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ABSTRACT

This paper is part of a dissertation defended in January 2001 as part of the author's Ph.D. requirement. The study investigated the effects of use of Mathematica, a computer algebra system, in learning basic linear algebra concepts, It was done by means of comparing two first year linear algebra classes, one traditional and one Mathematica implemented class. A total of fifty-five students participated. Each class had a different instructor; The traditional class was taught by a professor in the mathematics department, and the experimental class was taught by the investigator. Students enrolled in each section were used in the study. They were no told the nature of the experiment until after the enrollment was completed. The traditional section was in lecture format whereas the experimental section was in mostly discovery format; Students in the experimental group discovered definitions of basic abstract concepts mostly through visual-based Mathematica notebook demonstration, whereas the students in the traditional group were given the definitions. Data was collected through a background questionnaire, post questionnaire, pre-test scores, post-test scores, interviews and observation notes. This study discusses a variety of comparison between the traditional and the experimental classes. The data shed light on a range of differences in understanding basic linear algebra concepts. (Author)

A Comparison Study between a Traditional and Experimental Program

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Abstract

This paper is part of a dissertation defended in January 2001 as part of the author's Ph.D. requirement. The study investigated the effects of use of *Mathematica,* a computer algebra system, in learning basic linear algebra concepts. It was done by means of comparing two first year linear algebra classes, one traditional and one *Mathematica* implemented class. A total of fifty-five students participated. Each class had a different instructor; The traditional class was taught by a professor in the mathematics department, and the experimental class was taught by the investigator. Students enrolled in each section were used in the study. They were not told the nature of the experiment until after the enrollment was completed.

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1 INTRODUCTION

The purpose of this paper is to summarize some of the results of a study that investigated effect of use of *Mathematica* notebooks in a first year linear algebra class. This paper, which is part of a dissertation, addresses *Mathematica* notebooks used as part of the study and overall results on students' scores from the post-test and the post-questionnaire. The paper will start with a brief description of history of linear algebra and follow with a summary of studies addressing learning- and teaching-related difficulties students seem to be having in first year linear algebra classes. It will continue with a short description of the methodology used in the analysis of the data and then it will describe types of *Mathematica* notebooks used to enhance learning of basic linear algebra concepts. A summary of the results from post-questions and post-questionnaire along with a short conclusion will be the last section of the paper.

As a result of new advancements in technologies such as digital computers and the use of linear algebra in these technologies [9], linear algebra classes began to attract not only mathematics majors, but variety of students with different backgrounds and different majors such as economics, computer sciences and meteorology. The growing heterogeneity of linear algebra classes brought the question of how one can modify a "first linear algebra curriculum" so that it can respond to the needs of both mathematics and non-mathematics students.

The reform movement in undergraduate linear algebra courses started in a calculus-reform conference in Tulane [2],[3],[6] In the conference, a linear algebra study group was formed. In 1990, this group started working on a list of recommendations based on results of the surveys and questionnaires collected from faculty members in a variety of colleges, universities and client disciplines. Results of the surveys and questionnaires indicated a high demand from industry and client disciplines for making the first year linear algebra courses matrix-oriented courses. The group made the following recommendations:

1. The syllabus and presentation of the first course in linear algebra must respond to the needs of client disciplines.
2. Mathematics departments should seriously consider making their first course in linear algebra a matrix-oriented course

In addition to the recommendations, there have been a few studies attempting to investigate possible problems that occur due to the new structure of linear algebra classes. So far, the focus has been on possible correlation between the abstraction level of linear algebra concepts and students' learning difficulties. According to a study done by Dias, Artigue & Didirem [4], students seem to be having difficulties in recognizing different representations of the same concepts, which is defined as a level of abstraction by Dubinsky [5]. According to Dubinsky and Harel [6], students can achieve abstraction at this level if flexibility between the representations of the same concepts is established. They also indicated that abstraction can be established if concept images, defined as all mental pictures, properties and processes associated with the concept, and concept definitions, defined as a form of symbols used to specify the concept, are not contradicting each other. They suggested that if abstract definitions are introduced *visually*, it could help students have better mental images, and as a result better understandings.

