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ABSTRACT

The everyday mathematics processes of structural engineers were studied and analyzed in terms of abstraction. A main purpose of the study was to explore the degree to which the notion of a gap between school and everyday mathematics holds when the scope of practices considered "everyday" is extended. J. Lave (1988) promoted a methodology that treats person-in-activity as an integral whole. Lave's approach was used as the researcher conducted 70 hours of ethnographic observation of structural engineers in two firms as they went about their usual work. The four tasks observed involved multiple engineers of varying experience. Findings show that structural engineers practice in a world of quantities, units, procedures, and concepts, some of which exhibit concrete qualities, some of which appear more abstract, and some of which defy placement in either camp. Engineering expertise appears two-pronged, involving an increased amount of personal, concrete meaning associated with particular quantities and concepts, and at the same time, greater facility with abstract methods and theory. Findings support classroom methods that encourage students to construct their own meanings of mathematical concepts and quantities, and suggest that constructivism may describe the knowledge acquisition of adults no longer in classroom situations. (Contains 7 figures and 25 references.) (SLD)

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Abstraction and Concreteness in the Everyday Mathematics of Structural Engineers

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> *In school you get the theory and you think [a structure] should react in this direction, or this way. And then when you actually get to your job, you get a little bit of a better feel for the actual quantitative – yeah, it should react in this way but it should only go this much. And that's just the kind of learning experience that you pile on after the years.*
>
> – Carl, junior engineer, Firm 1

Overview

When the authors of the *Principles and Standards for School Mathematics* stress that, "The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater" (NCTM, 2000, p. 6), they reflect an international conviction that K-12 math education can and must play a central role in meeting that need. Many educators believe that "authentic learning experiences," mimicking what adults do in work and life, enhance students' learning of mathematical concepts as well as prepare students to be productive math users as adults in an increasingly mathematical world. Linking the math classroom to the world outside requires an understanding of the actual mathematical requirements of adults in their daily practices, and a growing body of research has been providing just that. Ironically, however, many "everyday mathematics" studies, and virtually all ethnographic ones (e.g., Lave, 1988; Scribner, 1984; Hall, 1999; Hutchins, 1995; Nunes, Schliemann, and Carraher, 1993) expose a striking contrast, or "gap," between mathematical behavior in out-of-school practices and that promoted by schools. While schools traditionally aim to teach formal, abstract, general techniques that derive their power by transcending situational and social factors, everyday math appears to be informal, situated (Greeno, 1997), reliant on contextual artifacts, culturally shaped, and locally invented.

Central to the arguments in these "gap-finding" studies is the abstract or concrete nature of the quantities, concepts, and methods involved in mathematical activity. Abstract knowledge is typically characterized as knowledge whose meaning is independent from any particular, real-world situations or artifacts, but which can be generally applied to many situations. Noss, Hoyles, and Pozzi (2002) define abstract knowledge as "lying in a separate realm from action, tools, language, or any external referential

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sign system [...], inside a system with its own objects and its own rules for transforming them" (p. 207). Davis and Hersh (1981) offer two meanings for abstraction: *idealization*, in which "all the accidentals and imperfections of the concrete instance have been miraculously eliminated" (p. 126), and *extraction* of the theoretical features common to a set of tangible objects. Everyday math researchers have identified, in particular, units involving ratios as abstract. For example, Carraher (1991) finds the profit per item purchased an "abstract notion" for Brazilian street sellers because it is "a derived quantity never directly encountered in their work" (p. 196). Similarly, Noss et al. (2002) consider the concept of concentration (the ratio of drug mass to solution volume) to be an abstraction in the practice of nursing because of its invariance across diverse dosage calculation problems.

Concreteness, usually positioned as the opposite of abstraction, generally refers to a close and direct correspondence with particular entities, not their class. Wilensky (1991) explains that it is natural for schools to want to move students "away from the confining world of the concrete, where they can only learn things about relatively few objects, to the more expansive world of the abstract, where what they learn will apply widely and generally" (p. 195). Despite the putative advantages of abstraction, some researchers have noted the tendency of "just plain folk" to avoid calculating with abstract units in everyday activity. Murtaugh (1985) found that grocery shoppers determining the best buy generally did not calculate the unit price, presumably because of its lack of direct relationship to the desired solution: "Shoppers apparently feel that it is not worth the effort to calculate the price per single ounce, when a single ounce is neither purchased nor consumed" (p. 192).

Besides units, solving processes have been separated in the research literature along abstract-concrete lines. Carraher and Schliemann (2002) found that, in school, students "commonly learn algorithms for manipulating numerical values without reference to physical quantities," while adults in the workplace use mathematics with "continuous reference to the situation and the physical quantities involved" (p. 135). The difference goes deeper than just the nature of the quantities used; the abstraction or generalizability of the methods themselves is considered. For decades, cognitive scientists have operated within the paradigm of knowledge "transfer" to explain how people know how to act in new situations. Transfer research has traditionally presumed the existence of cognitive mechanisms, such as general algorithms, analogies, isomorphism, and mental maps, that epitomize abstraction. According to transfer theory, people apply such decontextualized "tools" across divergent problem-solving situations; the tools remain unchanged by the process, retaining no feature of the situation (Lave, 1988). Lave criticizes transfer research for these presumptions as well as for its methodology, and she demonstrates through her own studies that everyday problem solving is inextricably grounded in its particular settings, with solution methods and ongoing activity shaping each other.

It is possible, however, that this dichotomous view, whereby school math is abstract and out-of-school math is concrete, is a byproduct of a limitation of the extant research. Most everyday-mathematics studies have examined practices known to involve only low-level math, or little math at all, for example, grocery shopping (Lave, 1988) and dairy factory work (Scribner, 1984)[2]. Investigating more mathematized practices might present a different picture of the abstraction or concreteness of quantities, methods, and concepts used by some people everyday. Such research, in fact, would hold more relevance for many math educators; at least by high school, math teaching aims beyond the arithmetic requirements of grocery shopping and factory work. Accordingly, I conducted an ethnographic study of the mathematical behavior of workers in a highly mathematized field: structural engineering. The texts, processes, and training programs of the engineering profession are rife with abstract quantities and units, idealized forms, and general theory, so engineering would seem to be as fertile a field for abstract thinking as any. In this paper, I explore the units, quantities, and problem-solving procedures that engineers actually use in practice, and I analyze these in terms of abstraction. Drawing on the conventional notions of abstract and concrete discussed above, I pay particular attention to the relationship of units and quantities to specific entities in the context; the engineers' efforts to make meaningful connections to physical phenomena; the use of general, decontextualized conceptual tools; and the influence of the setting on calculation methods. A main purpose of this inquiry is to explore the degree to which the notion of a gap between school and everyday math holds when the scope of practices considered "everyday" is extended.

My analysis resonates in some ways with prior findings about everyday mathematics. In my observations, the quantities with which the engineers calculate have meaning with respect to the current situation, and referring to that meaning can facilitate calculation and help steer it along a correct path. But implicit in earlier research is the notion that abstraction, generality, concreteness, and contextual meaning are static, inherent features of particular quantities, at least relative to their calculation settings, such that categorizing the quantities used in a practice along these lines serves as something of a litmus test for determining whether the mathematical behavior is school-like or everyday (e.g., Harris, 1991). My analysis leads me to a different interpretation, in which the degree of abstraction of a quantity is individual and dynamic. The following episodes will illustrate my contention that engineering experience includes the gradual, personal transformation of concepts from abstract to concrete as well as the ability to move fluidly between abstract and concrete perspectives. This interpretation weakens a key criterion for identifying a school-everyday math gap. Just as the level of formality of engineering calculations was

[2] An exception is the nursing study by Noss et al. (2002). While the calculations required of the nurses were not especially sophisticated, mainly using ratios, they were at least arguably in the realm of the early high school curriculum. Their findings support the notion that the gap metaphor might be less apt for more mathematized practices, as I discuss later in this paper.

seen in the previous chapter to be difficult or impossible to judge, so, too, is judging the level of abstraction of engineering quantities, concepts, and methods. Abstract and concrete might appropriately describe the elements of engineering when considered as entities in codes or textbooks. But I hope to show in this paper that, as these elements are used and understood by engineers in practice, abstract and concrete are far more relative terms.

