DOCUMENT RESUME

ED 468 386 SE 066 608

TITLE	High School Performance Indicators: Mathematics--Model 1 [and] Model 2.
INSTITUTION	Delaware State Dept. of Education, Dover.
PUB DATE	2001-00-00
NOTE	37p.
AVAILABLE FROM	For full text: http://www.doe.state.de.us/DPIServices/Desk_Ref/DOE_DeskRef.htm.
PUB TYPE	Legal/Legislative/Regulatory Materials (090)
EDRS PRICE	EDRS Price MF01/PC02 Plus Postage.
DESCRIPTORS	*Academic Standards; High Schools; *Mathematics Curriculum; Mathematics Education; Measurement; Spatial Ability
IDENTIFIERS	Delaware; *Performance Indicators

ABSTRACT

This document, from "The Teacher's Desk Reference to Standards and Performance Indicators for Curriculum Planning and Unit Development," is part of the Delaware Department of Education's ongoing efforts to provide assistance and support to local school districts in their development of a standards-based curriculum. This document explains high school performance indicator models 1 and 2 for mathematics education. Standards 1-10 are presented citing the skills and abilities for students to develop. (YDS)

High School Performance Indicators: Mathematics-- Model 1 [and] Model 2

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HIGH SCHOOL PERFORMANCE INDICATORS

MATHEMATICS – MODEL 1

2A

MATHEMATICAL PROCESSES

MATHEMATICS STANDARD ONE: Students will develop their ability to **SOLVE PROBLEMS** by engaging in developmentally appropriate problem-solving opportunities in which there is a need to use various approaches to investigate and understand mathematical concepts; to formulate their own problems; to find solutions to problems from everyday situations; to develop and apply strategies to solve a wide variety of problems; and to integrate mathematical reasoning, communication and connections.

MATHEMATICS STANDARD TWO: Students will develop their ability to **COMMUNICATE MATHEMATICALLY** by solving problems in which there is a need to obtain information from the real world through reading, listening and observing; to translate this information into mathematical language and symbols; to process this information mathematically; and to present results in written, oral and visual formats.

MATHEMATICS STANDARD THREE: Students will develop their ability to **REASON MATHEMATICALLY**, by solving problems in which there is a need to investigate significant mathematical ideas in all content areas; to justify their thinking; to reinforce and extend their logical reasoning abilities; to reflect on and clarify their own thinking; to ask questions to extend their thinking and to construct their own learning.

MATHEMATICS STANDARD FOUR: Students will develop their ability to make **MATHEMATICAL CONNECTIONS** by solving problems in which there is a need to view mathematics as an integrated whole and to integrate mathematics with other disciplines, while allowing the flexibility to approach problems, from within and outside mathematics, in a variety of ways.

The processes listed above will be reflected in mathematics assessment items in the Delaware Student Testing Program.

MATHEMATICAL PROCESSES

9th	10th	11th
Students will be able to:	Students will be able to:	Students will be able to:
9.201 develop and apply strategies to solve problems;	10.201 develop and apply strategies to solve problems;	11.201 develop and apply strategies to solve problems;
9.202 use mathematical notation and language to explain and defend their thinking;	10.202 use mathematical notation and language to explain and defend their thinking;	11.202 use mathematical notation and language to explain and defend their thinking;
9.203 make and test conjectures;	10.203 make and test conjectures;	11.203 make and test conjectures;
9.204 determine if a mathematical solution is reasonable.	10.204 determine if a mathematical solution is reasonable.	11.204 determine if a mathematical solution is reasonable.

It is expected that technology, particularly the graphing calculator, will be an integral part of instruction so that all students may become proficient in using technology as a tool toward meeting the Mathematics Content Standards.

MATHEMATICS STANDARD FIVE: Students will develop an understanding of ESTIMATION, MEASUREMENT, and COMPUTATION by solving problems in which there is a need to measure to a required degree of accuracy by selecting appropriate tools and units; to develop computing strategies and select appropriate methods of calculation from among mental math, paper and pencil, calculators or computers; to use estimating skills to approximate an answer and to determine the reasonableness of results.

END OF CLUSTER EXPECTATIONS

By the completion of **grade 8**, students will be able to:

- estimate and then measure angles, circumference, volume and surface area to the degree of accuracy required using standard and nonstandard units;
- convert measurement units within the same system;
- apply ratios, proportions and percents to real life situations;
- compute circumference; areas of triangles, parallelograms, trapezoids, and circles; and surface area and volume of cylinders, triangular and rectangular prisms and pyramids;
- apply order of operations;
- choose and explain an appropriate method for calculating an answer in a given situation;
- use multiple computational procedures with rational numbers;
- determine if an estimate is an over-estimate or an under-estimate.

By the completion of **grade 10**, students will be able to:

- compute permutations and combinations;
- compute areas and volumes by partitioning and indirect methods;
- compute with real numbers;
- compute with matrices;
- extend computation procedures to algebraic procedures;
- determine if errors are within tolerance limits;
- estimate and calculate derived measures;
- assess the error resulting from estimation;
- estimate algebraic solutions on a graphing calculator.

The areas listed above will be reflected in mathematics assessment items in the Delaware Student Testing Program.

ESTIMATION, MEASUREMENT, and COMPUTATION

9th	10th	11th
Students will be able to:	Students will be able to:	Students will be able to:
9.205 compute/count the possible outcomes of an event when order matters and when it does not (A);	10.205 compute areas and volumes by indirect methods (e.g., use strategies to find missing dimensions) (G);	11.205 develop and use formulas to compute permutations and combinations;
9.206 compute areas and volumes by partitioning (G);	10.206 add, subtract, multiply, and use scalar multiplication with matrices (A-G);	11.206 represent and analyze problem situations using finite graphs and matrices.
9.207 use multiple computational procedures with real numbers (A);	10.207 describe the effect on area and/or volume when linear dimensions are changed by a scale factor (G);	
9.208 extend computation procedures to algebraic procedures (A);	10.208 extend computation procedures to algebraic procedures (A).	
9.209 determine if measurement errors are within tolerance limits for given situations (G);		
9.210 estimate and calculate derived measures (e.g., rates) (A);		
9.211 explain the error resulting from estimation within a given context (e.g., π=3.14) (A);		
9.212 estimate algebraic solutions on a graphing calculator (A).		

