DOCUMENT RESUME

ED 476 101		SE 067 802

AUTHOR	Pegg, John
TITLE	Fundamental Cycles of Cognitive Growth.
PUB DATE	2002-07-00
NOTE	9p.; In: Proceedings of the Annual Meeting of the International Group for the Psychology of Mathematics Education (26th, Norwich, England, July 21-26, 2002); see SE 067 806.
PUB TYPE	Reports - Research (143) -- Speeches/Meeting Papers (150)
EDRS PRICE	EDRS Price MF01/PC01 Plus Postage.
DESCRIPTORS	*Cognitive Development; Elementary Secondary Education; *Learning Processes; Learning Theories; *Mathematics Education

ABSTRACT

Over recent years, various theories have arisen to explain and predict cognitive development in mathematics education. We focus on an underlying theme that recurs throughout such theories: a fundamental cycle of growth in the learning of specific concepts, which we frame within broader global theories of individual cognitive growth. Our purpose is to use our experience in such areas as the SOLO Model, van Hiele levels, process-object encapsulation and symbols as process/concept, to raise the debate beyond a simple comparison of detail in different theories to move to address fundamental questions in learning. In particular, a focus of research on fundamental learning cycles provides an empirical basis from which important questions concerning the learning of mathematics can and should be addressed. (Author)

FUNDAMENTAL CYCLES OF COGNITIVE GROWTH

John Pegg

Centre for Cognition Research

University of New England

Armidale, NSW, Australia

<jpegg@metz.une.edu.au>

Over recent years, various theories have arisen to explain and predict cognitive development in mathematics education. We focus on an underlying theme that recurs throughout such theories: a fundamental cycle of growth in the learning of specific concepts, which we frame within broader global theories of individual cognitive growth. Our purpose is to use our experience in such areas as the SOLO Model, van Hiele levels, process-object encapsulation and symbols as process/concept, to raise the debate beyond a simple comparison of detail in different theories to move to address fundamental questions in learning. In particular, a focus of research on fundamental learning cycles provides an empirical basis from which important questions concerning the learning of mathematics can and should be addressed.

The named author is the presenter of this paper. This paper was written as a joint paper by John Pegg and David Tall. It is presented here as a single authored paper to comply with a newly-implemented rule restricting the number of papers that can be accepted by a given co-author.

INTRODUCTION

We here focus on two kinds of theory of cognitive growth:

- **global theories of long-term growth** of the individual, such as the stage-theory of Piaget (e.g., Piaget and Garcia, 1983).
- **local theories of conceptual growth** such as the action-process-object-schema theory of Dubinsky (Czarnocha et al, 1999) or the unistructural-multistructural-relational-extended abstract sequence of SOLO Model (Biggs & Collis, 1982, 1991; Pegg, 1992).

Some theories (such as that of Piaget, the SOLO Model, or more broadly, the enactive-iconic-symbolic theory of Bruner, 1966) incorporate both aspects. Others such as the embodied theory of Lakoff and Nunez (2000) or the situated learning of Lave and Wenger (1990) paint in broader brush-strokes, featuring the underlying biological or social structures involved.

Our focus is on local theories, formulated within a global framework whereby the cycle of learning in a specific conceptual area is related to the overall cognitive structures available to the individual.

A range of global longitudinal theories each begin with physical interaction with the world and, through the use of language and symbols, become increasingly abstract.

Table 1 shows four of these theoretical developments. It is not our purpose to make a detailed stage-by-stage comparison of these theories here. Although reports on comparisons between SOLO and van Hiele can be found in Pegg and Davey (1998), and between SOLO and Piaget and SOLO and Bruner in Biggs and Collis (1982).

Piaget Stages	van Hiele Levels Hoffer,1981	SOLO Modes	Bruner Modes
Sensori Motor	I Recognition	Sensori Motor	Enactive
Preoperational	II Analysis	Ikonic	Iconic
Concrete Operational	III Ordering	Concrete Symbolic	Symbolic
Formal Operational	IV Deduction	Formal	
	V Rigour	Post-formal	

Table 1: Global stages of cognitive development

What stands out from such theories is the gradual biological development of the individual, growing from dependence on sensory perception through physical interaction and on, through the use of language and symbols, to increasingly sophisticated modes of thought. SOLO offers a valuable viewpoint as it explicitly nests each mode within the next, so that an increasing repertoire of more sophisticated modes of operation become available to the learner. At the same time, all modes attained remain available to be used as appropriate. As we go on to discuss fundamental cycles in conceptual learning, we therefore need to take account of the development of modes of thinking available to the individual.