Method

In everyday mathematics research, Jean Lave's *Cognition in Practice* (1988) is seminal. Lave promotes a methodology that treats "person-in-activity" as an integral whole. This analysis is part of a larger study that adopts Lave's methodology to examine the mathematical behavior of structural engineers. For this study, I conducted 70 hours of ethnographic observations of structural engineers in two firms as they went about their normal, everyday work. In order to increase my understanding of the contexts surrounding the engineers' mathematical activity – the overarching purposes and problems motivating it, background knowledge supporting it, and social and cultural influences – I took as my unit of analysis an extended work task. The four tasks I observed (two in each firm) were characteristic of design work and spanned several days each. They involved multiple engineers, both novice and expert, required the use of various technological tools, and presented a wide range of quantitative problems. Data collected included field notes, audiotape transcripts of nearly all dialogue among engineers or between engineers and me, copies of artifacts generated in practice (drawings, documents, calculation sheets, spreadsheets, etc.), and the engineers' written and oral answers to clarifying questions I posed afterwards (including 24 hours of interviews). While the study took a primarily etic perspective, particularly in the interpretation of what counted as mathematical behavior, I relied heavily on the engineers' explanations of their work and thinking during their activity as well as their later confirmations of my descriptions and understanding of their problem solving.

My analytic process involved multiple interpretive passes through the data. Initial passes were concerned with generating accurate accounts of the engineers' mathematical steps and my understanding of the engineering and mathematical nature of the problems and solutions. This early analysis enabled the construction of four cases – coherent accounts of the engineers' problem-solving activity within each major task that captured the social, cognitive, and resource contexts. Later passes, using the cases as the primary data form, aimed to distill themes and patterns relevant to the issue of the gap between school and everyday math. Qualitative analysis software was used to develop categories for the kind of math used and to code the cases for these categories. Larger themes and patterns were derived through repeated and comparative readings of the cases and close analysis of major problem-solving episodes in each task.

This paper presents the analysis of the data regarding one of the major themes identified in the larger study: abstraction and concreteness. First, I present a detailed interpretation of an extended vignette from one of the four cases. Next, I extend the analysis through shorter vignettes from the other cases and interview excerpts. Finally, I offer some implications for further research.

* * *

Vignette: Oldtown Parking Garage

Like many parking garages, the one that the engineers of Firm 1 are designing will be constructed from precast concrete elements, molded in a factory in a limited number of standard shapes. My observations occur early in the Oldtown project and focus on the analysis of the *lateral system*, the theoretical subset of the structure designed to resist lateral loads – in this project, a system of extra strong walls. Lateral analysis involves identifying physical features of the lateral system (e.g., the walls' locations, dimensions, and materials) and environmental features (e.g., wind and seismic loads expected in the region, soil type) and using these to calculate the lateral forces imposed on each member of the system. That data will subsequently inform the design of each member. Lateral analysis is something of a "bootstrapping" process: the engineer must make initial rough design assumptions in order to get started, then the design and analysis inform each other as each converges to a final state through repeated iterations. Outside factors, primarily frequent design changes from the architect, also impel and inform analysis iterations. Central to the lateral analysis is the calculation of the *base shear*, the force an external lateral load applies to the base of the building. *Base shear* is a function of the weight of the building and various environmental factors.

An unusual aspect of the Oldtown project is the use of multiple building codes. Firm 1's projects are usually subject to the 1997 Uniform Building Code (UBC)[3], the governing code for projects in the Western United States, with stringent requirements for seismic design. However, Oldtown is located in the east, and the official code is the Building Officials & Code Administrators (BOCA) National Building Code[4]. Meanwhile, the entire industry is slowly adopting the International Building Code (IBC)[5]. The formal documents and calculations for this project will obey the BOCA, but, anticipating the possible local adoption of the IBC mid-project, George, the owner of Firm 1, wants the lateral design to satisfy the IBC as well, to avoid redesign and reconstruction later to meet the IBC's higher seismic requirements. Also, because the junior engineers on the project, Carl and Francie, have never worked with either the IBC or BOCA, George wants them to find the UBC-required loads as a check on their BOCA and IBC

[3] International Conference of Building Officials, 1997.
[4] Building Officials and Code Administrators International, Inc., 1999.
[5] International Conference of Building Officials, 2000.

results. Firm 1 has a homegrown spreadsheet for finding the base shear according to formulas and factors from the UBC.

Except for the few rough, preliminary calculations George did before leaving town, the lateral analysis begins in earnest with this vignette. George has recruited Peter, a senior engineer from Firm 1's sister office, to come to town today to guide Carl, Francie, and Craig, the summer intern, through the use of the UBC-based spreadsheet to find the seismic base shear.

Concrete and abstract units

After entering project-identifying information into the spreadsheet, the team is prompted for the building weight, a value that requires some calculation. Most of the garage's weight is made up of "double T-beams": precast concrete elements integrating two beams with a wide flange above it, thus with a cross-section that looks like a double-stemmed T (Figure 1, p. 24). The 12'-wide top flanges interlock to form the floor. A *topped* T-beam has an extra 2" of concrete poured on top of the 2"-thick flange on the *untopped* version. Twelve-foot Ts are a new shape in the industry; until recently only 8'- and 10'-wide Ts were standard. The concrete manual that Firm 1 owns, the PCI Design Handbook[6], only lists specs for those older forms. In this first excerpt from the case, Peter leads the team through a rough weight calculation for the 12' T, based on the information they have for the 10' T.

Peter asks what a typical 12' T-beam weighs, and Carl goes for the PCI, though Peter reminds him that the 12' T won't be in it. Carl returns and opens the PCI to a spec table for a 10'x 32" T; Carl has guessed at the 32" depth. Out loud, Peter now proceeds to calculate a weight estimate for a 12' T. The PCI gives the beam weights per lineal foot, so users can calculate the weight of any length T. So some of Peter's calculations concern 1'-thick slices.

C: [Reading] Weight: 641 pounds per lineal foot, 64 pounds per square foot.
P: 64 pounds per square foot. Now if we add...is there a total weight? 64...641 pounds per lineal foot. And we're adding to that 2' of...This is going to be topped, right? And this is the topped weight [he looks at the PCI page; at this point he seems to notice and switch to the topped weight of 891 pounds per foot].

A 10' T is 10' wide, so there are 10 square feet in each 1' slice of beam. A rough approximation (that ignores the unequal distribution of concrete across the cross-section) is that each square foot would weigh 641/10 or about 64 psf. At first, Peter uses the weight given in the table for the untopped beam, 641 plf, but in the middle he switches to the topped weight of 891 plf. Peter now conceptually models a 12' T by adding a foot of extra flange to each side of a 10' T (Figure 2).

P: OK. So, I'm going to say 2 feet of 4" thick is...times...is 48, that's another 48 pounds per square foot. No, wait a minute, times [he pauses, then uses a calculator]. Is that right? Add a 4" slab, 96 pounds per foot, an extra 2' on top of the 891 pounds per foot? OK, so I'm going to...988 pounds per foot.
C: So 99 pounds per square foot?
P: Yeah, divided by 12...82 psf.

[6] Prestressed Concrete Inst, 1999.

The flange is 4" thick, and Peter uses a concrete weight of 12 pounds per inch per square foot, so a 4"-thick concrete slab weighs 48 pounds per square foot (psf). To each lineal foot slice of the T, then, Peter adds 2 sq. feet of slab, one on each side, at 48 pounds each. Now he must add this extra 96 pounds to the weight of a lineal foot of the beam: 96 + 891 = 987 pounds. Finally, he finds a psf weight for the 12' T by taking the weight of the lineal foot slice (which he for some reason calls 988 plf), and dividing it by the 12 square feet that are in the slice, to get 82 psf. Although this average poorly represents any single square foot of the T because of its irregular cross-section, it will work in the calculation of the total floor weight, typically found per square foot, since every square foot of the T-beam will be included. At first Carl and Peter are surprised by the fact that the weight of the 12' T is lower than that of the 10' T. Then they realize that this is not the total weight of a T-beam but of a square foot of the beam. This should be lower for the 12' T, since, at least as Peter has modeled it, the 12' T is not a proportionally scaled-up version of the 10' T but just a 10' T with extra flange – the lighter part of the beam.

