13 YEAR-OLDS' MEANINGS AROUND INTRINSIC CURVES WITH A MEDIUM FOR SYMBOLIC EXPRESSION AND DYNAMIC MANIPULATION.

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We explore how 13 year-olds construct meanings around the notion of curvature in their classroom while working with software that combines symbolic notation to construct geometrical figures with dynamic manipulation of variable. The ideas of curve as intrinsic dynamic construction, and curve as object with properties related to its positioning on the plane were some of those developed. The use of symbolic and graphical notation in conjunction with the dynamic manipulation played an important part in the generation of these ideas, which was interwoven with the activity and the use of the tools.

THEORETICAL FRAMEWORK

In this paper we report research aiming to explore how 13 year-olds construct meanings around the concept of curvature with 'Turtleworlds', a piece of geometrical construction software which combines symbolic notation through a programming language with dynamic manipulation of variable procedure values (Kynigos et al., 1997). The students worked in small collaborative groups in their classroom during a weekly computer-based project-work session established in their school. They were engaged in a project to build models of bridges by constructing, experimenting with and editing intrinsic arc procedures (Kynigos, 1993) and by manipulating their variable values to observe Cabri-style continual change of the constructed figures.

In our task design and research perspective, we adopted a constructionist approach to learning (Harel and Papert, 1991), focusing particularly on the notion of using both formalism *and* dynamic manipulation to construct mathematical meaning. Along with diSessa, 2000, we argue that symbolic language interfaces in computational expressive media provide rich opportunity for student engagement with *meaningful* formalism. We suggest that formalism is a powerful, inherently mathematical medium for expressing mathematical ideas. It has, however, been placed in the background of attention, given the well-established problem of 'meaning-less' formalism in understanding mathematical ideas (Dubinsky, 2000), coupled with the advent of dynamic manipulation interfaces which provide access to such ideas bypassing formal representation (Laborde and Laborde, 1995). In the results section, we adopt an instrumentalist view of the 'Bridge microworld' with 'Turtleworlds', i.e. we focus on the instrument constructed by the students rather than the artefact designed by the researchers (Marioti, 2002). Turning for a moment at the microworld *artefact*, however, it is important to say that the epistemological validity (Balacheff and Sutherland, 1994) and the pedagogical design of the software and the activity involved an integrated use of both formal mathematical notation and dynamic manipulation of variable values as part of a coherent available representational register. We were interested to study the ways in which the students interacted with these representations and the ways in which the meanings they constructed structured and were structured by them, in the sense of Noss and Hoyles, 1996. Study on the generation of mathematical meaning with microworlds based on constructions with symbolic notation seems to have been rather fragmented from that involving dynamic manipulation of geometrical figures (Arzarello et al., 1998). It may be that research interest in those two arose in different – almost sequential – times, or that the coherence and potential of properties of each type of corresponding software environment have been so exciting, that joining the two has not received much thought apart from a few exceptions (e.g. Clements and Sarama, 1995, Schwartz, 1997). Healy and Hoyles (1999) state that:

> 'the critical difference between programming environments and direct manipulation interfaces revolves around this emphasis in interaction on symbolic control, in the former case, *as opposed* to visual control in the later, p. 236 (our emphasis).

However, we suggest that geometry is a field where mathematical formalism and graphical representation of objects and relations are dynamically joined in interesting ways and that *joint* symbolic and visual control may have important potential for mathematical meaning-making processes. In Turtleworlds, what is manipulated is not the figure itself but the value of the variable of a procedure. Dragging thus affects both the graphics and the symbolic expression through which it has been defined, combining in that sense these two kinds of representations and corresponding epistemological validities.

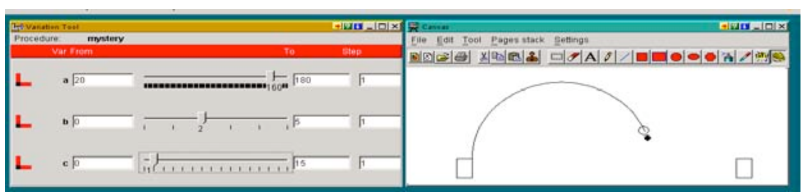
In order to study the meanings generated during the students' work with the 'bridge' microworld, we found Vergnaud's (1987) notion of 'conceptual field' particularly useful, even though it was originally articulated within a cognitive perspective and focused on mathematical concepts rather than student knowings. Vergnaud argued that it makes no sense to perceive of a mathematical concept on its own. Rather, it is more useful to see it in terms of a set of concepts tightly related to it, a set of situations in which it may be used and a set of available representations. Our interest was thus to keep a wide lens with respect to students' generation of meaning *around* curvature, in the sense that we were interested in their use of curvature-*related* ideas in their construction of bridges. Although we were interested in the concept of curvature through the epistemological domain and the representational repertoire of this particular piece of software, we were nonetheless prepared to keep an open mind in order to interpret the meanings students generated for themselves while we observed them constructing their bridge models (Balacheff, in press). We were particularly interested in connections made by the students between mathematical situations they were dealing with and the ways in which they used the available formalism and graphical representations to express them.