It is expected that technology, particularly the graphing calculator, will be an integral part of instruction so that all students may become proficient in using technology as a tool toward meeting the Mathematics Content Standards.

MATHEMATICS STANDARD SIX: Students will develop NUMBER SENSE by solving problems in which there is a need to represent and model real numbers verbally, physically and symbolically; to use operations with understanding; to explain the relationships between numbers; to apply the concept of a unit, and to determine the relative magnitude of real numbers.

End of Cluster Expectations

By the completion of **grade 8** students will be able to:

- connect physical, verbal and symbolic representations of **rational numbers**;
- apply multiple representations of numbers: **integers, fractions, decimals, percents, exponents, and scientific notation**;
- model **integer** representations using manipulatives;
- demonstrate an understanding of order relations for **rational numbers**;
- examine the relative effect of operations on **rational numbers**;
- use various forms of "one" to demonstrate the equivalence of **fractions**.

By the completion of **grade 10** students will be able to:

- connect physical, verbal and symbolic representations of **real numbers**;
- demonstrate an understanding of order relations for **real numbers**;
- examine the relative effects of operations on **real numbers**;
- recognize inverse operations: **powers** and **roots.**

The areas listed above will be reflected in mathematics assessment items in the Delaware Student Testing Program.

NUMBER SENSE

9th	10th	11th
Students will be able to:	Students will be able to:	
9.213 connect physical, verbal and symbolic representations of **real numbers** (A-G);	10.209 connect physical, verbal and symbolic representations of **real numbers** (A-G);	
9.214 demonstrate an understanding of order relations for **real numbers** (A);	10.210 recognize and use inverse operations: (e.g., powers and roots) (A).	
9.215 examine the relative effect of operations on **real numbers** (see Learning Event # 4, p 38 in Vol I) (A).		

It is expected that technology, particularly the graphing calculator, will be an integral part of instruction so that all students may become proficient in using technology as a tool toward meeting the Mathematics Content Standards.

MATHEMATICS STANDARD SEVEN: Students will develop an understanding of ALGEBRA by solving problems in which there is a need to progress from the concrete to the abstract using physical models, equations and graphs; to generalize number patterns; and to describe, represent and analyze relationships among variable quantities.

End of Cluster Expectations

By the end of **grade 8** students will be able to:

- represent situations with tables, graphs, verbal rules, and equations; and describe the interrelationships of the representations;
- model and solve real-world and mathematical problems using algebraic methods;
- evaluate algebraic expressions and formulas for given values of the variable;
- solve linear equations using concrete, informal, and formal methods;
- solve proportions;
- solve linear inequalities and non-linear equations using informal methods.

By the end of **grade 10** students will be able to:

- model relationships among quantities using symbols and expressions;
- develop appropriate symbol sense to use algebraic technology;
- use tables and graphs to interpret expressions, equations and inequalities;
- describe relationships between variable quantities verbally, symbolically and graphically;
- translate and make connections from narrative to table, graph and function;
- solve linear and quadratic algebraic problems using graphs, tables, equations, formulas and matrices;
- solve systems of equations algebraically, graphically and with matrices;
- solve inequalities graphically and symbolically;
- explore algebraic relationships using technology.

The areas listed above will serve as the basis for mathematics assessment items in the Delaware Student Testing Program.

ALGEBRA

9th	10th	11th
Students will be able to:	Students will be able to:	Students will be able to:
9.216 model relationships among quantities with tables, graphs and symbols (A);	10.211 model relationships among quantities with tables, graphs and symbols (A);	11.207 analyze tabular, graphic, and symbolic representations of real world data and apply to decision making;
9.217 interpret the meaning of the coefficients of an equation in real-world terms given the context of the problem (A);	10.212 make connections between tabular, graphic, and symbolic representations of non-linear equations (e.g., quadratic equations, cubic equations, inverse variation, direct variation) (A);	11.208 use the relationship between the correlation coefficient and the line of best fit to analyze a set of data;
9.218 make connections between tabular, graphic, and symbolic representations of linear and exponential equations (A);	10.213 solve equations and inequalities using tables, graphs, or symbols (A);	11.209 solve linear programming problems using a system of linear inequalities to determine the maximum or minimum value of an objective function;
9.219 use symbols, tables, and graphs as tools to evaluate expressions (A);	10.214 model situations involving systems of equations and solve using tables, graphs, symbols, or matrices (A);	11.210 solve simple quadratic equations by factoring and by using the quadratic formula;
9.220 solve equations and inequalities using tables or graphs (A);	10.215 use a table or graph to estimate the root(s) and determine the number of roots of a quadratic relationship (A);	11.211 use at least one strategy for solving quadratic inequalities.
9.221 model situations involving systems of equations and solve using tables or graphs (A).	10.216 solve equations of the form $ax^2 = b$ using a table of values, a graph, or by reasoning with the symbolic form (A).	

It is expected that technology, particularly the graphing calculator, will be an integral part of instruction so that all students may become proficient in using technology as a tool toward meeting the Mathematics Content Standards.

MATHEMATICS STANDARD EIGHT: Students will develop SPATIAL SENSE and an understanding of GEOMETRY by solving problems in which there is a need to recognize, construct, transform, analyze properties of, and discover relationships between geometric figures.

End of Cluster Expectations

By the completion of **grade 8** students will be able to:

- identify, describe, compare and classify two and three dimensional figures;
- use a compass and straight edge as tools for basic geometric constructions;
- investigate and discover geometric relationships through the use of manipulatives, constructions and computer graphic software;
- create models of nets of three dimensional figures such as a cube, rectangular prism, cylinder and square pyramid;
- visualize and draw orthographic projections;
- discover and apply geometric properties and relationships such as congruence, similarity, parallelism, perpendicularity and symmetry;
- apply geometric properties and relationships to make conjectures.