LOCAL CYCLES OF DEVELOPMENT

We now turn to the core of our study: the cycles of development that occur within a range of different theories. These have been developed for differing purposes. The SOLO Model, for instance, is concerned with assessment of performance through observed learning outcomes. Other theories, such as those of Davis (1984), Dubinsky (Czarnocha et al, 1999), Sfard (1991), and Gray and Tall (1994, 2001) are concerned with the sequence in which the concepts are constructed by the individual (see Tall et al, 2000, for a further analysis of these theories).

SOLO of Biggs & Collis	Davis	APOS of Dubinsky	Gray & Tall
			[Base Objects]
Unistructural	Procedure (VMS)[1]	Action	Procedure
Multistructural	Integrated Process	Process	Process
Relational	Entity	Object	Procept
Unistructural		Schema	

Table 2: Local cycles of cognitive development

[1] Davis used the term 'visually moderated sequence' for a step-by-step procedure.

As can be seen from table 2, there are strong family resemblances between these cycles of development. Although a deeper analysis of the work of individual authors will reveal discrepancies in detail, there are also insights that arise as a result of comparing one theory with another as assembled in table 3.

First, Gray and Tall (2001) note the role that objects operated upon play in concept formation which focus on actions of those objects. These *base objects* may be perceptual or conceptual, provided that the individual conceptualises them as entities. Gray and Tall see the configuration of the base objects (such as a set divided into three equal pieces and two of these being selected) as an embodied representation of the encapsulated abstract concept 2/3.

Second, Dubinsky's notion of action begins at a more primitive level than the notion of procedure, formulated in the following terms:

> **Action.** A transformation is considered to be an action when it is a reaction to stimuli which the subject perceives as external. This means that the individual requires complete and understandable instructions giving precise details on steps to take in connection with the concept. (Czarnocha et al, 1999)

The SOLO Model is in the Piagetian tradition. Here humans are seen as actively constructing their world as a result of processes of interaction between a person and his or her social and physical environment. The resulting development can be identified through 'logically' sequential, qualitatively different, levels each representing a coherent viewpoint of the world. This pattern of thought is revealed through the underlying cognitive structure of what the person says, writes or does. Growth in understanding is seen as an active change in patterns of thinking. Hence, cognitions are seen to be structures that are rules for processing information.

The central task associated with SOLO codings, and one that can be very demanding, is to analyse the pattern of responses provided by individuals and groups. In doing this, it is important to distinguish between the content and the cognitive structure. There is a tension here. The pattern of responses has to be sensitive to (i) those aspects that are cultural or individual and primarily associated with the subject content, and (ii) the reasoning structures of a student's thoughts that are part of our human heritage and true for all individuals.

The unistructural level of SOLO focuses on the appropriate domain but uses relevant data seen by the students as a single entity. This notion seems to encompass the levels of base object, action and procedure. On the other hand, the distinct SOLO level of 'multistructural' is not explicitly mentioned in any of the other theories. Analysing the Gray and Tall distinction between procedure and process, we find that it is possible for students to have more than one procedure to carry out a process, but yet may not have a flexible conception of process itself. For instance, in the analysis of Ali and Tall (1996) considering Malaysian students' flexibility in having different procedures to differentiate a given formula in calculus, there are examples of students having different procedures available for a given derivative, and yet their

thinking remains procedural, rather than conceptual. Thus the procedure-process-procept spectrum naturally expands to procedure-multiprocedure-process-procept.

The relational level of SOLO readily equates to the process level of the process-object encapsulation theories. Here the learner has an overview of the elements or procedures. The data known to the student are able to be woven into an overall mosaic of relationships. The whole has become a coherent structure with no inconsistency within the system known by the student.

