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## ABSTRACT

This study investigates whether the National Council of Teachers of Mathematics (NCTM) professional standards are appropriate for a two-year college developmental mathematics courses. The NCTM professional standards of worthwhile mathematical tasks, classroom discourse, and teacher reflection were all part of this preparatory mathematics course. Most students indicated that they learned the topics the college required and increased their disposition in doing mathematics problems. The project concludes that these standards are suitable for two-year college classrooms. (Contains 24 references.) (KHR)

*Professional Standards for Teaching Mathematics*  
In a Two-Year College Classroom

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November, 1999

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The undergraduate education I received in mathematics education certification provided me with an advantage that established, experienced mathematics teachers probably did not have. I was trained to teach secondary mathematics in a way that was very different from the way I learned mathematics. The pedagogy I learned was based on the National Council of Teachers of Mathematics (NCTM) (1989, 1991) new standards developed to improve the teaching and learning of mathematics. Courses emphasized ways to promote student understanding instead of just passing mathematical facts along to students. Once in the classroom the instructional goal was to help secondary school students construct knowledge for themselves with the purpose of understanding concepts rather than memorizing and applying disconnected rules. Life was challenging but good. In order to better prepare my students to become productive citizens I needed further education and support. I returned to the university to become a better mathematics educator.

The purpose in graduate classes was not to merely satisfy state requirements to teach mathematics but to learn the best ways to facilitate other students' learning. Again, the National Council of Teachers of Mathematics standards guided my professional development. Even though I was never trained to teach mathematics as a conduit of knowledge, I consider myself a "reformed" mathematics educator and believe students must construct knowledge for themselves in a variety of ways. I subscribe to the ideology that NCTM proposes to improve mathematics education in the United States.

NCTM's (1989) education goals include "(1) mathematically literate workers, (2) lifelong learning, (3) opportunity for all, and (4) an informed electorate. Implicit in these goals is a school system organized to serve as an important resource for all citizens throughout their lives" (p. 3). In addition, NCTM presents standards specifically for K–12

mathematics education reform with an understanding that “similar standards need to be developed for both preschool programs and those beyond high school” (p. 7). Even though NCTM does not elaborate on standards for post-secondary studies, extending the vision to two-year college curricula seems to be an appropriate next step.

Are the National Council of Teachers of Mathematics standards for mathematics education really suitable for post-secondary studies? The chance to investigate this question was presented in an opportunity to teach a summer course in a two-year college classroom. Considering the specific constraints placed on a college instructor I was not sure the reformed mathematics pedagogy was applicable to adult students’ learning. With the plan described below I returned to the classroom to investigate the applicability of the *Professional Standards for Teaching Mathematics* (1991).

My plan was to look at how I could incorporate NCTM’s (1991) professional standards for teaching mathematics in a two-year college setting. I was especially interested in encouraging communication and utilizing teacher reflection as tools for improving instruction and, therefore, providing students an opportunity to learn mathematics. Armed with knowledge, mathematical and pedagogical, I accepted a position teaching a class called Preparatory Mathematics, a pre-requirement for students who had not yet demonstrated enough skills to be successful in the college algebra course. The mathematics faculty expected the course to consist of whole numbers, fractions, mixed numbers, decimal fractions, ratios/rates, proportions and percent, geometry, measurements, and signed numbers. In addressing concepts in these areas I wanted to incorporate NCTM’s (1991) six standards for teaching mathematics: worthwhile mathematical tasks, teacher’s role in

discourse, students' role in discourse, tools for enhancing discourse, learning environment, and analysis of teaching and learning (p. 19).

My report of this endeavor includes a literature review to provide professional support for my classroom decisions, a description of my procedure to answer my question, a collection of student and teacher assessment results, a discussion of how I interpret the results, and a summary of where and how this particular project could lead to further research.

## Literature Review

The National Council of Teachers of Mathematics (NCTM) and the American Mathematics Association of Two-Year Colleges (AMATYC) have made concentrated efforts to reform mathematics education. NCTM's area of interest is grades kindergarten (K) through twelve while AMATYC's focus is two-year colleges. Both areas have unique characteristics requiring special attention yet also have much in common. AMATYC has attempted to follow NCTM's effort in making mathematics education reform a public concern. Both organizations believe change is necessary and quite possible.

Educators who believe in this vision of mathematics education reform, no matter what level, incorporate constructivist methodologies and encourage classroom communication. In meeting and responding to the challenges of teaching, reflection is also a key ingredient to success in implementing NCTM's professional standards. Using worthwhile mathematical tasks, encouraging classroom discourse, and attending to the learning environment brings NCTM's vision alive in the classroom. Monitoring the progress of the tasks, discourse, and environment requires reflection.

### *Curriculum and Evaluation Standards for School Mathematics*

In the early 1980s the mathematics education community realized a need for reform in the teaching and learning of mathematics (NCTM, 1989, p. 1). NCTM (1980) published a report called *An Agenda for Action* which emphasized problem solving and increased use of calculators and computers in mathematics education. The National Commission on Excellence in Education (1983) published *A Nation at Risk* which pointed out the mediocrity of mathematics education in the United States. These two documents provided information needed to stimulate public and government support for mathematics education reform. As a

result, NCTM formed the Commission on Standards for School Mathematics to accomplish two tasks: a) to create a vision of what mathematically literate means in the technological society of today, and b) to create a set of standards to judge the quality of working toward the vision (McLeod, Stake, Schappelle, Mellissinos, & Gierl, 1996).

Realizing schools were still based on the needs and goals of an industrial society, the commission established new goals for our modern information society including “1) mathematically literate workers, 2) lifelong learning, 3) opportunity for all, and 4) an informed electorate” (NCTM, 1989, p. 3). In order to meet the societal goals, new educational goals were aligned with the expectations. These are for students to “1) learn to value mathematics, 2) become confident in the ability to do mathematics, 3) become mathematical problem solvers, 4) learn to communicate mathematically, and 5) learn to reason mathematically” (p. 5). These two sets of goals, societal and educational, set the basis for the commission’s vision of a good mathematics education. To determine progress in achieving the vision, fifty-four standards for grades K-12 were established (NCTM, 1989).

These standards focus on what school mathematics are important, how the content should be taught to students, and the level of development of the students. In deciding important mathematics content in each standard, NCTM distinguishes knowing mathematics from doing mathematics. The standards emphasize students creating knowledge through participation in activities which have a genuine purpose. This includes understanding that mathematics is found in many disciplines beyond the traditional engineering and physical science courses, such as in business, medicine, and sociology. And in developing technology skills, understanding how various tools can aid in reaching appropriate answers emphasizes the basic fundamental skills necessary for students to learn. The best methods for teaching

all students include “activities that grow out of problem situations; and . . . active as well as passive involvement with mathematics” (NCTM, 1989, p. 8). Furthermore, NCTM emphasizes the belief that all students need to learn these important mathematical ideas in order for mathematics education reform to be successful.

NCTM (1989) organized the fifty-four standards into four categories of grades K through 4, grades 5 through 8, grades 9 through 12, and evaluation. The first four standards in the first three categories are titled Problem Solving, Communication, Reasoning, and Mathematical Connections. Yet, each grade level addresses the students’ developmental level, mathematical background, and specific mathematical content (p. 11). For example, early elementary grades should focus on concrete models as evidence in learning about mathematical concepts. As the students mature, during middle school years, abstract ideas can be introduced along with the physical representations. In high school the focus can shift to abstract ideas and language in mathematics, such as developing deductive proofs. All the standards elaborate on the importance of classroom discussion to help teachers adjust pacing of mathematical ideas with student maturity. The fourth category of standards, evaluation, is presented separately and looks at how well the students develop personal confidence and a disposition toward using mathematics to solve problems.

In preparing the *Curriculum and Evaluation Standards for School Mathematics*, NCTM clearly described its vision of the mathematical concepts necessary for high-quality mathematics education. However, the initial goal was to create a document describing standards for curriculum and for instruction (McLeod, Stake, Schappelle, Mellissinos, & Gierl, 1996). In fact, once it was decided to prepare separate documents for the two types of standards there was disagreement over which to do first. After much discussion the

curriculum and evaluation standards were completed first, with the teaching standards to be addressed after completion of the document. This latter document, *Professional Standards for Teaching Mathematics*, became the next major project NCTM felt responsible for developing.

*Professional Standards for Teaching Mathematics*

Once curriculum standards were released, the National Council of Teachers of Mathematics appointed a second committee, the Commission on Professional Teaching Standards, whose task was to concentrate on a vision for teaching, for evaluating teaching, for the professional development of mathematics teachers, and for the responsibilities of other professional groups in the development and support of mathematics education (NCTM, 1991, p. vii). The underlying theme of the vision is mathematical power for all students which includes developing ability to apply mathematical concepts in order to reason through, communicate about, and solve routine and non-routine problems. Mathematical power also requires enough self-confidence in that ability to develop a disposition for using quantitative and spatial information to make decisions. The commission's job was to provide guidance to those who were committed to their vision of reform in mathematics education.

This commitment to reform entails a great deal of change. The five major shifts are a) toward the mathematics classroom as a community rather than a collection of individuals; b) toward logic and evidence as verification rather than the teacher as the main source of information; c) toward reasoning rather than memorization; d) toward investigation and problem solving rather than drill; and e) toward mathematics as connected concepts (within mathematics and between other subjects) rather than as isolated concepts and procedures

(NCTM, 1991, p. 3). The intent of these shifts is to permeate all topics in mathematics for *all* students.

The National Council of Teachers of Mathematics stresses that “every child” must have an opportunity for a comprehensive mathematics education. To not provide this opportunity for every child, could form an intellectual elite and a polarized society which is counterproductive to our democratic and economic needs (NCTM, 1989, p. 9). The NCTM Board of Directors elaborated on the term “every child” with “students who have been denied access in any way to educational opportunities as well as those who have not,” and “students who have not been successful in school and in mathematics as well as those who have been successful” (NCTM, 1991, p. 4).

The *Professional Standards for Teaching Mathematics* (NCTM, 1991) outlines six standards to describe NCTM’s vision of high-quality mathematics teaching: mathematical tasks, teacher’s role in discourse, student’s role in discourse, tools for enhancing discourse, the learning environment, and analysis of teaching and learning. The guidelines continue with standards for evaluating teaching, professional development, and support and development of teachers. As previously mentioned, NCTM also believes “that similar standards need to be developed for both preschool programs and those beyond high school” (NCTM, 1989, p. 7). The American Mathematics Association of Two-Year Colleges undertook the “beyond high school” task.

*Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus*

According to the American Mathematics Association of Two-Year Colleges (AMATYC) “higher education is situated at the intersection of two major crossroads” which are a) society’s increasing demand for well-educated citizens to adequately supply the changing workforce; and b) the increasingly under-educated population entering post-

secondary education (Cohen, 1995, p. 3). In an effort to provide guidance to two-year college instructors to better prepare students to meet society's needs, AMATYC formed a steering committee including representatives from organizations such as the Mathematical Sciences Education Board (MSEB), the American Mathematical Society (AMS), the Mathematical Association of America (MAA), the National Association for Developmental Education (NADE), and the National Council of Teachers of Mathematics (NCTM). The steering committee appointed a task force to develop a standards document, and in 1995 AMATYC published *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus* in a concerted effort to improve mathematics education at two-year colleges and also to encourage more students to study mathematics. The focus of this document is post-secondary students who are not yet prepared to study college calculus.

Two-year college students are either planning to take calculus but are not prepared to do so, or are not planning to study calculus but are underprepared in mathematics for their chosen career paths. They may also be studying in spite of obstacles such as English as a second language and/or a lack of developmental work in a variety of disciplines and study skills. With very diverse backgrounds and life goals, students at two-year colleges may be recent high school graduates or may be "older students with faded mathematical backgrounds" (Cohen, 1995, p. 25). Some may be studying for a degree on a full-time basis while others need to attend on a part-time basis in order to accommodate working full-time and/or managing a household. Better jobs after education are these students' major objective; some also strive to earn two-year associate degrees and/or bachelor's degrees.

In order to accommodate this diverse population AMATYC suggests all students experience a core level of introductory mathematics, called the Foundation, to provide them

with enough confidence and background to study higher levels of mathematics. The Foundation should not be interpreted to mean a repetition of high school mathematics. Instead, courses should attend to the special needs of the students by delving deeper into fewer topics and providing appropriate exposure to technology. “Students may master the Foundation by successfully completing high school mathematics based on the NCTM Standards (NCTM, 1989), by earning a GED based on the emerging standards for adult mathematics education, by taking mathematics courses in college, or by some combination of the three” (Cohen, 1995, p. 24).

The Foundation is based on three categories of standards: standards for intellectual development, standards for content, and standards for pedagogy (see Table 1). The standards for intellectual development are seven goals for students’ mathematical skills. The standards for content are seven descriptions of topics to be investigated and explored by students in

Table 1

AMATYC Standards by Category

<b>Standards for Intellectual Development</b>	<b>Standards for Content</b>	<b>Standards for Pedagogy</b>
1 Problem Solving	Number Sense	Teaching with Technology
2 Modeling	Symbolism and Algebra	Interactive and Collaborative Learning
3 Reasoning	Geometry	Connecting with Other Experiences
4 Connecting with Other Disciplines	Function	Multiple Approaches
5 Communicating	Discrete Mathematics	Experiencing Mathematics
6 Using Technology	Probability and Statistics	
7 Developing Mathematical Power	Deductive Proof	

introductory college mathematics. Standards for pedagogy consist of five strategies for creating a constructivist classroom and promoting student-constructed knowledge. Similar to NCTM's (1989) format AMATYC follows the descriptions of the standards with a table outlining areas of increased and decreased attention. Overall, the standards call for increasing all students' mathematical power by developing conceptual understanding through problem solving and appropriate use of technology. Once the Foundation is successfully completed, students should have acquired the ability and confidence to study higher mathematics relevant to their chosen areas of study.

After completing the Foundation, additional mathematics courses may or may not be recommended, depending on individual educational programs and goals. Students who enter technical programs study higher mathematics with a focus in applications. Students enrolled in mathematics-intensive programs such as majors in science, engineering, computer science, economics, business, and secondary school or college mathematics teachers would continue studying mathematics. In addition, liberal arts students need to understand the impact mathematics has had on various fields and be confident in their own mathematical abilities. A fourth program which requires additional mathematical studies is elementary education.

*Crossroads in Mathematics* addresses all the programs mentioned here by describing content, pedagogy, and topics of increased and decreased attention particular to the students' needs. The document also addresses the importance of faculty development and the need for collaboration among schools and industry for successful implementation of the standards. This collaboration is one way that teachers can reflect on and evaluate how well the standards are working in the classroom.

## Reflective Teaching

The four areas that NCTM (1991, p. 5) identifies, and AMATYC (Cohen, 1995, p. 15) supports, as a framework of important teacher decisions are mathematical tasks; classroom discourse; the learning environment; and analysis of student learning, mathematical tasks, and the environment. The standards developed from this framework are accepted as critical to good mathematics teaching (Prichard, 1993, Brown & Smith, 1997, & Willoughby, 1996). They provide a basis for teachers to examine how their practical experience in the classroom fits in with what other people involved with education have developed into theory. By reflecting on theory mixed with personal values and beliefs, teachers can take responsibility for their professional development and continuous improvement of instruction.

According to Zeichner & Liston (1996) the history of reflection begins with John Dewey. Reflection, as described by Dewey, is the opposite of routine behavior. Any behavior that is routine is considered an impulse, following tradition, or complying with authority. Reflection, on the other hand, is a result of difficulty or unease from a situation. Reflection is a holistic approach involving knowledge, intuition, emotion, and passion in meeting and responding to the challenges of teaching. Dewey identified three characteristics of reflective teachers: open mindedness to errors, responsibility for consequences and unexpected outcomes of teaching, and wholeheartedness in striving to understand and improve teaching.

Schon (Zeichner & Liston, 1996) expanded on Dewey's ideas on reflection with identifying a problem setting. He described this identification as the process of naming what needs attention as well as describing the context in which the attention will be given. Schon

believes teachers encounter lots of “low ground” or knowledge which is never encountered in teacher education, yet the practice of teaching is the result of theories learned at the university. It is critical that teachers pay attention to the here-and-now with full awareness in order to frame, think about, and reframe problem settings.

Schon (Zeichner & Liston, 1996) further described reflective teaching as reflection-on-action and reflection-in-action. Reflection-on-action is the thinking a teacher does before and after a lesson, away from students in the classroom. Reflection-in-action is the spontaneous adjustments of lessons when unexpected ideas emerge in the presence of students. Both are essential to improving teaching. Educators learn many theories about teaching while studying at a university and classroom experiences provide them with practical knowledge. Schon believes it is the teacher’s job to bridge theory with practical knowledge-in-action. Through reflection teachers begin to define and articulate their own theories about how to best reach students. Zeichner and Liston (1996) criticized Schon for concentrating on teaching as a solitary practice and not considering the social interactions among teachers, a critical part of reflection and development of a perspective on behavior (p. 18).

Zeichner & Liston (1996) discuss how Norwegian educators, Gunnar Handal and Per Lauvas, examined Schon’s idea of teachers’ “practical knowledge.” They believe that educators’ personal experience, academic knowledge, and core values are the major influences in teaching. Perhaps the core values and beliefs are the most influential because all knowledge is interpreted through them. In addition, Feldt (1993) identifies several researchers who “have found that teachers’ conceptions about mathematics and mathematics instruction profoundly affect their teaching” (p. 400). NCTM’s (1991) suggestion that

teachers collaborate ideas through discussion with colleagues and supervisors to improve teaching supports the importance of teachers' beliefs and values. Greater awareness about beliefs is one step in improving the process of reflecting on teaching.

Hart, Schultz, Najee-ullah and Nash (1992) suggest the best way to begin reflective teaching is to ask questions which consider the nature of the mathematical tasks, classroom discourse, and the learning environment. For example, "Why did a lesson work?/Why didn't a lesson go over well?"(mathematical tasks), "What kind of questions do I ask?/How do I respond when [students] ask questions?" (classroom discourse), and "Is my classroom spontaneous or is it predictable?/Are my students involved?" (learning environment). Asking deliberate questions when planning and reviewing teaching experiences provides guides for needed changes in instructional practices. Videotaping, writing logs, talking with other teachers and the students, observing other teachers, and asking another teacher to observe a specific aspect of your teaching are excellent methods of realizing answers to the questions. When first learning to reflect Hart, Schultz, Najee-ullah, and Nash emphasize three things: change takes time, not everything needs to change, and focusing on one change at a time is good. The authors provide practical guidelines such as these to emphasize the importance and practicality of reflective teaching in the classroom.

#### Standards and Reflection in the College Classroom

Mary Kim Prichard (1993) wrote "Mathematics courses and programs of study in colleges and universities should share in [NCTM's] vision" (p. 744) of school mathematics. Although Prichard's classes—calculus, non-Euclidean geometry, and statistics and probability—are considered higher level college mathematics she incorporates many ideas from the *Professional Standards for Teaching Mathematics* (NCTM, 1991) which are also

found in *Crossroads in Mathematics* (Cohen, 1995). Selection of worthwhile mathematical tasks is an important teacher responsibility, and promoting classroom discourse and encouraging mathematics communication is essential to understanding mathematical concepts. Prichard also suggests going beyond memorized definitions by asking for answers that demonstrate student understanding, such as compare and contrast . . . or construct a model that . . . . She encourages participation and discourse to challenge students to generate their own answers to problems. She also suggests having students write paragraphs about what they predict will happen in a given situation and what they found out actually happens. Standards developed for K-12 are applicable to college level instruction.

Prichard (1993) warns that change is not easy especially with the obstacles encountered at college level. Some of these obstacles are that college professors probably did not learn mathematics as the vision suggests; that college level teachers are expected to participate in time-consuming research and publication for advancement; and that college class size is not conducive to many of the professional teaching standards. She offers ways to turn the obstacles into benefits. Many outstanding professors are available as good resources for ideas and support. Original research is a large part of scholarly activity and can be time-consuming yet it is also very important to step outside investigations and “build bridges between theory and practice” (p. 746). Students in large lecture halls can work in pairs or small groups with graduate assistants to encourage interaction and effective discourse. In conclusion, Prichard wrote “mathematics educators have the opportunity and the obligation to bring about changes in college curricula and instruction” (p. 747). In addition, improving the mathematics education of pre-service teachers will improve mathematics education for all mathematics students.

Feldt (1993) cites work done by Joyce and Showers investigating “the ability of teachers to acquire new teaching strategies or improve existing skills” (p. 401). They determined there are five essential elements to be included in training teachers: development of a strong theoretical basis or rationale; observation of an expert demonstration; classroom practice; opportunity for feedback regarding the practice; and some sort of support person who can provide feedback and companionship. Many authors (Feldt, 1993, Prichard, 1993, Leinwand, 1994, and Brown & Smith, 1997) agree with NCTM (1991) that “the kind of teaching envisioned in these standards is significantly different from what many teachers themselves have experienced as students in mathematics classes” (p. 2). Because most math teachers did not learn mathematics in a constructivist ideology as recommended by NCTM, it may first be necessary to change their conceptions of mathematics instruction. With appropriate training that includes the five elements above, educators will select tasks that actively engage students in important mathematics, promote classroom discourse that “emphasizes mathematical reasoning and evidence as the basis for deciding what makes sense,” model an environment that respects “intellectual and social aspects of the classroom,” and “examine how well the tasks, discourse, and environment are working to foster students’ learning” (Ball & Schroeder, 1992, p. 68). This type of concerted effort is easy to endorse yet often difficult to implement for many reasons. However, no student should be denied the chance to become mathematically empowered. College students are no exception.

