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ABSTRACT

Despite major differences in course delivery, the critical components of effective online mathematics teacher education may not be different from the critical components of effective face-to-face mathematics teacher education. Research indicates that components of effective mathematics teacher education include: aesthetic experiences with mathematics; confronting personal beliefs about mathematics; engaging in practical inquiry; discussing pedagogical implications in the context of mathematics education literature. This paper describes the case of an online mathematics education course for in-service elementary teachers, and discusses issues of design and effect of online experiences. (Contains 10 references and 1 figure.) (Author)

Mathematics Teacher Education Online

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Abstract: Despite major differences in course delivery, the critical components of effective online mathematics teacher education may not be different from the critical components of effective face-to-face mathematics teacher education. Research indicates that components of effective mathematics teacher education include (a) aesthetic experiences with mathematics, (b) confronting personal beliefs about mathematics, (c) engaging in practical inquiry, and (d) discussing pedagogical implications in the context of mathematics education literature. This paper describes the case of an online mathematics education course for in-service elementary teachers, and discusses issues of design and effect of online experiences.

Introduction

McGowen & Davis (2001a) suggest that we need an "antidote" to teachers' conceptions of mathematics as learning procedures and getting right answers. Findings show that such conceptions are consistently associated with observed practice of teachers (McGowen & Davis 2001a, 2001b; Stipek et al 2001) and that teachers who hold such conceptions of mathematics have lower teacher self-confidence and enjoy mathematics less than teachers who hold inquiry-oriented conceptions (Stipek et al 2001). Attempts to change teachers' mathematics conceptions and teaching practice through teacher education and professional development programs focusing on inquiry-oriented mathematics instruction "are minimally effective, in part because teachers filter what they learn through their existing beliefs" (Stipek et al 2001, 214). Teachers assimilate new ideas without substantially altering existing beliefs that drive their practice (Cohen & Ball 1990). In terms of affecting classroom practice, the new ideas are either ignored or they are interpreted and distorted through the lens of existing beliefs. In this paper we posit that instruction needs to provide critical experiences that lead teachers to pedagogical thinking and practice that enable them to move towards an inquiry-oriented conception of mathematics.

Critical Experiences

Critical experiences may be defined as those experiences that cause teachers to reflect on their knowledge, experience and beliefs and help them see mathematics and mathematics teaching in a new light. When moments of epiphany occur, mathematics education artifacts – such as past experiences, curriculum documents, classroom situations, ideas from professional development workshops, from education articles, and so forth – can be seen as inkblots in which the image (or idea) appears to shift and teachers see something new, something that was not apparent to them before. As one teacher in one of our studies commented, "I feel like [this experience] has cleaned my spectacles and I am reading the [curriculum] document with new vision". Similar findings are reported in a study by McGowen & Davis (2001b) where one teacher noted that the experiences provided in a course of study "opened my eyes to a new outlook on mathematics" (444).

In our research we have concentrated on aesthetic aspects of critical experiences. Contrary to the usual alignment of "critical" with "rigorously intellectual", participants in our study demonstrate that critical engagement occurs within an aesthetic context (Gadanidis, Hoogland & Hill 2002). This was true for teachers with diverse mathematical backgrounds and attitudes that ranged between positive and negative. It is interesting to note that although aesthetic qualities of critical experiences for mathematics teachers are not explicitly identified in related studies, aesthetic qualities are in some cases implied. For example, McGowen & Davis (2001b) make use of phrases such as “we focused on a [...] beautiful experience in establishing connections” (439) and “the atmosphere [...] was electric” (440) to describe aspects critical experiences for mathematics teachers.

Research on mathematics teacher development (Cohen & Ball 1990; Gadanidis, Hoogland & Hill 2002; McGowen & Davis 2001a, 2001b; Stipek et al 2001) indicates that the following are integral components of critical experiences:

- Teachers confront their beliefs about mathematics.
- Teachers have aesthetic experiences with mathematics.
- Teachers engage in practical inquiry.
- Teachers consider pedagogical implications in the context of relevant mathematics education literature.

Design and Effect of an Online Course

The discussion of the design and effect of a full-credit additional qualification online mathematics education course for elementary in-service teachers [referred to hereafter as Online Course] is organized around the four components of critical experiences outlined above. It should be noted that although the design of the course incorporated these components, the components were explicitly identified after the course was designed and taught. The original design of the Online Course was based on successful learning experiences for teachers that emerged from face-to-face professional development sessions. In the discussion below we highlight the design similarities between the Online Course and the face-to-face professional development sessions that inspired the design of the course. We hope to illustrate that the components of critical experiences for mathematics do not depend upon a particular environment – be that face-to-face or online. Rather, that critical experiences depend upon the quality and nature of instruction as identified above.