By the completion of **grade 10** students will be able to:

- explore, draw and construct three dimensional objects;
- construct geometric figures on a coordinate plane;
- identify congruent and similar figures using transformational, Euclidean, and coordinate geometries;
- deduce properties of figures apply similarity, congruence and proportionality using coordinate and Euclidean geometries;
- explore geometric patterns and generalize results using algebraic expressions;
- apply similarity, congruence and proportionality;
- use the properties and relationships in two and three dimensional figures including circles and spheres;
- apply right triangle trigonometry and the Pythagorean Theorem to problem situations involving right triangles.

The areas listed above will serve as the basis for mathematics assessment items in the Delaware Student Testing Program.

SPATIAL SENSE AND GEOMETRY

9th	10th	11th
Students will be able to:	Students will be able to:	Students will be able to:
9.222 explore, draw and construct three dimensional objects (G);	10.217 place a figure on a coordinate system and verify its properties algebraically (e.g., distance, midpoint, slope, parallelism, and perpendicularity) (G);	11.212 relate translations to vectors and use vectors to solve real world problems (e.g., use vector addition to find a resultant vector when 2 forces are applied to an object);
9.223 apply the Pythagorean Theorem (G);	10.218 use pictorial representations to solve problems involving symmetry and transformations (e.g., tiling patterns) (G);	11.213 explore periodic real-world phenomena using sine and cosine functions;
9.224 use appropriate technologies to model geometric structures and to develop conjectures about them (e.g., use compass and straight edge, graphing calculators, and geometry software such as Geometer's Sketchpad, Cabri Geometry, or Super Supposer) (G).	10.219 identify and apply geometric relationships such as angles with respect to lines and figures (including circles) as they exist in the real world (G);	11.214 solve problem situations involving non-right triangles by applying the Laws of Sines and Cosines;
	10.220 use similarity, congruence and proportionality to solve real-world problems (G);	11.215 design a project involving a three dimensional design to solve a real world problem;
	10.221 deduce properties of geometric figures and relationships between figures by applying definitions and postulates (e.g., every square is a rhombus since it has 4 congruent sides (G);	11.216 understand the difference between and recognize the use of inductive and deductive reasoning;
	10.222 use geometric models to represent problem situations and apply properties of figures to solve problems (G);	11.217 understand the role of conjectures, assumptions, conclusions, and counterexamples in formulating mathematical proofs.
	10.223 solve problem situations involving right triangle trigonometry and special right triangles (G);	
	10.224 use appropriate technologies to model geometric structures and to develop conjectures about them (e.g., use compass and straight edge, graphing calculators, and geometry software such as Geometer's Sketchpad, Cabri Geometry, or Super Supposer) (G).	

It is expected that technology, particularly the graphing calculator, will be an integral part of instruction so that all students may become proficient in using technology as a tool toward meeting the Mathematics Content Standards.

MATHEMATICS STANDARD NINE: Students will develop an understanding of STATISTICS and PROBABILITY by solving problems in which there is a need to collect, appropriately represent, and interpret data; to make inferences or predictions and to present convincing arguments; and to model mathematical situations to determine the probability

End of Cluster Expectations

By the completion of **grade 8** students will be able to:

- collect, organize, describe, and make predictions with data;
- construct and describe displays of data such as stem-and-leaf plots, scatter plots, box plots, and circle graphs;
- make and evaluate arguments that are based on data analysis;
- calculate and use mean, median, mode and range to interpret data;
- analyze a sample to make inferences about a population;
- compare and make predictions based on theoretical and experimental probabilities;
- construct a sample space to determine theoretical probabilities.

By the completion of **grade 10** students will be able to:

- explore and analyze sampling methods to collect data;
- collect, explore, compare and interpret one or two-variable real world data;
- use curve-fitting to model and draw inferences from real life data;
- summarize and interpret single-variable data by exploring and choosing measures of central tendency and dispersion;
- analyze the validity of statistical conclusions and the use, misuse, and abuse of data caused by choices of scale, inappropriate choices of central tendency, incorrect curve fitting, or inappropriate use of control groups;
- define a sample space using the fundamental counting principle;
- compare and determine the reasonableness of outcomes;
- model mathematical situations, using simulations or experiments, to determine probabilities of independent and dependent events.

The areas listed above will be reflected in mathematics assessment items in the Delaware Student Testing Program.

STATISTICS AND PROBABILITY

9th	10th	11th
Students will be able to:	Students will be able to:	Students will be able to:
9.225 collect, display, explore, compare and interpret one or two-variable real world data (A);	10.225 explore and analyze sampling methods to collect data (A);	11.218 design and conduct a statistical experiment to study a problem, and interpret and communicate the outcomes;
9.226 use curve-fitting to model and draw inferences from real life data (A);	10.226 collect, display, explore, compare and interpret one or two-variable real world data (A);	11.219 understand and apply measures of central tendency, variability, and correlation;
9.227 summarize and interpret single-variable data by exploring and choosing measures of central tendency and dispersion (A);	10.227 use curve-fitting to model and draw inferences from real life data (A);	11.220 describe, in general terms, the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed.
9.228 analyze the validity of statistical conclusions and the use, misuse, and abuse of data (e.g., choices regarding scale, measures of central tendency, curve fitting) (A);	10.228 analyze the validity of statistical conclusions and the use, misuse, and abuse of data (e.g., inappropriate use of sampling methods, control groups) (A).	
9.229 define a sample space using the fundamental counting principle (A);		
9.230 evaluate the reasonableness of conclusions drawn from the outcomes of experiments or simulations (A);		
9.231 use simulations or experiments to determine probabilities of independent and dependent events (A).		

It is expected that technology, particularly the graphing calculator, will be an integral part of instruction so that all students may become proficient in using technology as a tool toward meeting the Mathematics Content Standards.

MATHEMATICS STANDARD TEN: Students will develop an understanding of PATTERNS, RELATIONSHIPS, and FUNCTIONS by solving problems in which there is a need to recognize and extend a variety of patterns; and to analyze, represent, model and describe real-world functional relationships

End of Cluster Expectations

By the completion of **grade 8** students will be able to:

- recognize, analyze, create, extend, describe and generalize a wide variety of patterns and relationships;
- analyze functional relationships to explain how a change in one quantity results in a change in another;
- identify geometric patterns and relationships;
- detect patterns and functions from statistical data;
- use a calculator and computer software to explore number patterns and mathematical relationships;
- use patterns and functions to represent and solve problems.