The fourth level titled unistructural relates to a combined object-schema level. This level highlights the two possibilities of cognitive growth that exist within SOLO when development occurs past the relational level. This is summarised diagrammatically in figure 1. If the nature or abstractness of the thinking is the same as that identified in the previous three levels, i.e., U_1 M_1 R_1, then the next level is a new unistructural level, U_2. It is distinguished by the person seeing as a single concise entity what was previously an integration of several aspects.

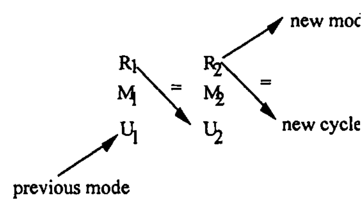

Figure 1: At least two cycles within concrete symbolic mode.

If, on the other hand, the new response represents a qualitatively different, more abstract way of thinking, then the response can be coded outside of the current mode and within the next acquired mode. This new level can be described as a new unistructural level, a U_1, in the next acquired mode. In the case of moving from the concrete symbolic mode into the formal mode, this new unistructural level can be written as U_{1F} and represents the start of a new Fundamental cycle. It is this cycle that has most to offer students in their late secondary and early tertiary education.

There is support from a range of papers in the literature to see the object-schema level as a coherent whole (which may be subdivided as appropriate). The first is Skemp's varifocal theory, which sees the duality of concepts and certain types of schema, depending on whether it is seen as a whole (concept) or in detail as a structure of relationships (schema). The second, in part, is Dubinsky's own perception that objects can be formed not only in terms of encapsulation of processes, but also of encapsulation of schemas (Czarnocha et al, 1999).

An in-depth discussion of the relationships between process, object and schema is given in Tall & Barnard (2001). This discussion shows a correspondence between these local theories and a fundamental cycle of conceptual development, see table 3.

The first stage involves some kind of action on one or more base objects with the focus of attention of the individual either on the objects, or on the actions. These can lead to different kinds of learning, one which focuses on the nature and properties of objects, the other on the nature and properties of the actions, involving symbols being introduced to represent the actions. Isolated actions are consolidated into (step-by-step) procedures, possibly with several procedures available to carry out essentially the same activity. The activity may then be seen as a single process that may be carried out by different procedures. At this stage, with the support of some kind of symbolism, the individual may construct a mental object that is both a schema within itself and becomes manipulable within a wider schema of activities.

SOLO	Davis	APOS	Gray & Tall	**Fundamental Cycle**
			Base Object(s)	Base Object(s)
Unistructural	VMS Procedure	Action	Procedure	Isolated Actions Procedure
Multistructural			[Multi-Procedure]	Multi-Procedure
Relational	Process	Process	Process	Process
Unistructural (Extended Abstract)	Entity	Object Schema	Procept	Entity Schema

Table 3: The fundamental cycle of conceptual construction

Example: Fractions

In the case of the fraction concept, the base objects may be a single object (a cake) or a collection of objects (the children in the class). The action is to divide the object (or separate the objects) into equal parts, by some kind of division or sharing process, then selecting a given number of the parts. Different actions (e.g., divide into 3 parts and take 2 or divide into 6 parts and take 4) lead to the same output. Different techniques for carrying out the activity may lead to procedures, then multi-procedures to carry out the action.

When the child realises that different procedures have the same output and focus on the effect of the procedures, then this moves to the process level where different procedures have the same effect (Watson & Tall, 2002). The introduction of the symbols 2/3 or 4/6 for the two distinct actions leads to the notion of equivalence of fractions that have the same effect. The notion of equivalent fraction is a schema in itself (as in Skemp's varifocal theory) which, conceived as a single entity, may become an element within the wider schema of arithmetic of fractions.

To expand on the above let us consider a particular problem asked of some several hundred students in the age range ten-to-fifteen years. This question was asked in mathematics lessons along with several other fraction-related questions. Students were asked how much would each person receive if nine apple pies were divide between sixteen people so that each person receives the same amount. In analysing

the student responses we can identify an initial fundamental cycle followed by several components of a second fundamental cycle. In terms of SOLO modes all responses were within the concrete symbolic mode. Early cycle responses, which attempt to deal with the problem, focus on the need (the action) to cut the pies equitably, usually in halves. However, a problem emerged for some students when their action of cutting did not result in each person gaining an equal amount: "cut the pies into halves and you have two pieces left over" (for these students there was one pie too many). These students did not resolve this issue.

