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ABSTRACT

In this paper I will trace a path through three different studies of mathematics learning in England and the United States. As different analytical lenses are cast upon students' experiences I will propose that notions of mathematical capability expand beyond knowledge, to include mathematical practices and the identities these encourage. Further that different classroom practices encourage students to develop different relationships with the discipline of mathematics that impact their capability in profound ways. (Author)

THE DEVELOPMENT OF DISCIPLINARY RELATIONSHIPS: KNOWLEDGE, PRACTICE, AND IDENTITY IN MATHEMATICS CLASSROOMS.

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In this paper I will trace a path through three different studies of mathematics learning in England and the United States. As different analytical lenses are cast upon students' experiences I will propose that notions of mathematical capability expand beyond knowledge, to include mathematical practices and the identities these encourage. Further that different classroom practices encourage students to develop different relationships with the discipline of mathematics that impact their capability in profound ways.

Researchers of mathematics education have, for many years, focused upon the knowledge students develop in classrooms, and the ways such knowledge is influenced by a number of different variables. Recently, situated theories of learning have led to the recognition that the practices of classrooms – the repeated actions in which students and teachers engage as they learn – are important, not only because they are vehicles for students' knowledge development, but because they come to *constitute* the knowledge that is produced (Cobb, 1998). Thus the field has moved to greater recognition of the intricate relationship between knowledge and practice and the need to study the practices of classrooms in order to understand students' mathematical capability in different situations. In this paper I will attempt to expand notions of mathematical capability yet further, to go beyond both knowledge and practice, to the identities students form in relation to different mathematical practices, and the *disciplinary relationships* that are afforded by different classroom experiences. Thus I will suggest that when students approach a new mathematics problem, the extent to which they are able to use mathematics depends partly on the knowledge they have developed, partly upon the practices in which they have engaged as they have learned, and partly upon the relationships they have developed with the discipline of mathematics. This focus on students' *disciplinary relationships* combines important mathematics education research on knowledge and practice (Cobb, 1998; Hershkowitz, 1999) with that on belief, disposition and identity (Christou & Philippou, 1998; DeBellis & Goldin, 1999).

The inter-relationship between the practices in which students engage in classrooms and the subsequent knowledge they develop, was demonstrated by a three-year study of students learning mathematics that I conducted in England

(Boaler, 1997). In that study I monitored 300 students who learned mathematics in very different ways. Students at one school learned through 'traditional' methods of watching and repeating standard algorithms and procedures, students at the other schools learned through open-ended projects. One of the findings of the research was that students' knowledge development in the two schools was *constituted* by the pedagogical practices in which they engaged (Boaler, 1997, 1999). Thus it was shown that practices such as working through textbook exercises, in one school, or discussing and using mathematical ideas, in the other, were not merely vehicles for the development of more or less knowledge, they shaped the forms of knowledge produced. One outcome, was that the students who had learned mathematics working through textbook exercises, performed well in similar textbook situations, but found it difficult using mathematics in open, applied or discussion based situations. The students who had learned mathematics through open, group-based projects were able to use mathematics in a range of different situations, partly because the classroom practices in which they had engaged were represented elsewhere.

In that study the students who had learned through open-ended projects outperformed the other students in a range of assessments, including the national examination. One conclusion that may be drawn from such a result, that would fit with cognitive interpretations of learning, would be that the students in the traditional school did not learn as much as the students who learned mathematics through open-ended projects, and they did not understand in as much depth, thus they did not perform as well in different situations. That interpretation is partly correct, but it lacks important subtleties in its representation of learning. A different analytical frame that I found useful, was to recognize that the students learned a great deal in their traditional mathematics classrooms. They learned to watch and faithfully reproduce procedures and they learned to follow different textbook cues that allowed them to be successful as they worked through their books. Problems occurred because such practices were not useful in other situations (Winbourne & Watson, 1998). When the students did not use mathematics effectively in different assessments, it was not because they had not met and 'learned' the mathematical knowledge, but because they tried to repeat the mathematical practices they had learned in the classroom and which shaped their knowledge (such as searching for cues and repeating standard methods) and these did not help them in non-standard assessments. Thus I came to understand the students' mathematical capability as an intricate relationship between mathematical knowledge and mathematical practices.

That study revealed an important dimension of mathematical capability that extended beyond knowledge, to the practices in which students engaged, but two subsequent studies that I will briefly summarise have raised further aspects of mathematical capability that extend beyond knowledge and practice. In the first study I and fellow researchers interviewed eight students from each of 6 Northern Californian high schools (Boaler & Greeno, 2000). The 48 students we interviewed were all attending advanced placement (AP) calculus classes. In that study four of the schools taught using traditional pedagogies – the teachers demonstrated methods and procedures to students, who were expected to reproduce them in exercises. In the other two schools, students used the same calculus textbooks, but the teachers did not rely on demonstration and practice, they asked the students to discuss the different ideas they met, in groups. In that study we found that students in the more traditional classes were offered a particular form of participation in class that we related to Belencky, Clinchy, Goldberger, & Tarule's notion of 'received knowing' (1986, p4). Mathematics knowledge was *presented* to students and they were required to learn by attending carefully to both teachers' and textbook demonstrations. The mathematical authority in the classrooms was external to the students, resting with the teacher and the textbooks, and the students' knowledge was dependent upon these authoritative sources. In these classrooms the students were required to receive and absorb knowledge from the teacher and textbook and they responded to this experience by positioning themselves as *received knowers* (Belencky et al, 1986).

