

Strengthening Mathematics Skills at the Postsecondary Level: Literature Review and Analysis

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**STRENGTHENING MATHEMATICS SKILLS AT THE
POSTSECONDARY LEVEL:
LITERATURE REVIEW AND ANALYSIS**

Prepared for:
U.S. Department of Education
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U.S. Department of Education

Margaret Spellings

Secretary

September 2005

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Abbreviations

AACC	American Association of Community Colleges
AASCU	American Association of State Colleges and Universities
AAU	Association of American Universities
ABE	Adult Basic Education
ACCUPLACER	College Placement Exam
ACES	Adult Classroom Environment Scale
ACT	American College Testing Program
AECF	Advanced Electronics/Computer Field
AFQT	Armed Forces Qualifications Test
ALEKS	Assessment and LEarning in Knowledge Spaces
AMA	American Management Association
AMATYC	American Mathematical Association of Two-Year Colleges
AP	Advanced Placement
AR	Arithmetic Reasoning
ARG	Association Review Group
AS	Auto Shop Information
ASI	Adaptive Style Inventory
ASSET	Assessment of Skills for Successful Entry and Transfer
ASTD	American Society for Training and Development
ASVAB	Armed Services Vocational Aptitude Battery
BLS	Bureau of Labor Statistics
BSEP	Basic Skills Education Program
CAI	Computer-Assisted Instruction
CAS	Computer Algebra Systems
CBE	Computer-Based Education
CBI	Computer-Based Instruction
CEI	Computer-Enriched Instruction
CMI	Computer-Managed Instruction
CNA	Center for Naval Analyses
CNAC	CNA Corporation
COMAP	Consortium for Mathematics and its Applications
COMPASS	Computerized Adaptive Placement Assessment and Support System
COTS	Commercial off the Shelf
CPT	Computerized Placement Test
CRAFTY	Curriculum Renewal Across the First Two Years
DOD	Department of Defense
DTIC	Defense Technical Institute Center
EI	Electronics Information
EIT	Employee- in-training
ESL	English as a Second Language
ETS	Educational Testing Service
FY	Fiscal Year

GED	General Equivalency Diploma
GS	General Science
GT	General Technical
HSDG	High School Degree Graduate
IEP	Individualized Education Plans
JDCC	Jefferson Davis Community College
KET	Kentucky Educational Television
KSAs	Knowledge, Skills and Abilities
MAA	Mathematical Association of America
MASP	Military Academic Skills Program
MC	Mechanical Comprehension
MK	Math Knowledge
NAEP	National Assessment of Education Progress
NALS	National Adult Literacy Survey
NCES	National Center for Education Statistics
NCLC	Navy College Learning Centers
NCLP	Navy College Learning Program
NCPACE	Navy College Program for Afloat College Education
NCRVE	National Center for Research in Vocational Education
NCTM	National Council of Teachers of Mathematics
NHSDG	Non-High School Diploma Graduate
OJT	On-the-job training
OVAE	Office of Vocational and Adult Education
PALS	Principles of Adult Learning Scale
PC	Paragraph Comprehension
RCT	Randomized Controlled Trial
ROI	Return on Investment
SAT	Scholastic Assessment Test
SOC	Servicemembers Opportunity Colleges
TA	Tuition Assistance
TABE	Test of Adult Basic Education
TiPS	Tutorials in Problem Solving
UAW	United Auto Workers
UVSC	Utah Valley State College
VE	Verbal Expression
VolEd	Voluntary Education
VTC	Video Conferencing
WASL	Washington State Assessment of Student Learning
WIA	Workforce Investment Act
WIB	Workforce Investment Board
WK	World Knowledge

Executive Summary

Background

The nature of America's workforce has changed dramatically in the past several decades, due in large part to the infusion of rapidly changing technology. This trend has resulted in an increased need for workers with greater mathematical skills and higher education.

The U.S. Department of Education Office of Vocational and Adult Education (OVAE) contracted with The CNA Corporation (CNAC) and its partners to identify promising strategies within community colleges, businesses, organized labor, and the military that enable adult learners to strengthen their math skills and abilities and to transition into higher-level math courses or work assignments requiring higher-level mathematics.

This literature review is the first step in this process. In order to establish a baseline understanding of postsecondary developmental mathematics programs we examine the following three issues:

1. What is the definition, or skill threshold, of adequate student preparation in mathematics at the postsecondary level?
2. What institutions provide developmental math education, and how does the education provided differ across these institutions?
3. What approaches and strategies appear to hold promise for enabling adult learners to strengthen their mathematical skills and to progress into college-level math courses or work assignments requiring higher-level mathematical abilities?

Strategies identified in this review will provide the basis for the second phase of this project, the purpose of which is to identify math programs in community colleges, business, labor organizations, and the military that have supporting evidence that such strategies are, indeed, successful.

Major Findings

Institutions providing developmental math instruction

Community colleges are the largest source of developmental math instruction, and virtually all public two-year colleges offer at least one developmental course. However, these colleges vary in the number of developmental courses they offer, how many of these courses each student may take, and the type of credit awarded. In addition, 56 percent offer developmental education to local businesses, and basic math courses are offered by 93 percent of the colleges that extend courses to businesses (NCES 2003).

The military services offer basic skills instruction to members who qualify on the basis of low entrance test scores or because they do not possess a General Equivalency Diploma (GED). Their remediation efforts vary across the services in both the length and method of instruction, yet more than 37,000 service members receive basic skills instruction each year (U.S. Department of Defense 2004).

Adult education and workforce development programs also provide basic skills remediation. There is growing interest in developing opportunities for people who lack skills and resources around career pathways that integrate education, training, and skill development in targeted high-wage, high-demand employment areas. Career pathways provide developmental, adult, or English as a Second Language (ESL) classes in the context of students' lives and the work-specific skills they need for employment in particular industries or sectors.

Central to federal government efforts to strengthen the skills of displaced or dislocated workers is Title I of the *Workforce Investment Act of 1998*. This program supports a network of One-Stop Career Centers that provide access to a full range of services pertaining to employment, training and education, employer assistance, and guidance for other types of assistance.

Businesses also are involved in the remediation of basic skills. Companies spend an average of 1.8 percent of payroll on training. Of this amount, 5 to 7 percent is in basic skills, including literacy, reading, comprehension, writing, math, ESL, and learning how to learn. By far, the largest category of training is in technical processes and procedures, which totals approximately 13 percent of all training expenditures (Van Buren 2001). The most often cited sources of external education and training used by business are community colleges, technical and vocational schools, business and industry associations, consultants, and universities.

What constitutes adequate math preparation?

We found that the knowledge necessary for successfully pursuing college-level math programs depends on the education and career goals of the individual. For instance, adult learners in community colleges would require somewhat different knowledge if the first college-level course were calculus rather than business math. Regardless of whether this is a contributing factor, we have found that no consistent definition of math standards for college-level preparation exists. However, a number of studies indicate the need to have a good foundation in arithmetic, geometry, trigonometry, and algebra I and II. Emerging work also indicates the increasing need for basic statistics and the ability to analyze data.

There is less uncertainty or ambiguity in the skills necessary to pursue college-level math and to succeed in the highest-paid and highest-skilled jobs. In particular, there seems to be agreement on the need to think critically, to solve problems, and to communicate mathematically. Both businesses and postsecondary institutions that were surveyed as part of large curriculum reform efforts indicate that they want people who can identify a problem, determine whether it can be solved, know which operations and

procedures are required to solve it, use multiple representations (such as graphs and words) to describe problems and solutions, and understand and apply mathematical modeling. However, these are the skills that are the most difficult to teach and to assess (American Diploma Project 2004).

Whether community colleges are universally adopting these recommendations—in terms of the specific knowledge, skills, and abilities—remains to be seen. It is also uncertain whether community colleges adequately assess the knowledge and skills necessary to pursue postsecondary level math or succeed in the workplace. Regardless, the majority of two-year colleges require incoming students to take and pass an assessment test before they are allowed to enroll in college-level math courses. Given their prevalence, this may be the most relevant benchmark for whether a student can successfully transition to college-level mathematics. While minimum scores vary, we note a range of score thresholds for the most common of these tests.

Best instructional practices

Our extensive search of the developmental education literature yielded only a limited number of studies pertaining to adult developmental mathematics instruction, the majority of which has been conducted in two-year colleges. Of these, we reviewed 15 studies of postsecondary institutions, with a majority based on programs in community colleges. Unfortunately, none were based on randomized controlled trial experiments, which have been elevated to a position of the “gold standard” for research because this experimental design is relatively unbiased in evaluating the effect of programmatic interventions in the field of education. To augment research on developmental mathematics programs for adult learners, particularly aspects of developmental math courses, we relied on a broader base of research to provide guidance as to what may hold promise for developmental mathematics specifically. However, we were not able to locate published research on developmental mathematics programs outside of academic institutions. In addition to scholarly sources, we searched Web sites of businesses and labor organizations. For example, the Web site of the AFL-CIO, with a membership of over 13 million, has a section concerning education issues and legislation, but it contains no information about specific education programs in general, or developmental mathematics in particular.

Although we did not identify existing studies containing scientifically based evidence of promising practices, salient themes concerning pedagogy emerged, suggesting promising but unproven instructional practices that are frequently implemented. These may warrant further study. Among the recommendations in the literature are: greater use of technology; integration of classroom and laboratory instruction; giving students the option to select from among different instructional methods; use of multiple approaches to problem solving; project-based instruction; low student to faculty ratios; assessment and placement of students into the appropriate mathematics courses; and integration of counseling, staff training, and professional development.

Underscoring these recommendations, our review found that a number of studies sought to evaluate the impact of various teaching delivery methods on student success, including traditional lecture, computer-assisted courses, self-paced instruction, Internet-based courses, and accelerated programs. No clear consensus of the effectiveness of technology-based delivery methods emerged. Using various metrics, some studies found no effects, some found higher levels of success, and some found lower levels of success for students using technology-based or technology-enhanced instruction versus traditional lecture. However, a number of research projects, such as those from the American Mathematical Association of Two-Year Colleges (AMATYC) (1995 and 2002) and the American Diploma Project (2004), conclude that all students should be familiar with technology, including graphing calculators, and spreadsheets, and should be able to understand the benefits and limitations of each. Further, there is general consensus that technology should be a supplement to, as opposed to a replacement of, more traditional delivery methods. However, given the inconsistency in study findings, we believe that this is one area that warrants further investigation.

Finally, we have found research that indicates that the types of problems used in teaching the material is important. In particular, it is important to use activities that engage students in the learning process, particularly in small collaborative group processes, most of which reflect the real-world problem solving done in businesses. These activities should require the student to actively plan, design, research, model, and report findings for projects or case studies. Some argue that students require contextual learning and real-world problems to help make coursework and training relevant and meaningful.

We summarize key components of best practice approaches to postsecondary developmental mathematics programs below:

- **Instructional and pedagogical:** adjuncts to traditional instruction; multiple delivery options from which students may choose; computer-assisted instruction; Internet-based; self-paced; distance learning; calculators; computer algebra systems; spreadsheets; laboratories; small group instruction; learning communities; contextual learning; linkages to and examples from the workplace; and career pathways.
- **Curriculum content:** nonstandard topics covered in developmental math courses or topics that vary by career path; length of instruction; and types of activities used to reinforce the material.
- **Professional development:** faculty training and development; full-time versus part-time instructors; and proportion of faculty that are adjuncts.
- **Supporting strategies:** counseling, assessment, placement, and exit strategies.
- **Learner and institutional characteristics:** full-time versus half-time community college student; socioeconomic attributes of learner; workplace versus academic learner; and having private or military employment versus preparing for a new career.

Introduction

The nature of America's workforce has changed dramatically in the past several decades, due in large part to the infusion of rapidly changing technology. This trend has resulted in an increased need for workers with greater skills and higher education. For instance, the Bureau of Labor Statistics (BLS) (BLS 2001) predicts that jobs requiring at least a bachelor's degree will grow 21.6 percent between 2000 and 2010, and those requiring an associate degree or vocational certificate will increase 24.1 percent. In contrast, jobs requiring only work-related experience will increase just 12.4 percent during the same time period.

In spite of these trends, employers are finding that their workforce is simply not prepared to meet even the most basic skill requirements, including reading, writing, and mathematics. The National Association of Manufacturers found in a 2001 survey sent to its members that 80 percent of manufacturers experience a moderate to serious shortage of qualified job candidates, and that 26 percent of employers listed inadequate math skills among the most serious skill deficiencies (National Association of Manufacturers Center for Work Success 2001). Further, 20 percent said they rejected applicants for hourly production positions due to inadequate math skills. A survey conducted in 2001 by the American Management Association (AMA), based on responses from 1,627 human resource managers in AMA member and client companies, found that 41 percent test job applicants in basic literacy and or math skills; of those tested, 34 percent lacked sufficient skills for the positions they sought (AMA 2001). Less than 9 percent of the respondents said that they hired those found to be deficient. If interested in hiring, respondents either assigned applicants to obligatory developmental training or offered voluntary developmental training, and this was true across business sectors. Manufacturing offered remediation to the most, 8.6 percent, while wholesale and retail offered remediation to the least, just 2.8 percent. Instead, the overwhelming majority of companies simply refuse to hire those who do not pass the basic skills requirements—the fate of over 80 percent of these applicants in all business sectors.

This growth in deficient skills is in large part a function of the rapidly increasing skill requirements and the changes in these requirements over the past few years. In other words, it is not just a matter of employers requiring more workers to know math; the type of math required is also changing.

The most recent statistics on adult literacy¹ confirm the deficit in skilled workers. The 1992 National Adult Literacy Survey (NALS), conducted by the National Center for Education Statistics (NCES), assessed the prose, document, and quantitative literacy

¹ A number of definitions exist for numeracy and mathematical, or quantitative, literacy. We prefer the definition provided in the 1994 Conference on Adult Mathematical Literacy. That source states that numeracy and mathematical literacy are interchangeable notions, and define them as “the aggregate of skills, knowledge, beliefs, patterns of thinking and related communicative and problem-solving processes individuals need to effectively interpret and handle real-world quantitative situations, problems, and tasks” (Gal and Schmitt 1995)

proficiency of adults (Kaestle et al. 2001). The study defined the following five hierarchical levels of quantitative proficiency (Kaestle et al. 2001, p. 11):

- Level 1: Tasks require performing single, relatively simple arithmetic operations, such as addition. The numbers to be used are provided and the arithmetic operation to be used is specified.
- Level 2: Tasks typically require performing a single operation using numbers that are either stated in the task or easily located in the material. The operation to be performed may be stated or easily determined from the format of the material (for example, an order form).
- Level 3: Two or more numbers are typically needed to solve the problem, and these must be found in the material. The operation(s) needed can be determined from the arithmetic relation terms used in the question or directive.
- Level 4: The fourth level requires performing two or more sequential operations or a single operation in which the quantities are found in different types of displays, or the operations must be inferred from semantic information given or drawn from prior knowledge.
- Level 5: Level 5 requires performing multiple operations sequentially. The features of the problem must be disembedded from text or based on background knowledge to determine the quantities or operations needed.

Their results indicated that 22 percent of adults demonstrated skills in the lowest level of quantitative literacy proficiency, and an additional 25 percent demonstrated skills at the second lowest level. In other words, almost half of all adults could not perform tasks at the level necessary as defined by the third level of quantitative proficiency, such as using a calculator to calculate the difference between the regular and sale price of an item in an advertisement (Kaestle et al. 2001, p. 208).

Kaestle and his colleagues (2001) concluded that there is a strong relationship between the level of literacy and education attainment. For instance, the quantitative proficiency of 79 percent of adults who were high school dropouts and completed nine to 12 years of school was below the third level, compared with 51 percent of those who completed a high school degree. They also found that those high school dropouts who went on to earn a GED fared as well as high school diploma graduates, with 54 percent scoring below the third level (Kaestle et al. 2001, p. 17).

Low levels of literacy have fairly serious negative economic consequences. The NALS study found employed workers who scored in the lowest two levels of literacy tended to be employed in the lowest wage occupations, such as food service, childcare, and maintenance (Kaestle et al. 2001, p. xxxviii). Other studies have also found that high school dropouts experience higher rates of unemployment and are more likely to receive

public assistance than high school diploma graduates who did not go on to college (National Center for Family Literacy 2003).

The fact that half of all high school graduates do not possess quantitative literacy skills at least at the third level, as defined by the NALS study, is a clear indication that a high school diploma is not enough to meet the increasing need for highly skilled workers. In part as a response to the needs of employers and the higher wages that high-tech jobs offer, the rate of college enrollment of graduating high school seniors has increased significantly since the last half of the 20th century, from 45 percent in 1960 to 62 percent in 2001 (NCES 2002, table 184). Even so, studies find that a large proportion of those who enroll in college are not prepared to pursue college-level courses. A recent study concluded that more than one million students entering postsecondary education each year require participation in developmental courses, representing 42 percent of the student population (McCabe 2000). This same research concluded that successfully remediated students do perform well in standard college-level courses, noting that 82 percent of a nonrandom sample of remediated students included in the study passed college-level mathematics classes. This is a striking finding considering that many developmental courses are described by students as dull and poorly taught, and emphasize low-level drill and practice (Grubb 1999).

