



JOURNALING AS A TEST PREPARATORY MEASURE IN SECONDARY  
MATHEMATICS: SUCCESSFUL STUDENT STRATEGIES

A thesis submitted in partial fulfillment  
of the requirements for the degree of  
Master of Education

By

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Lloyd Emery Nickerson II ENTITLED Journaling as a Test Preparatory Measure in Secondary Mathematics: Successful Student Strategies BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Education.

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## ABSTRACT

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Journaling as a Test Preparatory Measure in Secondary Mathematics: Successful Student Strategies.

This study examines a math journal writing assignment comparing how high school (grades 10 through 12) algebra students who performed well and students who performed poorly on traditional mathematics tests constructed their corresponding journal entries. Statistically significant differences found indicated that students who performed well on the tests were more likely to have originally composed the text and examples in their journal entries, and students who performed poorly were more likely to have copied much of the mathematical language and examples in their journal entries from their textbooks. Students who performed well on the test were also more likely to include examples accompanied by explanation for each step toward a solution. An assignment involving several such explanatory examples could perform a same or similar function as the longer journal assignment examined in this study.

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## DEDICATION

I dedicate this work *to my Lord and Savior Jesus Christ*, to whom I owe all my talents and abilities; *to my many teachers and professors*, who have provided countless examples from which to model my own teaching; *to my students*, especially those who, though they complain about my asking them to write in math class, eventually quietly admit that writing helps them learn; *to my wife, Allison*, who puts up with my countless hours of study, revision, frustration and perfectionist tendencies and still loves me despite my faults; and *to my parents*, who instilled in me during my childhood the value of education and the drive to work excellently in all endeavors.

## I. INTRODUCTION TO THE STUDY

### Introduction to the Investigation

Massive writing-across-the-curriculum and write-to-learn campaigns began during the 1970's and 1980's by organizations such as the National Writing Project encouraged schools to utilize writing in all core disciplines as an important mode of learning (Nagan, 2005). Many teachers have since incorporated writing into both primary and secondary math courses. The National Council of Teachers of Mathematics [NCTM] even lists communication in mathematics, with writing in mathematics as the principal goal, as one component in overall mathematics literacy. The NCTM holds that writing allows students to clarify and develop their understanding of mathematical concepts (NCTM, 2000).

Writing has been adopted and integrated into the mathematics curriculum for a variety of purposes. Some educators have used writing as a means to adjust attitudes toward and perceptions of math (Sewell, 2006; Furner & Duffy, 2002; and Mason & McFeetors, 2002). Others have assessed student thinking (Baxter, Woodward, & Olson, 2005; Burns, 2005; Gordon & Macinnis, 1993; and Countryman, 1992) or used writing as a learning mechanism for new concepts and skills (Marlow, 2006; O'Connell, Beamon, Beyea, Denvir, Dowdall, Friedland & Ward, 2005; Williams, 2003; Brandenburg, 2002; Cooley, 2002; Koirala, 2002, Burns & Silbey, 2001; McIntosh & Draper, 2001; Baker, 1999; and Fuqua, 1997).

## Definition of Terms

For the purposes of this study, writing in mathematics refers to any student-generated product involving the use of non-symbolic language, though not necessarily entirely exclusive of symbolic language, stemming from an assignment, activity, or assessment in a mathematics class or course. Writing in mathematics is the communication of mathematical ideas in words, and as such it can take many forms. Students can take notes from the board, create their own word problems, describe a mathematical concept, record entries in a journal about mathematical concepts, record entries in a journal regarding their feelings and perceptions of math class or a mathematical concept, write a formal report on a historical mathematician or other event involving mathematics, or write a report on a class or individual mathematics project.

When describing the specifics of this study, the math journal assignment (or math journal or journal assignment) is a loose-leaf notebook that students maintain with entries for each section of each chapter covered in the course. The sections and chapters are taken directly from the course textbook, *Algebra 2 for Christian Schools*, by Pilger and Tagliapietra (2000). A section math journal assignment (or section math journal or section journal) is a math journal assignment entry covering one textbook section. An acceptable section journal entry includes a verbal explanation of the ideas covered in the section and at least one example illustrating each major section idea.

Explanatory examples are illustrative examples of a mathematics concept integrating both verbal and symbolic explanation. Explanatory examples include a symbolic solution to the example as well as a line by line textual commentary explaining how and why the author performed each step toward the solution.

### Statement of the Problem

Despite the fact that there are an abundance of articles in the literature touting writing in mathematics, there are a very limited number of studies investigating any concrete link between student writing and student achievement of specific learning goals. Most articles pertaining to writing in mathematics either investigate the extent to which teachers use writing in mathematics and what form that writing takes (e.g. Quinn & Wilson, 1997; Silver, 1999; and Ntenza, 2006), or describe a specific use of writing in the mathematics classroom, usually the classroom(s) of at least one of the study's authors (e.g. Barlow & Cates, 2006; Brown, 2005; and O'Connell et al, 2005).

The literature is in need of studies investigating writing with less emphasis on blindly extolling its assumed benefits. While I am generally in agreement with the literature's faith that writing is helpful in learning mathematical concepts, this study investigates whether there is a method or strategy of writing that helps students learn more effectively than other methods or strategies of writing. Specifically, this study will primarily seek to answer the question of whether there are more and less efficacious styles, methods, and characteristics found in student created math journal writing assignments that are directly linked to high performance on a paper and pencil test covering the same mathematical concepts. An affirmative answer to this question could be considered indirect evidence that writing, or at least some types of writing, aides students in achieving specific learning goals in the mathematics classroom. This study will secondarily attempt to develop a more time-efficient writing assignment incorporating the style and characteristics of journal entries from students performing well on a traditional math test. Such a writing assignment, where all students are taught

to utilize the writing techniques of high performing students, could serve to increase the test scores of low performing students.

### Scope of the Study and Delimitations

This investigation examined a math journal writing assignment in a high school Algebra II class with the purpose of identifying ways of writing the journal assignment that seemed to help students when they were faced with a traditional paper and pencil test. I used as the subjects for this study the students from my Algebra II class at a Midwest Christian school. The results of this study may, therefore, generalize to other Christian high schools. However, further research in varied school contexts would be necessary to confirm the results of this study before applying them to the general high school population of the United States.

### Significance of the Study

It was of value to determine whether there were specific ways of writing the math journal assignment that were more and less effective for student learning for two reasons. First, my desire as a teacher was to encourage my students to write in a manner that best helped them succeed in the learning goals that I set for my classroom. While on a cursory level this may seem self-serving, I hoped that any significant results from this study could also be exported into other writing mathematics classrooms. Second, my math journal assignment has been critiqued by other math educators as being too lengthy and time-consuming for students to complete and teachers to grade. It is difficult to debate this point as it does require a significant amount of time to grade journal assignments. A study outlining the most effective ways of writing the math journal assignment leading to better student understanding of mathematical concepts could result

in redesigning the journal assignment into a more focused, less laborious, and more efficient project.

### Methods of Procedure

I used a grounded study approach to study the math journals from my Algebra II class. I wanted to approach the writing assignment with fresh eyes unpolluted by other researchers' ideas as to aspects of writing that aid students in learning mathematics. As such, I delayed performing a literature review until after I had analyzed the data from the journal assignments from my classes and had come to some conclusions regarding the aspects of my students' math journal assignments that seemed linked to students' achievement of learning goals. (Student learning goal achievement was assessed via a traditional paper and pencil chapter test. Specific test questions can be found in chapter three of this study.)

### *Assigning the Math Journals*

After my students and I had worked through the material in chapter one of our textbook, the class and I skimmed through the chapter together and developed a list of all the main ideas from each section. I then introduced the math journal assignment as a review mechanism for test preparation. I explained to my students that they would keep a math journal to help them make sure they had learned all the necessary mathematical concepts prior to my testing them over those concepts. I encouraged them to write their journals as though they were writing an explanation for another student who was struggling to learn the same material. I provided the students with several quality examples of section journal assignments from students in prior years who had given me permission to use their work in this manner. This gave the students some idea of

specifically what I was looking for in the assignment. The students then spent two class days writing the math journal assignment for chapter one in groups. I felt two days were justified for this initial assignment as this was the first time many of my students had been asked to write anything of significant consequence in a math class.

During all subsequent chapters I encouraged my students to maintain a running list of the main ideas in each section as we progressed through the chapter to use when writing their math journals. A couple students semi-regularly wrote their section journal assignments alongside their homework assignments (homework assignments were typically around twenty problems, half easy/medium and half medium/hard in difficulty) each night. I did not encourage or discourage this, but I did encourage them to at least discuss their list of main ideas with the other members of their three or four person mixed ability math group before writing their journal entry. I thought this practice might prevent students from accidentally ignoring important mathematical ideas in the chapter. For all chapters except chapter one, I provided only one review class period directly prior to test day for the writing of the math journal assignment. Students who did not write their section math journal assignments as we progressed through the chapter were generally unable to complete the entire assignment during one class period, so the math journal assignment was often completed as homework the night before the chapter test. Though one could certainly argue that this was procrastination on my students' part, I did not explicitly discourage it, as I knew that it would require them to engage the material in the chapter immediately prior to testing. In regards to the students who wrote their section journal assignments each night as they completed their nightly homework, I encouraged them to utilize their math journal assignments as study guides.

### *Grading the Math Journal Assignment*

I collected students' math journals once each chapter (approximately once every two or two and a half weeks) on the same day that students took their chapter tests. I read and commented on the journals and then returned them to my students generally within one or two class days. I did not accept any journals late as this would have violated the purpose of the journal assignment, a review mechanism for test preparation. I used a point system for grading, and students received twenty-five points for completing their math journal assignment with sufficient verbal explanation and at least one example illustrating each concept. Assuming students had a verbal explanation and example for each concept, this was largely a completion grade. In contrast to the points assigned to the journal assignment, a test was worth one hundred points, and most homework assignments, each coupled with a brief daily quiz, were worth five points. There were usually around ten homework assignments in each chapter translating into approximately fifty homework points per chapter.

