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ABSTRACT

This paper discusses how mathematics can be interpreted as an integral part of technological planning and decision making, and how mathematics therefore operates as an integral part of technology. Three aspects of mathematics in action are presented. The paper provides for the theoretical task of grasping mathematics in action and identifying how mathematics supports a technological imagination, how it establishes possibilities to investigate particular aspects of possible technological constructions, and how mathematics becomes installed in society and starts operating as part of technological devices. It is also pointed out that an understanding of mathematics in action is crucial for interpreting basic aspects of social development. (Contains 66 references.) (KHR)

Mathematics in Action: A Challenge for Social Theorising

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Mathematics: Insignificant or Crucial?

Is it true that mathematics—and we talk about 'real' mathematics and not about, say, school mathematics—has no social significance? Or does also 'real' mathematics provide a crucial resource for social change?

In *A Mathematician's Apology*, G.H. Hardy discusses the usefulness of mathematics, and his general conclusion is: "If useful knowledge is [...] knowledge which is likely, now or in the comparatively near future, to contribute to the material comfort of mankind, so that mere intellectual satisfaction is irrelevant, then the great bulk of higher mathematics is useless" (Hardy, 1967, p. 135). Could mathematics, nevertheless, do any harm? Hardy concludes: "[...] a real mathematician has his conscience clear; there is nothing to be set against any value his work may have; mathematics is [...] a 'harmless and innocent' occupation" (Hardy, 1967, pp. 140–141). In the final pages of his *Apology*, Hardy draws conclusions about his own work in mathematics: "I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or for bad, the least difference to the amenity of the world" (Hardy, 1967, p. 150). Hardy provides a picture of 'real' mathematics as an intellectual enterprise that cannot be judged by its effects on society, for the simple reason that there are no such effects.[1] Mathematics is *insignificant* in the sense that mathematics does not have any structuring impact on social development. Therefore, social theory is well justified in ignoring the possible social functions of mathematics.[2]

The philosophy of mathematics has been occupied by investigating the foundations of mathematics. What are the sources of this knowledge? What is the nature of mathematical objects and of mathematical truths? This preoccupation easily leads to the claim that an adequate understanding of mathematics can be obtained by studying the logical architecture of mathematics from 'within' the edifice of mathematics. The classical position in the philosophy of mathematics, thus, seems to align nicely with the assumption of insignificance: mathematics has no influence on social affairs and, therefore, it can be adequately interpreted in terms of its internal and logical structures alone.[3]

Let me contrast this perspective on mathematics with the following claim made by Ubiratan D'Ambrosio in 'Cultural Framing of Mathematics Teaching and Learning': "In the last 100 years, we have seen enormous advances in our knowledge of nature and in the development of new technologies. [...] And yet, this same century has shown us a despicable human behaviour. Unprecedented means of mass destruction, of insecurity, new terrible diseases, unjustified famine, drug abuse, and moral decay are matched only by an irreversible destruction of the environment. Much of this paradox has to do with the absence of reflections and considerations of values in academics, particularly in the scientific disciplines, both in research and in education. Most of the means to achieve these wonders and also these horrors of science and technology have to do with advances in mathematics" (D'Ambrosio, 1994, p. 443). D'Ambrosio strongly indicates that mathematics is positioned in the nucleus of social development. The role of mathematics is *crucial* and must be considered in the investigation of a wide range of social phenomena.

However, what is the response in the most overall social theories to the question of whether mathematics is indeed insignificant or crucial for social development? Naturally no simple answer is found, but if we study works such as *The Constitution of Society* and *Social Theory and Modern Sociology* by Anthony Giddens, and *The Theory of Communicative Action* by Jürgen Habermas, we do not find any reference to mathematics.[4] We do, of course, find suggestions for basic categories to interpret social development. So, judged by the silence about mathematics, the conception in much social theorising appears to be effectively that of Hardy's: The social impact of this science is insignificant. There is no reason to consider mathematics in particular in order to interpret social affairs.

In what follows, I shall discuss how mathematics can be interpreted as an integrated part of technological planing and decision making, and how mathematics therefore operates as an integrated part of technology. Therefore, I find that an *understanding of mathematics in action is crucial for interpreting basic aspects of social development*. This idea has recently been discussed with particular reference to critical mathematics education, but although it concerns social theorising, it has not got solid ground in sociology.[5]

Reflexivity

In *Reflexive Modernization*, Ulrich Beck, Anthony Giddens and Scott Lash present (in individual written chapters) a discussion of modernisation. According to Beck, we now face "the possibility of creative (self-)destruction for an entire epoch: that of industrial society. The acting 'subject' of this creative destruction is not the revolution, not the crisis, but the victory of Western modernization" (Beck et al., 1994, p. 2). In fact, it does not seem possible to identify more specifically any acting subject for this creativity. And Beck continues: "This new stage, in which progress can turn into self-destruction, in which one kind of modernization undercuts and changes another, is what I call the stage of reflexive modernization" (Beck et al., 1994, p. 2). So, *reflexive modernisation* is not about radical changes taking place as a result of certain critical dysfunction of modernity. Beck does not follow a variant of Karl Marx's analysis, that "capitalism is its own gravedigger"; instead he finds that it is "the victories of capitalism which produce a new social form" (see Beck et al., 1994, p. 2ff.). So this new social form is born within the existing social structures. Reflexive modernisation includes an unplanned change of industrial society which harmonises with existing political and economic orders. Nevertheless, reflexive modernisation breaks up the contours of industrial society and opens 'paths to another modernity'. Although there will be no revolution, there will be a new society.[6]

