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## ABSTRACT

This study used structural equation modeling and multilevel modeling approaches for purposes of simultaneous study of individual and group change patterns on three waves of two longitudinally assessed domains. This study illustrates a few of the dual approaches to the analysis of covariance structures as they relate to the same individual growth model and the same data. LISREL (structural equation modeling) and hierarchical linear modeling (HLM) were used. The investigation studied whether individual change over time in mathematics and language differs from student to student and whether the individual growth parameters of each of the two domains were related within domain. The study used panel data from the Louisiana State Department of Education for 3 waves of students tested in grades 4, 6, and 7: (1) 50,907; (2) 47,003; and (3) 50,157. Assessments from the state's testing program were administered in each of the three grades. Complete records for all 3 grades were available for 26,051 students, 11,627 of whom were African American. The study sheds light on the understanding of learners from the two ethnic groups and shows how they develop mastery in mathematics and language as they progress through school. The application of covariance structure analysis and growth curves to the study of growth in student academic achievement provides an avenue for an in-depth analysis of two academic areas in an available data set. The statistical techniques used in this research, LISREL and HLM methods, have a number of extensions that can be used in various research environments because they can accommodate any number of data points (waves) of longitudinal data. Four appendixes contain descriptive statistics from the study. (Contains 2 figures, 3 tables, and 50 references.) (SLD)

# A Study of Individual Patterns of longitudinal Academic Change: Exploring the Structural Equation Modeling (SEM) and Hierarchical Linear Modeling (HLM)

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## **A Study of Individual Patterns of longitudinal Academic Change: Exploring the Structural Equation Modeling (SEM) and Hierarchical Linear Modeling (HLM)**

While many theorists have presented models to describe growth and change, these models are infrequently tested with data (Magnusson, 1985). It is apparent that lack of familiarity with many quantitative methods for estimating learning growth curves appears to be a major obstacle to the empirical testing of growth models (Burchinal & Appelbaum, 1991). Bryk and Raudenbush (1992) amplified the same problem by noting that research on change has been plagued by inadequacies in conceptualization, measurement, and design and has long perplexed behavioral scientists. In many situations, instruments used to assess the subjects are developed for fixed points in time, yet individual academic growth is dynamic. These instruments have not adequately captured individual differences in the rate of change. The study of change requires more than two waves of data but frequent studies have utilized only two data points and are thus not able to adequately address the issue of growth (Bryk & Raudenbush, 1987; Bryk & Weisburg, 1977; Rogosa, Brandt, & Zimowski, 1982). When there are only two waves of data on each subject, there is no way to know the exact shape of individual growth over time (Willett, 1988). It has also been stressed that data from two time points and the difference score are less than optimal for the study of change but three or more waves of data are preferable (Olweus & Alsaker, 1991).

The difference score that was initially employed and continues to be used as a measure of change because of the concentration of two-waves measurement has restrictive assumptions and its continued use as a measure of change has been condemned by many researchers (Cronbach, Furby 1970; Lord, 1963; O'Connor, 1972; Thorndike, 1966). These researchers have instead recommended other statistical techniques of evaluating change.

Why study change in education? A focus on the study of change enables an in-depth investigation of how key elements of learning in and other variables exert an influence on student achievement outcomes. A study of change in education lends itself to an indepth evaluation of the extent differences in schooling experiences; in particular, differences in classroom environment and instructional quality, contribute to the development of interindividual differences in achievement.

Students are enrolled in schools so that they can grow academically and educationally, develop, and change. It is the measurement of these changes and the investigation of their relationship to supporting activities in the classroom and the resources provided by the school that empirical investigations ought to focus on (Willet, 1988). The study of this change in education is important because it is through change that the effectiveness of a curriculum can be assessed and improved.

The study of individual academic change has a relatively long history. The growth in the measurement of change has been gradual and the earlier problems that faced the adequate measurement of change continue to be addressed. In recent research on individual change, investigators have used individual growth modeling in order to make use of the enormous volume of multiwave data available in academic and related institutions, while providing better methods for investigating interindividual differences in change (Bryk & Raudenbush, 1987; Rogosa et al., 1982; Rogosa & Willett, 1985; Sayer & Willett, 1998; Willett, 1988; Willett & Sayer, 1994, 1996).

Recent studies (Raudenbush, 1995) have revealed that widely available software can be adapted to provide maximum likelihood (ML) estimates for a general class of multilevel covariance structure models if the data are balanced -- i.e., equal numbers of students in each of the many

schools, thus ensuring that every level-2 unit has same number of level-1 units. Studies of individual change are increasingly employing a combination of individual growth trajectories and structural equation modeling (SEM), while capitalizing on the unique strengths each of these procedures offers. SEM encompasses an entire family of models known by many names, among them covariance structure analysis, latent variable analysis, confirmatory factor analysis, and often simply LISREL analysis (Linear Structural RELations - the name of one of the more popular statistical software packages). SEM is a statistical methodology that takes a confirmatory perspective to the multivariate analysis of a structural theory bearing on some underlying phenomenon (Byrne, 1998). An equation which relates the dependent (Y) and independent(X) variable, such as  $Y = a + bX$ , is a *structural equation*, and the constants *a* and *b* are structural coefficients. When two or more equations simultaneously describe the set of variables under consideration, such equations are considered as *structural equation models*. SEM generally employs the maximum likelihood method, which is a large-sample procedure and is unlikely to behave well with small sample sizes in a multiple group perspective (Burstein, Kyung-Sung & Delandshere, 1989). Recently, pioneering researchers have shown how the analysis of change can be conducted conveniently by the methods of covariance structure analysis (Tisak & Meridith, 1990; Sayer & Willett, 1998; Willett & Sayer, 1994, 1996).