Unfortunately, contrary to the expectations, there has not been any scientific study on testing effects of visual-instructions on learning and teaching of abstract concepts. This study, through the use of *Mathematica* notebook demonstrations, is one of the first studies attempting to test the possible effects.

2 METHOD

A comparison method was used for the study. Data was collected from two fall 1999 first year linear algebra classes taught at a mid-size research university. One of the courses was taught traditionally, and the other was taught in a computer laboratory with the use of *Mathematica* notebooks that were created based on two- and three-dimensional demonstrations of basic abstract linear algebra concepts.

In both classes, the same textbook was used. The same types of homework problems were assigned, and similar quizzes were given. The traditional class had three more students than the experimental class, which had twenty-six students. Data collection included a background questionnaire including a pre-test, in-class observations, recorded interviews with a few volunteers from both classes, a set of exam and quiz questions, as well as a post-questionnaire. Background questionnaire was collected to see whether the two classes had students with similar backgrounds. As part of the background questionnaire, a pre-test was given to see if students in both classes started the semester with similar required mathematics knowledge. To test possible differences due to the implementation, students' scores on five common problems from the exams, the final, and a quiz were used. In an attempt to have a better insight on students' responses, one interview was given during the last week of fall 1999 semester.

3 *MATHEMATICA* NOTEBOOKS

Mathematica notebooks were written as interactive, guided supplements to the lectures. They were mostly composed of interactive cells of examples and non-examples of basic linear algebra concepts. Emphases were given mostly to two- and three-dimensional demonstrations of basic vector space concepts. Each cell in a notebook was labeled as the example corresponding to the example discussed in class.

As the concepts were defined in class, and their formal definitions were written on the blackboard, the related examples on the interactive *Mathematica* cells were run by the students. Students discussed, through the visual demonstrations, the outcomes of the cells by comparing the characteristics of the demonstrated concepts and their formal definitions already stated on the board. As more of similar interactive cells with different examples and non-examples of the same concept were run, students were asked to write their interpretations in proceeding cells.

Furthermore, the students in the experimental group were asked to answer the concept-related questions through analyzing related visual-outputs as seen in the corresponding *Mathematica* cells. For example, in one particular *Mathematica* notebook on the concept; linear independence, the students were asked to solve (for the coefficients; a, b, and c) homogeneous equations of the following type: a **v** + b **w** + c **u** = **0**, based on the interactive cells whose outcomes showed the positions of the vectors **v**, **w**, and **u** in different colors. Numerical values of the vectors were, purposely, not given so that, students, to be able to solve the equation, would be restricted to the two-dimensional outcomes of these vectors. The purpose of this activity was to get students have better understanding of the formal definition of a linearly independent set. The textbook uses solution types to the homogeneous equations as part of the formal (abstract) definition. The formal definition stated as:

> "*A set of vectors $S=\{v_1, v_2, ..., v_k\}$ in a vector space V is called* ***linearly independent*** *if the vector equation*
> $c_1 v_1 + c_2 v_2 + ... + c_k v_k = 0$
> *has only the trivial solution, $c_1=0$, $c_2=0, ..., c_k=0$. If there are also nontrivial solutions, then S is called* ***linearly dependent.*** "

4 RESULTS AND DISCUSSION

Previous studies suggested that linear algebra students experience difficulties understanding abstract definitions of basic linear algebra concepts [8], [1]. The present study, by including a new learning style, went one step ahead and investigated students' understanding on questions similar to those used in previous studies. *Mathematica* was implemented in one of the sections compared in this study. Students' responses in each group were compared to test the effect of the implementation on students' understanding of basic linear algebra concepts.

The investigator found that the implementation of the technology helped ease some of the learning difficulties. It seemed to help students conceptually understand the abstract definitions better. The investigator also found that students in the experimental group made fewer definition-related errors than those in the traditional group. These students in the experimental group seemed to make better judgments based on abstract definitions whereas the students in the traditional group seemed to repeat what was memorized. These students in the traditional group also seemed to insist on using the results of theorems, and mostly recall them incorrectly.

