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ABSTRACT

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A Methodology in the Teaching Process of Calculus and its Motivation.

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A Methodology in the Teaching Process of Calculus and its Motivation.

The development of calculus and science by being permanent, didactic, demands on one part an analytical, deductive study and on another a application of methods, rhochrematics, resources, within calculus which allow to dialectically confront knowledge in its different phases and to test the results. For the purposes of this study, the motivation in calculus, the characteristics of motivation, correlation between theory and practice, the concrete and abstract, creativity, and the systematization in the teaching of calculus, are presented.

Title: A Methodology in the Teaching Process of Calculus and its Motivation.

To form a first idea of what motivation represents, and at the same time detect the different parameters to which Motivation can respond, one thinks on the following question:

Why has Calculus been chosen and not another science?

Each student must think about his or her experiences and express his or her ideas.

The reasons which have been presented most frequently are these:

- One has EASE for this discipline.
- GOOD NOTES have been obtained.
- There is GOOD DEMAND for Calculus teachers.
- By COINCIDENCE (there was no professor who described my abilities).
- One wants to prove to the previous teachers that one IS CAPABLE.
- Because of the POWER TO PROVE WHICH CALCULUS HAS.
- Because Calculus is PLEASANT.

1.1 WHAT IS MOTIVATION.

Motivation can be understood as a psychological impulse in a person toward the achievement of his or her goals, taking into account that man by nature needs to be motivated to reach the full achievement of his abilities and his will. On the other hand, one must be aware that each person acts according to a motive, to a need and to a personal expectation; so one should not be surprised because a certain activity stimulates some and does not affect others. One must be also aware that motivation cannot be detached from the person's reality, and still less insist on forced motivations.

> "To think that the pleasure of contemplation and the search can be favored by means of coercion and the sense of duty is nothing but a large-bore mistake." EINSTEIN.

> "With such procedure, taking place without punishment nor rigor, light and softly, without any coercion and as in a natural manner..."
> COMENIUS

1.2 CHARACTERISTICS OF MOTIVATION

The following are the main characteristics of motivation:

1.2.1. The motivated behavior is cyclical. First appears a desire or impulse, then a means of satisfying or reducing such impulse, later comes the achievement of objectives and finally one returns to the initial state.

1.2.2. Motivation is selective. The person tends to satisfy in first place the needs which respond to a specific motive or to the strongest stimulus, to later satisfy others.

1.2.3. The motivated behavior is active and persistent. To more motivation always corresponds more activity and the person insists in the achievement of the objectives inherent to the type of motivation received.

1.2.4. Motivation is homeostatic, that is, conscious or unconsciously it is constantly bolstered up.

1.3 MOTIVATION IN TEACHING.

It is thought: How to motivate teaching?

That is in accordance with the studies and experiences, what methods would be used to motivate teaching? After listening and discussing the opinions of the students it is possible to conclude that motivation in teaching can be done by the application of the following didactic principles:

- Activity in teaching.
- Correlation between theory and practice.
- Unity between the concrete and the abstract.
- Systematization.
- Correlation with other sciences.
- Applicability.

1.3.1. These didactic principles do not function ones independently from the others, but they are intimately related; because of that, when speaking of one of them, one speaks direct or indirectly of the rest.

1.3.1.1. Activity in the teaching.

The infant is active and restless by nature, his organs develop and strengthen through physical and mental activity, but he must be concerned that such development is harmonic providing adequate activities, pleasurable for him and the development of his will and his creativity.

It is also said that activity in the teaching eliminates or minimizes the magisterial lectures which produce so much tedium in the students.

> "The magisterial lecture can be useful if the students solicit it as a means of elucidating issues already debated." La nueva pedagogía, Biblioteca Salvat, G.T.

> "The laws of Physics are not enunciated; one trusts that they are discovered by the students." GOODMAN

1.3.1.2. Correlation between theory and practice.

In the teaching there must be a biunivocal relationship between theory and practice. One cannot be without the other not risking falling into an incomplete and partialized teaching.