If we want to understand the dynamics of social development, then we should not seek for that understanding from within the institutions which represent this development. The mechanisms of reflexivity bypass the democratic institutions and operate as part of the social subconsciousness. This problem is significant to sociology: "The idea that the transition from one social epoch to another could take place unintended and unpolitical, bypassing all the forums for political decisions, the lines of conflict and the partisan controversies, contradicts the democratic self-understanding of this society just as much as it does the fundamental convictions of its sociology" (Beck et al., 1994, p. 3). Beck indicates that sociology has not been able to grasp the basic principles of reflexivity. In what follows, I shall try to explain in what sense I agree with this. However, before we embark on this analysis we need to follow Beck in one more step.

Beck introduces the notion of *risk society* which "designates a developmental phase of modern society in which the social, political, economic and individual risks increasingly tend to escape the institutions for monitoring and protection in industrial society" (Beck et al., 1994, p. 5).[7] Risk society is symbolised by many events such as the Chernobyl disaster, financial crises, pollution of food, etc. According to Beck: "Society has become a laboratory where there is absolutely nobody in charge" (Beck, 1998, p. 9). In this return of uncertainty a new frame of social life is established. Risk society is however formed by basic elements of industrialised society: "One can virtually say that the constellations of risk society are produced because the certitudes of industrial society [...] dominate the thought and action of people and institutions in industrial society. Risk society is not an option that one can choose or reject in the course of political disputes. It arises in the continuity of autonomized modernization processes which are blind and deaf to their own effects and threats" (Beck et al., 1994, pp. 5–6).[8] Industrial society accumulates its own products, including their effects and side-effects, and eventually this turns society into a new form. In particular, due to 'certitude', industrial society produces risks, which transform the industrial society into a risk society. But how might the nature and the process leading to the emergence of new risk structures be understood?

Mathematics! Let us take a look at the index of *Reflexive Modernization*: No reference to mathematics. However, we find the following sentence in Beck's chapter: "Risks flaunt and boast with mathematics" (Beck et al., 1994, p. 9).[9] In *Reflexive Modernization* this sentence is left as a passing remark. If reflexive modernisation can be discussed and analysed in depth, without any reference to mathematics, then the thesis of insignificance appears justified. But I want to illustrate that this is not the case. The recent development of the industrialised society—establishing a reflexive modernisation, a risk society, or maybe a network society—is linked to a mathematical resourced development. Mathematics makes part of that 'certitude', which transforms industrial society into a risk society.

Mathematics in Action

By means of a couple of examples, I hope to illustrate the importance of considering how mathematics may be operating as part of a technological planning and decisions processes, and how mathematics becomes part of technology itself.[10]

My first example of *mathematics in action*[11] refers to a model presented by Dick Clements in 'Why Airlines Sometimes Overbook Flights'.[12] Airlines deliberately overbook?! Why? Naturally, in order to maximise profit or, to put it more gently, to make sure that the prices of tickets are kept to a minimum. It is essential to try to prevent flying with empty seats. The costs associated with flying a full airplane or one with empty seats are approximately the same: "The airline must pay its pilots, navigators, engineers and cabin staff regardless of whether the airplane is full or not. The extra fuel consumed by a full airplane compared to that consumed by a half empty one is very little as a percentage of the gross fuel load [...] The take-off, landing and handling fees charged by airports are independent of the number of passengers carried by an aircraft" (Clements, 1990, p. 325). For every departure, it is most likely that some of the passengers who have already booked will fail to turn up ('no shows'): "The standard conditions of carriage for airline passengers allow full fare passengers to do this without penalty. They can turn up at the airport later and their tickets will be valid for another flight" (Clements, 1990, p. 326). As a consequence, it appears possible to overbook flights. Certainly, there must be an upper limit to this, as the company is going to compensate those passengers who might be refused, or 'bumped', if more than the expected number of passengers turn up. Furthermore, it must be considered that the probability of a passenger being a 'no show' depends on, for instance, the destination, the time of the day, the day of the week, and, as we shall see return to later, the type of his or her ticket.

All this can be incorporated into a mathematical model containing parameters such as the cost of providing a flight, the fare paid by each passenger, the capacity of the airline, the number of passengers booked on a flight, the costs of refusing a passenger who has booked, the probability of a booked passenger arriving being a 'no show', the surplus generated by a flight, etc.[13] With reference to the model, it becomes possible to plan the overbooking in such a way that revenue is maximised. Essential information, of course, is the probability, p, that a booked passenger will in fact be a 'no show'. If this probability is equal to 0, then there is no point in overbooking, but if p is greater than 0, then we can devise an overbooking strategy. The actual value of p for particular departures can be estimated by means of statistical records concerning previous departures, and in this way the degree of overbooking can be graduated according to a set of relevant parameters. For instance, the degree of overbooking the last evening flight from Copenhagen to London should be kept lower than that of an afternoon flight, as the compensation for bumping a passenger in the first case would include hotel costs.

This example illustrates that mathematics may serve as a basis for planning and decision-making. The traditional principle: 'Do not sell any more tickets than there are seats' becomes substituted with the much more complex principle: 'Overbook, but do it in such a way that revenue is maximised, considering the amount of money to be paid as compensation, the destination, the time of departure, the day of the week, as well as the long term effects of having sometimes to bump passengers who in fact have made valid bookings.' This new principle cannot be created or come to operate without mathematics. Its complexity presupposes that applications of mathematical techniques are 'condensed' into a booking programme. The principle illustrates what, in general, can be called *mathematics-based action design*.