### *Collecting Data*

I collected two types of data from my class in order to analyze the journal assignment's utility in student attainment of learning objectives. First, I retained all chapter tests so that I could evaluate whether my students had met my learning objectives. Second, I copied all my students' journal assignments prior to returning them to the students. I then qualitatively and quantitatively analyzed the section journal entries and the test questions corresponding to the same learning goals looking for similarities and differences in the ways that students who performed well on the test and the ways students who did not perform well on the test constructed their journal assignments.

## II. PLENARY LITERATURE REVIEW

### How Relevant Theory Affects this Study

Constructivism, a theoretical framework originating with Piaget that is concerned with the nature of learning, is a general term encompassing three different philosophies of learning. Endogenous constructivists are concerned with developing cognitive conflict in order to develop internal constructions in the learner. Dialectical constructivists view learning as a social exchange of ideas where each learner's knowledge is modified and adjusted when encountering other learners' knowledge. Exogenous constructivists hold that learners construct an internal reality, similarly to endogenous constructivists, but that the constructed internal reality reflects an objective external reality (Applefield, Huber, & Moallem, 2000).

I generally subscribe to an exogenous constructivist position that learners construct an internal reality, but in doing so they reference an actual, real, objective reality not determined by the learner him or herself. My exogenous constructivist leanings were the impetus for my developing and implementing the math journal assignment in my classes. The math journal assignment asked students to evaluate what they had learned over the past chapter (external reality) and construct an original representation of that knowledge including both written explanation and symbolic examples (representations of each student's internal understanding). I agree with Talman that "writing about mathematics forces construction of understanding, because we cannot

write coherently about something we do not understand” (Talman, 1992). I believe that writing is one of the best methods available to verify if each of my students has developed an internal construction that corresponds to each of the mathematical concepts covered in class. This view, that writing is a documentation of knowledge construction by a learner, since “writing involved deliberate analytical action on the part of the producer,” was a view originally held by Vygotski (Pugalee, 2001, p. 236).

#### Inductive Summary of the Relevant Literature

The literature regarding writing in math is fairly broad. Articles and studies range from discussing the effects of writing on students with disabilities to suggesting specific journaling prompts. There are two basic types of research about writing in math: teacher-centered studies and teaching-centered studies.

##### *Teacher-Centered Studies*

The first type of study concerned itself with the use of writing by math teachers and their attitudes toward and prevalence of using writing in mathematics. Quinn and Wilson (1997) investigated teachers’ attitudes toward writing in mathematics. They found that teachers on every level, primary, junior high, and high school, had similar, positive attitudes toward integrating writing into math classes. However, they also found that teachers tended to include writing assignments in their math class only once every couple of weeks. Teachers cited time limitations and students’ writing inadequacies as their principal reasons for not including writing more often in their lessons. The study concluded that more inservice training was necessary in order to cause teachers’ beliefs (writing in math is desirable) to match up with their practice (writing in math is infrequently assigned).

Seto and Meel (2006) agreed that many teachers were resistant to integrating writing into math because of time demands. They studied one teacher's journey toward implementing writing in her mathematics classes. Seto and Meel concluded that it was possible to creatively integrate writing into mathematics without too great an increase in time spent grading and/or reading the students' writing. They, along with the teacher-subject of their study, felt that the benefits of adding writing to the mathematics curriculum far outweighed the small amount of increased grading time.

Other researchers studied the prevalence of teachers using writing to teach math and the various types of writing produced in math classes. Silver (1999) performed a survey of mathematics teachers asking what types of writing assignments they utilize in their classrooms. She found that over half of the teachers she surveyed had either never heard of incorporating writing assignments into mathematics or had never or hardly ever used them. However, 37% of the teachers surveyed said that they used writing assignments frequently in their classes. Additionally, she noted that younger teachers, those less than forty years of age, were much more likely to include writing and other discovery learning assignments in their mathematics classes.

Davison and Pearce (1988) studied United States' junior high classrooms to determine the nature and amount of writing assignments in mathematics classes. They found that there were several distinct categories of student writing in mathematics. Specifically, mathematical writing could be copying or transcribing information, translating mathematical symbols into words, summarizing, writing test questions or word problems, or creative writing. Birken (1989) furthers this idea of multiple types of mathematical writing. He found that writing in math class could be expressive, informal,

in-class assignments; homework calling for interpretation, analyzation, or reflection; essay questions on exams; or formal reports. Shephard (1993) agreed that there were different forms of mathematical writing, but suggested that some forms of mathematical writing were more effective for student learning than other forms. Specifically, he held that transactional writing (informational and explanatory writing), when compared to other forms of writing in math classes, best actualized cognitive change.

Clarke, Waywood, and Stephens (1993) performed a case study of one school that had integrated journal writing into its math classes school-wide. They examined writing samples and conducted interviews with students and teachers from grades seven through twelve. The researchers found that students' journal entries could be classified into three progressively complex categories: recounting, summarizing, and dialoguing.

Finally, Ntenza (2006) studied types of writing produced in seventh grade South African classrooms. Ntenza found that the use of writing in mathematics was very sparse, and very rarely included any writing in the learners' own words. The researcher concluded that the primary reason for this in South Africa was the lack of appropriate resources for teachers, lack of teacher training, and in many cases the lack of basic needs such as the use of a photocopying machine.

#### *Teaching-Centered Studies*

The second major type of study on writing in mathematics in the research literature focused on ways of integrating writing into math class. The primary focus of these articles was a specific methodology leading to writing in math class. Various authors supported the use of writing prompts, poetry, creative writing, and journaling as successful ways of incorporating writing into math class.

### *Writing Prompts*

Several authors wrote about general ways to give student assignments requiring writing. Burns (2004), Silbey (2003), and O'Connell et. al. (2005) all advocated the use of partners and small group discussion prior to a writing assignment. This talking time allowed students to formulate their thoughts in an informal manner before formally writing them down.

Some authors specifically studied the use of writing prompts leading to writing assignments in math class. Sjoberg, Slavit, Coon, and Bay-Williams (2004) reported that the quality of writing they received from students was directly related to the quality of the writing prompt they used to ask the students to write. They found that when the writing prompt was clear and some modeling activities illustrating the teacher's expectations were performed in class, class writing improved dramatically. Ryan, Rillero, Cleland, and Zambo (1996) also gave general suggestions for writing prompts (e.g. avoid "yes" or "no" questions, use questions that are relevant to the students, and ask questions requiring opinion responses) as well as specific examples of good writing prompts (e.g. "Describe a practical career use for measuring the surface area of an object" [p.79].).

Both Kelly and LeDocq (2001) and Weber (2005) described specific assignments and courses that integrated writing into their undergraduate mathematics programs. Kelly and Ledocq described a writing intensive four-semester introductory college mathematics sequence where students were introduced to mathematical writing gradually and systematically rather than allocating mathematical writing to one writing-intensive course. Writing prompts and the details of each assignment were discussed. The researchers considered the length of the sequence of writing intensive courses the primary

strength of their program, as their students had plenty of time to develop their writing skills. They felt that their students showed significant growth as mathematical writers between the beginning and final semesters of the sequence. Weber (2005) discussed assignments that she used in her college geometry class to teach mathematical writing. She incorporated a variety of writing prompts, often related to required readings, into her course. She also claimed that her students dramatically improved as mathematical writers as a result of the intensive writing requirements of her course.

### *Poetry*

Labonty and Danielson (2004), Triandafillidis (2006), and Keller and Davidson (2001) all encouraged the use of poetry in mathematics class. Labonty and Danielson argued that there were parallels between the abstract language used in poetry and the abstract language of mathematics. They also provided an extensive list of sources from which to obtain poetry concerning mathematical topics. Triandafillidis (2006) also advocated the use of poetry in mathematics class, again suggesting that both mathematics and poetry were abstract subjects to many students, and combining them could help solidify both disciplines in students' minds. Additionally, Triandafillidis thought that students must thoroughly understand a mathematical principle before they would be capable of creating an original poem about it. Keller and Davidson (2001), on the other hand, discussed a poetry project in their classes that required students to use mathematical terms (from a provided list) in a poem about non-mathematical subject matter. They reported that the assignment helped their students relate mathematics concepts to the world outside of math class and that their students enjoyed and appreciated the assignment.

### *Creative Writing*

Uy and Frank (2004) related a creative writing method where students were asked to finish the beginning of a story in discussion groups. The students, preservice teachers, were to include mathematical principles or problem solving in their endings. The authors of the study noted that in creating original endings the students demonstrated higher order thinking processes.

Both Barlow and Cates (2006) and Munakata (2005) advocated children creating their own word problems to illustrate their math class's current mathematical concepts. Barlow and Cates gave elementary students the answer to a problem (e.g. 45 red cars) and had the students create a word problem that culminated in the given answer. They found that students were successful in the higher order thinking required in problem writing. They recommended the method for all elementary classrooms.

Munakata (2005), on the other hand, reported a cooperative learning method where a teacher placed students in groups and gave each student in the group one essential clue to the solution of a complex logic problem. Students could talk about their clue with their group but could not show the card containing their clue to the group (thus requiring group discussion and participation from all group members). After students successfully solved the group logic problem, the teacher gave a homework assignment requiring each student to write his or her own cooperative logic problem. Munakata purported that the method was a good way to incorporate cooperative learning into the math classroom and also cause students to use their mathematics skills in novel ways, encouraging thinking.