Recently, states have established higher standards for high school graduation, have increased admission requirements at colleges and universities, have structured open admissions programs at community colleges, and have used testing and evaluation to assess education outcomes (Bandy 1985; Fonte 1997; Merisotis and Phipps 2000; Thacker 2000). According to the U.S. Department of Education, however, only four states² required students to have four Carnegie Units (each unit is roughly equivalent to one academic year of study) in mathematics for high school graduation in 2001, seventeen of the remaining states only required two units, and the rest required three (NCES 2002, table 152).

Clearly, some students can take more math than what is mandated by state law for high school graduation, and some states recommend more math for college-bound students. According to Barth (2002), the percentage of students completing algebra II in high school, the minimum content typically required to enroll in college-level mathematics, has grown from 40 percent to 62 percent between 1982 and 1998.

Yet, according to the U.S. Department of Education, the average scores on the National Assessment of Education Progress (NAEP) of 17-year-olds whose highest level of mathematics is algebra II are at a level that enables them to perform reasoning and problem solving involving fractions, decimals, percents, elementary geometry, and simple algebra (NCES 2002, table 125). But the problem does not begin in high school. The most recent news of the math competency of the nation's schoolchildren shows that scores on the NAEP are up in mathematics, but a fairly large number still do not meet the proficiency standards set by the National Assessment Governing Board (Plisko 2003). Among fourth-graders, 77 percent are at or above a basic level of proficiency, up from 50

² Alabama, Georgia, North Carolina, and South Carolina.

percent in 1990. For eighth-graders, 68 percent are at a basic level of proficiency or higher, up from 52 percent in 1990. But as impressive as these gains are, they still show that almost one-third of eighth-graders are not at a basic level of math proficiency.

Adult learners who are not recent high school graduates who seek to improve their basic skills literacy, earn a GED, or pursue postsecondary education face more difficulties in obtaining higher level math skills than recent graduates do. In particular, they often face more financial (often as sole household earner) and logistical (such as daycare and time off from work) challenges. And in many cases, they have a history of education failure and of long-term functioning at low levels of quantitative literacy.

The *Adult Education and Family Literacy Act of 1998*, Title II of the *Workforce Investment Act of 1998*, authorizes a program of national leadership activities to enhance the quality of adult education and literacy programs nationwide, including collecting data and disseminating best practices information. The U.S. Department of Education Office of Vocational and Adult Education (OVAE) is sponsoring a number of studies that address the growing need for adult education in general and basic literacy in particular.

OVAE has sponsored this project to provide better information on current and new strategies under development in the field, and to use that information as guidance for future research efforts into promising practices in developmental mathematics for adult learners. One specific goal is to inform and enhance adult basic education programs to ensure that participants have the math knowledge and skills necessary to pursue college-level mathematics when they transition from Adult Basic Education (ABE) to postsecondary education or to workforce programs that require higher-level math.

The goals of this literature review—the first phase of this research—are to: (a) define the mathematical knowledge and skills necessary to pursue college-level mathematics and (b) identify the elements of developmental mathematics programs within community colleges, the military, businesses, and organized labor that enable adult learners to transition from developmental to college-level mathematics. To that end, we address the following three issues:

1. What is the definition and skill threshold of adequate student preparation in mathematics at the postsecondary level?
2. What institutions provide developmental math education, and how does the education provided differ across these institutions?
3. What approaches and strategies appear to hold promise in enabling adult learners to strengthen their mathematical skills and to progress into college-level math courses or work assignments requiring higher-level mathematical abilities?

We begin our literature review with a discussion of the knowledge and skills necessary to pursue college-level mathematics, and of common tests and their cutoff scores used to assess the ability of students to pursue college-level mathematics. Next we

discuss a review of the literature concerning components of successful developmental mathematics programs in postsecondary settings. Because of the paucity of research on many aspects of developmental mathematics instruction for adult learners, we also review literature of more common themes within developmental education.

We then review some common practices in colleges in terms of these components, to determine the progress of postsecondary institutions in addressing the recommendations from the literature. Finally, we examine what other organizations are doing in developing the mathematics skills of their workforce. In particular, we look at the military, businesses, labor organizations, and other adult education efforts.

What Skills and Knowledge Do Students Need to Pursue College-level Mathematics?

The primary goal in this literature review is to define the skill threshold necessary to pursue college-level mathematics and to discover promising strategies for helping students to progress to this threshold. We have discovered two salient themes in the literature concerning what this means precisely. The first is the knowledge, or content, required. This includes a detailed description of the specific math facts or subjects to be covered, such as ratios, decimals, or, more broadly, arithmetic. The second theme concerns the skills and abilities necessary to pursue college-level math. By skills we refer to observable competencies to perform a function. For instance, critical thinking, generating ideas, and determining which tool is necessary to do a job are considered skills. Abilities are attributes that affect the ability to perform a task, such as manual dexterity and inductive and deductive reasoning.

Based on our review, we conclude that there is less uncertainty or ambiguity in these necessary skills and abilities than there is in the required knowledge, or content. In fact, community colleges and businesses are in general agreement concerning these skills and abilities. However, skills and abilities are often more difficult than knowledge to teach and assess. In particular, there seems to be widespread consensus as to the need to think critically, to solve problems, and to communicate mathematically. Several studies provide more precise definitions of these skills, which we summarize below.

Crossroads

Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus (hereafter referred to simply as Crossroads), published in 1995 by the American Mathematical Association of Two Year Colleges, established goals and standards for preparation for college-level mathematics that are the most oft-cited of any study of developmental mathematics at the postsecondary level (American Mathematical Association of Two Year Colleges 1995). AMATYC developed on six guiding principles upon which it based its standards:

1. All students should grow in their knowledge of mathematics while attending college.
2. Students should study mathematics that is meaningful and relevant.
3. Mathematics must be taught as a laboratory discipline.
4. The use of technology is an essential part of an up-to-date curriculum.
5. Acquiring mathematics knowledge requires balancing content and instructional strategies recommended in the AMATYC standards along with the viable components of traditional instruction.

6. Increased participation in mathematics and in careers using mathematics is a critical goal in our heterogeneous society.

The standards are divided into three categories: intellectual development, content, and pedagogy. Because Crossroads is the seminal work in this area, we summarize its standards, which provide goals for introductory college mathematics and guidelines for selecting content and instructional strategies for accomplishing the principles.

Intellectual development standards

Students will:

- Engage in substantial mathematical problem solving;
- Learn mathematics through modeling real-world situations;
- Expand their mathematical reasoning skills as they develop convincing mathematical arguments;
- Develop the view that mathematics is a growing discipline, interrelated with human culture, and understand its connections to other disciplines;
- Acquire the ability to read, write, listen to, and speak on mathematics subjects;
- Use appropriate technology to enhance their mathematical thinking and understanding and to solve mathematical problems and judge the reasonableness of their results; and
- Engage in rich experiences that encourage independent, nontrivial exploration in mathematics, develop and reinforce tenacity and confidence in their abilities to use mathematics, and inspire them to pursue the study of mathematics and related disciplines.

Content standards

Students will:

- Perform arithmetic operations and will reason and draw conclusions from numerical information;
- Translate problem situations into their symbolic representations and use those representations to solve problems;
- Develop a spatial and measurement sense;
- Demonstrate understanding of the concept of function by several means (verbally, numerically, graphically, and symbolically) and incorporate it as a central theme into their use of mathematics;
- Use discrete mathematical algorithms and develop combinatorial abilities in order to solve problems of finite character and enumerate sets without direct counting;
- Analyze data and use probability and statistical models to make inferences about real-world situations; and

- Appreciate the deductive nature of mathematics as an identifying characteristic of the discipline; recognize the roles of definitions, axioms, and theorems; and identify and construct valid deductive arguments.

Pedagogy standards

Mathematics faculty will:

- Model the use of appropriate technology in the teaching of mathematics so that students can benefit from the opportunities it presents as a medium of instruction;
- Foster interactive learning through student writing, reading, speaking, and collaborative activities so that students can learn to work effectively in groups and communicate about mathematics both orally and in writing;
- Actively involve students in meaningful mathematics problems that build on their experiences, focus on broad mathematical themes, and build connections within branches of mathematics and other disciplines so that students will view mathematics as a connected whole relevant to their lives;
- Model the use of multiple approaches—numerical, graphical, symbolic, and verbal—to help students learn a variety of techniques for solving problems; and
- Provide learning activities, including projects and apprenticeships that promote independent thinking and require sustained effort and time so that students will have the confidence to access and use needed mathematics and other technical information independently, to form conjectures from an array of specific examples, and to draw conclusions from general principles.

The AMATYC is revising its Crossroads curriculum standards to “create a product that communicates a renewed vision and guidelines” (American Mathematical Association of Two Year Colleges 2002a). To that end, the AMATYC conducted two activities to assess the impact of the original Crossroads standards. It sent a survey to 150 AMATYC members and 250 potential members; respondents numbered 42 and 13, respectively. In addition, an Association Review Group (ARG) was established consisting of 63 AMATYC affiliates, academic committee chairs, and members. While the survey response rate is low, we note the findings because they may be indicative of a larger trend. In particular, the survey responses suggest that respondents made the following changes to mathematics curricula in response to the original Crossroads recommendations:

- Greater use of technology;
- More emphasis on contextual experiences, problem solving, or modeling;
- More collaborative work in the classroom; and
- Increased awareness of different teaching and learning styles.

Many respondents attributed these curricula changes to Crossroads. Those who did not said that either Crossroads reaffirmed such principles or the National Council of Teachers of Mathematics (NCTM) Standards had a greater effect on causing these changes. NCTM provides comprehensive guidelines covering curricula, professional teaching standards, and assessment standards targeted toward K–12 mathematics curricula (National Council of Teachers of Mathematics 1989, 1991, 1995, 2000).

Further, survey respondents noted that the most significant barriers to implementing the Crossroads recommendations were time, overcoming faculty resistance to change, money, scarcity of texts and materials, and lack of convenient and affordable professional development opportunities. Finally, the top six issues that faculty members believe should be addressed in their current reform efforts are instructional delivery, technology, pedagogy, content, adjunct faculty, and training new and retaining current faculty.

Standards for Success

Conley and Bodone (2002) provide one of the more comprehensive studies of mathematical content necessary for success for entry-level college students. This study reports findings from a collaborative group of 400 representatives from numerous universities that generated the so-called Standards for Success, in which they formulated key knowledge and skills necessary for university success in entry-level courses compared with just high school preparation. The Pew Charitable Trusts and the Association of American Universities (AAU) sponsored the work of Conley and Bodone, and the authors note that it is the first and only comprehensive statement of university entrance-level skills that is presented in terms of standards rather than simply in terms of course names or broad content statements.

The authors point out several important findings emanating from this work. For instance, academic content standards in the K–12 system are not set in consultation with higher education personnel, and no state’s standards correlate with college admission criteria. These criteria are expressed in terms of class rank, GPA, and required courses—but not in competencies. The authors suggest that it is important to align the K–12 standards with academic expectations so that there are not two distinct educational systems (K–12 versus postsecondary) with vastly different knowledge and skills expectations and outcomes. However, they also note that, while the basic content knowledge standards proposed by this group do align well with individual states’ standards for high school, a real divide exists in the types of intellectual development that should accompany the mastery of content knowledge.

Because the Standards for Success included the knowledge and skills necessary, which goes beyond the Crossroads standards, it is important to include the full set of standards in this report. However, because they are so detailed and lengthy, we provide the full list in Appendix A. In summary, the standards established by this group indicate that before pursuing college-level mathematics, students should have the following knowledge: basic arithmetic, including fractions and exponents; basic algebra, including

manipulation of polynomials and solutions for systems of linear equations and inequalities; basic trigonometric principles; basic pictorial and coordinate geometry, including the relationship between geometry and algebra; and statistics and data analysis.

Further, the standards stipulate abilities similar to those in Crossroads, under mathematical reasoning. For instance, they state that students should have the ability to: (a) use inductive reasoning and a variety of strategies to solve problems; (b) use a framework or mathematical logic to solve problems that combine several steps; and (c) determine mathematical concept from the context of a real-world problem, solve the problem, and interpret the solution in the context of a real-world problem.

Related to this literature, we note some emerging research below on math content and skill requirements of students in two-year colleges. This work has important implications. It may mean a shift in the knowledge and skills that are necessary to pursue math at two-year colleges. It also reinforces some common themes in the two studies cited, as well as other literature that we reviewed and that we discuss later.

The Vision Report

First, we note the work that is being conducted by AMATYC under the National Science Foundation grant, “Technical Mathematics for Tomorrow: Recommendations and Exemplary Programs.” At a recent national conference, over 80 educators, technical personnel from business and industry, and technical faculty from two-year colleges identified what they defined as exemplary practices in mathematics programs that serve highly technical curricula, such as biotechnology, computerized manufacturing, electronics, information technology, semiconductors, and telecommunications. Their work, summarized in *A Vision: Final Report from the National Conference on Technical Mathematics for Tomorrow* (AMATYC 2002b) and hereafter referred to as the Vision Report, built on that conducted by those of the Mathematical Association of America’s (MAA) subcommittee on Curriculum Renewal Across the First Two Years (CRAFTY), who also participated in this conference.

The recommendations that emerged from this conference cover several topics. Underlying the discussion of content is the necessity for students to possess certain abilities that the Crossroads and Standards for Success research highlighted, as well as those emphasized by other studies we discuss later. The recommendations are uniform, regardless of the learner’s age, level of mathematics, or organization noting the requirement, and are equally important as, or perhaps even more important than, the knowledge requirements themselves. These are critical thinking skills, the ability to communicate mathematics, and the ability to select an appropriate method to solve a problem—from fairly basic word problems to those that are much more complex and may require research, development of a new process, data collection, or use of technology to organize data.

The Vision Report’s discussion of content includes arithmetic, algebra, geometry, trigonometry, calculus, and statistics. While these topics and their subcategories refer to college-level mathematics, a level beyond what concerns us here, they indicate the types

of courses in which students should have some background before pursuing college-level math. It is worth highlighting that both the Vision Report's recommendations and those contained in the Standards for Success (Conley and Bodone (2002)) include some knowledge and skills in statistics, particularly the knowledge of analytic tools and the ability to analyze, interpret, and display real data. They also note the need to be able to use spreadsheets, graphing calculators, and Computer Algebra Systems. These studies suggest that, in preparing adult learners for college-level mathematics, it would be useful to introduce them simultaneously to the technology and the fundamental concepts of statistics.

American Diploma Project

The last study on which we report was conducted by the American Diploma Project (2004), a joint venture of Achieve, Inc., the Education Trust, and the Thomas Fordham Foundation, and supported in part by a grant from the William and Flora Hewlett Foundation. The National Alliance of Business also was an original partner of the project. We refer to this publication by project name rather than by title.³

This is a particularly relevant study for our work for a number of reasons. First, this research confirms the findings of the studies we have just cited concerning the knowledge, skills, and abilities that are necessary for high school graduates, and therefore adult learners, who want to pursue postsecondary education. It goes further by providing precise examples of the types of problems that students should be able to solve. In addition, these recommendations are not just based on feedback from postsecondary institutions; they have been developed in partnership with frontline managers in occupations with the highest projected pay and skill requirements for the next decade.

The goal of this work was to realign high school diploma requirements with the expectations of employers and postsecondary institutions to, in their terms, reestablish the value of a high school diploma. The starting point was to describe the English and math skills that high school graduates need to succeed in postsecondary education or in high-performance, high-growth jobs. The work of the American Diploma Project was based on a close collaboration with K–12, postsecondary, and business leaders in such occupations as healthcare, information technology, telecommunications, high-tech manufacturing, semiconductor technology, law, energy, retail, and financial services.

The math content necessary reflects what is typically taught in algebra I and II, geometry, and data analysis, the latter echoing the recommendations for statistics and data analysis skills noted previously. American Diploma Project partners also include analytic and reasoning skills that they suggest have been associated traditionally with honors or Advanced Placement (AP) courses, but that they now assert are considered to be essential skills by colleges and employers.

The American Diploma Project found a surprising amount of consistency in the skills and content standards established by business and postsecondary institutions

³ The title is *Ready or Not: Creating a High School Diploma That Counts*.

involved in this work, both within and across states. It concludes that this work confirms the notion that postsecondary and workplace expectations are converging.

Once again, the researchers note that it is not just specific knowledge that is important but the ability to think critically. They emphasize skills to develop and analyze an argument, to define and research a problem, to present a well-reasoned solution to the problem, and to apply basic knowledge and skills in new and unfamiliar contexts.

However, unlike many of the standards we have reviewed, the American Diploma Project differentiates the knowledge and skills necessary by whether the person intends to major in math or in math-dependent fields. This is an important distinction in terms of what defines adequate preparation to pursue college-level mathematics. Certainly, the answer to this question would depend on whether the person plans to pursue a career in, for example, electronics engineering versus law.

The project's research includes specific benchmarks and actual workplace tasks and postsecondary assignments that illustrate each of these benchmarks. Examples of these tasks can be found on its Web site at: www.achieve.org.

