

# TEACHERS' CONCEPTIONS OF MATHEMATICAL WORD PROBLEMS: A BASIS FOR PROFESSIONAL DEVELOPMENT

Olive Chapman

University of Calgary

*This paper reports on a study of mathematics teachers' thinking in the teaching of contextual or word problems [WP] with particular focus on teachers' conceptions of WP and the relationship to teaching. The 20 participants included Grades 1-12 preservice and inservice teachers. Data consisted of interviews and classroom observations. The findings indicated 8 ways in which the teachers conceptualized WP, e.g., WP as object and experience, and a model of WP as a nesting of mathematics and social contexts. These conceptions played a significant role in framing their teaching of WP in terms of 4 teaching perspectives, including a paradigmatic and a phenomenological approach. Implications for teacher development based on the findings are also discussed.*

Recent reform recommendations in mathematics education (e.g., NCTM 1989, 2000) assign a significant role to problem contexts in developing meaning for mathematics and to a problem-solving perspective of teaching and learning mathematics. Implementing such recommendations suggests an increase in importance in the use of a range of contextual problems or word problems [WP], "routine" or "non-routine", in the classroom. This paper considers the teacher as a basis for understanding the teaching of WP. The paper is based on a 3-year project that investigated teacher thinking in the teaching of WP. In particular, it reports on inservice teachers' conceptions of WP, the relationship to their teaching and implications for professional development.

## BACKGROUND AND THEORETICAL PERSPECTIVE

It would seem that, given their long history, WP should be easily recognizable and easily defined. But the definition/conceptualization of WP is not always clear-cut. For example, some consider WP as including problems normally in symbolic mode expressed in words, e.g., What is the sum of 8 and 5? Others consider them to be only those that are "story problems". This is reflected in the different ways WP are described in the literature, although the latter case tends to be the preferred view.

Leacock (1910), in his discussion of "the human elements in mathematical word problems", described WP as "short stories of adventure and industry with the end omitted" (p. 118). More recently, Verschaffel et al. (2000) offered the following:

Word problems can be defined as verbal descriptions of problem situations wherein one or more questions are raised the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement [p. ix].

Gerofsky (2000) in her study of WP from the perspective of genre theory concluded:

Literary analysis of word problems suggests that they are *like* religious or philosophical parables in their non-deictic, "glancing" referential relationship to our experienced lives, and in the fact that the concrete images they invoke are interchangeable with other images without changing the essential nature of the word problem or parable [p. 131].

WP have also been described in terms of the structural components that characterize them. Verschaffel et al. (2000, pp. x-xi) summarized these components as:

The mathematical structure: i.e., the nature of the given and unknown quantities involved in the problem, as well as the kind of mathematical operation(s) by which the unknown quantities can be derived from the givens. The semantic structure: i.e., the way in which an interpretation of the text points to particular mathematical relationships ... The context: What the problem is about ... The format: i.e., how the problem is formulated and presented. When WP are viewed as “genuine problems”, they are also associated with the position and state of problem solver required to solve them, e.g., the problem solver wants something but does not know immediately how to get it.

Studies on WP have traditionally focused on the learner to study cognitive and affective factors that aid or hinder his/her performance as a problem solver. For example, studies on arithmetic WP (Carey, 1991; Cummings, 1991; Debout, 1990; Reed, 1999; Verschaffel et al., 2000) have looked at the mathematical and linguistic structure of these problems in relation to the children’s performance; factors that affect the difficulty of the problem for children; strategies and methods children use; the errors children make in their solutions, and children’s suspension of sense making in doing WP. A similar situation has existed for studies on high school algebraic WP where, for e.g., the focus has been on students’ errors and methods in the translation process (Crowley et al., 1994; MacGregor & Stacey, 1993; Reed, 1999). Given this focus on the learner, studies on WP have generally ignored the classroom teacher. Thus, while these studies have enhanced our understanding of important issues associated with the learning of WP, they offer very little on the teacher and her/his role in teaching WP. This is likely to be a significant limitation in our understanding and means of improving the teaching of WP. The study in this paper is intended to make explicit aspects of teachers’ thinking and classroom behaviors that frame their teaching of WP.