Finally, Kasman (2006) described a writing assignment used in an undergraduate

modern algebra course where she required students to critique mathematical proofs containing errors. The author noted that over time her assignment evolved more specificity as she realized the quality of student response was significantly linked to the quality of her writing prompt, but she also found it interesting that in critiquing a fictional student's work, students were often able to diagnose and criticize errors that they themselves tend to make. She noted that student's enjoyed the creative aspect of critiquing fictional and error-laden proofs.

### *Journaling*

A plethora of researchers and authors recommended the use of journal writing in math class. However, stated reasons for incorporating a math journal and the purported benefits of incorporating journal writing into math class varied widely. Authors advocated the use of mathematics journal writing to adjust attitudes and behaviors, review concepts already learned, integrate true-to-life applications, examine student thinking, and learn new concepts.

#### *Attitude and Behavior Adjustment.*

Furner and Duffy (2002) suggested that using math journals where students can record their thoughts, feelings, and attitudes toward mathematics and mathematics class could be helpful in overcoming math anxiety in students. They claimed that this could be particularly helpful for students with learning disabilities. Sewell (2006) also supported the use of journaling in math class to promote positive attitudinal change toward mathematics. She studied thirteen seventh grade students and concluded that prompted journaling appeared to improve some of her students' preference for math class, self perception as math students, anxiety toward math class, and even comprehension of

mathematics concepts. Mason and McFeetors (2002), on the other hand, supported the use of journal writing as a method for requiring students to examine the way their study habits influence their progress in math class. The researchers reported that their students realized through prompted journaling the relationship between preparation/diligence in math class and the grades they received.

*Reviewing concepts.*

Brown (2005) suggested using a math journal for reviewing a test that had been handed back to students. He required students to take their graded tests home and complete his test aftermath assignment due the next day. The test aftermath was a journaling assignment where students wrote about their strengths and weaknesses on the test, demonstrated their understanding of a concept that was not included on the test, and re-worked one missed problem from the exam correctly. Brown felt that his assignment improved his students' ability to communicate mathematics.

Hartz (1992) and Meel (1999), on the other hand, took a more proactive approach toward journaling to review. Hartz gave weekly writing assignments asking his students to write a summarized form (an "abstract") of the week's lessons and their response to it. He claimed that the abstracts increased his students' interest in the course and forced them to review regularly, preventing them from falling irreparably far behind. Meel (1999) also required weekly writing assignments summarizing class material but additionally asked his students to write about their difficulties with the material and what they were planning on doing to address those difficulties. He claimed that these additional parts to the summary assignment led to candid communication between the teacher and student and promoted an atmosphere where contributions, suggestions,

opinions, and student ideas were welcomed.

*Real Life Applications.*

Albert and Antos (2000) used a math journal to help students connect the mathematical concepts in their classroom to their students' daily lives. At least two students took home notebooks each day where they recorded specific instances when they used math in their outside-of-school life. The subsequent day's class period began with a recounting of mathematics applications from students who had taken the journals home the previous day. The authors reported that this was an excellent and successful method of connecting math to the real lives of their students.

*Examining Student Thinking.*

A number of authors encouraged the use of journaling to examine student thinking. Baxter, Woodward, and Olson (2005) studied four low-achieving students and their journal entries for a year and found that journals were a valuable method for student-teacher communication. They felt that journals particularly increased the amount of communication between low-achieving students and the teacher. Journals, according to the authors, provided a private channel for encouragement and teacher-suggestions where students did not fear their peers' responses, and where teachers could examine students' thinking without fear of humiliating the student.

Gordon and Macinnis (1993) felt that dialog journals, in which the primary purpose was two way student-teacher communication, illuminated gaps and strengths in student knowledge, allowed students to assess their own learning and learning processes, and provided a means for teachers to understand students' thinking processes. Burns (2005) maintained that writing assignments in general were an excellent tool to delve into

her students' thinking. Drake and Ammspaugh (1994) also held that writing in math class, particularly if used regularly, would become a learning tool for students and would also give a teacher clear insight into students' thinking. They felt that this knowledge of student thinking would increase the effectiveness of the teacher. Hopkins (1997) concluded that journaling not only provided a window into student thinking, it performed essentially the same task as an individual diagnostic interview with each student, only requiring significantly less time. Schaffter (2002) also felt that journaling allowed for increased student teacher communication. She claimed that journal writing led to a better teacher-understanding of the students' understanding as well as improved students' organization, writing, and mathematical confidence.

Finally, Countryman (1992) authored a book on writing in mathematics and devoted two chapters to the subject of journaling. She felt journals allowed teachers to understand how their students were thinking. According to Countryman, journals also allowed students to clarify their own thinking.

*Journaling to Learn.*

Finally, journaling to learn mathematics was the most common suggestion in the literature. Some researchers have explored the use of journaling in learning elementary mathematics. Fuqua (1997) utilized a kindergarten class problem solving notebook where she encouraged students to record the results of various problem solving activities. She felt that the problem solving book helped her students develop problem solving skills. O'Connel et. al. (2005) discussed the conclusions of a group of elementary teachers who had introduced writing into their mathematics teaching. One of their conclusions was that writing about problem solving and listening to what other students

had written about problem solving helped their students learn to be better problem solvers. Baker (1999) qualitatively studied the use of math journals with eight second grade students and felt that the math journals effectively showed progress in each student. Burns and Silbey (2001) wrote that math journals were a good tool to foster thinking and develop problem solving skills. Finally, Marlow (2006) maintained that although writing is a necessity in the mathematics classroom, the writing must be meaningful and interesting to students. According to Marlow, teachers must construct writing assignments in a way that learners gain a sense of purpose in completing the assignment.

Other researchers have examined the use of journaling to learn in higher mathematics. Cooley (2002) studied journaling assignments integrated into an undergraduate calculus course. She found that student writing assignments improved over a semester of use. Students began the semester by simply describing concepts, but by the end of the semester, students gave more examples, made more connections between concepts, and made more generalizations in their writing. To Cooley, this demonstrated that the students had adopted the writing assignments as a way of constructing knowledge and demonstrating their newly constructed knowledge. Britton (1992) also wrote about his experience with journaling in undergraduate mathematics. He felt that students learned more effectively if they were given an opportunity to describe their learning.

Koirala (2002) analyzed over 1800 journal entries from a mathematics class directed toward preservice elementary teachers. Students used their journals to share both cognitive and affective ideas with the instructor. The math journals were, then, effective tools for promoting thinking and therefore learning. Koirala also recognized

that journaling allowed valuable communication between students and the teacher, and, even though such communication could be very time-consuming on the instructor's part, such communication improved students' mathematical understanding.

Williams (2003) performed a controlled study to determine whether having students write about their problem solving processes helped them become better problem solvers. He found that the students who wrote about their thinking processes improved in their problem solving ability to a greater degree than students who did not write about their thinking processes. McIntosh and Draper suggested using frequent "learning logs" to allow students to communicate using mathematics. They maintained that such frequent writing assignments encouraged students to "clarify, refine, and consolidate their thinking" (2001, p. 554).

Brandenburg (2002) wrote that including writing in her upper level high school math classes (precalculus and calculus) increased their comprehension, ability to explain math concepts, and ability to make connections between mathematical ideas. She suggested the use of journals and writing portfolios in upper level math classes. She required her students to present a typed, professional portfolio explaining each mathematical concept they had learned over the course of the semester.

Finally, Porter and Masingila (2000) compared students from two calculus classes, one including writing assignments and one not requiring writing assignments, in an attempt to determine if the act of writing actually improved student learning. The non-writing class included class discussions on topics similar to the writing class's writing assignment topics. Porter and Masingila found that there was no difference in learning between the two groups. They concluded that writing itself may not actually help in

learning mathematical concepts, but that the writing process, where students struggle to communicate mathematical ideas to others, may be the actual source of learning.

#### How the Literature Provides a Foundation for this Study

Various researchers have written numerous articles and performed studies supporting the use of writing in mathematics class. However, there was little research on the analysis of the texts of math writing; most articles and studies simply involved description of writing activities in math classes (Pugalee, 2001). This study sought to begin to rectify that situation by identifying characteristics of journal writing linked to successful performance on more traditional mathematics progress indicators, paper and pencil tests. Since there seemed to be no objection in the literature to using writing in mathematics class, I felt that it would be appropriate to investigate what types of writing led to good scores on traditional exams. Certain traditional style mathematics tests (e.g. the SAT, ACT, and state proficiency tests) will be a part of most high school students' educational experience for the foreseeable future, so such research was necessary as teachers continue to incorporate writing into more and more mathematics classrooms.

### III. METHODOLOGY

#### Introduction to the Method

A grounded theory design is a design used to inductively elucidate a theory or explanation directly from gathered data. A researcher using grounded theory methodology attempts to set aside any previously held beliefs about his or her subject(s) and look with untainted eyes at collected data with the explicit goal of forming a theory or explanation that encompasses all of it. However, the underlying hope of a researcher is that a novel explanation of a phenomenon may rise from the study that will drive future research to confirm or discredit the developed theory (Johnson & Christenson, 2004).

#### Rationale for the Method

Though I did not know for sure whether a math journal assignment was wholly or even partially helpful for students in their attainment of mathematical knowledge at the outset of this study, I believed that a math journal assignment was helpful for knowledge attainment. I based this belief primarily on personal experience in reading students' self-generated explanations of math concepts and my own exogenous constructivist views of learning. For the purpose of this study, I assumed that the math journal was at least partially helpful for learning, though I did not know what aspects of the math journal were helpful and what aspects were not helpful. I felt that a grounded theory study looking for differences between the math journals of students who experienced success and students who did not experience success on a test covering the same topics as the

math journal might uncover the specific aspects of the math journal assignment that are most helpful for learning. As I looked for different strategies and ways of writing the journals between the two groups, I thought that I could potentially use knowledge of these differences in order to refine the journal assignment to make it less burdensome and more helpful to all students.

