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AUTHOR	Nemirovsky, Ricardo; Tierney, Cornelia; Wright, Tracey
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ABSTRACT

This paper explores children's efforts to make sense of graphs by analyzing two students' use of a computer-based motion detector. The analysis focuses on the students' growing understanding of the motion detector which enables them to plan their movements in order to create graphs and interpret them in terms of kinesthetic actions. Students gradually began to talk about the tool in ways that did not distinguish between the symbol and the referent. They also changed how they used the motion detector. This paper contributes to a reconceptualization of the nature of symbolizing, the learning of graphing, and the links between children's and scientists' graphing. (Contains 52 references.) (MM)

Body Motion and Graphing

by Ricardo Nemirovsky
Cornelia Tierney
and Tracey Wright

TERC

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To Eleanor and Dina, two girls who taught us so much.

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A brief and preliminary version of this paper is in Tierney, Nemirovsky, Wright, and Ackerman (1993).

Contents

Preface

TERC is a nonprofit education research and development organization founded in 1965 and committed to improving science and mathematics learning and teaching. Our work includes research from both cognitive and sociocultural perspectives, creation of curriculum, technology innovation, and teacher development. Through our research we strive to deepen knowledge of how students and teachers construct their understanding of science and mathematics.

Much of the thinking and questioning that informs TERC research is eventually integrated in the curricula and technologies we create and in the development work we engage in collaboratively with teachers. In 1992 we launched the TERC Working Paper series to expand our reach to the community of researchers and educators engaged in similar endeavors.

The TERC Working Paper series consists of completed research, both published and unpublished, and work-in-progress in the learning and teaching of science and mathematics.

Introduction

This paper reports a study of how two girls, Eleanor and Dina, who were at the time 10 and 9 years old respectively, used a motion detector in the context of individual interviews. The episodes include the first 22 minutes after Eleanor started to use the motion detector for the first time and 35 minutes of Dina's initial use of the same tool.[1]

Our analysis unfolds three themes: tool perspectives, fusion, and graphical spaces. Through the theme of tool perspectives, in Part I, we attempt to capture Eleanor's and Dina's efforts in emulating the tool's capabilities to enact graphical responses to body motion. Eleanor and Dina develop different tool perspectives that enable them to plan how to move to create certain graphs, as well as to interpret graphs by the kinesthetic actions. The theme of fusion, in Part II, explores Eleanor's and Dina's emergent ways of talking, acting, and gesturing that do not distinguish between symbols and referents; by merging visual traits of the graph, qualities of their motions, and the intentions guiding their actions, terms like "I," or "here," indicate at once past and present events, (e.g., the computer screen graph and the previous movement of the hand, or an "elephant" that the graph was intended to depict with the "beak" that it turned out to be). The theme of graphical spaces, in Part III, reflects our account of episodes in which a change in how the motion detector was used seemed to prompt Eleanor and Dina to investigate anew how the tool works; it was as if the change of the setup (e.g., measuring the position of an electric train instead the hand) had turned the motion detector into something entirely different.

Our analysis of these learning episodes strives to grapple with two interrelated questions:

(1) What emerging resources do students bring to their making sense of graphing?

When we talk about "resources," we do not have in mind isolated rules or memorized examples, but experiential domains, that is, bodies of life experience that the student brings to the situation full of intuitions, expectations, and ways of talking. An example may clarify this sense of "resource." Eleanor and Dina brought to the interview situation their drawing experience. The drawing experience offered them a rich background of ideas, kinesthetic patterns of body motion, and ways of describing visual shapes, which enabled them to be surprised, articulate hypotheses, and organize their use of tools. We talk about *emerging* resources to emphasize that they are not just "retrieved" — as if they were modular ready-to-use objects — but continuously changed and transformed by the local history of the student's situation. For example, an important dimension of Eleanor's and Dina's learning about graphing was their exploration of the ways in which drawing feels similar and different from graphing.

(2) What patterns of significance do students develop to grapple with the surrounding situation?

Here is another way to pose this question: What aspects of the situation are significant to the student, and how do they shift? At a certain time, for

[1] The motion detector that we have used was provided by Lipman Co. (Israel).

example, Eleanor was intrigued that the motion detector seemed to become unresponsive to her whenever she went beyond a certain distance from the sensor. This aspect lost significance later, after she reckoned that the motion detector cannot "see" that far and that she must use the tool without trespassing those intangible boundaries. At times, aspects that Eleanor and Dina perceived as irrelevant, such as the graphs appearing on the screen from left to right, shifted to the focal point of their inquiries, such as when Dina wanted to draw a "simple shape," as a square might be.

We think of the interviews as open-ended mathematical conversations with students, in which the participants create fictional worlds and unexpected ways of using tools and symbols. We participate in these conversations to gain insights not only into how mathematical situations appear from students' shoes but also into our own mathematical understandings (Confrey, 1992). By striving to closely relate Eleanor's and Dina's shifting perspectives with our own questioning on the nature of graphing, we come to conclusions that might contribute to a reconceptualization of the nature of symbolizing, the learning of graphing, and the links between children's and scientists' graphing.

Contextual Remarks

Many studies on the nature and learning of graphing, within the literature on mathematics education, have been erected on the assumption of some fundamental dichotomies. For example, researchers have analyzed students' approaches by looking at misconceptions, or alternative conceptions, which are to be contrasted to the conception of the expert grapher (Janvier, 1978; Clement, 1989). Others focused on the polarity between pointwise and shape-based understanding of graphs, a polarity that has been presented as an instance of the more general dialectic between process/object which, according to several authors, underlies all mathematics learning (Dubinsky & Harel, 1992; Sfard, 1991). Other studies focused on the opposition between external and internal representations. External representations, in the form of tables, graphs, or equations, are viewed as corresponding to internal representations — cognitive versions of the symbol systems displayed on paper or on a computer screen (Goldin, 1992).

These foundational dichotomies connect with polarities that are often viewed as being at the essence of human nature itself. Opposing "misconceptions" to the socially accepted conception of the expert, for instance, is a way of dealing with the general dichotomy between the social and the individual; that is, the individual becomes "encultured" by replacing his idiosyncratic approaches with the socially sanctioned approach. Depending on how it is used, the antithesis between external and internal representations may reflect the polarities objective/subjective, matter/thought, or social/individual.

By focusing on one or another dichotomy, students' performance is analyzed in reference to a standard and expert approach that is well known to the researcher (e.g., misconceptions are compared to a canonical conception). Consequently, it is not surprising that the literature highlights students' mistakes and deficiencies; in other words, it stresses what the student lacks (e.g., "this student conceives the graph as a process but not as an object"). In consonance with an emerging research perspective (Cobb & Bauersfeld, 1995; Smith, diSessa, & Roschelle, 1993; Confrey, 1991), we avoid seeing students by their mistakes to be overcome, we look instead at the students' emerging resources and how they relate to the use of tools and at the conversations in which the students participate.

What we find questionable is *not* the use of opposed qualities or the making of distinctions, but taking them as essential dualisms underlying thinking and learning. Our perspective is influenced by writings characterized by an effort to overcome classic dichotomies, such as subject/object, signifier/signified, mind/body, or internal/external, as the constitutive elements of "reality."[2] From this perspective, the notion of experience encompasses at once all the aspects of the ongoing human existence. Those distinctions are meaningful ways of talking about and reflecting on past events, but they are not at the foundation of the human experience. When, for example, one participates in a conversation one does

[2]In our opinion, landmark works from this literature are Heidegger (1927/1962), Merleau-Ponty (1946/1989), and Wittgenstein (1953).

not distinguish a gesture as belonging to the body or to the mind, or when one reads a text one does not split the signifier (the letters on the text) from the signified (the story told by the text); actually, we sense these kinds of splits as a disruption in the flow of experience (e.g., suddenly the text is a sequence of letters). This philosophical tradition argues that the meaning of symbols is to be found neither in the specific thoughts that they express, nor in the objects to which they refer, but in their use, that is, in the practices they are part of.

Recently a number of studies have reported a renewed exploration on the learning of graphing (Bèguin et al., 1994; Boyd & Rubin, 1995; Carraher, Schliemann, & Nemirovsky, 1995; Chazan & Bethell, 1994; diSessa, Hammer, Sherin, & Kolpakowski, 1991; Kaput & Roschelle, 1995; Monk & Nemirovsky, 1994; Moschkovich, in press; Nemirovsky, 1994; Nemirovsky, 1996; Pratt, 1995; Stroup, 1994; Tierney & Nemirovsky, 1995; Vitale, 1995; Yerushalmy, 1996). The overall view that emerges from these studies is, according to our interpretation, one in which (1) learning graphing entails the enrichment of a broad range of experiential domains, involving the refinement of visual, kinesthetic, and narrative resources; and (2) the adoption of mathematical conventions does not diminish the open-endedness involved in their use. In a similar sense that no two Italians, Americans, or Brazilians are identical and that for each individual being Italian, American, or Brazilian is a particular ongoing coping with unforeseen events, no two person's graphing are identical or bring the same resources to a new situation. Instead of the uniform "internalization" of the graphical conventions described in textbooks, we witness the creative, conflicting, diverse, and always shifting ways in which students strove to make sense of graphs and express themselves with graphs.

The notion that thinking is an "embodied" activity — that our incarnate experience with the human body as a whole, not just the brain, is involved in how we conceptualize situations — has become fashionable. Besides the influential writing of Merleau-Ponty (1946/1989), the work of Johnson (1987) and of Lakoff (1987) has stimulated this trend. The bodily basis for understanding is central to this paper, but, in contrast to Johnson and Lakoff, our intent is not to identify core cognitive structures but to gain insights into how situations involving body motion and symbols look to the person in the shoes of Eleanor and Dina.

Researchers (Mason, 1987; Lave & Wenger, 1991; Meira, in press) invoke metaphors involving the act of seeing in the process of understanding symbolic expressions — such as "seeing through," "transparency," and "fields of visibility" — to talk about the ways in which people construct and interpret symbolic expressions. An aspect woven into the common experience of seeing that appears to be significant for the study of symbol use is the shift among visual focuses, and how some of them are mutually exclusive or inclusive, such as when one looks at a landscape through a window: seeing the landscape is, to a certain extent, not-seeing the window and vice versa, but, at the same time, the

sight of the landscape through the window fuses the landscape and the window within a unified experience encompassing how they relate to each other and to the viewer.

An example of a similar phenomenon emerging in symbol use is a situation in which a person traces a line on paper and says "this straight line..." The understanding that this person expects to share requires some degree of not-seeing the trace — for no actual drawn trace is a straight line. On the other hand, the trace makes pointable a field of geometric relationships which is the actual subject of the speaker's utterances and gestures. We are investigating how this kind of shared understanding grows, including episodes in which it does not occur, such as when the interviewer draws a curve intending to express the idea of an increase that rises continuously and the child strives to make sense of the interviewer's drawing by focusing on its unintended pictorial qualities, such as some pieces that look more straight than others or one ending that seems vertical.