Summary

The four studies that we reviewed—Crossroads, Standards for Success, the Vision Report, and the American Diploma Project—together provide consistent and comprehensive guidance on specific topics and the skill threshold necessary to pursue college-level mathematics. The consensus is that students should have a basic foundation in geometry, trigonometry, algebra I and II, and some basic statistics. All four studies emphasize the importance of mathematical skills, particularly critical thinking skills. This research also indicates the necessity of tailoring the preparation to the types of college math and career path that the person intends to pursue.

While the conclusions and recommendations of these studies are based on careful research and collaboration with postsecondary institutions or businesses or both, they have yet to be adopted universally by two-year colleges. Even where they are in use, assessment of incoming students' skills may not reflect this new approach to content or abilities. As we noted previously, it is much easier to assess in a paper-and-pencil test whether a person has command over certain basic skills that require memorization than it is to determine whether a person has the critical thinking skills to perform higher-level math that postsecondary education and the workplace require.

An examination of the adequacy of assessment tests in determining whether incoming students possess adequate skills and abilities required for these emerging trends is beyond the scope of this study. However, we turn to a discussion of assessment tests because, more than anything else, they are currently the most common requirement of students who wish to pursue college-level mathematics at community colleges.

Assessment and Placement Policies

The use and misuse of placement tests is central to this review for two reasons. First, according to a recent survey by the American Association of Community Colleges, 58 percent of its 400 respondents required adult learners transitioning from an ABE program to pass an assessment of basic skills in order to enroll in a college-level math course (Schults 2001). Studies that we reviewed suggest that mandatory student assessment and placement tests have a positive impact on student performance. Young (2002) argues that requiring mandatory placement tests is a good policy because numerous studies have shown that students who take mandatory placement and assessment tests and subsequently enroll in developmental courses perform better in college-level courses than similar students who do not take developmental courses. And according to Boylan and Saxon (2002), fewer than 10 percent of students who require remediation will be successful in college without getting it. Their work, based on a synthesis of over 200 studies on developmental education, finds that only the most motivated students will enroll when assessment and placement into developmental courses is voluntary. They conclude that placement and assessment should be made mandatory. However, we note that, if unmotivated students are not seeking remediation, making remediation mandatory will not necessarily increase their motivation level or their course performance.

The other reason for focusing on placement tests is that an understanding of test content and cutoff scores to bypass developmental mathematics may help to create curriculum guidelines for enhancing ABE programs. In particular, they may provide useful information not only about what students should know but what level of comprehension is required. Using other tests, such as standardized tests or high school exit exams, may not be sufficient in many cases. Several studies we reviewed argued that there is a disconnect between scores on standardized tests used by postsecondary institutions and scores on exit exams required for high school graduation, or on other measures of mathematics knowledge acquired in high school.

For instance, a study conducted in the mid-1990s looked at how high school preparation affected placement rates in developmental courses at Utah Valley State College (UVSC) (Hoyt and Sorensen 2001). In that study, the researchers surveyed high school transcripts from five high schools in two districts for 1995 through 1997 to determine the relationship between high school preparation and college placement test scores. Of those students who took algebra II and geometry, the nominal prerequisites for college algebra, the average score on the American College Test (ACT)⁴ math component was 20 in one district and 19 in another. During the time frame under study, UVSC required students to score 24 or higher on the ACT math component to be eligible to enroll in college algebra. In fact, over half of all students in these districts who had completed that level of high school math were subsequently placed into developmental

⁴ The ACT is one of the two most popular tests used for four-year college admissions. We discuss the various placement tests later in this section.

math at UVSC. Thus, taking the presumed prerequisites for college algebra is no guarantee that the student will acquire the level of competency to take college algebra.

Hoyt and Sorensen (2001) found that their results were consistent with those obtained from ACT for nationwide trends. The publisher of ACT (ACT, Inc.) reported that, for those in the class of 1998 who took the ACT, totaling nearly 1 million students, the average math ACT score was 18 for those who had completed algebra II—still below the standard used by most colleges for college-level mathematics.⁵

Similarly, in a study conducted by the State of Washington, scores on the 10th-grade Washington State Assessment of Student Learning (WASL) test were compared with scores on college placement tests for students taking both tests in the same spring term (Pavelchek, Stern, and Olsen 2002). The authors found that, while there was significant correlation in the content of the tests, the college placement tests tended to include some higher-level material than that in the WASL. Even so, WASL scores and college placement scores were moderately correlated. Further, while a score between 400 and 424 on the WASL is sufficient to “meet standards,” they found that only 33 percent of students who scored 400 on the WASL had placement scores high enough to place them in college-level math. Further, students who scored 442 on the WASL, considered to be exceeding standards, had only a 75-percent chance of being placed in a college-level math course based on their standardized placement test scores.

We turn now to a discussion of the tests most commonly used by community colleges for the purpose of placing students in mathematics courses.

Tests Commonly Used⁶

The most well-known tests are the two standard college entrance exams typically taken by students seeking admission to a four-year college: the ACT and the Scholastic Assessment Test (SAT). Each test assesses students’ verbal and mathematical reasoning skills, and virtually all four-year colleges require results from at least one of these tests for admission.

Community colleges do not require the ACT or the SAT for entrance to the institution. Instead, admission typically is based on high school GPA or, even less stringent, admission is open to all those with either a high school degree or GED. In the case of those who are not seeking a degree but want to take vocational courses or simply want to take a limited number of courses, admission is typically open to everyone.⁷ While two-year colleges do not require the ACT or the SAT, some will accept scores from either one of these tests in lieu of institutionally required tests to determine whether the student has the necessary basic skills in mathematics. In some cases, however, the

⁵ We discuss the ACT and commonly used cutoff scores later, but generally a score of 19 or higher on the math component of the ACT is required.

⁶ For reference, the maximum mathematics component score is 800 on the SAT, 36 on the ACT, 100 on the COMPASS test, 55 on the ASSET test, and 120 on the ACCUPLACER test.

⁷ For instance, in 1999-2000, 62 percent of public two-year institutions had a policy of open admissions (NCES 2002, table 308).

student may still be required to take the basic skills assessment test because the tests are required to place the student in the correct college-level mathematics course.

Three tests that are more widely used and have been developed specifically to assess basic skills are the Assessment of Skills for Successful Entry and Transfer (ASSET) and the Computerized Adaptive Placement Assessment and Support System (COMPASS), both of which are published by ACT, and ACCUPLACER, published by the College Board. A less commonly used test is the Test of Adult Basic Education (TABE), produced by CTB McGraw-Hill. We describe these tests next. We gathered the information reported in this section from telephone conversations with corporate representatives, from e-mail exchanges with the same, and from Web sites. The contact list is included in the bibliography.

ASSET and COMPASS

The ASSET and COMPASS tests cover much the same material, except that the COMPASS test is a computerized adaptive test, while ASSET is a paper and pencil test. The math component of the ASSET test covers numerical skills, elementary algebra, intermediate algebra, and college algebra, and COMPASS covers pre-algebra, algebra, college algebra, and trigonometry.

According to ACT, about 1,000 institutions use COMPASS, 85 percent of which are two-year colleges and the remaining 15 percent are four-year colleges. ASSET is used by about 500 institutions, some of which use both ASSET and COMPASS. And to their knowledge, no two-year college uses ACT exclusively to assess students' basic skills (Roth 2003).

ACT has compiled data that reflect the most typical cut scores that institutions report they are using to determine adequate preparation for various levels of mathematics, which we provide in Appendix C (Roth 2003). The table includes comparable scores for all three tests: ACT, ASSET and COMPASS. The courses at the top of the list are those that are usually considered to be developmental mathematics. ACT notes that colleges generally require a score of at least 23 on the mathematics component of the ACT to be allowed to enroll in a college algebra class. For reference, 66 percent of all people who took the ACT in 2003 scored 22 or below on the mathematics component (ACT 2003). However, as we note later, it appears that several states have established an ACT math score of 19 or 20 as the minimum score necessary for students to enroll in college-level mathematics courses. For reference, 44 percent of all who took the ACT in 2003 scored below 20 on the math component.

ACCUPLACER

According to the College Board, the publisher of ACCUPLACER, about 800 postsecondary institutions use this test, roughly 40 percent of which are four-year colleges; the rest are two-year colleges. In addition, some high schools use this test in conjunction with the local community college (Murphy 2003).

The College Board provided us with information pertaining to the level of proficiency in each of the three categories (arithmetic, elementary algebra, and college-level mathematics) that scores represent. They are reproduced in Appendix D.

TABE

The TABE battery encompasses five graduated levels of difficulty across five different content areas that include reading, language, and applied mathematics. In addition to its regular forms, the TABE is available in a special version suitable for use in a work environment and in a Spanish edition designed to measure the basic skills of Spanish-speaking adults in their primary language.

The TABE was developed through a review of adult curricula and follow-up meetings with content specialists to determine common educational goals, knowledge, and skills emphasized in the curricula. The TABE is available in both paper-and-pencil and software formats. The software is PC-based to permit electronic administration and scoring.

State-specific Policies⁸

Not all states mandate whether, or which, assessment tests must be administered to incoming students. We searched the Internet and contacted several officials in states that had policies governing the placement of incoming freshman to learn more about what tests and cutoffs they use. In correspondence with the Florida Department of Education, we learned that Florida requires all entering freshmen in a degree program at any public community college or state university to take a common placement test in math, reading, and writing (Sherry 2003). Florida uses a computerized, adaptive test, called the Computerized Placement Tests (CPT) that was developed by the College Board and is part of the ACCUPLACER System. The score required on the elementary algebra, a minimum of 72, is all that is required for a student to be enrolled in college-level mathematics. Among the various exemptions stipulated by the state are completion of an associate degree or higher, or a score of 440 or better on the math portion of the SAT or 19 or better on the Enhanced ACT.

The Oklahoma State Regents for Higher Education have a policy that governs entry-level assessments at all public institutions within Oklahoma, with only certain specifics and other requirements determined by each institution. Their policy concerning mathematics is that all students at all institutions are required to score a minimum of 19 on the math portion of the ACT prior to placement in a college-level math course. Students who do not meet that requirement are given the opportunity to take an additional test that is selected by the institution. In this system, 11 institutions use ACCUPLACER, 14 use COMPASS, one uses ASSET, and one uses the ETS for the secondary mathematics assessment (Moss 2003). Some of these institutions use combinations of tests, including locally developed instruments. The cut-scores are determined by the

⁸For reference, the maximum mathematics component score is 800 on the SAT, 36 on the ACT, 100 on the COMPASS test, 55 on the ASSET test, and 120 on the ACCUPLACER test.

institution and vary greatly among institutions. The Oklahoma state regents' policy has no requirement for either the method of assessment or the cut-scores.

The Tennessee Board of Regents requires an assessment of all students admitted with less than 19 on the ACT in math, or 460 on the math portion of the SAT. Those students who have not taken the ACT or SAT, or whose scores are below the cutoffs and want to challenge the results, must take the COMPASS test for appropriate placement (Bradley 2003).

In the University System of Georgia, students must take a placement test if they have not completed the college preparatory curriculum (four years of high school math) or have SAT math scores below 400. To be exempted from or exit required remediation, a student must have a COMPASS algebra score of at least 37. Institutions may set higher scores (Burk 2003).

All first-time entering freshmen at all state-supported colleges and universities in Arkansas who are admitted to enroll in all associate or bachelor's degree programs must be tested by the admitting institution for placement purposes. Students must score 19 or above on the math section of the Enhanced ACT, 460 or above on the quantitative portion of the recentered SAT, 39 or above on the ASSET intermediate algebra test, or 41 percent or above on the COMPASS algebra test to enroll in college-level mathematics courses. Students not meeting the standard must successfully complete a developmental math program demonstrating achievement at a level at least as sophisticated as intermediate algebra. These scores are considered to be the minimum math requirement, and individual institutions may elect to follow a higher standard (Bird 2004).

The South Dakota Board of Regents developed a standardized placement process that requires all entering freshmen seeking a bachelor's degree, or enrolling students in English and mathematics, to either score 20 or higher on the ACT or take the COMPASS exam. For COMPASS, they must score 45 or greater on the algebra test in order to enroll in a college-level math course.

According to the West Virginia Higher Education Policy Commission Web site, in order to enroll in college-level math, students must score 460 or above in the SAT math test, 19 or higher on the ACT math test, 40 or higher on the numerical and 38 on the elementary algebra ASSET tests, 59 on the pre-algebra and 36 on the algebra COMPASS tests, or 85 on the arithmetic and 84 on the elementary algebra ACCUPLACER tests (West Virginia Higher Education Policy Commission 2003).

The cutoff score required to enroll in college-level math course in Colorado is 440 or higher on the math SAT test or 19 or higher on the ACT math test (Colorado Community College Online Placement Testing 2003).

For comparison, in table 1 we summarize the minimum scores necessary for the ACT, SAT, ACCUPLACER, COMPASS, and ASSET tests provided as stipulated by

state policies or a test publisher. We note that, in all cases, the minimum scores provided by the test publisher are higher than those mandated by the states included in our survey.

Table 1: Summary of Required Minimum Mathematics Assessment Test Scores of Selected States

State	ACT	SAT	ACCUPLACER	COMPASS Algebra	ASSET
AR	19	460		41	Inter. Alg: 39
CO	19	440			
FL	19	440	Elem. Alg: 72		
GA		400		37	
OK	19				
SD	20			45	
TN	19	460			
WV	19	460	Arith: 85 Elem. Alg: 84	Algebra 36 Pre-Alg 59	Num. Skills: 40 College Alg: 23
Publisher	23		Arith: 93 Elem. Alg: 82 College Math: 63	66	

While the value of placement tests is known, some research has shown that not all assessment tests accurately place students in developmental mathematics courses. For instance, faculty at the United States Air Force Academy wanted to determine whether providing appointees to the academy with a practice placement exam before the actual placement exam would reduce the number of students required to take developmental mathematics, and whether those who took the practice exam and were subsequently placed into college-level calculus did worse in that course (Revak, Frickenstein, and Cribb 1997). Revak and his colleagues found that the placement scores for those who took the practice exam were significantly higher than for those who did not. Further, those who took the practice exam and were placed into calculus I were as successful in that course as those who did not take the practice exam. Their experiment also resulted in fewer students being placed into remediation math (precalculus at the Air Force Academy). Thus, the practice placement exam allowed more students to place into, and successfully pass, calculus I their first semester.

We turn now to a review of the literature pertaining to successful instructional strategies.

What Instructional Methods Work Best for Adult Learners?

Our review of the literature includes 15 studies of developmental mathematics programs in postsecondary institutions.⁹ We chose these studies because they addressed developmental mathematics in particular, they covered a number of different strategies, and they were representative of the body of literature in general. However, we were not able to identify any studies of developmental mathematics that were based on randomized controlled trial (RCT) experiments. RCT has become the gold standard for research (National Research Council 2002) and is the method that has been found to be the most unbiased in evaluating the effect of programmatic interventions in the field of education. Instead, the studies we found are based on nonscientific methods, with students self-selecting¹⁰ into the course, and no attempt is made to control for factors that are correlated with such choices. Even so, we cover these studies in some detail because we believe that their findings help to inform our initial understanding of programmatic structures and practices that may hold promise. We also include findings from studies of developmental education in general that serve to reinforce the findings from the developmental mathematics literature, or to suggest promising practices in areas that have not been addressed for developmental mathematics for adult learners specifically.

The Role of Technology

The literature that we reviewed pertaining to instructional methods in developmental education in general, and in developmental mathematics in particular, revealed that a significant body of research has been devoted to the question of the relative value of technology versus the traditional instructor-led modality, and the extent to which technology should be used as an instructional tool. We concentrate on this debate in this section, and review other classroom strategies in the section on pedagogy.

Research on education technology has increased tremendously in this arena in the past several years, with studies on the effectiveness lagging behind. For instance, while calculators have been around for decades, controversy still exists over the appropriate use of this far less complicated and far more pervasive technology.

⁹ See Appendix B for a summary of the studies we reviewed.

¹⁰ The difficulty with self-selection is that it can cause a statistical bias resulting in a misattribution of cause and effect. For example, consider a system that allows each student to select his or her own method of instruction. Those who are more motivated and more confident would be more likely to select computer-based instruction because they are more familiar with computers. In a simple statistical test, it would not be surprising to find that students using computer-based instruction have a higher pass rate in the course than those using an alternative method of instruction. It would be incorrect to conclude that it is the mode of instruction that is causing the better results unless it is possible to adjust for the motivation and confidence of the entering students. It is rare in this context to be able to make such an adjustment. In an RCT experiment, however, in which each student is randomly assigned to either an experimental or traditional class (the latter constitutes the control group), there is no reason to believe that students assigned to one mode of instruction are any different from students assigned to another mode. Thus, if there are differences in outcomes, they can be more confidently attributed to the mode of instruction.

Perhaps the greatest topic of debate and uncertainty in the effectiveness of various strategies in developmental education for adult learners is in the appropriate use of technology. The debate includes questions of how extensively it should be used, as well as the appropriate choice of technology. For instance, MacDonald et al. (2002) summarize the debate as follows: “The debate is over whether or not to utilize technology that is capable of conducting the very skill that the developmental mathematics student is trying to obtain” (p. 36). They note that one aspect of the debate concerns the type of calculator that students should use—scientific versus graphing. Unlike graphing calculators, scientific calculators do not allow the learner to see the connection between input parameters and output results. However, the debate cannot be conducted apart from the content of the course, as the authors point out. A number of developmental mathematics instructors are finding greater success in using calculators when they change their emphasis from basic skills to problem solving, using real-world problems or emphasizing development of critical thinking skills.