The next category of response also has the cutting of the pies fairly as its focus, but the problem is seen in two parts with a new procedure employed to deal with the left over pie. "Cut one pie into 16 pieces and the rest into halves." The third category of response considers the effect of the cutting. The response moves on past the focus on equitable cutting to provide a summative view on how much each person would receive. "Cut each apple pie into sixteen pieces and each person gets 9 pieces each." Missing from this response is any use of standard fraction notation.

The next level of response, a new unistructural level occurs as the response provided is more succinct and fraction notation is employed to express the different parts of the problem. "I would cut 8 pies in half and then cut the last pie into 16 pieces. So everyone gets 1/2 and 1/16 of a pie each." In this response students use fraction notation freely in their written language. This is the culmination of the first fundamental cycle and also represents the first element in a new cycle. This new Fundamental cycle represents a development of fractions as numbers.

The second level in this new cycle is on combining the fractions identified previously. The method employed is equivalent fractions whereby all fractions are rewritten with the same denominators before combining. "Cut 8 pies in half and give one half to each person. Then cut the last pie into 16 pieces and give one piece to each person. $1/2 + 1/16 = 8/16 + 1/16 = 9/16$ each person ends up with 9/16 of a pie." Here, work with fractions takes on a more systematic process.

This was the highest level of response identified from the responses received. One could predict that the next level is when students can approach the task with considerable versatility. They can use a number of written approaches and are usually able to provide the answer efficiently in verbal form. These options of response were not available to the students in this study.

THE BIOLOGICAL BASIS OF THE FUNDAMENTAL CYCLE

The biological basis of this phenomenon was discussed in Tall (1999), in which the stimulation of neurons places them on alert so that they will fire more easily for a while. They then react to a lower level of stimulation and, if this occurs, the link becomes even easier to make, until the neurons concerned fire together as a unit in a more subtle pattern. This *long-term potentiation* of neuronal links builds

sophisticated structures that act in consort, which are both complex (because they have many connections) but also simple (because many neurons fire as one unit).

However, this highly subtle mental development has a consequence. If the individual becomes aware that certain sequences of activity tend to occur, the individual can operate with them at a higher level. Thus, whilst individuals may need initially to be externally guided at the action level before contemplating procedures, then processes and then symbolise the activities to build a sophisticated mental schema, others are aware of the overall scheme of things. Thus a natural attribute of the fundamental cycle is the possibility of seeing the kind of outcome that might be possible and have a 'top-down' view of the need to build actions into processes into schemas to construct the wider vision in greater detail.

Here the different modes of operation may be of great advantage. For instance, an embodied combination of SOLO's sensori motor and ikonic modes may enable the individual to gain an insight into the overall plan *before* building the more powerful symbolic and deductive modes of thought. On the other hand, an embodied approach which may work for some (natural) thinkers (using the language of Pinto, 1998) may be less effective with (formal) thinkers who have highly developed symbolic and deductive cognitive structures.

CONCLUSION

A primary goal of teaching should be to stimulate cognitive development in students. Such development as described by these fundamental learning cycles is not inevitable. Ways to stimulate growth, to assist with the reorganisation of earlier levels need to be explored. Important questions about strategies appropriate for different levels or even if it is true that *all* students pass through all levels in sequence. Research into such questions is sparse. Nevertheless, the notion of fundamental cycles of learning does provide intriguing potential for research.

REFERENCES

Ali, M. & Tall, D. O. (1996). Procedural and Conceptual Aspects of Standard Algorithms, *Proceedings of PME 20*, Valencia, **2**, 19–26.

Biggs, J. & Collis, K. (1991). *Multimodal learning and the quality of intelligent behaviour*. In H. Rowe (Ed.), Intelligence, Reconceptualization & Measurement. New Jersey. Laurence Erlbaum.

Biggs, J. & Collis, K. (1982). *Evaluating the Quality of Learning: the SOLO Taxonomy*. New York: Academic Press.