Practice needs the enlightenment of theory and in turn, theory requires the sense that practice provides.

The importance of practice in the fixation of knowledge has been experimentally proven in the following way: A group of students is explained how to do a certain experiment; another group watches the teacher while he carries it out; a third group does the experiment by its own means. After six months the fixation level of the first groups is similarly low while the third group shows a high level of fixation.

1.3.1.3. Unity between the concrete and the abstract.

Following the didactic suggestion that in the achievement of knowledge one must go from THE NEAR TO THE REMOTE, FROM THE EASY TO THE DIFFICULT, FROM THE KNOWN TO THE UNKNOWN; the following steps are recommendable.

Contact with the objective reality.	→	Graphic representation of the objects and the relationships between them.	→	Deduction and analysis of properties. Formulation of hypothesis, postulates, theories...
-		-		-
-		-		-
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-				
CONCRETE	→	DIDACTIC	→	ABSTRACT

The recommendation of beginning the teaching in the concrete is based on the proven fact that the more SENSES are used in the acquisition of a knowledge, the more solid the learning is.

> "A good characteristic is worth more than entire volumes of political oratory." LANCELOT

> "There are things which having been presented clearly by Mechanics, have been later proven by Geometry, because the first method lacks the proving force which is inherent to the second. It is frequently easier, after having, thanks to the mechanical method, a first idea on the issue, to elaborate the demonstration than without having the preliminary mechanical experience." ARCHIMEDES...

One is aware, nonetheless, that a good learning does not remain in the concrete, but it moves forward toward abstraction in an organized and conscious manner on the individual's part.

1.3.1.4. Systematization in the Teaching.

The systematization is of vital importance in all human activities, especially in those of the educational kind. Calculus could be named the Science of Order and an exquisite organization of the contents is therefore essential to achieve greater clarity in the teaching as well as in research.

Some of the aspects that must be taken into account for a good systematization of the teaching of Calculus are the following:

Organize the global content into units, each unit divided into parts and each part with one or more gravity centers.

Link the known subject with the new one.

Constantly strengthen (revise, insist) the contents already seen.

Regularly control and evaluate the subject, the method, the teacher...

Have a sequential order.

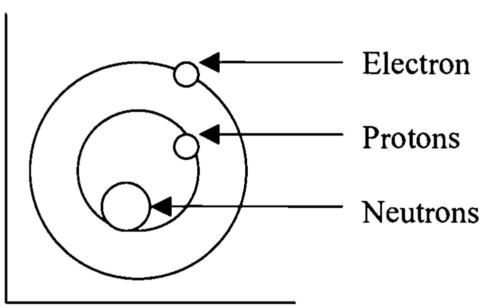
1.3.1.4. Correlation with other Sciences.

Every discipline that is a study object, must be related with other areas of knowledge so it does not lose sense, when appearing isolated from the curricular context and so it awakens the most diverse interests in the students. A way to achieve correlation or integration of different subjects is through applications.

Example: Calculus and Archaeology.

Who knows the C^{14} method to determine the age of materials of organic origin?

Discuss for about 15 minutes and afterwards present before the class.



When adding neutrons to the nucleus, the latter becomes unstable and emits energy in the form of radiations.

C_{14} ATOM

Fig. 1-1

In living beings the C^{14} percentage is of 0.002%. Upon death, the percentage of C^{14} decreases when emitting energy in the form of radiations, then the age is determined according to the amount of C^{14} in the remains.

Be:

m: mass of one radioactive atom in the moment "*t*"

"*m*" is a decreasing function of "*t*" since "*m*" decreases by the emission of radiations.