A mathematical booking-model does not only *describe* a certain situation, in this case, patterns of reservation, cancellations and 'no shows'. Mathematics does not only provide a 'picture' of reality, as suggested in several philosophies of mathematical modelling. In fact, many descriptions of mathematics as language assume a picture-like theory of what mathematics does. In this way the descriptions embark on the metaphysics from Ludwig Wittgenstein's *Tractatus Logico-Philosophicus*. However, should mathematics be compared with language, then the speech act theory, as suggested by John L. Austin and John R. Searle, invites the following question: What is in fact *done* by means of mathematics? This question introduces also the idea of linguistic relativism as presented by Edward Sapir and Benjamin Lee Whorf: What world view is provided by a specific language? Applied to the language of mathematics, the question becomes: What world views are made available by means of mathematics? Or: How is the world constructed, according to mathematics?[14]

A booking model does not just describe some principles of queuing. It actually establishes new types of queues. And it might create a situation in which some people suddenly have to make new travel plans. Mathematics becomes part of a technique, here represented by the management of booking of flights. But this is just a particular example illustrating the fact that mathematics of all possible kinds and complexities operates in a wide range of modern management systems. Mathematics becomes part of reality, as mathematics-based design is put in operation.

An adequate understanding of the actions carried out in the process of selling tickets is not possible unless we pay attention to the existence of the booking-model. What interpretation to make of the airline assistant's exclamation: "Oh, I'm so sorry, but unfortunately we have some problems with the computer system...." How would a sociological interpretation of this particular situation look like? Without awareness of the existence of a booking model, the assistant's explanation may appear plausible. But this explanation does not capture the fundamental rationality of the situation. In many cases, 'bumping' a passenger is not a computer mistake. Instead it is a well-calculated consequence, occurring when the passenger in question comes to represent a statistical 'deviation' from the expected norm. If we want to interpret the episode, we need to understand how mathematics operates behind the desk. This is the case as well with many other situations where mathematical models provides rationales (or pseudo-rationales) for decision making. The example of overbooking is not a unique example of mathematics-based action design. Instead it can be seen as paradigmatic of any (complex) business management. Without being aware of mathematics being in place, sociological explanations of such enterprises will become superfluous, if not misleading. To me sociology must be aware of mathematics-based action design in order to interpret a wide range of social phenomena.

Mathematics is certainly involved in grand scale economic management. This can be illustrated by the Danish macro-economic model ADAM (Annual Danish Aggregated Model), which is used by the Danish Government as well as by other institutions (private as well as public).[15] One of the principal aim of ADAM is to promote 'experimental reasoning' in political economy. In this way, ADAM provides a basis for political decision-making. One way of doing so is to provide economic prognoses. Another, maybe even more important application of the model, is to provide different scenarios. Experimental reasoning tries to address the question: If a certain set of decisions is made and the economic circumstances develop in a particular way, what would be the consequence? Implications of a scenario can be investigated by a comparison between applications of the model to different sets of values of the parameters in question. In this way it becomes possible to observe the implications of a political action without having first to carry out the action. Naturally, such reasoning is basic in politics. However, by relying on the model, the political discourse changes because the experimental reasoning which refers to the model acquires a new authority. Experimental reasoning can help to discover which economic initiatives are 'necessary' in order to achieve some economic aims, say, within a definite time limit. (Certainly, 'necessary' has to be put in inverted commas, as necessity refers to the space of possibilities produced by the model.)

As emphasised by the builders of the model, the quality of the scenarios provided by the model depends on the accuracy of the estimations of the variables providing the foundation for the calculations. It naturally has to be added that the quality of the presented scenarios also depends on the quality of the model itself. What, then, does a model like ADAM consist of? An awful lot of equations! These equations can be summarised in different ways, one possibility is to group them into seven clusters having to do with commodity demands, commodity supply, labour market, prices, transfers and taxes, balance of payments, and income. In fact, ADAM can be considered as a set of sub-models addressing certain aspects of the Danish economy. The system of equations in ADAM is constructed around different types of variables, exogenous and endogenous. The value of an exogenous variable is determined from outside the model; the population of Denmark is as an example of such a variable. To estimate the employment-unemployment ratio, this number is essential. Endogenous variables are those which are determined by the model itself, and many variables, which appear exogenous in some part of the ADAM-complex, are determined by other parts of the model, so when ADAM is considered in its totality, they become endogenous.

When such a system of equations is constructed and accepted, experimental political reasoning can be carried out. The problem is, of course, how to present such reasoning. Obviously, the detailed structure of the model cannot be presented, nor grasped, in actual political discussions. A possibility is to let experimental reasoning take the particular form of a multiplier analysis. Let us assume that the equation $y = f(x_1, \dots, x_n)$ belongs to the model. If the variable x_1 is multiplied by a certain factor c, the result would be $y_c = f(cx_1, \dots, x_n)$. By calculating $d = y_c / y$, we it can be claimed that when the input x_1 is multiplied by c, the output y will be multiplied by the factor d. Questions inviting multiplier analysis are raised everywhere in political discussions. For instance, if the government tries to carry out an expansive finance policy, and expand public demand, what effect would such a policy have on the degree of unemployment? In particular, if the government increases its public demand by 5%, how much would the unemployment then decrease? A multiplier analysis would provide an estimation.