On the other hand, Norwood’s (1994) research which focused on mathematics anxiety and mathematics achievement in a developmental arithmetic course at a community college<sup>1</sup>

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<sup>1</sup> Community college and two-year college may be used interchangeably. Webster defines community college as “a non-residential 2-year college that is usually government supported” (Mish, 1984, p. 267).

resulted in surprising data. Norwood states that 68% of the college population experiences high levels of mathematics anxiety and, therefore, programs that reduce anxiety are needed at the college level. In an effort to find approaches that reduce anxiety, she developed a study in which two developmental arithmetic courses were taught, each using a different teaching approach: a) an instrumental approach which “emphasized memorization of rules and formulas” (p. 248); and b) a relational approach which emphasized conceptual understanding, and “presented mathematics as a cluster of related concepts” (p. 248). Students in both courses were evaluated before and after completing the 14-week remedial course using the Fennema-Sherman Math Anxiety Scale and the Arithmetic Skills Test of the Descriptive Tests of Mathematics Skills of the College Boards. The topics presented in both courses were “operations with whole numbers, fractions, decimals, percent, ratio and proportions, metric system of measurement, English system of measurement, signed numbers, exponents (positive and negative), exponential notation, scientific notation, and equations” (p. 250). Norwood used ANCOVA statistics and found:

- Students taught in the instrumental approach “experienced a more significant reduction in mathematics anxiety scores” (p. 251) when compared to the students taught in the relational, conceptually-oriented approach.
- There was no significant difference in the mathematics achievement mean scores for both instructional approaches.
- There was only a very moderate negative correlation between mathematics anxiety and mathematics achievement.

Norwood’s (1994) data contradicted previous theories that mathematics anxiety develops because of traditional, lack of understanding teaching strategies. She concluded

that the population she studied cannot lead to generalizations for other populations. Most of the students in this population were only concerned with getting the “right” answer rather than understanding how to get the answer.

### Summary

The National Council of Teachers of Mathematics has been working since the early 1980s to improve mathematics education in the United States. The *Curriculum and Evaluation Standards for School Mathematics* sets forth important topics to be taught the first twelve years of a student's education. The *Professional Standards for Teaching Mathematics* provides guidelines on the most effective ways to reach all the students. The Council has done more than any other organization to prepare the students of this country to be mathematically empowered. The American Mathematics Association of Two-Year Colleges also subscribes to NCTM's vision of mathematics education. The document prepared by AMATYC, *Crossroads in Mathematics*, attends to the special needs of the population at a two-year college and describes ideals for educating older students trying to attain modern day career goals.

Many educators have worked at incorporating constructivist ideologies in the classroom. Communication among students and with other professionals has been given high priority. Self-examination of the progress of incorporating conceptual understanding and mathematics concepts as part of a greater whole has also been described as essential to improving instruction. Some of these educators have been involved with K-12 students, and others are post-secondary instructors. The people who have tried to incorporate NCTM's vision and have written about their experiences seem to appreciate the results and use them to improve further instruction.

Could worthwhile mathematical tasks and appropriate use of manipulatives create an environment conducive to classroom discourse in a two-year college classroom? Would reflective journaling provide a medium for improved instruction and, therefore, increased student disposition for solving mathematical problems in a developmental, pre-algebra, two-year college course? University-learned theory, practical knowledge, and personal reflection have stirred my interest in improving classroom communication and in reflective teaching as tools for increasing students' learning and mathematical disposition.

## Method

Current mathematics education literature suggests that two-year college students have special needs. It also suggests changes in mathematics education need to be made to improve instruction that meets their special needs. The changes must address developing critical mathematical skills of the 2-year college population. Because a major goal of this education reform is to increase students' willingness to study mathematics and to use the skills learned, educators need to attend to increasing student participation which requires self-examination of personal teaching strategies. This project utilized taped student interviews and examples of classroom discourse to investigate communication, and an ongoing teacher reflection journal as a tool for reflective teaching. Findings reveal the status of the students' disposition toward solving non-routine problems requiring pre-algebra mathematical skills.

### Setting

This project took place in a rural, open-enrollment, two-year college of 1300 students ranging in age from 17 to 80. At the time of this project the average age of the students was 27 and sixty-six percent were female. All students who enroll at this college take a college placement examination in which one section addresses pre-algebra skills. If a student's score on the mathematics section of the test is low, he or she is asked to take the course called Preparatory Mathematics.

### Participants

Six students enrolled for the summer session of Preparatory Mathematics. These six students were similar to the overall college body; the small differences could be attributed to the course taking place in the summer. The youngest student was 17 and graduated from high school early. The oldest student was 51 and was preparing to take the high school

equivalency test (G.E.D.). The average age was approximately 30. The 3 females and 3 males had a variety of career goals including medicine, elementary education, nursing, aviation, law enforcement and technical engineering. The students in the course were informed of the true intent of the project and consented to participate in the project by signing a letter of consent (see Appendix A).

### Procedure

The taped student interviews, classroom discourse, and an ongoing teacher reflection journal are three well-defined areas of the procedure developed for evaluating success of this Preparatory Mathematics course.

Interviews. In an effort to assess improvement in the students' willingness to attempt problem solving with mathematical concepts two parallel interview protocols were developed (see Appendixes B and C). Before starting the 5-week course, five of the six students were contacted by mail and asked to participate in an interview before beginning the course.<sup>2</sup> The interview questions were selected from mathematics literature (Kenney & Silver, 1997, and Bloomfield, 1994) to encourage student discourse about the college-prescribed topics. Each interview consisted of five problem-solving questions regarding fractions, rates, decimal fractions, geometry, and measurement.

The first interview took place before the course began and the second interview took place after the final examination. Both interviews were recorded on audio tapes and transcribed. Student responses were examined to identify specific comments expressing willingness to attempt to solve the problems.

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<sup>2</sup> Due to privacy requests one student could not be asked to participate until the first day of class. Because this particular student arrived late, her first interview took place after the second half of the first class.

Stenmark (1991) and NCTM (1989) suggest specific attributes for observing students' disposition toward learning. The attributes include "1. Confidence in using mathematics; 2. Flexibility in doing mathematics; 3. Persevering at mathematical tasks; 4. Curiosity in doing mathematics; 5. Reflecting on their own thinking; 6. Valuing applications of mathematics; and 7. Appreciating role of mathematics" (Stenmark, p. 34, and NCTM, p. 235). This organization of attributes was utilized for classifying improved disposition found in the interview transcriptions.

**Figure 1.** Rubric used to score answers for pre- and post-interview questions and journal entries.

Score	Qualifications
3	Explains one or more appropriate processes and carries out a process to get answers for all steps involved. Communicates all steps taken to solve for question.
2	Explains one or more appropriate processes and carries out a process to get answers for all steps involved. Omits explanations in process carried out.
1	Gives correct answer with no explanation of process.
0	No acceptable response given.

Each question was also given a score based on the rubric in Figure 1. The four-step rubric was used to score answers given for each question in both interviews and evaluated communication of ideas (rather than correct answers) as important. The overall purpose of the pre- and post-interviews was to evaluate improvement in each student's willingness to use mathematical concepts to solve problems. Stenmark's and NCTM's suggested attributes and the rubric provided two organizations of the data.

Preparatory Mathematics Class Sessions. The class met for 5 weeks, twice a week. Traditional concepts were introduced through worthwhile mathematical tasks intended to encourage student discourse. Hands-on activities, class discussions of the mathematics involved in the activities, and how the mathematics applied to activities the students

encountered in their lives monopolized time in the classroom. When students had questions, other students were encouraged to answer the questions. The classroom was organized to provide a comfortable learning environment where students could take risks in communicating their perceptions of mathematical concepts. Chairs were arranged in a circle, in groups, or pairs depending on the planned activities for the class. Time before and after class was also provided for students who were not comfortable asking their questions in class to see me on an individual basis.

Mathematical manipulatives for various topics were introduced and used by the students to model mathematical concepts. Some of the manipulatives used were Cuisenaire Rods™, fraction circles, M&M™ candies, and wooden cubes. Each of the manipulatives provided a concrete model of abstract mathematical concepts. Math Explorer™ calculators were introduced as appropriate technology, and were available throughout the course. The purpose of using manipulatives and calculators was to develop students' conceptual understanding to build their confidence for using mathematical strategies and, therefore, increase disposition for solving non-routine problems.

The mathematics department required D.I. Bloomfield's (1994) *Basic Mathematics* as the course textbook. The prescribed syllabus outlined teaching the required concepts of whole numbers, fractions and mixed numbers, decimals, ratios/rates and proportions, percent, geometry, measurement and, finally, signed numbers. The order was not prescribed except for signed numbers which were to be introduced last as a lead-in to algebra, the subsequent course.

Traditional-style homework from the required text was assigned after each class. These assignments were mainly exercises which provided practice on the course topics. The

purpose of the assignments was to prepare the students for success on the final examination which was the only required assessment and was prepared by the instructor (see Appendix D). To insure that the participants did the homework assignments they were graded on six class quizzes and one take-home quiz (see Appendix E). The quizzes consisted of short answers to questions about assigned homework.

Problem-solving journal entries were assigned after each class and collected on Tuesdays (see Appendix F). Students were encouraged to talk to other people in attempting to solve the journal entries to emphasize the importance of understanding problem solving strategies in non-routine problem situations. An increase in conceptual understanding leads to more communication in the mathematics classroom indicating an increase in mathematical disposition.

Students were encouraged to talk to each other about all assignments but were asked to work individually on quizzes and the final exam. Students' final grades were based on the quizzes, the final examination, the journal entries, and class participation (from peer and teacher evaluation).

Teacher Reflective Journal. Several methods can be used to reflect on how well a lesson went—videotaping, writing logs, talking with other teachers and the students, etc. (Hart, Schultz, Najee-ullah & Nash, 1992). Keeping a reflective journal was my personal, ongoing assessment of my teaching and the students' learning. After each class session, I sat in my car and wrote observations about the class. This activity provided an opportunity to verbalize what I had experienced. Through reflection I focused on student discourse to assess how well the students were communicating and understanding concepts presented. Journaling was my tool for reflecting on each class and assessing how well my lesson

objectives were being met. The purpose of the journal was to track observations of students' willingness to participate in learning mathematics; specifically, student and teacher actions and interactions that increased positive mathematical disposition of the students were recorded.