The variation in the amount of matter is proportional to the mass in a given moment:

$$\frac{-dm}{dt} = km$$

$$\frac{-dm}{m} = kdt$$

$$-Ln(m) = kt + c$$

$$Ln(m) = -(kt + c)$$

$$m = e^{-kt} - c$$

$$m = e^{-kt} \cdot e^{-c} \quad (1)$$

For $t = 0$. $m_0 - e$

Questions to pose:

1) How is e^X defined?

A: $e^X = 1 + X + \frac{X^2}{2!} + \frac{X^3}{3!} + \cdots + \frac{X^n}{n!}$

There is a reason for it being called exponential.

$$e = \underset{x \to \infty}{Lím} \left| x + \frac{1}{x} \right|^x ; \quad e = \underset{h \to 0}{Lím} (1 + h)^h$$

2) Why is that function important?

A: Because the change or variation of the function is proportional to the original:

$$\frac{de^x}{dx} = e^x$$

For what time "t", the mass will be reduced to half?

In (1)

$$\frac{m_0}{2} = e^{-kt_1} \cdot m_0$$

$$Ln\left(\tfrac{1}{2}\right) = -k\,t_1$$
$$k\,t_1 = -Ln\left(\tfrac{1}{2}\right) \qquad t_1 = \frac{Ln(2)}{k} = \frac{0.693}{k} : \qquad \text{Half-life period.}$$
$$k\,t_1 = Ln(2)$$

In the case of Radium, k = 0.00064/year.

$$t_1 = \frac{0.693}{0.00064} = 1.575 \; years$$

1.2.1.6. <u>Principle of Rochrematic Applicability</u>.

The rochrematic applications play a very important role in the teaching of science and very specially in the teaching of Calculus.

“Of everything perceived, consider at once what use it may have so nothing is learned in vain” COMENIUS.

Frequently one finds students who say: THAT WHAT FOR? Attitudes like this show lack of interest or motivation. The student is not able to see neither the importance nor the usefulness of the subject. To avoid these situations it is CONVENIENT to propose certain typical problems to the class and when the student finds out that he or she cannot solve them, then explain the necessary theory for it.

Some examples of how the above can be related:

THE FUNDAMENTAL LIMIT OF THE SEQUENCE

$\left\{\left(1 + \tfrac{1}{n}\right)^n\right\}$: THE NUMBER e.

The sequence $\left\{\left(1 + \tfrac{1}{n}\right)^n\right\}$ appears very frequently in the solution of pure and applied mathematics problems; in this part we will see some of these problems.

In this part we will demonstrate that $\{(1+1/n)^n$ is increasing but bounded, and by virtue of the limited growth principle it will have this limit which is represented by the letter *e* and is one of the fundamental limits of Mathematics. We will expound the demonstration in several stages with the intention of facilitating its comprehension.

1st) Applying Newton's formula to obtain the expansion of the power of binomial to the nth term, one has:

$$(1+1/n)^n = 1^n + n\cdot 1^{n-1}\cdot\frac{1}{n} + \frac{n(n-1)}{1\cdot 2}\cdot 1^{n-2}\cdot\frac{1}{n^2} + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}\cdot 1^{n-3}\cdot\frac{1}{n^3} + \cdots$$

$$\cdots + \frac{n(n-1)(n-2)\cdots[n-(n-2)]}{1\cdot 2\cdot 3\cdots(n-1)}\cdot 1\cdot\frac{1}{n^{n-1}} + \frac{n(n-1)(n-2)\cdots[n-(n-1)]}{1\cdot 2\cdot 3\cdots(n-1)\cdot n}\cdot\frac{1}{n^n}$$