The conception of symbol use as a complex seeing highlights another question: What does come into view? Latour (1986) made an important analysis of this question. He described how symbols are used by people and cultures to make present the absent. A conversation that uses a map, for instance, merges the here and now of the speakers with selected features of a distant land. The represented land is brought to the site of the conversation in ways that make it partially pointable and accessible. Latour talks about the constitution of a "common place" that is shared by symbol users. The common place is populated by absent experiences, objects, and events that come into view within the localized here and now of the symbol users. The idea of a common place is important to our research because we explore children's making sense of symbolic expressions — Cartesian graphs representing their body motion — as a creation of a common place, which we call "graphical space": a space that is gradually enriched by past experiences and which becomes part of conversations and gestures.

The Study

This paper reports a study of how children used a specific tool — a computer-based motion detector — to make sense of symbolic expressions in the form of Cartesian graphs of position vs. time. We exemplify our analysis with excerpts from individual interviews with two girls, Eleanor and Dina, who are 10 and 9 years old respectively. The interviews were conducted as part of the Students' Conceptions of the Mathematics of Change project.

We analyze the activity of Eleanor and Dina in their initial work with a motion detector. The motion detector we use consists of one small object — the "button" — whose position is measured, a sensor or "tower," and a computer. The children often hold the button or they may place it on a moving object, such as an electric train. For the interviews reported in this paper we set the interface so that the computer monitor displays in real time a graph of the changing distance between the button and the tower; that is, a graph of distance vs. time appears on the screen as one moves the button (see Figure 1).

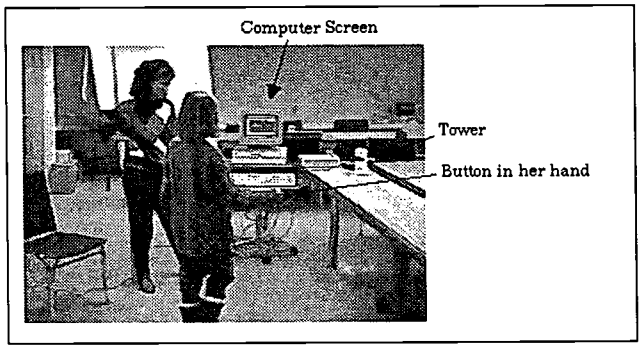

Figure 1. A computer monitor displays in real time a graph of distance vs. time.

Beyond a distance of three meters the detector does not measure and the computer graph flattens where the measurement was last taken. We set the graphical scales from 0 to 18 seconds and from 0 to 4 meters. The computer screen displays a grid of dotted lines (see Figure 2).

The interviews consist primarily of conversations between the interviewer and the child as the child uses the motion detector. During conversation the goal of the interviewer is to understand the child's thinking and learning. In planning these sessions, we do not know ahead of time what will be rich; they are deliberately open-ended. Although we plan a possible sequence of tasks for the interviews, each interview is a unique event. We ask the children to talk aloud about what they are thinking. The interviewer responds naturally with expressions of interest and also intervenes at times with restatements and questions, or to focus the child's attention on aspects of the situation or upon the child's own statements or actions. The interviewer cannot learn what the child thinks without affecting how she is thinking. We videotape the interviews and meet as a group to

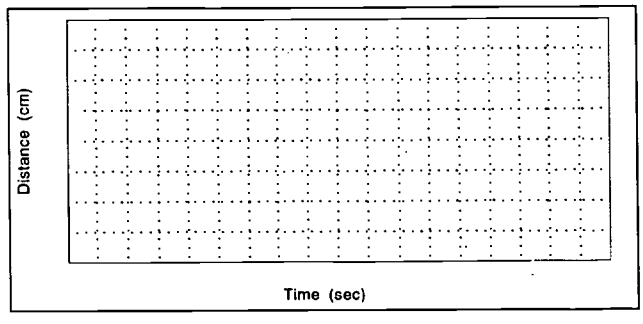

Figure 2. The computer screen display of a grid of dotted lines.

discuss them together as well as to plan the next interview. The questions we pursue when we analyze the videotapes often arise from the data we observe. We do not limit our interest to what the child knows; we also look at what is of interest to her, how she goes about finding out more, and how she relates to the interviewer.

We have interviewed individually 12 children ranging in age from 7 to 14. We have chosen to write this paper about Eleanor and Dina because the sessions with them helped us learn and clarify ideas that became extremely useful for our analysis of the interviews with all the students.

Dina and Eleanor first used the motion detector in the second of a series of hour-long individual interviews with the interviewer, Tracey Wright. In the first session they had drawn representations to instruct Tracey to move a handheld toy truck and an electric train in particular ways that each one established. Both children came to include instructions whether to go fast or slow, where to start or stop, and when to change direction.

In their second sessions the children began working with the motion detector. The interviewer posed no particular tasks to Eleanor and Dina, instead she encouraged them to take charge of the investigation. Both children began by investigating what motions elicited a response from the motion detector. Each in her own way came to see that the line on the screen rises higher when one moves away from the tower and lower when one moves toward it. They then made use of their findings to explore something of interest to them. Later Tracey suggested a change of context in the use of the tool, such as using two buttons instead of one, so as to generate two graphs simultaneously or to attach the button to an electric train.

The paper consists of three parts. With only one exception, the episodes are introduced in the paper following their chronological order, that is, whatever is presented later in the paper took place later in the interviews. Each part has an introduction, commentaries on the annotated transcript, and a discussion section. We indicate missing pieces of transcript with this symbol: (...). Almost all the missing pieces of transcript are short exchanges that refer to procedural details.

Part I: Tool Perspectives

As part of our everyday use of tools we often experience a shift between the tool as an implicit aspect of our goal-oriented activities (e.g., using a knife to cut vegetables as one is making a salad) and the tool as an object of reflection (e.g., trying to figure out what type of knife is best to cut vegetables). These familiar experiential shifts have become an important theme for 20th-century philosophy. As in the famous example of the blind man's cane, cited by Merleau-Ponty (1946/1989), Wittgenstein (1953), and others, the fluent use of a tool appears to involve its transparency in the service of accomplishing goals as well as its availability as a subject of analysis when, for example, the tool does not seem to perform as expected. If this is a suitable characterization of expertise in the use of a tool, what happens when the tool is unknown and unfamiliar to the user? How does expertise develop? Part I of this paper is a case study on these questions. In this instance the tool is a motion detector and the users are Eleanor and Dina as they encountered the motion detector for the first time.

Vygotsky's (1978) description of the development of expertise in tool use as a process of internalization, through which the individual becomes encultured, is well known. We avoid this language, however, to elude anchoring ideas on the external-internal dualism and because we want to leave open the question as to when and how fluency in the use of tools involves enculturation. Our analysis highlights the development of what we call a "tool perspective." Adopting a tool perspective involves emulating the tool's sensitivity to certain aspects and not to others, to enact conditions under which the tool is to be used, and to recognize patterns of significance in the tool's products.

A possible example of tool perspective is the interpretation of X-ray photographs. Many of us have participated in a conversation with a physician analyzing a X-ray photograph from a part of the body. Often the physician points to a small gray area on the image and asks "See this?" to help us focus on a little spot that otherwise might be absolutely unremarkable to us. The patterns of significance expressed by the physician can be very counter-intuitive to a lay person; what looks salient may be an irrelevant optical artifact of the X-ray device. Interpreting X-ray images — our example of adopting a tool perspective — involves not only knowing anatomy, pathology, and other biological sciences but also awareness of what is visible and invisible to the device and how idiosyncratic limitations of the tool produce visual traits to be ignored.

Just as the X-ray device or any other measurement instrument has certain limitations, so too does the motion detector (e.g., it cannot measure beyond a certain distance). Besides these limitations, the motion detector has been designed assuming that the user would conform to certain performance requirements, such as always facing the button toward the tower. Finally, the designers of the motion detector adopted as "natural" certain expectations; for example, that temporal graphs are always drawn from left to right or that connected lines are supposed to be seen as continuous even though the pixel structure of the computer screen may show them as bumpy.

When children begin to use the motion detector they are aware of neither its general limitations nor the kinds of actions that the user has to avoid (e.g., obstructing the optical link between the tower and the button). In addition, children do not necessarily come to the situation sharing cultural expectations as they are reflected in the tool (e.g., that time cannot reverse direction). Through their initial use of the motion detector, Eleanor and Dina began gradually to discriminate between what for each one of them were idiosyncratic aspects of the tool use to be ignored or just complied with (e.g., hold the button facing the tower, stay within the measurement range of the tower), and what is significant to interpret and play with (e.g., the graph goes always to the right, getting closer moves the graph down). It is not the case that every possible aspect of the tool use is either idiosyncratic or significant in a general sense; instead, the idea is this: anything that becomes an issue for the tool user (e.g., the impossibility of graphing a square) tends to gravitate toward being more or less significant in ways that are not necessarily the ones intended by the designers of the tool.

Note that we talk about developing a tool perspective and not the tool perspective. We avoid deterministic views according to which the use of a tool would necessarily force the tool user to develop a given perspective; tool perspectives are open ended. For this reason we emphasize the distinction between what is idiosyncratic and significant to the tool user. Blocking the button with one's hand to create vertical lines might be significant for someone, even though it is not intended to be so by the designer of the tool. Tool perspectives may be different for different individuals and also change for the same individual as she experiments and reflects on the use of the actual tool over time. Developing tool perspectives enabled Eleanor and Dina to anticipate how the graph should look, so that if the graph did not conform to their expectations they would try to recognize accidental mistakes in their motions or inherent limitations of the tool to explain the discrepancy.

The episodes included in Part 1 describe the first 10 minutes of each student's use of the motion detector. The first half focuses on Eleanor and the second on Dina. Each half is divided in three sections. Our central aim is to describe specific aspects in Eleanor's and Dina's process of developing a tool perspective for the motion detector, hoping that such an account might broaden our understanding of what is involved in becoming fluent tool users. Our analysis strives to articulate three contributions:

1. *The foremost quality of the tool was, for Eleanor and Dina, its responsiveness to their body motion. They began by striving to identify which elements of their body motion the motion detector was sensitive to. This is the focus of the first section of Part I ("Responsiveness of the Tool").*

2. *Eleanor and Dina gradually envisioned a tool's point of view from which certain actions were noticeable and others indiscernible. This is the subject of the second section of Part I ("The Tool as a Point of View"). They each strived to find out how what they perceived as significant and idiosyncratic traits in the behavior of the tool affected "its" perspective.*

3. *Finally, the third section of Part I ("Emergence of Logical Necessity") illustrates how Eleanor and Dina considered what types of graphical productions are possible, impossible, or difficult, expressing their growing sense of how the tool assumes a certain logic.*

Keep in mind that these three focuses of analysis do not attempt to separate distinct sequential phases. Eleanor's and Dina's views on the responsiveness of the motion detector to their body motions, for example, does not end with the episode included in the first section. Rather, we have chosen to highlight a certain aspect in our analysis of each set of episodes — even though it develops throughout the whole interview — because of its conspicuousness at that particular time.

