THE GENERATIVE ADOLESCENT MATHEMATICAL LEARNER

by

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Purpose

This research was embedded in a deconstruction of the field of Mathematics Education in order to reconstitute the mathematics student as a generative mathematical learner. The purpose of this paper was to consider the personal epistemologies of generative adolescent mathematical learners (GAMLs).

This work serves a larger agenda to understand how high school mathematics teaching might achieve frequently cited policy goals to develop productive mathematical dispositions within students (Boaler, 2000; NCTM, 2000; Schoenfeld, 1989; Yackel & Cobb, 1996) in light of the modernist epistemological principles that continue to invisibly guide the contemporary learning of mathematics (Steffe, 2007). This development of a productive disposition, an intellectual and social autonomy, is a “major goal in the current educational reform movement, more generally, and in the reform movement in mathematics education, in particular” (Yackel & Cobb, 1996, p. 473).

This goal for productive dispositions in learners has proven problematic in mathematics education. Given that “the educational system is the political means of maintaining or modifying the appropriation of discourse, with the knowledge and power it carries” (Foucault, 1972/1982, p. 46), to attend to assumptions about *What is mathematics? Why teach mathematics?* and *How do children learn mathematics?* allows serious consideration of the epistemological and ontological factors that influence the classroom ecologies in which students participate.

Theoretical Framework

A *generative* disposition is meant to characterize the learner as someone who operates mathematically in ways that reflect an internal sense of *authority for knowing* and a *constructive orientation to the knowledge* they come to know. For a student to be considered a GAML, the generative disposition must go beyond what an observer identifies as the activity of the child—an assumption inherent in a constructivist epistemology, but also what seems to constitute the child's own epistemological identity.

The generative learner is a producer rather than a receiver. Larochelle and Désautels (1991) considered the learner similarly, "A subject must be conceptualized as a producer, and not simply a reproducer of phenomena" (p. 375). Furthermore, they are inventors of knowledge rather than discoverers. While the notion of *learner as producer* speaks to the active learner aspect of the radical constructivist epistemology, the second notion locates an ontological viewpoint—that knowledge is constructed to negotiate the experiential world. Larochelle (2000) stated,

> Radical constructivism itself approaches an essentially undecidable question as though it were decidable, namely whether we are 'discoverers' or whether we are the 'inventors'. Radical constructivism does indeed take a position and opts for the latter view. (p. 61)

This second idea, emphasizing knowledge as invention, takes a significant role in this study when considering: What characterizes the student that perceives herself as generative?

A consideration of the epistemological identity of adolescent mathematical learners is the distinctive focus of this research. Previously, Grieb and Easley (1984) explored the social mechanism of primary schools regarding the development of an independent attitude in mathematically creative children. They found that white, middle-class males more often than females and minorities, can “survive pressures toward conformity in the early grades with their confidence in mathematical reasoning intact and thus preserve more courage to tackle new types of problems throughout their schooling” (p. 318). It appeared to Grieb and Easley that these boys have an advantage over girls and minorities of equal creativity because their teachers seemed to not expect them to conform to social norms of arithmetic.

Grieb and Easley (1984) hypothesized that socio-cultural norms dictate affordances and constraints in the powered social interaction between teacher and student. If the “authoritarian” (p. 355) role of mathematics teachers were reduced, this would “lead to more pupil thinking and less straining to reproduce procedures” (p. 355). Hence, more students could maintain a productive disposition in their mathematical learning.

Methods

Drawing upon the radical constructivist (von Glasersfeld, 1995; 2007) teaching experiment methodology (Cobb, 2000; Steffe, 2000), I conducted a qualitative inquiry of students’ experiences that may have informed a generative disposition and their orientation toward mathematical knowledge. The study was guided by the research question, “How do generative adolescent mathematical learners maneuver through their mathematics courses while maintaining such a disposition?”

I conducted this study in an urban high school of the American Northeast. With the help of the mathematics faculty, I identified six Grade 11 students in two classrooms who had potential to be characterized as GAMLs. I co-taught these mathematics classes, interacting with the students as both teacher and classroom researcher. Field notes served as a primary data source. Several classroom episodes and student interviews were videotaped to aid retrospective analysis. Co-teachers and their classroom environments served as both another ecological component of the study, that is another data source, as well as an opportunity to confer about data and my initial analysis of the students. Case studies of these GAMLs were developed, and then used to enrich an initial definition of the GAML.

Data

For the purpose of this paper, the personal epistemological identities of two students will be highlighted, focused on *authority for knowing* and *orientation to knowledge*.

Fisk

During an interview, I asked Fisk whether the ideas of mathematics were invented or discovered by mathematicians. He replied, “It wasn’t invented by mathematicians because a mathematician was somebody who was good at that position. Somebody else had to realize it at first, therefore they couldn’t have been a master at that position.” He continued,

> If something is discovered, that means it’s already there. Invented is somebody’s idea in their own head, or an innovation on another, something to make it better. I can’t say I discovered sand. But I can say I did invent a sand castle.