2nd) Let us simplify the terms of the above formula, suppressing the 1 factors, which do not affect the result; also, the fractions which appear in the formula have in their numerators products of decreasing factors, starting from *n*, and in their denominators powers of *n* whose exponent *coincides* with the number of factors of the numerator. Dividing each factor of the numerator by a *n* factor, contained in the power of *n* of the denominator, one reaches:

$$(1+1/n)^n = 1 + 1 + \frac{1}{1\cdot 2}\cdot(1-1/n) + \frac{1}{1\cdot 2\cdot 3}\cdot(1-1/n)\cdot(1-2/n) + \cdots$$

$$\cdots + \frac{1}{1\cdot 2\cdots(n-1)}\cdot(1-1/n)\cdot(1-2/n)\cdots(1-\tfrac{n-2}{n}) + \frac{1}{1\cdot 2\cdots n}\cdot(1-1/n)\cdot(1-2/n)\cdots(1-\tfrac{n-1}{n}), \quad (1).$$

3rd) The factors which figure contained between parentheses in the above formula are different and when *n* increases the subtrahends decrease with which the value of such differences *increases*; therefore, when *n* increases, the value of $(1+1/n)^n$ *increases*, for two reasons: first, because the number of terms of it's expansion increases, and second, because the terms become greater.

SUMMARY: *The sequence* $\{(1+1/n)^n$ *is increasing.*

4th) The parentheses of the second member of (1) are lesser than one and therefore are contraction factors (when a number is multiplied by a factor lesser than one the number is decreased, for example 8 · 0,5 = 4); therefore, if we suppress the parentheses in the second member of (1) we will obtain an expression of *higher value*, that is:

$$\left(1+\tfrac{1}{n}\right)^n < 1+1+\frac{1}{1\cdot 2}+\frac{1}{1\cdot 2\cdot 3}+\cdots+\frac{1}{1\cdot 2\cdot 3\cdots(n-1)}+\frac{1}{1\cdot 2\cdot 3\cdots n}, \quad (2).$$

5th) If in the second member of (2) we substitute the denominators of the fractions, by powers of base 2, which are *smaller* than such denominators, the fractions will increase; but we have that $1 \cdot 2 \cdot 3 > 2^2$, $1 \cdot 2 \cdot 3 \cdot 4 > 2^3$, ..., $1 \cdot 2 \cdot \ldots \cdot (n-1) > 2^{n-2}$, $1 \cdot 2 \cdot 3 \cdot \ldots \cdot n > 2^{n-1}$, with what one concludes that:

$$\left(1+\tfrac{1}{n}\right)^n < 1+1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots+\frac{1}{2^{n-2}}+\frac{1}{2^{n-1}}, \quad (3).$$

6th) The terms in the second member of (3), starting from the second, constitute a geometric progression of a ratio of ½: we know that the sum of infinite terms of one such progression comes given by the formula $\underset{n\to\infty}{Lím}\, S_n = \frac{a}{1-r}$, which we will simply write like this: $S = \frac{a}{1-r}$. This formula is of great use in Mathematics and is expressed as: The sum of the infinite terms of an undefined geometric progression, of a ratio lesser than one, is equal to its prime term divided by one minus the ratio. Applying such formula to the second member of (3) we come to, however large *n* may be,

$$\left(1+\tfrac{1}{n}\right)^n < 1+1+\frac{1}{1-\frac{1}{2}} = 1+\frac{1}{\frac{1}{2}} = 1+2 = 3.$$

CONCLUSION: All the terms of $\left\{\left(1+\tfrac{1}{n}\right)^n\right.$, however large *n* may be, are lesser than 3; therefore:

The sequence $\{(1+1/n)^n\}$ is bounded.

Having demonstrated that the sequence $\{(1+1/n)^n\}$ is increasing, but bounded, it will have a finite limit, Cantor's "limited growth principle". This limit is represented by the letter *e* (initial of it's discoverer's name, *Euler*).

The number *e* is an irrational number (moreover it is *transcendent*, that is that it is not the solution of an algebraic equation). The first ciphers of *e* are obtained by giving values to *n*, successive, in the expression $(1+1/n)^n$ whose limit is precisely the said number *e*.

The approximate value of *e* to the millionths, is:

e = 2,718281...

OTHER SEQUENCES WHICH HAVE THE NUMBER *e* FOR LIMIT.