ADAM is certainly not merely providing a description of some part of socio-economic reality. It also imposes certain theoretical assumptions about this reality. Taken together, ADAM "displays features which are characteristically Keynesian" (Dam, 1986, p. 31). Thus, the choice of the basic equations, which supply the model with a 'soul', does not simply reflect certain economic reality; it also prescribes a particular perception of economic affairs. Also in this case, the phenomena of linguistic relativism must be kept in mind. ADAM provides a new example of mathematics-based action design. By being a resource for actions, the model becomes part of economic reality. It even comes to dominate this reality, to the extent that its assumed economic linkages establish real linkages. ADAM was created by mathematics, but ADAM got life. And, as we all know, ADAM did not stay alone.[16]

Since 1981, ADAM has been connected to the international LINK project, through which a huge number of national macro-economic models are structured into a world model. In 1995, 79 nations and regions participated in the LINK Project, organised by the United Nations. The connection of different models makes it possible to estimate many of the exogenous variables of particular national, macro-economic models. With reference to a 'connecting structure of models' such exogenous variables can be regarded as endogenous variables. In this way, our world gets enveloped in calculations.

Human beings become part of a reality structured by economic principles formulated in mathematical terms. We observe the same phenomenon associated with the booking-model: the mathematical model becomes part of a social reality. Therefore, we must again raise the question: How is a sociological interpretation of economic decision-making possible without an understanding of the nature of the economic world as represented (and, therefore, reworked and constructed) by an ADAM or other macro-economic models? In *Social Theory and Modern Sociology*, Giddens discusses the problems of macro-economics in relation to social theorising. One of the issues he raises is that such models include assumptions, for instance in terms of a 'rational expectation theory', which may compromise the descriptive value of such models. I am sure Giddens is right: macro-economic models cannot be justified by their descriptive relevance for sociology. But this is not the point. Whatever the macro-economic models might do or not do, they are in fact *used*, and *this use* is of critical importance for social theorising, as understanding this example of mathematics in action is one of the conditions for understanding political and economic decision making.

Mathematics does not only influence the economic part of our reality. In 1995, the Danish Council of Technology (Teknologirådet) published the report, *Magt og Modeller* (*Power and Models*), discussing the increasing use of computer-based models in political decision making. The report refers to 60 models, which cover the following areas: economics, environment, traffic, fishing, defence, population. The models are developed and used by public as well as private institutions in Denmark.[17]

The authors of the report *Magt og Modeller* emphasise that political decision-making concerning a wide range of social affairs is closely linked to applications of such models. They also emphasise that this development may erode conditions for democratic life: Who construct the models? What aspects of reality are included in the models? Who have access to the models? Are the models 'reliable'? Who is able to control the models? In what sense is it possible to falsify a model? If such questions are not clarified in an adequate way, traditional democratic values may be hampered. As an illustration of this problem, I shall summarise the comments of the report related to traffic and environmental issues. In this case models are often used in support of decisions which cannot be changed, like the construction of a bridge between two major Danish islands. Decisions concerning traffic are almost exclusively based on models developed in private companies. It is not usual to develop more than one model to illuminate a certain issue. Finally, it happens that models are used in order to legitimate *de facto* decisions, in the sense that a model-construction provides numbers and figures which justify a decision already made.

Beck claims that the process of reflexivity, which leads to a risk society, occurs outside of democratic control, and that it eludes contemporary sociology. The extensive use of mathematical modelling, as discussed in *Magt og Modeller*, exemplify this claim. How to obtain a democratic access to decision-making, which refers to mathematical modelling processes? The conditions for democratic life may be eroded by the spread of mathematical based action design.[18] Thus, it becomes difficult to ignore the role of mathematics, if we want to establish a sociological discussion of conditions for democracy regarding the nature of technological development.

Three Aspects of Mathematics in Action

The philosophy of mathematics has interpreted mathematics as abstract and has tried to study sources for abstraction. By talking about mathematics in action, I concentrate on the inverse process: seeing how mathematical abstractions are projected into reality. When we use mathematics as a basis of technological design, we bring into reality a technological device that has been conceptualised by means of mathematics. First, it exists in the world of mathematics, later it is brought into reality by an actual construction. A mathematical 'speech act' has been carried out.

In order to specify aspects of this particular act, let us consider the notion of *sociological imagination*, which expresses a capacity to separate what is necessary from what is contingent and, therefore, possible to change. A fact is not only a fact but also a (social) necessity, when it is impossible to imagine that the fact is not the case. If we consider a particular culture where a certain work process is carried out in a particular way (maybe obeying some ceremonial traditions), and no alternative to that approach is identified, then this process would appear to be a (social) necessity.[19] The existence of an imagination that describes alternatives to an actual situation makes a difference. In this case, the fact is 'reduced to' a contingent fact. The experienced necessity is revealed as an illusion when an alternative is conceived. This is the power of sociological imagination: A social given has been identified as available to change.[20]

A process of design includes the identification and the analysis of hypothetical situations, and mathematics helps by providing material for constructing such situations. By means of mathematics, we can represent something not yet realised and therefore identify technological alternatives to a given situation. Mathematics provides a form of technological freedom by opening a space of hypothetical situations. In this sense, mathematics becomes a resources for *technological imagination* and, therefore, for technological planning processes including mathematical based action-design. However, as we shall come to see, all the attractive qualities associated to sociological imagination are not simply transposed to technological imagination. This is important to keep in mind.