It seems almost trivial (not to mention a bad pun) to point out that many of the quantities Peter and Carl use in this episode are concrete. Outside of pure mathematics, any practice will need to employ quantities that measure actual entities in the setting in order for math to be of any use in resolving quantitative problems. Here, the values of 12' (the width of the T) and 4" (its flange thickness) correspond fairly literally to specific physical phenomena, though not quite as directly as do the size of a box of noodles described by Lave or the numbers of bottles in a crate described by Scribner; unlike those situations, Peter and Carl cannot see or touch the actual T-beam. But the issue of concreteness becomes grayer with the values 641 pounds per lineal foot (plf) and 64 pounds per square foot (psf). These quantities still make reference to an obvious phenomenon (weight) associated with particular physical entities (T-beams) and as such seem much more concrete than abstract. Arguably, however, they move a step towards abstraction by introducing a ratio in their units. While an inch (as in the 4" thickness value) is simple to picture, a pound per lineal foot is not so easily tied to a concrete artifact or image. In fact, the units of psf and plf seem conceptually equivalent to the ratio units that Carraher and Murtaugh, cited in this paper's introduction, used to exemplify abstraction. This suggests it is more useful to consider abstraction a matter of degree. After all, it is possible to picture a lineal foot of a T-beam: one can imagine the beam being cut, like bread, into 1'-thick slices. Peter may be picturing a lineal-foot slice of beam when he mentally models a 12' T as a 10' T with its flange extended by a foot on each side. Since he carries out this weight calculation by lineal foot, a mental image of a 1' slice of beam may help him keep his work in the appropriate units.

There is no reason to presume, however, that engineers depend on concrete mental images of the units with which they calculate, particularly ones as common as plf and psf. In manuals and in calculations, most weight specifications for beams and columns are expressed per unit length or per unit area, since the lengths and areas of these elements are variable and determined by the project. In my observations, the engineers usually operate with these ratio units with ease and familiarity. When they do encounter confusion, they rely on dimensional analysis to keep their calculations straight. Once units become as complex as pounds per inch per square foot, the unit on the concrete weight that Peter uses, it would be surprising indeed if the engineers bothered to try to conjure up a mental image of them in the course of calculation (though it can be done). The fact that Peter and Carl always speak the full unit names on each quantity, when in most other respects their dialogue is informal and vague, suggests that dimensional analysis is an important tool for guiding calculation steps where reference to physical meaning cannot, and that its use is anticipated.

Though Peter and Carl are comfortable working with quantities and units that they may not directly relate to concrete artifacts or images, it is important to note that they do not stay in the abstract realm for long. Exhibiting behavior I see throughout my observations, Peter and Carl do not go far in their calculation before attempting to make a direct connection to the real situation. At the end of the above excerpt, when they arrive at a weight for the 12' T, they try to assess its reasonableness by comparing it to the weight of the 10' T. Here, they become temporarily confused by the abstraction of their quantity. Their initial surprise that the 12' T would weigh less than the 10' T indicates that, fleetingly and perhaps unconsciously, they may have mentally simplified the psf unit to pounds in order to tie their result to a more tangible physical phenomenon. But they quickly remember they are working in ratio units and realize that the way they have modeled the 12' T would indeed make it lighter, per square foot, than the 10' T. This instance also demonstrates that tying results to concrete phenomena – in this case absolute weight – in order to judge reasonableness or make sense can be of limited benefit. The per-square-foot weight of a T-beam has little concrete, practical meaning because the square-foot chunks in a slice of T-beam have widely varying shapes and, therefore, absolute weights. The chunks that include the stems of the T are far deeper and heavier than those out in the flange, and probably no actual square foot chunk has the same weight as the psf weight of the slice (Figure 3). Attempting to connect to a concrete image of the psf weight momentarily hinders Peter and Carl's effort to judge their result.

Guidance by an abstract, general concept

Having found a psf weight for the T-beams, the team continues the weight calculation for the entire garage and then checks it against another calculation that George, the firm's owner, had calculated before leaving town. This excerpt and the previous one make clear that structural engineering, unlike many practices previously studied, can involve a considerable amount of calculation activity. What is not always clear, however, is how the engineers know what calculations to perform; the team does not follow a set of steps in a text and many calculation decisions are open. This excerpt illustrates one means of guidance: an overarching, abstract concept.

Using dimensions from the architect's drawing, Carl calculates the square footage of a floor – 434 by 242 or about 105,000 sq' – then multiplies by the 82 psf weight they have just calculated for the T-beams, gets 8610 *kips*[7], and exclaims, "Wow!" Peter estimates the remaining weight of a story, to take the columns and walls into account along with the T-beams:

P: What do you think for the columns and wall panels? Just for, lemme just throw in another 10%, and you guys'll have to go back and make that right. So why don't you say times 1.1 for walls and columns?

Peter multiplies 8610k by 1.1 and gets approximately 9500k for the story weight. Immediately, he wants to check this figure against a value that George had calculated days ago: 340k for the load on each foundation column. Peter and Carl make out a 5-by-12 array of columns in the drawing. Peter calculates the load on each column by first multiplying the story weight by 4 for the number of floors, then dividing by 60 columns. This gives (9500)(4)/(60) or 633k per column, almost twice George's 340k.

Now, they systematically try to reduce the discrepancy. First, Carl remarks that "George was spouting out numbers" and might have taken different loads into account. Peter reworks his calculation but gets the same result. Carl looks at the drawing and finds 6 additional columns outlying the 5-by-12 array. Carl then asks Peter to explain his calculation. As Peter does so, he realizes that there are only three, not four, stories actually supported by columns, since the lowest floor rests on the ground. Peter revises the calculation for three stories and gets (9500)(3)/(60) or 475k. Carl changes this calculation to reflect 66 columns, which reduces the load to 430k. Finally, they decide to subtract off the weight of the holes that will be cut in the floors at the top of each auto ramp. Peter ascertains from the drawings that these holes are 11,000 sq.'. Carl multiplies this area by the 82 psf floor weight and reports they can subtract roughly 1000k per story. Carl completes the calculation, first reducing each story weight from 9500k to 8500k, next multiplying by 3 stories, and finally dividing by 66 columns to get 386k per column. They accept this as close enough to George's estimate to justify their weight calculation.

In this excerpt, Carl and Peter apparently won't feel comfortable with their own weight calculation until it comes close to the one George had calculated days earlier. The iterative process by which they close in on their target is typical of the engineers' mathematical behavior. Non-engineers might read this excerpt with horror, seeing evidence that engineers decide almost capriciously what factors to include in their calculations. After all, if there are in fact ramp holes in the floors, shouldn't Peter and Carl have subtracted them right off the bat? But there is method to their process. Peter and Carl are guided by *conservatism*, a key theoretical concept that pervades the work I observed (and the language; this is an engineering term, not my own). To understand a value or procedure as conservative is to attribute to it a broadly general and widely applicable quality, defined informally as "safe." To be conservative, a value must be larger or smaller than some critical value (perhaps given by the code), below or above which, respectively, failure (or code violation) occurs. Similarly, a conservative method relies on assumptions expected to produce conservative values. Besides ensuring physical safety, conservatism is something of a trick that makes much of structural design work possible, by narrowing down the vast number of potential solutions or by enabling approximation where exact solution would be impossible. In the excerpt above, the team cannot possibly calculate the exact weight of a story; they neither have enough information about the building (for one thing, they lack the actual weight of a 12' T), nor can they take into account every detail (for example, the weight of every bolt). But an exact building

[7] One *kip* equals 1,000 pounds.