If we observe the *nth* term of the sequence that defines *e*, we see that it is composed of: "*One plus an infinitesimal raised to the reciprocal of the infinitesimal.*" The sequences whose *nth* term is constructed in accordance to this rule, have the number *e* for a limit. Let us see some examples.

The sequences $\left\{\frac{1}{2^{n-1}}\right\}$, $\left\{\frac{1}{n^2+1}\right\}$, $\{\sin a_n\}$, when $a_n \to 0$, $\left\{\frac{1}{n^3-9}\right\}$, are infinitesimal; in accordance with the previous rule we will have:

$lím\left[1+\frac{1}{2^{n-1}}\right]^{2^{n-1}}=e$, $lím\left[1+\frac{1}{n^2+1}\right]^{n^2+1}=e$, $lím[1+\sin a_n]^{\frac{1}{\sin a_n}}=e$, when $a_n \to 0$, $lím\left[1+\frac{1}{n^3-9}\right]^{n^3-9}=e$.

Sometimes the structure "one plus an infinitesimal raised to the reciprocal of the infinitesimal" cannot be seen very clearly, but by simple algebraic transformations one can reach it.

EXAMPLE. If we are asked to find the limit for $\left\{\left[\frac{3n+4}{3n}\right]^{\frac{3n}{4}}\right\}$ (1), being that $\frac{3n+4}{3n}-1=\frac{3n+4-3n}{3n}=\frac{4}{3n}$, the expression (1) could be written like this: $\left[1+\frac{4}{3n}\right]^{\frac{3n}{4}}$ and in this form one sees that it is "one plus an infinitesimal raised to the reciprocal of the infinitesimal" and therefore: $lím\left[1+\frac{4}{3n}\right]^{\frac{3n}{4}}$ is *e*.

SOME PROBLEMS WHOSE SOLUTION DEPENDS ON THE NUMBER e

FORMULA OF CONTINUOUS UNIFORM GROWTH

Explanation of the problem:

Let us suppose that a substance grows continuously. It can be a living being, the wood of a tree, a substance being formed through a chemical reaction, a capital to which its interests were being continuously accumulated, a ball rolling on a snow-covered road with uniform velocity.

Furthermore let us suppose that the growth ratio is uniform, this is, constant along time, which means that, if a unit of substance turns into $1 + r$ in a certain time period, the same will occur along the entire time period through which the phenomenon is considered. This condition is only observed approximately in natural phenomena; for example, the growth of a tree is not uniform, for there are seasons in which the growth is faster than in others.

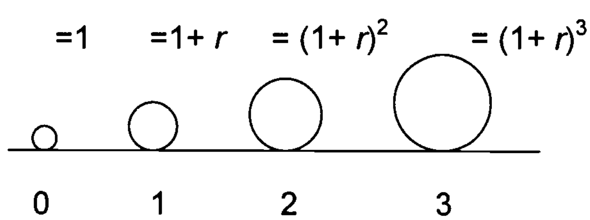
Now we will pose the following fundamental problem: Given an initial amount of substance, *c*, and a time period, *t*, (*t* time units), *what will be the final amount of substance,* C, *that will* *have been formed from the initial amount, c, through a continuous uniform growth, being **r** the rate per one of growth in the time unit, for **t** time units?*

To solve this problem, typical of Infinitesimal Calculus, we will consider three stages: In the first one we will assume that the growth of the substance occurs *every fixed time period*, that is, that the substance remains constant until the period expires, instant in which an instantaneous growth occurs, to remain constant once more until the period expires again, another instantaneous growth happening, at the end of the second period, and so on.

In the second stage we will divide the time period into *n* equal parts and we will assume that there is growth at the end of each *nth* part of the initial period, and finally, in the third stage, we will go to the limit when *n* tends to infinity, with which the *nth* part of the period considered initially will tend to zero and the accumulation of substance will happen by instants, infinitely small, with which we will have the continuous growth. This last stage, in which a step to the limit is taken, corresponds to a typical process of the Infinitesimal Calculus and it will allow us to obtain the formula of continuous uniform growth.