The study, then, is framed in the context of WP and in the theoretical perspective of teacher thinking in which teachers are viewed as creating their own meaning to make sense of their teaching, i.e., a constructivist orientation of knowledge construction. The importance of researching the teacher is associated with the view that teachers are the determining factor of how the curriculum, mathematics in this case, is taught. This validates the importance to learn from teachers what they do and how they make sense of what they do in the classroom. This is reflected by the increased focus on researching the mathematics teacher in recent years. There is a growing body of literature on mathematics teachers’ content knowledge, beliefs, conceptions, classroom practices, learning, professional development and change (e.g., Lampert & Ball, 1998; Chapman, 1997; Fennema & Nelson, 1997; Leder et al., 2003; Schifter, 1998; Thompson, 1992; Tzur et al, 2001). These studies have provided us with insights on, for example, the relationship between beliefs and teaching, deficiencies in teachers’ content knowledge, and the challenges of teacher education and change. However, ongoing research to understand the teacher’s perspective of specific topics like the teaching of WP is important as we try to reform the teaching of mathematics.

## **RESEARCH PROCESS**

The 3-year study on teacher thinking in teaching WP followed a humanistic research approach (Chapman, 1999) framed in phenomenology (Creswell, 1998), i.e., the focus is on the participants’ meaning, what they value, and how they make sense of their

experiences. The participants were 20 teachers from different local schools. They included 6 pre-service and 14 in-service teachers at elementary, junior high and senior high school levels. The main criterion for selecting the teachers was willingness to participate. However, a subset of the Grades 7-12 inservice teachers were included because in addition to satisfying this criterion, they were considered to be excellent teachers in their school systems. All of the participants were articulate and open about their thinking and experiences with WP.

The main sources of data for the study were open-ended interviews, classroom observations and role-play (Chapman, 1999). Role-play scenarios allowed the teachers to act out, instead of talk about, a situation, e.g., presenting a WP to the class. Interview questions were framed in a phenomenological context to allow the teachers to share *their* way of thinking and to describe their behaviors as lived experiences (i.e., stories of actual events). The interviews examined the participants' thinking/experiences with WP in three contexts: (i) past experiences, as both students and teachers, focusing on teacher and student presage characteristics, task features, classroom processes and contextual conditions, (ii) current practice with particular emphasis on classroom processes, planning and intentions, and (iii) future practice, i.e., expectations. The interview did not suggest particular attributes of WP to talk about. Questions were often in the form of open situations, e.g., telling stories of memorable, liked and disliked classes involving WP that they taught, giving a presentation on WP at a teacher conference, and having a conversation with a preservice teacher about WP. Classroom observations focused on the teachers' actual instructional behaviors during lessons involving/related to WP. Special attention was given to what the teachers and students did during instruction and how their actions interacted. Complete teaching units over a 2-week period were observed for each teacher. Post-observation discussions with them focused on clarifying their thinking in relation to their actions.

The data (audio-taped transcripts and field notes) were thoroughly reviewed by the researcher and two research assistants working independently to identify attributes of the teachers' thinking and actions that were characteristic of their perspective of teaching WP. In particular, conceptions about WP were deduced from the data based on significant statements and actions that reflected judgements, intentions, expectations, and values of the teachers regarding WP that occurred on several occasions and in different contexts. These attributes were grouped into themes and validated by comparison of the findings by the three reviewers and triangulation of the findings from interviews, classroom observations and role-play.

## **FINDINGS**

The findings are presented only for the inservice teachers. There were three dominant features of all of the teachers' thinking that played a significant role in framing their teaching of WP – their conceptions of WP, their perceptions of students in relation to doing WP, and their conceptions and intentions of the WP-teacher-student relationships. The focus here is on their conceptions of WP. The following eight ways of characterizing WP emerged from the teachers' thinking and classroom behaviors. All quotes in the following sections are the teachers' thinking taken from the data.

## Conceptions of WP

(I) *WP as computation/algorithm*: This view is associated with the simplicity of the WP based on their transparent semantic structure, i.e., they have language that explicitly suggests the solution to the situation, e.g., “take away”, “putting together”. One high school teacher explained, “They're extra, they're not necessary, they're trivial and they do little, most of the time I think to enhance a topic.”

(II) *WP as problem*: This is viewed in three ways. (a) The relationship between student and problem: E.g., “All word problems are real problems if students have not encountered them before. ... I don't think there's anything in the problem that makes it necessarily routine or non-routine. ... No problem is routine if you've never seen it before.” (b) The nature of problem/solution: This is viewed in terms of two situations. First, there are “problems for which students must deduce a structure to determine a solution.” Second, there are “problems for which students must impose a structure on problem to create a solution.” (c) The teacher's intent: This relates to when and how a WP is introduced to students by the teacher, for example, a teacher could take a potentially routine WP and problematize it. So, “If they are given to students at the right stages as something beyond their level of experience at this time... [they] could be used to practice their problem solving skills.” “If you want it to be a problem solving type of question, it's all in how you present the question.”

