DOCUMENT RESUME

ED 466 959   SE 066 544

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TITLE   What Mathematical Knowledge Do Pre-Service Elementary
                Teachers Value and Remember?
PUB DATE   2001-10-00 .
NOTE   11p.; Paper presented at the Annual Meeting of the North
                American Chapter of the International Group for the
                Psychology of Mathematics Education (23rd, Snowbird, UT,
                October 18-21, 2001).
PUB TYPE   Reports - Research (143) -- Speeches/Meeting Papers (150)
EDRS PRICE   EDRS Price MF01/PC01 Plus Postage.
DESCRIPTORS   Elementary Education; *Elementary School Mathematics;
                *Knowledge Base for Teaching; Mathematics Instruction;
                *Preservice Teachers; Teacher Attitudes

ABSTRACT
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Pre-service elementary teachers want to get mathematical answers right. They
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they see as different? (Contains 21 references.) (Author/MM)

What mathematical knowledge do pre-service elementary teachers value and remember?[1]

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Introduction

This study addresses the question of how pre-service elementary teachers can be induced to change their attitudes to mathematics. We know that pre-service elementary teachers want to get mathematical answers right. They want to know which formulas to use, and how to get the correct answer. We present transcriptions of student writing that illustrate and support this point. Changing what students value in mathematics is a much harder challenge than teaching them mathematical procedures and application of formulas. We need an antidote to a severely procedural orientation to mathematics focused on 'correct answers' that prospective teachers have learned to value above all. How can we explicitly emphasize connections, and assist students to construct relationships between parts of mathematics that they see as different?

Theoretical perspective

We could approach the issue of pre-service teachers' attitudes about mathematics through the extensive literature on change, and teacher change in particular. However, our approach in this study is a different one. It is to focus on students' long-term declarative memories and the relation of these memories to individual student gain in test scores over the course. Thus we are attempting to build a bridge between what student teachers themselves recall as important in their learning of mathematics with what test scores tell us.

Declarative memories are those we are capable of expressing in words, drawings or gestures. They are to be distinguished from *implicit*, or non-declarative, memories that assist us to carry out routine procedures and habits (Squire, 1994). Long-term declarative memories are mediated by protein formation, following gene expression, to stimulate novel neuronal connections (Squire and Kandell, 2000). The relevance of this neurological fact is that long-term memory formation is an energetic, committed process for an individual. Long-term memories—certainly those sustained over a period of several months—are therefore a good indicator of what a student

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values. Using long-term declarative memories as an indicator of what pre-service teachers valued about the mathematics they engaged with, we focused on connections between combinatorial problems (Maher & Speiser, 1997) to promote the formation of such memories. The model of Davis, Hill and Smith (2000) emphasizes a role for a mathematics teacher in assisting students to turn implicit, procedural memories into declarative memories. We used that model to guide us in designing a series of interrelated combinatorial problems for the beginning of a 16-week mathematics content course for pre-service elementary teachers.

Episodic memory is the system of memory that allows us to explicitly recall events in time or place in which we were personally involved. (Tulving, 1983; Tulving & Craik, 2000). Semantic memory is the memory system that deals with our knowledge of facts and concepts, including names and terms of language. (Tulving, 1972, p. 386; Tulving, 1983; Tulving & Craik, 2000). *Explanative memory* is that part of declarative memory dealing with explanations for facts. Davis, Hill, Simpson, & Smith (2000) present a case that explanative memory is a separate memory system, linked to, but different from episodic or semantic memory. Most psychological studies of memory are oriented to memory for language. Studies of memory for mathematics are much less common. A semantic memory such as *"Paris is the capital of France"* has quite different content to one such as *"the square of the length of the hypotenuse in a right angled triangle is the sum of the lengths of the squares of the other two sides"*. The first is a linguistic convention, the other expresses a non-obvious fact. Note that this is the way *we* see these semantic facts: we do not claim that students see them similarly. In fact, they often interpret mathematical facts in a similar way to simple linguistic conventions. There are, at least, the following distinctions in memory for mathematical facts:

- Labels, customs, and conventions. For example: A prime number is a whole number with exactly 2 factors.
- Things sensed, or done. For example: The proportion of prime numbers less than 500 is 19%.
- Things believed. For example: There are infinitely many prime numbers.
- Things explained. For example: A proof that there are infinitely many prime numbers.

