

MISTAKE-HANDLING ACTIVITIES IN THE MATHEMATICS CLASSROOM

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The handling of mistakes in the mathematics classroom is an area in which only little research is done so far. In mathematics education there are, in fact, several investigations about students' mistakes from a diagnostic perspective, but hardly any studies on the question how teachers react in concrete mathematics lessons. In this article theoretical aspects and results of different video studies are summarized. Moreover, a student survey based on a questionnaire is presented in detail. The findings indicate that particularly low achieving students are not aware of the learning opportunities within mistake situations.

INTRODUCTION

In the German language there exists typical proverbs like “Aus Fehlern wird man klug.” (You will become clever from mistakes) or “Aus Fehlern lernt man.” (You will learn from mistakes). Though it seems that mistakes are generally accepted as a natural part of the learning process, it is unpleasant for individuals to make mistakes or to be caught when making mistakes. This fact is mainly based on two reasons: on the one hand, there is an affective component, i.e. to make a mistake means to be embarrassed. On the other hand, mistakes give in some sense evidence that there are some deficits in a certain individual competence, which in school is a disadvantage when being judged¹. Thus, when approaching the topic of mistake in the classroom we must have in mind that there are different facets of mistake situations. In this contribution we will focus on cognitive and affective aspects of the mistake-handling in the classroom processes from the student and observer perspective.

THE ROLE OF MISTAKES IN THE LEARNING PROCESS

Within the last decades, in particular in the 1970s and 1980s, there was a lot of research in mathematics education concerning the question of students mistakes (e.g., Radatz, 1979). The main aim of this research was the identification of reasons for different typical students' mistakes and, consequently, the development of didactical ideas, material etc. to prevent these mistakes or to use these mistakes as a learning opportunity. Beyond this research approach which follows a diagnostic perspective, there is less research on the question what teachers know about typical student mistakes and what is a good way to deal with them in the lessons. Examples are the intervention program of the Italian group for detecting and overcoming conceptual mistakes (e.g., Garuti, Boero, Chiappini, 1999) or the study of the Israeli group on

¹ This led to the fact that the word “mistake” has a negative connotation and is often avoided. We use the word mistake anyway and stress the positive aspects of mistakes in the learning process.

teacher awareness of students' difficulties when manipulating with rational numbers (e.g., Klein, Barkai, Tirosh & Tsamir, 1998). However, if we address the issue of mistake-handling activities of teachers in mathematics lesson, particularly in the students' perception, only very few studies are known. Thus, we do not really know how mathematics teachers and their students handle mistake situations.

Learning from mistakes – the theory of negative expertise

As already mentioned in the introduction it is generally accepted (in the sense of a public opinion) that one can learn by mistakes. However, we rarely find a detailed description or idea how this learning process takes place. Mainly, we find the diffuse idea which can be described as “a mistake is only made once”, which is obviously not true. A deeper theoretical approach to the topic “learning by mistakes” is given by the research group of the educational psychologist Fritz Oser which worked on this question in a three year research project. The basic idea of Osers group is the theory of negative expertise.

For the theory of negative expertise the notion “mistake” was defined as follows: *A mistake is a process or a fact that does not comply with the norm. It is necessary for the identification of the line of demarcation to the correct process or fact that complies with the norm.* In other words: Mistakes are necessary to sharpen the individual idea about what is false and what is correct (according to a given norm). If we compare this with the role of examples and counterexamples for the learning of mathematical concepts, then mistakes play the role of counterexamples. Thus, it is not enough that an individual knows what is correct. She/he also has to know what is incorrect, because otherwise it is not possible to identify at which point the correct ends and the incorrect begins. From this point of view the knowledge of incorrect facts and processes is necessary and this negative expertise completes the knowledge of correct facts and processes, i.e. it completes the positive expertise (cf. Oser, Hascher & Spychiger, 1999).

Mistakes are essential for the acquisition of negative knowledge and, consequently, mistakes are necessary components of the learning process. However, to make a mistake does not mean automatically to acquire negative expertise, which can be used to prevent further mistakes. For using a mistake in a productive way by Oser et al. (1999) it is necessary that an individual is able to realize, to analyze and to correct the mistake and, moreover, that she/he uses the mistake to develop a strategy for the prevention of further mistakes. At this point we are facing a lot of open questions. For example, the question if an individual has to make a mistake by itself or if it is sufficient to participate in a mistake situation of another person. Moreover, the question is how to foster the individual productive use of mistakes in the mathematics classroom. And finally, what about different types of mistakes? If a mistake is part of the procedural knowledge of a student, then often she/he will make this mistake again and again and it might be more difficult to learn from this mistake than from mistakes which are part of non-procedural knowledge.