The space opened by a technological imagination might very well contain hypothetical situations which are not accessible via common sense. A mathematical framework provides us with new alternatives. For instance, when a booking model is established, it becomes possible to specify 'special fare schemes' like the APEX.[21] Thus, the model makes clear the relevance of creating certain groups of passengers, where it becomes easy to predict the probabilities of 'no show'. In order to do a more detailed planning (How many APEX are going to be offered? By how much should the price be reduced?), it becomes essential to have a booking model available. The set of equations in ADAM also constitutes hypothetical situations. The ADAM makes it possible to establish political thought experiments; this means conceptualising details of situation, which is not possible to identify by common sense. In other respects, the space of hypothetical situations might be very limited. Certainly, ADAM does not support political thought experiments which contradict the political priorities, installed in ADAM in terms of its basic equations. When a technological imagination relays on mathematics, it may provide a very particular space of hypothetical situations.

Political and economic interests can express themselves in the set of technological alternatives that are established as mathematically well-defined. Therefore, mathematics as part of a technological imagination can interact with other power structures. As mentioned previously with reference to models for traffic planning, the set of alternatives established by mathematics can be so limited that the modelling in fact serves as a legitimisation of a *de facto* decision. By providing one and only one alternative, this alternative appears to be a necessity within the space of hypothetical situations provided by the model. This situation helps to establish credibility in the political claim that a certain political decision is a 'necessary' decision.

Thus, the first aspect of mathematics in action concerns technological imagination: *By means of mathematics, it is possible to establish a space of hypothetical situations in the form of (technological) alternatives to a present situation. However, this space may contain serious limitations.*

Mathematics provides the possibility for *hypothetical reasoning*, by which I refer to analysing the consequences of an imaginary scenario. By means of mathematics we seem to be able to investigate particular details of a not-yet-realised design. Thus, mathematics constitutes an important instrument for carrying out detailed thought experiments. Because of ADAM, it is possible to carry out hypothetical reasoning related to economic policy. This reasoning is counterfactual, as it address implications of the form: 'p implies q, although p is not the case'. A representation of p is provided by ADAM in terms of equations including the values of the relevant parameters. The hypothetical reasoning can then address a particular situation 'realised' by ADAM. Some conclusions of the hypothetical reasoning can then be simplified and expressed in multipliers that are easily included in the common political discussion. Without mathematically based hypothetical reasoning, the political discussion would take a completely different form. It would lose a great deal of so-called 'precision'. Hypothetical reasoning represents an essential element in the mathematics-bases analysis of *particular* implications of *particular* actions.

The strength of the hypothetical reasoning is demonstrated by the level of details to which the hypothetical situation is specified. However, hypothetical reasoning, supported by mathematics, also lays a trap, because we are investigating details represented only within a specific mathematical construction of a given alternative. Furthermore, the actual hypothetical reasoning is limited by the fact that the reasoning itself is supported by mathematics. As clearly illustrated by ADAM, the weakness of the hypothetical reasoning is that the decisions made on the basis of hypothetical reasoning will operate in a real life situation, not grasped by ADAM. So when q is found attractive, and p is realised, we will see that the ADAM-supported hypothetical reasoning, does not operate straightforward in a real life context. The hypothetical situation, p, is an imaginary situation created only by the model, and it need not have much in common with any actual situation. The problem of hypothetical reasoning is caused by the 'gap' between the model-constructed virtual reality and the 'complexity of life'.

The second specification of mathematics in action concerns hypothetical reasoning: *By means of mathematics, it is possible to investigate particular details of a hypothetical situation, but mathematics also cause a severe limitation of the hypothetical reasoning.* This means that the quality of mathematically-based thought experiments might be highly problematic. Here we touch upon an aspect that can help to explain the emergence of risks.[22]

A particular aspect of carrying out investigations of hypothetical situation concerns the choices between alternatives. One option is to let 'formal' reasoning do the job. This is based on the assumption that, in some way, we can measure 'pain' and 'pleasure' ('cost' and 'benefit') related to the realisation of each of the alternatives in question. This utilitarian assumption makes it possible to transform a political discussion into a management discourse. This transformation can be illustrated by the use of ADAM to provide justification for political actions. If, say, it is a political aim within five years to decrease unemployment by a certain percentage, then a multiplier analysis could indicate necessary political actions. The 'necessity' of such actions of course refers to the model, but when this model-reference is forgotten, and this reference seems immediately forgotten when policy is discussed in public, then the political actions can be referred to as merely 'necessary'. And then there is only a small step to be taken in order to introduce a 'technological necessity' in politics: We have to do so and so, because this is the only possibility feasible! When 'technological necessity' is acted out this way, reality becomes structured in accordance with the perspective of ADAM. The gap between model and reality tends to diminish. The distinction between 'reality' and the 'virtual reality' of the model becomes blurred.