RESEARCH SETTING AND TASKS

Although curves are often found in many areas of the curriculum (circles, arcs, trigonometry, function graphics etc.), at least in Greece where the study took place, there is not much focus on the nature of curvature as a context for generating mathematical meaning. For instance, the importance of arcs seems to lie on the angular and trigonometric properties within the corresponding circles rather than on the nature of curve. Function curves are in a sense even more dissociated from curvature investigations in the sense that they are more abstract representations of mathematical relations rather than geometrical figures themselves, even though students have been reported to treat them as such (Ainley et al., 2000).

The activity was part of a wider study involving five schools situated in England, Italy and Greece, each using their own Logo-based software. The task was to engage students in building the model of a bridge[1] and to then bring them into email contact to discuss their construction methods and techniques and to provide information about the specific bridges they modelled. The research reported here involved a primary school in a town called Larissa, where a weekly small group project work session, taught by the students' normal teacher, had been established involving the use of 'Turtleworld' microworlds amongst other exploratory software. At the time of the study, the students had already had experience with traditional Logo constructions including variable procedures. During the 'Bridge' microworld project they were introduced to the dynamic manipulation feature of the software called 'variation tool'. After a variable procedure is defined and executed with a specific value, clicking the mouse on the turtle trace activates the tool, which provides a slider for each variable. Dragging a slider has the effect of the figure dynamically changing as the value of the variable changes sequentially. The graphics, the tool and the Logo editor are all available on the screen at all times. In the corresponding classroom activity, the students were engaged in trying to build the arcs of models of bridges of different sizes and shapes using the following Logo procedure called 'mystery', which was given to them from the beginning.

```
to mystery :a :b :c
repeat :a [fd :b rt :c]
end
```



In this procedure the first variable changes the length of the arc, the second its width and the last its curvature. Dragging any of the three sliders corresponding to the respective variable causes an effect resembling continual change of the curve. During the activity, which lasted for 6 hours in total over 3 weeks, we took the role of participant observers and focused on two groups of students, recording their talk and actions and on the classroom as a whole recording the teacher's voice and the classroom activity. Our aim was to gain insight into a) the kinds of mathematical meanings constructed around the notion of curvature and arcs, b) the ways in which meaning generation interacted with the use of the available tools.

METHOD

In our analysis we used a generative stance, i.e. allowing for the data to shape the structure of the results and the clarification of the research issues. We read the data looking for incidents where mathematical meaning was discussed amongst students or where we identified ideas in use. Classroom observations were conducted in all five schools (6 hours in each one) as well as interviews with teachers. Here we use data from one classroom using the variation tool. A team of two researchers participated in each data collection session. Two video cameras were used, one for each group. A microphone captured all that was said in the groups under study. Background data was also collected (i.e. observational notes, students written works). Verbatim transcriptions of all audio-recordings were made. The researchers occasionally intervened to ask the students to elaborate on their thinking, with no intention if guiding them towards some activity or solution.

RESULTS

Semi-circles as dynamic constructions

In our focus group of three students, the use of the available dragging modality during their exploration oriented the students to play with the idea of discrete versus continuous changes in variable values perceiving continuous curve change as the 'limit' of them reducing the dragging step of the slider. They firstly constructed two rectangular bases for their bridge model with a distance of 500 turtle steps between them, as instructed by their teacher. They were to investigate how they could find a method to make a semicircle join the bases together if they wanted to be able to make bridges of varying widths with their model. At first they seemed to move the sliders of the variation tool at random just because the microworld allowed them, making comments stemming from the observation of the visual feedback of the continuing changes in the variables. They engaged in dragging, which gradually moved from this mode (equivalent to that of 'wandering dragging', Arzarello et al., 1998) to become more systematic and focused in their attempt to create curvy – like semicircles. In the end they decided that a) in order to have a curvy shape they needed small turn and step measures and b) in order to get a semicircle the values of the iteration and turn variables would have to be constant. They then decided to edit their procedure and substitute variables (:a) and (:c) with constant values of 180 and 1. They executed the new procedure with a step value of 1 and noticed that it fell short from joining up the two bases. While dragging the slider of variable (:b) (the length of step) they observed the differences in the moving semicircle each time. After a while they saw that the appropriate value would be between 4 and 5. In fact, they saw that for (:b)=4 the moving edge of the semicircle fell before the upper left vertex of the rectangle representing the opposite base while for (:b)=5 it went past it. One of the students then suggested the use of decimals for the slider step. They then dragged the variation tool in the scale of 4 to 5 using decimals for the step but the arc still did not link the two bases neatly. Observing these changes one student suggested changing the step again from 0,1 to 0,01.