### Population of the Study

I drew my sample from a small, suburban high school in the Midwest during the 2006-2007 academic year. The subject school was a kindergarten through twelfth grade, private, non-denominational Christian school where I was employed as a high school science and math educator. The K-12 student body during the 2006-2007 school year was 378 students, of whom eighty-seven percent were Caucasian, ten percent were Black, one percent were Asian/Pacific Islander, and two percent were multi-racial. The overall student body was forty-six percent female and fifty-four percent male. The high school (grades 9-12) student population was ninety-two students of whom eighty-nine percent were Caucasian, seven percent were Black, three percent were Asian/Pacific Islander, and one percent were multiracial. Sixty percent of high school students were male and forty percent were female.

### Sample

My specific sample was my 2006-2007 Algebra II class. This class was twenty-one students in size (sixteen male students and five female students; nine sophomores, ten juniors, and two seniors). There was one student who was repeating the class. One student was an Asian foreign exchange student, one student was multiracial, and the remainder of the class was Caucasian. Two students had been previously diagnosed with

ADHD but received no special accommodations in Algebra II. Additionally, all students had earned at least a C- or above in Algebra I and a passing grade in Geometry (Algebra II prerequisites at the subject school).

The sophomores enrolled in Algebra II had enrolled in Algebra I during their eighth grade year and were considered to have been taking an accelerated high school mathematics sequence. The juniors and seniors in Algebra II had enrolled in Algebra I in the ninth or tenth grade and were considered to have been taking a normal high school mathematics sequence. The textbook used for Algebra II was *Algebra 2 for Christian Schools*, second edition, published by Bob Jones University Press (Pilger & Tagliapietra, 2000).

#### *Criteria and Rationale*

While my class at a Christian school in the Midwest was certainly not a representative sample of high school students in general, I was limited in my choice of a sample because I was studying an assignment that, as far as I knew at the beginning of my grounded study, was unique to my math classes. I did not know any other math teachers who employed an identical or even similar assignment in their math classes. In addition, on the two occasions that I had discussed the math journal assignment at length with other mathematics educators, their response had been that the assignment seemed a good idea on the surface, but it also seemed to be too much work for both the students and the teacher grading the math journals. [These conversations echoed what I later found in the literature regarding teachers' attitudes toward including writing in mathematics (e.g. Quinn & Wilson, 1997 and Seto & Meel, 2006).] Per these discussions, I thought that I would have had a difficult time convincing other

mathematics educators to incorporate the math journal assignment into their curriculum for the purpose of my study. As a result, I decided to study my own class in an effort to identify any helpful parts of the journal assignment with the goal of making the assignment more manageable for both students and teachers.

### *Methods of Sampling*

I included in my sample all students who enrolled in Algebra II at the subject school during the fall semester of the 2006-2007 school year. I did not have the luxury of randomly sampling out of this group as this would have greatly diminished my sample size.

### *Statistical Power*

As I conducted a grounded theory study, I did not concern myself with ensuring a high experimental power as much as I would have if I had been conducting a more strict, quantitative experiment. My overarching goal was the production of a descriptive explanation of the differences between journals created by students performing well on the test and journals created by students performing poorly on the test. This descriptive explanation contained statements and conjectures that could be tested in the future under strict, quantitative experimental guidelines.

I did, however, recognize that my limited sample size diminished my study's power, making type I and II errors more probable. I sought to mitigate the possible power effects of my small sample size by taking journal samples at four different times over ten weeks and analyzing the textbook sections individually. This allowed me to separate the math journals into twenty-eight sections for analysis. Of those twenty-eight sections, eleven were selected for detailed analysis due to their clear differences between

the upper and lower quartile student test scores (c.f. *Statistics Utilized with Rationals* in this chapter). Hence, while it is true that I only used the math journal assignments of twenty-one students in my study, I actually analyzed 157 separate math journal sections from eleven different textbook sections: eighty-five upper quartile journals and seventy-two lower quartile journals (c.f. Table 2 in chapter four of this study). These numbers include eleven journal assignment sections from students who scored in the upper quartile and eleven assignments from students who scored in the lower quartile who did not turn in a journal assignment for the given section.

#### Procedure

I collected two forms of data from my Algebra II class: chapter tests and math journal assignments.

#### *Instruments*

The chapter tests were a combination of the textbook publisher provided tests (*Tests for Use with Algebra 2*, 2000), selected problems or slightly modified problems from student homework assignments (from Pilger & Tagliapietra, 2000), and problems of my own creation. Listed below are the test problems and their accompanying directions that led me to analyze certain sections' journal assignments. For information on the key ideas covered in each textbook section, see Table 1 in chapter four of this study.

Aside from one exception as described below on the chapter three test (Section 3.4), I graded the test problems according to a rubric of either four or five points. I graded the problems on the chapter two and four tests with a five point rubric where students received five points for a correct response, four points for a correct response except for one arithmetic/copy error, three points for a correct response except for two

arithmetic/copy errors or one small algebraic/procedural error, two points for multiple arithmetic/copy errors and incorrect procedure, one point for multiple arithmetic/copy errors and significant procedural errors evidencing only a vague understanding of the correct procedure, and zero points for either an incorrect response without any work provided by the student or a blank response. I graded the problems on the chapter three and five tests with a four point rubric where students earned four points for a correct response, three points for a correct response except for one or two arithmetic/copy errors, two points for multiple arithmetic/copy errors or small algebraic/procedural errors, one point for significant procedural errors (could also include arithmetic/copy errors), and zero points for either an incorrect response without any work provided by the student or a blank response.

*Sections 2.5 and 2.6 Test Problems*

Solve the following word problems:

- 1) [2.5] Two airplanes leave at the same time from airports that are 880 miles apart and travel toward each other. If one plane travels 126 mph faster than the other and they pass each other in 2 hours, how fast is each plane traveling?
- 2) [2.6] Mary Auburn makes three investments. The investment that pays 12% is twice the amount of the account that pays 9%. The amount invested at 10% is \$500 more than the amount invested at 9%. If the annual interest income is \$1555, how much money is invested at each rate?

*Sections 2.7 and 2.8 Test Problems*

Solve for the variable:

- 1) [2.7]  $3 < x + 2 \leq 9$

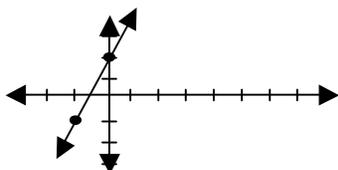
- 2) [2.7]  $4x - 9 \leq 7$  and  $-3x + 4 \leq 16$
- 3) [2.7]  $3y + 9 < 4y + 7$  or  $7y - 2(y - 4) > -2y + 43$
- 4) [2.8]  $|3x + 4| \leq 7$
- 5) [2.8]  $\left| \frac{5x}{2} - 5 \right| < 1$
- 6) [2.8]  $|2x - 1| \leq 0$

*Section 3.4 Test Problems*

- 1) Give the equation of the line that passes through the points  $(-4,3)$  and  $(2,6)$ .

Put the equation in slope intercept form. (Note: This problem was only worth two points rather than four. Students received two points for a correct response, one point for either a correct slope or correct y-intercept, and no points for incorrect slope and y-intercept.)

- 2) Find the equation of the following graphed line.



- 3) What is the slope of the line parallel to  $2y = 5x - 3$  that passes through  $(343, 799)$ ?

*Section 3.6 Test Problems*

Consider the following functions:  $f(x) = x^2 + 3$ ;  $g(x) = 4x - 9$ ;  $h(x) = \sqrt{7x^2 + x}$

- 1) Find and simplify if possible  $(f - g)(x)$
- 2) Find and simplify if possible  $(fg)(x)$
- 3) Find and simplify if possible  $(h / f)(x)$
- 4) Find and simplify if possible  $(f \circ g)(x)$

5) Find and simplify if possible  $\{h \circ [(g \circ f)(x)]\}(x)$

*Sections 4.1 and 4.2 Test Problems*

Solve by factoring: (No credit will be given unless work is shown.)

1) [4.1]  $3x^2 = x$

2) [4.1]  $x^2 + 14x + 49 = 0$

3) [4.1]  $14x^2 - 29x - 15 = 0$

Solve by any method: (No credit will be given unless work is shown.)

4) [4.1]  $4x^2 - 9 = 0$

5) [4.1, 4.2]  $x^2 + 4x = 10x - 9$

6) [4.2]  $(x + 5)^2 = 6$

7) [4.2]  $5 - x^2 = x$

Solve by completing the square: (No credit will be given unless work is shown.)

8) [4.2]  $2x^2 + 20x + 1 = 0$

All students worked problems one through three above by factoring and using the zero product property (section 4.1 objective), and all students worked problem eight by completing the square (section 4.2 objective). Problems four through seven were part of a larger set of test problems where students could choose any method to solve the quadratic so long as they showed their work. Some students used factoring and the zero product property to solve problems four and five, and some students completed the square to solve problems five through seven. Some students also used the quadratic formula (section 4.3 objective) to solve problems four through seven above.

*Section 4.6 Test Problems*

Solve the following quadratic inequalities:

1)  $x^2 - 8x + 7 \geq 0$

2)  $2x^2 + x - 3 < 0$

3)  $3x + 40 \leq x^2$

4)  $(x + 5)(x - 2) < 0$

*Sections 5.2 and 5.6 Test Problems*

Use the function  $y = -5(x - 2)^2 - 7$  to answer the questions below:

1) [5.2] Which way would the graph open? Why?