By the completion of **grade 10** students will be able to:

- model real-world phenomena with appropriate functions;
- search for and describe algebraic, geometric and statistical patterns using mathematical models;
- classify relationships between variable expressions as linear, quadratic, inverse, direct or exponential;
- use technology to explore transformations of functions caused by parameter changes;
- identify and interpret maximum and minimum values of functional relationships graphically.

The areas listed above will be reflected in mathematics assessment items in the Delaware Student Testing Program.

PATTERNS, RELATIONSHIPS AND FUNCTIONS

9th	10th	11th
Students will be able to:	Students will be able to:	Students will be able to:
9.232 compare and classify relationships expressed in tables, graphs, or functions as linear or exponential (A);	10.229 recognize power relationships expressed in tables, graphs, or functions (e.g., quadratic, cubic) (A);	11.221 understand the basic principals of iteration, recursion, and mathematical induction;
9.233 extend a pattern to determine a given term in an algebraic, geometric, and statistical patterns (e.g., recursive patterns) (A-G);	10.230 classify relationships expressed in tables, graphs, or functions as inverse variation or direct variation (A);	11.222 represent problem situations using discrete structures (e.g., series, sequences, recurrence relations);
9.234 use a graphing calculator to compare transformations of linear and exponential functions caused by parameter changes (A).	10.231 generalize algebraic, geometric, and statistical patterns (A-G);	11.223 classify sequences as arithmetic, geometric, or neither and write a formula if one exists;
	10.232 use a graphing calculator to compare transformations of functions; caused by parameter changes (A);	11.224 explore a variety of infinite sequences and informally evaluate their limits (e.g., compound interest formula);
	10.233 identify and interpret maximum and minimum values of functional relationships graphically (A).	11.225 model situations using composition of functions (e.g., percent off a sale item);
		11.226 use polynomial, rational, trigonometric, and exponential functions to model real-world phenomena.

It is expected that technology, particularly the graphing calculator, will be an integral part of instruction so that all students may become proficient in using technology as a tool toward meeting the Mathematics Content Standards.

HIGH SCHOOL PERFORMANCE INDICATORS

MATHEMATICS – MODEL 2

MATHEMATICAL PROCESSES

MATHEMATICS STANDARD ONE: Students will develop their ability to **SOLVE PROBLEMS** by engaging in developmentally appropriate problem-solving opportunities in which there is a need to use various approaches to investigate and understand mathematical concepts; to formulate their own problems; to find solutions to problems from everyday situations; to develop and apply strategies to solve a wide variety of problems; and to integrate mathematical reasoning, communication and connections.

MATHEMATICS STANDARD TWO: Students will develop their ability to **COMMUNICATE MATHEMATICALLY** by solving problems in which there is a need to obtain information from the real world through reading, listening and observing; to translate this information into mathematical language and symbols; to process this information mathematically; and to present results in written, oral and visual formats.

MATHEMATICS STANDARD THREE: Students will develop their ability to **REASON MATHEMATICALLY**, by solving problems in which there is a need to investigate significant mathematical ideas in all content areas; to justify their thinking; to reinforce and extend their logical reasoning abilities; to reflect on and clarify their own thinking; to ask questions to extend their thinking and to construct their own learning.

MATHEMATICS STANDARD FOUR: Students will develop their ability to make **MATHEMATICAL CONNECTIONS** by solving problems in which there is a need to view mathematics as an integrated whole and to integrate mathematics with other disciplines, while allowing the flexibility to approach problems, from within and outside mathematics, in a variety of ways.

The processes listed above will be reflected in mathematics assessment items in the Delaware Student Testing Program.

MATHEMATICAL PROCESSES

Algebra	Geometry	Year 3
Students will be able to:	Students will be able to:	Students will be able to:
9.201 develop and apply strategies to solve problems;	10.201 develop and apply strategies to solve problems;	11.201 develop and apply strategies to solve problems;
9.202 use mathematical notation and language to explain and defend their thinking;	10.202 use mathematical notation and language to explain and defend their thinking;	11.202 use mathematical notation and language to explain and defend their thinking;
9.203 make and test conjectures;	10.203 make and test conjectures;	11.203 make and test conjectures;
9.204 determine if a mathematical solution is reasonable.	10.204 determine if a mathematical solution is reasonable.	11.204 determine if a mathematical solution is reasonable.

It is expected that technology, particularly the graphing calculator, will be an integral part of instruction so that all students may become proficient in using technology as a tool toward meeting the Mathematics Content Standards.

MATHEMATICS STANDARD FIVE: Students will develop an understanding of ESTIMATION, MEASUREMENT, and COMPUTATION by solving problems in which there is a need to measure to a required degree of accuracy by selecting appropriate tools and units; to develop computing strategies and select appropriate methods of calculation from among mental math, paper and pencil, calculators or computers; to use estimating skills to approximate an answer and to determine the reasonableness of results.

END OF CLUSTER EXPECTATIONS

By the completion of **grade 8**, students will be able to:

- estimate and then measure angles, circumference, volume and surface area to the degree of accuracy required using standard and nonstandard units;
- convert measurement units within the same system;
- apply ratios, proportions and percents to real life situations;
- compute circumference; areas of triangles, parallelograms, trapezoids, and circles; and surface area and volume of cylinders, triangular and rectangular prisms and pyramids;
- apply order of operations;
- choose and explain an appropriate method for calculating an answer in a given situation;
- use multiple computational procedures with rational numbers;
- determine if an estimate is an over-estimate or an under-estimate.

By the completion of **grade 10**, students will be able to:

- compute permutations and combinations;
- compute areas and volumes by partitioning and indirect methods;
- compute with real numbers;
- compute with matrices;
- extend computation procedures to algebraic procedures;
- determine if errors are within tolerance limits;
- estimate and calculate derived measures;
- assess the error resulting from estimation;
- estimate algebraic solutions on a graphics calculator.