When an alternative is chosen and *realised*, our environment changes. What is the nature of this new situation? The point here can be illustrated by the ADAM and also by many micro-economic models. As already emphasised, the model that structures airline bookings is certainly not simply a description of what takes place when tickets are booked and sold. When introduced, the model becomes part of the passengers' reality. And this story can be continued: Insurance companies also offer insurance for APEX tickets. They, therefore, need a model telling about the likelihood that a 'sure passenger' will in fact become a 'no show'. In this sense, models create models, and one layer after another of mathematics sinks into our social reality.[23]

Thomas Tymoczko has summarised this point in the following point:

> Business does not just apply various already existing mathematical theories to facilitate an activity that is, in principle, independent from such mathematical application (although it

> can do that). Business could not exist in anything like its historical form without some mathematics. Certainly we cannot imagine a modern economy struggling along without mathematics then suddenly becoming more efficient because of the introduction of mathematics! (Tymoczko, 1994, p. 330)[24]

That mathematics becomes part of reality is a general phenomenon. At his lecture at the 7th International Congress on Mathematical Education in Québec, Tymoczko mentioned the relationship between mathematics and war. His point was that war and mathematics are interrelated in an intimate way. We may talk about modern warfare as constituted by mathematics. Not in the sense that mathematics is the cause of war; but we cannot imagine a modern warfare to take place without mathematics as an integrated part. The same statement can be made if we, instead of 'war' or 'business', talk about 'travel', 'management', 'communication', 'architecture', 'insurance', 'marketing', etc. In their present form such types of social phenomena are modulated if not constituted by mathematics.[25]

Whenever we talk about mathematics-based design, we have to remember that the realised situation need not have much in common with the hypothetical situation presented and investigated in mathematical terms. Any technological design has implications not identifies by the hypothetical reasoning. This is a basic problem related to any kind of mathematical based investigation of counterfactuals. When p is represented by a mathematical based vision, and the implications of p is identified by a hypothetical reasoning as q, and found attractive, then the realisation of p may nevertheless contain heavy surprises. Risks emerge in the gap between the mathematical based reasoning related to the hypothetical situations and the really functions of the contextualised realisation. Certitude turns into risk.

Still, the realisation maintains mathematics as an operating element. In this sense we come to live in a environment, produced by integrating a model-supported virtual reality with an already constructed reality.[26] For instance, much information technology materialise in 'packages'. Such packages can be installed and come to operate together with other packages, and they contain mathematics as a defining ingredient. In particular, Hardy's research has made a significant contribution to the area of cryptography, which addresses the question of 'trust' and security of electronic communication. Knowledge about the distribution of prime numbers and about the efficiency of mathematical algorithms, is essential for estimating the likelihood of maintaining privacy. Also in this case mathematics has become inseparable from other aspects of society.[27]

This bring us to the third aspect of mathematics in action which concerns realisation: *Mathematics modulates and constitutes a wide range of social phenomena, and in this it becomes part of reality.*

Put together, the three aspects of mathematics in action send the following message: By means of mathematics it is possible to establish a space of hypothetical situations in the form of possible (technological) alternatives to a present situation. However, this space may have serious limitations. By means of mathematics, in the form of hypothetical reasoning, it is possible to investigate particular details of a hypothetical situation, but this reasoning may also include limitations, and therefore also uncertainties for justifying technological choices. As part of the realisation of technologies, mathematics itself becomes part of reality and inseparable from other aspects of society. Being part of this process, mathematics is positioned in the centre of social development, in the production of wonders as well as of horrors.

Social Theorising

In his study 'The Information Society', Daniel Bell emphasises that "information and theoretical knowledge are the strategic resources of the postindustrial society, just as the combination of energy, resources and machine technology were the transforming agencies of industrial society" (Bell, 1980, p. 545). In his impressive work, *The Information Age: Economy, Society and Culture I-II-III*, Manuel Castells both develops and modifies this idea. He describes knowledge and information as "critical elements in all modes of development, since the process of production is always based on some level of knowledge and in the processing of information" (Castells, 1996, p. 17). Such statements are certainly crucial to understanding the information age. However, the significance of these statements rests upon an specification of what can be understood as information and as knowledge. Castells adds a footnote to this part of his text: "For the sake of clarity of this book, I find it necessary to provide a definition of knowledge and information, even if such an intellectual satisfying gesture introduces a dose of the arbitrary in the discourse, as social scientists who have struggled with the issue know well." Following these preliminaries he characterises knowledge as set of organised statements, which includes some kind of justification, and which is transmitted to others. 'Information' he described as a concept even broader than knowledge. It is clear that Castells does not take this intellectual gesture seriously, and he does not apply this definition in any profound way later in his work. Instead he lets 'knowledge' and 'information' stay as cloudy concepts throughout his whole study of the information age. (I am sure that Castells has realised this.) But I find that it is essential to make a much stronger specification of the notion of knowledge in order to get a deeper understanding of some of the basic social process of the information age (and I am afraid that Castells has not realised this).

By being kept on a general level, the discussion of knowledge and information makes it difficult to raise questions about the particular roles different types of knowledge and information might play in the construction of new technologies. In this way, the thesis of mathematics being insignificant regarding social affairs becomes incorporated in the sociological discussion of the information age. However, I simply do not think that any kind of knowledge and information operate as 'strategic resources'. Quit contrary, I find that particular types of knowledge operate in particular ways as resources for developing and realising technologies. Thus, the use of 'knowledge' and 'information' as dummies obstruct the possibility of an interpretation of social development. Beck did emphasise that the risk society is produced because the certitudes of industrial society dominates thought and action. As I have tried to argue, this phenomenon is related to mathematics-based action design and, in particular, to the application of mathematics in investigating counterfactuals. To me, *a basic challenge to social theorising* is to grasp the nature and scope of mathematics in action. I conceive this as a condition for any adequate interpretation of the basic processes which brings about reflexive modernisation, and for interpreting how 'certainty' turns into free growing risk structures, which are going to accompany us into the future.