There is even a debate about the precise definition of certain terms, including computer-based education (CBE), computer-based instruction (CBI), computer-assisted instruction (CAI), computer-managed instruction (CMI), and computer-enriched instruction (CEI). For the purpose of this review, the term CAI will be used to refer to instruction that is typically a supplement to traditional instructor-led instruction and most commonly includes drill-and-practice, tutorial, or simulation activities.

A number of studies evaluate the impact of various teaching delivery methods on student success, such as traditional lecture, computer-assisted courses, self-paced instruction, Internet-based courses, and accelerated programs. Several studies reported inconsistent conclusions as to whether technology-assisted or technology-based instruction is superior to instructor-led approaches. Conclusions are based on different definitions of success in each study, such as receiving a passing grade in the course, persistence to higher-level mathematics, or scores on final exams. However, there is general agreement that, while students may not necessarily be more competent with one particular type of instructional mode, their persistence in developmental math and beyond may be enhanced by the option of instructional choice. Several researchers contend that allowing students to choose the instructional method that they feel best suits their particular learning style makes them more likely to complete the course and perhaps take higher-level mathematics.

Computer-Assisted Instruction

Our first studies on this topic include computer-assisted instruction as an option to the traditional instructor-led modality. Carnal (1999) examined success, retention, and persistence in several math courses between those offering traditional instructor-led methods and computer-assisted courses. The study found that students who took computer-assisted courses in elementary algebra and intermediate algebra had higher retention rates, but students in the traditionally taught courses had a higher success rate, defined as receiving a passing grade (C or higher). However, of those who successfully completed the computer-assisted algebra courses, a greater percentage went on to take higher math, including trigonometry and precalculus. Because the study did not control

for self-selection, and results are not robust, Carnal suggests the need to do further research in this area.

McClendon and McArdle (2002) conducted a study of the effectiveness of three delivery modalities of developmental instruction in mathematics in the Mathematics Department of the Winter Park Campus of Florida's Valencia Community College: traditional lecture, Academic Systems (an Internet-accessed software curriculum that combines lecture, practice programs, and self-administered assessment tests), and Assessment and LEarning in Knowledge Spaces (ALEKS) (a nonlinear, nontraditional Internet-based course). Students were aware of which modality was used in each of the classes, and their advisors were told by the researchers which modality was optimal for various learning styles. However, students self-selected into each course, and not all students used advisors when selecting courses. In addition, ALEKS was the only modality available for students who registered late.

Using raw percentages of students who completed the course (defined as a grade of C or higher), the study found that students who attended the traditional lectures had the highest completion rate, while those who attended ALEKS had the lowest. Because of the significantly higher withdrawal rate in the ALEKS courses versus the traditional lecture method, which they believe was due to the inability of a large number of students to self-select out of ALEKS, the researchers recalculated the completion rate only of those who did not withdraw from the course. Netting out withdrawals, the authors found no significant difference in outcomes by modality. Nevertheless, the study concluded that institutions should consider making various learning modalities available.

Similar to the study by McClendon and McArdle, Kinney (2001) analyzed the difference in effectiveness of various approaches to teaching elementary algebra and intermediate algebra at the University of Minnesota–General College. Two methods were used, with students choosing the one that they believed would meet their learning preferences best: direct instruction classes or a computer-mediated instruction using what is known as a full implementation model. In this latter model, students met at the same time and followed the same schedule, but the software delivered the instruction while the instructor provided individual or small group assistance on request. The advantage of this type of instruction for developmental education is that it provides students with an alternative to direct instruction, and gives them more control over their learning. However, this is not a self-paced model.

The study found no significant difference in scores on common final exams between the two methods of instruction. However, students in both groups reported an increase in confidence to succeed in math, and their attitudes toward math had improved. And, similar to McClendon and McArdle, Kinney also concluded that it is important to provide students with alternative instructional formats and to give them guidance in choosing the format that best suits their particular learning style.

In a similar vein, a study conducted by Creery (2001) compared the outcomes of developmental math students taught basic math using lecture, self-paced, and online

methods. Not all students were aware of the different modalities when they signed up for the course, although descriptions were available in the bulletin. They also conclude that many of the students in the nontraditional modalities were in those classes because the traditional ones were already closed, again implying that these types of students were enrolling relatively late. Creery noted that many of the students in the online courses had relatively few computer skills.

Creery uses grades at the end of the semester for those who did not drop out by the end of the first week of class to evaluate the outcomes of the three methods. No statistically significant difference in the outcomes for the three different delivery methods was found. Following students into the next level math course, elementary algebra, the results were the same—no statistically significant difference for the three delivery methods. Even so, Creery argues that it is important to offer various methods of instruction.

A seven-year study conducted by Waycaster (2001) in five Virginia colleges examined 10 instructors and 15 developmental math classes whose primary instruction was either lecture with lab or individualized computer-aided instruction. The goal was to determine the most effective ideas and teaching methods being used in developmental mathematics, looking specifically at such factors as course credit hours, class size, attendance, student and teacher gender, class participation rates, method of instruction, success rates in developmental and subsequent college-level mathematics courses, and retention and graduation rates.

Results indicated that the success rate in the developmental classes, defined as a passing grade, was independent of the manner of instruction used, although no test for statistically significant difference in proportion passing was conducted. It also was found that students who took developmental mathematics had higher retention rates (although retention is not defined, it often means persistence at the college from one semester to another), again by simply looking at the retention rates across programs for those who took developmental mathematics versus those who did not.

Several studies included in our review addressed the effectiveness of specific software in developmental mathematics programs. Two of the studies were based on a popular computer-based instructional tool, the PLATO Adult Learning Technologies. Each study found the use of software to be effective, either as a self-paced program, or as a computer-assisted component of an instructor-led course.

Quinn (2003) reported on the success of adult learners using this system at Miami-Dade Community College. As part of the admission process, all students at Miami-Dade Community College must take a CPT, developed by the College Board as part of its ACCUPLACER system, to assess their competency in reading comprehension, sentence skills, elementary algebra, and arithmetic. Students must achieve a certain score to be able to take college-level courses. If students do not pass the CPT, they are required to meet with counselors and receive guidance as to what they must do to pass the CPT,

which includes assignments to complete certain PLATO Adult Learning Technologies courseware modules.

Following 82 students who used the PLATO software for arithmetic skills, and 79 who used it for elementary algebra, Quinn found that student scores on the CPT showed a statistically significant increase for both subjects, with an average gain of about one standard deviation.¹¹ Quinn also was able to correlate the time spent on the software to increases in CPT scores and found that, for every hour students spent on the courseware instruction, they gained 0.61 percent to 1.86 percent on the CPT retake test.

Lancaster (2001) studied the effectiveness of using PLATO computer-assisted instruction in developmental courses at Jefferson Davis Community College (JDCC). All students are placed in the appropriate class based on scores on the COMPASS test, which they take at enrollment. Students whose scores place them into two or more developmental courses are required to take a study skills class.

JDCC's developmental courses use computer-assisted instruction in combination with classroom instruction, and some instructors choose to use additional Web-based programs to enhance other skills or for career awareness purposes. For instance, the career awareness software allows students to explore career opportunities, conduct job skills assessments, and develop job preparation skills.

In the beginning of each term, students take an initial assessment on the PLATO software program, which then forms the basis for their Individualized Education Plans (IEPs). These form the basis for specialized modules that are established to remediate those deficiencies. To increase mastery of the material, instructors also often conduct traditional classroom instructions and group activities.

The CAI approach was initiated during the summer 2000 term, and the study compared the performance of those in developmental courses the previous summer term, using the traditional classroom format, with those using the CAI approach during the summer 2000 term. This is a quasi-experimental design since students could not necessarily self-select into the academic term based on instructional modality differences. The study found that using CAI resulted in a 7-percent decrease in the number of withdrawals in elementary algebra, a 12-percent increase in the number of satisfactory grades, and an 11-percent decrease in the number of unsatisfactory grades in elementary algebra.

The U.S. Navy recently evaluated the effectiveness of Tutorials in Problem Solving (TiPS), an intervention for training arithmetic and problem-solving skills in adult populations, using adult learners requiring developmental mathematics instruction at Mississippi State University (Atkinson 2003). The material in TiPS is provided in the context of word problems, providing students with a set of diagrammatical tools to

¹¹ Approximately 68 percent of all student scores fall within plus or minus one standard deviation of the mean.

analyze the problem. It has an interactive interface tool, a help system, and diagnostic and feedback capabilities.

This study used a rigorous approach to evaluate the effectiveness of the software, using pretest and posttest measures, and program-comparison group strategies in which students were chosen for the program based solely on a pretest cutoff score. The author notes that this approach is useful when courses are given on the basis of need or merit, and it is considered to be as robust in inferences as those drawn from randomized designs.

Their metric was test performance change across time for students using TiPS, compared to the test performance change of a control group of students in developmental mathematics who had similar scores on the pretest, but who were not chosen to use TiPS. They also sought to determine whether the length of instruction varied systematically according to a student's mathematical ability before using TiPS. They found that the average posttest score for TiPS participants was significantly higher than those of their peers in the control group. TiPS was originally designed for use in training of enlisted sailors, but the results indicate that it has much wider applicability for middle schools, adult literacy programs, and workplace training programs.

Computer Algebra System

Another type of technology that is the subject of some debate is the Computer Algebra System (CAS), which in general refers to a system or software that is used in manipulating mathematical formulae in both symbolic and numeric form, unlike traditional calculators that only allow manipulation of numeric equations. In addition, CAS automates some of the more tedious or difficult algebraic manipulations, with the intent to reduce the amount of time spent on drill exercises, allowing more time to spend on greater comprehension of the subject matter.

Livingston (2001) investigated the impact of a computer algebra system on six intermediate algebra classes at Orange Coast College in California. Using a quasi-experimental nonrandomized control group pretest-posttest design, he examined whether classes taught using a graphing calculator with a computer algebra system (the CAS TI-89, produced by Texas Instruments) performed as well as classes taught using traditional methods with scientific calculators. Livingston does not mention how the control and experimental groups were chosen, though he notes that it was a nonrandomized design. Also, he does not control for student characteristics, thereby further reducing the value of his results. The findings indicated that there was no statistically significant difference in the pretest and posttest scores of the two groups, nor was there a difference in the ability to perform mathematics by hand. However, the group taught using the computer-based system did perform better at solving higher-order reasoning skills by hand.

In general, CAS has not met with overwhelming success. Leinbach, Pountney, and Etchells (2002) argue for the value of such systems. Their argument is that for technology such as CAS to be successful, it must be a tool used in otherwise good pedagogy that allows students to become active participants in their learning experiences

and to plan and carry out problem-solving strategies, echoing conclusions drawn by MacDonald, Vasquez, and Caverly (2002).

The Standards for Success study cited earlier (Conley and Bodone 2002) did not provide much detail in its recommendations concerning technology, but the authors did state that students should understand both the use and the limitations of calculators, including graphing calculators. The Crossroads standards (American Mathematical Association of Two-Year Colleges 1995), described earlier, stated that the use of technology is an essential part of an up-to-date curriculum; students should use appropriate technology to enhance their mathematical thinking and understanding and to solve mathematical problems.

The most important point concerning the role of technology is that it appears to be most useful as a supplement to, rather than a replacement for, regular classroom instruction. This point is reiterated by Boylan and Saxon (2002), as well as by Leinbach, Pountney, and Etchells et al. (2002) and the AMATYC Vision Report (American Mathematical Association of Two Year Colleges 2002b). The latter report also concludes that, for technology to be effective, instructors must have adequate professional development in the appropriate use of technology. This conclusion applies to adjunct as well as full-time faculty. The authors suggest that all types of technology, including graphing calculators, spreadsheets, and CAS, should be used to give students the chance to become familiar with the technology and to understand its benefits and limitations.

Even so, the studies we reviewed that specifically addressed the effectiveness of technology have found that, relative to the traditional instructor-led format, CAI and CAS resulted in higher, lower, or no difference in pass rate, no difference or higher rates of persistence to higher-level math, and no difference in final grades. Clearly, this is an area ripe for further study.

Pedagogical Issues

We turn now to addressing research on other pedagogical techniques. We begin with a discussion of the research concerning how people learn. This field of study provides a context for much of the literature that we reviewed on pedagogy.

How people learn

The AMATYC (American Mathematical Association of Two-Year Colleges 1995, 2002b), Standards for Success (Conley and Bodone 2002), and American Diploma Project (2004) research that we discussed previously defined the knowledge necessary to pursue college-level mathematics. Their recommendations also included several skills and abilities, as well as pedagogical strategies. These strategies, as well as the skills and abilities themselves, are best understood in the context of how people learn. The National Research Council compiled an important body of research on this topic. According to *How People Learn: Bridging Research and Practice* (Donovan et al. 2000), because students have certain preconceived notions, they will fail to grasp new concepts if their initial understanding is not engaged. Further, students need a deep foundation of factual

knowledge and a strong conceptual framework if they are to develop competence in a particular area, and they need to monitor their own understanding and progression in problem solving. If learners are able to make analogies to what is known when confronted with new material, they can better advance their understanding of new material. While the study was initially intended for primary and secondary education, the authors indicate that the larger design framework for children's learning environments applies to adult learning as well.

Other research on learning confirms that there are new ways to introduce adult students to traditional subjects, such as mathematics. In concert with strategies outlined by the National Research Council (Donovan et al. 2000), these new approaches make it possible for the majority of individuals to develop a deep understanding of important subject matter (Bransford et al. 2000).

The National Research Council has developed a notion that there are four perspectives on learning environments. These seemingly separate perspectives, however, should be interconnected to mutually support one another (Brown and Campione 1996). Specifically, the perspectives are as follows:

1. **Learner-centered environment.** Accounting for the perspective of the adult learner requires paying careful attention to the learner's knowledge, skills, attitudes, and beliefs. Adult learners need to be treated as adults who are responsible for their own lives and who are capable of self-direction (Knowles 1989). Also, adults acquire knowledge about things they need to know—that is, to cope effectively with their real-life situations. Postsecondary programs for adults should be designed with the understanding that adults undertake most learning efforts in response to life transitions (Aslanian and Brickell 1980).
2. **Knowledge-centered environment.** Adult learners require a well-organized structure of concepts, such as those defined by AMATYC (1995), that organize the presentation of subject matter and help students: (1) develop substantial mathematical problem-solving abilities; (2) learn to develop models involving real-world situations; (3) expand their mathematical reasoning skills; and (4) use technology. Students need help to become metacognitive by expecting new information to make sense and asking for clarification when it doesn't (Palinscar and Brown 1984; Schoenfeld 1983, 1985, 1991). Students should learn to compute, but they should also learn other things about mathematics, especially the fact that it is possible for them to make sense of mathematics and to think mathematically (Cobb, Yackel, and Wood 1992).
3. **Assessment-centered environment.** Postsecondary programs should have clear learning goals and assessment methods, procedures, and items that are congruent with those goals. The primary element of importance is that assessment be used for feedback and revision of the program, including teaching and learning. In many classrooms, opportunities for feedback appear to occur infrequently, resulting in grades on tests, papers, worksheets, homework, and final reports that

represent summative assessment only, since students typically move on to a new topic without opportunity to revise their thinking—in particular, higher-order thinking. Assessment of higher-order thinking in mathematics includes among factors: methods of examining nonalgorithmic (not fully specified in advance) problem solving; opportunities for multiple solutions (each with costs and benefits); nuanced judgment and interpretation; application of multiple criteria that sometimes conflict with one another; self-regulated thinking processes; and imposing meaning and structure in situations with apparent disorder (Romberg, Zarinnia, and Collis 1990). Critics argue that in contrast to assessments built around higher-order thinking, many of the typical assessments in postsecondary mathematics developed by teachers emphasize memory for procedures and facts (Porter et al. 1993).

4. **Community-centered environment.** The extent to which students, teachers, or even administrators feel connected in several aspects of community is reflected in classrooms as communities, institutions as communities, and even larger communities such as those within the military and businesses. The importance of communities in learning cannot be emphasized strongly enough with adult learners. In the development of higher mental functions, such as planning and numerical reasoning, “internalization” of self-regulatory activities first takes place in the social interaction between adults and more knowledgeable others (Vygotsky 1978). Studies of mathematical problem solving, for example, by Noddings (1985), Pettito (1984a, 1984b), and Schoenfeld (1985), indicate how useful dialogues among mathematics problem solvers can be in learning to think mathematically. Small group dialogues prompt disbelief, challenge, and the need for explicit mathematical argumentation; the group can bring more previous experience to bear on the problem than can any individual, and it highlights the need for an orderly problem-solving process. In addition, computers can serve to enhance communities of learning by functioning as mediational tools that promote dialogue and collaboration on mathematical problem solving.