From the previous description it becomes clear that the theory of negative expertise with its positive role of mistakes corresponds to a constructivist view of learning. While the behaviouristic approach avoids mistakes and tries to stress only successful students activities (i.e. only positive knowledge is important), this approach realizes mistakes as learning opportunities which are necessary and unavoidable.

Mistake-handling in mathematics classroom – empirical results

There are only a few studies which focus on the question of mistake-handling activities in mathematics lessons from an empirical perspective. Until now we do not know much about teacher and student behaviour within mistake situations in the real mathematics classroom. It is obvious that there is a difference between mistake-handling activities in so called private and public situations. For example, if an individual mistake is discussed with the whole class, then the student may feel to be exposed. In a private situation, i.e. the teacher discusses a mistake with one student, there may be a better chance for an individual learning progress. Thus, before presenting findings of previous investigations on this topic from Germany, Switzerland, USA and Italy, I will give a short overview about basic data on the organisation of the mathematics lessons in these countries (cf. Stigler et al, 1999; Hiebert et al, 2003; Santagata, in press).

As one can see in Table 1 in all four countries most of the lesson time is devoted to public work, i.e. phases in the lessons in which the whole class is working together.

Generally, in this phase the teacher is talking to the students or tries to progress with the content of the lesson by asking questions to the students. Private work means that students are working on their own (individual) or in small groups/pairs. However, these phases are mainly used for practicing routine procedures, making drawings etc. and rarely for explorative work or the introduction of new content.

	private work		public work
	individual	group/pairs	
Germany	23%	3%	73%
Switzerland	33%	11%	54%
Italy	10%		82%
USA	29%	9%	61%

Table 1: Percentages of lesson time for private and public work

The following results are from three video-based investigations of mistake-handling in mathematics classroom. The German study by Heinze (2004) comprises 22 lessons from grade 8, the Swiss study by Oser et al. (1998) ten lessons (different grades on the lower secondary level) and the study from USA and Italy by Santagata (in press) comprises 30 lessons from grade 8 in each country. All studies analyzed the lessons on the basis of mistake situations which were identified by the teacher reaction to a student contribution. A further characterization of the mistake situations was then made in different ways.

Table 2 shows the average number of mistakes in the mathematics lessons of the different countries in public and private situations (duration of the lessons: 45-50 min-

utes). For the German lessons we do not have data for the private situations, because the sound recording of the students' voices during private work is of bad quality.

From the table we can see that the number of mistakes in the public teacher-students interaction in Germany, Switzerland and the USA is comparatively low. Even if we take into account that in Italy 82% of the lesson time is used for public class work

Mistakes per lesson	Public	Private
Germany (N=22)	4.7	no data
Switzerland (N=10)	3.5	1.6
Italy (N=30)	10.7	0.3
USA (N=30)	4.6	3.2

Table 2: Average number of mistakes per lesson in private or public situations.

and in Switzerland only 54% of the lesson time (cf. Table 1), we can state that in Italian mathematics lesson proportionately more public mistakes appear than in Swiss lessons. The reason for the comparatively high number of public mistakes in Italy goes back to the so called blackboard activity: nearly half of these mistakes occur when an individual student is asked to solve a problem at the black board

(Santagata, in press). Such activities do not play an important role in the other three countries. If we take a closer look to the relation between students mistakes and students contributions in a lesson, then we have for the German sample an average number of 47.3 contributions of students in a lesson, i.e. in 10% of the students' contributions the teacher identified a mistake.

The public mistake-handling activities in Italian, US and German mathematics lessons are clearly directed by the teacher (93% USA and Italy, 88% Germany, no data for Switzerland). As shown in Table 3 a quarter to a third of the mistake situations

	Teacher corrects	Teacher asks students
Germany	26.9%	48.5%
Italy	31.6%	53.6%
USA	25.1%	62.2%
Remaining cases: Ignoring, student corrects directly etc.		

Table 3: Mistake-handling by Teachers.

are directly solved by the teacher; between 48% and 62% are returned by the teacher to the students as a challenge. In the German lessons 15.4% of the mistakes were directly corrected by students without a teacher activity and about 10% were ignored, i.e., no corrections could be

observed though the mistakes were identified by the teacher or students.

A last topic that should be addressed here is the question how the mistake situations in the German mathematics lessons are finally solved. In 12.5% of the cases there was no clear correction, because the mistake was ignored or the correction of the teacher/students was obviously not sufficient. For 46% of the mistakes there was a simple correction and for 41.3% a detailed explanation was given. A second rating of the mistake situations showed that 44% of the mistake-handling activities mainly focus on the individual learning progress and also 44% were mainly oriented to the continuation of the intended course of the lesson (12% other or undecidable).