weight is not needed, only one that is larger, or more conservative, than the exact weight. In so far as the team's task is to design a building that will withstand a code-specified seismic demand, any building whose capacity (strength) exceeds that demand is acceptable. The more a building weighs, the greater the base shear (i.e., the greater the force imposed by a given level of seismic demand) and, in response, the more capacity the engineers must design in. Overestimating a building's weight, therefore, is safe. Nearly all quantitative design problems take advantage of the idea of conservativism. Mathematically speaking, structural engineers usually solve inequalities.[8]

Consequently, Peter and Carl's process is not at all capricious. They begin with a very conservative (heavy) weight calculation, ignoring holes in the floor and, when finding a per-column load, ignoring the six odd columns outside the regular array. These omissions simply save time and, had they led to a weight value the team could feel good about, the team would have stayed with that value during this early phase of analysis. But because the team deems the value too big, based on George's estimate, they take steps to reduce it, being less conservative each time but only in ways they can justify: there really will be 66 columns, and concrete really will be cut out at the top of the ramps. This exemplifies one of central tensions of structural design: safety versus cost. The team does not simply stick with its first weight calculation even though it offers a very comfortable safety margin. Peter and the others know that the high cost of the extra-strong lateral system they would have to design as a result of an overconservative weight estimation would surely dissatisfy the owner and architect. Structural engineers usually try to reduce their margin of conservatism, while making sure to preserve some degree of it. Operating in this fashion means the engineers must apply the concept of conservatism to nearly every procedure and type of element in the trade. In this light, the concept of conservatism is highly abstract and widely generalizable, invariant among rather than specific to particular situations, building structures, or calculation processes, and reminiscent of (but even more general than) nurses' concept of drug concentration. Applying conservatism to a calculation, of course, requires the engineer to attend to the particular features of the problem, as he attempts to maintain a constant sense of the direction in which design values lie relative to the critical value and of the potential quantitative effects of changing certain factors. This skill seems to map in a general sense on to the school-math topic of understanding functions (in this particular situation, mostly involving proportional relationships). For example, Carl knows that increasing the number of columns would reduce the weight on each, and because of this knowledge he knows to look for extra columns in the drawing that they might have missed.

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[8] Despite this fact, the engineers' written calculations rarely use the symbols > or <, except when stating a given limit from the code (e.g., a formula that Carl uses later to find the upper limit on the base shear, $V \geq 0.44\ S_{DS} I_E W$). More typically, calculations for a design value, such as base shear, or an analytic value, such as deflection, are carried out as equalities. Rather than calculate with inequalities throughout the problem, as school texts would have them do, the engineers work with equalities, but maintain a sense of whether the variable they are evaluating be greater or less than the numerical result they find.

Making sense of abstract quantities

Having settled on a weight approximation for the building and entered it into the spreadsheet, the team moves on. The next box in the spreadsheet is called "1997 UBC Factors," and the first prompt asks for r_{max}, the *redundancy factor*, which quantifies the redundancy of the walls.

Peter decrees r_{max} an important but possibly problematic calculation. He asks for the UBC codebook and Carl looks up the factor. Later, I see that the UBC says r_{max} is the maximum of the element-story shear ratios, r_i, calculated on each story. r_i is the ratio of the shear force in the most heavily loaded wall to the total shear for the story. The UBC says to find this by taking the maximum value of (*wall shear*)(10/*wall length*)/(*total story shear*). Peter counts 8 north-south shear walls on a typical floor of the garage, but he does not know the total story shear, as this is a function of the total base shear that the team is currently trying to calculate. He gets around this problem by assuming that each wall will be equally loaded. With 8 walls in the N-S direction, each takes 1/8, or .125 of the total story shear. This gives him the (*wall shear*)/(*total story shear*) ratio. Now he only needs to multiply this ratio by (10/*wall length*). He does not explain his shortcut to the team, just calculates out loud with Carl:

P: So we decided we're going to have 2, 4, 6, 8 walls...
C: Shear walls...
P: Right? So that's .125, and then there's a, 10...
C: 10 over the length of the wall...
P: 10 over the length of the wall... 10 times, and then I divide by whatever the length of that wall is.

After estimating from the architect's drawing that all walls are about 15' in length, Peter uses his calculator to find r_{max}=(.125)(10/15) or .083.

The formula for r_{max} is one of the least transparent I observe in use. The meaning of its factors and their contributions to the result are not nearly as obvious as in most other structural engineering formulas. Peter clearly understands two of the factors – wall shear and total story shear – well enough to replace their quotient with a third quantity – the reciprocal of the number of walls. This transformation requires knowledge about these theoretical concepts as well as knowledge about the particulars of this building. As Peter describes this replacement later: "[This] would only be a good assumption if all the walls were the same length and height, and uniformly arranged around the center of rigidity[9]." It is not apparent in the above excerpt, however, what sense Peter makes, if any, of the remaining factor, (10/*wall length*), since he carries out its calculation unproblematically. In a follow-up e-mail, I probe his understanding of this factor, and Peter replies:

> In my mind, this factor increases the importance of redundancy for walls less than or near 10 feet in length, and reduces the importance of redundancy for walls much longer than 10 feet. Not a bad concept, but I have no idea where 10 came from. I do not know the research upon which it was based. Consequently, I interpret it as a nearly arbitrary, empirical factor, invented during a code committee meeting by cigar-smoking old engineers.

[9] The center of rigidity is the theoretical "center" of the building's resistance to lateral load, and is a function of the locations and stiffnesses of the walls in the lateral system.

At some point in his career, Peter has dissected the r_{max} formula and made a significant amount of sense of its parts. In his conceptualization of this formula (or one of his conceptualizations, tailored for today's calculation), he isolates the factor (10/*wall length*) and envisions it as a scalar operating on some redundancy measure composed of the other factors. He knows the mathematical behavior of this scalar fraction as its denominator decreases or increases from 10, which allows him to predict the effect of those different denominators on the redundancy. Specifically, he understands that 10' walls have no effect on the redundancy, walls shorter than 10' will reduce the redundancy, and longer walls will increase it. Peter's efforts to make sense of this formula extend to its most opaque part: the mysterious, dimensionless 10. Despite his claim that he has "no idea where 10 came from," he fills in a story that sounds as plausible as it is humorous.

The spreadsheet uses r_{max} and the floor area to automatically calculate the next factor, ρ. According to the UBC, ρ is the Reliability/ Redundancy Factor, given as $\rho = 2 - [20/(r_{max}\sqrt{A_b})]$. This factor measures the system's capacity to survive the failure of walls based on the presence of redundant walls, i.e., a back-up system. It is a function of r_{max} as well as A_b, the ground-floor area of the building. A ρ of 1 is considered a high degree of redundancy (good) and the target for design, with ρ of 1.5 or more considered unacceptable. In the next excerpt, the team reacts to the ρ value that results from having just inputted an r_{max} of .083.

When the team sees the value 1.26 appear for ρ, Peter says, "Whoa!" and announces that they're "busted" for having too many little walls. Carl wonders how many walls they'd need to add to get ρ "back to 1." But Peter says the real question is, "Is that damn factor in the IBC or the BOCA?" Craig goes off to hunt for the BOCA codebook as Peter and Carl move on to the next two prompts, the Importance and Zone Factors, both of which they know off the tops of their heads to be 1. Once entered, these factors cause the spreadsheet to go back and automatically change ρ to 1 (I read in the UBC that in Seismic Zones 0, 1,and 2, ρ should simply be set to 1.) Craig's search for the BOCA is moot, now that ρ has been brought down to an acceptable level.

Based on its formula, ρ appears highly arcane and abstract. Like r_{max}, it is dimensionless, and its formula makes even less reference to obvious structural features. Yet the team has enough familiarity with ρ to feel immediately unsettled about attaining a result of $\rho = 1.26$. Peter knows they want ρ to be close to 1.00 and that values greater than 1.00 indicate decreasing redundancy in the system. I presume he also knows what I discover later about the spreadsheet: in the final step, the base shear is scaled up by ρ, forcing the design of a stronger building overall for ρ-values greater than 1, as a penalty for the lack of redundancy. Peter even knows which actual feature of the garage's design accounts for the high ρ value: not enough sizable walls. What is arguably the most abstract quantity in the base-shear calculation, therefore, still seems to hold a significant amount of concrete meaning for Peter and the others. Initially, Peter's understanding of ρ seems to be more general, not specific to this building, namely that it measures the capacity of the building to survive the failure of some elements and that the more it exceeds 1.00 the worse. Later, however, Peter makes situation-specific sense of ρ when he interprets its resulting value in terms of physical characteristics of this particular building.