## Results

This project investigated whether or not NCTM's professional standards are appropriate for a two-year college, developmental mathematics course. Two types of data were collected: qualitative and quantitative. Qualitative data were taken from the pre-interview and post-interview transcripts, from the teacher reflection journal, and from the student course evaluation. Qualitative data from the interviews were organized into a quantitative summary, and in following the rubric for scoring individual interview questions (p. 24) mathematical disposition is quantified. In addition, summarizing the four aspects of the college's requirements of a final grade provides information indicating level of success in this two-year college, pre-algebra course.

### Qualitative Data

Interview tapes were transcribed and responses were analyzed using Stenmark's (1991) categories of disposition: 1. Confidence in using mathematics; 2. Flexibility in doing mathematics; 3. Persevering at mathematical tasks; 4. Curiosity in doing mathematics; 5. Reflecting on their own thinking; 6. Valuing applications of mathematics; and 7. Appreciating role of mathematics (p. 34). A summary of the results tallies each students' comments that represent a lack of the category (-) and for those that represent evidence of the category (+) (see Table 2). All cells with no number entered means there was no apparent evidence in any of that student's comments regarding that category.

The teacher reflection journal and the course evaluations (see Appendix G) were also analyzed for evidence of student communication. Each student's comments tell a different story related to Stenmark's (1991) categories of disposition in doing mathematics.

Table 2

Tally of Student Comments in Pre- and Post-Interviews by Category

Category	Student									
	A		B		C		D		E	
	-	+	-	+	-	+	-	+	-	+
	Pre-interview									
1. Confidence	3		9		1		5		4	
2. Flexibility	1									1
3. Perseverance	1		2		1		1		1	
4. Curiosity										
5. Reflection			4							
6. Value									1	1
7. Appreciation			1	3						
	Post-interview									
1. Confidence		1		4		2		1	2	
2. Flexibility		1					1			1
3. Perseverance		2		1		3		3		1
4. Curiosity				1						
5. Reflection		1		2		2		4		2
6. Value		1		3				1		1
7. Appreciation		1		1				1		2

Student A. At first Student A lacked confidence in herself. During the first interview she made comments such as:

These are hard . . . That's a problem, that is a major problem that I have. I make everything three times as hard as it should be.

In my journal I noted:

She did arithmetic fine and seemed to use common reasoning skills when attacking the interview problems. . . . she stated that she doesn't like math topics.

These comments related to a lack of Stenmark's category of confidence in using mathematics.

During the first interview Student A did not attempt to answer the question about the area outside a circle inscribed in a square but still inside the square (see Appendix B, number 3). After completing all the problems that she felt she could answer, Student A looked at this problem a second time and said:

I'll look at the first one again. I really don't know if I . . . [silence]

Zero and the radius of the length is three. I really don't know.

However, in attempting the same type of problem in the post-interview (see Appendix C, number 3). Student A quickly stated:

Well, I'm thinking that this is a square. And if you found the area of the square and this [the circle] and subtract the two, you'd find the shaded area.

One of the other problems not attempted in the pre-interview was followed up in the post-interview with:

OK, I'm trying to remember from my notes.

This quotation was followed by one full page of transcription relating Student A's attempt to solve the previously unsolvable problem.

In my reflection journal I noted that Student A appeared to have confidence:

One of the students asked why " $3/8 + 2/8 \neq 5/16$ " The whole class was willing to discuss why. Student A offered help in multiplication to get common denominators. She (voluntarily) went to the board to show writing whole number next to fraction

$$2 \quad \frac{1}{3} + \frac{1}{2} \quad 3$$

Student A inserted the 2 and the 3 to show algorithm.

Her confidence was apparent in two ways. She not only was willing to explain an algorithm to another student, but she also felt comfortable enough to go to the board without being asked.

In a course evaluation Student A wrote:

It [communication] makes the math much more clear. I seem to finally understand it.

And when asked “What is your confidence level when it comes to solving problems?”

Student A responded:

Much more confident!

Student B. Student B’s increase in disposition toward mathematics was apparent in reflecting on his own thinking category. Throughout the pre-interview Student B made comments such as:

I’m just going to start to feel like an idiot.

I’m really nervous.

I’m going to be honest with you, I haven’t got a clue. I absolutely do not know.

At one point I thought Student B might be attempting to think about his work when he said:

How about if I do it for just go by one day. Maybe that’d be easier for me. Yeah, I’m going to break it down to one day at a time.

However, when I asked “Does that sound right to you?” he replied:

No, but I’m really getting embarrassed and want to leave.

At this point in the pre-interview Student B was unwilling to explain his thinking even if he was willing to think.

During the post-interview Student B expressed himself with:

Let me think.

Yeah, it does. [Sound reasonable] But I'm trying to figure out in my head why.

OK. That makes sense. . . that just clicked.

These comments are indicators of Stenmark's category of reflecting on their own thinking.

My reflective journal describes a vignette regarding Student B's reflecting on his own thinking.

Student B was making a factor tree on the board (risk). He wrote

$$\begin{array}{c}
 36 \\
 / \quad \backslash \\
 2 \quad 18 \\
 \quad \quad / \quad \backslash \\
 \quad \quad 2 \quad 9
 \end{array}$$

and concluded that prime factorization was  $2 \bullet 2 \bullet 9$  or  $2^2 \bullet 9$ . Student A and Student F helped him realize 9 could be broken down to  $3 \bullet 3$ .

Student B looked at his work until he agreed. Then corrected his tree and the factorization. Nice work by all 3.

In the course evaluation Student B responded to the question regarding communication with:

it a lot easier to remember if I draw picture. help me to remember. [sic]

Student B has reflected on his own thinking and what helps him to understand mathematics.

Student C. This student's pre-interview lasted 10 minutes and the transcription has no responses longer than one line. The post-interview lasted somewhat longer, but this

student had no interest in communicating her thought processes as she solved problems.

There were comments such as:

I don't know. Quit screwing with my mind.

Student C was absent from four classes and would not participate when she was present. Her negative disposition toward mathematics and my concerns about her were evident throughout my reflective journal.

I'm challenged by Student C's quietness.

Everyone is communicating but Student C.

Student C spent 10 minutes on quiz. At 20 minutes everyone else still working.

Student C showed up today but did not pay attention in class. When I checked on her work there was nothing written down. I said "How are you doing with proportions?" She said, "I took trig. I can do this."

Student C refused to sit in the circle [with the rest of the class] and left at 11:00.

She told me she didn't turn in her journals because she "had no idea."

The course evaluation asked "How has this math course been similar to other math courses you have taken?" Student C replied:

i know most of it

And her answer to "How does communicating mathematical ideas help you to understand mathematics?" was:

it doesn't really, i enjoy working them out in my head.

She also indicated that her confidence had not changed.

Student D. Student D improved in his reflection, but his most apparent improvement was in his persevering at mathematical tasks. For the pre-interview this student told me he left his glasses in the car and could not read the questions. As the questions were read to him, Student D attempted to answer only two of the questions. His transcription has sentences like:

I have no idea.

This is what you get for not paying attention in school.

I wish I did know, but. . .

However, near the end of the pre-interview Student D said something contrary to his initial message of not wanting to try. He said:

I never did use this math very much. I always loved math, I was never very good at it. I hated English.

During the post-interview Student D tried all of the problems. He wasn't always on the right path to a conclusion but in his own words:

Uuh, give me a piece of paper. Put it on here. I'll figure something out here. I might not be right. Let's see . . .

He kept working at each of the problems, asking questions of the interviewer, until something made sense. In trying to decide which fraction was closest to .73 he said:

You can forget this. As a matter of fact I think that was the class I missed.

The transcription for that problem continues for several pages and through further probing the interviewer discovered this student

changed it into inches. Yeah, so half would be like that. And then you got quarters. And then, let's see, eighths. And then sixteenths. So I changed it into inches.

And when the interviewer said, "It wasn't as hard as you thought it was going to be, once we started talking about it," the student replied:

No. It's just trying to figure out what you want. What you want to put down there. How, what you're asking. Are you after money or inches?

This example also indicates Stenmark's (1991) category of valuing applications of mathematics which seemed to be another of Student D's strengths. At another point in the second interview he said

Of course in some problems that you run across you would have to be able to know how to do that. Of course, you wouldn't have pepperonis. You might have some kind of wiring or whatever. You have to know how much area you've got.

His applications of fractions to inches and pepperonis to wiring helped Student D persevere in solving non-routine problems. It also indicated he was able to make connections to real world problems which were meaningful to him.

Throughout the 5-week course Student D's perseverance kept him working hard, even when I thought he might not continue. Journal entries included comments such as:

Student D having trouble keeping up. He offers discoveries minutes after we've [the rest of the class] discussed the same thing.

Student D gave me his entire homework p. 58 + 59 so I looked at that. He can do 2-step problems. Words seem to be his block. Something gets mixed up.

Student D told me I said we would study 2 hours but it's more like 6 or 7.

Student D was very active today. Offered suggestions on work of others even. He really likes Math Explorer [calculator].

Some things are getting organized for Student D. It's a miracle he isn't giving up!

In evaluating the course three of Student D's comments were:

[communicating about mathematical ideas] helps me to understand math better

Have more confidence in math

Making it longer [would make this particular math course better]

Overall, this student missed only one class due to an overtime work assignment.

Other indicators of Student D's perseverance were related to homework. He was allowed to take homework quizzes home to finish them and returned every one. He did more homework than was required and turned in all journal entries.

Student E. Of the seven Stenmark (1991) categories, valuing applications of mathematics seems to describe Student E. This was indicated during the pre-interview when Student E was asked if she liked working with others on math problems. Her response was:

Yeah, I like that. Because a lot of times, if I can't see it in a certain way, somebody's able to explain it in that way then that's just helping me learn, more different ways.

In being flexible about strategies Student E expressed an appreciation for applications she had not thought about before. It appeared that Student E valued understanding problems but one of her comments:

You have to do an equation to do this.

offers a foreshadowing of her definition of understanding. During the class on percents Student E was one of the first students to arrive at correct answers using the formula  $\text{Percent} = \text{rate} \times \text{base}$ . This was an algorithm that she could memorize. Yet when given situations appropriate for the formula's application Student E had great difficulty deciding which variable represented which number. After grading the quiz I entered the following comment in my journal:

Student E had a perfect quiz score but misunderstood the practical application [bonus point] - did great on memorized algorithm.

Her subsequent struggles in understanding how to apply the formula indicated a value of applications.

In the post-interview Student E was contemplating doubling the length of a rectangle and halving the width. Would the area be the same as the original rectangle?

Yeah, it would be cuz if I flipped this [the right half of the new rectangle] up on top of there [the left half of the new rectangle] it would make the square.

Through further questioning this student was very confident that this could work for any size rectangle, no matter how big the rectangles are. The interviewer was more impressed with this response than the interviewee; it was not a complicated application for her, because she really understood it.