Mistake-handling activities from the perspective of students were investigated in the Swiss study (c.f. Spychiger, Mahler, Hascher, Oser, 1998). A questionnaire with 27 items (four-point Likert scale) was developed and administered in a sample of 295 students from grade 4 to 9. A factor analysis yielded three main components which describes the teacher behaviour in mistake situations, the individual use of mistakes in the learning process and the individual emotions in mistake situations. The results can be summarized as follows: The teacher behaviour and the individual emotions in mistake situations were rated comparatively positive by the students (cf. Table 4). In contrast to this the finding for the second component indicates that the students hardly use mistakes as an individual learning opportunity. Spychiger et al. (1998) consider this result as a possible starting point for an intervention to improve students' individual mistake handling.

RESEARCH QUESTIONS AND DESIGN

Based on the theoretical approach and the empirical results described in the previous sections we addressed in our research the student perception of the mistake-handling activities in the German mathematics classroom. Our study was guided by the following research questions:

1. What is the perception of the students regarding mistake-handling activities in the mathematics lessons? What do the students think about the teacher behaviour and are they afraid of making mistakes in public situations?
2. How do students individually deal with (their own) mistakes? Do they use mistakes as a learning opportunity?
3. Which kind of mistakes are “permitted” or “forbidden” in the perception of students (in the sense of negative teacher reactions)?

The sample of the study comprises 85 students from grade 8 and grade 9 from three different classes of one school. For the data collection the questionnaire from the Swiss study described above (Spychiger et al, 1998) was adapted. At the end of the questionnaire two open questions were added: (1) *In which situations (except exams, tests etc.) are mistakes forbidden?* and (2) *In which situations (except exams, tests etc.) are mistakes permitted?.* Moreover, the grade for mathematics of the last school report was asked.

RESULTS

Like in the Swiss study a factor analysis yielded three main components which basically coincide with the components described above. However, in our study some of the items of the Swiss component *Teacher behaviour* also loaded on the component *Individual emotions in mistake situations*. These were particularly items like “I have the feeling that it is not allowed to make mistakes, because our mathematics teacher doesn't like this.” Thus, we denoted the emotions component in our study *Fear of making mistakes* (10 items). The other two components were denoted like in the Swiss study *Teacher behaviour* (7 items) and *Individual use of mistakes*

(8 items). The reliability turned out to be satisfactory (Cronbach’s α between 0.73 and 0.83).

Table 4 gives the mean values for the scales for the German and the Swiss study. Regarding the component “Individual use of mistakes for the learning process” and

	Teacher behaviour	Individual use of mistakes	Fear of making mistakes
German study	2.93	2.86	1.74
Swiss study (Spychiger et al. 1998)	3.38	2.75	1.82*
Likert scale: 1=strongly disagree, 2=disagree, 3=agree, 4=strongly agree			
*Calculated from the reverse component “Individual emotions in mistake situations” with mean value 3.18.			

“Fear of making mistakes” the values are similar: On the one hand, the students are rarely afraid of making mistakes and on the other hand, they do not use their mistakes very productively.

Table 4: Mistake-handling from students’ perspective (mean values).

The rating of the teacher behaviour is in the German sample worse than in the Swiss sample; however, from the students’ perspective the teacher behaviour is acceptable.

If we consider the histograms of the three components in Figure 1, we can see that the positive results for the teacher behaviour and the fear of making mistakes is shared by most of the students in the sample. Moreover, we can learn that there is a slightly broader range regarding the individual use of mistakes for the learning process (e.g. nearly 20 students show values about 2.5, the mean of the Likert scale).

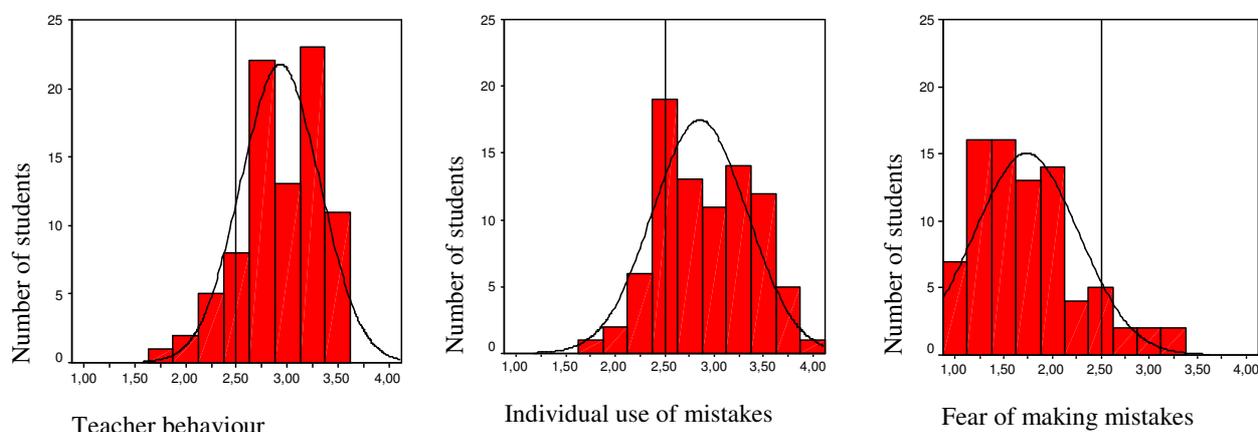


Figure 1: Distributions of students for the three components regarding mistake-handling.