Having entered all required input, the team now looks at the results in the spreadsheet box called Results Summary.

The first result is the base shear, 2,202k, and the team immediately tries to assess the adequacy of the current walls to resist this demand. Peter divides to find the shear force on each of the 8 walls; he gets 275k. Using 15' as the length of each wall, he divides again to find that each foot of wall, longitudinally, would have to carry 18.3k of lateral force. He continues on to find the force on each square inch of the wall's footprint. He and Carl guess that the walls will be 8" thick (the architect has not specified), and they divide the 18.3k per lineal foot by 12" per foot and by the 8" thickness to find .191 ksi or 191 psi. This is a value Peter can insert into a memorized concrete-strength formula: the required strength of the wall is $\alpha\sqrt{f'c}$, where $f'c$ is the strength of the concrete. Peter wants to find α, a factor that will tell him if his walls are adequate or if more, longer, or reinforced walls will be required. Peter assumes a concrete strength of 4000 psi and performs the calculation: $\alpha = 191/\sqrt{4000} = 3$. He reports this result to the others, and Carl announces what his teammates presumably already know: an α over 2 means reinforcement is required.

In this last excerpt, the team continues to demonstrate facility with units and dimensions. One way Peter can judge the reasonableness of the overall base shear is to see if the concrete in the walls is strong enough to handle it, and concrete strength is conventionally expressed in per-square-inch units. Starting with the total building's shear force (2,202k), Peter quickly proceeds through multiple conversion steps to attain the force on a single square inch of the base of a wall. The intermediate results generated during the calculation, such as 18.3k of load on a wall or .191 ksi, appear to hold no concrete meaning for Peter. He has no overt reaction to any of these as he mumbles them and he gives no indication of stopping to try to judge or make sense of them. Only the final result, $\alpha = 3$, seems to allow the team to reconnect with physical meaning. Because α exceeds 2, the concrete must be reinforced with steel. Here again, an abstract quantity has become, over time, quite meaningful in a concrete way for these engineers. Alpha has no units and no formula to define it, unless one counts the established formula explicitly solved for required strength ($V_c = \alpha\sqrt{f'c}$), usually used when α is known. According to this formula, α is a factor that expresses the scalar difference between the strength needed in the structure and the square root of the strength that the concrete alone can provide – not a particularly helpful description in which to ground one's sense making about α. Rather, α has probably gained meaning for the engineers over time and through experience, as they have played out the structural implications of various α values and developed a sense of what constitutes a normal, feasible range.

* * *

The Dynamism of Abstraction

In finding the base shear of the Oldtown garage, Peter and the team demonstrate the dynamism of the degree of abstraction of the quantitative concepts used in structural engineering work. In my observations, I found different forms of movement of concepts along an abstract-concrete spectrum. In the remainder of this paper, I describe three kinds of conceptual movement:

- from abstract to concrete
- fluidly between abstract and concrete
- from both abstract and concrete poles towards the center.

From abstract to concrete

The Oldtown vignette confirms the conventional wisdom: structural engineers work with many quantities, units, and formulas that, at least on paper, would be characterized as highly abstract. At the same time, however, these abstractions have in a sense become quite concrete for the engineers. Some nominally abstract factors conjure up images of specific structural causes or implications; some prompt nearly emotional responses when they exceed certain values. In the process of their work over time, engineers appear to engage, sometimes quite actively, in continually assigning meaning to all factors, regardless of how broadly these factors apply or how abstract their definitions or formulas for derivation.

The following excerpt offers a glimpse into this meaning-making process. It occurs a few days after the opening vignette, as Carl hand calculates the Oldtown base shear by the IBC, a code he has never used before. Following a chain of formulas in the codebook, Carl, at one point, is directed to look in tables to find two values, F_a and F_v, to which he refers as "amplification factors."

Carl looks up the amplification factors and remarks that one is "pretty big," implying some knowledge of these variables despite the fact that he has never seen them before. When I ask about this, he replies:

C: ...Basically, if you look at the equation, there're a whole bunch of factors. These are all modifications to how much percentage of the base shear you're gonna take. And so some...in the UBC [the code with which he is familiar], they have something called C_a and C_v, which are near-field effects for faults and it's kind of like F_a and F_v. C_a and C_v. This [referring to F_a] is acceleration-based, and this [referring to F_v] is, um, this is probably factor...acceleration-based factor, velocity-based, and that's just...They have something similar in the UBC...But if you look, you get penalized for soft soil 'cause you get E, and it goes way up.

He points to a column in a table showing that, from Site Class A to E (having to do with soil type), the associated F_a values rise. I ask whether his judgment that one of the factors was "big" was based on a comparison with the other values in the table or on his knowledge of the UBC.

C: It's relative to the other values here, and I also know from my schooling that soft soil is bad. And, it's actually not bad as much as it is...If you put the improper structure on soft soil you'll hit resonance and everything will fall apart.

Me: So you weren't surprised by what you just saw?

C: No. They want to guard against that, so they're going to make you overdesign.

Carl has never seen the factors F_a and F_v before, yet he immediately assigns some meaning to them through association with similar factors from the code with which he has much experience. Though the range of values of F_a and F_v differs from that of C_a and C_v, simply making the connection that they are all measures of seismic acceleration tells Carl that F_a and F_v will range from low, for less serious seismic conditions, to high, and that, consequently, soft soil will be associated with high F values. Notably, Carl does not just blindly substitute F values from the table into the appropriate formula with no attempt to understand their meaning, although the calculation would be possible if he did. In addition, Carl reveals that over time he has developed a fairly deep understanding of the UBC counterparts, C_a and C_v. He understands these factors' role in the intermediate formulas and ultimately in the base shear, and he can even articulate the code-writers' rationale for setting their values. This kind of meaning making would be accomplished through experience with calculations, a process Carl demonstrates here in its early stages as he tries to learn about the new factors. But here and in the next excerpt, Carl also implicates school as a source of his understanding.

After his calculation is complete, I probe Carl's understanding of other new IBC factors. I ask what he might have known previously about the S factors that appear in intermediate formulas with F_a and F_v:

C: It's kind of complicated 'cause I know where the theory of this is derived from because I took it for my Masters and they show you the *response spectrum*[10] that they're using. And I know what...Basically, I'm not sure what they're assigning values to 'cause they say the S_{DS} is 2/3 of the S_S times F_a. [He jumps a step here: his calc sheet and the IBC say $S_{DS} = 2/3\ S_{MS}$ and $S_{MS} = F_a\ S_S$. He seems to have memorized the algebraic link during his calculation.] I know the F_a is a factor of acceleration; I know the S_S is probably the response spectrum, could be displacement, could be...it's most likely acceleration based. Basically it's a portion of the curve...However, I couldn't tell you for certain that's what they mean, you know? That means that S_{DS} is a made-up number. It just a made-up empirical number...

Me: ...that maybe UBC doesn't use, necessarily?

C: ...that they had factor, yeah, and, like I said, I've done in my Masters...UBC has something like it, in '97, and it's for a dynamic analysis, you can see the response spectrum. So I know where they come from but I can't say for sure that that's exactly what they're doing because I don't know where they got the 2.5 [the value for F_a]. It's a near-fault factor and they've calculated somewhere that, through testing and whatnot, that this is the number we're gonna use.

Me: So it's a product of two concepts that you're very familiar with, but it's sort of their own synthesis?

C: For their actual numbers, you know, their little peak acceleration graph, you know, I don't know for sure that that is the peak ground acceleration at that point. These could all be empirical things they've just come up with. Which is what the code is, basically. It's an empirical way to try to rationalize earthquakes, and, you know, that's why it changes every time there's a major quake, because they decide that it wasn't good enough the last time, they're going to change it.