The application of covariance structure analysis techniques in research subsumes more traditional approaches to the analysis of panel data, such as repeated measures analysis of variance (ANOVA) and multivariate analysis of variance (MANOVA) (Jöreskog & Sörbom, 1989; Meridith & Tisak, 1990; Rao, 1958; Tucker, 1958). For this study, the individual change aspect was represented through a two-level hierarchical model (“multilevel”). At level 1, each person’s

development is represented by an individual growth trajectory that depends on the unique set of parameters. In level 2, the level 1 growth parameters become the outcome variables, where they depend on some person-level characteristics. The multiple observations recorded for each individual in the study provide a ‘hierarchy’ which can be adequately processed by a multilevel data analysis technique.

Multilevel analysis involves estimating growth curves for multiple observations in the first phase and testing the covariation between the estimated indices of growth curve analysis and hypothesized predictors or outcomes of the change process in the second phase (Bryk & Raudenbush, 1992; Muthén, 1997; Sayer & Willett, 1998; Willett & Sayer, 1994, 1996). The multilevel covariance structure analysis model is a flexible procedure and as such an attractive analytical tool for a variety of SEM analyses that can be used to investigate growth and development among variables of interest with multilevel data. This study, with availability of panel data, and particularly for studies of individual growth, will demonstrate how covariance structure models can be set up for hierarchical data (observations nested within person) and how these models can be analyzed by traditional SEM software, such as LISREL.

Longitudinal studies occupy an important place in the psychological and social sciences research realm. In these studies the same individuals are repeatedly measured on a number of targeted variables over a series of important time points (Hedleker, Gibbons, 1997). However, there has been a real struggle among researchers for some time over concepts such as hierarchical nested observations, intra-class correlations, the unit of analysis, and random rather than fixed effects (Duncan, Duncan, Alpert, Hops, Stoolmiller, & Muthén, 1997). The study of change, like other complex studies, has been slowed by lack of a complete and stand-alone statistical package that has

the capacity and capability of handling all the univariate and multivariate statistical data analysis requirements. Most data for the analysis of change must go through a series of preprocessing stages before they can be utilized to analyze change.

The conceptual framework guiding this study is based on the recent emerging approach to the measurement of change, the latent growth curves within structural equation modeling such as LISREL (Duncan et al. 1999; Duncan & Duncan, 1990; McArdle & Epstein, 1987; Patterson, 1993; Sayer & Willett, 1998; Stoolmiller, 1994; Stoolmiller, Duncan, Bank & Patterson, 1993; Willett & Sayer, 1994, 1996). McArdle and Epstein (1987) defined the latent growth curve model (LGM) as a “longitudinal model that includes correlations, variances, and means.” The inclusion of means in these models make them more similar to repeated-measures ANOVA and MANOVA. Latent growth model and general multivariate growth models try to describe the way the individual develops. Studies about change have been cross-sectional, with the center of investigation being mean level changes across different groups. Over the past twenty five years, researchers have called for the development of appropriate methods for the analysis of longitudinal data (Mehta & West, 2000).

In longitudinal research, investigators often measure multiple variables at multiple points in time and are interested in investigating individual differences in patterns of change on those variables (MacCallum, et al., 1997; McArdle & Aber, 1990). MacCallum, et al. (1997), McArdle and Aber (1990) study focused on the relationships between patterns of change on different variables while showing how the multilevel modeling framework, which is often used to study univariate change, can be extended to the multivariate case to yield estimates of covariance of parameters, which represent aspects of change on different variables.

This study utilizes structural equation modeling and multilevel modeling approaches for purposes of simultaneous study of individual and group change patterns on three waves of two longitudinally assessed domains. Recent studies (Raudenbush, 1995) have revealed that widely available software can be adapted to provide maximum likelihood (ML) estimates for a general class of multilevel covariance structure models if the data are “balanced”-- i.e., equal numbers of students in each of the many schools, thus ensuring that every level-2 unit has same number of level-1 units.

Schmidt (1969) developed software to compute maximum likelihood (ML) estimates for two-level data for a balanced design. McDonald & Goldstein (1989) provided theory for ML estimation for unbalanced models that incorporate both level-1 and level-2 variables showed that software such as EQS (Bentler, 1983), LISCOMP (Muthén, 1987) and LISREL (Jöreskog & Sörbom, 1996) can be implemented with little or no modification to the software for the analysis of multilevel data. Bryk and Raudenbush (1987) advanced strengths of HLM in its ability to make predictions and the fact that the HLM model draws on strengths are available in the data.

### **Purpose**

The purpose of this study was to demonstrate a few of the dual approaches to the analysis of covariance structures as they relate to the same individual growth model and based on the same data. More specifically, using both the LISREL 8 (including PRELIS 2) and SAS PROC MIXED (SAS Institute, 1996). The investigation was based on a study of whether individual change over time in mathematics and language differs from student to student and if the individual growth parameters of each of the two domains were related within domain. The study was guided by the following specific research questions: (a) are the growth parameters (intercepts and slopes) in mathematics and language related within each domain? (b) is the pattern of interrelationships, among the individual

achievement growth parameters, the same for African American and White students? (c) are there discernible patterns in variability in academic growth parameters within each ethnicity?