Before starting the analysis of the episodes we need to clarify what we mean by "actions" and "responses" because we make repeated use of these terms. By actions we mean situated and intentional attempts to affect one's circumstances (Suchman, 1987). Actions are situated because they incorporate, on an ongoing basis, local and contingent aspects of the actor's setting. Actions are intentional because they express desires to accomplish shifting and at times conflicting purposes that emerge from the history of the situation itself. We think — as in the famous Ryle's (1963) discussion on the difference between winking and closing one's eye as a result of a nervous tic — that understanding actions involves grasping the lived situations, intentions, and histories surrounding their occurrence. Suppose, for example, that a man is hammering a nail on a wall. Understanding what he is doing is not just a matter of producing a detailed description of his arm movements; it may be crucial to know that he is planning to hang a painting on the wall, that he has not much time to do it, and so forth. In other words, what enables us to make sense of his actions is to discern his local intentions and circumstances.

Schön (1987) and Bamberger and Schön (1991), through their description of the design process as a "conversation with the elements of the situation," suggested that actions and responses constitute each other continuously. Responses often bring to the fore unexpected issues and questions. Let us briefly go back to our man hammering a nail on the wall. He hits the nail expecting that it will penetrate the wall a bit more, but, instead, the nail bends; this is to him an unintended response. He then needs to cope with the unintended, by, for example, starting all over with a new nail. Some responses prompt the actor to adopt a more reflective attitude. For example, the nail might, instead of getting deeper into the wall, stay at the same depth and produce a metallic noise. The man would then wonder what the nail could be hitting and whether another location on the wall would be more suitable. Responses emerge within the actor's interpretive activity; moreover, actors are often trying to figure out whether a certain event counts as a response or not. It may matter to our man trying to hang the picture on the wall whether a sudden break on the wall was due to his hammering or whether it would have happened anyway. We chose to exemplify our notion of actions and responses with a person using a tool because it might be seen

as obvious that in a human interaction, such as in a conversation, actions and responses constitute each other on a continuous basis.

Eleanor

Introduction

These episodes took place during the second session with Eleanor during the initial 10 minutes of Eleanor's use of the motion detector. Previously Tracey and Eleanor exchanged ideas about ways of representing the motion of a handheld truck. The three sections include episodes according to the time scale shown in Figure 3.

Responsiveness of the Tool

Tracey introduced the motion detector, highlighting the button, the tower, and their mutual relationship.[3]

<1> *Tracey: This is a button, and this is a tower. And this is how this works. If I push F1 it'll start, and you can move that [indicating the button in Eleanor's hand]. And it's going to respond to the tower. So [as she moves her hand a horizontal line begins to appear on the computer screen, (Figure 4, Segment 1)].*

<2> *Eleanor: [producing Figure 4. (See Figure 5 to recognize the different types of movements that Eleanor tried.)] [She moves her hand laterally,*

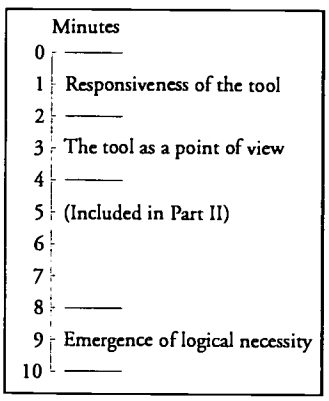

Figure 3. Time scale of Eleanor's initial 10-minute use of the motion detector.

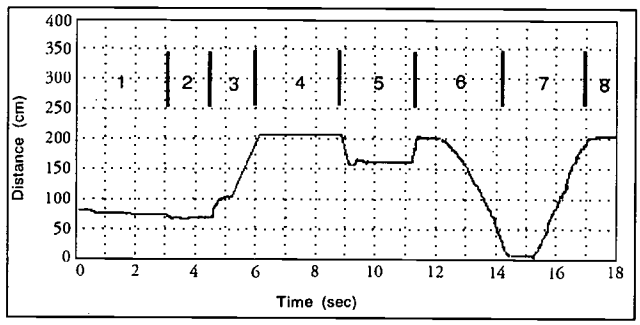

Figure 4. Eleanor's initial graph with the motion detector.

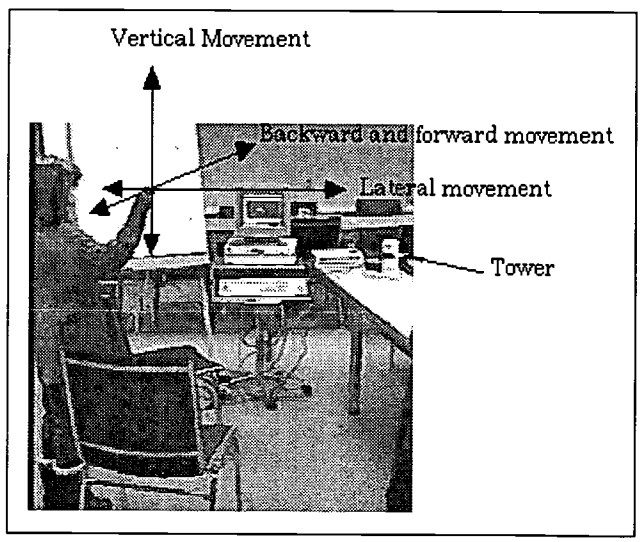

Figure 5. Eleanor tests the graphing capabilities by using different types of arm movements.

[3]This was Tracey's first interview using the motion detector.

in a counterclockwise circle, the line lowers a little bit, Figure 4, Segment 2.] Let me move it farther away. [She walks backward away from the sensor.] The line goes up [as she produces Segment 3, then she continues walking backward and the line flattens, Segment 4]. Maybe this is the farthest it can go. What if I move it up higher? [She walks forward and moves her hand up high, Segment 5. Then she walks backward and walks forward all the way to the sensor, Segment 6]. The closer to the tower it gets, the lower, I think. [She walks backward away from the sensor producing Segment 7]. And I think it gets. It gets higher until this line. [She walks in and points to the screen where the line has flattened in Segment 8]. And then it [the graph reaches the end of the scale and stops]. Wait, oh, I can't do it anymore. How do you clear it [the computer screen]? (...)

Eleanor began her use of the motion detector to investigate the graphical responsiveness of the tool to her kinesthetic actions. Her first counterclockwise movement of her hand did not engender a significant response on the computer screen (Figure 4, Segment 2). As soon as she tried a movement back with her hand, however, the tool seemed to wake up (first curvy piece in Segment 3). This graphical response encouraged Eleanor to further pursue the movement back by walking steadily away from the tower. Beyond a certain distance the line flattened even though she continued to walk back (Segment 4). The silence of the tool, as Eleanor stepped outside an unmarked region, made her aware of apparent boundaries and inspired this comment: "Maybe this is the farthest it can go." The "it" of her comment was probably not the button she was holding in her hand — there were no limitations for the location of the button — but the engagement between the button and the tower. Striving to break the passiveness of the tool, Eleanor held the button up and also walked a bit closer. The graphical jump down of Segment 5 and its subsequent flattening out did not provide Eleanor any clues about a possible significance of the vertical movement. Her walking forward instead, toward the end of Segment 6, prompted Eleanor to express a key insight: "The closer to the tower it gets, the lower, I think." This utterance announced Eleanor's sense that her distance to the tower gets expressed in the graphical height of the line.

We want to highlight two interrelated aspects of Eleanor's sense that closeness to the tower is expressed by the height of the graph: (1) Eleanor accounted for her experience of the most remarkable features of the tool's responsiveness by articulating actions — getting closer, getting lower; and (2) Eleanor envisioned a qualitative relationship between the two measures, namely, that a decrease in one (getting closer) corresponds to a decrease in the other one (getting lower). More than specific correspondences (e.g., certain heights on the graph corresponding to certain distances to the tower) Eleanor pointed out a relationship between changes along the two continuous magnitudes. We shall see later how Eleanor began to incorporate quantitative and pointwise elements. This action-based and qualitative graphical expression of her closeness to the tower was important to

Eleanor because it seemed to her a way of accounting for the responsiveness of the tool.

As Eleanor produced Figure 6 she wondered about the tool's response to stillness.

<3> *Eleanor: OK, so let's see. So. [Eleanor moves slowly away from the sensor, Segment 1 (see Figure 6). Then she stops part way back and holds the button still in front of her.] What if I just leave it here? Will it keep on going? [The graph is a horizontal line, Segment 2] Yeah. What if I move it up [Eleanor moves her arm up, Segment 3], wait, let me see. So it goes... [She walks closer to the tower, Segment 4], the closer I go, the lower it goes...*

Grasping the responsiveness of the tool involves an awareness of what aspects are not responses, in other words, what the tool does on its own. Pursuing her systematic investigation, Eleanor found an example of graphical activity (going to the right) that occurs in the absence of kinesthetic actions: the graph "keeps on going" even if she "just leaves it here."

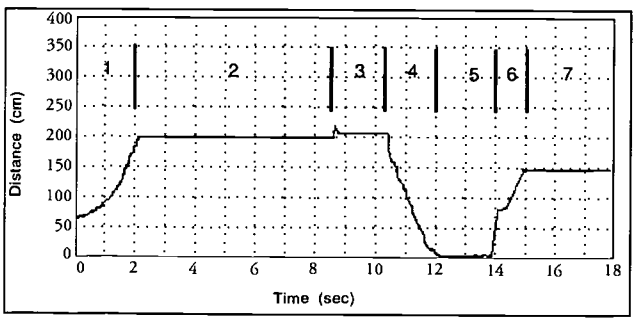

Figure 6. Eleanor's graph reflects her curiosity about the tool's response to stillness.

Then Eleanor tried holding the button up high again, she had not ruled out the tool's sensitivity to vertical motions. Our work with Eleanor and other children showed us that a sense of responsiveness, such as "the closer I go, the lower it goes," does not preclude other possibilities; the tool might respond in that way and still be sensitive to other body actions that do not seem to affect closeness to the tower, such as vertical hand motion.

During her initial interaction with the motion detector Eleanor tried to grapple with two issues: What aspects of one's body motions are "visible" to the tool? (Here is another way to formulate this question: How should one move to elicit responses from the tool?) and What does the tool do on its own, that is, independent of one's actions? Through her exploration she began to perceive some significant aspects in the tool's responsiveness: it seemed particularly sensitive to the action of getting closer to or farther from the tower, and the graph seemed to go to the right on its own, even when she remained still.