We refer to these remembered facts as *semantic conventions, semantic actions, semantic beliefs,* and *semantic explanations*, respectively. Episodic memories and semantic actions—memories of things sensed or done in mathematical settings—are easily confounded. The reason, of course, is that a semantic action, by its very nature, involves memory of sensing or doing some-

thing. However, the critical difference is that one senses or does some non-trivial fact—calculating the number of primes less than 100, for example. The episode, remembered as such, is overlaid with an act or sense of a fact that goes beyond a mere episode. The distinction between episodic memories and semantic actions is one that distinguishes memory for mathematics from more everyday memories, as does, in part explanative memory—memory of mathematical explanations.

In this paper we will be concerned almost exclusively with semantic conventions—the linguistic conventions of mathematics. As we remarked, above, these are the simplest form of mathematical factual memory: the memories carrying the least mathematical information. They are non the less interesting. The main point of this paper is that the number of semantic conventions—memories of conventional mathematical language—a student has and can express at the end of a mathematics content course is an indicator of the degree of engagement with the mathematics content. That engagement, in turn, is indicative of a change in attitude as to what mathematics is about. Our principal aim in relating semantic conventions to student test scores is to generate hypotheses as to what motivates student change of attitude in mathematics classes, and to the nature of mathematics. Our hypothesis generation uses a combination of heuristic methods advocated by McGuire (1997) and is "shamelessly eclectic", in the spirit of Rossman & Wilson (1994), in that it mixes both qualitative and quantitative methods in formulating a relevant hypothesis about student growth.

Method

We set a number of connected combinatorial problems in the first 3 weeks of a 16 week course mathematics content course for pre-service elementary teachers. A detailed description of all the problems set in the first weeks of the course is seen at: www.soton.ac.uk/~plr199/algebra.html These combinatorial problems were specifically designed to:

(a) be relatively straightforward to begin working on, given the students' backgrounds;

(b) not be susceptible to easy solution by known or remembered formulas;

(c) set up strong episodic memories as a result of students discussing their solutions in class.

For example, after students had attempted the problem of finding how many towers of heights 4 and 5 they could build using blocks of 1 or 2 colors, they were shown, and discussed, a video clip of four grade 4 students attempting the same problem. This problem and its connections

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with algebra, which we utilized, has been reported on by Maher (1998), Maher & Speiser (1997), Maher, C. A., & Martino, A. M. (1996).

Students worked on the problems in groups. We asked them to write reflectively after each of the combinatorial problem sessions, and re-writes were encouraged. After completing the sequence of problems, they explained connections between the problems as a homework exercise. Opportunities for making connections with their earlier work were provided during the semester in questions students hadn't seen previously on three group and two individual exams. Students also wrote mid-term and end-of course self-evaluations.

The prompts provided to students to generate their initial written statements included the following:

Reflections on Towers Building I.

Reflect on building 4-high and 5-high towers.Complete the following:

1. When I first read this problem, I thought....
2. I first attempted the problem by....
3. After working with my group I realized....
4. I know I have found all possible towers because....

Reflections on Towers Building II

Write a brief paragraph describing your observations and reactions to the video of the "gang of four "students working the Towers problem. Complete the following sentences:

1. Before seeing the video of the 4th graders working the Towers Problem, I thought....
2. After seeing the video, I....
3. As a result of the class discussions, I now realize....
4. I have the following additional comments and/or reflections....

These questions placed value on what students saw and thought, before and after various activities, what their reasons were for making various statements, and their reactions to the problems and issues raised by them. Other questions asked of students placed value on seeing and explaining connections between apparently different problems.