For Fisk, sand existed prior to experience. Fisk considered his mathematical activity akin to inventing the sand castle. He hadn't invented the materials with which he crafted meaning, but it was of his own work to craft that meaning. In this way, his mathematical activity was inventing ways of doing *a* mathematics. His "sand castle" (mathematics) was not so much a generalized sand castle, but each particular; hence the potential for invention.

Fisk's activity suggested he located authority internally. He was dissatisfied with mathematical work until things made "sense", in his mind. For example, as the class worked to convert standard-form quadratics to vertex form, Fisk was wrestling with a problem, thinking aloud, "It don't make no sense".

He did not ask for the teacher to confirm his accuracy or conclusions, but sought to share his knowing with the teacher to either demonstrate that he was on track or to show off his understanding. If a teacher or classmate disagreed, or hinted of an error through their tone, Fisk would think further. Interestingly, ideas recorded to the whiteboard, either by the teacher or by other students, seemed to be granted a heightened authority, as though sanctioned.

Fisk's teacher respected him for his mathematical thinking. In other mathematics classes, Fisk shut down. The swings in perceived mathematical abilities by his teachers spoke to some need for ego protection.

Jack

Jack's teacher also exhibited an openness to this GAML as a learner. Her simple belief in him impacted belief in himself, and thus affected his classroom activity. Jack wrote about this impact, "Since I been in the math class my teacher has confidence in me that I know what I am doing and can help others." Again later, "My teacher also has a lot of confidence in me when I am doing math because she know I can do math. She like the way I do math that she wants me to share my ideas with other students in the class."

Jack did much of his mathematical work in his head, never recorded to paper. Also, what appeared to be detachment from the classroom would often and unforgettably conclude with a surprisingly insightful connection to a topic being discussion. I asked him during an interview about what prompted his breaks from intense mathematical work. He replied, "I had to take a break 'cause there was wearin' on my mind." Jack thought hard about mathematics. "There's plenty of times when I been daydreaming. Then like sometimes I daydream and I come right back in and I answer a question and then I'm right and I'm like, how was I right?" Jack was aware that although he disconnected from what was going on, he was not so disconnected; he provided solutions to questions asked. This quality surprised him.

For Jack, the ideas of mathematics were invented, and then frequently rediscovered; invented *and* discovered by people, including both mathematicians and himself. His initial dispositional survey and follow-up interview bore out these findings. Jack's orientation toward the invented quality of mathematics suggested that he would see himself as a mathematical author, and hence an authority for his knowing. While interactions reported above support an internal locus of authority, other episodes suggested contradictory evidence. For example, I asked him to share an idea with the class. His immediate response was to ask me, "Is it right?". However, considering Jack's frequent verbal participation in classroom discussion, rather than seeking affirmation, this query more likely marked surprise to the possibility that what he had offered in private to me was correct.

Results

The following considers two ideas related to the initial goal in this inquiry, the GAML's perception of self as a mathematical learner, and the GAML and school structures, drawn from the analysis of data that spoke to the GAML's locus of authority and ontology of mathematics.

The students of this study indicated that the GAML sees mathematics as activity, rather than as a static entity. The nature of mathematical activity and knowing was highly social for the subjects of my study. While mathematics had an important social role, the GAML maintained a strong notion that it was for them to determine a truth to the knowing, particularly evident in Fisk. The disagreement of another served as a catalyst for further inquiry and drive to make meaning, rather than passive acceptance of proof from external authority. The GAML drew heavily on interaction with others to confirm their own knowing, to feel justified to speak of confirmed facts (Glasersfeld, 1995).

Occasional conflict between reported confidence in knowing and externalized artifacts of such knowing had its roots in the need to communicate personal ways of thinking in conventional manners. GAMLs demonstrated moderate dysfunction in relation to classroom structures. The two teachers I worked with in the study were open and flexible with students, but other teachers reported a variety of trouble with these students. The GAMLs of my study could have been regarded as lazy, uninterested in doing well in school, talkative, too confident in their ability, disrespectful of rules and classroom norms, and even aggressively defiant of authority.

Significance

The modernist orientation to mathematics education focuses on knowledge-as-information; it is discipline-centered and culturally reproductive. Such an institution could only further cement the inequalities found in a society. This research project begins to wonder what if the learner's relation to knowledge and perception of self as a knower could reconstruct unjust school structures? Mathematics education purports to seek the curious, self-directed, productive learner; one who constructs knowledge, and who feels empowered; a contributing author of mathematics and a responsible authority for their knowing.

These findings about generative adolescent mathematical learners' knowledge of, relationship to, and manners of experiencing mathematics suggest that granting learners a more active epistemological status may be an important locus of emphasis with regards to design, intent, and practice of mathematics education if the goal to create students with productive mathematical dispositions.

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