First Stage: Let us assume that the growth of the substance occurs at the end of each time period whose duration we will take as time unit. In the intermediate instants it is assumed that the amount of substance remains constant. Let us see into what does a unitary amount of substance transform itself after *t* time periods in which there is accumulation.

See the following Figure 1-2.



(Fig. 1-2)

Figure 1-2 shows us that when the process begins, in instant zero, the amount of substance is equal to 1; in the instant of expiring a period the substance unit grows transforming itself into $(1 + r)$, being r the "*rate per one of uniform growth*", in the considered period. In the instant of expiring the second period a new growth occurs, and it is clear that, if a substance unit transforms itself into $(1 + r)$, the units we had at the end of the previous period, that is $(1 + r)$, will transform themselves into what one unit transforms itself multiplied by the existing units, that is, into $(1+r) \cdot (1 + r) = (1 + r)^2$, because it is a growth proportional to the substance, that is to say, that double, triple, etc., amount of substance, grows double, triple, etc.; and generally, when multiplying the amount of existing substance by any real number the growth is multiplied by the same number. Similarly, upon ending the third period a new growth takes place; the substance there was at the end of the second period transforms into what one unit transforms itself, that is, $(1 +r)$, multiplied by the existing units, $(1 + r)^2$, reaching $(1 + r) \cdot (1 + r)^2 = (1 + r)^3$; so following to the end of the tth period, the initial substance unit will have become transformed into $(1 + r)^t$.

Now one easily understands, by the proportionality previously alluded, that if one substance unit transforms itself in t periods into $(1 + r)^t$, c units will transform themselves in the same time into $c \cdot (1 + r)^t$.

That way one comes to the famous formula for discrete or discontinuous growth (performed by jumping).

If we represent by C the final amount of substance, after t time periods, we will have:

$$C = c \cdot (1 + r)^t \qquad (1),$$

being r the rate per one of growth in the considered time unit.

When c is interpreted as the initial capital, r as the rate per one of annual interest (what one dollar produces in one year), and t as the number of years, formula (1) is the famous compound interest formula; in this case C represents the final capital which is attained placing a capital c for t years at a compound interest of r per one annual rate, with accumulation of interests every year. But, by the explanation we have given here of this formula, it is understood that it can be applied to other cases of growth.

Second Stage: In this second stage we divide the time unit from the previous stage into n equal parts, and assume that the growth occurs every nth part of the unitary period of the previous stage.

Since r represents the rate per one of growth in the time unit, and we assume that the growth is uniform, that is, proportional to time, in a period equal to one nth of the unit, the rate per one of growth will be $\frac{r}{n}$. Now, in t time units there are t · n nths of a unit, or, t · n accumulation periods. By a reasoning analogous to that of the first stage, except that here the accumulation periods (of substance growth) happen every nth of the period of the first stage and that the rate per one of growth in this new period is of $\frac{r}{n}$, and that instead of t accumulation periods there are $t \cdot n$ periods, one reaches the formula:

$$\mathrm{C} = \mathrm{c}\left(1 + r/n\right)^{nt} \quad (2)$$

Formula (2) corresponds also to a discontinuous growth, but the growths of the substance occur every nth part of the time in the first stage.

When c represents the initial capital and t the number of years, formula (2) is that of compound interest with interest accumulation every nth of years; r still represents the rate per one of annual interest. For example, if n = 4, one has: $\mathrm{C} = \mathrm{c}\left(1 + r/4\right)^{4t}$, which is the formula for the compound interest with quarterly accumulation of interests. It is easily understood that the final capital which is attained by this type of interest accumulation to the capital every fraction of a year, is greater than that which would be attained by annual accumulation.

Third Stage: If we assume now that $n \to \infty$ (reads: n tends to infinity), the growth will happen every instant, for $\frac{1}{n} \to 0$, and we will have the continuous growth.