In the post-interview Student E also expressed an appreciation for manipulatives in solving problems.

I have to practice more with those little cutouts . . . then I can see what I'm doing and create it and be able to visualize it and figure it out as I'm doing it.

Most noticeable about Student E was her change in attitude about calculators. She refused to use a calculator in the beginning of the course and as calculators were introduced in the classroom, Student E used them minimally. In the post-interview Student E was given pencils, manipulatives and calculators; her comment was:

OK. Yeah, I gotta get used to those calculators.

And when she was trying to figure out the number of minutes in the month of June she said:

There's 30 days in June. This is when I need a calculator.

This change in attitude demonstrates Stenmark's (1991) flexibility category.

Student E participated during every class and completed all assignments. Sometimes the assignments were turned in late because she needed more time to think about them.

Student E progressed from an algorithm-dependent student to valuing how the math topics could be applied in her world.

When asked about how communicating mathematical ideas helps with understanding mathematics Student E wrote:

It ables me, to find different ways of solving the problems.

And when asked about her confidence level she wrote:

My ability in finding different approaches is more open minded.

Her favorite aspect of the course was:

Learning more about everyone's way of solving problems.

Apparently flexibility in doing mathematics made quite an impression on this student.

Student F. Student F did not participate in the interviews but his curiosity in doing mathematics was evident in his participation in the classroom. He often asked more thought-provoking questions than the rest of the class. This was first noted in my reflective journal after a class discussion on decimal place value.

Student F was discovering things that left the class behind. He asked how is .04% so different from 86%. Then he posed questions about .086%. Decided it was twice as big as .04%. Then he asked what part .00086 was. I pointed out place values tenths, hundredths, thousandths and he said "hundred thousandths. That's small!" His questions and answers definitely communicated understanding.

Later in the course the students used graph paper to make boxes that had the largest volume.

My journal entry said:

Student F tried to make a box out of 4x3x2. Didn't work but I think he learned from it.

Another example of Student F's curiosity occurred on the day of the final examination. After completing his test Student F showed me a car manual and a formula in the manual that determined how fast a car could go depending on particular numerical ratings

of the car. Student F realized this was a mathematical application he could use, but because he had not studied algebra before he wanted help. With a few directions on how to substitute variables using values with which he was well-acquainted, Student F had great fun exploring potential driving speeds.

Student F was planning to attend a technical school following this class. His evaluation indicated this course would help him follow his interest in car engines.

It [the course] is nothing like any math course I have ever taken. I used to dread going to math class but this course didn't bother me at all.

[Regarding communicating mathematics] I can't just look and do, I have to talk about the problem to really understand it.

### Quantitative Data

Quantitative data were organized through the use of a rubric which describes scores for each of the questions in the pre- and post-interviews. The final exam scores and the final grades also provide quantitative data indicating student disposition toward solving mathematical problems.

Rubric Scores. Five of the six students who completed Preparatory Mathematics were interviewed before and after the course. Responses to each of the questions were assessed and assigned a score based on the rubric shown in Figure 1 (p. 24). Scores increased for all five students (see Table 3). Although mistakes were still made, especially in the use of algorithms, four of the five students demonstrated an increased disposition for using mathematics to solve problems.

Table 3

Rubric Scores on Pre- and Post-Interviews by Student

	Student				
	A	B	C	D	E
Pre-Interview	9	8	5	3	4
Post-Interview	14	12	7	11	12

Final Examination. The final exam (Appendix D) was required by the college and, therefore, assessed the topics set forth on the prescribed syllabus. The questions, however, were in problem solving format except for the very last problem about order of operations. There were 15 questions on the test and the grade represented the 10 best answered questions. This score was worth 15% of the final grade.

Final Grade. The students decided on the weighting of journal entries, class participation, quizzes, and the final examination in determining the final grade. Table 4 summarizes all scores for each student. The final grades are indicators of mastering the topics required by the college.

Table 4

Final Report Card Grades

Student	Journal Entries (25%)	Class Participation (25%)	Quizzes (35%)	Final Exam (15%)	Final Grade (100%)
A	90	100	100	100	A
B	95	100	88	100	A
C	14	50	57	80	F
D	76	90	60	70	C
E	91	95	86	100	A
F	90	100	96	100	A

## Discussion

“Are the National Council of Teachers of Mathematics standards for mathematics education really suitable for post-secondary studies?” was my question at the beginning of the project. The results of this particular project seem to indicate they are applicable for this 2-year college, pre-algebra course. The goals of NCTM’s ideology are for students to learn to value mathematics, to become confident in the ability to do mathematics; to become mathematical problem solvers; to learn to communicate mathematically; and to learn to reason mathematically (1989). Five of the six students who participated in Preparatory Mathematics showed evidence of moving toward NCTM’s ideal.

Student A’s self-confidence in solving mathematical problems increased. Student B realized his thinking about problems enabled him to understand the mathematics that escaped him before taking the course. Student D learned to communicate his valuable thoughts by persevering in solving problems. Student E listened to new ideas about mathematical problems to reason through, rather than memorize rules governing mathematical problems. Student F, through a natural curiosity of mathematics, found real world applications that made sense to him. These 5 students experienced the same constructivist classroom, yet learned through a diverse collection of backgrounds.

Student C, my antithesis, was not ready or willing to develop mathematically. Norwood (1994) may say “I told you so,” but I think Norwood would have come to the same conclusion. This student was not open to learning additional mathematics at this point. The reasons could be hypothesized, however, her situation is beyond the scope of this project. Because her results were so different from the majority of this class, it is more productive to keep in mind that she was atypical among five success stories.

### Worthwhile Mathematical Tasks (WMTs)

The students' journal entries (see Appendix F) were worthwhile mathematical tasks. The most productive journal entry involved proportions (Geddes, 1994). The activity began with a sheet of paper with pictures of six common objects. It seemed that insufficient data were presented to answer the repeated question "How tall is she/he [the person who lost the item in the picture]?" As a class, each student was given the item pictured but in a size they were not used to experiencing. The ring that someone lost was a bangle bracelet, the tablecloth was a cloth napkin, the teacup was a doll's coffee cup, the car was a toy car, the teaspoon was a caterer's serving spoon, and the thimble was a plastic drinking cup. Each student had one week to determine what size person would use the item in the same manner that we would use a regularly-sized item. One week later, when we discussed each method, some were challenged and some led to alternative methods. Our class discussion also led to differences in individuals which pointed the way to differences in solving problems. This particular WMT provided a marvelous source for student discourse and brought the class together as a community of reasoners and investigators. It worked quite well in a room of adult learners. Other journal entries were not accepted as well. And still other journal entries were pondered and discussed even more, depending on the individual student's questions and concerns.

### Classroom Discourse

Emphasizing classroom discourse was difficult at first. The students seemed somewhat reluctant to open up. At one point it even backfired with one student interpreting my requests for the class to discuss problems as match-making. With time and appropriate WMTs the students communicated mathematically.

The discourse provided observable expressions of thoughts and growing dispositions of my students. It was my foundation for reflection-on-action (my teacher reflection journal) and reflection-in-action (on the spot adjusting of the lesson) (Zeichner & Liston, 1996). Through classroom discourse the students learned from the instructor and the instructor learned from the students.

### Learning Environment

The 2-year college learning environment is different from secondary education and 4-year higher education. My students came from diverse backgrounds and had varying plans for the future. In our classroom, however, five of the students came together as a community of learners interested in preparing for additional mathematics courses, job promotions, success in standardized tests, and conceptual understanding of mathematical topics. From the comments on the students' course evaluations it appears that these students appreciated the chance to learn mathematics in a way different from their past experiences.

### Analysis of Learning and Teaching

The teacher reflection journal provided a medium for reflecting on the day's class; it was my record of reflection-on-action (Zeichner & Liston, 1996). I was able to sort through situations that could be considered uncomfortable. It also provided a place to verbalize what I experienced and how I planned to deal with that experience in the future. Writing specific questions in the journal, as suggested by Hart, Schultz, Najee-ullah and Nash (1992), provided necessary guidance in improving my instruction for that class.

### Conclusion

In summary, the students who participated in class and made an effort to do the journal entries achieved the objectives of the course. Most of the students eventually

appreciated the communication that took place in class. I learned that no matter what teaching strategies are used, some students may not be ready to learn just because they enroll in a class.

The NCTM professional standards (1991) of worthwhile mathematical tasks, classroom discourse, and teacher reflection were all part of this Preparatory Mathematics course. All but one of the students indicated that they learned the topics the college required and increased their disposition in doing mathematical problems. These two indications equate to success in this two-year college classroom through use of the NCTM professional standards.

Many educators may not subscribe to NCTM's standards as whole-heartedly as this researcher does. What they are doing may be the best situation for their students. If, however, an educator is interested in self-assessment, this project indicates high endorsement for increased classroom discourse/communication in a 2-year college classroom and a teacher reflection journal as an excellent source for analysis of learning and teaching in any classroom. And if these two aspects of the NCTM standards work, other aspects may work, too. At this point there is plenty of literature supporting the professional standards in primary and secondary education. Support for higher level education is not as abundant, yet AMATYC supports the ideology.

This project addressed a very small number of students in a two-year college. Perhaps other college educators will publish results of college classroom experiences which may support or add to what I have found. Do worthwhile mathematical tasks and appropriate use of manipulatives create an environment conducive to classroom discourse in a two-year college classroom? Does reflection improve instruction and in turn improve learning in

terms of increasing student disposition for solving mathematical problems in this setting?

Teaching standards of worthwhile mathematical tasks; classroom discourse; the learning environment; and analysis of tasks, discourse, and environment are a product of a council concerned with improving mathematics education. This project indicates these standards are suitable for two-year college classrooms.

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Appendix A

Letter of Consent

July 24, 1998

Dear Student,

As a portion of the requirements for a Master of Education Degree at the University of Cincinnati, I am required to submit a project that involves the curriculum and instruction of mathematics. My personal goal for this project is to find an area of math education that will most directly and immediately benefit my students. I have decided to concentrate on communication in mathematics.

All data I collect will remain confidential and will not be traceable to any individual student. You may choose not to participate in my project. The subject matter represents no departure from the regular course of study. Only a different method of attack and an increased focus on mathematical communication will be emphasized. Students will be encouraged to work together and to share and discuss their solutions with each other. Data for the entire group will be submitted as part of my project. I will also share group information with any of the participants.

I am very excited about this upcoming project. I hope it will benefit my students as they prepare to enter the workforce in their chosen area of study.

Please sign the top of the attached consent form if you wish to participate. Sign the bottom portion of the statement only if you do not wish to participate.

Thank you,

Teri Rysz

I, the undersigned, have understood the above explanation and have given consent for participation in Teri Rysz's masters' project. I understand that this project represents no departure from the standard course of study objectives for Preparatory Math 104, and that scores and results of the project will not be individually traceable.