The areas listed above will be reflected in mathematics assessment items in the Delaware Student Testing Program.

ESTIMATION, MEASUREMENT, and COMPUTATION

Algebra	Geometry	Year 3
Students will be able to:	Students will be able to:	Students will be able to:
9.205 compute/count the possible outcomes of an event when order matters and when it does not (A);	9.206 compute areas and volumes by partitioning (G);	11.205 develop and use formulas to compute permutations and combinations;
9.207 use multiple computational procedures with real numbers (A);	9.209 determine if measurement errors are within tolerance limits for given situations (G);	11.206 represent and analyze problem situations using finite graphs and matrices.
9.208 extend computation procedures to algebraic procedures (A);	10.205 compute areas and volumes by indirect methods (e.g., use strategies to find missing dimensions) (G);	
9.210 estimate and calculate derived measures (e.g., rates) (A);	10.206 add, subtract, multiply, and use scalar multiplication with matrices (A-G);	
9.211 explain the error resulting from estimation within a given context (e.g., π= 3.14) (A);	10.207 describe the effect on area and/or volume when linear dimensions are changed by a scale factor (G).	
9.212 estimate algebraic solutions on a graphing calculator (A);		
10.206 add, subtract, multiply, and use scalar multiplication with matrices (A-G).		

It is expected that technology, particularly the graphing calculator, will be an integral part of instruction so that all students may become proficient in using technology as a tool toward meeting the Mathematics Content Standards.

MATHEMATICS STANDARD SIX: Students will develop NUMBER SENSE by solving problems in which there is a need to represent and model real numbers verbally, physically and symbolically; to use operations with understanding; to explain the relationships between numbers; to apply the concept of a unit, and to determine the relative magnitude of real numbers.

End of Cluster Expectations

By the completion of **grade 8** students will be able to:

- connect physical, verbal and symbolic representations of **rational numbers**;
- apply multiple representations of numbers: **integers, fractions, decimals, percents, exponents, and scientific notation**;
- model **integer** representations using manipulatives;
- demonstrate an understanding of order relations for **rational numbers**;
- examine the relative effect of operations on **rational numbers**;
- use various forms of "one" to demonstrate the equivalence of **fractions**.

By the completion of **grade 10** students will be able to:

- connect physical, verbal and symbolic representations of **real numbers**;
- demonstrate an understanding of order relations for **real numbers**;
- examine the relative effects of operations on **real numbers**;
- recognize inverse operations: **powers** and **roots.**

The areas listed above will be reflected in mathematics assessment items in the Delaware Student Testing Program.

NUMBER SENSE

Algebra	Geometry	Year 3
Students will be able to:	Students will be able to:	
9.213 connect physical, verbal and symbolic representations of **real numbers** (A-G);	10.209 connect physical, verbal and symbolic representations of **real numbers** (A-G).	
9.214 demonstrate an understanding of order relations for **real numbers** (A);		
9.215 examine the relative effect of operations on **real numbers** (see Learning Event #4, p 38 in Vol I)(A);		
10.210 recognize and use inverse operations (e.g., powers and roots) (A).		

It is expected that technology, particularly the graphing calculator, will be an integral part of instruction so that all students may become proficient in using technology as a tool toward meeting the Mathematics Content Standards.

MATHEMATICS STANDARD SEVEN: Students will develop an understanding of ALGEBRA by solving problems in which there is a need to progress from the concrete to the abstract using physical models, equations and graphs; to generalize number patterns; and to describe, represent and analyze relationships among variable quantities.

End of Cluster Expectations

By the end of **grade 8** students will be able to:

- represent situations with tables, graphs, verbal rules, and equations; and describe the interrelationships of the representations;
- model and solve real-world and mathematical problems using algebraic methods;
- evaluate algebraic expressions and formulas for given values of the variable;
- solve linear equations using concrete, informal, and formal methods;
- solve proportions;
- solve linear inequalities and non-linear equations using informal methods.

By the end of **grade 10** students will be able to:

- model relationships among quantities using symbols and expressions;
- develop appropriate symbol sense to use algebraic technology;
- use tables and graphs to interpret expressions, equations and inequalities;
- describe relationships between variable quantities verbally, symbolically and graphically;
- translate and make connections from narrative to table, graph and function;
- solve linear and quadratic algebraic problems using graphs, tables, equations, formulas and matrices;
- solve systems of equations algebraically, graphically and with matrices;
- solve inequalities graphically and symbolically;
- explore algebraic relationships using technology.

The areas listed above will serve as the basis for mathematics assessment items in the Delaware Student Testing Program.

ALGEBRA

Algebra	Geometry	Year 3
Students will be able to: 9.216 model relationships among quantities with tables, graphs and symbols (A); 9.217 interpret the meaning of the coefficients of an equation in real-world terms given the context of the problem (A); 9.218 make connections between tabular, graphic, and symbolic representations of linear and exponential equations (A); 9.219 use symbols, tables, and graphs as tools to evaluate expressions (A); 10.212 make connections between tabular, graphic, and symbolic representations of non-linear equations (e.g., quadratic equations, cubic equations, inverse variation, direct variation) (A); 10.213 solve equations and inequalities using tables, graphs, or symbols (A); 10.214 model situations involving systems of equations and solve using tables, graphs, symbols, or matrices (A); 10.215 use a table or graph to estimate the root(s) and determine the number of roots of a quadratic relationship (A); 10.216 solve equations of the form $ax^2 = b$ using a table of values, a graph, or by reasoning with the symbolic form (A).		Students will be able to: 11.207 analyze tabular, graphic, and symbolic representations of real world data and apply to decision making; 11.208 use the relationship between the correlation coefficient and the line of best fit to analyze a set of data; 11.209 solve linear programming problems using a system of linear inequalities to determine the maximum or minimum value of an objective function; 11.210 solve simple quadratic equations by factoring and by using the quadratic formula; 11.211 use at least one strategy for solving quadratic inequalities.