2) [5.2] Is the vertex a maximum or a minimum point?

3) [5.2] Is the graph narrower or wider than the graph of  $y = x^2$ ? Why?

4) [5.6] Would the graph be solid or dotted? Why?

Graph each equation or inequality:

5) [5.6]  $y < -x^2$

6) [5.6]  $y > 2x^2 + 3$

*Data Collection Methods*

I gathered the raw data from my 2006-2007 fall semester Algebra II class by photocopying the math journal assignments before returning them to my students and retaining my students' chapter tests. I then determined which students scored at or above the upper and at or below the lower quartile limit for each section on the exam. I analyzed these scores and found eleven sections where the students in the upper and lower quartiles performed quite dissimilarly (c.f. *Statistics Utilized with Rationale* in this chapter). Finally, I analyzed the journal assignments for those students identified as performing in the upper and lower quartiles of the eleven identified sections looking for differences. I took notes on the journal assignments as I analyzed them and created a

quantitative database summarizing and recording the differences I found.

### *Objectivity of Scoring*

Concern regarding inter-rater reliability was not an issue as I was the only person grading the tests and analyzing journal entries. I did think, however, that my personal knowledge of my students, particularly their study habits and academic histories, could influence my analysis of their journal entries, so I attempted to eliminate the distraction of personally knowing my students by using a point deduction rubric for each problem on the chapter tests and numerically coding the students' names on their journal entries. In this way, I was confident that the section test scores were consistent, and I usually did not know the identity of the student whose journal entry I analyzed until after all the journal entries had been analyzed. Unfortunately, I did recognize the hand writing of seven (out of twenty-one) of my students, so all I could do when analyzing those sections was try my best to remain completely objective.

### *Relevant Ethical Considerations*

Confidentiality of academic test scores was of ethical concern, but since I was using data from my own students, I already had access to their tests and test scores and did not need to apply through special channels to access the information I needed. My students were aware that I was copying their journals for research purposes, and I took care to present the data in this study either in aggregate (e.g. Tables 3 and 4 in chapter four of this study) or in a manner where individual test scores could not be traced back to individual students (e.g. Table 2 in chapter four of this study).

### *Independent and Dependent Variables*

The independent variable was student performance as measured by the quartile a

student belonged to on a given section of the chapter test, specifically whether a student scored in the top or bottom quartile. The dependent variables in this study were the characteristics of the journal entries. Continuous dependent variables identified in this study were the total number of words, the number of original words, the number of copied words, the total number of examples, the number of original examples, and the number of copied examples in a journal entry. I also compared the percentage of original and copied words to total words within the upper and lower quartiles. Categorical dependent variables identified were whether a student submitted a journal entry, whether the student had included all section ideas in words and examples; whether a student included any original examples, whether the student had included any explanatory examples, and whether the student had included any *original* explanatory examples.

#### *Methods of Data Analysis*

I used Microsoft Excel (2003) to analyze all quantitative data in this study.

#### *Hypothesis Statement*

My hypothesis was that I would find differences in how students who performed well on tests and students who performed poorly on tests created their math journals. I believed that I would find both qualitative and quantitative differences. I expected to find differences (e.g. number of words in the journal entry) indicating that the students who performed well on the test put more effort into the math journal assignment. I did not know what other potential variables to expect when I began analyzing the math journals.

#### *Statistics Utilized with Rationale*

I used 99% confidence intervals around the class mean percent score for each test section as an objective standard for upper and lower quartile scores that were different

enough to warrant analyzing the students' math journals. In other words, I only analyzed journal sections for test sections where all the upper quartile and lower quartile scores fell outside the 99% confidence interval. Since my section test questions were not standardized or statistically evaluated for their efficacy, I only analyzed journals corresponding to textbook sections that my test questions had strongly distinguished between the upper and lower quartiles. I felt that such strong differentiation demonstrated that the test questions effectively discriminated between those who thoroughly understood the subject matter and those who did not. Using the confidence intervals in this way prevented me from analyzing journals from a textbook section where all students (or at least a vast majority of students) mastered the subject matter. I was primarily interested in sections where at least a quarter of the students mastered the material and at least a quarter did not learn it well so I could compare the journals of those two groups of students. I thought that in this way I might be able to link students' performance on the test with the way that they created their math journals.

After collecting quantitative data on the variables identified as possibly different between the journals from students who scored in the upper quartile and students who scored in the lower quartile, I used a two-tailed independent samples t-test to compare the amounts of each potentially significant continuous variable. An independent samples t-test is appropriate because the upper and lower quartiles contain different numbers of students and differ only in their performance on the section test. All students experienced the same class, same notes, same practice problems, and took the same chapter tests. All students also did the same review exercise (the math journal), and all chose their own methodology in creating their math journal. Therefore, all students had the same

opportunity to perform well on the chapter tests. Before running the t-tests, I first performed a two sample F-test for each dependent variable to see if the variance in the upper and lower quartile was significantly different ( $df=155$ ,  $p<0.05$ ). If the F-test revealed that there was a significant difference in the variance of the upper and lower quartiles, then I used a heteroscedastic t-test to examine the means. If the F-test showed that there were not significant differences in the variances of the upper and lower quartile, then I used a homoscedastic t-test to look for differences between the means. A two-tailed test was necessary because I did not know whether to expect more or less of an identified independent variable in either the upper or lower quartile math journals. I also computed the point biserial correlation coefficient for each dependent variable and the classification as either upper or lower quartile. This statistic was appropriate because the point biserial correlation coefficient described the degree of relatedness between a continuous variable (e.g. the number of words), and a categorical one (Heiman, 2001), in this case a student's quartile.

Finally, I used two-way Chi-square analysis to determine whether the categorical variables in my study were over or under represented in the upper or lower quartile journals. Chi-square analysis compares participants across two categorical variables. It allowed me to determine whether my data was categorically independent or if there were likely to be some categorical interactions (Heiman, 2001).

#### *Level of Significance*

I used an  $\alpha$  of 0.05 for all significance testing. This level is generally considered the basic, most accepted value in educational research (Johnson & Christenson, 2004). I saw no reason to deviate from this typical level.

### *Safeguards to Internal and External Validity*

Internal validity in this study was threatened somewhat by possible confounding variables. There was the possibility that any differences I found between the journals from the upper and lower quartile students could be due to a confounding variable, a difference or multiple differences already existing between students who tended to score in the upper quartile and students who tended to score in the lower quartile. However, I did not consider this a likely possibility as the students included in the upper and lower quartiles varied for each section analyzed. Though I could not fully eliminate the threat of confounding variables to the internal validity of this study, and my small sample size exacerbates it, I did not feel that the threat was significant enough to preclude performing the study.

External validity was severely threatened in this study because I used a convenience sample from a population that was available to me rather than randomly sampling from a larger population. I could not be sure that my findings would hold true among high school students in general, but I did not consider that fact a major drawback since the goal of this grounded study was simply to identify differences in journal composition between students who do well on a test and students who do not do well. As such, any findings from this study need to be further tested for external validity under more strict experimental conditions and a much wider sample. In short, I was not too concerned with the possibility that my findings might not be externally valid as the goal of a grounded study is to produce ideas to be tested later in a larger population.

## IV. RESULTS AND ANALYSIS

### Introduction

Data Collection took place over ten weeks during the first and second quarters of the 2006-2007 academic year. I explained the math journal assignment to my Algebra II students when the class was nearing the time for the chapter one test, and I heavily commented on the chapter one journals, encouraging my students to put effort into the assignment as it would help them on the tests (my personal belief). I considered the chapter one journal as practice for my students in understanding my expectations for the assignment, and I began collecting data beginning with chapter two. I photocopied all submitted journals for chapters two through five and retained these and all chapter tests from chapters two through five for analysis.

### Description of the Data

Before grading most chapter exams, I identified which questions corresponded to the objectives from each textbook section. For chapter four only, I graded the exams first and then determined which textbook section each student had used for each problem in the test area labeled "Solve by any Method," (c.f. *Sections 4.1 and 4.2 Test Problems* in chapter three of this study). Table 1 shows the number of test points for each section along with the key ideas of each textbook section. Further information regarding specific test questions can be found in chapter three of this study.

After calculating each student's percentage of possible section points, I calculated

**Table 1**

Text sections with their corresponding test points and subject matter.

Text Section	Number of Possible Test Points	Key Ideas/Learning Objectives in Section
2.1	15	Solving simple linear equations
2.2	15	Solving simple linear inequalities
2.3	20	Solving absolute value equations
2.4	10	Finding distance and midpoint on a number line
<b>2.5</b>	<b>5</b>	<b><i>Word problems: rate <math>\times</math> time = distance</i></b>
<b>2.6</b>	<b>5</b>	<b><i>Word Problems: simple interest and mixture</i></b>
<b>2.7</b>	<b>15</b>	<b><i>Compound linear inequalities (conjunctions/ disjunctions)</i></b>
<b>2.8</b>	<b>15</b>	<b><i>Solving absolute value inequalities</i></b>
3.1	16	Ways of representing relations
3.2	24	Definition of a function
3.3	6	Graphing lines (slope and y-intercept)
<b>3.4</b>	<b>10</b>	<b><i>Finding equations of lines [<math>y=mx+b</math> and <math>y- y_1=m(x- x_1)</math>]</i></b>
3.5	0 <sup>a</sup>	Absolute value functions, the step function
<b>3.6</b>	<b>20</b>	<b><i>Composition function and combining two funcs (+,-,<math>\times</math>,<math>\div</math>)</i></b>
3.7	4	Graphing linear inequalities
3.8	12	The distance formula, 2D/3D midpoints and graphs
<b>4.1</b>	<b><i>Varied<sup>b</sup></i></b>	<b><i>Factoring to solve quadratic equations</i></b>
<b>4.2</b>	<b><i>Varied<sup>b</sup></i></b>	<b><i>Completing the square: solving quadratic equations</i></b>
4.3	Varied <sup>b</sup>	Quadratic formula: solving quadratic equations
4.4	Varied <sup>b</sup>	Choosing the fastest method of solving a given quadratic
4.5	0 <sup>a</sup>	Word problems: quadratic equations
<b>4.6</b>	<b>20</b>	<b><i>Solving quadratic inequalities</i></b>
5.1	4	Graphing $f(x)=x^2$ and parabolic terms/definitions
<b>5.2</b>	<b>12</b>	<b><i>Graphing <math>f(x)=ax^2</math>; effects of changing the "a."</i></b>
5.3	16	Horizontal and vertical translation; $f(x)=a(x-h)^2+k$
5.4	4	Graphing quadratic functions of the form $f(x)=ax^2+bx+c$
5.5	4	Word Problems: quadratic equations
<b>5.6</b>	<b>12</b>	<b><i>Graphing quadratic inequalities</i></b>
5.7	16	Applying the remainder and factor theorems to polynomials
5.8	8	Graphing cubic, quartic, and quintic polynomials

Note: Sections analyzed for the purposes of this study are bold and italicized.