We focus on the *individual gain factor: (final test% – initial test%)/(100-initial test%), a* statistic we use to measure student growth over time. (Davis & McGowen, 2001; McGowen & Davis, 2001; Hake, 1998). Individual gain factors are important in measuring student growth because they tell us what the student achieved in tests, given what was possible for them to achieve. We also coded the written work of students for instances of statements of semantic con-

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ventions. One student's written work was omitted from the analysis: this student had recently suffered brain trauma due to a car accident and had some difficulty with verbal and written communication, and with memory.

Results

Students' written statements at the beginning of the course indicated wide-spread instrumental views of mathematics. Typical statements include:

- "Coming into this class, I was under the impression that finding a formula to solve a problem was, in reality, the answer to the problem."
- "All throughout school, we have been taught that mathematics is simply just plugging numbers into a learned equation. The teacher would just show us the equation dealing with what we were studying and we would complete the equation given different numbers because we were shown how to do it."
- "As long as I could remember I have seen math as getting the right answer, and that being the only answer."

Students' written comments indicate that more than three-fourths of the students (15 of 19 or 79%) experienced changed attitudes towards mathematics and what it means to learn mathematics. The final write-up carried out individually by all students at week 16 included many examples of positive feelings about mathematics and the course. Typical of these comments are the following:

- "When our class finally concluded that the towers, tunnels, grids and Pascal's Triangle were all about 'choices', everything seemed to fall into place. ..."
- "... my perspective of mathematics changed over this semester... learning that my mathematical understanding was instrumental and not relational. I had to re-learn basic math in order to eventually teach it to children."
- "Not only did we get the answers, we made connections with other ideas.... That is a true way of learning mathematics and what it means."
- "This class helped me realize you will get nowhere (sic) knowing equations ... you have to understand what you are doing. ... I have learned more in this class than any other class that I have ever taken."
- "I feel this was the most productive experience I have ever had in my educational career. ... I deeply feel that I will be a better educator because of it."
- "I have learned that mathematics is indeed a series of interrelated ideas. I was challenged to extract these connections from our daily work while acquiring new skills in mathematics."

Typical semantic conventions in the student's written work relating to mathematics were:

- "Prime numbers are counting numbers with exactly and only two different factors."

- "A bit is the smallest term of measurement."
- "Factors are numbers that can be multiplied together to get another number."
- "Triangular numbers are the cumulative sum of counting numbers."
- "... I discovered how each relates. Both present two choices, or a 'binomial'."
- "In our class discussion, we defined algorithm as a systematic procedure that one follows to find the answer."
- "The relationship between addition and subtraction is an inverse relationship and can be demonstrated with a four-fact family."

Individual student gain factors ranged from 0.14 through 0.91. The mean gain factor for the class was 0.56 with standard deviation of 0.19. Fourteen of the 19 students (74%) who completed the course had a gain factor of 0.5 or higher.

FIGURE 1: Individual Student Gain
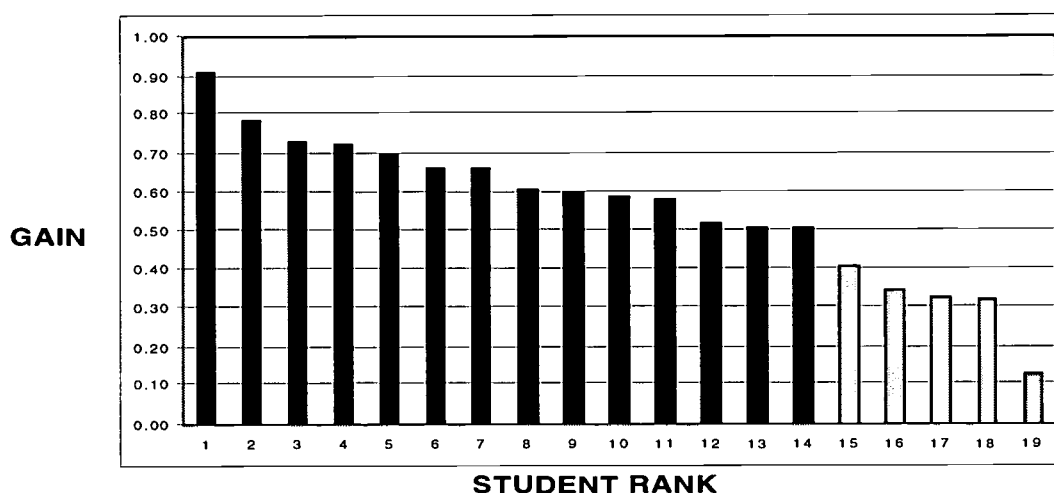
With the exception of the students with highest and lowest gain factors (0.91 & 0.14), the number of statements of mathematical convention generally increased with gain factor. We argue that the student, LJ, with the highest gain factor, is an outlier in that LJ made very explicit statements about the way she approached the course. LJ was the only student to make remarks related to organization of her personal life and we infer that this attitude played a role in her significant gain. Student NM, with the lowest gain factor (0.14), was also the student with the highest pre-test score: 77% (the pass mark was 80%). NM made many written statements about seeing the mathematics through the eyes of a child, and how that mathematics could be taught to a child. We infer that NM was reasonably satisfied with the level of mathematical understanding indicated by her test scores and concentrated, instead, on how that mathematics could be better taught to children.