It is expected that technology, particularly the graphing calculator, will be an integral part of instruction so that all students may become proficient in using technology as a tool toward meeting the Mathematics Content Standards.

MATHEMATICS STANDARD EIGHT: Students will develop SPATIAL SENSE and an understanding of GEOMETRY by solving problems in which there is a need to recognize, construct, transform, analyze properties of, and discover relationships between geometric figures.

End of Cluster Expectations

By the completion of **grade 8** students will be able to:

- identify, describe, compare and classify two and three dimensional figures;
- use a compass and straight edge as tools for basic geometric constructions;
- investigate and discover geometric relationships through the use of manipulatives, constructions and computer graphic software;
- create models of nets of three dimensional figures such as a cube, rectangular prism, cylinder and square pyramid;
- visualize and draw orthographic projections;
- discover and apply geometric properties and relationships such as congruence, similarity, parallelism, perpendicularity and symmetry;
- apply geometric properties and relationships to make conjectures.

By the completion of **grade 10** students will be able to:

- explore, draw and construct three dimensional objects;
- construct geometric figures on a coordinate plane;
- identify congruent and similar figures using transformational, Euclidean, and coordinate geometries;
- deduce properties of figures apply similarity, congruence and proportionality using coordinate and Euclidean geometries;
- explore geometric patterns and generalize results using algebraic expressions;
- apply similarity, congruence and proportionality;
- use the properties and relationships in two and three dimensional figures including circles and spheres;
- apply right triangle trigonometry and the Pythagorean Theorem to problem situations involving right triangles.

The areas listed above will serve as the basis for mathematics assessment items in the Delaware Student Testing Program.

SPATIAL SENSE and GEOMETRY

Algebra	Geometry	Year 3
	Students will be able to:	Students will be able to:
	9.222 explore, draw and construct three dimensional objects (G);	11.212 relate translations to vectors and use vectors to solve real world problems (e.g., use vector addition to find a resultant vector when 2 forces are applied to an object);
	9.223 apply the Pythagorean Theorem (G);	
	10.217 place a figure on a coordinate system and verify its properties algebraically (e.g., distance, midpoint, slope, parallelism, and perpendicularity) (G);	11.213 explore periodic real-world phenomena using sine and cosine functions;
	10.218 use pictorial representations to solve problems involving symmetry and transformations (e.g., tiling patterns) (G);	11.214 solve problem situations involving non-right triangles by applying the Laws of Sines and Cosines;
	10.219 identify and apply geometric relationships such as angles with respect to lines and figures (including circles) as they exist in the real world (G);	11.215 design a project involving a three dimensional design to solve a real world problem;
	10.220 use similarity, congruence and proportionality to solve real-world problems (G);	11.216 understand the difference between and recognize the use of inductive and deductive reasoning;
	10.221 deduce properties of geometric figures and relationships between figures by applying definitions and postulates (e.g., every square is a rhombus since it has 4 congruent sides (G);	11.217 understand the role of conjectures, assumptions, conclusions, and counterexamples in formulating mathematical proofs.

It is expected that technology, particularly the graphing calculator, will be an integral part of instruction so that all students may become proficient in using technology as a tool toward meeting the Mathematics Content Standards.

MATHEMATICS STANDARD EIGHT: Students will develop SPATIAL SENSE and an understanding of GEOMETRY by solving problems in which there is a need to recognize, construct, transform, analyze properties of, and discover relationships between geometric figures.

End of Cluster Expectations

By the completion of **grade 8** students will be able to:

- identify, describe, compare and classify two and three dimensional figures;
- use a compass and straight edge as tools for basic geometric constructions;
- investigate and discover geometric relationships through the use of manipulatives, constructions and computer graphic software;
- create models of nets of three dimensional figures such as a cube, rectangular prism, cylinder and square pyramid;
- visualize and draw orthographic projections;
- discover and apply geometric properties and relationships such as congruence, similarity, parallelism, perpendicularity and symmetry;
- apply geometric properties and relationships to make conjectures.

By the completion of **grade 10** students will be able to:

- explore, draw and construct three dimensional objects;
- construct geometric figures on a coordinate plane;
- identify congruent and similar figures using transformational, Euclidean, and coordinate geometries;
- deduce properties of figures apply similarity, congruence and proportionality using coordinate and Euclidean geometries;
- explore geometric patterns and generalize results using algebraic expressions;
- apply similarity, congruence and proportionality;
- use the properties and relationships in two and three dimensional figures including circles and spheres;
- apply right triangle trigonometry and the Pythagorean Theorem to problem situations involving right triangles.

The areas listed above will serve as the basis for mathematics assessment items in the Delaware Student Testing Program.

SPATIAL SENSE and GEOMETRY (CONTINUED)

Algebra	Geometry	Year 3
	Students will be able to: 10.222 use geometric models to represent problem situations and apply properties of figures to solve problems (G); 10.223 solve problem situations involving right triangle trigonometry and special right triangles (G); 10.224 use appropriate technologies to model geometric structures and to develop conjectures about them (e.g., use compass and straight edge, graphing calculators, and geometry software such as Geometer's Sketchpad, Cabri Geometry, or Super Supposer (G).	

It is expected that technology, particularly the graphing calculator, will be an integral part of instruction so that all students may become proficient in using technology as a tool toward meeting the Mathematics Content Standards.

MATHEMATICS STANDARD NINE: Students will develop an understanding of STATISTICS and PROBABILITY by solving problems in which there is a need to collect, appropriately represent, and interpret data; to make inferences or predictions and to present convincing arguments; and to model mathematical situations to determine the probability

End of Cluster Expectations

By the completion of **grade 8** students will be able to:

- collect, organize, describe, and make predictions with data;
- construct and describe displays of data such as stem-and-leaf plots, scatter plots, box plots, and circle graphs;
- make and evaluate arguments that are based on data analysis;
- calculate and use mean, median, mode and range to interpret data;
- analyze a sample to make inferences about a population;
- compare and make predictions based on theoretical and experimental probabilities;
- construct a sample space to determine theoretical probabilities.