Though Carl cannot explain how specific values were generated for the F and S factors, he has a deep grasp of what physical phenomena these factors measure, the general source of their derivation (empirical tests), and even the social process by which their values are officially revised. Again, he arrives at this understanding of factors he has never seen before by relating them to familiar factors, and

[10] The *response spectrum* is a code-provided plot of seismic ground motion versus the building's oscillating period. It supplies factors, such as the C and F factors discussed here, used to determine the base shear. The response spectrum reflects the geologic, tectonic, seismological, and soil characteristics of the building site.

he attributes much of what he knows about those familiar factors to theory he was taught in school, specifically the general concept of a response spectrum. Clearly experience also plays a role in an engineer's construction of meaning for a factor; in the process of his calculation, Carl has taken the first step in building up his understanding F_a and F_v. Most importantly, this episode illustrates the dynamic nature, in terms of abstraction, of the quantities used in engineering work – the result of the engineers' personal transformation of initially abstract quantities into ones that hold concrete meanings. During my time in these two firms, I came to see this transformation as a hallmark of engineering expertise. The more experience an engineer has with certain factors, the more meaning the factors accrue and the more fluently and the more ways he can predict their practical implications. Peter's conceptualization of the factors comprising r_{max} is a case in point.

Fluidly between concrete and abstract

Despite the engineers' active and continual efforts to make abstract concepts meaningful, it is not the case that, over the course of an engineer's experience, concepts only travel down a one-way street from abstract to concrete or from general to particular. Expertise at the same time appears to include an enhanced ability to recognize the applicability of methods and concepts in new situations. During interactions between junior and senior colleagues, I repeatedly observed the same pattern: the senior engineer would suggest the use of a general method or theoretical concept the junior had not considered (this occurs in both of this paper's remaining examples). Furthermore, denotations of abstract and concrete are confounded by the fact that in an important sense all artifacts in structural design work are abstract. The entire design process operates on symbolic, idealized representations of physical elements and structures that are inaccessible to the engineer and usually don't even exist. At best, the engineer can tie abstract factors or concepts to other (possibly less) abstract factors or concepts, such as drawings or mental pictures. During an interview about a different project of Firm 1's, George offers some insight into his mental representation of theoretical concepts. The fuzziness between real and symbolic artifacts is apparent in his explanation. So, too, is evidence against the notion that quantities and concepts might evolve irreversibly from more to less abstract, or vice versa.

My interview with George concerns the design and analysis of a single large beam, which had become the major issue in the task I had observed. When Susan, a junior engineer, and George had analyzed this beam, three variables had been central. V conventionally represents the major concept of *shear* – the tendency of two parts of an element to slide past each other. When a beam deflects by sagging in the middle, two horizontal forces develop internally: compression (C) and tension (T) (Figure 4). Since V also refers to a horizontal force in the beam, and because I had observed George and Susan frequently setting C or T equal to V in their calculations, I had misinterpreted C and T as specific instances of *V*. Later, in this interview, George corrects me and explains that *V* is a distinct behavioral phenomenon from *C* and *T*. They had only been set equal as a calculation strategy, to express that the system was in equilibrium. George emphasizes the difference between the concepts:

> There are behaviors associated with each of these letters you have there, and when I do engineering I think first about the beam, about the elements, and I know how it's going to behave. Then I can control that behavior or prevent that behavior. That's how I do engineering, is by thinking about them on a fundamental level of their behavior – first principles, we call it. And so all of these things have slots in my mind. When you say *V*, that's a behavior, right? That's a slipping behavior.

Here, George appears to think and speak about theoretical and concrete concepts simultaneously. "First principles" – a handful of fundamental laws expressing relationships among space, time, mass, and force that include Newtown's first three laws – are the most general and fundamental concepts in structural engineering, the ultimate theoretical source of virtually every structural analysis method (Beer and Johnston, 1996). Yet George associates them with the behavior of the specific beam in the project, suggesting that his understanding of the concept of *V* depends, at once, on highly abstract and highly concrete ideas. I ask George if he pictures *V*, *C*, and *T*:

> Absolutely. I can't get away from it. It is what it is, so when you say *V*, I can't see anything else. There's no way that I can imagine *V* being *T* or *C*. [...] The picture is things sliding...I go so far...I have this arrow, just like that, opposite directions [he draws two parallel arrows pointing opposite directions (Figure 5)]. Things sliding relative...This is the slip plane, that's why the arrow only has half a head, 'cause it's sliding...and I actually have that in my AutoCAD database as an element. That's a real element to me. That's something I use as a thought process and I have this little guy [he draws a curved arrow] that's a *moment*.[11]

Again, George seems to think at once in both real and abstract terms, as he describes his mental representation of *V* as "things sliding" and as a simple diagram, which he goes on to call "a real element to me." According to George, both images – one of actual elements in motion and one comprising drawn symbols – prevent him from confusing *V* with *C* or *T*. During a calculation for this project, George had coached Susan to apply the principle of equilibrium and set *C* equal to *T*. Recalling this episode, and suspecting that Susan may not be as facile as George with these concepts, I now ask George what he thinks Susan pictures. He laughs and says, "I don't know what Susan pictures. Something different! Because we always have to negotiate a connection when we're talking about...!" I ask why he thinks they imagine different pictures, and he replies:

> All of these come to play in the educational process, OK? And when you're going through...being taught engineering, you get diagrams that are basically distributions on sections of members, and forces that are balancing each other, and you get equations. I mean, all this beam stuff, you know, if you're a mathematician, you'd probably be really comfortable if you looked at the fourth derivative of load is deflection, and everything could be done in derivatives and integrals without nary a picture or an arrow or

[11] The *moment* about a given point is the product of a force acting at a distance from the point and that distance, thus with units of, for example, kip-feet. The distance is called the *moment arm*.

a V or anything else. And so there are just a lot of different languages for just describing the same thing. But the language that appeals to me is the picture language, and the subset of the picture language that I think in is the arrows and the diagrams of the things in equilibrium.

Daniel, George's counterpart at Firm 2, also prefers to visualize. Like George, Daniel recognizes math as another form of representation, one more accessible to people more mathematically trained than he.

> I happen to do this visually. There's a professor at [university] who I met and saw and he could do it mathematically. So he could see even...he could see higher, I mean, things that I could never see. [...] I remember him describing *base isolation*[12] -- and it was a base isolation course – and writing this very kind of [summary] equation down, and I think...I don't know, it was an integral or something like that. And he would point to a part of it and he would say, "That's...there you can do. You can see how base isolation works." And he's pointing out part of an equation. And he actually could see how it works. [...] So, he's seeing with math, and I'm seeing with visualization.

When I ask George if he thinks his junior engineers should picture concepts as he does, he admits, "Well, actually I probably am saying that, because people who understand the mathematics tend to make big mistakes because they don't think about the real world." I challenge his implication that his arrows are the real world and elicit this response:

> Well, no, I will grant you that that is not a piece of concrete and a piece of steel. That's beauty of it, because, you know, when I said that, three images of the real world pop into my mind of where those two arrows apply. One is at the center of a wood beam...a rectangular wood beam. And I know instantaneously that the value of the arrow at the centroid of that beam is 1½ times the average over the cross section. You know, another one of those things from mechanics of materials. And that's because it's the integral of the stress in the area above that. And, you know, if you do all those integrals, you find out it's 1½ at the centroid.

George understands that the symbols, pictures, and equations engineers learn in school are merely different means of describing concepts like V, C, and T, and he suspects that different people are comfortable with different mental representations, depending on their training. But George also believes that, for structural analysis, mental pictures like his "arrow and diagrams of the things in equilibrium" are the most effective forms of representation because they force him to "think about the real world." Though the actual beams and studs are unavailable to be directly experienced, George feels that his mental arrows facilitate connections with real physical behavior and therefore help prevent calculation errors. George's self-described system of representation apparently supports a form of understanding of these engineering concepts that permits fluidity between abstraction and concreteness. This fluidity enables widespread application of these concepts and, at the same time, immediate grounding in particular situations.