\_\_\_\_\_ signature

\_\_\_\_\_ date

\*\*\*\*\*

I do not wish to be a part of this project. I understand that I will be expected to participate in class activities to complete the course in Preparatory Math 104.

\_\_\_\_\_ signature

\_\_\_\_\_ date

Appendix B

Pre-interview

Question

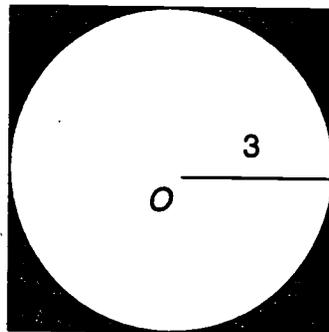
1 Of the following, which is closest in value to 0.52? Explain how you know.

- A.  $\frac{1}{50}$                       C.  $\frac{1}{4}$                       E.  $\frac{1}{2}$   
 B.  $\frac{1}{5}$                           D.  $\frac{1}{3}$

(Kenney & Silver, 1997, p. 128)

2 TRUE OR FALSE? In a triangle, if you double the length of the base and cut the altitude in half, the area will be the same. WHY? (Bloomfield, 1994, p. 383)

3



In the figure above, a circle with center O and radius of length 3 is inscribed in a square. Explain how to find the area of the shaded region. (Kenney & Silver, 1997, p. 156)

4 Explain how you could find the number of seconds there are in a week. (Bloomfield, 1994, p. 394)

5

- |   |   |
|---|---|
| <u>Video Store A</u>                    | <u>Video Store B</u>                      |
| \$2.65 per tape for one night           | \$3.00 per tape for 2 nights              |
| \$1.50 charge for each additional night | 1 credit if tape returned after one night |
| Every 10th tape free for one night      | Every 10 credits = one free rental        |

The Peterson family rents 30 videotapes yearly, of which 23 are rented for one night only and 7 are rented over a period of two nights. Given the rental fee structures shown above, fill in the chart below with the total yearly costs for the Petersons at each store. (Note: The 30 tapes include the free tapes earned.)

Store	Total Cost
A	
B	

(Kenney & Silver, 1997, p. 135)

Appendix C

Post-interview

Question

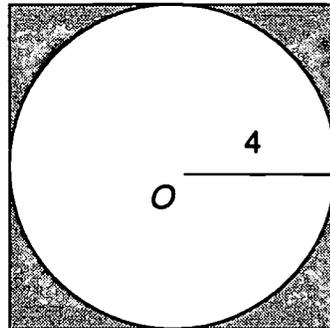
1 Of the following, which is closest in value to 0.73? Explain how you know.

- A.  $\frac{3}{50}$                       C.  $\frac{3}{4}$                       E.  $\frac{3}{10}$   
 B.  $\frac{4}{5}$                       D.  $\frac{4}{3}$

(Kenney & Silver, 1997, p. 128)

2 TRUE OR FALSE? In a rectangle, if you double the length and cut the width in half, the area will be the same. WHY? or WHY NOT? (Bloomfield, 1994, p. 383)

3



In the figure above, a circle with center O and radius of length 4 is inscribed in a square. Explain how to find the area of the shaded region. (Kenney & Silver, 1997, p. 156)

4 Explain how you could find the number of minutes there are in the month of June. (Bloomfield, 1994, p. 394)

5

Video Store A

\$3.75 per tape for one night  
 \$2.00 charge for each additional night  
 Every 10th tape free for one night

Video Store B

\$4.00 per tape for 2 nights  
 1 credit if tape returned after one night  
 Every 10 credits = one free rental

The Peterson family rents 30 videotapes yearly, of which 23 are rented for one night only and 7 are rented over a period of two nights. Given the rental fee structures shown above, fill in the chart below with the total yearly costs for the Petersons at each store. (Note: The 30 tapes include the free tapes earned.)

Store	Total Cost
A	
B	

(Kenney & Silver, 1997, p. 135)

## Appendix D

### Final Examination

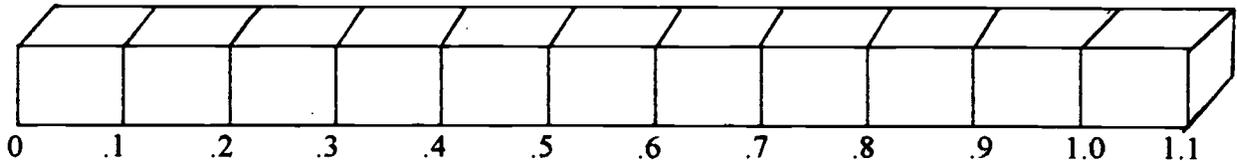
Final Exam

Name \_\_\_\_\_

**Directions**

Answer the following questions and circle your final answer. Be sure to show all your work so that if the circled answer is incorrect I will be able to follow your reasoning and determine if partial or full credit is still appropriate. Your grade will be based on the 10 best answers out of all 15 questions. For example, if you answer 11 questions and miss 2 your score will be 9 out of 10. If you only answer 8 questions and miss 3 your score will be 5 out of 10. Good Luck!!

1. Put the letters on the right box.



R      $\frac{3}{4}$

T      $\frac{13}{12}$

O      $\frac{5}{9}$

A      $\frac{1}{20}$

L      $\frac{1}{4}$

R      $\frac{5}{8}$

E      $\frac{7}{8}$

C      $\frac{19}{20}$

C      $\frac{4}{9}$

L      $\frac{1}{8}$

2. Consider three consecutive pages in a book. The sum of the page numbers is 534.

What are the three page numbers?

3. A child's sandbox has a square base that measures 4 ft by 4 ft, and it is 1 ft high. How many cubic feet of sand are required to fill the sandbox half full?



8. Maurice has \$43.50 in his checking account. If he writes a check for \$51.20, what is his new balance?
9. Margaret and Darryle meet each other in a state park. Darryle dives to 30 feet below sea level, and Margaret climbs to 250 feet above sea level. How far apart are Margaret and Darryle?
10. Ashley's Little Tots soccer team won 8 out of 12 games. What percentage of the games did her team win? What percentage of the games did her team lose?
11. You are at mile 47 on the Garden Parkway. If there are gas stations at mile 59 and mile 33, which is closer to you?

12. Betsi earns  $\frac{3}{5}$  of her college expenses with a scholarship. What percent of her expenses is her scholarship?

13. Paul invested \$1500 and earned \$184 in interest. How much would Paul have earned if he had invested \$7300?

14. A wheel of cheese was put out for an appetizer at a party. One third of the wheel was eaten in the first hour, and one half of the wheel was eaten in the second hour. How much is left?

15. Evaluate using the correct order of operations:

$$6 - 3^4 + (\sqrt{49} + 9) \div 4$$

and

$$16(-\sqrt{144}) \cdot 0 - 1$$

## Quiz Chapter 1

Homework Quizzes (7) Name \_\_\_\_\_

Answer the following questions with explanations for your answers. If you need more room use the back of this paper.

1) Mexico covers 760,373 square miles. Round this number to the nearest one hundred thousand. (1 point)

2) Anissa started the day with \$1,247 in her checking account. She wrote checks for \$89, \$150 and \$143. Anissa also deposited \$345 into her account the same day. Write a number expression that could be used to determine the balance after the day's transactions. (2 points)

3) If a mathematician earns \$39,676 per year, How much does the mathematician earn per week? (1 point)

4) Identify which property of whole numbers the following illustrates:  $6(3+5)=6 \times 3 + 6 \times 5$  (1 point)

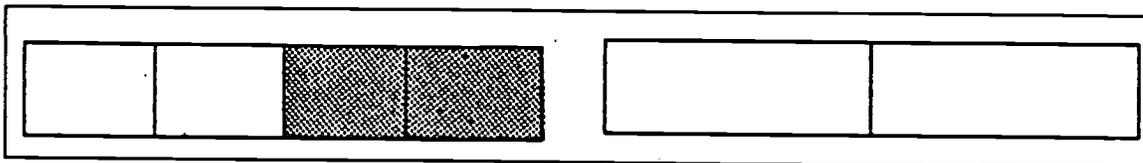
5) Would you rather earn  $2^7$  dollars or  $7^2$  dollars? Why? (1 point)

6) Maria and her friends were on a hike in the woods. They stopped to listen to some crickets. Maria said she knew a way to estimate the temperature from the number of cricket chirps in one minute. She uses this formula.  $T = 40 + c \div 4$  where  $c$  represents the number of chirps a cricket makes in one minute. They counted 88 chirps in one minute. Maria wrote the expression  $40 + 88 \div 4$  on a sheet of paper and said, "The value of this expression is the number of degrees Fahrenheit!" Joan said the temperature was  $32^\circ\text{F}$ . Leah said the temperature was  $62^\circ\text{F}$ . What did Joan and Leah do differently from one another? Who is correct? Why? (3 points)

7) From your homework assignment only, copy here what you wrote for question number 21, page 58. (1 point)

Answer the following questions with explanations for your answers. If you need more room use the back of this paper.

- 1) Use the divisibility tests discussed in class to explain why 126 is, or is not, divisible by 2, 3, 4, 5, 6, or 9. (2 points)
- 2) Shade the second figure to make it represent a fraction that is equivalent to the fraction represented by the shaded part of the first figure. (1 point)



- 3) Assign a fraction to all the parts of both figures. Explain your answer. (1 point)
- 4) Which fraction is larger  $\frac{5}{8}$  or  $\frac{5}{12}$ ? Find the difference between the two fractions. (2 points)
- 5) On an English exam,  $\frac{1}{6}$  of the students received an A,  $\frac{1}{5}$  of the students received a B,  $\frac{1}{4}$  of the students received a C, and  $\frac{1}{5}$  of the students received a D.
- What fraction of the class received a B or better? (1 point)
  - What fraction of the whole class received C's and D's? (1 point)
  - If the remainder of the class received an F, what fraction of the class is this? (1 point)
- 6) From your homework assignment only, copy here what you wrote for question number 31, page 103. (1 point)

Answer the following questions with explanations for your answers. If you need more room use the back of this paper.