S2: We need it [the arc] to be further along.

S1: It doesn't fit [i.e. the other side]. Why?

S2: It needs one more step [i.e. a different step] / 0,01.

The way in which pupils 'see' the need to extend the shape of the arc is still in the stream of their exploration through dragging. The control of meaning is ascending (Arzarello et al. 1998), i.e. they are manipulating the variation with precise intent and taking control on the continuity of the designed curve. The students' decision to change the slider step reflects the way in which the computational setting provided a web of structures which pupils could control and exploit at a particular moment, shaping the available resources to suit their purpose (Noss and Hoyles, 1996). Furthermore, as a result of their dynamic manipulation, they decided to go back to the formal description of the figure and express one of their findings by changing the code from variables to constants even though this was actually not necessary since they could have simply not moved the sliders corresponding to variables (:a) and (:c).

Curves as Geometrical Figures

This episode is about a different group of students who caught our attention when they decided they wanted to make horizontal arcs, which were not so intensely curved as these created by semicircles. Their goal was to create a bridge which looked like one they had found in a book about bridges of Thessalia, the area in which their school was located. The idea that changing the curvature was possible, however, was brought up in the context of their initial dragging to create different curves. They used the mystery procedure with all three variables and they were dragging the sliders trying to discover some rule or invariant property so that they could change the curvature of the arc without loosing its fit onto the bases. When they experimented by giving corresponding values to the iterations variable (:c), they realised by observing the screen outcome that the shape seemed to be tilted towards the left and decided to insert a command to turn the turtle towards the right before it began to make the arc. They then edited their mystery procedure to look like this.

```
to arc :a :b :c
rt :a repeat :c [rt 1 fd :b]
end
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In this sense, the students inserted a new feature influencing their construction and began to investigate weather there is some underlying property. In contrast to the tendency of the students in the first section to substitute variables with constants, they decided to insert a variable value for the initial turn so that they could investigate by changing it with the use of the variation tool. It is interesting, however, that in effect, they did not change the curvature in their investigation since they substituted the variable turn by a constant value of 1. In that sense, what was changing was the length of the arc of the same circle. They inserted the value of 45 degrees for (:a) and then began to change the others so as to get a 'differing curvature' for their arc.

S2: Just a minute. I've got an idea. Instead of having 180 degrees here, since we don't want to draw a semicircle… Yes, let's have 45 degrees here and 45 here and the rest of it 90, here.

Firstly, they moved the corresponding slider to a value of 45 for variable :a and then the slider for variable (:c) to 90. By dragging the variable (:b) slider, they 'found' the value which would give the right size for their arc. The researcher asked them how they knew which values to give for variables (:a) and (:c) and one student's response was:

S2: It's all part of the semicircle, i.e. the semicircle has 180 repeats inside it, let's say at the beginning we turned 45 degrees from the one side and assuming that there would be 45 on the other side, 45 plus 45 we have 90 degrees, and subtracting from 180 of the semicircle we have the part of the semicircle, the half / and we have 90 repeats.

The students seem to have taken into consideration the symmetrical nature of the arc ('assuming that there would be 45 on the other side') and to have built on the previous properties they discovered. The researcher did not rest with this explanation, asking for a more elaborate one where he probed whether the students were able to generalize the description of their 'rule' to other values for variable (:a). He found out that they in fact had already tried other values for (:a) and brought them in as examples to their explanation.

R: If I didn't turn 45 degrees and turned 30, what would have happened?

S2: Yes, 30 plus 30 sixty, 120 times repeat, 120 times. We tried this here. The more we turn at the beginning, the less we repeat. And the step changes.