The Vision Report (American Mathematical Association of Two Year Colleges 2002b) also provides an extensive discussion of what participants considered to be the best teaching and learning methods for the first two years of college, and it specifically addresses the needs of adult learners. It recommends that lectures be supplemented by a number of student-centered methods, such as computer simulations and collaborative learning activities, including working in teams. The need to be able to work collaboratively, particularly in teams, is consistent with requirements of businesses as stated in the American Diploma Project (2004). The Vision Report also contends that it is not just the pedagogy but the curriculum content that can be effective in teaching mathematics to adults. In particular, the report states that it is important to use activities that engage students in the learning process, such as the use of case studies and projects that require designing, modeling, researching, and presenting findings.

Several studies note similar factors as being effective in adult education programs. Alamprese (1998), Alamprese, Labaree, and Voight (1998), and Alamprese (2001) detail

program-level factors in adult education and their relation to student outcomes in the review of the literature. According to their review, exemplary adult education programs feature the following five characteristics:

- Effective program management and instructional leadership;
- A commitment to staff development;
- Conscious attention to appropriate instructional strategies;
- A focus on learner assessment; and
- Extensive supports for learning, especially for students with low levels of literacy proficiency.

The themes discussed above are present in many of the studies that we reviewed. We summarize a number of these studies that specifically address the role that learner-centered instruction and metacognition strategies, small group instruction, and collaborative learning play in developmental math education. In addition, we summarize a few studies that suggest additional strategies or highlight important considerations.

Learner-centered environment

Lending support to the potential benefits of learner-centered instruction, Miglietti, Strange, and Carney (2002) investigated the relationship between learning and teaching styles in developmental English and mathematics courses in a two-year branch of a four-year Midwestern college. Instructors rated their own teaching styles using a specifically designed instrument for this purpose, the Principles of Adult Learning Scale (PALS) tool. Students assessed their learning styles with two tools: the Adult Classroom Environment Scale (ACES), which the authors state is the only scale designed to measure adult students' perception of the classroom environment in general, and the Adaptive Style Inventory (ASI), which measures students' emphasis on styles of learning. A total of 185 students chose to participate, but only 159 completed the courses. Of these, 59 percent were enrolled in a developmental math course.

The study found no age or gender effects on classroom environment and learner style preferences. In terms of age and teaching style on classroom outcomes, the authors note that they could conduct an analysis of the effects of teaching style on developmental English classes only because, within the mathematics sections, none of the five mathematics instructors reported a learner-centered teaching style. The authors' findings in terms of developmental English led them to conclude that adult underprepared students in learner-centered classrooms achieved higher grades than similar students in teacher-centered classrooms.

Higbee and Thomas (1999) reviewed the literature concerning important factors pertaining to a learner-centered environment, and how they relate to achievement in

mathematics. The authors note that the following affective variables are important: student's academic self-concepts, attitudes toward success in mathematics, confidence in their ability to learn mathematics, math anxiety, text anxiety, perceptions of the usefulness of math, motivation, self-esteem, and locus of control. Further, these researchers also have examined the relationship between performance in mathematics and cognitive factors, such as preferred learning styles, visual and spatial ability, the use of specific cognitive strategies, and critical thinking skills. Based on this body of research, educators have begun to research various techniques to reduce or eliminate some of the barriers so far identified, including the use of collaborative learning and verbalization during the problem-solving process. Finally, Higbee and Thomas note that there is an increasing shift from a focus on learner characteristics to a more integrated and holistic approach, incorporating the role of the teacher and course content, including different types of tests, grading systems, the use of mathematics applications, and collaborative learning.

Higbee and Thomas also explore the relationship between noncognitive variables and success in a two-quarter developmental algebra sequence designed for high-risk students at the University of Georgia. The two-quarter sequence covered the same material as a one-quarter course, except at a slower pace. One day per week a counselor taught with the math instructor and introduced special learning-promotion topics, such as relaxation exercises and metacognition strategies, as well as strategies for solving word problems in collaborative groups. Students also were required to attend the mathematics laboratory weekly to take computer tests that paralleled those administered in the core algebra course.

The results indicated a significantly lower test anxiety and an increase in students' confidence to succeed in learning math as measured at the beginning and end of the two-quarter class sequence. In terms of course outcomes and affective variables, they found negative correlations between pretest scores on general test anxiety and math test anxiety. They note that there was no relationship between posttest scores on tests of anxiety and any of the test, homework, or final GPAs. In other words, on average, students experienced a reduction in math and test anxiety over the two-quarter sequence, but the reduction in anxiety was not correlated with greater math competency, as measured by a variety of course outcomes.

Consistent with these findings and with a learner-centered classroom approach in general, Boylan and Saxon (2002) conclude that remediation programs require counseling as an integral part of the program. They find that remediation programs with counseling that is integrated into the entire remediation program have better results. They report that counseling should be based on stated goals and objectives of the program and undertaken early in the program. The counseling should use sound principles of student developmental theory, and should be carried out by counselors who are trained to work with developmental students.

Different sets of student perceptions were the subject of a recent study by Wheland, Konet, and Butler (2003). They looked at five perceived inhibitors to student

success in intermediate algebra at a public university with an undergraduate enrollment of 24,000 students. Student-perceived factors inhibiting success were as follows:

- (a) Nonnative English-speaking instructors had a detrimental impact on their success;
- (b) Teaching assistants resulted in lower success than adjunct professors;
- (c) Student performance in intermediate algebra was not reflective of overall performance in nonmathematics courses;
- (d) Student success in intermediate algebra did not affect performance in subsequent math courses; and
- (e) Attendance had no significant impact on performance.

Faculty, however, perceived that factors (c), (d) and (e) all had a potential negative impact on performance.

Using midsemester tests, final exam scores, and GPA to investigate the effect of each of these factors, the study found, contrary to students' perceived notions, that nonnative instructors and teaching assistants did not have a negative impact on their success in intermediate algebra, but their performance in intermediate algebra correlated quite highly with overall GPA that semester, their attendance also was highly correlated with success in the course, and their grade in intermediate algebra did have a fairly high predictive value on their performance in subsequent math courses. However, these conclusions are based on examination of the effect size (the difference in means of two groups divided by the standard deviation) for the various metrics under study; they do not attempt to control for self-selection or other potentially confounding effects. For instance, students who do not perceive nonnative instructors as having a negative impact on their learning may disproportionately select into courses that have these types of instructors.

The authors conclude by noting that a misconception of many students (i.e., that factors contributing to their failure in intermediate algebra are in large measure perceived to be out of their control) only serves to make the course material more difficult to master.

Small-group instruction

DePree (1998) examined differences in outcomes for students at a large urban community college who were instructed in preparatory algebra classes delivered either by instructor or by small-group instruction. Small-group instruction is one strategy in community-centered learning environments that is specifically recommended in the Crossroads and Vision documents (American Mathematical Association of Two Year Colleges 1995, 2002b). Students were not aware of which type of instruction would be used at the time they enrolled, enabling a quasi-experimental design.

The results indicated that those taking the course via small-group instruction had statistically higher confidence in their mathematical ability, as measured by the Fennema-Sherman Mathematics Attitude Scales. Improvements were greatest for students who have been traditionally underrepresented in mathematics: Hispanic, Native American, and female students. Further, students who received the small-group instruction were more likely to complete the course than students in the instructor-led course. However, Depree did not find any difference in achievement between the two teaching methods. Even so, the fact that students increased their confidence and were more likely to complete the course when administered by small-group instruction led the author to conclude that a larger number would ultimately be successful in this type of class.

Contextual learning

Consistent with other literature that we have cited concerning the need for adults to have contextual learning experiences, Mazzeo, Rab, and Alssid (2003) describe the efforts of five community colleges that have created bridges between basic skills development and entry-level work or training in high-wage, high-demand career sectors. Mazzeo and his colleagues argue that contextualized basic skills instruction is often more successful than traditional models of adult education for engaging disadvantaged individuals and linking them to work. Each program they describe uses contextualized teaching and learning experiences, which means that courses incorporate material from specific fields into course content, and employ projects, laboratories, simulations, and other experiences that enable students to learn by doing. Further, they contend that workforce and education systems should be reorganized around “career pathways” that integrate education, training, and work, and are targeted to high-wage, high-demand employment to address the growing needs for skilled workers and workers’ needs for economic self-sufficiency. However, they state that further research is needed to determine whether contextualized basic skills instruction is more effective than more traditional instructional approaches.

We have found that other authors also suggest that a strong connection between education and employment increases earnings and placement rates (Grubb 1996; Jenkins and Fitzgerald 1998). Both Grubb (1999) and Murphy and Johnson (1998) argue that one essential characteristic of effective programs is a focus on employment-related goals through instruction that integrates basic and occupational skills training with work-based learning. Rogoff (1990), Lave and Wenger (1990), Lave (1991), and Wenger (1998) emphasize the important role of context in shaping student learning. Greeno et al. (1999) describe one of the most important contexts for adult learning as the world of work itself and the specific tools, practices, and social relations embedded in the work setting.

The five programs that Mazzeo and his colleagues reviewed also exhibited these additional characteristics:

- Integration of developmental and academic content.
- Development of new curriculum materials and provision of professional development to learn to teach in a new way.

- Maintenance of active links with employers and industry associations.
- Identification of resources to fund the programs, at least in the short run.
- Production of promising program outcomes, especially in terms of job placement and earnings.

We highlight their finding that these programs emphasize professional development for faculty. Boylan and Saxon (2002), a study by the Education Commission of the States (Spann 2000), and the reviews of adult programs conducted by Alamprese (1998), Alamprese, Labaree, and Voight (1998), and Alamprese (2001) cited earlier all conclude that staff training and ongoing professional development are very important components of successful adult developmental efforts.

“Systems Thinking”—DevMap Project

We turn now to a project that addresses the National Research Council’s notion that students must be able to make analogies to what is known when confronted with new material. In a 1997 National Center for Research in Vocational Education (NCRVE) workshop, “Beyond Eighth Grade,” industry representatives emphasized the need for “systems thinking” that allows workers to recognize complexities in various situations subject to multiple inputs, and for those in certain fields to formulate a problem and design experiments to determine the influence of various factors (Consortium for Mathematics and Its Applications 2003). The DevMap Project at the Consortium for Mathematics and Its Applications (COMAP), which is funded by the Adult Technology Education Program of the National Science Foundation, is intended to address the disconnect between traditional college developmental programs in math, that have been just a replication of high school math, and the needs of industry as stated in the 1997 workshop. To that end, COMAP is developing a one-year sequence, *Developing Mathematics Through Applications*. This program, based in large part on the Crossroads recommendations, includes the following unique features:

- While all the major components of algebra, geometry, and trigonometry will be included, the program will not be divided into these distinct and separate topics.
- The program will be based on applications, which should be particularly relevant to adult learners who can use applications that draw on areas in which they may be working or in which they aspire to work.
- Problem solving will require integrating technology in a natural way, as opposed to the “drill-and-practice” use of technology found in many developmental mathematics programs.

Accelerated courses

Lastly, we note an interesting approach that is not based on any of the other strategies or theories that we have covered so far but may be a new strategy that holds promise. The University of Maryland, College Park (Adams 2003) began a program in the fall of 2001, in which students who required math remediation (representing 20 to 25 percent of entering freshman, or about 1,000 students) and scored in the top 60 percent of the placement tests were placed into a combination course that met five days a week and covered the material for both the developmental mathematics and the introductory college-level course. At the end of the intensive first five weeks of the course, in which all of the developmental material was completed, students retook the placement test. If they passed the test, they continued in the course, which had as many contact hours during the remainder of the semester as those students who enrolled in the regular college-level course. Students successful in the latter part of the class were then able to complete both their developmental and first college-level math requirement in just one semester. Those who did not pass, only 11 percent, were placed into the regular developmental course, which is a more traditional six hours-per-week self-paced course, using a computer platform.

Both the accelerated developmental course and the regular college-level course used the same final exam. Adams found that the final test scores were about the same for the two groups; in fact, they were often higher for the students in the accelerated developmental course. Adams also found that the two classes had about equivalent A, B, and C grade rates. Precise statistics are not cited, however, nor is it noted which statistical tests were conducted.

The study also followed those students who successfully completed the accelerated program into higher math courses. The pass rate (grade of C or higher) of those students in elementary calculus was about 7 percentage points higher than regular students. The results were not as good for those students who took the engineering calculus, however; their pass rate was sometimes far worse than that of regular students.

In summary, we note that the research we reviewed consistently indicates that certain skills and abilities are as important as the specific knowledge to successfully pursue college-level math and to succeed in the workplace. These include the ability to:

- Understand the connection of math to other disciplines;
- Perform inductive reasoning;
- Communicate math orally and in writing;
- Model real-world problems; and
- Work collaboratively.

Some pedagogical strategies to achieve these skills and abilities include:

- Addressing students' perceptions;
- Incorporating counseling into the program;
- Using small-group instruction;
- Using collaborative learning;
- Using contextual learning and real-world examples; and
- Requiring students to conduct research and modeling exercises.

Finally, professional development of developmental math educators is necessary to ensure that they keep current with technology and pedagogical research. Absent gold-standard research on the value of these strategies for adult mathematical literacy education, we believe that they warrant further study.

Metrics of Program Effectiveness

In this section, we discuss metrics of program effectiveness noted in the literature. We do so because, in our search of promising practices, it may be difficult to locate programs that track the metric of greatest interest to this study—that is, the ability of adult learners to successfully progress through and out of a developmental mathematics program and into their first college-level mathematics course. It is important to understand whether this is in fact a well-established goal or stated objective of programs in order to understand whether the strategy is aimed at improving this metric or at some other equally valid outcome.

Our review has found that numerous metrics are used in evaluating the effectiveness of college developmental mathematics courses or programs. However, no clear consensus emerged concerning optimal metrics for impact evaluations. This lack of consensus may be due to a more general lack of consensus about the ultimate role of developmental courses. In other words, should the goal be to ensure that those who complete the course achieve some heightened mathematics competency, or is it better to ensure that a larger number complete the course, but at a slightly lower yet acceptable level of competency?

We have already discussed that some researchers, for instance, when looking at whether a particular type of instruction or material is more effective, examine the pass rate (typically a C or higher), or average exam scores, or final class GPA of the various approaches. These studies are intended to determine whether students who ultimately complete the course have learned more of the material or are more competent in the subject. This may be at the expense of a higher withdrawal rate, however.

Others are concerned with the completion rate of the course for which the Grade Point Average (GPA) does not count since only data for those students who completed the course can be included in the GPA. These studies are concerned less with whether those who pass the course are more knowledgeable in the subject than with whether more students are able to pass the course. This is an important consideration because, even if the particular approach does not improve the overall understanding of those who pass, a larger number of underprepared students may be able to succeed in the basic skills instruction.

Still other research concerns the level of anxiety or satisfaction with the developmental course because that directly relates to the willingness of students to either persist in the developmental program or pursue higher-level mathematics. Again, this is important not so much because each “successful” student is more knowledgeable but because more developmental students are able to pursue even higher mathematics in order to ultimately achieve their career goals. Even in these cases, as we have noted, researchers rarely follow students beyond the immediate semester or, at best, they follow them one additional semester.

We also note that, because of the subjective nature of grading, the use of grades and pass rates are not necessarily valid or reliable measures of the knowledge and skills of students. This means that a comparison of pass rates or the average GPA of students in institutions using different strategies, both across institutions and even within institutions that do not use common exams, often is not useful. It is more meaningful when common placement exams and cutoff scores are used, say, as a requirement to transition out of developmental, or into college-level, mathematics courses or when performance within the same college-level math course at a particular institution is compared for students subjected to various strategies in lower-level developmental math courses.

For instance, two studies we reviewed looked at differences in the GPA of students in college-level mathematics between those who took developmental courses and those who did not. One study investigated whether students who successfully complete an exit-level developmental course and enroll immediately in a college-level course in the same subject do better than those who do not enroll immediately afterward (Sinclair Community College 2003). It found that, in the case of developmental math, average GPA is significantly lower when students delay three terms, but the course pass rate is not affected. While this study uses two metrics—GPA and passing the course—the analysis fails to take into consideration factors that influence a student’s choice in delaying additional math courses. In other words, students who are not as confident in math, and may not have completed their last developmental math course with a high grade, may be more likely to delay taking a subsequent math course. Their lack of confidence, as well as low GPA in the developmental math course, may have a direct impact on their performance in subsequent math courses.

The University of Wisconsin–Madison Mathematics Department has been publishing results of its developmental math program for the past several years (University of Wisconsin 2003). In 1999, five criteria department staff developed to evaluate the overall program effectiveness in quantifiable terms:

- Success in developmental math, defined as percentage of students who remained after the initial add/drop period and earned a grade of C or better in the course.
- Progress from developmental math to degree credit math courses, defined as the proportion of students who completed elementary algebra who enrolled in the first college-level math course, intermediate algebra, within one year.
- Success equal to other students overall, as defined by three metrics: average number of course completion attempts, grade of C- or better, and average grade of students enrolled in intermediate algebra for those who took developmental math courses versus those who did not.
- Better preparation for college credit courses than other students, defined as the cumulative GPA of students currently enrolled in intermediate algebra of those who took developmental math courses versus those who did not.

- Achievement of designated course proficiencies, defined as percentage meeting expectations in seven different proficiency goals for elementary algebra.

These metrics are examples of what may be commonly tracked by community colleges. Some are good proxies for the metric of greatest interest to us, while others are not. For instance, in the University of Wisconsin example, the fourth metric is not necessarily a good proxy for persistence in developmental education, whereas the second metric is precisely that in which we are most interested. Yet we have found few studies that specifically use that metric as a measure of program effectiveness.