## **Methodology**

### **Sampling Procedures**

This study used panel data drawn from the Louisiana State Department of Education (LDE) school data files. The subset of students involved was obtained as follows. Of all the elementary school students in the LDE data files, only those who attended public schools and were of African American and White ethnic group origins were sampled. The sampled students were tested on the Norm Referenced Tests (NRTs) in grade 4, 6 and 7. Wave one had 50,907 students (African Americans=24,030, Whites=26,872), wave two had 47,003 students (African Americans=22,262, Whites=24,741) while the third and last wave had 50,157 students (African Americans=23,982, Whites=24,536). The subsets of students who had complete records for grades 4, 6 and 7 were 26,051 (African Americans=11,627, Whites=14,424).

### **Instrumentation and Measurement**

The Iowa Tests of Basic Skills (ITBS) and Norm-Referenced Tests (NRTs) as part of the Louisiana Educational Assessment Program (LEAP), was utilized. The two domains utilized in this investigation (language and math) were average composites of their respective constituents. Math subscales were math concepts/ estimation and math problem solving/data interpretation while language subscales were spelling, capitalization, punctuation, and usage and expression. The NRT measure is a multiple choice scale for mathematics and language domains and allow the educators to compare individual and group performance results with a national norm. These tests indicate how a given student's knowledge or skill compares with others' in the norm group. Reliability data for

the ITBS meet stringent psychometric standards where the ITBS Complete Battery average test reliabilities (K-R 20) for grades 3 through 8 are 0.86 and 0.87 for the fall and spring, respectively.

### **Data Analysis Procedures**

This study adopted a two-stage road-map of the data analysis procedure as provided in the individual growth curves analysis of Singer (1998) and covariance structure analysis technique of both Sayer and Willett (1998) and Willett and Sayer (1994, 1996) for single and double populations. First, a series of preliminary data analyses was conducted to check on the normality, skewness, and kurtotic nature of each of the three waves of data to gain familiarity and knowledge of the data at the individual level (See Appendix A). Ordinary least squares (OLS) fitted trajectories summarizing observed growth patterns for both mathematics and language between grade 4 and 7 for the subsample of 27 (See Appendix B) selected students from both ethnic groups was completed (Appendices, C and D).

In the study of change patterns in student academic achievement, over time, the analysis was conducted in two levels. At level 1 (within person), the curve fitting techniques to describe growth events such as the effect of student grade level on mathematics and on language achievement were applied. This level involves fitting, to each individual, a particular curve that is a function of time (grade). In the second level (between-person), comparison of the patterns of the growth parameters was made. The different student background characteristics was presented through the summary descriptions of means of the individual curve coefficients gleaned from the first level analysis. The multilevel data analysis techniques carry out such analysis at two levels simultaneously (Bryk & Raudenbush, 1992; Kaplan & Elliott, 1997; Yang & Goldstein, 1996). The individual growth model was evaluated in line with the tenets of the classical test theory approach where the observed score is distinguished from the true score. Table 1, presents the sample mean vectors and covariance

**Table 1:** Estimated Means and Covariances for Three waves of Mathematics and Language Achievement Scores at grades 4, 6, and 7 for (a) African American (AA) students (n=10,724), (b) White students (n=13,578).

AA	Mathematics			Language			
	Grade	4	6	7	4	6	7
Means		186.28	208.06	216.76	189.72	213.53	226.84
Covariances		256.24					
		176.34	358.01				
		225.85	316.87	553.50			
		214.68	185.73	238.85	404.99		
		222.64	319.37	369.70	355.48	726.40	
		226.07	299.66	427.87	349.59	545.47	811.31

N=10,724

WHITE	Mathematics			Language			
	Grade	4	6	7	4	6	7
Means		204.50	231.01	246.01	206.66	237.53	251.73
Covariances		411.12					
		329.49	541.72				
		372.02	481.18	687.09			
		322.37	308.73	358.42	574.12		
		355.65	446.36	492.71	517.45	901.85	
		350.66	421.75	541.20	488.67	682.69	959.18

N=13,578

matrices for language and mathematics and for the two groups of learners--African American and White students respectively. The data in the table was utilized in the computation of individual growth parameters for the two groups of learners.

Random Coefficient Regression Analysis (Hierarchical Linear Modeling)

The results of the covariance structure analysis were computed and compared with those derived from a hierarchical linear modeling approach, utilizing the SAS PROC MIXED routine, as detailed in the works of Littell, Milliken, Stroup, and Wolfinger (1996), and Singer (1998). In utilizing this approach, individual growth models for mathematics and language were treated as linear functions of time with the individual intercepts and slopes treated as random. Using this technique (hierarchical/random coefficient modeling), “an unconditional linear growth model” with a simple two-level model was considered, in which the level-1 model is a linear individual growth model, and the level-2 model expresses variation in parameters from the growth model as random effects unrelated to any person-level covariates/predictors. The parameters in level-1 (within person) model used  $\pi$  and the parameters in the level-2 (between person) model used  $\beta$ . The level-1 and level-2 models were then written as:

$$Y_{ij} = \pi_{0j} + \pi_{1j}(\text{Time})_{ij} + r_{ij}, \text{ where } r_{ij} \sim N(0, \sigma^2) \text{ and}$$

$$\pi_{0j} = \beta_{00} + u_{0j},$$

$$\pi_{1j} = \beta_{10} + u_{1j}, \text{ Where } \begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} \sim N \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix} \right]$$

which were written in combined form as:

$$Y_{ij} = [\beta_{00} + \beta_{10} \text{Time}]_{ij} + [u_{0j} + u_{1j} \text{Time}]_{ij} + r_{ij},$$

As can be seen above, the multilevel model was expressed as the sum of two parts: a fixed part, which contains two fixed effects (for intercept and for the effect of TIME) and a random part,

which contains three random effects (for the intercept, the TIME slope, and within person residual  $r_{ij}$ ). The time variable for this study was grade level and it appeared in the model line as the predictor for both mathematics and language. The treatment of the intercept and slopes as random effects can be changed, and also the covariates (predictors) of the level-2 components can be introduced depending upon the nature of the particular research question.