The students who were learning in these traditional classrooms were generally successful, but we found that many students experienced an important conflict between the practices in which they engaged, and their developing identities as people (Wenger, 1998). Thus many of the students talked about their dislike of mathematics, and their plans to leave the subject as soon as they were able, not because of the cognitive demand, but because they did not want to be positioned as received knowers, engaging in practices that left no room for their own interpretation or agency. The students all talked about the kinds of person they wanted to be – people who used their own ideas, engaged in social interaction, and exercised their own freedom and thought, but they experienced a conflict between the identities that were taking form in the ebb and flow of their lives and the requirements of their calculus classrooms:

K: I'm just not interested in, just, you give me a formula, I'm supposed to memorize the answer, apply it and that's it.

Int: Does math have to be like that?

B: I've just kind of learned it that way. I don't know if there's any other way.

K: At the point I am right now, that's all I know. (Kristina & Betsy, Apple school)

Most of the students who told us about their rejection of mathematics in the 4 didactic classrooms – 9 girls and 5 boys, all successful mathematics students – had decided to leave the discipline because they wanted to pursue subjects that offered opportunities for expression, interpretation and human agency. In contrast, those students who remained motivated and interested in the traditional classes were those who seemed happy to 'receive' knowledge and to be relinquished of the requirement to think deeply:

J: I always like subjects where there is a definite right or wrong answer. That's why I'm not a very inclined or good English student. Because I don't really think about how or why something is the way it is. I just like math because it is or it isn't. (Jerry, Lemon school)

The students in didactic classes who liked mathematics did so because there were only right and wrong answers, and because they did not have to consider different ideas and methods. They did not need to think about 'how or why' mathematics worked and they seemed to appreciate the passive positions that they adopted in relation to the discipline. For the rest of the students in the traditional classes, such passive participation was not appealing and this interfered with their affiliation and their learning.

In the other two calculus classes in which teachers engaged students in mathematical discussions, a completely different picture emerged. In the discussion oriented classes the students had formed very different relationships with mathematics that did not conflict with the identities they were forming in the rest of their lives. The students in these classes regarded their role to be learning and understanding mathematical relationships, they did not perceive mathematics classes to be a ritual of procedure reproduction. This lack of conflict was important – it meant that the students who wanted to do more than receive knowledge, were able to form plans for themselves as continued mathematics learners. The type of participation that is required of students who study in discussion-oriented mathematics classrooms is very different from that required of students who learn through the reception and reproduction of standard methods. Students are asked to contribute to the judgment of validity, and to generate questions and ideas. The students we interviewed who worked in discussion-based environments were not only required to contribute different aspects of their selves, they were required to contribute *more* of their selves. In this small study we found the notion of identity to be important. Students in the different schools were achieving at similar levels on tests but they were developing very different relationships with the knowledge they encountered (Daskal & Simpson, 2000). Those students who were only required to receive knowledge described their relationships with mathematics in passive terms and for many this made the discipline unattractive. Those who were required to contribute ideas and methods in class described their participation in active terms that were not inconsistent with the identities they were developing in the rest of their lives.

That was a small interview study but it served to illuminate the importance of students' relationship with the discipline of mathematics that emerged through the pedagogical practices in which they engaged. In the final study that I will describe, a team of researchers is monitoring the learning of approximately 600 students as they go through three different high schools in the US. Two of the schools offer a choice of mathematics curriculum, which they describe as 'traditional' and 'reform' oriented. In the 'reform' classrooms we observe very different patterns of interaction than those in 'traditional' classrooms and as we work to understand the capabilities that are being encouraged by these examples of classroom interaction we are finding notions of identity and agency to be important.

The students in the reform classrooms we are studying, as in the project based school in England, are given the opportunity to use and apply mathematics, a process which confers upon them considerable amounts of human agency. Students are required to propose 'theories', critique each other's ideas, suggest the direction of mathematical problem solving, ask questions, and 'author' some of the mathematical methods and directions in the classroom. We are finding that the *nature* of the agency in which students engage in these classrooms is related to the discipline of mathematics and the practices of mathematicians in important ways. Such insights have emanated from an analytic frame proposed by Andrew Pickering (1995). Pickering studied the work of professional mathematicians and concluded that their work requires them to engage in a 'dance of agency' (1995, p116). Pickering considers some of the world's important mathematical advances and identifies the times at which mathematicians use their own agency – in creating initial thoughts and ideas, or by taking established ideas and extending them. He also describes the times when they need to surrender to the 'agency of the discipline', when they need to follow standard procedures of mathematical proof, for example, subjecting their ideas to widely agreed methods of verification. Pickering draws attention to an important interplay that takes place between human and disciplinary agency in mathematical work and refers to this as 'the dance of agency' (1995, p116).