A Survey of Community Colleges' Practices

In our survey of community colleges in the second phase of this project, we need to be aware of certain policies or relevant components of programs that may have a direct impact on the success of their particular strategies. Such factors as professional development for developmental mathematics instructors or the organizational location of developmental math (e.g., whether it is part of the mathematics department, in a separate developmental education department, and so on) may be highly correlated with the value that the institution places on developmental education in general. This section summarizes key components of community college practices that are relevant to developmental education in general and developmental mathematics specifically.

A recent study by the American Association of Community Colleges (AACC) involved sending surveys to over 1,100 community colleges, with a nearly 40-percent return rate (Schults 2001). The study covered a number of the most important features of developmental education, including the student body involved, and the colleges' approaches and policies. Many of the questions concerned approaches that have been made as recommendations for optimal developmental training that we have outlined in previous sections. In some of these cases, it is clear that community colleges have not universally adopted the recommendations. Below are some highlights of their findings:

- Every college responding to the survey offered at least one developmental course. Math, reading, and writing were offered by 94 to 96 percent, and adult basic education was offered by less than half.
- Of the institutions that responded, 33 percent of their faculty at public community colleges who teach developmental education classes were full-time, roughly equivalent to the overall proportion of faculty that are full-time.
- The majority, or 58 percent of responding institutions, required assessment of basic skills for all students. Many allow exemptions from these tests; 76 percent of those that allow exemptions use college entrance exam scores instead. Other criteria for waiver of the tests include high school GPA, statewide high school exam scores, advanced placement scores, and transfer from another postsecondary institution.
- The most commonly used tools for assessing skills were a computerized test (63 percent) and a paper-and-pencil test (60 percent). Other measures included college entrance tests (36 percent), institutionally developed measures (24 percent), and state-developed measures (16 percent).
- A large percentage of the institutions—77 percent—set their own cutoff scores on the assessment tests, while the state sets the standards in the remaining 23 percent.

- Of the 58 percent of institutions that mandate assessment, 75 percent require placement in courses based on the testing. Of these, almost two-thirds set this policy, and the remaining one-third reports that the standards are set by the state.
- Developmental courses are predominantly offered within relevant departments (61 percent), while 25 percent report that developmental courses are housed in a separate developmental department. The remaining 13 percent report that courses were offered through just one academic department.
- The majority of institutions responding indicated that ESL and ABE courses were typically offered by departments that are separate from those offering developmental education, typically through a noncredit department.
- The median number of levels of developmental math offered by colleges is three. More levels of remediation were offered in institutions located in large cities, and enrollment in developmental education also was typically higher in these institutions.
- Three-fourths of the institutions offered only institutional credit (not toward graduation but counting toward full-time status for the purpose of financial aid) for all developmental courses, 5 percent offered degree credit only, and 5 percent offered no credit. The remaining institutions offered multiple forms of credit.
- Developmental courses in math had the highest median class size—25—of any other type of developmental class surveyed. For comparison, the median class size for developmental reading and developmental writing were both 20. However, almost two-thirds of the institutions report having a policy concerning limits to class sizes with 95 percent of these reporting that the state did not mandate such limits.
- Partly because of limits on federal student financial aid that a person may receive for developmental education, almost one-quarter of institutions use various methods to limit the number of developmental courses a student may take. Of those, 20 percent increase tuition after students attempt multiple times to take developmental courses, 32 percent simply restrict students from taking additional developmental courses, 30 percent cease nonfederal funding, and 19 percent use other methods. Of those that set limits, 45 percent do so by state mandate.
- Virtually all colleges surveyed report that students could take college-level courses not related to a degree or certificate program while in developmental courses.
- Slightly over half (56 percent) of institutions report using more than one measure to assess whether a student can transition out of developmental work. The largest percentage (91 percent) use successful completion of the developmental course for assessment.

- Almost half (45 percent) of institutions offered self-paced developmental courses to students, and 26 percent offered distance education for developmental courses.
- Approximately 80 percent of institutions responding indicated that they sometimes or frequently use computers in instruction.
- Forty-five percent of institutions provide contract developmental training to business and industry, with 65 percent of these reporting that they do not award college credit for those classes.

The U.S. Department of Education recently released an updated summary of remedial education in postsecondary institutions for the year 2000 (NCES 2003). Many of its findings are similar to those of the AACC report, but the Department surveyed all postsecondary institutions, not just two-year colleges, and it also measured other phenomena. The major findings include:

- Seventy-one percent of all institutions surveyed and 97 percent of public two-year institutions that enrolled freshmen, offered developmental mathematics.
- Of those that offered developmental mathematics, 60 percent offered between two and four courses, with an average of 2.5 courses. The average for public two-year colleges was 3.4 courses.
- Of the institutions that did not offer any developmental courses, 34 percent said that they did not because either institutional or state policy or law prohibits them from offering such courses. This is an increase from 27 percent in 1995.
- Fifty-six percent of all public two-year colleges provided remedial education services to local businesses and industry, an increase from 50 percent in 1995. Twenty-one percent of all institutions did so.
- Of the public two-year institutions that offered remedial education to employers, 93 percent offered math skills; 85 percent offered instruction on site, 80 percent offered instruction at the business or industry, and 16 percent offered instruction via distance learning.
- Sixty-one percent of all institutions required all entering freshmen to be given placement tests in mathematics, while 64 percent of all public two-year colleges did so.
- Twenty-six percent of all institutions limit the amount of time a student may spend in developmental courses. Of those, 71 percent state that the policy is set by the institution, while 24 percent say that it is set by state policy. For public two-year colleges, 20 percent have limits, and their reasons are roughly divided between state law or policy (46 percent) and institutional policy (43 percent).

- Forty percent of all two-year colleges stated that computers were used frequently, and 44 percent said students used them occasionally as a hands-on instructional tool for on-campus developmental mathematics courses. This compares to 31 percent of all institutions reporting that they were used frequently, and 40 percent stating they were used occasionally.

Community colleges and four-year institutions are not the only source of developmental mathematics or even postsecondary education. All of the military services provide training and education, as do many businesses and business organizations. In addition, numerous basic skills services offered in adult education and in employment training efforts are outside the traditional postsecondary arena. We turn to these next.

The U.S. Military

General Eligibility Requirements

All of the services have basic enlistment eligibility criteria that include age, weight and height, moral background (e.g., arrest histories and prior drug use), medical conditions, education credentials, and mental ability. Among the eligibility criteria is a cap on recruiting non-high-school-diploma graduates (NHSDGs). No service is allowed to have more than 10 percent of its total accessions¹² per year in this category. The reason for the cap is that there is a significant body of evidence showing that NHSDGs are much less likely to complete their enlistment contract than are high school diploma graduates. Also for the purpose of this cap, recruits with GEDs are considered in the NHSDG category because GED graduates perform more like dropouts than like graduates in terms of their attrition. Within the cap, each service is permitted to manage its own mix of recruits. While the Army and the Navy strive to cap their NHSDG accessions at 5 percent, during the very difficult recruiting period of the late 1990s, both services had to increase their cap to 10 percent. In recent years, they have decreased their caps steadily. The Marine Corps has consistently accessed 5 percent or fewer NHSDG accessions, and the Air Force typically accesses less than 1 percent NHSDGs, and almost all of these possess GEDs.

Armed Services Vocational Aptitude Battery

The services assess the mental ability of potential recruits with a test known as the Armed Services Vocational Aptitude Battery (ASVAB). According to the ASVAB Web site (2003):

The ASVAB is the most widely used multiple aptitude test battery in the world. It was originally designed to predict future academic and occupational success in military occupations. Since its introduction in 1968, the ASVAB has been the subject of extensive research. Numerous validation studies indicate the ASVAB assesses academic ability and predicts success in a wide variety of military and civilian occupations.

The test consists of eight components: General Science (GS), Arithmetic Reasoning (AR), Word Knowledge (WK), Paragraph Comprehension (PC), Math Knowledge (MK), Electronics Information (EI), Auto and Shop Information (AS), and Mechanical Comprehension (MC). The ASVAB scores are standardized to a nationally representative sample of American youths (18- to 23-year-olds) who took the ASVAB in 1980,¹³ with each test normalized¹⁴ to have a mean of 50 and a standard deviation of 10.

¹²The term "accession," as used by the military, is reserved for those who begin military active duty. This is a smaller number than those who are recruited, since the entire recruiting process has a number of phases. A certain proportion of recruits never make the transition to active duty. To accession is to successfully make that transition to active duty.

¹³The services are in the process of renorming the ASVAB based on a sample from the late 1990s.

¹⁴Normalized means related to a normal or bell-shaped curve distribution.

The scores on various subtests are used to screen recruits for specific occupations, and four of the tests are used for basic enlistment eligibility. Specifically, the Armed Forces Qualifications Test (AFQT) is composed of the standardized scores on the Arithmetic Reasoning test plus the Math Knowledge test, plus two times the Verbal Expression (VE) measure, with VE being the standardized score for the sum of the Paragraph Comprehension and Word Knowledge components. The AFQT is then expressed as a percentile.

Congress has stipulated that the services cannot recruit people who score in the lowest 10 percentile of AFQT scores, and only 20 percent may be in the 10th through 30th percentile. Further, NHSDGs must score above the 30th percentile (Title 10 United States Code, Section 520). However, most of the services impose even higher standards. For instance, the Navy currently does not access anyone who scores below the 31st percentile on the AFQT.

Thus, all of the services do assess the basic skills competency of their recruits in the areas of reading, writing, and mathematics, and they establish certain criteria for admission. Beyond the screens for admission, however, the services also screen recruits for their military occupation, based on ASVAB scores. For instance, to enlist in one of the Navy's most technical programs, the Advanced Electronics/Computer Field (AECF), in addition to stricter screening criteria that pertain to citizenship, color perception and moral background, they impose guidelines in terms of mental ability. The sum of a recruit's scores on the MK, EI, and GS components of the ASVAB must meet a minimum threshold, and scores must be above a prescribed minimum for the AR and MK components (U.S. Navy 2002). These thresholds have been established to ensure that the recruit will have a satisfactory chance of successfully completing the training, which takes as long as 18 months.

Because of these ASVAB requirements, the services typically do not recruit people who are significantly deficient in their math skills who also will require higher-level math to perform their job. Instead, those with inadequate math skills either are not recruited, or are chosen for occupations that do not require those skills.

Servicemembers who need minimal remediation in math may receive some as part of their training, but this would typically involve only very short courses of a few days in length. Instead, servicemembers with deficient math skills would pursue developmental mathematics education only to enhance their own personal goals, such as to improve their ASVAB scores in order to requalify for another occupation, or to earn a GED, or to pursue a college education. All of these pursuits would be under what the military terms Voluntary Education (VolEd). VolEd in the military consists of numerous programs, each leading to various education outcomes and each with a variety of financial support to the servicemember. In fiscal year (FY) 2003, Department of Defense (DoD) spent \$477.5 million on all VolEd programs (U.S. Department of Defense 2004).

Basic skills

The lowest level of VolEd consists of basic skills training—that which helps servicemembers (primarily enlisted personnel) master reading, writing, and mathematics skills necessary to either do their job or accomplish their personal education goals. In FY03, 37,346 servicemembers, or 3.1 percent of the total, enrolled in noncredit basic skills courses:

- 18,025 in the Army,
- 11,768 in the Navy,
- 2,392 in the Marine Corps, and
- 5,161 in the Air Force.

By service these numbers range from 4.4 percent for the Army to 1.5 percent for the Marine Corps (U.S. Department of Defense 2004).

All of the services pay 100 percent of the cost of studies and testing that lead to a GED. In FY03, 84 soldiers, 64 sailors, 22 Marines, and 36 airmen received high school diplomas or GEDs while on active duty (U.S. Department of Defense 2004).

Each of the services provides basic skills education independently. The Army's program is called the Basic Skills Education Program (BSEP), designed to help soldiers master the functional reading, writing, and mathematical skills required of their jobs. In 1999, the undersecretary of the Army directed that BSEP should be automated and fully deployed (Bilodeau 2003). The Army found in an evaluation of three commercial off-the-shelf (COTS) products,¹⁵ students performed as well on TABE posttests as soldiers who took traditional BSEP courses.

BSEP is open only to soldiers who score below the 10th-grade level on TABE, or whose General Technical (GT) scores are below 100 (U.S. Army 2005). GT scores are computed as the sum of VE plus AR standard ASVAB scores (each has a mean of 50 and a standard deviation of 10 in the youth population). All services except the Navy then standardize this sum to have a mean of 100 and a standard deviation of 20.

The Marine Corps basic skills program is called Military Academic Skills Program (MASP), and is offered in a variety of delivery methods: via videoconferencing classroom, traditional classroom, or online. This is a four-week program targeted toward Marines who score 99 or below on the GT and 10.2 or below on TABE; there is an additional MASP preparatory course for those who score 8.5 or below on the TABE. In addition, Marines who have been referred by their commander also may enroll (U.S. Marine Corps 1998).

Thus, both the Army and the Marine Corps restrict enrollment in their basic skills education courses to servicemembers who score below a threshold of the TABE that is addressed by our research—that is, below the 10th-grade level.

¹⁵ Lifetime Library, Passkey, and PLATO.

The Navy's VolEd program is called Navy College Program. Basic skills are available either through online instruction in the Navy College Learning Centers (NCLC) located on numerous bases or in teacher-led instruction under the Navy College Learning Program (NCLP). NCLC instruction is provided using PLATO software, and sailors are administered a pretest to determine proper placement. The content of the PLATO instruction covers math beginning with second grade material, through the twelfth grade, including algebra (U.S. Navy 2003).

We consulted with members of the Navy and the Army research staff, and conducted academic literature searches and a search in the Defense Technical Institute Center (DTIC) repository (DTIC 2003) and were not able to identify research that has been conducted concerning the effectiveness of the services' basic skills education in developmental mathematics. The only study that emerges on this topic in general is one that was conducted in the late 1990s by the Center for Naval Analyses (CNA) (Garcia 1998). The Navy wanted to conduct an analysis of the effectiveness of VolEd in general because so little was known about the returns to this program, including basic skills and postsecondary education. The study found that, relative to nonparticipants, participants in the VolEd program got promoted faster and farther, had fewer disciplinary problems, and had higher retention even after controlling for relevant factors, such as military specialty and demographic factors. In sharp contrast to other studies in this review, this study uses an econometric technique that adjusts for the self-selection bias inherent in a voluntary program. Furthermore, this study calculates a return on investment (ROI) for various components of the program by comparing the implied reduction in recruiting and training costs from high retention to the cost of the program. The ROI on the adult basic education component of the program was even higher than the ROI on more advanced skill components.

Postsecondary VolEd

For servicemembers who want to pursue postsecondary education, there are a number of avenues, and they vary by service. Tuition Assistance (TA), which is offered by all the services, provides 100 percent of the mandatory tuition and fees at accredited institutions of higher education, up to \$250 per semester hour. All of the services except the Navy cap this assistance at \$4,500 per year. The Navy limits funding to 12 semester hours per year (Military.com 2003).

The Army, Navy, and Marine Corps each have a group of colleges that together compose the Servicemembers Opportunity Colleges (SOC), a consortium of approximately 1,700 colleges, which is cosponsored by the American Association of State Colleges and Universities (AASCU) and the AACC. Among other requirements, members of the consortium agree to accept credit awarded from other members. This feature is an important consideration for servicemembers who move frequently and so may not be able to complete a degree at the institution that originally granted credit. The Air Force does not have a comparable consortium because it has the Community College of the Air Force that serves a similar function.

Servicemembers may take courses at any accredited college, regardless of membership in SOC, on their own time. For many, this includes distance learning or Video Conferencing (VTC), but for those located near large bases, a number of community colleges will actually provide instruction on base.

The cost of these courses is subsidized for servicemembers through TA, including developmental mathematics courses that confer credit (including institutional) and are required by the college. Hence, any developmental mathematics that would be required by the institution would be similar to those for other adult learners enrolled at that institution and are, therefore, included in our analysis of community colleges.

Businesses and Organized Labor

Many corporations and moderate-sized companies support continuing education and training of their workforce. A study conducted by the American Society for Training and Development (ASTD), in which they surveyed 365 organizations in 1999, found that companies spend an average of 1.8 percent of payroll on training (Van Buren 2001). Of the amount spent on training, between 5 and 7 percent is in basic skills, including literacy, reading comprehension, writing, math, English as a second language, and learning how to learn. By far, the largest category of training was in technical processes and procedures, totaling approximately 13 percent of all training expenditures. The National Association of Manufacturers survey cited earlier found that 7 percent of employers offered advanced math education opportunities, while 15 percent offered basic math training. The study also found that just 4 percent of respondents in companies with at least one union said that the union offered training to employees.

In concurrence with the NCES (2003) findings previously cited, the most often cited sources of external education and training in the survey were community colleges (45 percent), technical and vocational schools (46 percent), business and industry associations (45.9 percent), consultants (39 percent), and universities (17 percent).