<sup>a</sup>Section objectives not tested.

<sup>b</sup>Possible points varied as I gave my students several quadratics to solve by a method of their choosing. All students attempted at least 4 points from each of the sections.

99% confidence intervals around the class mean percentage for each textbook section. I found that there were eleven sections where the upper and lower quartiles lay completely

above and below, respectively, the 99% confidence interval: 2.5, 2.6, 2.7, 2.8, 3.4, 3.6, 4.1, 4.2, 4.6, 5.2, and 5.6. Since these section test scores were so different between the upper and lower quartiles, I decided to use the math journals corresponding to these sections to look for differences in the way the students created their journals. I was confident that the upper and lower quartiles for these sections were truly different since they both performed so differently from the mean, even with my relatively small sample size. The test data for these sections is in Table 2.

After I had identified the textbook sections where the upper and lower quartiles of students had together performed the most dissimilarly from the mean, I analyzed the journals of the students in the upper and lower quartiles for the eleven identified sections looking for differences in journal composition. I focused first on chapters three and four, and after reading the student journals and taking and organizing detailed notes on the upper and lower quartile journals for sections 3.4, 3.6, 4.1, 4.2 and 4.6, I had identified several potential differences. Specifically, it seemed that the upper quartile journals generally were of greater length, included all the main ideas from the section instead of just the most prominent idea, contained more student generated material and less copying from the textbook, contained more examples, and contained more explanatory examples (symbolic examples accompanied by textual commentary alongside explaining each step in solving the problem). There also seemed to be a higher rate of completion of the journal assignment among the upper quartile. I then set to work going back through the 3.4, 3.6, 4.1, 4.2 and 4.6 upper and lower quartile journals compiling numeric data on the variables listed above to see if I could verify these impressions quantitatively.

Though I did find that the quantitative data for chapters three and four backed up

**Table 2**

Student test scores for sections where the upper and lower quartiles were completely outside the 99% confidence interval for the group mean.

Section:	2.7	2.8	2.5	2.6	3.4	3.6	4.1	4.2	4.6	5.2	5.6
Student Scores at or above the upper quartile	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	100.0	100.0	100.0	100.0	100.0	90.0	100.0	100.0	100.0	100.0	100.0
	100.0	100.0	100.0	100.0	100.0	85.0	100.0	100.0	95.0	100.0	100.0
	93.3	86.7	100.0	100.0	80.0	80.0	96.7	100.0	90.0	100.0	100.0
	93.3	86.7	100.0	100.0	80.0	80.0	92.0	100.0	90.0	100.0	100.0
	86.7	86.7	100.0	100.0	80.0	80.0	88.0	100.0	85.0	100.0	100.0
	86.7	86.7	100.0	80.0	80.0	80.0	86.7	90.0	85.0	100.0	100.0
	86.7	86.7	100.0	80.0	70.0	75.0	84.0	90.0	75.0	83.3	100.0
	73.3	80.0	100.0	80.0	70.0	70.0	84.0	90.0	65.0	83.3	100.0
	73.3	80.0	100.0	80.0	70.0	60.0	80.0	80.0	60.0	83.3	100.0
	73.3	73.3	100.0	60.0	60.0	55.0	75.0	70.0	55.0	83.3	100.0
	66.7	66.7	100.0	40.0	50.0	55.0	68.0	70.0	50.0	83.3	91.7
	60.0	60.0	80.0	20.0	50.0	55.0	68.0	60.0	50.0	83.3	91.7
	60.0	60.0	80.0	20.0	50.0	50.0	65.0	60.0	40.0	66.7	91.7
	60.0	53.3	50.0	0.0	40.0	45.0	56.7	60.0	25.0	66.7	75.0
Student Scores at or below the lower quartile	53.3	46.7	50.0	0.0	40.0	40.0	45.0	46.7	25.0	66.7	66.7
	46.7	40.0	50.0	0.0	30.0	30.0	44.0	45.0	10.0	66.7	66.7
	46.7	33.3	20.0	0.0	20.0	25.0	44.0	40.0	10.0	66.7	66.7
	33.3	33.3	0.0	0.0	20.0	20.0	30.0	40.0	5.0	50.0	66.7
	33.3	13.3	0.0	0.0	10.0	15.0	10.0	20.0	5.0	50.0	50.0
	6.7	6.7	0.0	0.0	10.0	0.0	0.0	0.0	5.0	50.0	33.3
<u>M</u>	68.3	65.7	72.9	50.5	57.6	56.7	67.5	69.6	53.6	80.2	85.7
SD	25.5	28.1	38.2	43.6	29.0	27.4	29.3	29.5	34.6	18.0	19.9
Quart. 1	53.3	46.7	50.0	0.0	40.0	40.0	45.0	46.7	25.0	66.7	66.7
Quart. 3	86.7	86.7	100.0	100.0	80.0	80.0	88.0	100.0	85.0	100.0	100.0
UL <sup>a</sup>	82.6	81.5	94.4	75.0	73.8	72.1	84.0	86.2	73.1	90.3	96.9
LL <sup>b</sup>	54.0	49.9	51.4	26.0	41.4	41.3	51.0	53.0	34.1	70.1	74.5

Note: Columns are arranged in descending order for the visual identification of quartiles, so no row represents a single student's scores.

<sup>a</sup>Upper limit of 99% confidence interval (n=21).

<sup>b</sup>Lower limit of 99% confidence interval (n=21).

my impressions (data not shown), I did not think that journals from textbook chapters three and four alone were enough to be sure that any differences identified were consistent with more general reality, so I also analyzed the upper and lower quartiles of 2.5, 2.6, 2.7, 2.8, 5.2, and 5.6. I thought this would add to my confidence that any differences I identified were real differences between students scoring in the upper and lower quartiles on the chapter tests. This time, instead of inductively studying the journals from chapters two and five looking for patterns, commonalities, and differences as I had for the journals from chapters three and four, I gathered only quantitative data on the same variables I thought might be significant from my previous study of the chapter three and chapter four math journals. After I had added the quantitative data from textbook chapters two and five to the data from textbook chapters three and four, I used independent samples t-tests to statistically compare the averages of the continuous variables and Chi-square analysis to compare the frequencies of the categorical variables in the upper and lower quartiles. In addition, I computed point-biserial correlation coefficients for each continuous variable. T-tests and point-biserial coefficients for the continuous variables are in Table 3, and Chi-square results for the categorical variables are in Table 4.

#### Data Analysis

The quantitative data from chapters two, three, four, and five confirmed my suspicions on many, though not all, of the differences I had subjectively noticed in studying the chapter three and four journals. There were significant differences between the upper and lower quartiles in the total number of words in the journal entry ( $t=3.768$ ,  $p<0.001$ ), the number of original words ( $t=5.241$ ,  $p<0.001$ ), the proportion of original

**Table 3**

Comparison of means between the lower and upper quartiles for quantitative qualities of student journal entries.

	<u>Lower Quartile</u> (N=72)		<u>Upper Quartile</u> (N=85)		<u>t</u> (155)	<u>r<sub>pb</sub></u>
	<u>M</u>	<u>SD</u>	<u>M</u>	<u>SD</u>		
Number of Words <sup>a</sup>	37.8	33.7	70.4	70.4	3.768***	0.290***
Number of Original Words	21.0	25.6	65.3	71.1	5.241***	0.388***
Percent Original Words <sup>a</sup>	51.0	45.4	76.7	39.7	3.788***	0.291***
Number of Copied Words	16.8	27.0	5.15	15.0	3.224**	-0.251**
Percent Copied Words	29.6	40.4	8.00	22.6	3.976***	-0.304***
Number of Examples	1.19	1.02	2.16	1.81	4.211***	0.320***
Number of Original Examples	0.50	0.87	1.41	1.73	4.228***	0.322***
Percent Original Examples <sup>a</sup>	28.8	43.7	47.6	46.1	2.598*	0.204*
Number of Copied Examples	0.69	0.88	0.75	1.07	0.376	0.030
Percent Copied Examples <sup>a</sup>	43.4	48.2	37.1	44.3	0.847	-0.068
Cumulative Student GPA	2.88	0.46	3.50	0.56	7.575***	0.520***

<sup>a</sup>Equivalent variance allowed a homoscedastic t-test. All other t-tests were heteroscedastic as the variances in the upper and lower quartiles were not equivalent.