[12] Base isolation is a design technique for seismic resistance that decouples or isolates the motion of the foundation from the rest of the building, which can then remain relatively still. This is sometimes accomplished by placing a layer of natural or synthetic rubber between the structure and its foundation.

From both abstract and concrete poles towards the center

In the above discussion with George, equilibrium emerges as another abstract principle useful for guiding calculations, much like conservatism. The frequent occurrence of non-routine calculation methods that I witnessed in structural engineering work raises the question of how the engineers are able to generate them. My observations revealed that an important resource –though not the only one – for deciding how to proceed in uncertain situations is the corpus of abstract, general concepts or principles, with which all the engineers were deeply familiar. Besides *conservativism*[13] and *equilibrium*, such theoretical concepts include *moment*, *redundancy*, *oscillating mode*, and *base shear*. A final excerpt illustrates how the principle of equilibrium, as well as some basic concepts from statics theory, guides two Firm 2 engineers through the creation of an original calculation method.

Larry, under the guidance of his supervisor, Daniel, is performing a state-of-the-art lateral analysis of a large, complicated building built in 1948. He employs several analysis software packages, including one called ETABS[14], in which he models the entire building. Larry's analysis depends on the internal forces generated at each intersection between piers (major columns) and spandrels (major beams). ETABS calculates these forces for elements of uniform thickness. The 4th-floor spandrel, however, has two thicknesses, so Larry has modeled it in ETABS as two separate spandrels, a thin one lying along the top of a thicker one. As a result, ETABS has returned two separate moments and two separate axial, or horizontal, forces for this spandrel where it intersects a pier (Figure 6a). In the following excerpt, Daniel and Larry struggle together to figure out a way to calculate the single moment force on the end of this odd spandrel from the available ETABS data (Figure 6b).

Initially, Larry and Daniel believe the total moment should be made up of the two separate moments reported by ETABS plus the moment generated by the discrepancy between the axial forces acting on the thin and thick parts. So they proceed to co-invent a way to calculate that axial force differential. Larry reads the two axial forces from the ETABS Spandrel Forces table: 676k on the lower part and –180k on the upper. This means a total axial force of 676 + (-180) or 496k has developed here. For the area to be in equilibrium, an "external" force of 496k must be acting axially on this section but in the opposite direction, i.e., -496k. (Here, "external" refers not to the seismic force applied to the entire building but to the force ultimately translated to this section of spandrel through the immediately surrounding elements.) Distributing this external load evenly to the two parts of the spandrel assigns each -248k. Now the total force on the lower part is 676 + -248 or 428k and on the top is -180 + -248 or -428k. Axial equilibrium is thereby attained: 428 + -428 = 0. But because they are separated by the distance between the centers of the upper and lower parts of the spandrel, these axial forces produce a moment – the tendency for the spandrel to rotate.

[13] Bucciarelli (1994) found that the principle of conservation "reigns supreme" across the engineering fields he studied. He describes (p. 85) how energy conservation provides "the main theme" for one engineer's model of a photovoltaic system.

[14] Produced by Computers & Structures, Inc., ETABS is a software utility for modeling and analyzing structures, based on a mathematical technique called *finite-element analysis*.

Larry and Daniel's development of the method to this point, however, is far messier than the above explanation, as their dialogue illustrates:

L: What could you add to both of them to make'em all the same? That's your force that you're adding in. So if you took that out, it'd just be a moment. Right?
D: 180 and 677 [I do not know why Daniel says 677 rather than 676]. That guy is the one that's locked [to him?].
L: So, then, we're getting 428.
D: So if you...If you add what you think the axial load through the system is, you get 428?
L: I think...No, what I get is 240.
D: If you add 240...
L: If you add 248...
D: Mmhmm. Times 2, right here. So 248 times 2...
L: The other way. Not times 2, just times one.
D: OK, 248...
L: Right, to each of these...
D: Mmhmm.
L: ...you end up with the same.
D: [Writing calculations] And this...So this one becomes what?
L: 180 add 248. 428?
D: So this minus 124? 1...
L: What?
D: Negative 180 minus 124 equals negative 428.
L No! Negative 180 minus 248 equals negative 2...
D: OK, so...
L: Oh, I guess, yeah, times 2 [missing].
D: Times 2. Times 2. 248 [missing] and this...
L: Plus...Minus 248.
D Minus 248 equals 428.
L: Right.
D: OK.
L: So 428 is causing your couple.

Proceeding on, the men multiply this 428k by 3.2', the *moment arm* or distance between the central axes of the top and bottom parts, to find a moment of 1370 k'. They add this to the two separate moments given by ETABS, 107k' and 1065 k', to finally get the single moment value of 2542 k'. For now, they accept this as the moment on the fourth-floor spandrel at this intersection and deem the method suitable for calculating the rest of the spandrel's moments.

Daniel and Larry work on another problem, but after several minutes Daniel suddenly announces he knows a better way to calculate the 4th-floor moments "using statics." On scratch paper, he draws a cross, representing the intersecting pier and spandrel, and at each end of the cross he draws a curved arrow representing a moment (Figure 8). ETABS does give accurate moment forces for the piers at this intersection, and Daniel writes these values on the top and bottom arrows. Then, he explains to Larry, the sum of the top and bottom pier moments must also be the sum of the left and right spandrel moments.[15] Although the men cannot calculate the particular moment on each side, they can at least find their sum and, subsequently, their average. Daniel says he feels confident about this method.

In some ways, Daniel and Larry's methods exemplify the kind of situation-specific, contextually shaped mathematics described by Lave and others. Their first, co-invented method in particular is highly non-standard, probably unique, and, considering that they ultimately reject it, possibly even wrong.

[15] More accurately, the sums must be opposites. Daniel is obeying Newton's first law, which says that, for a system to be in equilibrium, the sum of all moments about any point must be zero.

(They never bother to test it by comparison with their presumably correct second method.) Even if it were correct, it would never be found in a textbook or curriculum because it has no use beyond this very specific and unusual situation. Nevertheless, both of the calculation methods invented in this episode are deeply rooted in abstract, theoretical, school-taught principles, including Newton's first law, the concept of equilibrium, and the standard algorithm for calculating the moment produced by offset horizontal forces. Larry and Daniel appear to enact a process of working inwards, simultaneously, from opposite ends of the concrete-abstract spectrum – from the specific constraints of the situation (namely the limited information provided by ETABS) and from the overarching theory that the men know must apply – to arrive at a method that works here and now but obeys the fundamental principles of their practice.

* * *

Conclusion

As the analysis in this paper illustrates, structural engineers practice in a world of quantities, units, procedures, and concepts, some of which exhibit concrete qualities, some of which appear more abstract, and some of which defy placement in either camp. Quantities whose "book" versions would qualify them as abstract and difficult to relate to real phenomena are, through experience and association, imbued with meaning by individual engineers. Units lacking clear physical referents are used fluently in calculations. Highly theoretical and general concepts are combined with contextual constraints to guide or generate calculation methods. Thus, engineering expertise appears two-pronged, involving an increased amount of personal, concrete meaning associated with particular quantities and concepts and, at the same time, greater facility with abstract methods and theory.

This two-pronged image of expertise stands somewhat in contrast with Lave's (1988) portrayal of expert dieters. As they spent more time in the Weight Watchers program, her subjects replaced calculations for apportioning food with invented measurement strategies that relied on artifacts in the setting and past experience with similar situations. Dreyfus and Dreyfus (1986) similarly characterize the transition from novice to expertise as a one-way trip, an "evolution from the abstract toward the concrete," which entails:

> ...the progression from the analytic behavior of a detached subject, consciously decomposing his environment into recognizable elements and following abstract rules, to involved skilled behavior based on an accumulation of concrete experiences and the unconscious recognition of new situations as similar to whole remembered ones (p. 35).