Eleanor's ensuing experimentation with the motion detector pursued these same questions in new ways, portraying her remarkable refinement of what is to be taken as significant or idiosyncratic.

The Tool as a Point of View

Let us resume the episode when Eleanor was in the middle of creating Figure 6.

<4> *Eleanor: But what if I go, like, far and, like, no [she walks away from the tower creating Segment 6, then Eleanor sees that Segment 7 is*

flat even though she still moved farther away from the tower]. Hmm, why isn't it going as high? Maybe I went too far. (...)

As Eleanor walked away from the tower the graph displayed an increasing line (Segment 6); however, contrary to her expectations that it should continue to do so, at 1.5 meters the graph flattened out (Segment 7). This flattening happened because Eleanor turned the button away from the tower, so that the optical link between tower and button did not work anymore. In producing the graph, Eleanor encountered two kinds of flatness (Segments 2 and 7). Segment 2 was the graphical response to her stillness ("What if I just leave it here? Will it keep on going?"), but Segment 7 was puzzling to her. She did not perceive Segment 7 as a response to her actions because she was not aware of doing anything distinctive when the graph went horizontal; the computer had done it on its own. Segment 7 could not mean stillness because she had moved the button further away during its creation. Eleanor resorted to the idea of the maximum ("I went too far"), but this explanation could not account for Segment 7, because Segment 2 had been even higher. The result was Eleanor's feeling that there was something strange around Segment 7 and the maximum height. To Eleanor, this reopened the issue of the maximum height reachable by the graph.

Eleanor started a new graph (not shown here). She turned the button to face the tower again and the graph became horizontal around 1.5 meters. She struggle to interpret it ("Oh, I know it goes from...I think it goes from, like...") but Tracey decided to make explicit the issue of the button orientation.

<5> *Tracey: I just noticed one problem, and I think it has to do with the way you're holding it [the button]. If you hold it [the button] like this [toward the tower], it [the tower] can see better than if you turn it [the button] that way [away from the tower], it [the tower] can't see what you're doing. [Eleanor continues to generate the graph, and Tracey notices that she had accidentally turned the button away from the tower again]. You're turning it away again.*

Tracey's explanation made Eleanor aware that the orientation of the button is a factor to consider in how to use the motion detector. Holding the button toward the tower became an aspect of good performance with the tool, an action to conform with, rather than to play with. This is a central trait of the idiosyncratic aspects of tool use. Tracey wanted to eliminate turning the button as a component in the graphical production or interpretation; she did so by talking about the button bearing in the context

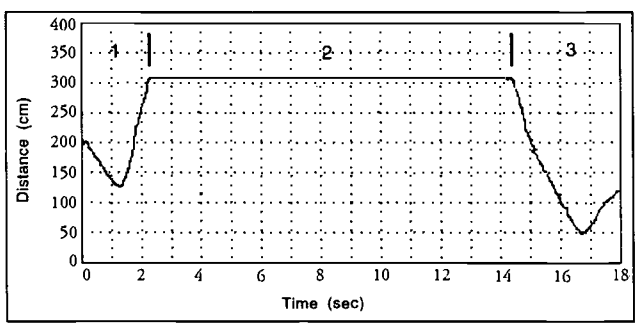

Figure 7. Eleanor's graph reflects what happened when she walked far away as she created a new graph.

of the tower seeing the button, an image that Eleanor would use later.

Eleanor continued to investigate what happens when she walked far away as she created a new graph.

<6> *Eleanor: (...) Let's see. What happens when you're really far away? (...) [begins Figure 7.]*

<7> *Eleanor: [She walks back as far as she can in the room. The line flattens in Segment 2 before she gets all the way back.] I think I'm going to leave it like this for a minute. [She then reaches further back, holding the button as far away from the tower as possible.] Hmmm, I wonder if I hold it up high? [She reaches high up in the air while standing way back, then she lowers the button to shoulder height and, producing Segment 3, walks in then slightly back.]*

The flatness of Segment 2 was the key aspect that she experimented with in Figure 7. As Segment 2 appeared on the screen Eleanor tried out different actions to break its uniform horizontality (staying still, reaching as far back as possible, and reaching high up). The computer failure to respond to her actions moved her to conclude initially that various movements did not matter.

<7> *Eleanor: No. I think that the... this is the... maybe this [points to the horizontal line, Segment 2] is the highest it can go. I don't know. But, um, so if you get closer to it [the tower], it [the graph] gets... it goes down low [points to the low part of Segment 1]. And if you get far away it goes, um, well, when you're walking farther away it goes up high [traces the upward piece in Segment 1]. Then it... then it goes to about here [Segment 2]. And then, but I don't think it really makes any difference if you go like that [bending knees and moving button by reaching down low and moving it up high].*

<8> *Tracey: It [up and down movement] doesn't make a difference?*

<9> *Eleanor: Well, maybe it does. I don't know. I'll try it. [Eleanor begins another graph, she holds the button down on the floor and notices that the table might block the tower from seeing the button]. Maybe it [the tower] can't see [the button]. Wait, can I hold this tower like [on the floor], or should I just leave it there [on the table]? Because it [the tower] might not be able to see it [the button] right? If I put it [the button] down here [under the table] (...)*

Her subsequent use of the motion detector was mostly within the space visible to the tower, preserving the button bearing so that it could be sensed by the tower.

In this episode Eleanor incorporated idiosyncratic aspects of the tool to her performance. Eleanor learned to recognize and enact peculiar conditions (not too far from the tower, the button always visible to the tower, etc.) within which the meaning of graphs is to be discussed and envisioned. It is from this context that we talk about Eleanor's envisioning the tool as a "point of view." Eleanor learned to recognize aspects of her body motion that were visible to the motion detector and conditions to be preserved so that the tool remains sensitive to them.

The Emergence of Logical Necessity

We have reviewed how Eleanor came to perceive the graphical height on the computer screen as expressing the movements toward and away from the tower. To Eleanor this was a significant aspect of her tool use. It allowed Eleanor to imagine sequences of actions to achieve a certain graph; she could run thought experiments to anticipate graphical productions. When an actual graph did not seem to validate their expectations, it was important for Eleanor and Tracey to recognize whether it was an issue of performance and adjustment — the management of the idiosyncratic — or a misunderstanding of what the graph means as a reflection of actions, that is, of how they grasped the significant. In this section we will examine how Eleanor conceived graphs that are possible, impossible, and difficult. This episode took place after 8 minutes of her experimentation with the motion detector. Eleanor wondered aloud about the possibility of creating a vertical line. Throughout this episode "it" refers to the graph line.

<10> *Eleanor: I wonder if you could get it to go straight up? Not like diagonal? Probably you couldn't because if it would go straight up it would have to just be the same time, because it's moving along, no matter what you do.*

<11> *Tracey: Moving along in time?*

<12> *Eleanor: Yeah, so you'd have to kind of stop the time and go like that [gesturing a vertical line on the computer screen]. And go like this [moving as if to rush away from the tower, still holding the button]. Because, because it's moving along this way [to the right on the screen] the same time it's going that way [straight up] (...)*

Eleanor immediately expressed the impossibility of the vertical line ("Probably you couldn't"), based on the notion that the graph moves to the right "no matter what you do." Her utterance ("no matter what you do") indicates that the displacement to the right is something that the tool does on its own, regardless of her actions. Eleanor recognized this feature as a significant aspect of the tool: it is not a matter of limitation, performance, or inaccuracy, but a crucial element in imagining the creation of graphs.

Eleanor inferred that the only way to stop the line in its inexorable shift to the right side is "to kind of stop the time." The movement of the graph from left to right is as "unstoppable" as the progress of time. Eleanor began to develop a perspective from which she could emulate the tool's view of motion, space, and time. The next exchange expresses Eleanor's growing sense that the vertical line is not just an isolated case, but one extreme along a continuum of possibilities.

<13> *Tracey: Do you think you could make a steeper line than this? Maybe you can't make it go straight up, but maybe you can make it a little bit.*

<14> *Eleanor: Maybe, maybe if you do it faster.*

<15> *Tracey: OK, shall we try that?*

<16> *Eleanor: [She starts a new graph by running a short distance toward the tower and back and then stands still moving her arm quickly forward and back.] (...) That's almost straight up.*

<17> *Tracey: Yeah, it is almost straight.*

<18> *Eleanor: So you can make it kinda go like this [holds finger slightly slanted to the left] even though you can't make it go like straight up. You can get pretty close if you do it faster. And when you do it slower you can get like [gestures a line close to horizontal] slower.*

Eleanor expressed her sense of what "should" necessarily happen. Eleanor judged that one could be more or less close to the vertical line by going faster or slower. She incorporated into her tool perspective a way of showing speed: by closeness to the vertical. By doing so she enriched her appreciation of the significant; it made more likely that if she were to see a graph that becomes more horizontal when she moved more quickly, she might assume that the tool did not work properly since a sense of how it should work has become compelling to her.

Dina

Introduction

Dina's approach to using the motion detector was different from Eleanor's. She started assuming that graphical height corresponded to the speed of her motion ("low is slow"). She consistently interpreted the graphs as indicating the speed of her motions. Gradually, Dina began to incorporate elements of distance in her graphical interpretations, leading her to construct a new meaning for the graphical height. Dina was especially interested in the figurative and aesthetic analysis of the graphs. Such an interest was reflected in her many spontaneous comments on finished graphs and plans for the production of new ones. This section includes the episodes that took place in the second session, as she started to use the motion detector, according to the time scale shown in Figure 8.

Responsiveness of the Tool

Tracey introduced the motion detector.

<19> *Tracey: Here is a button, and you can move the button and make. The computer will make lines on the graph. It's [the tower's] looking for the button now. Its eye is looking for it. And as you move the button [a graph starts to appear on the computer screen, Tracey moves the button generating a small wavelike graph, Segment 1, Figure 9] something's happened. Here [handing button to Dina], why don't you try?*

<20> *Dina: I've never heard of something like that. [She takes the button and completes the graph moving her hand slowly forward, backward, and forward again, Segment 2.] (...)*

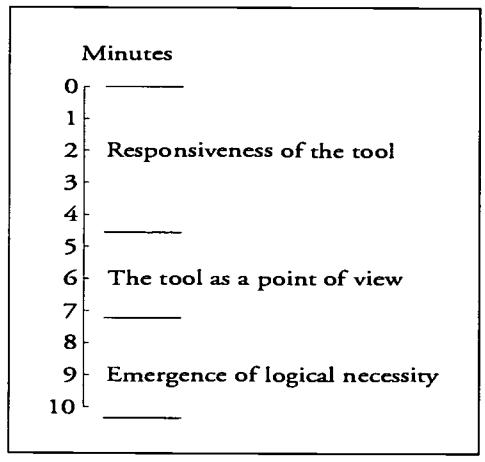

Figure 8. Time scale of Dina's initial 10-minute use of the motion detector.