From the interviews and the analysis of the post questions, the investigator found that even though students in the experimental group had better conceptual understanding of the basic concepts and definitions, they were not as good at items requiring procedural knowledge defined as knowledge of symbols and syntax of mathematics that implies only awareness of surface features, not a knowledge of meaning [7]. However, these students, compared to the students in the traditional group, did equally well on the procedural questions. Some students in the experimental group expressed that they would like to have little more time on learning procedures. They also indicated that they had to spend little more time to learn the procedures by themselves, which, students stated, was frustrating at times.

Use of correct terms among the experimental group seemed to be another issue that should be mentioned here. These students seemed to come up with their own terminology, and use them correctly. To the investigator, this result is not surprising. Since these students were mostly exposed to *Mathematica* notebook demonstrations of the basic definitions, it was expected that the students would adopt terms they saw happening in demonstrations. These terms seemed to stay with them longer than the book notations that were introduced afterward. Comparing both groups, letting students make their own terminology, as long as it is done correctly, seemed to be more helpful on understanding the concepts involved than asking students learn the terms as the related definitions are introduced.

The experimental group also indicated that the textbook used in this class did not go parallel with the *Mathematica* notebooks. If a right book, they stated, were used, *Mathematica* activities would be more helpful on learning the basic concepts. Both groups, however, expressed that homework assignments were harder than the examples given in class. The investigator thinks that this should not be considered as an effect of the implementation. Responses to the question on whether there should be a lab section for *Mathematica* activities were mixed. Half the class stated that activities should be covered during class time as the concepts are introduced, and the other half stated that having activities during class time was destructive so they should be covered in a laboratory environment where there is no lecturing involved.

Implementation of *Mathematica* (it should be noted that two different instructors taught the classes. Thus, this fact may have also had influence on students' motivation) seemed to have positive effect on students' motivation. More students in the experimental group indicated that they enjoyed the class than the number of traditional students with the same opinion. The experimental group thought that *Mathematica* activities were more helpful than lectures. The investigator does not find the result surprising since most of the learning was done through *Mathematica* activities, and lecturing was done at a minimum level.

The interviews indicated that *Mathematica* activities may have long-term effect on remembering basic concepts. The experimental group indicated that they would remember basic definitions in long-term (The investigator feels that this should be further investigated) whereas the traditional group could not remember the definitions during the interviews even though they had an exam the next day based on these definitions. These students indicated that they would, a night before the exam, sit down and memorize the definitions. It should be noted here that these students were mostly "B" students. Here is an outline of some of the indications the investigator observed.

- The notations and symbols do not seem to be affecting students' learning of the basic concepts, assuming the concepts are learned first, not their abstract definitions.
- Students in the group with technology implementation still seem to need little more in-class time on learning procedures.
- Over all, technology seems to have positive effect on learning concepts.

4.1 Future Research Questions

While this study addressed several issues in its area of concern, many issues remain to be addressed. These include:

- There is a need for further study with a change of the textbook to one that goes hand in hand with the implementation.
- Some of the questions used in the study did not seem to reflect students' conceptual understanding, hence there is a need to repeat the study with questions reflecting students' conceptual understanding better.
- There is a need to investigate the long-term effect of the implementation. The present study addressed only comparatively short-term effects.

5 CONCLUSION

The present study found evidence that the experimental group performed significantly better than the traditional group in tasks involving only conceptual knowledge defined as knowledge that is rich in relationships [7]. The most noticeable differences in understanding were found in applying basic vector space concepts ($p=0.04$) into linear transformations, also found in writing bases ($p=0.02$), by recognizing objects of the subspace as vectors, for subspaces. No significant evidence was found to support the belief that the experimental group performed less well than the traditional group in questions that required procedural knowledge or in questions that required both procedural and conceptual knowledge.

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