It is particularly interesting in the above excerpts that the student's descriptions switch from referring to specific sets of values in concrete cases to attempting a more generalized kind of language explaining the interdependence represented by the variables of the construction. In that sense, they refer to the constructed objects from a detached point of view (Marriotti et al., 2000) mentioning qualitative properties of them such as interdependence. This type of generalization is in accordance with Noss and Hoyle's (1996) notion of 'situated abstractions' since mathematical invariants that underpinned student's actions in the course of interaction were rooted in action and articulated – quasi-mathematically – in the operational terms of the available tools. The mathematical idea tapped by pupils through this 'theorem in action' (Vergnaud, 1987) is that of co-variation between two values as a relational property of an evolving object when the value of a variable changes.

CONCLUSIONS

Some interesting meanings around curvature seemed to have emerged in this classroom activity, through the students' engagement with the graphical and symbolic interdependence of Turtleworlds. Amongst these were the idea of continuity, the dynamic nature of mathematical relations, using the curve as starting point in generating arc properties, discovering unexpected properties of arc positioning and engaging in experimental and formal maths in the same activity. It is worth mentioning here that in contrast to the ways curve is presented in school, pupils have attached a variety of meanings to the notion during their exploration with the provided tools. The use of symbolic and graphical notation in conjunction with the dynamic manipulation of the way the figures evolved as variable values changed, played an important part in the generation of these ideas which was interwoven with the activity and the use of the tools. The kinds of understandings supported by such media in varying mathematical activities warrants further research. Hershkowitz and Kieran (2001) wonder if the use of advanced computational tools in algebra signals the beginning of the loss of the algebraic representation from our mathematical classes at a secondary level. It is interesting to reassess which – if any – aspects of this kind of representation of mathematical ideas are important for the generation of mathematical meaning.

Notes

1 NETLogo: the European Educational Interactive Site, European Community, Educational Multimedia Taskforce, Joint Call on educational Multimedia, MM1020, 1998-1999.

References

Ainley, J., Nardi, E. and Pratt, D. (2000) The construction of meanings for trend in active graphing, *Int. Jnl. of Computers for Mathematical Learning, 5:* 85-114.

Arzarello, F., Micheletti, C., et al. (1998) *Dragging in Cabri and Modalities of Transition from Conjectures to Proofs in Geometry*, Procs of the 22nd PME Conf., Stellenbosch, South Africa, Vol. 2., 32-39.

Balacheff, N. (in press) Meaning: a Property of the Learner–Milieu System. Kilpatrick,J.,Hoyles,C,Skovsmose,O. *Meaning in mathematics education.* Kluwer.

Balacheff, N. and Sutherland, R. (1994) Epistemological domain of validity of microworlds: the case of Logo and Cabri-geometre, *Lessons from learning*, Lewis, R. and Mendelsohn, P. (Eds), Elsevier Science B.V. (N. Holland), IFIP, 137-150.

Clements, D. H. & Sarama, J. (1995) Design of a Logo environment for elementary geometry. *Jnl. of Mathematical Behaviour, 14*, 381-398.

DiSessa, A. (2000) *Changing Minds: Computers, Learning and Literacy.* MIT Press.

Dubinsky, E. (2000) Meaning and formalism in mathematics, *Int. Jnl. of Computers for Mathematical Learning, 5:* 211-240.

Harel, I. and Papert, S. (eds) (1991) *Constructionism: Research Reports and Essays*, Ablex Publishing Corporation, Norwood, New Jersey.

Healy, L. and Hoyles, C. (1999) Visual and Symbolic Reasoning in Mathematics: Making connections with computers? *M/al Thinking and Learning, 1*, 1, 59-84.

Hershkowitz, R. & Kieran, C. (2001) *Algorithmic and meaningful ways of joining together representatives within the same mathematical activity: An experience with graphing calculators*, Proceedings of the 25th PME Conference, Utrecht, The Netherlands, 1, 96-107.

Kynigos C. (1993) Children's Inductive Thinking During Intrinsic and Euclidean Geometrical Activities in a Computer Programming Environment, *Educational Studies in Mathematics, 24*, 177-197.

Kynigos, C., Koutlis, M., Hadzilacos, T. (1997) Mathematics with Component Oriented Exploratory Software, *Int. Jnl. of Comps for M/al Learning 2*, 229-250.

Laborde, C. and Laborde, J-M. (1995). What about a learning environment where Euclidean concepts are manipulated with a mouse? In A. diSessa, C. Hoyles and R. Noss with L. Edwards (Eds), *Computers for Exploratory Learning* (pp. 241–261). Springer-Verlag.

Mariotti, M. A. (2002) Influence of Technology Advances on Students' Math Learning. In L. English (Ed), *Handbook of International Research in Mathematics Education* , Mahwah, Lawrence Erlbaum.

Mariotti, M. A. (2000) Introduction to proof: The mediation of a dynamic software environment, *Educational Studies in Mathematics 44*: 25-53.

Noss, R. & Hoyles, C. (1996) *Windows on Mathematical Meanings*, Kluwer.

Schwarz, B. B. (1997) Understanding symbols with intermediate abstractions: An analysis of the collaborative construction of mathematical meaning, in C. Pontecorvo, R. Saljo and L. B. Resnick (eds.) *Tools, Discourse and Reasoning,* Springer Verlag, Berlin, pp. 312-335.

Vergnaud, G. (1987) *About constructivism,* Proceedings of the 11th PME Conference, Dordrecht, 43-54, Montreal.