Our search of the literature revealed no definitive research on business or organized labor education programs in general. In addition, a search of the Web sites of major unions and large corporations also did not reveal more information about specific education programs in developmental mathematics. For example, the Web site of the AFL-CIO, with a membership of over 13 million, has a section concerning education issues and legislation, but it contains no information about specific education programs in general, or developmental mathematics in particular (AFL-CIO 2003). We did find one Web site that mentions an education program offered by a joint partnership between the United Auto Workers (UAW) and General Motors Corporation (GM), although little information about the specific program is provided (UAW-GM Center for Human Resources 2004). They do note, however, that there are more than 97 UAW-GM Skill Centers across the country that operate within a local education agency, and offer opportunities in: Adult Basic Education, General Educational Development, educational enrichment services, English as a second language, academic advising services, and high school completion.

Wolfe (2001) described a business-based program in Kentucky that is a collaboration with the local community college, Kentucky Educational Television (KET), and Hospitality Television. This School-to-Work program brings basic math and developmental classes to the workplace via television, thereby bringing the education to the worker rather than requiring them to come to the college. The program was launched in 1998 for workers in the hospitality sector, but by 2001 it had expanded to include retail, manufacturing, and local government, especially school districts. Although no evaluation of effectiveness of the program was noted, Wolfe stated that one official envisioned this program as being a feeder for community colleges and technical schools because of its potential to get learners started on the right path.

Dougherty and Bakia (2000) discussed the role that business and organized labor have in contract training to community colleges. They describe contract training in the five areas they investigated: auto manufacturing, construction, apparel marketing, banking, and auto repair. They note that entry-level skills training across the five industries they studied were focused primarily on machinists, carpenters, and auto repair technicians. They found far less contract training for entry-level semiskilled workers.

These entry-level skills training programs combine a large amount of classroom and on-the-job (OJT) training. Among the most significant of these programs are the apprenticeships in auto manufacturing and the nonapprenticed employee-in-training (EIT) programs that are primarily conducted at two-year postsecondary institutions, the control of both being dominated by employers and the UAW. In fact, Dougherty and Bakia note that many are joint union-management apprenticeship programs and union members play a key role in decisions concerning curriculum, choice of providers, and evaluation of the program.

Because of the absence of unions in the auto repair industry, they note that training in this area is primarily sponsored by GM, Ford, Chrysler, and Toyota, with the community college as a much more equal partner.

Also, construction contract training tends to be shorter in duration if it is sponsored by industry than if it is sponsored by joint union-management apprenticeship training committees.

Finally, they note that contract training has the potential to change not only the content of courses to meet the particular needs of the business or union but also the pedagogy, particularly when the corporate customer uses newer instructional techniques or technology than the college uses. In those cases, college instructors who teach corporate-sponsored courses often pick up these technologies and then import them back into their regular courses.

Corporate Math Skill Trainers

Much of our review so far has focused on developmental college programs. Businesses also report using business and industry associations staff, and corporate math skills trainers to remediate the math skills of their workforce. We gathered information via Web sites and exchanges with personnel from a number of corporate math skill trainers, including four of the largest—MathWorks Inc. (Maher 2004); Thomson NETg (Rollins 2004); SkillSoft Inc. (Jordan 2004); and Mathsoft (Schindler 2004)—and two smaller organizations—Math Learning Institute Inc. (Charles 2004) and the Alinea Group (Martin 2004) in order to obtain a more comprehensive representation of corporate training.

The research revealed that a variety of businesses rely on the contractual services of corporate math skill training companies to provide math skill development tools, products, and training for their employees. Although most companies cannot give

detailed information about their clients, they indicated that clients have included U.S. government agencies, major auto manufacturers, aerospace companies, major airlines, IT companies, and large discount chain stores. This inquiry yielded several key observations:

- A large number of companies rely primarily on the contractual services of local colleges and universities, particularly services provided by community colleges, for basic math skill (addition, subtraction, division, multiplication, fractions, etc.) training.
- Of the four largest math skill training companies interviewed and researched, all required basic math skill knowledge before taking classes, or using their products, or both. Although courses were offered to those with little to no advanced level math knowledge, in almost all cases, the training companies' clientele consisted primarily of individuals with a strong mathematics background, at least a baccalaureate degree, or both, often in the areas of mathematics, engineering, the sciences or technology. Many of their clients also have advanced degrees.
- The two smaller companies did provide customized basic math skill training.
- Several of the larger math skill training companies provide referral services to clients to local colleges and universities that provide basic math skill training to employees to ensure that their clients are adequately prepared to succeed in their training classes, or use their math software, or both.
- Several corporations that provide some math skill training have partnered with educational institutions (e.g., University of Phoenix) to give employees the opportunity to receive college credit for courses they take through their training programs.
- Most corporate consulting and training organizations provide coursework through e-learning (Web-based), CD ROMs, instructor lead training, onsite training, or training at their own facilities.
- The math skill courses are designed to meet the needs of the client and therefore vary in length and format. Some classes are as short as one to two hours or may meet regularly over the course of several months.
- Math skill training is generally only one component of the services provided by companies that provide math skill training through various course offerings (finance, budget, etc.). Most of the companies that provide training also have developed products and tools that they market, train others to use, and sell.

Adult Education and Workforce Development

New economic realities have created a need for workforce and education policies that better meet employer demands for skilled workers. Current efforts to develop opportunities for people lacking skills and resources are focusing on career pathways that integrate education, training, and skill development in targeted high-wage, high-demand employment areas (Mazzeo, Rab, and Alssid 2003). Career pathways provide developmental, adult education, or ESL classes in the context of students' lives and the work-specific skills they need for employment in particular industries or sectors. With this type of approach, courses such as mathematics are modified to incorporate materials from specific fields into the actual course content. Programs that promote contextual teaching and learning make heavy use of projects, laboratories, simulations, and other experiences that enable students to learn by doing (Jenkins 2002). By integrating instruction in basic skills with instruction in technical content, contextualized teaching and learning also enable academically unprepared students to obtain career training at the same time that they enroll in basic education.

Central to federal efforts to remediate displaced or dislocated workers is the *WIA*. Title I of *WIA* is the cornerstone legislation in the federal arsenal as it provides workforce investment services and activities through a network of One-Stop Career Centers and strategic planning and oversight by business-led workforce investment boards (WIBs). Available workforce development activities provided in local communities can benefit job seekers, laid off workers, youths, incumbent workers, new entrants to the workforce, veterans, persons with disabilities, and employers. The purpose of these activities is to promote an increase in the employment, job retention, earnings, and occupational skills improvement by participants. This, in turn, is intended to improve the quality of the workforce, reduce welfare dependency, and improve the productivity and competitiveness of the nation.

Adult and laid-off worker services are provided through locally based One-Stop Career Centers. Comprehensive one-stop centers provide access to a full range of services pertaining to employment, training and education, employer assistance, and guidance for obtaining other assistance. While *WIA* requires one-stop centers to provide specific services, local areas may design programs and provide services that reflect the unique needs of their area. *WIA* Title I funds may be used to support adult education and other literacy activities only if this instruction is provided in combination with occupational skills or on-the-job training. Title II of *WIA* authorizes the *Adult Education and Family Literacy Act*, which provides formula funding to states to support adult education and literacy services (including workplace literacy services), family literacy services, and English literacy programs. Eligible individuals may access adult education programs funded by *WIA* Title II through One-Stop Career Centers.

Community colleges are also key players in adult education and workforce development. A recent study by the Education Commission of the States (Jenkins and Boswell 2002) found that community colleges frequently offer training to upgrade the skills of workers, which is often provided under contract to employers and typically does

not confer college credit. They note that a study done by Columbia University's Teachers College (Bailey et. al. 2004) estimated that, in 1999, 2.3 million students were enrolled in noncredit, job-related training programs at community colleges. In addition, community colleges play a key role in helping unemployed and underemployed adults in basic skills training, such as ESL, in programs linked to training for jobs. The authors also offer extensive programs to help welfare recipients enter the workforce.

The study notes that community colleges are designated as the lead agency to provide workforce training in at least 19 states: Alaska, Alabama, Arkansas, Colorado, Delaware, Iowa, Kansas, Kentucky, Maine, Missouri, Mississippi, Nebraska, New Hampshire, Nevada, North Carolina, North Dakota, Virginia, Washington, and Wisconsin.

The Education Commission of the States (ECS) administered a survey in 2001 to the state agency responsible for community colleges in all 50 states. Five states did not respond: Hawaii, Idaho, Maryland, Montana, and South Dakota. According to the ECS (2002) report, the majority of states indicated that the lack of workforce development funding is a challenge, particularly in terms of making investments in technology to prepare a technically competent workforce. Further, several states pointed out the inconsistency in the decreased willingness of policymakers to support workforce development programs while they stress the growing importance of a skilled labor force. Major highlights of the findings were:

- Eighteen states provide state funding (in addition to federal funding) to support occupational training of disadvantaged students by community colleges. In all but one of these states, welfare recipients are targeted specifically for such training. Other targeted groups include low-income adults, displaced workers, veterans, the disabled, and at-risk youth.
- Thirty-two states provide state funding to support customized training for employers, with most states imposing some restrictions on the use of these funds. The level of funding ranged from under \$1 million in several states to \$50 million in New Jersey. In general, community colleges compete with other training providers for these funds.
- Twenty states fund noncredit occupational training (separate from funding for customized training) at community colleges.

Summary and Conclusions

This literature review set out to examine research on promising strategies for strengthening math skills at the postsecondary level. Our work has indicated that research into instructional practices and curriculum content methodologies that are specific to developmental mathematics is largely flawed, lacking in the scientific rigor necessary to make sound inferences. Most of the studies we reviewed are methodologically limited by the absence of control or comparison groups, which makes it virtually impossible to gauge the interventions' true impact on learning.

In terms of the knowledge necessary for successfully pursuing college-level math, we have found that no consistent definition of math standards for college-level preparation exists. However, a number of studies indicate the need to have a good foundation in arithmetic, geometry, trigonometry, and algebra I and II. Emerging work also indicates the increasing need for basic statistics and the ability to analyze data.

We found that there is less uncertainty or ambiguity in the skills necessary to pursue college-level math and to succeed in the highest-paid and highest-skilled jobs. In particular, there seems to be widespread agreement on the need to think critically, to solve problems, and to communicate mathematically. Both businesses and postsecondary institutions indicate that they want people who can identify a problem, determine whether it can be solved, know which operations and procedures are required to solve them, use multiple representations (such as graphs and words) to describe the problems and solutions, and understand and apply mathematical modeling. However, these are the skills that are the most difficult to teach and to assess.

It remains to be seen whether community colleges are adopting these recommendations, in terms of the knowledge, skills, or abilities. It is also uncertain whether community colleges adequately assess the knowledge and skills necessary to pursue postsecondary-level math or succeed in the workplace. Regardless, the majority of two-year colleges require incoming students to take and pass an assessment test before they are allowed to enroll in college-level math courses. Given their prevalence, this may currently be the most relevant benchmark for whether a person may successfully transition into college-level mathematics.

While we did not identify existing studies based on gold-standard research in developmental mathematics at the postsecondary level, salient themes concerning pedagogy emerged, suggesting promising but unproven instructional practices that are frequently implemented. These may warrant further study. We summarize promising key components of strategies or approaches to developmental mathematics programs at the postsecondary level into the categories and topics below:

- **Instructional and pedagogical:** traditional instruction; multiple delivery options for students to choose from; computer-assisted instruction; Internet-based; self-paced; distance learning; calculators; computer algebra systems; spreadsheets;

labs; small-group instruction; learning communities; contextual learning; linkages to and examples from the workplace; and career pathways.

- **Curriculum content:** nonstandard topics covered in developmental math courses or topics that vary by career path; length of instruction; and types of activities used to reinforce the material.
- **Professional development:** faculty training and development; and full-time versus part-time instructors.
- **Supporting strategies:** counseling; and assessment, placement, and exit strategies.
- **Learner and institutional characteristics:** full-time versus half-time community college student; socioeconomic attributes of learner; workplace program; and servicemember.

Implications for Further Research

Additional research is necessary to understand what works in developmental mathematics. In particular, we need to understand more precisely why students drop out of developmental math courses. Is it because of the material covered, the instructional methods used, challenges outside of the classroom such as financial or family constraints, or some combination of all of these factors? As a first step in enhancing that understanding, researchers need to gather more information concerning (a) a variety of outcomes, such as developmental math course pass rates, persistence to and pass rates of developmental mathematics students in higher-level math courses, transfer rates to other institutions, and graduation rates, (b) student characteristics (e.g., race and ethnicity, age, gender, highest education credential, socioeconomic status), and (c) the relationship of each of these characteristics to the various outcomes. This information could then begin to address important questions, such as whether a particular pedagogical approach benefits all students equally regardless of their education credential, age, and other characteristics, and whether the benefits persist to higher-level math courses and ultimately, to graduation.

Appendix A: Mathematics Knowledge and Skills for Success From Conley and Bodone (2002)

The key knowledge and skills considered to be necessary for success in mathematics include the following:

- Computation
 - The student will know basic mathematics operations by being able to:
 - Use arithmetic operations with fractions;
 - Use exponents and scientific notation;
 - Use whole numbers to perform all basic arithmetic operations, including long division with and without remainders;
 - Use radicals correctly;
 - Understand relative magnitude and absolute value;
 - Know terminology for real numbers, such as irrational numbers, natural numbers, integers, and rational numbers; and
 - Use the correct order of arithmetic operations;
 - The student will know and carefully record symbolic manipulations.
 - The student will know and demonstrate fluency with mathematical notation and computation by being able to:
 - Perform addition, subtraction, multiplication and division;
 - Perform appropriate basic operations on sets; and
 - Recognize alternative symbols (e.g., Greek letters).
- Algebra
 - The student will know and apply basic algebraic concepts by being able to:
 - Use the distributive property to multiply polynomials;
 - Multiply and divide polynomials;
 - Factor polynomials;
 - Add, subtract, multiply, divide, and simplify rational expressions including finding common denominators;
 - Understand properties and basic theorems of roots and exponents; and
 - Understand properties and basic theorems of logarithms.
 - The student will use various techniques to solve basic equations and inequalities by being able to:
 - Solve linear equations and absolute value equations;
 - Solve linear inequalities and absolute value inequalities;
 - Solve systems of linear equations and inequalities using algebraic and graphic methods;
 - Solve quadratic equations using various methods and recognize real solutions by being able to:
 - Use factoring and zero products;
 - Use completing the square; and

- Use the quadratic formula.
- The student will be able to recognize and use basic algebraic forms by being able to:
 - Distinguish between expression, formula, equation, and function and recognize when simplifying, solving, substituting in, or evaluating is appropriate;
 - Determine whether a relation is a function;
 - Understand applications;
 - Use a variety of models to represent functions, patterns, and relationships;
 - Understand terminology and notation used to define functions; and
 - Understand the general properties and characteristics of many types of functions (e.g., direct and inverse variation, general polynomial, radical, step, exponential, logarithmic, and sinusoidal).
- The student will understand the relationship between equations and graphs by being able to:
 - Understand slope-intercept form of a equation of a line and graph the line;
 - Graph a quadratic function and recognize the intercepts as solutions to a corresponding quadratic equation; and
 - Know the basic shape of the graph of an exponential function.
- The student will know how to use algebra both procedurally and conceptually by being able to:
 - Recognize which type of model best fits the context of a situation.
- The student will demonstrate ability to algebraically work with formulas and symbols by being able to:
 - Understand formal notation and various applications of sequences and series.
- Trigonometry
 - The student will know and understand basic trigonometric principles by being able to:
 - Know the definitions of the trigonometric ratios—sine, cosine, and tangent—using right triangle trigonometry and position on the unit circle;
 - Understand the relationship between a trigonometric function in standard form and its corresponding graph;
 - Know and use identities for sum and difference of angles;
 - Recognize periodic graphs;
 - Understand concepts of periodic and exponential functions and their relationships to trigonometric formula, exponents, and logarithms;
 - Solve problems using exponential models; and
 - Understand and use double and half angle formulas.
- Geometry
 - The student will know synthetic (i.e., pictorial) geometry by being able to:

- Use properties of parallel and perpendicular lines in working with angles;
 - Know triangle properties;
 - Understand the concept of mathematical proofs, their structure and use;
 - Use geometric constructions to complete simple proofs, to model, and to solve mathematical and real-world problems; and
 - Use similar triangles to find unknown angle measurements and lengths of sides.
- The student will know analytic (i.e., coordinate) geometry by being able to:
 - Know geometric properties of lines;
 - Know the equations for conic sections;
 - Use the Pythagorean Theorem and its converse and properties of special right triangles to solve mathematical and real-world problems;
 - Use transformations of figures to graph simple variations of equations for basic graphs;
 - Set up appropriate coordinate system for applications; and
 - Understand vectors in mathematical settings.
- The student will understand the relationships between geometry and algebra by being able to:
 - Know how to manipulate conics;
 - Understand that objects and relations in geometry correspond directly to objects and relations in algebra; and
 - Solve real-world problems using three-dimensional objects.
- The student will demonstrate geometric reasoning by being able to:
 - Prove congruency of triangles; and
 - Use inductive and deductive reasoning to make observations about and to verify properties of and relationships among figures.
- The student will be able to combine algebra, geometry, and trigonometry by being able to:
 - Understand and use the law of sines and the law of cosines; and
 - Use properties of and relationships among figures to solve mathematical and real-world problems.
- Mathematical Reasoning
 - The student will demonstrate an ability to solve problems by being able to:
 - Use inductive reasoning;
 - Demonstrate ability to visualize;
 - Use multiple representations to solve problems;
 - Use a framework or mathematical logic to solve problems that combine several steps;
 - Use a variety of strategies to understand new mathematical content and to develop more efficient solution methods or problem extensions; and