\*p<0.05, \*\*p<0.01, \*\*\*p<0.001

**Table 4**

Number of student journal entries in the upper and lower quartiles with specific categorical properties.

	<u>Lower Quartile</u>	<u>Upper Quartile</u>	$\chi^2(1)$
	<u>N=72</u>	<u>N=85</u>	
Submitted a Journal Entry	61	74	0.1766
Included all Major Section Ideas in Words	24	52	12.10***
Included all Major Section Ideas in Examples	21	51	14.93***
Included $\geq 1$ Original Example(s)	24	49	9.263**
Included <i>any</i> Explanatory Examples	21	30	0.6673
Included $\geq 1$ <i>Original</i> Explanatory Example(s)	12	30	6.903**
Number of Females	21	25	0.03437
Number of Sophomores	22	50	13.10***

\*\*p<0.01, \*\*\*p<0.001

words to total words ( $t=3.788$ ,  $p<0.001$ ), the number of words copied from the textbook ( $t=3.224$ ,  $p<0.01$ ), the proportion of words copied from the textbook to total words ( $t=3.976$ ,  $p<0.001$ ), the number of students who included explanation in words for all key section ideas ( $\chi^2(1)=12.10$ ,  $p<0.001$ ), the number of examples ( $t=4.211$ ,  $p<0.001$ ), the number of original examples ( $t=4.228$ ,  $p<0.001$ ), the proportion of original examples to total examples ( $t=2.598$ ,  $p<0.05$ ), the number of students including at least one original example ( $\chi^2(1)=9.263$ ,  $p<0.01$ ), the number of students including examples for all key section ideas ( $\chi^2(1)=14.93$ ,  $p<0.001$ ), and the number of students including *original* running commentary explaining how to solve at least one example ( $\chi^2(1)=6.903$ ,  $p<0.01$ ). There were also significant point-biserial correlations between the quartile placement and the number of words ( $r_{pb}=0.290$ ,  $p<0.001$ ), the number of original words ( $r_{pb}=0.388$ ,  $p<0.001$ ), the proportion of original words to total words ( $r_{pb}=0.291$ ,  $p<0.001$ ), the number of copied words ( $r_{pb}=-0.251$ ,  $p<0.01$ ), the proportion of copied words to total words ( $r_{pb}=-0.304$ ,  $p<0.001$ ), the number of examples ( $r_{pb}=0.320$ ,  $p<0.001$ ), the number of original examples ( $r_{pb}=0.322$ ,  $p<0.001$ ), and the proportion of original examples to total examples ( $r_{pb}=0.204$ ,  $p<0.05$ ). There were not significant differences ( $p>0.05$ ) in the rates of submission of the math journal assignment for the upper and lower quartiles, the number of copied examples in the journal entry, the proportion of copied examples to total examples, or in the number of students including any explanatory examples (original or copied from the textbook's examples) explaining at least one example.

I also made several demographic calculations to determine if there were differences between the upper and lower quartiles. I used an independent samples t-test to compare the average GPA of the students who scored in each extreme quartile. I

included a student's GPA every time he or she was a member of the upper or lower quartile. There was a significant difference ( $p < 0.001$ ) between the GPA's in the upper ( $M = 3.50$ ,  $SD = 0.56$ ) and lower quartile ( $M = 2.88$ ,  $SD = 0.46$ ) journal samples. I also performed Chi-square analyses to determine if boys or girls and sophomores or upper class-members were disproportionately represented in either the upper or lower quartiles across all eleven identified textbook sections. I found that male and female students were not disproportionately represented between the upper and lower quartiles ( $\chi^2(1) = 0.0344$ ,  $p > 0.05$ ). However, Chi-square analysis revealed that the sophomores were disproportionately represented in the upper quartile, and the juniors/seniors were disproportionately represented in the lower quartile ( $\chi^2(1) = 13.10$ ,  $p < 0.001$ ).

#### Conclusions

There were statistically significant differences in how students who performed in the upper and lower quartile of the section tests created their journals. Most notably, students performing in the upper quartile on the section test wrote journals of greater textual length, more of the text length was written in their original words as opposed to copying from their textbooks, more examples were given to illustrate the ideas of the section, more student-originated examples were given, and there was a higher incidence of student-generated running commentary explaining in a step by step fashion the examples given. However, simply turning in a journal to be graded did not seem to be related to placing in either the upper or lower quartile on the section test, since a statistically equivalent proportion of both the upper and lower quartiles did not turn in a journal. In addition, the act of copying examples from the textbook did not seem to affect student test performance.

## V. DISCUSSION AND IMPLICATIONS

### Introduction

The results of this study can be expressed simply in one overarching, educationally significant theme illustrating the primary difference between the math journals of students in the upper and lower test quartiles. Original journal entries, as opposed to text and examples copied from the student textbook, were directly linked to high test performance. This was true in regards to written explanations of mathematical concepts in words, symbolic examples, and explanatory examples.

### Interpretation of the Results

Students scoring in the upper quartile used significantly more original language in describing mathematical concepts than their fellow students who scored in the lower quartile on the test. Out of all the variables in this study, the number of original words in a student's journal entry was the most striking difference between the upper and lower quartiles (Table 3,  $t=5.241$ ,  $p<0.001$ ). Additionally, students scoring in the lower quartile on a section test used more words copied from the textbook in writing their journals (Table 3,  $t=3.224$ ,  $p<0.01$ ). The above relationships taken together indicate that there was a direct link between a student's ability to rephrase mathematical concepts in his or her own original words and the student's performance on a traditional, paper-and-pencil test covering the same concepts.

The data also show that students who took the time to include in their journal

entries all the major ideas in each textbook section tended to perform superior to their peers on a section test. This was true both when applied to the inclusion of all section ideas in words (Table 4,  $\chi^2(1)=12.10$ ,  $p<0.001$ ) and the provision of examples for all section ideas (Table 4,  $\chi^2(1)=14.93$ ,  $p<0.001$ ). This observation may seem intuitively obvious, but it is illustrative of the possibility that one of the reasons all students did not receive the same benefit from writing the math journal assignment was that they put different amounts of effort into writing the journal assignment. It took more effort to analyze a textbook section and determine what all the major section ideas were than to simply pull the biggest idea out of each section and include only that idea in the math journal assignment.

Copying examples from the student textbook did not seem to have the same effect on test performance that copying text from the textbook had. In fact, there was not a significant difference between the upper and lower quartiles in the number of examples copied from the textbook. Students who scored well on the test and students who did not score well on the test were equally likely to include examples copied from their textbook in their section journal entries. However, this is easily resolved by noting that the upper quartile group used significantly more examples (copied and original examples counted together) than the lower quartile group (Table 3,  $t=4.211$ ,  $p<0.001$ ). In other words, while both groups of students tended to include copied examples at similar rates, the group of students scoring well on the chapter test did more than just copy from the textbook. Successful students also created their own, original examples. The data directly support this by showing that students performing in the upper quartile created significantly more original examples than students who performed poorly (Table 3,

$t=4.228$ ,  $p<0.001$ ). Once again, the data suggest that originality in explaining a mathematical concept with words or examples was a key factor linked to high test performance.

Continuing even further with the above theme, while there was not a significant difference between the upper and lower quartile journal entries in the percentage of students including explanatory examples, there was a significantly greater occurrence of *original* explanatory examples within the upper quartile group (Table 4,  $\chi^2(1)=6.903$ ,  $p<0.01$ ). Since I never mentioned or suggested using explanatory examples in my Algebra II class, I was a bit surprised at the number of students scoring in both the upper and lower quartiles utilizing this strategy in writing their journal entries. I attribute this high rate of use to the fact that the student textbook often included line by line explanation alongside its examples. That may be where the explanatory example idea first occurred to some students. Indeed, forty-three percent of the explanatory examples in journals from students scoring in the lower quartile were directly copied from the line by line explanation in the student textbook. By way of contrast, *every* time a student scoring in the upper quartile utilized the explanatory example format the student used an original example with original running commentary alongside it. Copying an explanatory example was clearly not a good strategy for test preparation. In keeping with the developing theme, explanatory examples were more helpful if the student originated the line by line explanation him or herself.

Though the categorical demographic variables in this study showed that there was no difference in the gender composition of the upper and lower quartiles, the class-year composition of the two groups was significantly different. Sophomores were

overrepresented in the upper quartile when compared to juniors and seniors. I believe this difference was actually a reflection of the fact that the upper quartile was composed of academically superior students when compared to the lower quartile. At the subject school students could take Algebra I in the eighth or the ninth grade. Students who were gifted in mathematics tended to enroll in Algebra I in the eighth grade. They could then take Geometry in the ninth grade, Algebra II in the tenth grade, Trigonometry in the eleventh grade, and Calculus in the twelfth grade. However, eighth grade students enrolled in Algebra I who did not earn at least a B- in the course were required to repeat Algebra I in the ninth grade before moving on to more advanced math courses. This policy was intended to prevent students from advancing in math courses more quickly than their individual math competencies allow. Students who chose to take a fundamental math course in the eighth grade instead of Algebra I followed the same course sequence as above but typically could only progress to Trigonometry by graduation. Since the sophomores enrolled in Algebra II were enrolled in (and so far were succeeding at) the most advanced math course sequence available, it made sense that as a group the sophomores would be of better than average mathematical ability and more likely to score in the upper quartile on the section tests. The academic superiority of the upper quartile is further validated by their significantly higher GPA when compared to the lower quartile.

This difference in academic ability between the two groups does not invalidate the findings of this study. The primary purpose of this study was to identify differences in the way students scoring in the lower and upper quartiles constructed their journal assignments. This study identified several differences between those two groups of

students. It is natural to assume that the student group tending to score in the upper quartile would be of a higher academic caliber than students who tend to score in the lower quartile. It is also natural to assume that students in the lower quartile could benefit, quite possibly with improved test scores, by imitating the writing behavior of students in the upper quartile.