If we consider the relation between the students’ achievement (measured by the last grade² in mathematics on the school report) and the students’ rating for the three mistake-handling components, then we find only one correlation. The grade in mathematics correlates significantly with the rating for the scale *Individual use of mistakes*

² School grades in Germany: 1 = very good, 2 = good, ..., 6 = insufficient.

($r = -.235$, $p=0.006$, Kendall- τ). This correlation indicates that high achieving students are better in using mistakes for their learning processes than low achieving students (cf. Figure 2).

The last part of the student questionnaire was dedicated to typical situations in mathematics lesson, in which mistakes are (1) forbidden or (2) permitted. The students' responds were categorized by a bottom-up analysis and we got the results of the following Table 5.

From the students point of view basic knowledge and repetitions are areas in which an individual

forbidden	students	permitted	students
basic knowledge	28	new content	39
repetition	19	challenging tasks	18
hardly any / none	9	homework	17
at the blackboard	7	exercises	15
other	6	(nearly) always	14

Table 5: Forbidden / permitted situations for mistakes

different opinions on that question, because 14 students think that mistakes are (nearly) always permitted.

DISCUSSION

The findings of the different studies described in the previous sections discover positive and problematic aspects of the mistake-handling activities in the mathematics classroom. Regarding the German situation the video analysis shows that there are comparatively few mistake situations in the lessons (the same problem is observed in Switzerland and the USA, cf. Table 2). If mistakes are considered as an essential part of the learning process, like the theory of negative expertise states, then this small amount of mistakes is problematic. As Santagata (in press) supposes the mistake-handling activities by the US teachers is still influenced by the behaviouristic approach concerning learning, i.e. avoiding mistakes and reinforce correct answers. A similar explanation may be valid for the German situation since 44% of the mistakes-handling activities aimed at continuing the indented course of the lesson. Nevertheless, we could observe that in the German lessons for 41.3% of the mistakes a detailed explanation was given which led to a productive solution of the mistake situation. Thus, there are only few mistakes in the lessons, but if a public mistake appears, then there is a good chance that it will be explained and corrected. Until now we do not know which types of mistakes are treated more deeply by the teachers.

From the students perspective we got a comparatively positive image regarding the mistake-handling activities in the German mathematics classroom. Generally, the students do not fear to make mistakes and in their opinion the teacher behaviour is acceptable. However, there is some potential to improve the students' individual

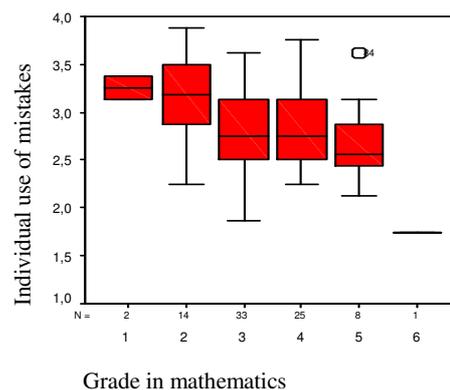


Figure 2: Grade and individual use of mistakes for the learning process.

cannot afford to make mistakes. In contrast, new content, challenging tasks and the official opportunities to practise the acquired knowledge are areas in which mistakes are permitted. However, the students have

different opinions on that question, because 14 students think that mistakes are

mistake-handling activities. Thus, the findings of the German study replicate the results of the Swiss study (cf. Table 4). Moreover, the correlation between the achievement and the individual mistake-handling activities indicates that this is a particular problem of the medium and low achievement students (Figure 2). Here we can think about the development of special teaching elements.

The students' views on forbidden and permitted mistake situations must be seen critical. In particular, the fact that mistakes are considered as forbidden when basic knowledge or repetitions are treated in mathematics lessons, contradicts the idea that teachers should assist students in their individual knowledge construction. Since knowledge construction substantially involves the pre-knowledge, these "forbidden mistakes" give the opportunity to detect critical gaps. Thus, a starting point for productive mistake-handling activities by teachers is to avoid the students' impression that certain mistakes are forbidden.

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