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ABSTRACT

We examined elementary school teachers' justifications to number- theoretical "for all" propositions and existence propositions, some of which are true while others are false. We also assessed whether teachers regarded their justifications as mathematical proofs. About half of the teachers produced formal algebraic proofs. A smaller number of teachers produced non-formal proofs appropriate for presentation in elementary school classes. However, a substantial number of teachers applied inadequate methods to validate or refute the propositions. Finally, many teachers were uncertain about the status of the justification they gave. (Author)

LITTLE CHILDREN, BIG MATHEMATICS: LEARNING AND TEACHING IN THE PRE-SCHOOL

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Three-, 4-, and 5-year-old children (in the U.S., we call them pre-schoolers) are little. They are short. They do not know as much as adults. As Piaget suggested, their thought may be different from adults'. Mathematics is big. It is one of the crowning intellectual achievements of humanity. It is abstract, symbolic, and difficult. Many believe that mathematics is too big to teach to little children because they cannot understand it a meaningful way.

I strongly disagree, and in this talk will present a different view of early mathematics education. I will show that little children are more competent than we usually think they are. They have a great interest in mathematical ideas and even in mathematical symbolism. They can learn mathematics and can benefit from teaching. I will also describe how research can both stimulate and be stimulated by the development of a rich program of early childhood mathematics education.

COUNTING AND EVERYDAY MATHEMATICS

In this talk, I will focus on one small area of young children's mathematical knowledge, namely their acquisition of counting words, the language of counting—not enumeration, not understanding of cardinality, not any of the other important topics on which researchers have focused.

Why place so much emphasis on counting words? At first, the topic appears to be relatively uninteresting. Counting is often seen as a mechanical process; it is not considered to entail genuine mathematical thinking. Children seem to acquire counting through a process of rote learning. In this view, although children clearly need to learn to count, mathematics educators and researchers should devote attention to deeper mathematical topics.

I think these views are wrong. First, counting is not simply a matter of rote memory; it is an interesting cognitive activity. And second, counting can present young children with opportunities for rich mathematical learning. It is worth thinking about how children learn to count and how it can be taught. Moreover, the example of counting suggests some general lessons about research on the psychology of mathematics education and on curriculum development.

We know several things about counting language. First, young children like to count, even to high numbers. Walkerdine (1988) reports that "... children initiate a counting sequence and often count spontaneously" (p. 105). Irwin and Burgham (1992) describe how children at 5 years of age enjoy counting up to relatively large numbers, like 100. Similarly, Gelman (1980) observes that some children exhibit a spontaneous interest in large numbers. "I remember a five-year-old who told me that she had been trying to count to a million, had been at it for two days and said it would still take her a long time to reach her goal!" (p. 65).

I have observed similar phenomena.

> One day during "free play" Kasheef, a 5-year-old African-American boy, was sitting at a table alone. He poured a large number of small beads from his hand onto the table. "Oh, man! I got one hundred," he proclaimed. Then, he began to count the beads as he picked them up one by one, although grasping them was difficult because they were so small. When he reached "ten," Barbara joined the activity. She said the numbers but did not actually count out the beads. As he progressed through the twenties, Kasheef made a mistake. He dropped the bead as he said "twenty-six," but ignored it and continued counting. Barbara kept pace with him. When he got to "thirty," Kasheef dropped the bead again. He paused for a second and said, "Wait! I made a mistake."

So here is a child who spontaneously decided to enumerate a large collection of objects, estimated their number, said the numbers correctly through 30, and performed a 1-1 matching of the spoken numbers to the objects. No one told him to undertake these activities. He made two mistakes, but recognised the second. He was highly motivated, persisting over a period of time despite the physical difficulty of the task. And he was joined by a peer who also found the activity of some interest.