One more aspect of the challenge to social theorising has to be mentioned. This also concerns the philosophy of mathematics. Implicitly, in our discussion of mathematics in action and of the apparatus of *reason*, we have been dealing with reason. Following the 'modern condition' and the spirit of the Enlightenment, reason can be interpreted as a powerful resource for progress. Reason, in the shape of science and of mathematics, represents an 'ultimate good'. Following logical positivism the trust in rationality evolves into a trust in scientific methodology. However, critical voices have indicated that reason, in the shape of instrumental reason, reveals its problematic nature. In *One-Dimensional Man*, Marcuse tried to show how instrumental reason, associated with logical positivism and instrumentalism and specified by a scientific methodology, could increase in scale and manufacture social development in a particular form. Operating outside its proper domain, the natural sciences, instrumental reason becomes problematic. It comes to exercise an illegitimate power. It facilitates suppression and social manipulation. However, instead of concentrating on instrumental reason as basis for an interpretation of how science becomes involved in social affairs, I find it necessary to broaden the scope of investigation considerable. We have to study the role of reason, in particular as manifested by mathematics.

Do we like mathematics-based action design? For instance, do we like the booking-model? If we think of the situation as a passenger who has just been bumped, then we will surely have a negative impression. The principle of not selling anything more than you have seems to represent 'honest business'. But it is also possible to see the model in a different light. It ensures that the total number of flights are kept to a minimum, ensuring that, as far as possible, airplanes do not travel with empty seats. By a slight reformulation of

'Kranzberg's First Law', my claim is: *What mathematics is doing is neither good nor bad, nor is it neutral.*[28]

According to classic philosophy of mathematics, mathematical thinking was a model for human thought. However, this glorification of the queen of science is no longer the object of all philosophies of mathematics.[29] In particular, *aporism*, as a philosophy of mathematics, acknowledges that 'pure reason', in terms of mathematics, can turn into 'disastrous reason'.[30] Aporism sees mathematics as an essential element in social and technological development; at the same time aporism realises that the presence of mathematics does not provide any guarantee for the 'quality' of this apparatus. Therefore, the certainty of mathematics can transform into uncertainty regarding the construction of our future. Wonders mix with horrors.

Previously it might have been appropriate for sociology to ignore the social role of mathematics. Mathematics might *recently* have disarmed social theorising from grasping the basic processes of reflexivity. The theoretical task now is to provide a framework for grasping mathematics in action, in particular to identify how mathematics supports a technological imagination (which might be problematic and narrow), how it establishes possibilities to investigate particular aspects of possible technological constructions (and ignores other aspects), and how mathematics becomes installed in society and starts operating as part of technological devices. The functioning of mathematics cannot be ignored by social theorising. In order to cope with this, sociology may get inspiration from recent studies of mathematics and of mathematics education, which have tried to reconsider mathematics in action.

Notes

1. Hardy makes a distinction between 'real' mathematics and practical applied—or trivial—mathematics which may have such effects. I do not make a sharp distinction between pure and applied mathematics, or between real and trivial mathematics. All areas contribute to the mix, which I call mathematics, and in the rest of this paper, I will simply talk about mathematics.
2. Many studies have revealed that a social structuring of mathematics takes place. See, for instance, Wilder (1981). However, this issue is not going to be discussed in the following.
3. As an illustration of classical concerns in the philosophy of mathematics, see, for instance, Benacerraf and Putnam (eds.) (1986).
4. See also, for instance, Giddens (1990, 1998) and Habermas (1987). Surprisingly, mathematics is not referred to in Castells (1996, 1997, 1998). However, Lyotard (1984) includes mathematics in his discussion of the post-modern condition.
5. For a discussion of critical mathematics education and related ideas see Borba and Skovsmose (1997); Keitel et al. (1989); Niss (1994); Skovsmose (1994); and Skovsmose and Nielsen (1996).
6. The notion of reflexive modernisation has come to play a crucial role in recent sociology. By this concept, Giddens emphasises that the consequences and the implications of any action become part of the process of acting itself. Giddens seems to rephrase reflexivity as part of the 'conscious' level of social dynamics, while Beck relegates reflexivity to a deeper level of social processes.
7. See also Beck (1992, 1995a, 1995b); Franklyn (1998); and Hiskes (1998).
8. Richard P. Hiskes expresses this as follows: "Risk is the product of our lives together, and to fully understand risk's emergent character is to realize that most of the efforts to either explain risk or to cope with it within an individualistic political framework are doomed to failure because they do not acknowledge the 'togetherness' of our risky present" (Hiskes, 1998, p. 13).
9. In other parts of his work, Beck refers to mathematics. See, for instance, Beck (1995b, 20–22) where he talks about the calculus of risks. See also the discussion of 'hazards' in Beck (1995a, 73–110).
10. I include a variety of aspects within the notion of technology: the artefacts of technology (be it a car, a computer or any other device) as well as strategies for action (a plan of production or any other product of 'systems development'). Tailorising is one classic example, and computer-based systems development has produced all kinds of examples.