## Results

The major findings of the study showed that: 1) students vary significantly in knowledge of mathematics at entry into grade 4 and that White students overall initial status in mathematics was higher than that of African American students, 2) language intercepts for the two groups were statistically significant, signifying language knowledge differences at grade 4, 3) the mathematics overall slopes for the two groups of learners were positive and significantly different from zero, 4) language overall rates of learning within ethnicity were significantly different from zero, 5) the correlation coefficients of the slope and initial status for each domain and within each ethnicity were not statistically significant, 6) variance estimates for language and mathematics slopes were significantly different from zero and showed variance increases at lower grade levels as students advance in school from grade 4 through grade seven, and 7) the results of LISREL analyses and those of Hierarchical Linear Model (random coefficient regression analysis) were similar in intercepts and slopes both within domains and across learning groups. A LISREL model that reasonably fit the data well showed marked different results from those of HLM. The results described in the above section are summarized in Tables 2 and 3 presented in the following section.

In this study, LISREL model 2 maximum likelihood estimates of the population means of true intercept and true slope in both mathematics and language for both the African American and

White students groups were reported.

The entries in the first two rows of Table 2 for Model 2 estimate the African American population means of true intercept (188.96,  $p < 0.05$ ) and true slope (10.01,  $p < 0.05$ ) for mathematics. The estimated population means of true intercept and true slope for language are 189.02 ( $p < 0.05$ ) and 12.42 ( $p < 0.05$ ), respectively. The true intercept and true slope for the respective domain describe the average trajectory of true change in the dependent variable. On average, African American students' true mathematics scores increase by 10.01 per year while true language scores increase by 12.42 per year.

Table 3 presents parameter estimates and model fitting for mathematics and language scores for the White students. As was the case with the African American model fitting, model 2 was adopted for each domain. An inspection of the parameters in the table show that all the intercept parameters were statistically significant. Entries in the first two rows of Table 4.10 for Model 2 estimate the White population means of true intercept to be 204.04 ( $p < 0.05$ ) and true slope to be 13.80 ( $p < 0.05$ ) for mathematics.

The estimated population means of true intercept and true slope for language were 207.09 ( $p < 0.05$ ) and 15.00 ( $p < 0.05$ ), respectively. These growth parameters describe the average trajectory of true change in the dependent variable. On average, White students' true mathematics scores increase by 13.80 per year while true language scores increase by 15.00 per year. Students' knowledge in both mathematics and language improved over time, and more rapidly in language than in mathematics.

In both the African American and the White samples, slope parameters were all positive and statistically significant. The domain respective intercepts are initial average achievement

**Table 2: Fitted Models For Interindividual differences in Change in Mathematics and Language in the African American Sample**

<u>Maximum Likelihood Estimates</u>				
Parameter	Mathematics		Language	
	Model 1	Model 2	Model 1	Model 2
$\mu_{0p}$ (Intercept [I])	186.59*	188.96*	189.52*	189.02*
$\mu_{1p}$ (Slope [S])	10.26*	10.01*	12.31*	12.42*
$\sigma_{\pi_0}^2$ (Intercept Variance)	312.33*	215.86*	619.72*	354.22*
$\sigma_{\pi_1}^2$ (Slope Variance)	51.03*	27.77*	103.58*	62.60*
$\sigma_{\pi_0\pi_1}$ (I-S Covariance)	-53.31*	0.12	-104.35*	0.48
df	3	1	3	1
$\chi^2$	324.78*	8.56*	742.40*	41.45*
Goodness of Fit Index (GFI)	.985	1.000	.960	1.000
Normed Fit Index (NFI)	.960	.999	.952	.997
Comparative Fit Index (CFI)	.960	.999	.952	.997
Root-Mean-Square Error of Approximation (RMSEA)	.100	.026	.152	.061

Note: N=10,724. Descriptions of the models are given in the text below

\*  $p < .05$

scores at grade 4 adjusted for measurement error (Kline, 1998). The intercept is a characteristics of the whole sample while the variance of the same, reflects the range of individual differences in the domain of interest around the intercept. The mean rate of change, on the other hand, reflects a group-level characteristic– its value indicates the average amount of occasion-to-occasion change in mean levels of the domain of interest (also adjusted for measurement error). The statistics provided by the slope(rate of change) presents information about the rate of individual differences in linear occasion-to-occasion changes over time.

Tables 3 and 4 summarize the maximum likelihood estimates of the growth parameter matrix of individual differences in true change that were detected in mathematics and language. The variances of both mathematics and language were statistically significant. Thus, there is evidence of interindividual heterogeneity in true change in mathematics and language. Thus, students differed in their growth trajectories in these two domains. Correlation coefficients of intercepts and slopes within each domain were not statistically significant but they were both positive in direction. These coefficients show that students who had high initial language achievement scores showed greater rates of subsequent change. They tended to progress more rapidly in language over time. The same could be said about mathematics. However, the intercept was unrelated to the slope changes in the respective domain. Thus, where a particular student starts in an achievement domain is not necessarily related to his or her future growth (mean level) in the domain of interest. This conclusion is supported by the individual sample growth trajectories provided in Figures 1, 2, 3, and 4 (Appendices C, D).