Pickering's framework seems important for our analyses of the different practices of teaching and learning we observe. 'Traditional' classrooms are commonly associated with disciplinary agency, as students follow standard procedures of the discipline. 'Reform' classrooms, by contrast, are associated with student agency, with the idea that students use their own ideas and methods. The idea that students use their own ideas instead of learning standard methods is part of the reason that many are concerned about 'reform' approaches in the US, but we do not see students failing to learn standard methods in our observations of 'reform' classrooms. Rather than a group of students wandering unproductively, inventing methods as they go, we see a collective engaged in the 'dance of agency'. The students spend part of their time using standard methods and procedures and part of the time 'bridging' (Pickering, 1995, p11) between different methods, and modifying standard ideas to fit new situations. In many of the traditional classrooms I have studied, in this and previous years, students have received few opportunities to engage in the 'dance of agency', and when they need to engage in that 'dance', in new and 'real world' situations, they are ill prepared to do so. When I interviewed a class of students in the fourth year of the reform program at one of the schools, the students all described an interesting relationship with mathematics that contrasted with the students who had learned calculus in traditional classes. As part of the interviews we asked students what they do when they encounter new mathematical problems that they cannot immediately solve. In the extracts below the students give their responses:

K: I'd generally just stare at the problem. If I get stuck I just think about it really hard and then just start writing. Usually for everything I just start writing some sort of formula. And if that doesn't work I just adjust it, and keep on adjusting it until it works. And then I figure it out. (Keith)

B: A lot of times we have to use what we've learned, like previous, and apply it to what we're doing right now, just to figure out what's going on It's never just, like, *given*. Like "use this formula to find this answer" You always have to like, change it around somehow a lot of the time. (Benny)

These students seem to be describing a 'dance of agency' as they move between the standard methods and procedures they know and the new situations to which they would apply them. They do not only talk about their own ideas, they talk about adapting and extending methods and the interchange between their own ideas and standard mathematical methods. The student below talks in similar terms:

E: Like, if nothing else, it's breaking out of the pattern of just taking something that's given to you and accepting it and just going with it. It's just looking at it and you try and point yourself in a different angle and look at it and reinterpret it. It's like if you have this set of data that you need to look at and find an answer to, you know, if people just go at it one way straightforward you might hit a wall. But there might be a crack somewhere else that you can fit through and get into the meaty part. (Ernie)

Many of the students in the traditional classrooms I have studied frequently 'hit a wall' when they were given mathematics problems to solve. They tried to remember standard procedures, often using the cues they had learned. If they could remember a method they would try it, but if it did not work, or if they could not see an obvious method to use, they would give up. The students we interviewed in the reform classes described an important practice of their mathematics classroom – that of working at the interplay of their own and disciplinary agencies – that they used in different mathematical situations. Additionally the students seemed to have developed identities as mathematics learners who were willing to engage in the interplay of the two types of agency. The students had developed what we are regarding as a particular *relationship with the discipline of mathematics* that meant that when they met new mathematics problems they *expected* to adapt and apply methods to solve the problems. It seemed that their capability in different situations depended partly upon the knowledge they had learned, partly upon the practices in which they engaged and partly upon the relationships they had developed with the discipline of mathematics that emerged through the practices of their classrooms.

A number of researchers have written about the importance of productive beliefs and dispositions (Schoenfeld, 1992; McLeod, 1992) but the idea of a 'disciplinary relationship' serves to connect knowledge and belief in important ways. Herrenkohl and Wertsch (1999) have suggested a notion that addresses this connection, that they call the 'appropriation' of knowledge. They distinguish between mastery and appropriation, saying that too many analyses have focused only upon students' mastery of knowledge, overlooking the question of whether students 'appropriate' knowledge. They claim that students do not only need to develop the skills they need for critical thinking, they also need to develop a disposition to use these skills. In claiming that students need to 'appropriate' knowledge, they suggest a connection between the content students are learning and the ways they relate to that knowledge. The fact that the students who learned through open-ended projects in England were able to use mathematics in different situations may reflect the similarity in the practices they met in different places, but it also reflects the fact that they had developed a positive, active relationship with mathematics. They expected to be able to make use of their knowledge because of the opportunities they had received to engage in a disciplinary dance. Thus they were able to 'transfer' mathematics, partly because of their knowledge, partly because of the practices in which they engaged, and partly because they had developed an active and productive relationship with mathematics. This idea seems to pertain to theories of learning transfer and expertise in important ways, expanding notions of capability beyond knowledge and practice to the dispositions they produce and the relations between them.

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