By the completion of **grade 10** students will be able to:

- explore and analyze sampling methods to collect data;
- collect, explore, compare and interpret one or two-variable real world data;
- use curve-fitting to model and draw inferences from real life data;
- summarize and interpret single-variable data by exploring and choosing measures of central tendency and dispersion;
- analyze the validity of statistical conclusions and the use, misuse, and abuse of data caused by choices of scale, inappropriate choices of central tendency, incorrect curve fitting, or inappropriate use of control groups;
- define a sample space using the fundamental counting principle;
- compare and determine the reasonableness of outcomes;
- model mathematical situations, using simulations or experiments, to determine probabilities of independent and dependent events.

The areas listed above will be reflected in mathematics assessment items in the Delaware Student Testing Program.

STATISTICS AND PROBABILITY

Algebra	Geometry	Year 3
Students will be able to:		Students will be able to:
9.225 collect, explore, compare and interpret one or two-variable real is world data (A);		11.218 design and conduct a statistical experiment to study a problem, and interpret and communicate the outcomes;
9.227 summarize and interpret single-variable data by exploring and choosing measures of central tendency and dispersion (A);		11.219 understand and apply measures of central tendency, variability, and correlation;
9.229 define a sample space using the fundamental counting principle (A);		11.220 describe, in general terms, the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed.
9.230 evaluate the reasonableness of conclusions drawn from the outcomes of experiments or simulations (A);		
9.231 use simulations or experiments to determine probabilities of independent and dependent events (A);		
10.225 explore and analyze sampling methods to collect data (A);		
10.227 use curve-fitting to model and draw inferences from real life data (A);		
10.229 analyze the validity of statistical conclusions and the use, misuse, and abuse of data (e.g., inappropriate use of sampling methods, control groups) (A).		

It is expected that technology, particularly the graphing calculator, will be an integral part of instruction so that all students may become proficient in using technology as a tool toward meeting the Mathematics Content Standards.

MATHEMATICS STANDARD TEN: Students will develop an understanding of PATTERNS, RELATIONSHIPS, and FUNCTIONS by solving problems in which there is a need to recognize and extend a variety of patterns; and to analyze, represent, model and describe real-world functional relationships

End of Cluster Expectations

By the completion of **grade 8** students will be able to:

- recognize, analyze, create, extend, describe and generalize a wide variety of patterns and relationships;
- analyze functional relationships to explain how a change in one quantity results in a change in another;
- identify geometric patterns and relationships;
- detect patterns and functions from statistical data;
- use a calculator and computer software to explore number patterns and mathematical relationships;
- use patterns and functions to represent and solve problems.

By the completion of **grade 10** students will be able to:

- model real-world phenomena with appropriate functions;
- search for and describe algebraic, geometric and statistical patterns using mathematical models;
- classify relationships between variable expressions as linear, quadratic, inverse, direct or exponential;
- use technology to explore transformations of functions caused by parameter changes;
- identify and interpret maximum and minimum values of functional relationships graphically.

The areas listed above will be reflected in mathematics assessment items in the Delaware Student Testing Program.

PATTERNS, RELATIONSHIPS AND FUNCTIONS

Algebra	Geometry	Year 3
Students will be able to:	Students will be able to:	Students will be able to:
9.232 compare and classify relationships expressed in tables, graphs, or functions as linear or exponential (A);	9.233 extend a pattern to determine a given term in an algebraic, geometric, and statistical patterns (e.g., recursive patterns) (A-G);	11.221 understand the basic principals of iteration, recursion, and mathematical induction;
9.233 extend a pattern to determine a given term in an algebraic, geometric, and statistical patterns (e.g., recursive patterns) (A-G);	10.231 generalize algebraic, geometric, and statistical patterns (A-G).	11.222 represent problem situations using discrete structures (e.g., series, sequences, recurrence relations);
10.229 recognize power relationships expressed in tables, graphs, or functions (e.g., quadratic, cubic) (A);		11.223 classify sequences as arithmetic, geometric, or neither and write a formula if one exists;
10.230 classify relationships expressed in tables, graphs, or functions as inverse variation or direct variation (A);		11.224 explore a variety of infinite sequences and informally evaluate their limits. (e.g., compound interest formula);
10.231 generalize algebraic, geometric, and statistical patterns (A-G);		11.225 model situations using composition of functions (e.g., percent off a sale item);
10.232 use a graphing calculator to compare transformations of functions, caused by parameter changes (A);		11.226 use polynomial, rational, trigonometric, and exponential functions to model real-world phenomena.
10.233 identify and interpret maximum and minimum values of functional relationships graphically (A).		

It is expected that technology, particularly the graphing calculator, will be an integral part of instruction so that all students may become proficient in using technology as a tool toward meeting the Mathematics Content Standards.

A

Algorithm: A rule or procedure for computing or solving a certain type of problem.

B

Box and whiskers plot: A graph that summarizes data using the median, upper and lower quartiles, and the extreme values. A box is drawn around the quartile values and the whiskers extend from each quartile to the extreme data points.

C

Combination: The total number of groups of n items taken r at a time when the order of the items is not important, $C(n,r) = \frac{n!}{r!(n-r)!}$

Composition of function: The operation of first applying one function then another, denoted by $f \circ g = f(g(x))$.

Conceptual knowledge: Understanding of mathematical ideas that are abstracted through repeated experiences and reflection.

Conjecture: A generalization or hypothesis made by observing data and recognizing patterns (inductive reasoning) without sufficient evidence for proof.

Correlation: The degree of relative correspondence between two sets of data.

D

Deductive (logical) reasoning: Process of demonstrating that if certain statements (axioms, postulates, theorems) are accepted as true, then other statements can be shown or proved to follow from them.

Dependent events: Two events for which the occurrence of one affects that of the other.

Derived measures: Measures that cannot be calculated directly but are determined by finding other direct measures i.e., miles per hour.

Dispersion: The spread of data around a central value for the set of data.

E

Event: In probability, any possible outcome.

Experimental (empirical) probability: A probability formulated using the results of an experiment or series of trials.

F

Finite graphs: Any drawing or diagram that represents the relationships among tasks, choices, or destinations. Graphs include digraphs (directed graphs), tree diagrams, Gant charts, Vertex-Edge Graphs, Euler and Hamiltonian paths and circuits.