When they can, expert engineers surely take advantage of their "accumulation of concrete experiences" to avoid calculations in favor of estimations or judgments about which they feel certain. But their increased experience also appears to facilitate their recognition and application of abstract concepts and algorithms

– behavior necessitated by the sociopolitical conventions of structural engineering and arguably just as "skilled" as the more intuitive and unprincipled action elevated by Dreyfus and Dreyfus. This is not to say that the terms abstract and concrete are unhelpful for describing aspects of the mathematics involved in engineering work. The episodes presented here, however, problematize an abstract-concrete dichotomy in which quantities and concepts are permanently relegated to one of two poles according to static, inherent qualities. Instead, abstraction may be more aptly imagined as a dynamic, continuous quality, with engineers displaying some degree of control over the level of abstraction of the quantities and concepts they use. In this view, the movement of concepts along this continuum can vary in time and direction. An important implication of such a view would be that research about everyday and workplace mathematical behavior would stand to yield more insights by adopting a conceptual framework that admits a continuum, not a chasm, between abstraction and concreteness.

Some scholars have recently explored such continuous frameworks. Wilensky's (1991) redefinition of concreteness well describes my findings of the dynamic quality of abstraction and the engineers' agency in determining a concept's degree of abstraction:

> [C]oncreteness is not a property of an object but rather a property of a person's relationship to an object. Concepts that were hopelessly abstract at one time can become concrete for us if we get into the 'right relationship' with them. [...] The more connections we make between an object and other objects, the more concrete it becomes for us. The richer the set of representations of the object, the more ways we have of interacting with it, the more concrete it is for us. [...] Once we see this, it is not difficult to go further and see that any object/concept can become concrete for someone (p. 198).

In their nursing study, Noss, Hoyles, and Pozzi (2002) develop the notion of "situated abstraction" to describe a conceptualization (like drug concentration) that is "finely tuned to its constructive genesis – how it is learned, how it is discussed and communicated – and to its use in a cultural practice, yet simultaneously can retain mathematical invariants abstracted within that community of practice" (p. 205). The efforts of Carl, Peter, and George to endow abstract quantities with concrete meaning but retain the ability to apply them widely are reflected in Noss et al.'s observations of the nurses' behavior:

> Abstraction in the form of conceptualizing relationships of mass and volume were made by the nurses but did not necessarily involve abstracting away from the situation. In fact, the reverse was true: the noise of the practice appeared to be decisive in generating meaning (p. 226).

Similarly, Carraher and Schliemann (2002) propose that "Abstract thinking is not antagonistic to the idea of reasoning in particular contexts" (p. 142), and elaborate, "Symbols and representational systems are abstract not because they are removed from contexts, but because they can be employed in a very wide range of contexts" (p. 141). These authors coin the term "situated generalization" to capture the phenomenon of abstract thinking with contextual ties.

Dynamic perspectives of abstraction have allowed scholars to resuscitate, yet transform, the notion of transfer. According to Carraher and Schliemann, "developing flexible mathematical knowledge depends on our ability to recontextualize problems – to see them from diverse and fresh points of view and to draw upon our former experience, including formal mathematical learning" (p. 146). Lobato (2003) draws on the ideas of recontexualization and "dynamic production" to develop a new model of transfer. She defines *actor-oriented transfer* as "the personal construction of relations of similarity across activities, (i.e., seeing situations as the same)" (p. 20). Echoing Wilenski, Lobato emphasizes "learners' personal perceptions" of what is similar across situations, as well as the influence of artifacts, language, and social structures on these perceptions. These reconceptualized versions of transfer resonate with my depiction of engineering expertise and would provide helpful frames for inquiry into how engineers develop the ability to recognize and apply theory.

The static associations found by previous scholars – of school math with the abstract and real-world math with the concrete – gave rise to pessimism about the potential of school to prepare students for solving everyday problems. Rejecting that dichotomy and readmitting transfer (albeit in a new form) could be cause for greater optimism. Further ethnographic research into other workplace practices would be needed to ascertain whether a dynamic capacity for abstract and concrete conceptualization is a general aspect of adult mathematical expertise. To the degree that it is, a new goal might be implied for school math curricula: to immerse students in opportunities to develop both abstract and concrete understandings of concepts and quantities and to develop students' ability to move fluidly between them. Investigating the learning of practicing engineers was not an explicit goal of this study, but the engineers' process of building both abstract and concrete meanings – whether called learning or not – evokes the constructivist view of how children learn. Thus, the findings of this study not only lend support to classroom methods that encourage students to construct their own meanings of mathematical concepts and quantities, but they suggest that constructivism may describe equally well the knowledge acquisition of adults.

Figures

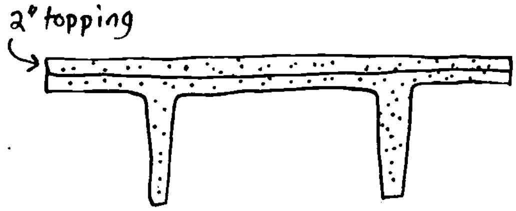

Figure 1 Cross-section of a double T-beam

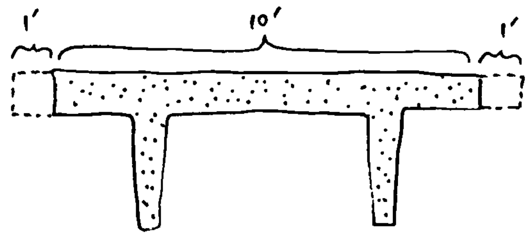

Figure 2 Peter's mental model of a 12' T beam

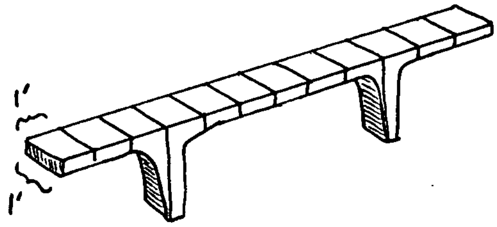

Figure 3 A lineal-foot slice and square-foot chunks of a T-beam

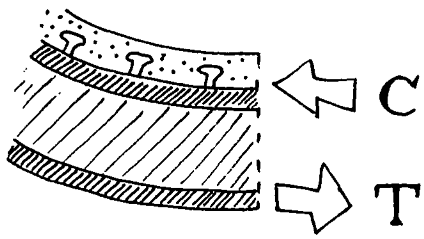

Figure 4 Compression and tension in a free-body diagram of a composite beam

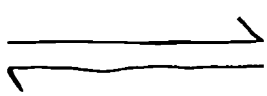

Figure 5 George's symbolic representation of V (shear)

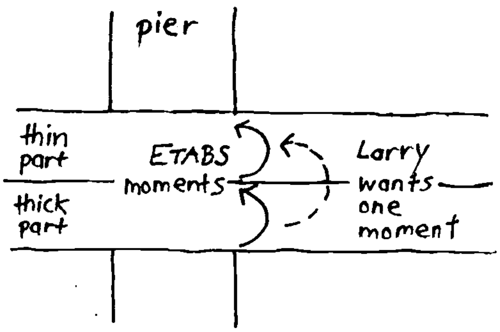

Figure 6a ETABS models the dual-thickness 4th-floor spandrel as two spandrels

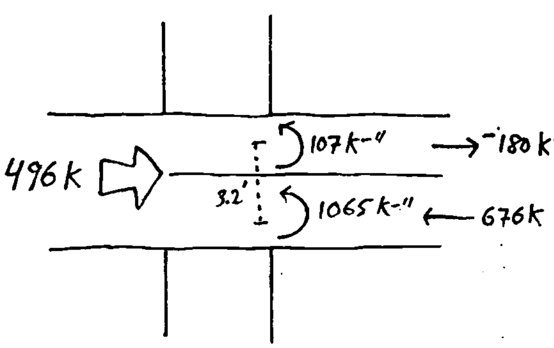

Figure 6b ETABS-reported forces on the 4th-floor spandrel

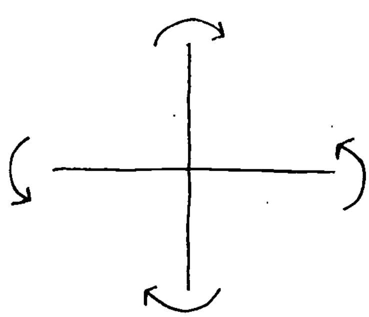

Figure 7 Moments at the spandrel-pier intersection

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