> Kasheef began to count the beads again. "One, two, three...." When he counted "three," Barbara picked out one bead, showed it to him, and said, "I have one." But Kasheef ignored her and continued to count. Barbara joined him at "five." When he counted "ten," Barbara again showed her beads to Kasheef, "I got, look...." Kasheef again ignored her and continued counting. Barbara kept pace with him, uttering the same number words. When he counted "twelve," Barbara shouted in his ear, as if she wanted to distract his attention. Kasheef ignored her again. When Kasheef reached "forty-seven," another girl came over and asked, "What do you count?" Again, he ignored the distraction. After the "forty-nine", Kasheef paused, apparently not knowing what came next. Barbara said, "fifty" and Kasheef followed her with "fifty-one, fifty-two...."

Kasheef had clearly decided to count the whole collection of beads and was not about to be distracted from the task. He persisted despite several interruptions. He counted very well, making almost no mistakes until he reached "forty-nine." At this point, he needed and benefited from Barbara's help. But once given "fifty," he knew what to do with it: he knew the rule for constructing numbers from decades and units.

> When they reached "fifty-two," Ruthie came to the table, picked out one bead, and joined their counting, "fifty-two, fifty-three..." Madonna also joined the counting, "fifty-six, fifty-seven..." The female chorus said the counting words but sometimes picked out

> several beads at once or sometimes didn't pick out a single bead. After "seventy-nine," Kasheef again paused. After the girls said, "eighty," Kasheef continued, "eighty-one, eighty-two..." When they reached "eighty-five," the girls fought with one another to grab beads, which then scattered on the table, dropped on the floor and rolled in every direction. Despite the disturbance, Kasheef continued, "eighty-six, eighty-seven... ninety-four."

Kasheef's counting drew a crowd of girls who found the activity of interest. They were competent in saying the counting numbers, but were not careful in enumerating the objects. They were disruptive. Kasheef largely ignored them, except when they provided him with the numbers he did not know. Kasheef showed remarkable persistence in his attempt to count each bead and to say the numbers correctly. He was almost always accurate, getting help on only two occasions.

> After he picked out the ninety-fourth bead, there were no more on the table. He picked up some beads from those on the floor, and continued, "ninety-five, ninety-six, ninety-seven..." What he said next was inaudible. Madonna shouted, "One hundred!" raising her arm. Kasheef, Barbara, and Ruthie said, "One-hundred!" right after her. And Kasheef concluded, "We all got one hundred." In the course of the celebration, the girls got down under the table and began to squabble over picking up the beads. Kasheef joined the fight. At this point the teacher came over and yelled at the children for their misbehaviour.

We see then that Kasheef continued to count very well, at least to "ninety-seven," and like the other children was very proud to reach the goal, "one-hundred." What the teacher observed was a group of children squabbling under the table.

Many features of this example stand out. Kasheef and the other children were able to count, making only a few errors, to 100. They showed great interest both in the counting numbers themselves and in counting as a means to determine a quantity. Also, Kasheef persisted, despite frequent distraction; he was excited, interested, enthusiastic, hard-working.

Kasheef's behaviour was not unique. In a large naturalistic study—which I will not describe in detail here—we found that during free play, 4- and 5-year old American children from various social class and ethnic groups spontaneously and frequently engaged in "everyday mathematics" (Ginsburg, Pappas, & Seo, 2001). This refers to the mathematical skills and competencies children employ in the ordinary environment. Young children's everyday mathematics is usually intuitive (like the idea that adding makes more), (Brush, 1978); it is informal (not acquired through formal instruction); and it is free of written symbolism. In our study, children exhibited everyday mathematics in a little more than 40 percent of the minutes during which we observed them. Further, we failed to uncover statistically significant social class differences in the overall frequency of everyday mathematics. For example, the average percentages of minutes in which lower-, middle-, and upper-SES children engaged in everyday mathematical behaviour were 44, 43, and 40 respectively.

We also found that three types of everyday mathematics occurred most frequently in children's free play: enumeration, magnitude, and pattern. Enumeration included saying the counting words, enumerating objects (like Kasheef), and observing the quantity of a set of objects (e. g., "There are five here"). Magnitude involved such activities as judging that one set had more than another or that a quantity was very large. And pattern was shown when, for example, children devised elaborate symmetries, in three dimensions, during block play. Again, we failed to uncover statistically significant social class differences in the frequency or complexity of these three types of everyday mathematics.