11. The expression 'mathematics in action' is inspired by the title of Latour's book, *Science in Action*. However, while Latour follows scientists and engineers through society, I try to follow mathematics into society. In other contexts I have developed this idea in terms of the *formatting power of mathematics*. See, for instance, Skovsmose (1994).
12. Clements (1990) does not claim that his model is identical to any actually used model (such models are 'commercial in confidence'), but certainly it is similar to such models: "The purpose [...] is to develop a model of the decisions facing an airline and, from this, to acquire an understanding of why it may indeed be beneficial to an airline to book more passengers onto a particular flight than the capacity of the flight that is to make the flight" (Clements, 1990, p. 324). Booking strategies may have developed considerably since Clements constructed his model; nevertheless this model illustrates several basic aspects of mathematics in action. Clements's model has been further discussed by Hansen, Iversen and Troels-Smith (1996).
13. For more details, see Clements (1990, p. 325).
14. See Austin (1962, 1979); Sapir (1929); Searle (1969); and Whorf (1956).
15. ADAM is presented in Dam (1986) and Dam (ed.) (1995). For a critical examination of ADAM, see Dræby, Hansen and Jensen (1995).
16. The Institute for Learning and Research Technology, Bristol University has provided a Virtual Economy, which is an on-line model of economy based on the Treasure's model: "Users can try out policies [...] The program provides extensive feedback on how the economy would perform over the next ten years if those policies were actually implemented. Users can also see the impact of their policies on a range of sample families" (*Newsletter*, University of Bristol, 22 April 1999). The Virtual Economy can be found at: http://www.bized.ac.uk/virtual/economy.
17. The authors are Per Kongshøj Madsen, Bent Andersen, Jørgen Søndergaard, Ruth Emerek, Hans Frost, Poul Lübcke, Kim Viborg Andersen and Rolf Ask Clausen. Besides ADAM, the economic models referred to in *Magt og Modeller* include: the SMEC (Simulation Model of the Economic Council), which operates in a similar way to ADAM but is used first of all by the Economic Council; GEMIAE (General Equilibrium Model of the Institute of Agricultural Economics), which emphasises economic aspects related to agriculture; GESMEC (General Equilibrium Model of the Economic Council); HEIMDAL (Historically Estimated International Model of the Danish Labour Movement), which emphasises Nordic relationships; MONA (Model Nationalbank), which is used by the Danmarks Nationalbank as a tool of forecasting and analysis making; and MULTIMOD (Multi-region Econometric Model). The environmental models referred to in *Magt og Modeller* include: ARMOS (Areal Multiphase Organic Simulator For Free Phase Hydrocarbon Migration and Recovery); HST3D, which provides simulation of heat and solute transport in three-dimensional groundwater flow system. Among the models related to defense is SUBSIM (Small Unit Battle Simulation Model).
18. For a discussion of how mathematics may influence different spheres of practice, see, for instance, Appelbaum (1995); Dorling and Simpson (1999), and Porter (1995).
19. It is, naturally, possible to specify further the notion of necessity by distinguishing between 'logical necessity', 'physical necessity', 'social necessity', etc., depending on the possibilities of conceptualising alternatives. Thus, a fact constitutes a physical necessity, if it is impossible to imagine it to be different without also imagining some physical laws to be different. Similarly, a fact constitutes a social (or cultural) necessity if it is impossible to imagine it to be different without also imagining some (deeply rooted) cultural traditions and social norms to be different.
20. The importance of sociological imagination to sociology has been emphasised by Wright Mills (1959) and repeated by Giddens (1986).
21. With the APEX "... the passenger is offered tickets valid only for a specified flight but at a reduced fare. If the passengers fail to arrive for that flight the ticket is void and the passengers lose their money. Obviously some passengers (chiefly business travellers requiring some flexibility in their planning [and not paying for the tickets themselves]) will still be prepared to pay full fare to retain that flexibility, whilst others (chiefly holiday makers) will accept the restriction in return for the reduced fare. The second category of passengers will not miss their flight lightly so we can assume that their 'no show' probability is virtually zero. These passengers then form a solid base of passengers who can be relied on to turn up for the flight" (Clements, 1990, pp. 335-336).
22. For an indication of how risks can be related to mathematical formalisation, see Booss-Bavnbek (1991).

23. In a similar way many other services, public and private, are based on models linking to models. For instance, the many new forms of services and special offers provided by telecompanies cannot be established without careful mathematically based planning.
24. A historical study of how mathematics constitutes and modulates economic affairs is discussed in Swetz (1987).
25. See Tymoczko (1994) and Højrup and Booss-Bavnbek (1994).
26. The notion of 'frozen mathematics', which refers to mathematics as part of social and cultural life, has been discussed in, for instance, Keitel (1989, 1993). The prescriptive use of mathematics, also illustrating mathematics in action, is discussed in Davis and Hersh (1988).
27. For a discussion of mathematical foundation for 'trust' and security in the electronic transmission of information, see Skovsmose and Yasukawa (2000).
28. See Kranzberg (1997).
29. See, for instance, Bloor (1976); Ernest (1998); Hersh (1998); and Kitcher (1984). The social role of mathematics in technology has been discussed by many authors, for instance, Booss-Bavnbek (1995); Højrup and Booss-Bavnbek (1994); Keitel (1989, 1993); Keitel, Kotzmann and Skovsmose (1993); and Restivo et al. (1993).
30. The Greek word *aporia* refer to 'being without direction' or 'being lost'. In the present *aporia* refers to the basic uncertainty in identifying the role of rationality, as exercised by mathematics in action. Aporism has been presented in Skovsmose (1998, 2000). It can serve as a working philosophy of critical mathematics education.

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