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A plot of $\log_2$(# semantic conventions) versus gain factor gives a correlation with $r^2 = 0.70$ when the student with highest and lowest gain factors are excluded ($r^2 = 0.48$ when they are included). The number of semantic conventions did not correlate significantly with either pre- or post-test scores.
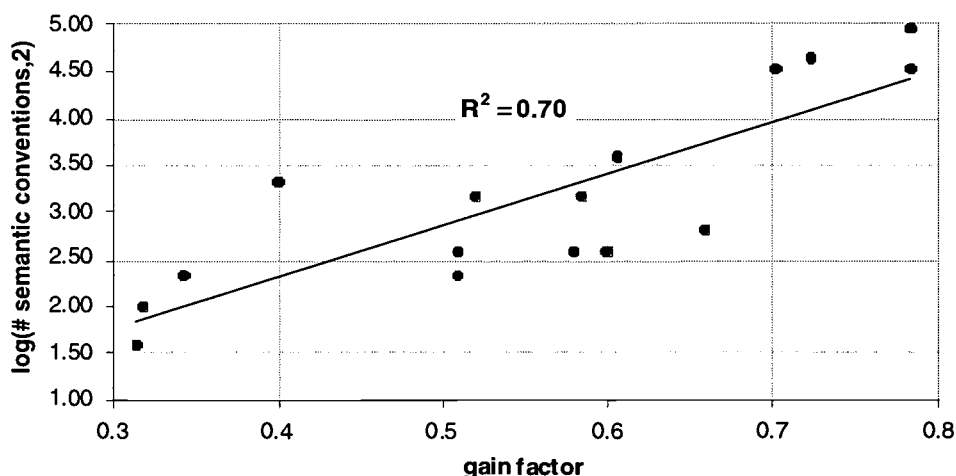
FIGURE 2:   Gain vs. # of Semantic Conventions

Figure 2. Plot of the logarithm (base 2) of the number of semantic labels in the writing of 16 out of 19 students, versus the gain factor ($r^2 = 0.70$).

The students with the highest and lowest gain factors are omitted, as is one student who had difficulty with expression and memory due to brain trauma.

Discussion

On the basis of the data presented here we hypothesize that there are at least two distinct factors associated with a student obtaining a high gain factor (≥ 0.5). First there is a conscious determination to organize one's personal life and involvement in the course so as to give oneself the maximum chance of growth. This is illustrated by LJ, whose growth factor was the highest in the class at 0.91 (z-score 1.81). LJ wrote:

> "By immersing myself in the mathematics through group study sessions, timely arrival to class, and the completion of homework, I was able to give myself a solid foundation in order to delve deeper into mathematics. Throughout this semester, I have missed one class, and have been tardy (five-minutes) once."