<21> *Dina: That [Figure 9] would kind of be at a low speed.*

<22> *Tracey: How did you know that?*

<23> *Dina: Because I think that low is slow.*

<24> *Tracey: That's like on ours [she referred to the system that they had previously invented to represent motion on paper for the electric train, in which higher meant faster], that's what we did.*

Probably expressing her previous experience creating ways to represent motion, Dina accounted for the graphical response to her actions by assuming that graphical height expressed speed ("low is slow"); accordingly, Dina incorporated the numbers along the vertical axis as referring to speed.

<25> *Dina: (...) [pointing to the 50 mark on the vertical axis] And here it also shows, it says 50 down below.*

<26> *Tracey: So you think if you moved it really fast then...*

<27> *Dina: It would be at about 350 [on the vertical scale].*

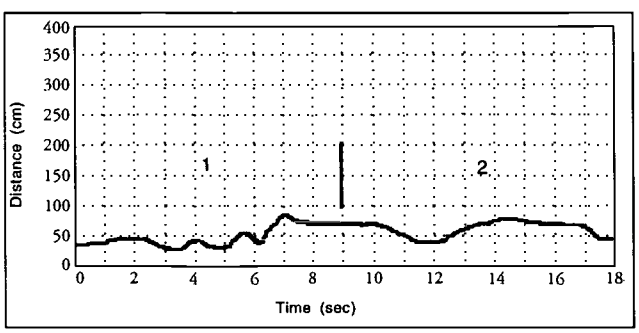

Figure 9. Dina's initial graph at "low speed."

<28> *Tracey: OK, why don't you try that [to reach the 350 mark]. See what happens.*

<29> *Dina: [Producing Figure 10, she moves her hand away from the tower, Segment 1]. Well, you have to get far [she walks slowly back, Segment 2].*

Despite Dina's speed-based interpretation of the graph, in her subsequent attempt to actually reach a high mark on the graphical scale Dina encountered the need to walk further back. In producing Figure 10, Segment 1, Dina moved her arm quickly but the graph did not seem to go high enough; this failure prompted her to "get far." This is the first example of a kinesthetic differentiation that she would enact several times: on the one hand, her overall distance from the tower; on the other hand, the back and forth movement of her arms; both get expressed in a graphical up-and-down curve. As soon as she finished Figure 10, Dina started to comment on it, appraising the graph's appeal as a visual object.

<30> *Dina: That one [Figure 10] looks like an interesting one. It's straight, but I don't know*

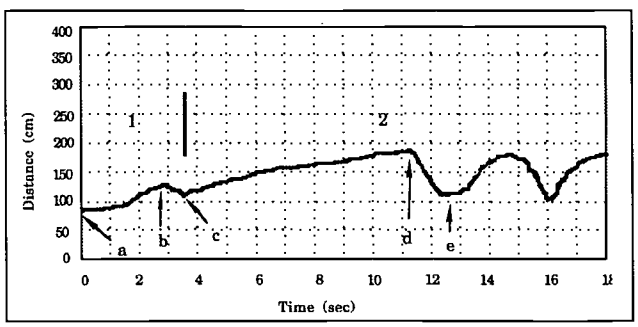

Figure 10. Dina's graph reflects her attempts to reach a higher mark on the vertical scale by using a different speed.

why it just gets my attention. (...) when I think about it, I would think the other one [Figure 9] is more attracting because of all the bumps, but. When I look at it [Figure 10], I don't know, I think of this one [Figure 10] more attracting [than Figure 9].

It is likely that Dina's preference for Figure 10 over Figure 9 was based on her experience in producing them. When Dina created Figure 10 she had more control, sensing the power to generate significant graphical responses. Tracey shifted the conversation toward the kinesthetic meaning of Figure 10. Dina made specific remarks that expressed her notion that the graphical height showed the speed of her motion.

<31> *Tracey: Hmmm. You were thinking about speed, like if you moved it [the button] fast what might happen.*

<32> *Dina: Yeah, I was wondering how the speed would change.*

<33> *Tracey: So you were moving slowly some times and then fast some times. [Dina: Yeah.] Do you remember which part of the graph you were moving slowly and which part you were moving fast?*

<34> *Dina: Well, it was a very short time but here I was a little slow [point c, Figure 10] in here [Point e, Figure 10] and here [Point a, Figure 10]. And the fast times were all the tips. [Tracey: The fast time were all the tips.] I don't think this one really [Point b], but it's faster than the other parts [around Point b]. I think the fastest part is right at this tip right there [Point d]. So.*

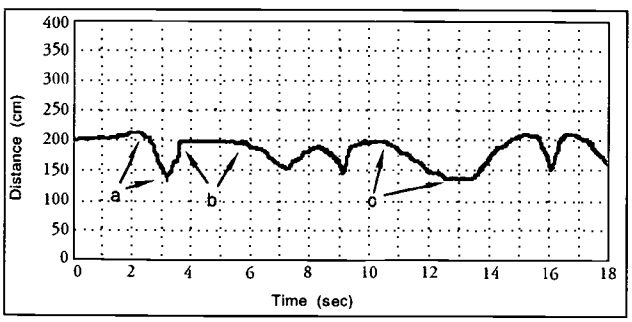

Figure 11. Dina's graph reflects her experimentation with swiftness of arm movements.

Instead of reviewing her actual actions Dina interpreted that the low and high points of the graph must have been where she went slow and fast respectively. Dina's analysis of maxima and minima included details: maximum speeds lasted for "a very short time," Point b is just a local maximum, and Point d is a global maximum. Dina's interpretation of the graph as an expression of her motion's speed led her to reconstruct how fast she must have moved at different times during her past creation of the graph. Nevertheless, the following exchange included again an element of distance to the tower in Dina's sense of the graphical responsiveness to her body motion.

<35> *Tracey: Ah-ha. So, what could you do to make it even faster?*

<36> *Dina: Get further. (...) [Dina creates Figure 11.]*

To create Figure 11 Dina stood at a "far" distance (around 2 meters), and moved her arm back and forth. Sometimes, as in Segment a, her arm movement was swift and some other times, as in Segment c, it was gentle and gradual. These differences were

clearly intentional — Dina was experimenting with the swiftness of arm movements. Observing Figure 11, Dina noticed how the graph responded to the abruptness of her arm motion by the sharpness of the curve, which she interpreted as expressing different ways of slowing down, that is, different ways of getting low on the graph.

<37> *Dina: On this one [Figure 11, Segment a] I kind of tried to think on how fast it would take to get slower. That's why I kind of let this one get slower slow [Segment c], and this one [Segment a] kind of fast. [Tracey: Yeah.]*

<38> *Dina: They look like birds, kind of.*

<39> *Tracey: They do. And this wing [Segment c] gets slower slowly, and this one [Segment a] gets slower fast. And how about here [Segment b]?*

<40> *Dina: Well, there it kind of stays at the same speed. So once it gets here [beginning of Segment b], it keeps the same speed until there [end of Segment b].*

Dina began making sense of the graphical responsiveness to her actions by interpreting the graphical height as an expression of speed. This view may have grown out of her previous experience in inventing representations on paper with Tracey, in which Dina used vertical height to indicate speed. She repeatedly reconstructed how she had moved the button to be consistent with this interpretive framework. Gradually, Dina began to include elements of distance to the tower as a significant aspect; her failure to produce a "high" graph with just fast arm motion, for example, led her to introduce a distinctive kinesthetic action, that is, getting farther from the tower, from which she remained bodily still except for her arm motion. Her interpretation of Figure 11, consistent with speed being shown in vertical height, accounted for the quickness of her arm motion by noticing how fast or slow one "gets slow" (moves down on the graph); but note that, in doing so, she did not compare the bird's wings merely as a difference of height — which is the same for both wings — but as a comparison between their slopes.

The Tool as a Point of View

Eleanor had a need to know what the boundaries of the tool were before she was ready to work within them. It seemed important for her to know from the beginning what space in the room she could work within or what motions she could do that would elicit a visual response from the graph. Dina, on the other hand, worked more often within the limits of the tool and did not move out of range or bump against other constraints. She was more tacit in her discovery of the motion detector's constraints.

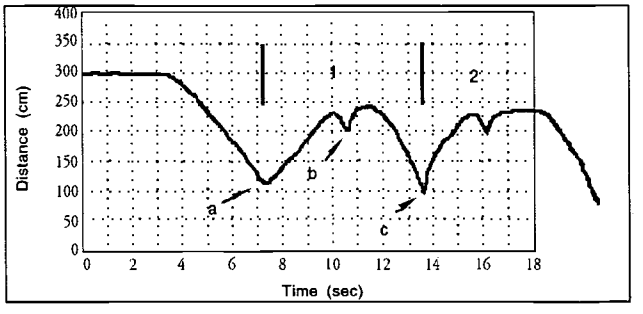

Figure 12. Dina's graph shows her work within the limits of the tool, not a discovery of its range or other constraints.

As Dina was getting ready to produce Figure 12, she expressed a blending of being far and fast as well as a kinesthetic "mixing" of walking and moving her arm.

<41> *Dina: (...) OK. I'm just going to get far. I really want to get it fast. [She walks back to start far from the tower and produces Figure 12.]*

<42> *Dina: That's a little different. Well, different. These two [Segments 1 and 2] are the same, but anyways, it's different, all over.*

<43> *Tracey: Which part was different? You mean the whole thing was?*

<44> *Dina: Well, there's, these two parts [Segments 1 and 2] are the same except this one [Segment 2] doesn't have a stick going down [at the end]. It just ends there. They cut it.*

The motion detector had been set up to register 18 seconds of movement. After 18 seconds it stopped displaying more measurements. Dina never got to finish the second symmetric bird, and, in fact, stopped her movement when she saw that her motion was no longer being graphed. Note the mysterious "they" in "They cut it." Dina was aware that it was not her movement that had caused the motion detector to stop; it was "they" (whoever is behind the doings of the computer) who created the constraint.

In analyzing the picture, however, it was as if she were seeing the whole shape of the bird. Dina completed the graph by imagining the missing piece. The time interval of the computer screen was an idiosyncratic trait for Dina in the sense that she saw it as a matter of performance. Had she started the bird earlier she could have fit them within the screen. That the computer "cut" her graph did not cause her to question her approach to interpreting graphs; she could easily imagine an elongated screen. Emphasizing the idiosyncrasy of the screen limit, Tracey repeated Dina's assertion ("Yeah, they cut it"), and immediately reframed their conversation with other aspects of the graph that were more notable to Tracey, such as the sharpness of Point c.