In brief, our research clearly reveals that children frequently and spontaneously engage in an everyday mathematics that includes not only counting but also work with magnitudes and patterns. The research also shows that teachers were almost never involved in these mathematical activities.

We also know something about the cognitive processes underlying counting (Miller & Parades, 1996). To count successfully, learners (children or adults) must first memorise a relatively small set of meaningless sounds in an arbitrary sequence. In English, these sounds—a kind of meaningless "number song" (Ginsburg, 1989)—include "one, two... ten." Second, learners must induce and then use rules for generating subsequent number words. In English, the rules for generating 21 through 99 are reasonably clear: take the decade word (twenty, thirty, and the like) and add to it the memorised sequence "one... nine" and then go to the next decade word. Use of rules like these was evident in the fact that once given a decade word he did not know, Kasheef easily attached to it the unit numbers.

Unfortunately, English presents learners with several difficulties. One is that a few number words above "ten," namely "eleven" and "twelve," appear to be arbitrary. A second is that generating the decade words is not so simple. "Seventy" bears a clear relation to "seven" whereas the relation between "twenty" and "two" is not entirely transparent. A third difficulty is really serious. The number words from "thirteen" to "nineteen" put the units (e. g., "thir") first and the tens ("teen") second, whereas all number words after 20 do the reverse, as in the case of "twenty-three."

Counting need not be as odd as occurs in English. East Asian languages based on the Chinese are perfectly regular. To be sure, their learners must memorise the equivalent of "one" through "ten," but after that they can benefit from a consistent set of rules. After "ten," comes "ten-one... ten-nine," and after that, with perfect regularity, come "two-ten, two-ten-one, ... nine-ten-nine." Perhaps the coherent design of Chinese counting contributes to its speakers' mathematical understanding.

In brief, learning the language of counting is a key aspect of everyday mathematics. Children enjoy counting and often try to count to large numbers. Counting requires more than memorisation; like other aspects of language, it is based on a system of rules.

Consider next how counting can be taught and what the teaching of mathematics teaches us.

A MATHEMATICS CURRICULUM

For many years, the message taken from Piagetian research was that young children are egocentric and cannot understand important mathematical ideas. Certainly young children's thought suffers from various limitations. At the same time, the recent research literature on everyday mathematics paints a different portrait, namely that young children possess greater competence and interest in mathematics than we ordinarily recognise.

Given this view of young children's abilities, I decided to develop a comprehensive mathematics curriculum for 4- and 5-year-olds. Why is it useful to do so? One might argue that a better policy would be let young children enjoy and grow their everyday mathematics. In this view, young children learn from play and cannot profit from formal instruction; indeed, early lessons will only have the effect of turning them against mathematics at an earlier age than usual.

My answer to this argument is that play is not enough for several reasons. First, in the American context at least, many children, particularly poor children, are at risk of school failure and failing schools. They need a kind of mathematical head start to succeed in school. Second, mastering challenging tasks fosters children's feelings of confidence and competence (Stipek, 1997). Third, many teachers are not aware of young children's mathematical competence. Pre-schoolers' successful learning of formal mathematics may help teachers appreciate the children's competence and raise expectations of how much they should be learning in school in the later grades. And most importantly, our observations show that in their everyday play, the children already engage in and enjoy significant mathematical learning. It is great fun for them and of great intellectual significance.

As John Dewey (1976) advised: "Abandon the notion of subject-matter [like mathematics] as something fixed and ready-made in itself, outside the child's experience..." (p. 278). As we have seen, the child's experience includes an everyday mathematics that is more substantial and enjoyable than much of what is typically taught in school. Given this fact, as Dewey puts it, "Guidance is not external imposition. It is freeing the life-process [that is, children's everyday mathematics] for its own most adequate fulfilment" (p. 281). We therefore choose to engage in early mathematics education to help children achieve the fulfilment and enjoyment of their intellectual interests.