(III) *WP as Enigma*: This view is associated with WP students cannot relate to contextually and/or mathematically. Such WP are “intimidating”, “threatening”, and can erode “students' confidence”. One high school teacher explained, “You are fearful of those problems because you don't understand where they're coming from. ... So it really becomes a problem because you can't make sense out of the wording.”

(IV) *WP as object*: This view treats WP as consisting of universal properties independent of the student. The key idea is that a WP “is more of a declarative statement” and has a unique or pre-determined interpretation of the mathematics and semiotic structures/ contexts established by the author or equivalent authority (eg. the teacher) of the WP. The goal of the student is to uncover/identify the designated meaning and solution of the WP.

(V) *WP as contextualized mathematics*: This view treats WP as a way to frame mathematics and “not seen as a separate topic”. The WP “should be done with every topic” and “not [the] end of a unit” or “not [as] a separate unit”. They should form the “basis for presenting each concept” and be intertwined with other concepts. “You always introduce a new concept or idea in a context of a WP.” “I come to realize that everything about mathematics is framed within word problems.” “It should be something that is integrated throughout the year and throughout each of the lessons.”

(VI) *WP as experience*: This view considers WP in terms of a phenomenological relationship between WP and student – e.g., a lived experience, real, dependent on student, linked to intention/interest/value. The meaning of the WP is personally determined and justified, i.e., it is dependent on the student and not the author of the WP. The meaning is what it calls forth in the mind of the student, the particular association or images it excites. In order for students to accomplish this in the context of a positive experience, the WP should: “Capture their attention.” “Invite them, intrigue them and prod them to want to solve it.” “[Be] the students' story.”

(VII) *WP as tool*: This view deals with the utility of WP for students' learning. In this context, there are two levels of WP – (a) those that are: “A means to apply concepts or practice a skill they have seen most recently in class.” (b) Those that are a means of exploring new mathematics and fostering mathematical thinking, e.g., “to get the kids to handle a new situation where it does not seem like anything that we’ve done before.”

(VIII) *WP as text*: This view considers WP as conveyors of knowledge. For example, “[A way] to transfer information to somebody else.” “A way to share mathematical experience with another.” “Stories from which you can extract mathematics.”

In addition to these 8 ways of viewing WP, figure 1 is a schematic model of a WP that emerged from the teachers' thinking and classroom behaviors. This model considers the WP in terms of its mathematics context and perceived social contexts. The mathematics context of the WP is situated in the problem situation, which is situated in a social context, which is (or should be) situated in the student's experiences, knowledge and ability. The problem situation is a specific case of the social context of the problem and acts as a bridge between personal and impersonal aspects of the problem, i.e., together they make the mathematics context meaningful. To illustrate the model, consider the example:

The perimeter of a pool table is about 7.8 m. Four times the length equals nine times the width. What are the dimensions of the table, in meters?

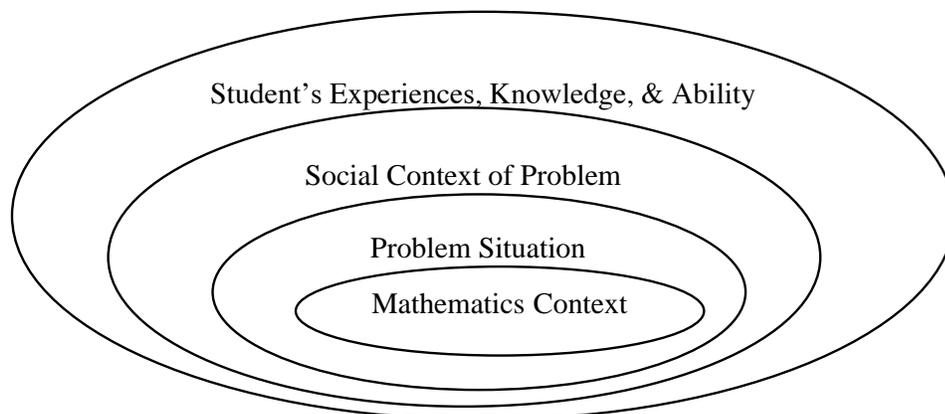


Figure 1

The social context could be, for example, playing pool, sport equipment, sports, and/or games. The problem situation is the pool table, i.e., a particular case of the social context. The mathematics context consists of two parts, “the math information” – perimeter, arithmetic/algebraic relationship and “the goal” – finding the dimensions of the table in meters. The students' experiences determine the aspects of the social context of the problem that emerge.

