in coherence and focus, there are critical differences between the CCSS and the *Focal Points*. These differences are generally the result of the different purposes between the two documents, with the CCSS providing greater specificity in what students should know and be able to do, while the *Focal Points* provides more of an overview of the mathematical topics at each grade.

- ✓ The CCSS provide more detailed and specific expectations of students. For example, the CCSS indicate that fourth graders should compare base ten numbers: "Generalize place value understanding for multi-digit whole numbers: Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right." On this topic the *Focal Points* is less specific, indicating that students should be able to "...use place value and properties of operations to write, compare, and order multi-digit numbers." The greater level of detail and specificity in the CCSS provides additional guidance to educators regarding learning expectations.
- ✓ While both documents make connections among topics, the *Focal Points'* presentation of the connections is particularly useful to educators. For example, the layout of the *Focal Points* includes a column in each grade that explicitly calls out the connections among topics. In nearly all cases, these connections apply in the CCSS as well, but they are not as explicit. As such, the *Focal Points* can be an important tool in guiding the implementation of the CCSS, particularly with respect to drawing important connections between and across the content.
- ✓ Like the standards of many high-performing countries including Hong Kong and Japan, the CCSS begin with a focus on the properties of operations<sup>1</sup> as a way of gradually preparing students for algebraic thinking. In contrast, the *Focal Points* introduces algebraic thinking through the creation, description, extension and explanation of patterns. The CCSS include this content somewhat later in Grades 4 and 5 after development of the core knowledge associated with numbers and operations in Grades 1-3. Because of this different progression, some states that adopt the standards will need to be vigilant as topics will be introduced at different grade levels than at present, and teachers will need to focus on different content that may be less familiar to them.

In short, while the Common Core State Standards and the *Focal Points* share some traits of coherence and focus, important differences exist. The *Focal Points* more clearly describes connections among topics, yet the CCSS provide greater detail and specificity regarding learning expectations.

<sup>1</sup> Associativity and commutativity of addition and multiplication, distributivity of multiplication over addition, the additive identity property of 0, and the multiplicative identity property of 1.

## Conclusion

Overall, the CCSS are well aligned to the *Focal Points*. Policymakers can be assured that in adopting the CCSS, they will be setting learning expectations for students that are similar to those set by the *Focal Points*. There are, however, a number of important differences in the placement and priority of topics as detailed in this document. These differences will require careful consideration in building curriculum, planning instruction, providing professional development and developing assessments during the implementation of the CCSS in states. The CCSS, like the *Focal Points*, place a priority on focus and coherence, seeking to set forth rigorous learning outcomes that will prepare students for later success.

*Achieve is a bipartisan, nonprofit education reform organization that has worked with states, individually and through the 35-state American Diploma Project, for over a decade to ensure that state K-12 standards, graduation requirements, assessments and accountability systems are calibrated to graduate students from high school ready for college, careers and life. Achieve partnered with NGA and CCSSO on the Common Core State Standards Initiative and a number of its staff and consultants served on writing and review teams. Achieve thanks the Brookhill Foundation for its generous support in making this brief available, and providing educators and policymakers across the nation with a way to more deeply understand the CCSS through comparison to other well-known mathematics expectations.*