In collaboration with my colleagues Carole Greenes and Robert Balfanz, and with the generous support of the National Science Foundation, I have been engaged in the development of a comprehensive mathematics curriculum for little children, Big Math for Little Kids (Ginsburg, Greenes, & Balfanz, 2003). Our goal is to help children to think mathematically and to explore mathematical ideas in depth, over a lengthy period of time and through extended activities. Building on children's everyday mathematics, the curriculum consists of what we think are exciting and enjoyable activities, games, and stories, organised into six major strands, namely number, shape, measurement, patterns and logic, operations, and spatial relations. Our program presents the study of mathematics both as a separate "subject" and as an integrated part of other pre-school activities. Sometimes, the curriculum presents "math activities" like counting or studying shapes. Sometimes, it blends the mathematics into such activities as stories, songs, and block building. The program employs large group activity, small groups, and individual exploration. We believe that young children need to learn how to behave and learn in large groups. But they also profit from the greater degree of teacher attention possible in small groups. And they need time for individual learning and exploration. We characterise the program as offering playful but purposeful learning.

TEACHING COUNTING WITH PERSONALITY AND PIZZAZZ

I would like to describe what we learned from one small but I think important part of the program, namely learning the counting words. Our first counting activities were in good measure based on the research already described. We knew that children enjoy counting and like to count to high numbers. We also knew that the first 12 numbers in English are meaningless sounds that need to be memorised, that the numbers from 13 to 19 are "backwards," and that the numbers thereafter are very regular. In most curricula, 4-year-olds are generally taught to count to about 20, and 5-year-olds to 31 (the highest number of days on the calendar). Just think what this means. We ask young children to learn the harder part of counting before the easier part. They must learn the memorise the first 12 meaningless sounds, then learn the backwards numbers that violate the rules that later become evident, and finally are not allowed to count far enough into the larger numbers so that they could detect the only rational part of the English counting system. It is as if we wanted to make learning to count as hard as possible for young children.

Our solution was to make counting easier by helping young children to count to one hundred rather than 20 or 31. Counting from 21 to 99 makes sense, and then it's fun to shout "one hundred" at the end. Counting from 1 to 20 does not make sense and is hard to do. So we devised several activities to promote counting. In "Numbers with personality," children assign each "decade" a distinctive personality. They decide that the numbers from "one" to "nine" will be timid, and say them in a quiet, mousy voice. Later they might describe the twenties as "roaring like a mouse" or the forties as whining like a baby. We almost always refer to the numbers between "ten" and "nineteen" as the "yucky teens," because they suffer from ugly design features.

In another variant of this activity, "Counting with Pizzazz," the children choose different body movements for the decades. They may jump the twenties and hop the thirties. And when they get to the end of a decade, they always say, "cut," or "think," to indicate that they have reached the end of a decade and that they must decide what new decade term comes next.

Teachers using our program generally choose to do these activities every day throughout the school year. In New York, our wonderful pre-school teacher, Suzanne Mir, does counting at the beginning of every day during "circle time" (which in her room morphs throughout the year into square, rectangle, and pentagon time). The children enjoy the activities, they seem to be learning some interesting things about counting that I will soon describe, and we all get a good workout.

INTRODUCING NUMERALS

For some time, we taught counting with the activities described above. But then we developed another aspect of the counting curriculum. The story begins with an observation of a 3-year-old's behaviour. One of our activities is a game in which children step from one written numeral, printed on a large cardboard card, to another. They may start on 1, then have to locate 2, step on it, and then proceed in order to 5 or 10. This obviously helps them recognise the sequence of written numbers. For our present purpose, the game itself is not important. What is interesting is that one day, 3-year-old Arlo went to the place where the numerals were stored, took them out, and during free play tried to arrange them in order. He was successful in arranging the first several numerals, but skipped one of them. He then put a few numerals in the wrong order. After receiving some help from one of the graduate students working on the project, Angelika Yiassemides, he quickly learned to order the numerals to 10. Then he continued, and within a few days, got up to about 30 with few mistakes. He kept asking for more numerals to arrange in order, and we had to keep making them. We eventually made numerals up to 100 and he was well on his way to mastering the entire sequence.