**Table 3: Fitted Models For Interindividual differences in Change in Mathematics and Language in the White Sample**

<u>Maximum Likelihood Estimates</u>				
Parameter	Mathematics		Language	
	Model 1	Model 2	Model 1	Model 2
$\mu_{0p}$ (Intercept [I])	204.25*	204.08*	206.84*	207.09*
$\mu_{1p}$ (Slope [S])	13.75*	13.80*	15.08*	15.00*
$\sigma_{\pi_0}^2$ (Intercept Variance)	493.38*	328.74*	840.48*	516.95*
$\sigma_{\pi_1}^2$ (Slope Variance)	60.69*	35.19*	113.76*	63.96*
$\sigma_{\pi_0\pi_1}$ (I-S Covariance)	-64.40*	0.29	-126.35*	0.20
df	3	1	3	1
$\chi^2$	199.55*	39.24*	584.95*	23.32*
Goodness of Fit Index (GFI)	.992	1.000	.974	1.000
Normed Fit Index (NFI)	.981	.999	.965	.999
Comparative Fit Index (CFI)	.981	.996	.965	.999
Root-Mean-Square Error of Approximation (RMSEA)	.069	.053	.120	.041

Note: N=13,578. Descriptions of the models are given in the text

\*  $p < .05$

Table 4 presents a summary of the results of random coefficient regression analysis (hierarchical linear modeling) for both mathematics and language. The results show the presence of variability in the intercepts and slopes and the covariance-variances for the two groups of learners and within each domain. When compared with the earlier results of for the LISREL analysis, the (HLM) intercepts and slopes are exactly the same both within domains and across learning groups as was the case with LISREL model 1. However, there are marked differences in the variance-covariance estimates for both language and mathematics, as presented in Tables 2, 3 and 4. This

**Table 4:** A Summary of the Random Coefficient Regression Analysis (Hierarchical Linear model-HLM) Results for both Mathematics and Language.

	Intercept (I)	Slope (S)	Variance (I)	Variance (S)	$\sigma^2$
<b>AA<sup>a</sup></b>					
Math	186.60	10.26	123.19	10.69	125.30
Language	189.52	12.31	221.07	9.75	202.24
<b>WHITE</b>					
Math	204.25	13.75	278.00	10.89	127.42
Language	206.84	15.08	378.81	9.02	227.93

<sup>a</sup> African American

Note: All entries in table are significant at  $p < 0.05$

may be due to computational improvements within each software, differences in how each software handles missing values that may not be missing at random, the problem of multicollinearity or the differences could simply be an artifact of scale scores and equating processes.

## **Discussions/Significance**

This study is important in a number of ways. First, the study contributes to the expansion of literature brought about by research on the key components of the model of interest and the associated findings. Secondly, it sheds light on the understanding of two groups of learners from two ethnic groups and how the two learners develop mastery in mathematics and language as they graduate through school. Thirdly, the application of covariance structure analysis and growth curves to the study of growth in student academic achievement provides an avenue for an indepth analysis of two academic areas in an available data set. The use of longitudinal assessments in the identification process and assessment of outcomes offer several advantages over the traditional static cross-sectional assessment of learning outcomes. The employment of this technique shifts the focus from the assessment of mathematics and language achievement to learning and in effect leads to a more refined definition of learning problems and measurement of outcomes, which is a conceptual advantage over the current traditional approaches.

The study about growth in academic achievement is significant in that a better understanding of the cognitive abilities of different groups of learners in different academic fields is realized. Achievement outcomes are normally collected at the end of a specified period in the student academic career. The use of longitudinal assessments to measure growth in academic achievement makes early detection of learning problems a reality in that the rates of learning can simultaneously be measured in mathematics and language to assess the degree to which skills are differentially developing. Ultimately, the value of the study pertains to the following:

a) It provides a close examination of the trends and individual differences in mathematics and language and explores the effects of ethnicity on developmental trends that could go undetected due

to insufficient power by more traditional analyses such as ANOVA, MANOVA, (M)ANCOVA, etc,

b) It draws similarities in model formulation between the traditional methods, such as regression analysis, and covariance structure analysis with a view of lessening the burden inherent in SEM and HLM technical aspects that would naturally close out potential users of important research findings of student academic growth, c) it provides a potential base for further research on the measurement of change in student academic achievement which eventually may lead to schools and school systems adopting of measures tailored to meeting specific needs of specific students or groups of students, d) it provides the research findings on academic growth to educators, parents, and Louisiana department of education, among other school stakeholders, for the benefit of the education in the state.

From the measurement theory, research design and future practice perspectives, the statistical techniques employed in this research (the LISREL and HLM) methods have a number of extensions that can be utilized in various research environments due to their abilities to accommodate any number of data points (waves) of longitudinal data with more data leading to higher precision for the estimation of the individual growth parameters and greater reliability of the measurement of change.

### Missing Data

The results of this investigation should be interpreted with some caution because of a number of factors that were beyond the control of the researcher. Important among these was missing data. More often than not, loss of subjects in longitudinal studies of students may result in the pattern of data loss that may not be random. Due to the rather large data set utilized in this study, a test of whether the patterns of missing data were random or systematic was not completed but an

assumption was made that the missing cases in the data set were purely random and that missing data would not adversely affect the sample size. However, students who dropped out of school at each wave are perhaps more likely to come from families with particular characteristics (e.g., low SES, job instability of parents). This obviously can create problems with reliability of the data and the generalizability of the results. Further, the growth parameters computed may not be adequately representative of the true change in achievement for the ethnic groups compared over time. It is also important to be cognizant of the fact that when the missing pattern is not random, there is no adequate statistical fix to remedy this problem.