Prior to developing the Online Course the instructor was a mathematics consultant for a large school district, dealing mostly with elementary mathematics program design and teacher development. This experience reinforced research findings that many elementary teachers view mathematics as procedures to be learned for getting right answers (McGowen & Davis 2001a, 2001b; Stipek et al 2001). One of the goals of district-wide professional development was to help teachers become aware of these beliefs and to examine them critically. Bringing teachers face-to-face with their unexamined beliefs about mathematics involved more than simply telling or showing teachers what mathematics is really like or how it may be different from their personal beliefs. Teachers were provided with opportunities to personally experience aesthetically-rich mathematical contexts, which were different than the teachers' historical experiences with mathematics or the experiences they may have been providing for their own students. Likewise, teachers in the Online Course were provided with similarly aesthetic mathematics experiences. Such experiences created a reflective context for examining personal in both the face-to-face and the online environments.

Experiencing aesthetically-rich mathematics does not have to involve complex mathematics, especially for elementary teachers. Some of the mathematics experiences in the Online Course involved teachers in mentally solving arithmetic problems such as 16 x 24 and 156 + 78 + 9 (see Activities 1 & 2 in Figure 1). These activities were chosen based on their positive effect in previous face-to-face workshops conducted for elementary teachers and parents. In such workshops, typically half the people in each group were asked to solve a problem like 16 x 24 or 156 + 78 + 9 in their heads and half the people to use pencil and paper. After a few minutes, people shared and explained the methods they used in their groups. Then the discussion was opened up and people shared and explained other methods. It quickly became apparent that the people who used paper and pencil methods had little to say. One reason for this was that most people used the same procedure. Another reason was that although they were able to describe the procedure they followed, they often were not able to explain why. Some people reverted to statements like “this is how it works – it's just a rule”. On the other hand, people who solved the problem in their heads shared a variety of methods and they understood what they were doing and why they were doing it. They displayed pride in their individual approaches to problem solving. There was an excitement about mathematical thinking in the room, with people eager to share their personal methods and quick to express surprise and praise for unique methods that others shared. A palpable energy was created in this exchange.

We suggest that the experience of mentally solving 16 x 24 or 156 + 78 + 9 is aesthetically rich in that the mental processes involved do not demand rule-based procedures. How people solve 16 x 24 depends greatly on how they personally interpret the problem. For example, some people may multiply 16 and 25 and then subtract the extra 16. Others may deconstruct the problem as 10 x24 + 6 x 24. Many other solutions processes are possible – even ones that use algebraic structures like (20 - 4)(20 + 4). Given such problems, people are eager to share their solutions, they express interest and sometimes surprise in the solutions of others, and are motivated to try to come up with different solution processes. Open-ended inquiry, interest, surprise and motivation are characteristics of an aesthetic approach.

Teachers in the Online Course noticed that their mental solution processes were "different than when I did it with paper and pencil because I solved my problem by starting with the bigger numbers first (left to right, not right to left!)". Such experiences appear to have helped teachers move towards questioning traditional views of mathematics and developing a deeper understanding of what constitutes mathematical activity and mathematical understanding in the context of addition and multiplication. "To me, the implications are that doing arithmetic mentally requires real understanding. The traditional way (on paper, doing the "ones" first) is more of a procedure to be memorized that requires little understanding".

In face-to-face professional development, practical inquiry was facilitated through a double-session structure. Between sessions tried out new ideas in their classrooms and shared their experiences and reflections in the second session. Teachers were encouraged to bring to the second session samples of student work. Many of the insights that teachers gained and shared arose from observations of students doing mathematics and thinking mathematically in the context of the new ideas that teachers tried in their classrooms. The Online Course involved teachers in practical inquiry in that teachers were asked to explore the thinking of others, including the thinking of their own students, in mentally solving problems like 16 x 24 and 156 + 78 + 9 (see Activity 3 in Figure 1). They shared and reflected on these observations in online discussions. Many of the teachers tried the problems with their students and discovered that they too used a variety of methods, and usually not standard paper and pencil procedures. This helped teachers realize that their mathematical thinking as adults was similar to that of their students and different from the standard paper and pencil procedures. Teachers were impressed by the creativity of student answers and questioned their reliance on paper and pencil procedures.

Asking teachers in the Online Course to mentally solve problems like 16 x 24 and 156 + 78 + 9 and to share their solution processes also offered opportunity for practical inquiry into the nature of mathematics and doing mathematics. This set the context for discussions of related pedagogy. However, one would expect that practical inquiry would also involve experimenting with teaching practice, which was not a requirement of the Online Course. Unlike the face-to-face professional development described above, the Online Course did not explicitly ask teachers to experiment with new teaching ideas in their own classrooms. This is something that will be reconsidered when redrafting the Online Course.