### Relationship to Teaching

Each of the preceding conceptions of WP had an impact on teaching in terms of how it was or was not enacted in the teachers' teaching. Most of the conceptions were present in all of the teachers' thinking, but they were emphasized to different degrees in each teacher's teaching of WP. This contributed to the uniqueness of each teacher. While a description of each case is important to understand the relationship between the

conceptions and teaching, it cannot be adequately provided here, thus the focus will be on those features that can be generalized to groups of the teachers.

<u>Paradigmatic [Pa]</u>	<u>Paradigmatic-Narrative [PaN]</u>	<u>Phenomenological [Ph]</u>	<u>Paradigmatic-Phenomenological [PaPh]</u>
<ul style="list-style-type: none"> <li>- Universal interpretation</li> <li>- Mathematical solution</li> <li>- Social context of WP may or may not match personal experience</li> <li>- Suppresses students' resonance in social context of WP</li> </ul>	<ul style="list-style-type: none"> <li>- Universal interpretation</li> <li>- Mathematical solution</li> <li>- Context of WP reflects personal experience</li> <li>- Allows resonance in social context of WP to share personal stories; to "socialize"</li> </ul>	<ul style="list-style-type: none"> <li>- Subjective interpretation</li> <li>- Mathematical and social solutions of WP</li> <li>- Social context of WP is direct match to experience</li> <li>- Allows resonance in social context of WP as basis for solution to WP</li> </ul>	<ul style="list-style-type: none"> <li>- Subjective interpretation</li> <li>- Mathematical solution</li> <li>- Social context reflects personal experience</li> <li>- Allows resonance in WP social context to critique/ revise context, examine assumptions, rule out social solutions of WP</li> </ul>

Figure 2. Teaching Perspectives of WP [*All items in table refer to WP.*]

There were 4 perspectives of teaching WP in which the teachers could be grouped to show the relationship of the more dominant features of their conceptions of WP on their teaching, i.e., *paradigmatic*, *paradigmatic-narrative*, *phenomenological*, *paradigmatic-phenomenological* [figure 2]. These perspectives are characterized by the degree to which the teachers viewed WP as object versus experience, which also influenced how the other conceptions of WP were enacted in their teaching. Figure 2 summarizes some of the key attributes of each perspective that relate to how the teachers presented WP and/or allowed students to interact with the WP in their teaching. The Pa approach emphasizes WP as object, i.e., the WP has a universal interpretation, a particular mathematical solution and is not influenced by personal social context. Thus Pa suppresses students' interaction with the social context of the WP. The Ph approach emphasizes WP as experience, i.e., what is meaningful from the student's perspective, thus multiple interpretations and both mathematical and social solutions of the WP are recognized. Ph allows students to interact with the social context of the WP.

The PaN and PaPh approaches integrate aspects of Pa and Ph. PaN is unique in that it allows the social context of the WP to provide a basis for students to share real-life stories triggered by it during teacher-led discussion of the WP, but with no link to the problem solution. PaPh allows students to critique the social context of the WP and to revise it, if necessary, to make it more meaningful to them as part of the solution process. In relation to the WP model (figure 1), Pa recognizes the mathematics context, problem situation, and student's knowledge while the three others recognize all of the components of the WP model, but in different ways. The Grades 1-2 teachers were more Ph, the Grades 3-6 teachers were more PaN, the Grades 7-12 teachers who were recommended as excellent teachers were more PaPh while the others were more Pa. Most of the PaPh teachers started as Pa and grew into the other based on their perceptions of the students,

in particular what motivated them and helped them to learn. The Pa and PaN teachers taught the WP the way they solved them, while the Ph and PaPh teachers did not, but focused on the way the students were interacting with them. In general, the Ph and PaPh teachers were also more flexible, student-centered and inquiry oriented in their teaching than the others.