<45> *Tracey: Yeah, they cut it. I was noticing too that you've got a really sharp point down here [Point c].*

<46> *Dina: I didn't really want to stay down at the bottom. I don't know, because I get bored watching it go slow.*

<47> *Tracey: (...) Do you remember what you did with your hands here [Point c]?*

<48> *Dina: I just moved it up really fast, but close. Here [point b], I also moved it fast, but not that close.*

<49> *Tracey: Close to what?*

<50> *Dina: My little tower.*

<51> *Tracey: Oh, I see. Yeah, you have a point here too [Point b]. But this one [Point a] looks more rounded.*

<52> *Dina: I wasn't. There I wasn't trying to go really fast. I don't know why.*

Sharpness of the vertices was for Dina a visual attribute that expressed her quick motion which, in turn, reflected her "feeling bored." The aesthetic

analysis pervaded Dina's analysis of the graphs, including aspects like feeling bored or interested when watching a graph. Dina introduced for the first time an explicit account of graphical height showing closeness to the tower. She began to articulate a new point of view from which closeness to the tower as well as fastness were, for the motion detector, noticeable aspects of her body motion. Emphasizing the idiosyncrasy of the temporal limits of the graph, Dina commented on the first 3 seconds of Figure 12 that look horizontal.

<53> *Dina: Here [initial horizontal piece of Figure 12] I didn't mean to make it go straight. It's that I didn't know it [the computer] was on already. Because when I get, I think the closer I am [to the tower], the longer it [the computer] takes [to start] or something. Because it [the computer] took longer [to start] last time [in producing Figure 11].*

Dina, as well as Eleanor, developed a sense for the tool's point of view, that is, for aspects of the body motion that can be sensed by the motion detector (e.g., distance to the tower, quickness of hand motion), and for particular conditions that one has to enact to preserve the sensitivity of the tool (e.g., move within the time frame demarcated by the horizontal completion of the graph on the computer screen). Critical to her insights were (1) her interpretations of the differences between the actual graph displayed on the computer screen and the graph that would correspond to her purposeful actions (e.g., the complete birds in Figure 12), and (2) Tracey's utterances suggesting central and accessory aspects of the situation through the conversation, such as when she confirmed to Dina "Yeah, they cut it" and then she made an observation ("I was noticing too that you've got a really sharp point down here") that moved the conversation onto the kinesthetic analysis of the graph.

The Emergence of Logical Necessity

After Dina's comment about her uncertainty as to when the computer starts, Tracey decided to press the start key when Dina was ready to begin a new graph. Dina created the graph in Figure 13.

<54> *Dina: (...) Well, here [pointing somewhere near the last part of Figure 13] it kind of came out messy, but I was trying to make an N. It didn't fit. I wonder if you could draw something trying that.*

<55> *Tracey: That would be interesting. What would you like to try to draw?*

<56> *Dina: Something easy. OK, maybe a box. That's easy. Pretty easy. Can you get this to go back? Like once it's over here [in the right*

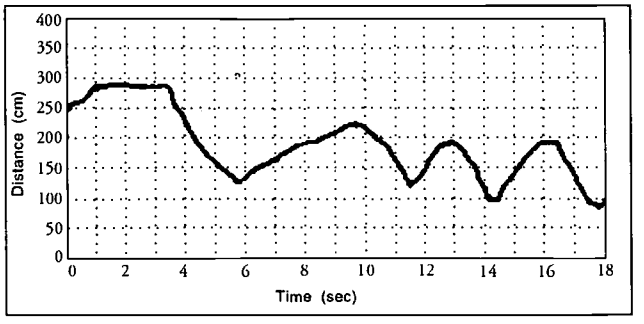

Figure 13. Dina's graph reflects her discovery that something may be easy to draw but impossible to graph.

hand part of the screen] to go back [motioning with her hand toward the left on the screen]? I don't think you can, but...

<57> *Tracey: Yeah, not the way it's set up right now.*

In this episode Dina discovered that something may be easy to draw and impossible to graph. Note how a characteristic that was present in the graphical productions from the very beginning, the movement to the right, only now became an issue for Dina. As soon as she intended to draw a square the directionality of the graph presented itself as something to cope with. Expressing the significance of this new awareness, Dina started to think about what letters are possible and impossible to write using the motion detector.

<58> *Dina: Should I try another one?*

<59> *Tracey: What other drawing could you make?*

<60> *Dina: Hmmm. Could I try to write something? I don't know what, but... "the"? [Tracey: OK.] Or "at," as long as it's easy. Except how to make an "a" I don't know how, because it doesn't finish the circle [small "a"]. It would only be a bump.*

<61> *Tracey: Yeah. Huh. So the "N" works [Dina: Yeah.], but the "a" doesn't.*

<62> *Dina: A "T" I don't think would work either. It [capital T] would be like an upside down L. It wouldn't have the stick in the middle of the straight one at the top. So it can't be a "T" or an "a."*

Dina's insight about the constraint of the graph going always to the right was part of her construction of the significant because it constituted a general principle underlying the relationship between the computer graphs and her actions. No matter what one does, the computer graph keeps moving in the same direction. On the one hand, Dina saw this behavior as an intrinsic attribute of the motion detector, part of what the computer does by itself; on the other hand, she took from it a strong sense of how she should read a graph (from left to right without changing direction) and imagine the sequence of actions that would produce it (left to right is first to last). By creating a new taxonomy of possible and impossible drawings, Dina used the directional principle to examine under a fresh light a broader domain of hand drawn shapes; in other words, her realization went beyond the mechanics of the tool enabling her to a adopt an alternative view — the view of the tool — toward familiar shapes. Note that in this exchange Dina analyzed possible, impossible, and difficult graphs without actually trying to produce them, but just envisioning their creation. It is in this sense that we talk about an evolving tool perspective which incorporates whatever is significant to the tool user against a background of idiosyncrasies to be complied with in the actual performance with the motion detector. Her tool perspective, incorporating the logic "principle" of always moving to the right enriches her imagining of what is likely, impossible, or necessary.

Our analysis of Dina's views on the logical necessity making some letters possible and others

impossible, strives to identify how it was lived by her. Expert graphers may recognize, for the same taxonomy, elements of cultural value that are not necessarily shared by Dina. For example, the impossibility for a temporal graph to change direction may seem to a knowledgeable adult, such as Tracey, a natural consequence of the irreversibility of time. But later in this episode Tracey had the following exchange with Dina.

<63> *Tracey: And can you make it [the line graph] come back [to the left]?*

<64> *Dina: I don't think you can.*

<65> *Tracey: How come?*

<66> *Dina: I have no idea. I would think just because the computer isn't made to do it.*

That Dina used the graph's rightward movement repeatedly to imagine possible, difficult, and impossible graphs in the form of letters while she attributed this trait to a designer's choice suggests that a competent use of the motion detector does not always imply sharing the cultural background of the choices made for its design.

Discussion

Responsiveness of the Tool

We see Eleanor's and Dina's initial encounter with the motion detector as an exploration of the responsiveness of the tool to their own actions. For example, during Eleanor's production of Figure 6, she asked herself: What if I just leave it [the button] here? Will it [the graph] keep on going? She posed her question as she stayed still, then she tried to account for the tool's response to her stillness. Noticing that the graph kept "on going," she remarked "Yeah": the tool had affirmatively replied to her.

Eleanor's and Dina's initial "problematic" (Confrey, 1991) was to find ways of body motion to provoke salient graphical responsiveness. For instance, during the creation of Figure 4, Eleanor walked backward as she commented that the "the line goes up"; at a certain point, however, the graph suddenly flattened. At that instant she had not performed any distinctive motion; that is, she just kept walking backward. She then perceived the graph's flattening as something that the tool did by itself: "Maybe this is the farthest it can move." The local circumstances surrounding the moment in which the graph became horizontal (such as that she did not "do" something distinctive at the time, that it happened as she was walking farther and farther from the tower, or that the graph seemed to go "with her" up to that point) were crucial aspects of how Eleanor made sense of the event. Then, to break the tool's silence she raised up her arm and said, "What if I move it up higher?" Intending to recover the lost responsiveness, Eleanor changed her body motion from walking backward to vertical arm displacement.

Grappling with the responsiveness of the tool involved, most of all, dealing with the concord and discord between their intended graphs and the resulting graphs. For instance, Dina reckoned that the flat first 2 seconds of Figure 12 were not inten-

ded: "I didn't mean to make it go straight. It's that I didn't know it [the computer] was on already." One way in which they dealt with disagreements between intentions and responses was by overlooking or denying them. Dina, for instance, reviewed her Figure 10 by composing body actions consistent with her "low is slow" (see <23>) interpretation even though, from our observer's point of view, this is not what she had done (see <33>). At other times Eleanor and Dina accounted for a divergence between intentions and responses by noticing what qualities of their body motion really count. For example, as part of her creation of Figures 4, 6, and 7, Eleanor attempted to get the graph line up higher by moving the button up and down, but in all cases her vertical movement failed to elicit specific responses on the computer screen. Later she accounted for the tool's lack of responsiveness to the button's vertical displacement by asserting that this is not a kind of body movement that one is supposed to play with (see the end of <7>). Pursuing a different avenue, Dina began to use the motion detector to highlight speed as the body motion quality that counts. However, after failing to reach the upper part of the computer screen, Dina began to include her distance to the tower in the foreground of her activity: "Well [as she produces Figure 10], you have to get far."

The Tool as a Point of View

The metaphor of the tool as a point of view invokes the familiar experience of trying to imagine an object from a different viewer's position (not the one from which we are already seeing the object). Envisioning an object from an alternative point of view involves recognizing what aspects of the object will be significant, salient, or notorious from the new angle, as well as to assume a background of idiosyncratic conditions to preserve the object visually available (e.g., the object must be appropriately illuminated, it should not be covered). Similarly, Eleanor and Dina began to recognize what qualities of one's body motions must be highlighted and played with in the creation of graphical expressions because they are visible to the tool (e.g., moving one's arm toward and away from the tower), and what aspects have to be part of a performance style (e.g., holding the button in a certain way, keeping oneself at a reasonable distance from the tower).

The process of sorting out what is idiosyncratic (what one needs to comply with for the use of the tool), and significant (what kind of actions one can enact to control graphical responses) embraced not only Eleanor's and Dina's experimentation with the tool but also their interaction with Tracey. For example, turning the button away from the tower might be used as a practical method to create horizontal segments in a graph, hence it could be a significant aspect in the creation of graphs. The interaction with Tracey, however, induced Eleanor to perceive the button's orientation as a peculiar feature to comply with. Tracey advised Eleanor to account for the flattening of the graph at the end of Figure 6 by an aspect that Eleanor had been unaware of: "I just noticed one problem, and I think it has to do with the way you're holding it [the button]." Tracey's contribution was not only

to bring up the button's orientation to Eleanor's attention, but also to treat it as a matter of good performance: "If you hold it [the button] like this [toward the tower], it [the tower] can see better than if you turn it [the button] that way [away from the tower], it [the tower] can't see what you're doing." By talking about the tower having to "see" the button, Eleanor understood that she should always keep a relative orientation of the button, a feature that she would comply with after a bit of practice ("You're turning it away again," Tracey said after Eleanor had accidentally turned the button away from the tower). Developing and adopting the tool's point of view seem to entail an anthropomorphization of the tool, such as the talk about the tower seeing the button or the sense that the tool is partially responsive to one's actions, as other human beings are.