Though this study did not attempt to model the problem of missing data, it employed listwise deletion. The covariance matrix generated by listwise deletion will always be consistent, that is, positive semi-definite (Anderson and Gerbing, 1984). However, if the pattern of missing data is not random, an inconsistent matrix – not positive definite, can result (Rovine , & Delaney, 1990). Despite the fact that listwise deletion can result in a positive semidefinite matrix, it is also known that this technique can present problems for tests of goodness of fit, unless the missing data are missing completely at random (Kaplan, & Elliott, 1997; Muthèn, Kaplan, & Hollis, 1987).

Though there have been advancements in statistical computing power, multivariate data are frequently hampered by missing values. The traditional and relatively old methods of dealing with incomplete data, that is, deletion (listwise, pairwise) for cases with incomplete information, substituting plausible values such as means, or regression prediction for missing values continue to be utilized. In this study, listwise deletion was used. With listwise deletion cases with missing observations on any variable in any analysis are excluded from all computations—thus a final sample includes only cases with complete records. Though the recent advances in theory and computational

statistics have produced flexible and powerful procedures with sound statistical bases (Likelihood-Based Estimation–Efficient Estimation--EM, Multiple Imputations–MI) (Cohen, & Cohen, 1983; Kline, 1998; Schafer, & Olsen, 1998; Rovine, & Delaney, 1990), the statistical processes involved are above the reach of many researchers who are faced with the problem of missing data on a daily basis. These computational statistical techniques are quite involved and may require equally demanding data preparation procedures which many users of secondary data analysis may see as a nuisance that should be avoided as much as possible.

Furthermore, many techniques for handling missing data rarely account for the patterns of missing observations—whether random or systematic. This is a much bigger problem and compounds that of the proportion of the missing data. There is no clear guideline about how much missing data is too much. Cohen and Cohen (1983) suggested that 5% or even 10% missing data on a particular variable is not large. Irrespective of the method utilized in imputing missing values, the data set would still fail to provide accurate measures of variability if it does not account for missing-data uncertainty (Schafer, & Olsen, 1998).

As discussed earlier, intercept changes in both language and mathematics and for the two groups of learners were unrelated to their respective slopes. This suggest that where a student starts in domain achievement is not necessarily related to his or her future growth in the domain of interest. Though this study did not investigate poverty among the two groups of interest, it is worth noting that poverty in the African American sample in Louisiana is much higher than that of White sample. This imbeddedness of poverty within any particular group translates into differential learning environments in terms of per capita learning resources made available at home, which subsequently impacts school learning and achievement. Though a number of individual growth patterns over time

were shown in this study with each group, and when comparisons were made within group by SES levels, the total group effects of home and schooling were shown to sustain over time. Recent large scale reviews of the literature to identify both proximal and distal factors impacting student learning and achievement clearly document the importance of proximal factors that include both the school and the educational quality of the home environment (Wang, Haertel, & Walberg, 1993).

African American and White students enter grade 4 with language and mathematics achievement differences. These differences are more than influenced by differing rates of poverty associated with race. However, the results reported here also suggest that proximal factors associated with school (i.e., differing teacher expectations, access to educational resources) may also differentially affect African American and White students. Both the mathematics and language intercept and slope variances were higher for White students than for African American students. These differences suggests that the effects of home and school learning environments within groups differ. The White sample in this study remained approximately normally distributed with both low, median and high achievers persisting through the schooling years. This may not be the case with African American students over a greater number of years when differential dropout rates might be expected. These rates might well be predicted by difficulties associated with early childhood learning experiences. Thus, shrinkage in differences in achievement between White and African American groups in the later years of schooling might well be expected by differential dropout rates. As well, greater variation in SES within these two groups might account for the greater heterogeneity in White student samples in later school years than in African American student samples (as shown in this study).

It also seems important that factors that directly relate to proper and reliable assessment of student achievement in mathematics and language be observed. Royer (1990), stated that test using

multiple-choice items were measuring offline reasoning processes rather than online comprehension processes and extreme care must be observed when using these tests to make grade placement decisions, diagnosing reading difficulty, or assessing educational gain. Royer (1990) argued that standardized reading comprehension tests that utilize multiple-choice questions do not measure the comprehension of a given passage, but rather measures a reader's world knowledge and his or her ability to reason and think about the content of the passage. For mathematics and language educators need to use multiple data points and multiple forms of assessments of students' knowledge of mathematics and language other than relying only on the scores of standardized tests to evaluate students' learning growth. Both reliability and validity of inferences about student learning and academic progress are enhanced with analyses of longitudinal data.

It is important also that teachers have a better understanding of their students' literacy development. This helps teachers to recognize patterns of behavior which suggests aspects of students' development behavior out of what is provided in the curriculum. Knowledge of student's literacy development accords teachers an opportunity to develop more flexible curricula to meet the changing needs of specific students or groups of students.

The Louisiana School Effectiveness study (Teddlie, 1994; Teddlie & Stringfield, 1993) discussed areas in which school policies can positively affect teachers behaviors such as appropriate teacher selection and replacement, frequent personal monitoring of classroom behavior, support for teachers through direct assistance and in-service programs, and overall instructional leadership. These strategies lay a fertile ground for effectiveness in classroom instruction and management. Mendro (1998) discussed equity in student access to a quality education as regards the type of help to provide to students who have had an ineffective teacher in the past. Mendro (1998) stated that

students who are placed with an ineffective teacher suffer long-term negative effects and there needs to be a policy issue put in place to allow for more equitable distribution of resources to enhance the quality of teaching and learning. In a recent study that aggregated data at the student level, Sanders and Horn (1998) found that ineffective teachers were ineffective with all students regardless of students' prior levels of achievement while teachers of the highest effectiveness were generally effective with all students. Though Sanders & Horn (1998) found teacher effectiveness to be a dominant factor affecting student gains in academic achievement when compared to other classroom context variables (.e.g, class size, classroom heterogeneity), it seems important that schools recognize socioeconomic differences among students in the early years in considering more equitable distribution of educational resources, particularly good teachers.