### **IMPLICATIONS FOR PROFESSIONAL DEVELOPMENT**

The findings indicate that teachers' conceptions of WP are not limited to a simple definition based on structural features, but have scope and depth in a pedagogical context. The findings also offer a possible range of ways of thinking about WP that teachers could hold and suggest that there is an important relationship between the teachers' conceptions of WP and their teaching that could limit or enhance how WP are perceived, experienced and learnt by students. This has implications for teacher development in the teaching of WP and problem solving, e.g., explicit consideration of the teachers' conceptions of WP in professional development activities may be necessary particularly when fundamental changes in teaching are the desired goal.

The findings offer information of ways of thinking and teaching that could be used to enhance how we work with teachers on two levels. First, although these ways are not intended to state how things should be but how they are and could be, they could form a basis for helping teachers to broaden their perspectives of WP. Second, and more importantly, the findings offer particular structures against which other teachers could examine their own perspectives and assumptions, either through reaction against or resonance with them, to gain understanding of their thinking and teaching.

Some activities that could facilitate such use of the findings are: (1) Ask teachers to individually make up WP and to reflect on and describe what they thought about to do so. They then share and reflect on their thinking in small groups. (2) Given a WP, ask teachers to make up one with a similar structure/context and a different structure/context, then reflecting on and discussing their thinking. (3) Given a set of WP that reflects the structure of figure 1 in different ways, e.g., different mathematics context and WP situation, ask teachers to determine and discuss the nature of each WP without actually solving them, then discussing if, when, and how they will use them. In these activities, the 8 conceptions of WP and figures 1 and 2 can provide a basis for interpreting what the teachers do, to pose questions to facilitate depth in their reflection and to allow them to become aware of alternative ways of thinking of and teaching WP.

### **References**

- Carey, D. (1991). Number sentences: Linking addition and subtraction word problems. *Journal for Research in Mathematics Education*, 19, 4.
- Chapman, O. (1999). Researching mathematics teachers' thinking. *PME23*. Haifa, Israel: The Technion - Israel Institute of Technology, Vol. 2, 185-192.
- Chapman, O. (1997). Metaphors in the teaching of mathematical problem solving. *Educational Studies in Mathematics*, 32, 201-228.
- Creswell, J.W. (1998). *Qualitative inquiry and research design*. London: Sage Publications.

- Crowley, L., Thomas, M., and Tall, D. (1994). Algebra, symbols, and translation of meaning. *PME18*, Vol. 2. Lisbon: University of Lisbon.
- Cummings, D. (1991). Children's interpretation of arithmetic word problems. *Cognition and Instruction*, 8, 261-289.
- Debout, H. (1990). Children's symbolic representation of addition and subtraction word problems. *Journal for Research in Mathematics Education*, 21, 123-131.
- Fennema, E. and Nelson, B.S. (Eds.) (1997). *Mathematics teachers in transition*. Mahwah, NJ: Lawrence Erlbaum.
- Gerofsky, S. (2000). The word problem as a Genre in mathematics education. *CMESG/GCEDM Proceedings 2000 Annual Meeting [pp.129-134]*. Edmonton, AB: University of Alberta.
- Lampert, M. & Ball, D. (1998). *Teaching, multimedia, and mathematics: Investigations of real practice*. New York: Teachers College Press.
- Leacock, S. (1910). *Literary Lapses: A book of sketches*. Montreal: Gazette Printing Co.
- Leder, G.C., Pehkonen, E., and Torner, G. (Eds.) (2003) *Beliefs: A hidden variable in mathematics education?* Dordrecht, The Netherlands: Kluwer Academic Press.
- MacGregor, M. and Stacey, K. (1993). Cognitive models underlying students' formulation of simple linear equations. *Journal for Research in Mathematics Education*, 24, 3, 217-232.
- National Council of Teachers for Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.
- National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM
- Reed, S.K. (1999). *Word problems: Research and curriculum reform*. Mahwah, NJ: Lawrence Erlbaum.
- Schifter, D. (1998). Learning mathematics for teaching: From a teachers' seminar to the classroom *Journal of Mathematics Teacher Education*, 1 (1), 55-87.
- Thompson, A. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D.A. Grouws (Ed.) *Handbook of research on mathematics teaching and learning*. NY: Macmillan.
- Tzur, R., Simon, M, Heinz, K., Kinzel, M. (2001). An account of a teacher's perspective on learning and teaching mathematics: implications for teacher development. *Journal of Mathematics Teacher Education*, 4 (3), 227-254.
- Verschaffel, L., Greer, B. and DeCorte, E. (2000). *Making sense of word problems*. Lisse, The Netherlands: Swets and Zietlinger Publishers.