1) Sort the following fractions into one of three groups. **Explain your reasoning.** (3 points)

Fractions to be sorted:  $\frac{4}{7}$     $\frac{2}{12}$     $\frac{8}{9}$     $\frac{6}{11}$     $\frac{1}{8}$     $\frac{9}{11}$

Group 1: Close to 0      Group 2: Close to  $\frac{1}{2}$       Group 3: Close to 1

2) Joan can work up to 15 hours per week at her job in the library. If she worked  $5\frac{1}{2}$  hours on Monday,  $1\frac{1}{4}$  hours on Tuesday, and 3 hours on Wednesday, how many more hours can she work that week? (2 points)

3) Write 180 as a product of primes (prime factorization). (1 point)

4)  $2\frac{1}{4} + 5\frac{1}{4} \div 1\frac{2}{3} =$  (1 point)

5) Brendan earns \$7.32 per hour for 3 hours on Friday and \$9.50 per hour for 2 hours overtime on Saturday. How much did he earn for these two days? (2 points)

6) From your homework assignment only, copy here what you wrote for question number 25, page 180. (1 point)

For each of the following exercises set up an equation in the form of  $\mathbf{P} = \mathbf{r} \times \mathbf{b}$  showing which number is missing. Solve for the missing number. Make sure decimal points are in the correct position and use your estimation skills to see if your answer is reasonable. (2 points for each exercise, 2 points for the BONUS)

1) 20% of what number is 24?

2) 45% of 124 is what number?

3) 60 is what percent of 240?

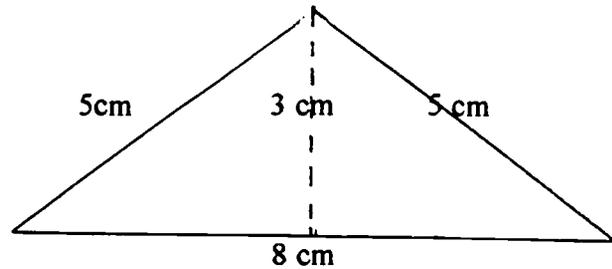
**BONUS:** If your salary is going to be increased by 10% next year and then reduced by 10% the following year, how will your final salary compare with your present salary? Why? (Hint: Start with a \$100/week salary and see what happens.)

Solve the following problems using proportions. Each problem is worth 2 points: 1 point will be given for the correct proportion set up and 1 point for solving for the proportion's missing number.

- 1) A 30 lb bag of clover seed is used to cover a 4000 square foot plot of land. How much land would a 50 lb bag of clover seed cover? Round your answer to the nearest tenth. (2 points)
  
  
  
  
  
  
  
  
  
  
- 2) On a map of New York, one inch represents 16 miles. If two cities are seven and one half inches apart on the map, how many miles apart are they? (2 points)
  
  
  
  
  
  
  
  
  
  
- 3) If you earn 28,000 working 8 months of the year, how much would you earn if you worked 12 months of the year at this salary rate? (2 points)
  
  
  
  
  
  
  
  
  
  
- 4) The price of a bicycle is \$625. If the sales tax is \$22.50, what is the tax rate? (2 points)
  
  
  
  
  
  
  
  
  
  
- 5) Fifteen percent of Mrs. Trumbull's yard is flower gardens. If her flower gardens cover 600 square feet how big is her yard? (2 points)

For each of the following exercises show how you would answer the question and then answer the question. (2 points each)

- 1) Find the area and perimeter of:



- 2) Find the volume of a tea mug which has a diameter of 4 in and a height of 6 in.
- 3) What is the cost to carpet a room which measures 9 ft by 30 ft if the carpet costs \$22 per square yard?
- 4) Find the area and circumference of a round swimming pool with a diameter of 24 m.
- 5) From your homework assignment only, copy here what you wrote for question number 21, page 382.

**Take home quiz**  
**Chapter 11**

Name \_\_\_\_\_

Answer the following questions. You may use your notes and/or your textbook. It will be considered a take home quiz and is due Tues, September 1, 1998 at 10 am.

1. Change 53 kilograms to centigrams.
2. Convert 502 decimeters to hectometers.
3. Convert 15 liters to pints.
4. 450 grams is equal to how many pounds?

5. Add and simplify:

$$\begin{array}{r} 11 \text{ yd} \quad 4 \text{ ft} \quad 7 \text{ in} \\ \phantom{11 \text{ yd}} \quad 9 \text{ ft} \quad 3 \text{ in} \\ + \quad 3 \text{ yd} \phantom{00} \quad 5 \text{ in} \\ \hline \end{array}$$

6. Subtract:

$$\begin{array}{r} 2 \text{ yr} \quad 14 \text{ wk} \quad 3 \text{ days} \\ - 1 \text{ yr} \quad 45 \text{ wk} \quad 10 \text{ days} \\ \hline \end{array}$$

7. Multiply and simplify:

$$\begin{array}{r} 10 \text{ gal} \quad 3 \text{ qt} \\ \times \phantom{10 \text{ gal}} \quad 9 \\ \hline \end{array}$$

8. Divide (25 yd 3 ft 10 in) by 3
9. Convert 15 square inches to square centimeters.
10. Which is greater 35 square feet or 15 square meters? How much greater? Give your answer in square meters.

## Journal Entries (8)

**Journal Entry 1, due Tues, August 4, 1998**

Name \_\_\_\_\_

**Exponents and Growth Patterns****Problem:**

In Bugsville, USA, at Fly By Night University, Tara, a first year student, decides to start a rumor that the town of Bugsville is going to declare September 14 as National Bug Day and is going to close all schools for the day. She tells two students the rumor with the instructions that each of these students is to repeat the rumor to two more students the next day and that each of these new students is to repeat the rumor to two more students on the third day, and so on. For example, on the first day two students know the rumor; on the second day four more students will know the rumor; on the third day eight more students will know the rumor, and so on. How many new students will be told the rumor on day 10? If Tara starts her rumor on September 1 and there are 8000 students in the district, what are the chances that all students will hear the rumor before September 14 and will stay home from school on that day?

(Phillips, 1991)

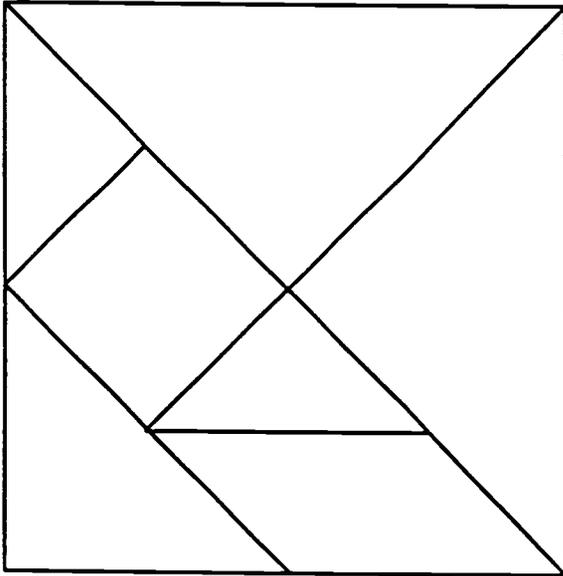
**Journal Entry 2, due Tues, August 11, 1998**

Name \_\_\_\_\_

**Rational Numbers**

**Problem:**

In the following diagram, the small, inside square is what fraction of the large, outside square? Explain the steps you would take to determine the fraction and why you would take the steps.



(Curcio & Bezuk, 1994)

## Journal Entry 3, due Tues, August 11, 1998

Name \_\_\_\_\_

### Rational Numbers

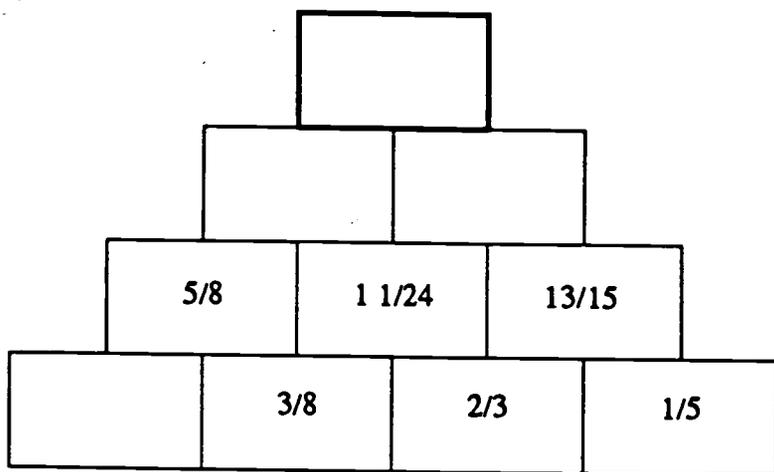
#### Problem:

The Ancients of Noitcarf (“fraction” spelled backward) is a rule-finding game involving fractions. You must find the rule in order to complete the pyramid and find the key.

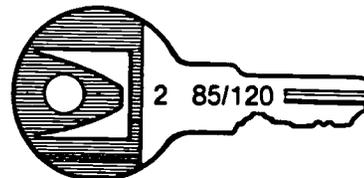
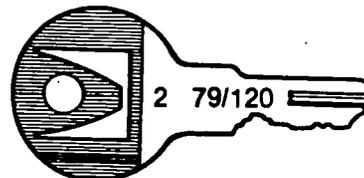
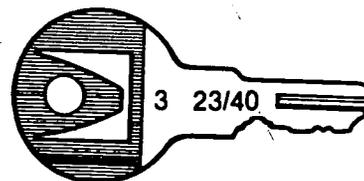
#### The Situation:

While exploring the ruins at the city of Noitcarf, you come upon a chamber. You know that a treasure and many wonderful secrets are inside. You find three keys with writing on them, which proves they were made by the Ancients of Noitcarf! Only one of the keys will open the chamber. If you use the wrong key, the chamber will vanish forever. You notice a mysterious drawing on the wall. You realize that the top block on the drawing will tell which key to choose.

#### The Drawing:



#### The Keys:



#### Do This:

- Find the rule that the Ancients of Noitcarf used in the above pyramid.
- Once you figure out the rule, complete the pyramid.
- The answer to the top block will tell you which key to choose. Circle the correct key.
- Be sure to write down all steps taken to determine the correct key.