For future research, this study raised a number of important points to consider for future research. First, student language and math achievement change need more research to pinpoint exactly where differences arise within each domain and across ethnicity. Second, lower math achievement scores and rates of change, particularly for African American students needs more intense study. The National Center for Educational Statistics study showed that, on average blacks and Hispanics score lower than Whites on reading and mathematics at the end of grade 8 and that these differences do not increase over the high school years. Sanders and Horn (1998) showed that, regardless of race, students who are assigned disproportionately to ineffective teachers are severely academically handicapped relative to students with other teacher assignment patterns. More research that links students' academic records to those of their teachers seems in order.

Third, the methodology of this study needs to be extended to ethnically diverse samples to further demonstrate its utility for investigating individual change over time. Studies using multi-

domain analyses to further investigate the nature of differences that were observed in language and math parameters in this study, and whether these differences are maintained across different groups of learners are needed.

Fourth, a replication of this study that uses the same measuring instrument across all measurement occasions, and a greater number of occasions, is recommended. This is preferred to using equating procedures such as vertical equating, with different tests. A greater number of data points (more waves) might also be quite informative. Such studies can yield information that has implications for understanding academic growth differences both within and between differing groups, and information that might be used for educational policy making, resource allocation and school intervention and improvement programs as well. In an era of educational policy making for greater school accountability, longitudinal studies can be used to better understand patterns of school change (or lack of change) over time. This seems particularly the case when such procedures are compared to more traditionally used procedures (i.e., pre and post test analyses from year to year). The data analysis procedures used in this study, and the attained results, also suggest the importance in future research, and in educational policy making as well, of understanding initial status differences and the cumulative effects of schooling among groups of students that differ by race and socioeconomic status.

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## APPENDIX A

**Table 2: Descriptive Statistics for African American (AA) Students**

AA	Mean	Variance	Skewness	Kurtosis
Domain--- Time				
Mathematics T1	186.28	256.24	0.27	-0.31
Mathematics T2	208.06	358.00	0.20	-2.80
Mathematics T3	216.77	553.50	0.34	-0.38
Language T1	189.72	404.99	0.25	-0.22
Language T2	213.53	726.40	0.38	-0.36
Language T3	226.84	811.31	0.28	-0.42

N=10,724

**Table 3: Descriptive Statistics for White Students**

WHITE	Mean	Variance	Skewness	Kurtosis
Domain--- Time				
Mathematics T1	204.50	411.12	0.19	-0.41
Mathematics T2	231.01	541.72	0.12	-0.47
Mathematics T3	246.01	687.09	-0.17	-0.69
Language T1	206.66	574.12	0.25	-0.38
Language T2	237.53	901.85	0.02	-0.59
Language T3	251.73	959.18	-0.02	-0.63

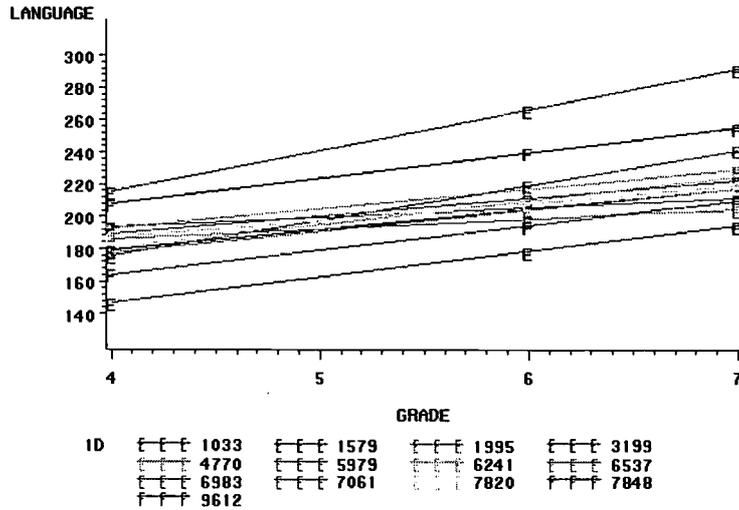
N=13,578

## APPENDIX B

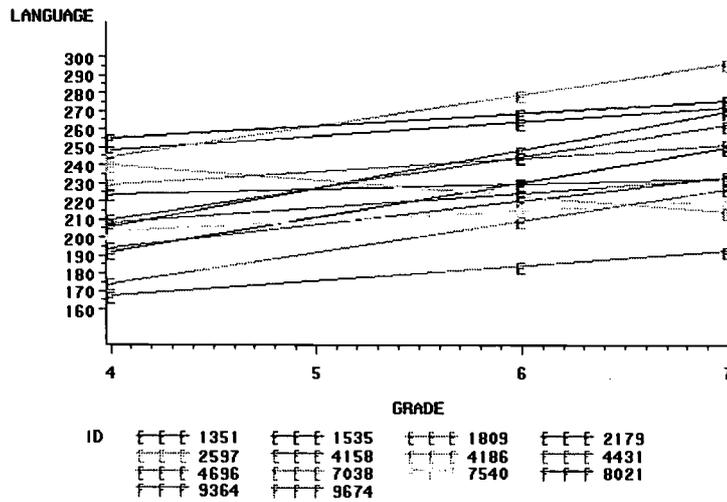
**Table 3: Longitudinal Data on Stratified Random Subsample of 27 Students with: (a) 3 Waves of Language Scores at Grades 4, 6 and 7 (b) 3 Waves of Math Scores at Grades 4, 6 and 7 ( c ) Values of the Indicator (AA=African American; W=White).**