The Emergence of Logical Necessity

As Eleanor and Dina each enriched their conception of the tool's point of view, they articulated a sense of logical necessity which allowed them to discern between possible and impossible graphs. For Eleanor, the inexorable graphical displacement to the right made vertical lines impossible unless you "kind of stop time." For Dina, it implied that some letters are possible and others impossible to graph. Letters are familiar objects to Dina; however, in her interest to draw them, she needed to look at them from a new perspective; she developed a taxonomy in which "T" and "a" are impossible, whereas "L" or "n" are possible (see <62>). By grappling with how to use the tool to draw a box or letters, Dina encountered logical constraints that made apparent, to her, differences between drawing and graphing. It is this perception of how the tool "should" work that first suggested to us the notion of "adopting a tool perspective." Eleanor and Dina strived to emulate not only the tool's responsiveness to body motion but also an inherent logic that they began to envision — a logic that enabled them to imagine what is possible and impossible to graph. Their emerging sense of logical necessity expressed simultaneously notions about the behavior of the graph (e.g., that it always goes to the right) and about the nature of body motion (e.g., that one cannot get farther away from the tower in no time).

The three ideas that we have discussed — responsiveness of the tool, the tool as a point of view, and the emergence of logical necessity — were all important aspects of Eleanor's and Dina's development of a tool perspective. Our intent is not to talk about tool perspectives as mental objects, but as interpretive frameworks that one can use to reflect on graphs and actions. The experience with the motion detector can help someone to imagine how a graph of distance vs. time corresponding to a car on a highway would look, regardless that, say, the car moves much beyond the range of distances that the tower is sensitive to. Tool perspectives emerge from one's experiences with the tool — experiences that include conversations with others, successes and failures in achieving intended results, and styles of performance.

Part II: Fusion

Graphic displays thus not only provide physicists with a cognitive domain to inhabit and wander in, they also transport physical phenomena into the perceptual presence of physicists and serve as a locus in which physicist and physical phenomenon can be brought into symbolic contact with one another. Cognitive and gestural orientation to a graphical representation, therefore, make possible for physicists to symbolically participate in the physical events represented by the graphic space.

— Ochs, Gonzales, & Jacoby (in press)

Introduction

A common way of talking about the understanding of symbols suggests that knowing the meaning of a symbolic expression is being able to recognize its correspondence to whatever it may refer to; for example, knowing the meaning of the word "table" implies the ability to point to the objects that table refers to. This image of correspondences between symbols and referents insinuates that understanding a graph of position vs. time would be grounded in the ability to establish links between points on the graph and positions of the moving object at a given time or between the slope of the graph at a certain time and the velocity of the object. Although Eleanor and Dina learned to establish some of these correspondences (e.g., "the closer to the tower it gets, the lower, I think" said Eleanor as she created her first graph), we see this emergent ability as part of a broader process that we call "fusion." Fusion is merging qualities of symbols with qualities of the signified events or situations, that is, talking, gesturing, and envisioning in ways that do not distinguish between symbols and referents. Let us review some examples from Part I.

From <45>:

> *Tracey: I was noticing too that you've got a really sharp point down here [Point c, Figure 12)].*
>
> *Dina: I didn't really want to stay down at the bottom.*

Who is "I" in Dina's answer? Dina knew that she, in the sense of her body in the room, had never been "down at the bottom," and yet her utterance makes sense to Tracey and to herself. The "I" that Dina invoked simultaneously moved the button back and forth and the graph up and down the computer screen and did not want to stay down at the bottom.

From <47>:

> *Tracey: Do you remember what you did with your hands here [after Point c, Figure 12]?*
>
> *Dina: [referring to the upward piece] I just moved it up really fast, but close.*

Tracey asked Dina about her hand motion. Dina's response describes a movement up and fast. What was the "it" that moved up and fast? Her description fuses a quality of the graph (upness) with qualities of her hand motion (fastness and closeness). Dina's statement is ambiguous, but her ambiguity is not a matter of vagueness or uncertainty; rather, it is an expression of connectedness and understanding.

From <37>:

> *Dina: On this one [Figure 11, Segment a] I kind of tried to think on how fast it would take to get slower. That's why I kind of let this one get slower slow [Segment c], and this one [Segment a] kind of fast. They look like birds, kind of.*
>
> *Tracey: They do. And this wing [traces Segment c] gets slower slowly, and this one [traces Segment a] gets slower fast. And how about here [traces Segment b]?*

The conversation between Dina and Tracey reflects a shared growth and enrichment of the fusion experience. Dina's remark about her intentions on the graph resembling a bird prompted Tracey to describe the wings as getting slower at different rates. Invoking Dina's interpretation (at that time Dina thought of the graphical height as showing speed), Tracey gestured the downness of the Segments 'c' and 'a' as displaying their "getting slower." Note the complexity of Tracey's pointing act; in a single utterance she integrated the bird, the directionality of the graph, and a kinesthetic pattern. Look at her sentence: "This wing gets slower fast"; and try this exercise: replace "wing" by "graph" or "motion," and "slower" by "down" or "inclined," and "fast" by "steeply." Every combination of these substitutions still makes sense to the situation. The issue is that Tracey was pointing at all of them and that Dina not only made sense of Tracey's utterance, but she also generated, as in former examples, her own utterances involving similar degrees of complexity and multiple dimensions.

Fusion involves the construction of a discourse in which indexical terms, such as "I," "it," or "here," are constantly revised dissolving boundaries among all the aspects that are relevant to the graphers' experience. The episodes with Eleanor and Dina exemplify that this discourse incorporates not only body movements and graphical shapes but also intentions ("I didn't really want to stay down at the bottom"), evoked objects ("They look like birds, kind of"), and past events ("Do you remember what you did with your hands here?").

The idea of fusion is, we believe, more appropriate to the former examples than the metaphor of transparency. Learning graphing was not, for Eleanor and Dina, a matter of the graphs becoming "transparent," because the visual attributes of the graphs remained present and salient to them (e.g., "they look like birds," "I didn't really want to stay down at the bottom"). Transparency implies that the graphs disappear from sight enabling the grapher to grasp what is "behind" them. Fusion, instead, suggests that the qualities of the graph *merge* with the qualities of the represented events in ways that they cease to be distinct.

In previous presentations in which we included the notion of fusion, we have noticed two reactions that we want to dispute here. The first one is that fusion expresses students' inability to separate symbols and referents; to put it more bluntly, that students' fusion indicates their conceptual confusion. We believe that this view is simply false. It is not true that Eleanor or Dina could not distinguish between the shape of the graph on the computer screen and their actual body motions across the room. When-

ever the ambiguity of their words was an obstacle they could spell it out. Fusion, in our view, does not constitute a misunderstanding but an achievement. By coming to see body motions, past actions, and evoked objects in their graphs, Eleanor and Dina crafted a rich background of intuitions and insights nurturing their interpretations. This conclusion does not imply, obviously, that whatever form of fusion they attained was error-free and correct. For example, initially Dina mistakenly saw graphical height as an expression of speed. This surmise did offer her an interpretive framework; it was in the context of recognizing variations of speed in graphical shapes that Dina began to sense the need to account for the distance to the tower in her graphical productions ("Well, you have to get far," Dina said in the middle of creating Figure 10; see <29>).

The second reaction we have seen is that what we see as fusion is "just a shortcut" with little relevance. For instance, instead of "I didn't really want to stay down at the bottom," Dina could have said "I didn't really want to stay close to the tower because the graph would have stayed down at the bottom of the computer screen." From this point of view the former utterance was preferred because it is shorter and, presumably, easier to produce. However, this "shortcut" theory overlooks what we think is the most crucial quality of the fusing experiences: playfulness. Playfulness is at the root of symbol use (Piaget, 1962; Winnicott, 1982). Imagine that one sees a child who is playing and using a stick as a horse; the child then lets the stick fall on the floor while saying "my horse is tired." Would one say that the child's utterance is merely a shortcut because he could have stated "I let this stick fall down because it stands for a horse which is tired"? The child is aware that the stick is not a real horse and yet acting, talking, and gesturing, as if the stick were truly a horse, is at the essence of his play, of his ability to "get lost" in the make-believe situation. Moreover, unless the inherent ambiguity in the play encumbers communication or one wants the child to do something else, it is important to avoid disrupting the illusion; a question such as "How come you believe that this stick is a horse?" is likely to interrupt his play.

Fusion is not an exceptional or anomalous phenomenon. On the contrary, it is ordinary and pervasive. It may take place when someone explains to a friend how to get somewhere on a map, in reading a poem where the sound of the words is a crucial aspect to what they come to mean, in a religious ritual, or in a conversation about the characters of a cartoon. But the experiences of fusion can be radically diverse. The type of fusion that emerges in the interpretation of a graph is not necessarily similar to the ones experienced in discussing directions on a map, reading a poem, or imitating a cartoon character.

The goal of Part II is not just to show that fusion took place in the episodes with Eleanor and Dina, but, more important, to gain insights on the specific forms of fusion that they experienced which suggest important features inherent in the learning and use of graphing. Through the analysis of the ensuing episodes we will characterize three traits which are specific to the form of fusion developed by Eleanor and Dina and, we conjecture, to graphing:

1. *The interplay between the graph as a shape and the graph as a response to actions.*
2. *The interplay between graphing and drawing.*
3. *Imaginary traveling along trajectories on the graphical plane.*

The study of physicists' use of graphs by Ochs et al., (1994, 1996), and which we consider to be seminal, suggests that these characteristics of the form of fusion enacted by Eleanor's and Dina's graphing are ingrained in the practices of all novice or experienced graphers:

"Through verbal and gestural (re)enactments of constructed physical processes, physicist and physical entity are conjoined in simultaneous, multiple constructed worlds: the here and now of the interaction, the visual representation, and the represented physical processes. These indeterminate grammatical constructions along with gestural journeys through visual displays constitute physicist and physical entity as coexperiencers of dynamic processes" (1994).

Eleanor: "I'm going to try to make a pattern."

This section describes four minutes of Eleanor's experimentation with the motion detector, following the time scale shown in Figure 14.

As soon as Eleanor feels that she has a sense of how the motion detector responds to her actions, she expresses her desire to use the tool for a purpose.