I was surprised at his intense interest in these symbols. But then it occurred to us that use of numerals could aid counting. Numerals after all are perfectly regular with respect to place value and the base ten system and hence may be easier to deal with than English counting. The tens are always on the left and the units on the right. There are no irregularities like "thirteen." We write "thirteen" as 13, not 31, (which is similar to the way we say it). Of course, knowledge of counting can help children learn to read and give meaning to the numerals. In brief, if children are interested in counting and they are also interested in numerals, then the two activities might reinforce each other. Further, from the child's point of view, reading numerals is even more "grown-up" than counting them.

So we devised a number chart in which starts with numerals ending in 0 on the left and 9 on the right. We developed activities in which the teacher pointed to the numerals as children counted out loud. "Cut" or "think" was always said when children reached a numeral ending in 9. The new decade could be found by going down a row and to the end at the left. At first the teacher said nothing about the numerals. Gradually she pointed out various features, for example that the 3 in 34 referred to thirty; that 20, 30... are just like 2, 3...; and that as you go straight down a column all numbers end with the same numeral. She sometimes left out a numeral in the chart and asked the children to tell what was missing. In short, we felt that linking written numerals to spoken numbers could make underlying patterns relatively transparent and thereby facilitate their discovery.

One of our reasons for doing this is that our previous research showed that discovering and using patterns is one of children's most frequent everyday activities. Children are pattern detectors—in language, perception, and mathematics. This is not really a constructivist idea; it is a "realist" notion. Patterns exist in the world—sometimes the constructed world of mathematics—and can be detected. The detection may provide the necessary conditions for subsequent construction.

Of course the discovery of mathematical patterns in charts is not new. For example, Trivett (1980) offers a wonderful description of the mathematics that can be discovered in the multiplication table. But what is new, I think, is using this method at the pre-school level to teach counting, and potentially even related notions revolving around place value and the base ten system.

WHAT CAN WE LEARN FROM THE LEARNING OF COUNTING?

These then are some of the ways we teach counting. What can we learn from our experiences? How successful are the activities and what do children learn from them? Although we have not yet done traditional evaluations of Big Math, we have had the opportunity to conduct, over a long period of time, many informal observations of children's learning in our schools in New York, Baltimore and Chelsea, Massachusetts. In my experience, these "anecdotal observations" often point to important phenomena, can provide useful insights, and lead to interesting speculations and topics for further research.

First, almost all children thoroughly enjoy counting activities (and other aspects of mathematics). At the outset, young children seem highly motivated to learn school mathematics—at least in our Big Math curriculum. They are not turned off by early mathematics and do not feel that learning it is an imposition. There is no reason for early childhood educators to fear that little children will find mathematics instruction to be unpleasant. The issue is rather how and why the initial motivation gets undermined over the course of the first few years in school.

Second, almost all children are proud of their success. Little children see counting prowess as a sign of being a "big kid" and being "smart." Probably their teachers and their parents share this belief. It is important for little children to achieve big results. Real achievement makes for self-confidence and heightened motivation (Stipek, 1997). It is important to present children with challenging work so that they can master it. We need to allow children to succeed.

Third, it appears that by the end of the year many of the 4-year-olds do indeed succeed. Many can count to at least 100 and can count by tens to 100 as well. (They can also achieve at a high level in many other areas of our curriculum.) Yet I do not want to exaggerate their success. It is clear that not all children succeed at this level. For example, one child I recently observed (after about 7 months of counting activities) claimed that "twenty" comes not only after "nineteen" but also after "thirty-nine" and any other number she was asked about. Clearly, careful evaluations with pre- and post-tests and control groups are necessary to make firm statements about children's achievement.