- Construct logical verifications or counter examples to test conjectures and to justify algorithms and solutions to problems.
- The student will understand various representations by being able to:
 - Understand abstract mathematical ideas in word problems, pictorial representations, and applications.
- The student will demonstrate a thorough understanding of mathematics used in applications by being able to:
 - Understand the concept of a function.
- The student will demonstrate strong memorization skills by being able to:
 - Know a variety of formulas and short proofs.
- The student will know how to estimate by being able to:
 - Understand the relationships among equivalent number representations;
 - Know when an estimate or approximation is more appropriate than an exact solution for a variety of problem situations; and
 - Recognize the validity of an estimated number.
- The student will understand the appropriate use of technology by being able to:
 - Know the appropriate uses of calculators and their limitations;
 - Perform difficult computations using a calculator;
 - Know how to use graphing calculators;
- The student will be able to generalize (e.g., to go from general to abstract and back and to go from specifics to abstract and back) by being able to:
 - Determine the mathematical concept from the context of a real-world problem, solve the problem, and interpret the solution in the context of the real-world problem.
- The student will be willing to experiment with mathematics by being able to:
 - Understand that math problems can have multiple solutions and multiple methods to determine the solution(s).
- Students will emphasize process over mere outcome(s) by being able to:
 - Understand the various steps to a solution.
- The student will show ability to modify patterns and computations for different situations by being able to:
 - Compare a variety of patterns and sequences.
- The student will use trial and error to solve problems by being able to:
 - Find the way(s) that did not work to solve a problem and finally find the one(s) that do work.
- The student will understand the role of mathematics by being able to:
 - Know the relationship between the various disciplines of math; and
 - Understand the connections between mathematics and other disciplines.
- The student will use mathematical models by being able to:
 - Use mathematical models from other disciplines.
- The student will understand the need to be an active participant in the process of learning mathematics by being able to:

Appendix A (cont'd)

- Ask questions throughout multistep projects, recognizing natural questions arising from a mathematical solution;
 - Use appropriate math terminology; and
 - Understand that mathematical problem solving takes time.
 - The student will understand that mathematics is a symbolic language and that fluency requires practice by being able to:
 - Translate simple statements into equations; and
 - Understand the role of written symbols in representing mathematical ideas and the precise use of special symbols of mathematics.
- Statistics
 - The student will understand and apply concepts of statistics and data analysis by being able to:
 - Select and use the best method of representing and describing a set of data;
 - Understand measures of central tendency and variability and their application to specific situations; and
 - Understand different methods of curve-fitting and various applications.

Appendix B: Summary of Studies Related to Developmental Mathematics Reviewed in This Report

Study	Approach	Metrics	Outcome	Research Issues
Adams 2003	Combined two-semester course in one semester	<ul style="list-style-type: none"> • Top 60 percent of scorers in assessment test • Comparison of final test scores for those in developmental/regular course vs. those in regular course not requiring remediation • Pass rates in higher math courses compared for two groups 	<ul style="list-style-type: none"> • Final test scores similar • Developmental students had 7 percentage points higher pass rate in elementary calculus • Results not as good for engineering calculus 	<ul style="list-style-type: none"> • No control for student characteristics • Precise measures of differences not stated
Atkinson 2003	Tutorials in Problem Solving (TiPS) for arithmetic and problem-solving skills in adults	<ul style="list-style-type: none"> • Pretest and posttest measures • Relationship between length of instruction and assessed mathematical ability prior to using TiPS 	<ul style="list-style-type: none"> • Average posttest score was significantly higher for students using TiPS than similarly assessed students in an untreated control group • While originally intended for Navy sailors, results indicated TiPS has much wider applicability to adult literacy programs and workforce training programs 	<ul style="list-style-type: none"> • Use of regression-discontinuity design ensures validity of results
Cartnal 1999	Traditional vs computer-assisted instruction in elementary and intermediate algebra	<ul style="list-style-type: none"> • Success • Retention • Persistence 	<ul style="list-style-type: none"> • Retention and persistence highest in computer-assisted instruction • Success highest in traditional • Recommends further research 	<ul style="list-style-type: none"> • Self-selection bias • No control for student characteristics • No statistical test of significance reported
Creery 2001	Lecture vs. self-paced vs. online instruction	<ul style="list-style-type: none"> • Final grades • Persistence 	<ul style="list-style-type: none"> • No difference in final grades or persistence among the three methods • Recommends offering multiple instructional modes 	<ul style="list-style-type: none"> • Self-selection bias • No control for student characteristics • No analysis of differentials in withdrawals
DePree 1998	Instructor vs. small-group instruction	<ul style="list-style-type: none"> • Math confidence • Achievement • Course completion 	<ul style="list-style-type: none"> • Small-group instruction had statistically higher confidence, more likely to complete the course • Improvements were greatest for traditionally underrepresented students in mathematics: Hispanics, Native Americans, and females • No difference in 	<ul style="list-style-type: none"> • Quasi-experimental design with students not aware of modality used at time of registration

Study	Approach	Metrics	Outcome	Research Issues
			achievement	
Higbee and Thomas 1999	Relationship between noncognitive variables and success in two-quarter algebra sequence	<ul style="list-style-type: none"> • Test anxiety • Confidence to succeed in learning math 	<ul style="list-style-type: none"> • Math and test anxiety decreased for students in course with relaxation exercises and metacognition strategies • Posttest measures of anxiety were not correlated with higher course grades 	<ul style="list-style-type: none"> • Authors acknowledge that design does not allow for separating out effects of individual treatments, teaching ability, and personality traits of professor • No control group • Withdrawals are not accounted for
Kinney 2001	Traditional vs. computer-mediated instruction in elementary and intermediate algebra	<ul style="list-style-type: none"> • Scores on final exams • Confidence in math 	<ul style="list-style-type: none"> • No difference in final exam scores • Both groups reported increased confidence • Recommends offering multiple instructional modes 	<ul style="list-style-type: none"> • Self-selection bias • No control for student characteristics • No information pertaining to metrics and tests conducted
Lancaster 2001	PLATO Software vs. traditional lecture in elementary algebra	<ul style="list-style-type: none"> • Course withdrawals • Satisfactory grades 	<ul style="list-style-type: none"> • Computer Assisted Instruction (CAI) resulted in 7-percent decrease in number of withdrawals • 12-percent increase in number of satisfactory grades • 11-percent decrease in unsatisfactory grades 	<ul style="list-style-type: none"> • Quasi-experimental design with control group consisting of students in prior year • No control for student characteristics
Livingston 2001	Computer Algebra System (CAS) vs. traditional method with calculators in intermediate algebra	<ul style="list-style-type: none"> • Ability to perform mathematics by hand • Ability to solve higher-order reasoning by hand 	<ul style="list-style-type: none"> • No statistically significant difference in ability to perform mathematics by hand • CAS group performed better in ability to solve higher-order reasoning by hand 	<ul style="list-style-type: none"> • Quasi-experimental nonrandom control group design • No control for student characteristics • Self-selection bias
McClendon and McArdle 2002	Traditional vs. lecture/computers vs. Assessment LEarning in Knowledge Spaces (ALEKS)	<ul style="list-style-type: none"> • Course completion (letter grade of C or better) 	<ul style="list-style-type: none"> • Without eliminating withdrawals, traditional had highest completion • Netting out withdrawals, no difference • Recommends offering multiple instructional modes 	<ul style="list-style-type: none"> • Self-selection bias • No control for student characteristics • No analysis of why ALEKS had higher withdrawal rate
Miglietti et al. 2002	Relationship between age and gender on learning styles, and between teacher style and classroom outcomes	<ul style="list-style-type: none"> • Principles of Adult Learning • Adult Classroom Environment Scale • Adaptive Style Inventory 	<ul style="list-style-type: none"> • Adult underprepared students in learner-centered developmental English classrooms achieved higher grades than similar students in teacher-centered classrooms • No age or gender effects on classroom environment or learning style preference • Effect of age and teaching style on developmental mathematics was not possible 	<ul style="list-style-type: none"> • Withdrawals are not accounted for

Study	Approach	Metrics	Outcome	Research Issues
Quinn 2003	PLATO Software for arithmetic and elementary algebra	<ul style="list-style-type: none"> Absolute increase in scores on computerized placement test (CPT) Relationship between time spent on software and increase in CPT scores 	<ul style="list-style-type: none"> Statistically significant increase for all students in pretest and posttest CPT scores Each additional hour spent results in 0.61 to 1.86 percent improvement in CPT posttest score 	<ul style="list-style-type: none"> No comparison group Does not control for student characteristics
Sinclair Community College 2003	Relationship between time since last developmental mathematics course and first college-level math course	<ul style="list-style-type: none"> Average GPA in course Success rate 	<ul style="list-style-type: none"> Average GPA is significantly lower when students delay 3 terms Success rate is not affected 	<ul style="list-style-type: none"> Factors affecting choice to delay subsequent math courses are not measured or controlled for
Waycaster 2001	Lecture with lab vs. individualized computer-aided instruction	<ul style="list-style-type: none"> Pass rate Retention Graduation rate 	<ul style="list-style-type: none"> Success was independent of instruction method Developmental mathematics students had higher retention than regular students 	<ul style="list-style-type: none"> Self-selection bias No control for student characteristics No statistical test for differences conducted Metrics not well-defined
Wheland et al. 2003	Perceived inhibitors to student success in intermediate algebra	<ul style="list-style-type: none"> Midsemester tests Final exam scores GPA 	<ul style="list-style-type: none"> Nonnative instructors do not have negative impact on success Performance in intermediate algebra correlated with overall semester GPA Attendance highly correlated with success Grade in intermediate algebra predictive of performance in higher math 	<ul style="list-style-type: none"> Self-selection bias Does not control for student characteristics

Appendix C: Comparable ASSET,¹⁶ ACT,¹⁷ and COMPASS¹⁸ Cutoff Scores For Student Placement into Mathematical Courses

This table provides information concerning the most typical cutoff scores used by colleges on the ASSET, ACT, and COMPASS tests for placement into various levels of mathematics courses. For instance, reading across the top row, colleges that use the ASSET test generally place a student who scores between 23 and 40 on the Numerical Skills component of that test into a course that provides a basic arithmetic review. Institutions that use the ACT Mathematics subtest instead would place students scoring below 18 into a similar course, and institutions that use the COMPASS would place a student into a similar course if he or she scored less than a 44 on the Pre-algebra component.

Table C-1: Asset, Act, and Compass Cutoff Scores for Student Placement

ASSET Scores	ACT Math	COMPASS Scores	Course Recommendations
Numerical Skills		Pre-algebra	
23–40	0–17	0–43	Arithmetic review
41–55	18–20	44–100	Elementary algebra or courses with arithmetic prerequisite
Elementary Algebra		Algebra	
23–40	18–20	0–45	Elementary algebra or courses with arithmetic prerequisite
41–55	21–22	46–65	Intermediate algebra or courses with elementary algebra prerequisite
Intermediate Algebra			
23–40	21–22	46–65	Intermediate algebra or courses with elementary algebra prerequisite
41–55	23–25	66–100	College algebra or courses with intermediate algebra prerequisite
College Algebra		College Algebra	
23–40	23–25	0–45	College algebra or courses with intermediate algebra prerequisite
41–55	26–27	46–100	Trigonometry or business calculus or courses with college algebra prerequisite
	26–27	Trigonometry 0–45	Trigonometry or business

¹⁶ Assessment of Skills for Successful Entry and Transfer

¹⁷ American College Testing Program

¹⁸ Computerized Adaptive Placement Assessment and Support System

ASSET Scores	ACT Math	COMPASS Scores	Course Recommendations
	28–36	46–100	calculus or courses with college algebra prerequisite Calculus 1 or courses with college algebra and trigonometry prerequisites

Source: Roth 2003

Appendix D: Level of Proficiency Associated With ACCUPLACER¹⁹ Cutoff Scores

Level of Proficiency Associated With ACCUPLACER Cutoff Scores

Arithmetic Proficiency	Elementary Algebra Proficiency	College-level Mathematics Proficiency
<p>Score of 38--64 Students at this level have minimal arithmetic skills. These students can:</p> <ul style="list-style-type: none"> perform simple operations with whole numbers and decimals (addition, subtraction, and multiplication) calculate an average, given integer values solve simple word problems identify data represented by simple graphs 	<p>Score of 28--43 Students at this level have minimal pre-algebra skills. These students demonstrate:</p> <ul style="list-style-type: none"> a sense of order relationships and the relative size of signed numbers the ability to multiply a whole number by a binomial 	<p>Score of 39 or less These students should take the elementary algebra test before any placement decisions are finalized.</p>
<p>Score of 65--92 Students at this level have basic arithmetic skills. These students can:</p> <ul style="list-style-type: none"> perform the basic arithmetic operations of addition, subtraction, multiplication, and division using whole numbers, fractions, decimals, and mixed numbers make conversions among fractions, decimals, and percents 	<p>Score of 44--81 Students scoring at this level have minimal elementary algebra skills. These students can:</p> <ul style="list-style-type: none"> perform operations with signed numbers combine like terms multiply binomials evaluate algebraic expressions 	<p>Score of 40--62 Students scoring at this level can:</p> <ul style="list-style-type: none"> identify common factors factor binomials and trinomials manipulate factors to simplify complex fractions <p>These students should be considered for placement into intermediate algebra. For further guidance in placement, have these students take the elementary algebra test.</p>
<p>Score of 93--109 Students at this level have adequate arithmetic skills. These students can:</p> <ul style="list-style-type: none"> estimate products and squares of decimals and square roots of whole numbers and decimals 	<p>Score of 82--108 Students at this level have sufficient elementary algebra skills. By this level, the skills that were beginning to emerge at a "total right score" of 57 (i.e. 57 correct) have been developed. Students at this level can:</p>	<p>Score of 63--85 Students scoring at this level can demonstrate the following additional skills:</p> <ul style="list-style-type: none"> work with algebraic expressions involving real number exponents factor polynomial

¹⁹ College Placement Exam

Arithmetic Proficiency	Elementary Algebra Proficiency	College-level Mathematics Proficiency
<ul style="list-style-type: none"> • solve simple percent problems of the form $p\%$ of $q = ?$ and $?%$ of $q = r$ • divide whole numbers by decimals and fractions • solve simple word problems involving fractions, ratio, percent increase and decrease, and area 	<ul style="list-style-type: none"> • add radicals, add algebraic fractions, and evaluate algebraic expressions • factor quadratic expressions in the form $ax^2 + bx + c$, where $a = 1$ • factor the difference of squares • square binomials • solve linear equations with integer coefficients 	<ul style="list-style-type: none"> expressions • simplify and perform arithmetic operations with rational expressions, including complex fractions • solve and graph linear equations and inequalities • solve absolute value equations • solve quadratic equations by factoring • graph simple parabolas • understand function notation, such as determining the value of a function for a specific number in the domain • a limited understanding of the concept of function on a more sophisticated level, such as determining the value of the composition of two functions • a rudimentary understanding of coordinate geometry and trigonometry
<p>Score of 100+ Students at this level have substantial arithmetic skills. These students can:</p> <ul style="list-style-type: none"> • find equivalent forms of fractions • estimate computations involving fractions • solve simple percent problems of the form $p\%$ of $? = r$ • solve word problems 	<p>Score of 109+ Students at this level have substantial elementary algebra skills. These students can:</p> <ul style="list-style-type: none"> • simplify algebraic expressions • factor quadratic expressions where $a = 1$ • solve quadratic equations 	<p>Score of 86–102 Students scoring at this level can demonstrate the following additional skills:</p> <ul style="list-style-type: none"> • understand polynomial functions • evaluate and simplify expressions involving functional notation, including composition of functions • solve simple equations

These students should be considered for placement into college algebra or a credit-bearing course immediately preceding calculus.

Arithmetic Proficiency	Elementary Algebra Proficiency	College-level Mathematics Proficiency
<p>involving the manipulation of units of measurement</p> <ul style="list-style-type: none"> • solve complex word problems involving percent, average, and proportional reasoning • find the square root of decimal numbers • solve simple number sentences involving a variable 		<p>involving:</p> <ul style="list-style-type: none"> • trigonometric functions • logarithmic functions • exponential functions <p>These students can be considered for a pre-calculus course or a non-rigorous course in beginning calculus.</p>
		<p>Score of 103+</p> <p>Students scoring at this level can demonstrate the following additional skills:</p> <ul style="list-style-type: none"> • perform algebraic operations and solve equations with complex numbers • understand the relationship between exponents and logarithms and the rules that govern the manipulation of logarithms and exponents • understand trigonometric functions and their inverses • solve trigonometric equations • manipulate trigonometric identities • solve right-triangle problems • recognize graphic properties of functions such as absolute value, quadratic, and logarithmic
		<p>These students should be considered for placement into calculus.</p>

Source: Murphy, S. 2002

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