Functional relationship: A relationship that pairs each object in a given set with exactly one object in a second set. (e.g., the area of a circle is a function of the radius - if the radius is 4", the area is 16π sq. ")

Fundamental counting principle: If one event can occur in n ways and, for each of those, a second event can occur in m ways, then the two events can occur in mn ways.

G

Geometric transformation: Process of changing the position, size, or shape of a figure according to a given rule such as rotation, translation, dilation, and reflection.

H

Histogram: A graphic picture of a frequency distribution using rectangles with class intervals as the bases and areas proportional to the frequency of that interval. The intervals include all possible values of the data; therefore there are no spaces between the bars of the graph.

I

Independent events: Two events for which the occurrence of the one event does not affect that of the other.

Inductive reasoning: Process of observing data, recognizing patterns, and making generalizations from the observations.

Interquartile range: The range of data within the ìboxî of a box plot determined by subtracting the lower quartile from

the upper quartile.
Inverse operations: Operations that undo each other; addition and subtraction are inverse operations.
Irrational number: A real number that cannot be written as the quotient of two integers, (e.g., π or $\sqrt{2}$; a non repeating, non-terminating decimal).
Isometric paper: Dot paper used to assist in drawing isometric views or three-dimensional drawings.
Iteration: Calculation which is made up of repetitive steps acting upon the previous answer.

L
Line graph: a graph using points and line segments to show both amount and direction of change over a period of time.
Line plot: A graph using marks (e.g., X,l) on a number line to show the frequency of data.

M
Manipulatives: Concrete models useful in representing various mathematical concepts. They are used to experiment with and explore various mathematical ideas.
Mathematical knowledge: The context in which the mathematical processes are used.
Mathematical processes: Procedures or actions that students engage in when doing mathematics - problem solving, reasoning, communicating and connecting and then relate the action to specific mathematical knowledge.
Mathematical tools: Aids for solving problems. Students should learn how to use these tools and also when to use them.
Mathematical modeling: A process in which students represent real world problem situations mathematically using concrete, oral, written, pictorial, graphic, numeric and/or algebraic methods.
Matrix: A rectangular array of terms written between parentheses or double lines on either side of the array. A matrix does not have a quantitative value.
Measures of central tendency: The mean, median, and mode of a set of data.
Model: (verb) To write an algebraic expression or equation to represent a real-life situation.
Model: (noun) An equation, expression or graph that can be used to study the real-life situation it is used to represent.

N
Net: A two-dimensional pattern for a three-dimensional object.
Nonroutine problem: A situation for which a student does not have a previously established procedure for finding a solution.

O
Orthographic or orthogonal: **See** projection.
Outlier: Data values so large or small that they stand apart from the rest of the distribution.

P
Parameter change: Change in constants or coefficients of an equation or function.
Partitioning: Separating into distinct parts for the purpose of finding the total area or volume
Permutation: The number of arrangements of n items taken r at a time when order matters. $P(n,r) = \frac{n!}{(n-r)!}$
Power model: Any equation that could be used to model a situation where a variable quantity may be raised to an exponent. For instance a quadratic equation would represent a second degree power model.
Probability of an event: The ratio of the number of favorable outcomes to the number of possible outcomes of an event, if all outcomes are equally likely.
Procedural knowledge: Understanding of a method for solving a problem.
Projection (orthographic or orthogonal): Two-dimensional sketch of a three-dimensional object from different perspectives (top, sides, and front, back).

Q
Quantitative: Relating to measuring and counting data.
Quartile: One of four equal divisions of a set of data.

R

Rational number: A real number that can be written as a quotient of two integers $\frac{a}{b}$ where b does not equal 0; a repeating or terminating decimal.
Real number: Any number that is either rational or irrational.
Recursive pattern: A pattern that can be extended by relating each term to the preceding term.
Reflection: Replacing each point in a figure by a point symmetric with respect to a line. (Flipping a figure across a line)
Rotation: Movement of a figure in a circular motion about a point. (Turning an object about a point)

S
Sample space: The set or collection of all possible outcomes of a probability experiment.
Scatterplot (scattergram): A graph consisting of ordered pairs possibly showing a relationship between two variable quantities.
Sequence: The result of evaluating a function for consecutive positive integers
Series: An expression representing the sum of the terms of a sequence
Simulation: The process of modeling the outcomes of a particular experiment in probability.
Single variable: data that is collected and represents only one thing.
Spatial reasoning: The ability to interpret and make drawings, forms mental images, and visualize movement or change in those images.
Stem and leaf plot: A method of organizing data using the digits of the greatest place value to group the data.
Strategies: Methods used to solve problems.
Symbol sense: Having understanding of the relationship between a good numbers and symbols, the representation of quantities using symbols, and how to operate with mathematical symbols.

Symmetric to a line: A figure is symmetric with respect to a line if one half of the figure **is** the mirror image of the other half across the line (e.g., **V** is symmetric to a vertical line) (see reflection).
Symmetric to a point: A figure is symmetric with respect to a point if you can rotate the figure about the point 180° and the new image coincides with the original figure (e.g., **Z** is symmetric to the midpoint of the diagonal segment) (see rotation).

T
Tessellation: A repetitive pattern of congruent figures that fit together with no overlaps or gaps.
Theoretical (a priori) probability: A probability formulated using prior or intuitive knowledge of the sample space before an experiment is performed.

Tolerance: The greatest range of variation that can be allowed in measurement to maintain necessary correctness.
Translation: Changing the coordinates of points to coordinates referred to new axes parallel to the old. (Sliding a figure from one location to another)

Tree diagram: A branching diagram used to show all the possible outcomes in an experiment.

V
Validate: Substantiate or confirm.
Variability: A measure of the spread of the data around the center of the data. Measures of variability include range, interquartile range, mean absolute deviation, average distance from the mean, and standard deviation.
Vector: A physical quantity having both magnitude and direction such as force or velocity, represented by a directed line segment or ray
Verify: To prove the truth of or justify.
Vignette: A short literary sketch.

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