Bruner, J. S. (1966). *Towards a Theory of Instruction*, New York: Norton.

Czarnocha, B., Dubinsky, E., Prabhu, V., Vidakovic, D. (1999). One theoretical perspective in undergraduate mathematics education research. In O. Zaslavsky (Ed.), *Proceedings of the 23rd Conference of PME*, Haifa, **1**, 95-110.

Davis, R.B. (1984). *Learning Mathematics: The cognitive science approach to mathematics education*. Norwood, NJ: Ablex

Gray, E.M. & Tall, D.O. (1994). Duality, ambiguity and flexibility: a proceptual view of simple arithmetic. *Journal for Research in Mathematics Education, 26*, 2, 115-141.

Gray, E.M. & Tall, D.O. (2001). Relationships between embodied objects and symbolic procepts: an explanatory theory of success and failure in mathematics. In Marja van den Heuvel-Panhuizen (Ed.) *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education* 3, 65-72. Utrecht, The Netherlands.

Hoffer, A. (1981). Geometry is more than proof. *Mathematics Teacher*, 74, 11-18.

Lakoff, G. & Nunez, R. (2000). *Where Mathematics Comes From.* New York: Basic Books.

Lave, J. & Wenger, E. (1990). *Situated Learning: Legitimate Peripheral Participation.* Cambridge, UK: Cambridge University Press.

Pegg, J. (1992). Assessing Students' Understanding at the Primary and Secondary Level in the Mathematical Sciences. In J. Izard & M. Stephens (Eds), *Reshaping Assessment Practice: Assessment in the Mathematical Sciences Under Challenge* (pp. 368-385). Melbourne: ACER.

Pegg, J., & Davey, G. (1998). A synthesis of Two Models: Interpreting Student Understanding in Geometry. In R. Lehrer & C. Chazan, (Eds), *Designing Learning Environments for Developing Understanding of Geometry and Space.* (pp.109-135). New Jersey: Lawrence Erlbaum.

Piaget, J. & Garcia, R. (1983). *Psychogenèse et Histoire des Sciences.* Paris: Flammarion.

Pinto, M. M. F., (1998). *Students' Understanding of Real Analysis.* Unpublished PhD Thesis, Warwick University.

Sfard, A. 1991. On the dual nature of mathematical conceptions: reflections on processes and objects as different sides of the same coin, *Educational Studies in Mathematics*, **22**(1), 1–36.

Tall, D.O. (1999). Reflections on APOS theory in Elementary & Advanced Mathematical Thinking. In O. Zaslavsky (Ed.), *Proceedings 23rd Conference of PME, Haifa, Israel*, **1**, 111–118.

Tall, D.O. & Barnard A.D. (Tony) (2001). Cognitive Units, Connections and Compression in Mathematical Thinking. (submitted for publication, available from http://www.warwick.ac.uk/staff/David.Tall.)

Tall, D.O., Gray, E.M., Ali, M.B., Crowley, L., DeMarois, P., McGowen, M., Pitta, D., Pinto, M.M.F., Thomas, M.O.J. & Yusof, Y.B. (2001). Symbols and the Bifurcation between Procedural and Conceptual Thinking, *Canadian Journal of Science, Mathematics and Technology Education*, 1, 81–104.

Tall, D.O., Thomas, M.O.J., Davis, G., Gray, E.M., Simpson, A.P. (2000). What is the object of the encapsulation of a process? *Journal of Mathematical Behavior*, **18** (2), 1–19.

Van Hiele, P.M. (1986). *Structure and Insight: a theory of mathematics education.* New York. Academic Press.

Watson A. & Tall, D. (2002). Embodied Action, Effect and Symbol in Mathematical Growth (submitted for publication, available in draft from www.davidtall.com/papers.)

NOTICE

Reproduction Basis

- [x] This document is covered by a signed "Reproduction Release (Blanket)" form (on file within the ERIC system), encompassing all or classes of documents from its source organization and, therefore, does not require a "Specific Document" Release form.

- [ ] This document is Federally-funded, or carries its own permission to reproduce, or is otherwise in the public domain and, therefore, may be reproduced by ERIC without a signed Reproduction Release form (either "Specific Document" or "Blanket").