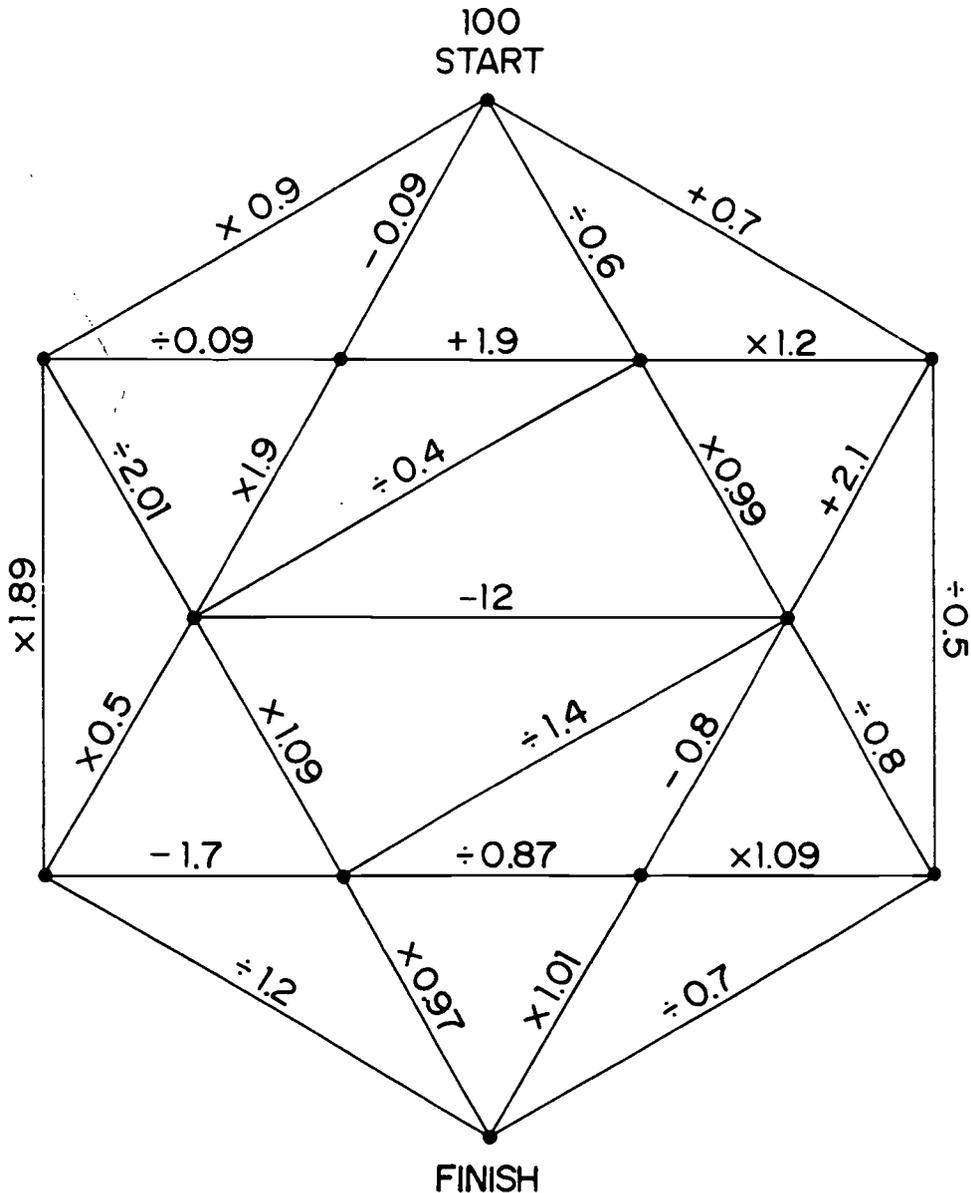
(Texas Instruments, 1988)

**Journal Entry 4, due Tues, August 18, 1998**

Name \_\_\_\_\_

**Decimals**

Using the following MAZE, find a path from start to finish. Begin with the number 100. For each segment you choose on the MAZE perform the assigned operation and number. The goal is to choose a path that results in the **largest** value at the finish of the MAZE. Do not retrace a path or move upward in the MAZE. Once you have decided how to find the largest value start over and determine how to find the path that leads to the **smallest** finish number. Don't forget to write down everything you are thinking while finding the two paths.



(Reys, 1991)

Name \_\_\_\_\_

**Percents**

Read and reflect on each statement below and then complete each statement by choosing an answer from the Answer List. Explain each choice for each statement under the completed statement.

**Answer List:**

- 0 percent
- Fewer than 25 percent but more than 0 percent
- About 25 percent
- Fewer than 50 percent but more than 25 percent
- About 50 percent
- More than 50 percent but less than 75 percent
- About 75 percent
- More than 75 percent but less than 100 percent
- 100 percent

1. \_\_\_\_\_ of the students in my classroom are left-handed.
2. \_\_\_\_\_ of the students in my classroom have red hair.
3. \_\_\_\_\_ of the students in my school like hamburgers.
4. \_\_\_\_\_ of the students in my school like baseball.
5. \_\_\_\_\_ of the students in my school are wearing tennis shoes today.
6. \_\_\_\_\_ of the people in my town are over ninety years old.
7. \_\_\_\_\_ of the people in my town own a car.
8. \_\_\_\_\_ of the people in my state are female.

(Reys, 1991)

Proportions

Answer the questions found in the box pertaining to the object given to you in class. Be prepared to share in class how you answered the questions. Also, think about how your classmates may answer their questions so you will be prepared to challenge their reasoning if necessary.



**Someone lost her ring.  
How tall is she?**

What measurements should be taken to find the height of the person who lost the object?

How did you find your answer?

Do you think that other people might have a very different answer?



**Someone just got out of her car.  
How tall is she?**

What measurements should be taken to find the height of the person who lost the object?

How did you find your answer?

Do you think that other people might have a very different answer?



**Someone lost a tablecloth for his card table.  
How tall is he?**

What measurements should be taken to find the height of the person who lost the object?

How did you find your answer?

Do you think that other people might have a very different answer?



**Someone dropped his teaspoon.  
How tall is he?**

What measurements should be taken to find the height of the person who lost the object?

How did you find your answer?

Do you think that other people might have a very different answer?

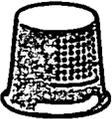


**Someone lost her teacup.  
How tall is she?**

What measurements should be taken to find the height of the person who lost the object?

How did you find your answer?

Do you think that other people might have a very different answer?



**Someone lost a thimble.  
How tall is he?**

What measurements should be taken to find the height of the person who lost the object?

How did you find your answer?

Do you think that other people might have a very different answer?

(Geddes, 1994)

**Journal Entry 7, due Tues, August 25, 1998**

Name \_\_\_\_\_

Look at the attached pictures from the movie, *Honey, I Shrunk the Kids!* The middle left picture shows one of the kids in a cereal bowl hanging on to a Cheerio. The bottom left picture is children playing near a bolt or screw. The big picture shows four children running from the dog's nose onto their father's nose. After looking at the pictures, answer the four questions above the pictures. Explain why you answer the way you do for all four questions.

**ACTIVITY 21B**

**HONEY, I SHRUNK THE KIDS**

Get set for the adventure of a lifetime in the #1 comedy hit of the year, *Honey, I Shrunk the Kids!* Rick Moranis stars as the preoccupied inventor who just can't seem to get his electromagnetic shrinking machine to work. Then, when he accidentally shrinks his kids down to 1/4 inch tall and tosses them out in the trash, the real adventure begins! Now the kids face incredible dangers as they try to make their way home through the jungle of their own backyard! Hurricane sprinklers! Dive-bombing bees! A runaway lawn mower and much, much more! Directed by Academy Award-winner Joe Johnston (Visual Effects, *Raiders of the Lost Ark*, 1981), this record-breaking smash is full of amazing special effects, hilarious comedy, wild chases, and nonstop surprises!

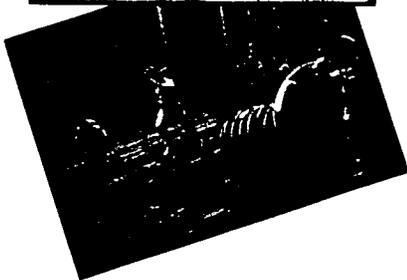
About how tall are the shrunken children in these pictures?

How could you find out?

Are the answers the same for each picture?

Do your results agree with the statement that the kids have been shrunk to 1/4 inch?

**RICK MORANIS**  
IN  
**HONEY, I**  
**SHRUNK**  
**THE KIDS**



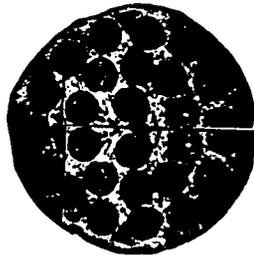
© Walt Disney Pictures

## Journal Entry 8, due Tues, September 1, 1998

Name \_\_\_\_\_

### Geometry

Last Saturday night I ordered a pepperoni pizza for dinner. The whole pizza was 15 inches in diameter and had 44, one-inch diameter pepperonis on it. While waiting for my food to cool, I noticed the pepperonis had the same shape as the large pizza but weren't nearly as big. I wondered what part of the pizza the pepperonis covered. I also wondered what part of the pizza was not covered by pepperonis. Explain what method you would use and help me find the actual area of the whole pizza that was not covered by a pepperoni.



## Appendix G

### Students' Course Evaluation

#### Preparatory Math 34-025-104 Course Evaluation

Please answer the following questions as honestly as you can about Preparatory Math 34-025-104, second half quarter, summer 1998. If you need more room, use the back.

1. How has this math course been different from other math courses you have taken?
2. How has this math course been similar to other math courses you have taken?
3. How does communicating (talking and/or writing about) mathematical ideas help you to understand mathematics?
4. What is your confidence level when it comes to solving problems? Are you more or less confident after completing Preparatory Math?
5. What was your favorite aspect of Preparatory Math?
6. What did you dislike about Preparatory Math?
7. How could the instructor make this particular math course better?
8. Additional comments:

The following activities were completed during the 5-week period. Please rate (4 to 0) each one as to how valuable an experience you consider it to be. Feel free to insert comments about particular activities.

4 = very valuable      3 = pretty helpful      2 = pretty lame      1 = complete waste of time  
 0 = Did not participate in this experience or else I can't remember it.

	<u>Circle a number for each</u>				
Exchanging phone numbers	4	3	2	1	0
Working/talking with classmates	4	3	2	1	0
Opportunities to ask questions	4	3	2	1	0
Handshaking: How many handshakes occurred?	4	3	2	1	0
Prime number "Sieve of Eratosthenes"	4	3	2	1	0
One color trains with Cuisenaire Rods	4	3	2	1	0
Fraction calculator keys	4	3	2	1	0
Factor "trees"	4	3	2	1	0
Fraction Circle	4	3	2	1	0
Four Decimals in a Row Game	4	3	2	1	0
M&M Investigation	4	3	2	1	0
Presentation of Journal Entry 6	4	3	2	1	0
Creation of the Giant	4	3	2	1	0
Measuring Circles & Diameters ( $\pi$ )	4	3	2	1	0
Changing a circle into a rectangle	4	3	2	1	0
Making a box with biggest volume	4	3	2	1	0
Counting # of squares inside a circle	4	3	2	1	0
Journal Entry 1: Bugsville rumor	4	3	2	1	0
Journal Entry 2: Fraction of small square to large square (tangram)	4	3	2	1	0
Journal Entry 3: Key of Noitcarf pyramid	4	3	2	1	0
Journal Entry 4: Decimal Maze	4	3	2	1	0
Journal Entry 5: Percent reasoning	4	3	2	1	0
Journal Entry 6: Individual Proportion Investigations	4	3	2	1	0
Journal Entry 7: Proportional pictures comparison	4	3	2	1	0
Journal Entry 8: Pizza Area	4	3	2	1	0



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