Finally, there was no statistical difference between the upper and lower quartiles in the percentage of students who turned in their math journal assignment. One plausible explanation for this lack of a difference in completion rates for the two quartiles is that the math journal assignment was not the only way for students to learn math concepts in my classroom. I also required students to take notes during lecture, work homework problems, and take frequent short quizzes. These other ways of engaging the material obviously were enough for some of my students to learn the concepts since they were able to perform in the upper quartile without completing the math journal assignment. Some students who scored in the upper quartile but did not turn in math journals were, then, students who were willing to sacrifice the math journal portion of their grade to avoid a lengthy assignment that they saw as having no real value to them, since they already knew the material. One could argue that they had a point if they scored in the upper quartile, but I still believe the math journal assignment would have been a valuable exercise for all my students.

#### Relation of the Results to Educational Theory

The principal finding above, that original student work was consistently associated with high student achievement, was not surprising considering relevant educational theory. The much acclaimed *Taxonomy of Educational Objectives*

commonly attributed to its primary editor, Benjamin Bloom, gave six levels for educational goals. The handbook classified educational goals in a hierarchy of increasing complexity with categories of knowledge, comprehension, application, analysis, synthesis, and evaluation (Bloom, 1956). Students originally composing their journal entries in this study were generally interacting with the mathematical objectives at least at the analysis and synthesis levels. The higher order thinking about the objectives demonstrated by these students in their journal entries translated into higher test scores. On the other hand, students copying the majority of their journals from the student text were only relating to the objectives at the knowledge or perhaps comprehension level and, accordingly, scored at a lower level on a test over those objectives.

Marzano, in an attempt to update Bloom's taxonomy in light of new understanding about learning from the field of cognitive psychology, proposed a new taxonomy of educational objectives. He proposed six hierarchical levels – retrieval, comprehension, analysis, knowledge utilization, metacognition, and self-system thinking – that apply to three non-hierarchical domains of knowledge: information, mental procedures, and psychomotor procedures (Marzano, 2001). Students in this study who originally composed the text of their journal entries were often relating to the mathematical knowledge at the comprehension, analysis, knowledge utilization, and sometimes metacognition/self-system thinking levels, while students who did not originally compose their journal entries were functioning at the retrieval or at best comprehension level. Just as in relating the results of this study to the original *Taxonomy of Educational Objectives* above, students who engaged the material at higher levels of thinking tended to score higher on a test covering the same material.

### Potential Applications of the Findings

Teachers should encourage students to rephrase knowledge in their own words or thoughts prior to a formal learning assessment. I recognize that teachers could accomplish this goal using mechanisms other than a writing assignment (as shown in Porter & Masingila, 2000). However, the results of this study show that students who have rephrased mathematical concepts in their own written words tend to perform better on a test covering those mathematical concepts than students who have not constructed their own version of the mathematical concepts under consideration.

Teachers should also consider encouraging the use of original explanatory examples as a means to promote original framing of mathematical concepts leading to successful achievement of learning goals. While the results of this study do not show that original explanatory examples are a characteristic of *most* students who perform well on a test (such as was the case for sufficient, originally written explanation of a mathematical concept in the previous paragraph), students performing well on the tests in this study were approximately twice as likely to include original explanatory examples when compared to students performing poorly on the test. I believe that original explanatory examples may have been more common in the upper quartiles because such examples represent the essence of the intended purpose of the math journal assignment: reviewing the types of and methods for solving mathematics problems similar to the ones students would be asked to solve on an impending formal assessment of learning. Original explanatory examples require a student to both solve a math problem (good review in itself) and also explain how and/or why he or she solved the math problem by the chosen method (effectively requiring a student to put a mathematical concept into his or her own

words).

One of the stated aims of this study was the development of a simplification of the math journal assignment, as several fellow educators had expressed doubtfulness toward the utility of the assignment because of the large amount of work required for both student and teacher in creating and grading the journals. One way a teacher could streamline the math journal assignment, possibly without losing many of its benefits, would be to assign several explanatory examples instead of separate verbal and symbolic explanations of every main idea in the chapter (the requirements of the math journal assignment as described in this study).

In other words, a teacher could create several explanatory examples that each contained several main ideas in the chapter (so that the explanatory examples together contain all the learning objectives for the chapter). Three to five well crafted and complex explanatory examples could cover the material in most chapters of the Algebra I and II courses I teach. The explanatory examples could even serve as an anticipatory set if they were first discussed in an exploratory manner as the class began to study the chapter and before most of the class were capable of solving the problems. Such a procedure might be even more effective if the explanatory examples involved problem solving in some way connected to things the students were interested in studying or learning (Marlow, 2006). In this way the journal assignment could be distilled down to a handful of explanatory examples that might draw students into the material as well as help them master it.

I believe that the shortened assignment outlined above makes sense in light of the data gathered in this study. The continuous variable showing the greatest difference

between the two groups was a high number of original words in explanation of a mathematical concept (Table 3,  $t=5.241$ ), and the inclusion of examples from all major section ideas was the categorical variable exhibiting the greatest difference between the upper and lower quartiles (Table 4,  $\chi^2(1)=14.93$ ,  $p<0.001$ ). Both of these qualities could be included in an explanatory example assignment as it would require students to originally phrase mathematical concepts in their own words, and the teacher could ensure that the examples covered all the ideas in the chapter.

Another possible side-benefit of requiring students to give explanatory examples could be in introducing a beginning concept of a formal mathematical proof. Requiring explanatory examples in Algebra I or II using a “Statements” column for the symbolic steps and a “Reasons” column for the verbal explanation might give students a basic concept of a proof for later development in Geometry. Most students first encounter formal mathematical proofs in a high school Geometry class, a course typically following Algebra I and/or Algebra II. I think students might have greater success in learning to construct beginning proofs (a discipline often causing much consternation among high school students at my school) if they already had an idea of the necessity of supporting symbolic statements with reasons.

Additionally, explanatory examples in two column format could prevent some students from developing an understanding that mathematics is something that one “does” to solve math problems. A two-column explanatory example could force students to explain how they are solving the problem. In so doing, students may realize that mathematics is more than purposeless processes; there are logical reasons behind the things one does to solve a math problem.

### Relation of the Results to Biblical Wisdom Literature

This study demonstrated that diligent and complete academic work often paid dividends of good grades on academic tests. It also showed that shortcuts to avoid work (e.g. solely copying from the textbook) did not pay off with good test scores. Students who created longer journal entries (more words indicated more time invested in the assignment), and more specifically created longer sections of student-originated material in their journal entries (again a reflection of time investment on the student's part: it is more difficult and time consuming to create original explanation and examples than to copy the explanation and examples out of the book) tended to perform at a higher level on the test. This corresponds directly with the injunction in Proverbs 6:6 for the lazy man to look to the ant as an example of diligence leading to life instead of persisting in his laziness which leads to death (New International Version). Analogously, students in the lower quartile should look to and learn from the example of students in the upper quartile. The overall results of this study illustrate the general biblical truth that diligent work pays rewards.

### Relation of the Results to Other Literature

While the literature held that there was a great diversity of types of mathematical writing (Davison & Pearce, 1988; Birken, 1989; and Shephard, 1993), the findings of this study suggest that certain types of mathematical writing are more valuable than other types of writing for student learning in math class. Specifically, students in this study seemed to receive very little benefit from simply transcribing information from their textbook to their journal assignments. Rather, the results of this study confirmed Shephard's assertion that transactional writing, where a student attempts to inform or

explain, is the type of writing most likely to lead to cognitive change (1993).

#### Strengths of the Study

The primary strength of this study was its internal validity. The data supported, from a variety of angles, the primary conclusion of this study, that student-originated work was linked to high test performance. This was true when considering written text explaining mathematics concepts, illustrative symbolic examples, and explanatory examples.

#### Limitations of the Study

Though my data exhibited solid differences between the methods of journal composition in the upper and lower quartiles, there was one major remaining concern with external validity. The primary threat to external validity was known before I conducted this study: I did not use a random sample of high school students. Instead, I used a convenience sample from my own classroom. While recognizing this was a problem if I were to attempt to apply the results of this study to the general population of high school students, I did not set out to perform a necessarily generalizable study. This study was intended to be a grounded study identifying differences between students performing in the upper and lower quartiles. Any differences noted in this study should be tested in the general high school population before being adopted as generally true. However, I suspect that many of the findings will be applicable to the general population as the primary conclusion in this study held true from a variety of standpoints (i.e. original text, original examples, and original explanatory examples all were linked to the upper quartile) and aligns well with applicable theories of learning (c.f. Relation of the Results to Educational Theory in this chapter).

### Suggestions for Future Research

Since I did not perform this experiment with a representative or random sample of high school students, such a study needs to be carried out before the results of this study can be generalized to the general high school population. I would also be interested in performing an experiment comparing randomly selected experimental and control classes where the experimental variable was the presence and absence of the math journal assignment. Additionally, I think it would be interesting to incorporate explanatory examples into my Algebra II class and compare their effectiveness in helping students study for tests to the effectiveness of the math journal assignment described in this study. Such a study could test some of the conjectures I have made above regarding the possible benefits of using explanatory examples, particularly whether their use could raise the test scores of the lower quartile by requiring them to utilize the natural learning strategies of the upper quartile.

Finally, I would like to see a study that incorporated explanatory examples in formal, two column proof format into a math class (such as Algebra I or II) prior to the first time students enroll in a course where formal proofs will be required (typically Geometry for most high school students). It would be interesting to test whether such an early introduction into “algebraic proofs” made it easier for students to later successfully master formal proof writing.

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