Fourth, we have observed that some 3-year-olds in the classroom participate in the counting activities for many months without saying anything, but after a period of time may suddenly start counting to a reasonable level. They seem to be absorbing a great deal without exhibiting their learning in external behaviour. It will be important to investigate the kinds of learning in which 3-year-olds engage during systematic mathematics instruction.

Fifth, we have observed that children's learning seems to be very inconsistent. Some children seem to "get it" on one day but not another. Others, like the 3-year-olds, seem not to "get it" at all for long periods of time but then seem to catch on very quickly. Most often we assume that that learning is more or less orderly. But writers like Siegler (1996) have pointed to the inconsistencies and general messiness of learning, and researchers like Dowker (1998) have highlighted the important role of individual differences in mathematical abilities. Clearly, greater insight into these matters will help us calibrate instruction to young children's needs. Our teachers sometimes worry that the children do not seem to be learning immediately. (Some parents are even more worried about this than the teachers.) We have learned to ask teachers (and parents) to be patient and not to worry about whether the children "get it" right away. I think our advice is generally sound; but it would be useful to have a deeper understanding of these issues and some firmer evidence upon which to base the recommendations.

Sixth, what mathematical ideas, if any, do children learn when they are learning to count in the Big Math program? Our observations suggest that they learn not only to say the counting words, but to discover the rules that underlie counting and to explore patterns that can be discovered in the counting numbers. Even though it is a highly symbolic and abstract activity, learning to count can be an exciting pattern detection and generation activity. Exploring patterns in mathematics need not involve manipulating concrete things. Instead, counting involves the manipulation of words and of ideas about words. The children also seem to learn that counting and numerals are related and have a similar structure, which, of course, involves ideas about place value and the base ten system. At the same time, the children probably do not learn from our counting activities very much about cardinality. But that does not necessarily imply that the children are failing to do interesting mathematical work when they count. Cardinality is not the only important thing there is for young children to learn about mathematics. (Of course, other parts of our program deal directly with cardinality). In brief, even learning a counting language and relating it to abstract written symbolism can present little children with important intellectual challenges and enable them to do some interesting mathematics. There's a conjecture for further research. And of course, studying many other topics at the pre-school level can also lead to genuine mathematical learning.

Seventh, although I have been stressing young children's competence, it is also clear that there are many things they cannot do and learn. We have not had great success in teaching ordinality, for example, and I can once again confirm that ideas of equivalence are very difficult for young children. Little children cannot learn everything. But the central message is that they can learn a great deal and we don't yet have a clear idea of the limits on their learning when they are engaged in an exciting curriculum with a good teacher.

Eighth, group instruction appears to be very effective in teaching counting and almost all aspects of our program. Most of our Big Math activities are introduced in the group setting, later extended to small groups, and then to individual work. I was never a believer in group instruction for little children. In fact, I think I have written some nasty things about this practice. Yet I have been surprised at how effective it can be. But I don't understand how it works and how effective teachers operate. Little is known about group and individual pedagogy in the early childhood setting; learning more about it would no doubt lead to many practical benefits, including effective teacher education.

And finally, our experiences teach us something about the processes of research and curriculum development. We began with some research on young children's abilities. Drawing upon this research, along with other resources, like our informal experiences with young children, we developed a curriculum. In the process of trying it out, we observed that children were engaged in some surprising learning. Indeed, it is fair to say that the children would not likely have exhibited this learning and the competencies it implies had we not developed the curriculum. The observation of the new learning then led to the revision of our educational goals and the creation of new curriculum, which in turn led to new and perhaps unexpected behaviour that needs to be studied and explained. So research leads to curriculum and curriculum leads to research. Exciting mathematical environments create new expressions of competence, which is not static. As Vygotsky (1978) maintained, we need to study the "...dynamic mental state, allowing not only for what has been achieved developmentally but also for what is in the course of maturing" with adult assistance (p. 87). Observing children within a challenging curriculum allows us to do this.

CONCLUSIONS

Yes, mathematics is big, but little children are bigger than you might think. Early mathematics education is a great opportunity for children, teachers and researchers alike.

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