In face-to-face professional development sessions, ideas from mathematics education literature were shared and discussed. The Online Course gave teachers the opportunity to read such literature. Two articles about children inventing personal algorithms for arithmetic operations (Burns, 1994; Kamii et al, 1993) provided a context for teacher reflections on their thinking when mentally solving problems like 16 x 24 and 156 + 78 + 9 and for considering pedagogical implications. "To guide your thinking" questions (see Figure 1) directed teacher attention to pedagogical issues. In contrast to the face-to-face professional development sessions where ideas verbalized may be forgotten, an advantage of the Online Course was the 'permanent' record of discussions. Many teachers revisited past discussions and created scrapbooks of 'good ideas' by copying sections of online transcripts in word processing documents. As was the case in the study by McGowen & Davis (2001b), teachers made important connections between their experiences and ideas in the articles they read.

> I do agree with Kamii and Burns' points of view. I think that by having the student discover a successful method they will be more likely to internalize and understand the concept. In coming up with their own methods they are doing the thinking the way their mind works. We can see [in our discussion] that everyone processes things differently.

Discussion

Research indicates that many teachers are unaware that an alternative to a procedural view of mathematics exists (McGowen & Davis 2001b). The interplay between the mathematics experiences, journal readings, online reflections, and discussions created for many of the teachers in the Online Course a critical experience that helped them 'see' mathematics and mathematics teaching in a new light. As one teacher commented,

> After seeing how different people calculate, I better understand this last overall expectation in [the curriculum document]. Hmph! It is not until I do something myself, do I more fully understand the language and what the curriculum is really driving at. Thus students need to explore different ways of doing calculations, talk about it, communicate their ideas in a variety of ways. I feel like [this experience] has cleaned my spectacles and I am reading the document with new vision.

A critical aspect of the Online Course was that teachers confronted their personal beliefs about mathematics and mathematics teaching in the context of practical inquiry with aesthetically-rich mathematics problems. In solving such problems, in observing students solve them, and in sharing, comparing and discussing solutions teachers realized that mathematics problems might be solved in many different ways. Another critical aspect was the reading and discussing journal articles that placed such experiences in the broader context of mathematics pedagogy. Online discussions allowed the personal kind of sharing that created community. This is not to suggest that such critical experiences created changes in teachers' perceptions of mathematics and mathematics teaching that were comprehensive or permanent or that significantly affected their classroom practice. How teachers teach is also greatly affected by accepted teaching practices in the wider school community (Buzeika, 1999; Ensor, 1998) and by conflicting priorities (Skott, 1999). However, such critical experiences, whether they are in online or face-to-face teacher education or professional development settings, may be important starting points for change in classroom practice.

Conclusion

There are important characteristics of online learning such as text-based communication and asynchronous discussion that distinguish it from face-to-face learning. However, the cases of the Online Course and the face-to-face professional development discussed above indicate that the components of critical experiences transcend such differences. These components include (a) aesthetic experiences with mathematics, (b) confronting personal beliefs about mathematics, (c) engaging in practical inquiry, and (d) discussing pedagogical implications in the context of mathematics education literature. We believe that in the design of online courses for mathematics teachers, the primary focus of teacher educators should be not on the technological differences but, rather, on the quality and the components of the critical experiences.

A. How do you think mathematically?

Let's take a close look at how your mind performs arithmetic operations. Try the following activities:

Activity #1:

- Multiply 16 and 24 in your head.
- Record the process you followed.
- Now multiply 16 and 24 on paper using the procedure you were taught in school.

Activity #2:

- Add 156, 78 and 9 in your head.
- Record the process you followed.
- Now add 156, 78 and 9 on paper using the procedure you were taught in school.

Activity #3:

- Ask someone else (a student, a teacher, a friend) to solve the following and then tell you the processes they used:
  - Multiply 16 and 24 in their head.
  - Add 156, 78 and 9 in their head.

To guide your thinking:

How are the processes you use to multiply or add in your head similar to or different from the paper and pencil procedures you were taught in school for multiplying or adding?

B. Jean Piaget

Jean Piaget, who spent a lot of time researching the mathematical thinking of young children, said that there are three types of knowledge (Kamii et al 1993):

1. **Physical knowledge:** This is the knowledge of our physical environment. For example, colours of objects, or the fact that an apple falls when you release it from your hand.
2. **Social knowledge:** These are, in part, the rules and conventions that we make up to make our social life run smoothly. For example, we all agree that a red traffic light means stop. Social knowledge is to some degree arbitrary. For example, we could all agree to stop on a green light and go on a red light.
3. **Logico-mathematical knowledge:** This is the knowledge of relationships. This knowledge allows us to see patterns and make connections in situations. For example, we can see two circles as representing a pair of glasses, the number eight, or a snowman. Or, when considering the following question, we can see different relationships that lead to different answers: Which one of the numbers 3, 4, 5, and 7 does not belong?

To guide your thinking:

Think about the procedures you learned in school for multiplying or adding numbers on paper. Which type of knowledge do they represent? Why?

Think about the processes you used earlier for multiplying or adding numbers in your head. Which type of knowledge do they represent? Why?

Think about the mathematics you learned in school. How would you classify it in terms of Piaget's knowledge categories? Can you identify some examples in each category?

Figure 1. Online course experiences.

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