Formula (2) can be written like this:

$$C = c\left[\left(1+\frac{r}{n}\right)^{\frac{n}{r}}\right]^{r \cdot t} \quad (3).$$

If in formula (3) we reach the limit when $n \to \infty$, one notices that the expression contained between the brackets is composed of "one plus an infinitesimal raised to the reciprocal of the infinitesimal", and therefore, the said expression tends to the number e when $n \to \infty$, reaching in this way the formula for continuous uniform growth:

$$C = c \cdot e^{r \cdot t} \qquad (4).$$

To reach this formula we have used the theorem: "The limit of a power is equal to the limit of the base raised to the exponent of the power".

Formula (4) is broadly used, especially in Physics and Chemistry.

References.

- Ackerman, T.A.; Evans, T.A. A didactic example of the influence of conditioning on the complete latent ability space when performing DIF analyses. Paper presented at the Annual Meeting of the National Council on Measurement in Education. Atlanta, GA. April 13-25, 1993.
- Artigue, M.; Perrin-Glorian, M.J. Didactic engineering, research and development tool: Some theoretical problems linked to this duality for the learning of Mathematics an International Journal of Mathematics Education; VII n.1 pp. 13-18. 1991.
- Berg, K.F. Scripted cooperation in high school Mathematics: Peer interaction and achievement. Paper presented at the Annual Meeting of the American Educational Research Association. New Orleans, LA. April 4-8, 1994.
- Burscheid, H.J.; Struve, H.; Walther, G. A Survey of Research Zentralblatt für didaktik der Mathematik. 24(7). 296-302. 1992.
- Brazil, R.D. A covenant for change. The Paideia manual: A guide to the re-training of America's teaching force. Illinois Univ., Urbana Coll. of Education; Paideia Inst. of Hyde Park, Chicago, IL. 1992.
- Brown, S.I.; Cooney, T.J.; Jones, D. Mathematics teacher education. In Houston, W.R., Haberman, M. and Sikula, J., (Eds.) handbook of research on teacher education. Macmillan. New York. 1990.
- Damarin, S.K. Rethinking science and Mathematics curriculum and instruction: Feminist perspectives in the computer era. Journal of education; V173 n.1 pp. 107-23. 1991.
- Ellerton, N.F.; Clements, M.A. Mathematics in Language: A review of language factors in Mathematics learning. Deakin University. Geelong, Victoria. 1991.
- Graebar, A.; Baker, K. Curriculum materials and misconceptions concerning multiplication and division. Focus on learning problems in Mathematics. Vol. 13. pp. 25-37. 1991.
- Grouwns, D.A. (Ed.). Handbook of research on Mathematics teaching and learning. Macmillan. New York. 1992.

- Guttman, C. All children can learn: Chile's 900 Schools Programme for the Underprivileged Education for all: Making it work. Basic Education Division, UNESCO, 7 Place de Fontenoy, 75351 Paris 07, sp France. 1993.
- Hasselbring, T.S.; and others. An evaluation of specific videodisc courseware of student learning in a rural school environment. George Peabody Coll. for Teachers, Nashville, TN. Learning Technology Center, 1991.
- Hoyles, C.; Noss, R. (Eds.). Learning Mathematics and Logo. MA. MIT. Cambridge. 1992.
- Leithold, Louis. El Cálculo con geometría analítica. Ed. Harla. Mexico. 1992.
- Swokowski, E.W. Cálculo con geometría analítica. Grupo Editorial Iberoamericana. Mexico. 1990.
- Tall, D. Advanced Mathematical thinking. Dordrencht: Kluwer Academic Publishers. 1991.
- Von Glasersfeld, E., Ed. Radical constructivism in Mathematics education. Kluwer Academic Publishers, 101 Philip Drive, Norwell, MA 02061. Mathematical education library series. Volume 7. p. 229. 1991.

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