Second, and not obvious until we counted occurrences of conventional mathematical terms in student writing, most of the students we studied who had above average gain factors used

a number of conventional mathematical terms in their writings that increased exponentially with their gain factors. These two things taken together—the increase in gain factor correlated with an increase in the use of conventional mathematical terms—suggest to us that for these students mathematical growth was associated with an acceptance of and a long-term memory for the language of mathematics. It seemed to us that these students were "buying in" to a mathematical ethos, accepting that newly encountered, or revisited, mathematical words mean something other than an everyday connotation might suggest. Along with that change in attitude went a corresponding growth in mathematical achievement.

For most pre-service elementary teachers an important first step in coming to grips with mathematics, and developing mathematical maturity, is a decision to take the language of mathematics seriously, and to work on developing long-term memories for the words and terms they meet in content courses. The role of the instructor is paramount in setting up experiences for students that (a) establish strong episodic memories and (b) allow and encourage opportunities for students to express themselves using mathematical language.

We utilized a combination of methodologies—qualitative and quantitative—to formulate our main hypothesis, which is that, for a majority of students, higher gain factors are correlated exponentially with higher use of conventional mathematical terms. Further, both these factors are indicators of student growth in mathematics, accompanied with a changed attitude toward mathematics. Progress on this hypothesis would require a larger cohort of students, and pre- and post-administration of a mathematics attitude inventory such as that of Sandman (1980).

We also hypothesize that the structure and form of the early weeks of the course, focusing as it did on seeing connections and building strong episodic memories, assisted students in changing their attitude to the nature of mathematics. Written comments indicate that for approximately three-fourths of the class this was the case. There was little in the written comments of the remaining students, or in their individual gain factors, that indicate a long-term change in attitudes to mathematics or substantial increase in basic skills.

We believe, but do not provide evidence here for the belief, that it is through an emphasis on developing strong episodic memories of procedural aspects of mathematics that the implicit procedures gain a touch of "emotional color", and through discussion and reflection are shaped into long-term semantic memories. This was a guiding principle of the construction of the material for the course and the conduct of the sessions.

Finally, we hypothesize that it is an individual student's flexibility of thought in mathematical settings, that is largely a determinant of their mathematical growth in understanding and basic skills throughout the course. (Krutetskii, 1969; Dubrovina, 1992; Shapiro,1992; and Gray & Tall, 1994). Krutetskii (1969) and Shapiro (1992) characterize flexible thinking as reversibility: the establishment of two-way relationships indicated by an ability to "make the transition from a 'direct' association to its corresponding 'reverse' association" (Krutetskii, 1969, p. 50). Gray and Tall (1994) characterize flexible thinking in terms of an ability to move between interpreting notation as a process to do something (procedural) and as an object to think with and about (conceptual), depending upon the context. For the students reported on here, flexibility of thought encompasses both Krutetskii's and Gray & Tall's ideas as facets of a broader notion of flexibility . In addition to reversible associations, and proceptual thinking focused on the use of syntax to evoke both mental process and mental object, we are interested in connections between various representations of problem situation, which we refer to as conceptual. An inability to use syntax flexibly creates the *proceptual divide* (Gray & Tall, 1994) and is, in a broader sense, part of *a conceptual divide* (McGowen, 1998) in which flexibility is compounded by student difficulties in using and translating among various representational forms. Our hypothesis is that while flexibility is modifiable over the longer term it is probable that setting expectations for growth in flexible thinking at the beginning of a course plays a major role in determining how students grow mathematically.

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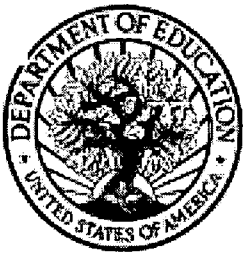
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Title: What mathematical knowledge do pre-service elementary teachers value and remember?

Author(s): McGowen, Mercedes A. and Davis, Gary E.

Corporate Source: R. Speiser, C.A. Maher, & C. N. Walter (Eds.) *Proceedings of the XXIII Annual Meeting, North American Chapter of International Group for the Psychology of Mathematics Education.* Snowbird, Utah, Vol. 2, 875-884.	Publication Date: 2001

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