Subject ID	Language			Mathematics			Race/Ethnicity
	Lang_4	Lang_6	Lang_7	Math_4	Math_6	Math_7	
6983	186.25	220	216	202.0	220	196	AA
5979	182.00	200	254	155.0	180	227	AA
6241	185.00	194	235	182.0	200	212	AA
1579	218.25	257	298	224.5	244	260	AA
1033	181.75	196	223	181.5	202	198	AA
7061	187.00	194	207	190.5	220	238	AA
1995	191.50	211	207	178.5	202	196	AA
7848	201.75	258	243	186.0	224	215	AA
3199	146.50	178	194	165.5	186	204	AA
4770	189.50	226	223	195.0	212	236	AA
6537	175.75	210	213	214.5	218	226	AA
7820	188.75	202	223	221.0	235	249	AA
9612	164.50	193	210	182.0	218	224	AA
2597	240.25	292	288	233.5	274	294	W
4186	237.75	229	210	199.5	254	268	W
4696	209.00	248	260	204.0	223	242	W
1535	222.25	235	229	202.0	206	213	W
4431	177.00	202	232	218.0	215	264	W
7540	208.50	198	232	179.5	188	221	W
2179	245.00	274	265	215.5	220	221	W
9674	201.00	246	218	187.0	196	212	W
8021	256.00	266	278	214.5	251	262	W
1351	194.00	224	254	185.0	217	240	W
9364	212.50	228	283	217.5	237	280	W
1809	166.25	188	190	167.5	168	196	W
4158	201.75	199	248	200.0	216	239	W
7038	219.50	273	232	213.5	228	250	W

## APPENDIX C

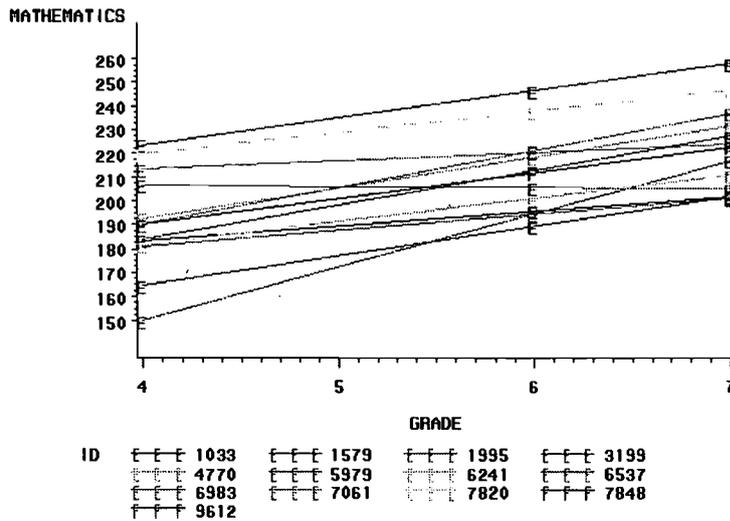


**Figure 1:** OLS Fitted Trajectories Summarizing Linear Growth in Language between Grades 4 and 7 for a Subsample of 13 Randomly selected African American Students whose associated Empirical Growth Records are provided in Table 4.5.

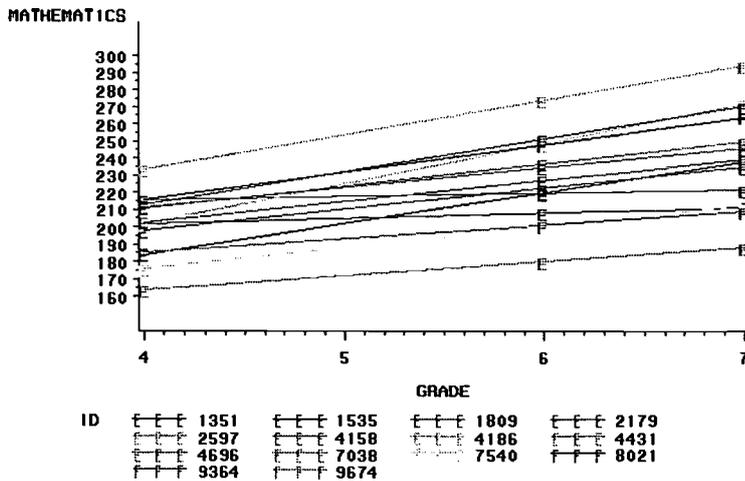


**Figure 2:** OLS Fitted Trajectories Summarizing Linear Growth in Language between Grades 4 and 7 for a Subsample of 14 Randomly selected White Students whose associated Empirical Growth Records are provided in Table 4.5.

## APPENDIX D



**Figure 3:** OLS Fitted Trajectories Summarizing Linear Growth in Mathematics between Grades 4 and 7 for a Subsample of 13 Randomly selected African American Students whose associated Empirical Growth Records are provided in Table 4.5.



**Figure 4:** OLS Fitted Trajectories Summarizing Linear Growth in Mathematics between Grades 4 and 7 for a Subsample of 14 Randomly selected White Students whose associated Empirical Growth Records are provided in Table 4.5.



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