<67> *Eleanor: OK, I'm going to try and make a pattern. [She alternately walks forward with her hand toward the sensor and then walks back away from the sensor. She doesn't back up as far each time. As she moves back and forth, Eleanor watches Figure 15 forming.] Actually [right after producing seven peaks] this is not exactly the same pattern. [As she produces each of the last three peaks, Eleanor goes a shorter and shorter distance away from the sensor.]*

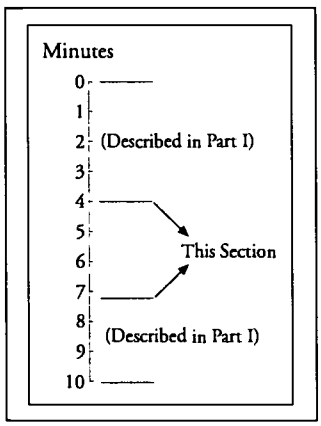

Figure 14. Time scale of Eleanor's efforts to make a pattern with the motion detector.

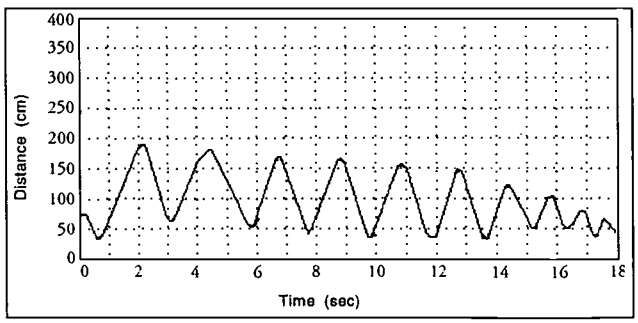

Figure 15. Eleanor's graph reflects her attempt to make a pattern.

During the production of Figure 15 Eleanor intended to create a regular wavelike pattern. As the graph came into view she noticed that it showed a shape different from the one she intended ("Actually, this is not exactly the same pattern"). This type of comparison between the actual graph and the pattern that she intended to create is a prevalent element that Eleanor, as well as all the other children we have worked with, brought to their graphing experience. These are moments in which the graph is seen as a shape which is assessed on its closeness to a "target" shape (e.g., the regular wave in her previous graph). We call this mode of graph interpretation "graph as a shape"; we now trace how the graph as a shape interrelates with another focus of graphical interpretation: the graph as a response to actions.

<68> *Tracey: Wow. Oh, I like that one. Wait, don't clear it yet. Let's stop it and look at it for a minute. [Eleanor traces it with her finger as if redrawing the graph.] What was happening?*

<69> *Eleanor: Well, I was going far. I was going, like, far, and a little bit closer but still far away then. I was really going like this [Eleanor moves her arm in a back and forth motion] but kind of changing a little [not as far away each time].*

Tracey's question ("What was happening?") shifted the conversation to the graph as a response to actions; that is, it prompted Eleanor to reflect on the body movements with which she had created Figure 15 and how they related to the graphical response on the computer screen.

<70> *Tracey: So the line up was when you were walking [how]...? [Tracey points to one of the upward slanting lines.]*

<71> *Eleanor: When I was walking backward [gestures back wall], and the line forward [she gestures a line with downward slope] was that way [looking toward the tower].*

This exchange elicited a kind of talk and gesturing that makes explicit the phenomenon of fusion. Eleanor said "and the line forward was that way [she gestures a line with downward slope]." "The line forward" is the downward line on the computer screen as well as her walking forward. Tracey and Eleanor talk and gesture an entity that is at once on the computer screen and in her walking across the room; an entity that is simultaneously present and past: the line is presently there as Eleanor points at it, but it also "*was* that way." Eleanor could have said, adopting Tracey's language, "the line down was when I was walking forward," which is, seemingly, a "clearer" expression. But we think that Eleanor's spontaneous use of "the line forward" expresses a common and significant aspect in graphing. The graph, the symbolized events, and the multiple graphers' interpretations are all experienced within a "graphical space," in which they coalesce. Words, gestures, and images evoke at once qualities of shapes, body motion, tool responses, the past, and the future. The emerging space that Eleanor and Tracey experience, the one that they talk and gesture about, is common (borrowing from Latour's common place) in several senses: it is shared by both of them, it is populated by events of which they are mutually

aware (like Eleanor's previous walk toward the tower), and it blurs boundaries between the graph as a shape and the graph as a response to actions.

<72> *Tracey: The whole thing has sort of a shape too, doesn't it?*

<73> *Eleanor: Yeah, it's all, like zigzags through the side [she turns her head sideways and does a zigzag motion with her finger], but I mean they're all. They look like, kind of like mountains or something.*

<74> *Tracey: They do.*

By alluding to the overall shape of "the whole thing," Tracey made salient Figure 15 as a shape. Note how Eleanor incorporated a figural resemblance in her analysis of the shapelike qualities of the graph: "They look like, kind of like *mountains* or something." In the section on Dina we will see more examples of the use of figural resemblance as part of the graph as a shape perspective.

<75> *Eleanor: At first I was going to have it stay on this line [tracing her finger on a horizontal line from the top of the highest zigzags], but they got. They kept on getting smaller, so.*

<76> *Tracey: Why did they get smaller?*

<77> *Eleanor: Because I. I didn't walk as far.*

Tracey's question turned again the conversation toward the graph as a response to actions. Tracey's "why" prompted Eleanor to analyze causes, that is, what in her actions had caused the shrinking of the wave.

Eleanor figured out a way to avoid the lowering of the peaks by using pads of note paper to mark places on the floor.

<78> *Eleanor: Maybe I can mark where I walked to. And then like. Maybe I can like just put this like right here. [She places a little note pad on the floor around 2 meters from the tower.] OK. Can I make a new one [a new graph on the computer]? (...) Maybe if I, like, start here. [She goes near the sensor.] No, if I start here [she walks back and stands on the pad] it will be like up [the beginning of the graph]. [Eleanor creates Figure 16 by walking six times at a steady pace up to the sensor and back, stepping on the pad each time. The seventh time she steps back beyond the pad.] (...)*

Eleanor decided to mark on the floor the point at which she would change direction. This action suggests how Eleanor conceived another way in which height on the graphical space shows location in the room. She did not look at the number scale

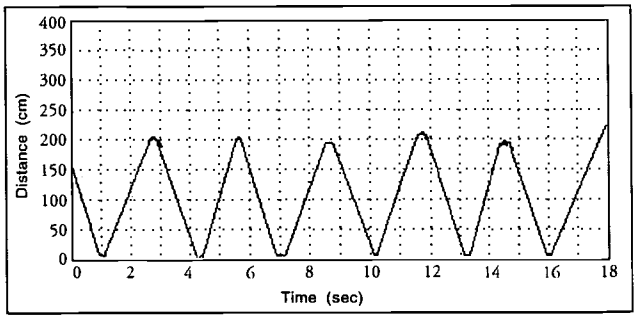

Figure 16. Eleanor's graph confirms her idea to make a more uniform pattern by marking on the floor the distance she needed to move.

for a certain distance; instead, she came to expect from her previous experience with the tool what location of the mark on the floor accords with a middle height on the screen. Eleanor's approach to how specific places on the floor correspond to specific heights on the computer screen was not a unit-based measurement, but an experiential sense of "being here" in which "here" meant simultaneously a place on the floor and on the computer screen.

<79> *Eleanor: [looking at Figure 16] That one [the last peak] went up a little too high, but... [Touching each of the peaks on the graph]. That one [this graph] was kind of more the same [each peak is almost the same as each other] but a little bit different [from each other]. Because I didn't always, also I didn't always go in, in the same place [pointing to bottoms of each peak which are slightly different], but it goes, tsh, tsh, tsh, tsh [touching the tops of the peaks].*

Eleanor's comments on Figure 16 combined aspects of the graph as a shape (e.g., "That one [this graph] was kind of more the same") and of the graph as a response to actions (e.g., "also I didn't always go in, in the same place").

Throughout Eleanor's explorations both perspectives were intertwined and under mutual influence; in this episode Eleanor first wanted to make a regular wave, she then envisioned a way of walking that would engender such a shape and created Figure 15; noticing that the pattern was not regular she thought of ways of adjusting her body actions to achieve the missing regularity in the shape, an issue that she assessed after producing Figure 16.

Dina: "What if we just doodled?"

This section describes the activities with the motion detector that directly follow the episodes included in Part I. Immediately after pondering which letters are possible and impossible to create on the computer screen (described at the end of Part I), Dina tried twice to draw an "N" on the computer screen.

<80> *Tracey: Well, I could just turn it on and you could experiment with the drawing idea.*

<81> *Dina: [as she creates a graph not shown here] OK. Oopsies. Well, there starts an N, I think. It's kind of big, but...just gonna make waves, because I kind of messed up on it. Hmm, I don't know.*

<82> *Tracey: So that was a drawing of waves.*

<83> *Dina: I don't really like it.*

<84> *Tracey: But you don't like it because you were trying to do something different.*

<85> *Dina: I wanted to make an N, but it came out kind of flat there. It had to be taller.*

<86> *Tracey: So do you want to try a taller N this time?*

<87> *Dina: Yeah, this time I want to start from the beginning. [While she creates another graph not shown here] that's an OK "N." That's an "M." Then here goes an "N." When I did the "M" [she says looking at the completed graph] I was trying to do an "N," but it kind of got mixed up. Well, it didn't get mixed up I just kind of did it.*

<88> *Tracey: Yeah, you changed your plan, and you turned your "N" into an "M." (...)*

Dina started with a plan, but then she altered the plan according to her shifting interpretation of the graph appearing on the screen. Once the graph was completed Dina reinterpreted the result and how it diverged from her original intentions. This dynamic was a constant in Dina's approach.

Dina then suggested interpreting graphs as they are produced instead of planning them in advance.

<89> *Dina: What if we just doodled and then thought of something it looked like?*

<90> *Tracey: Yeah, that's another way to do it.*

<91> *Dina: That's the way, sometimes when I'm bored I do that with the clouds.*

<92> *Tracey: Oh, I know what you mean.*

Intending to "doodle," Dina created Figure 17.

<93> *Dina: [after finishing Figure 17] When I started to make these two bumps [marking Figure 17 between seconds 6 and 10] I started to make a beak, kind of like a weirdo animal or something. I don't know. But I didn't...*

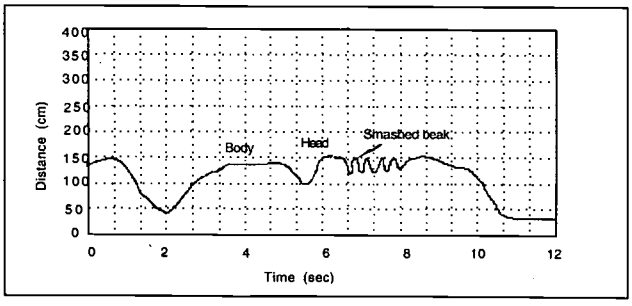

Figure 17. Dina's graph reflects her doodling as a way to interpret graphs as a shape while they are being produced.

<94> *Tracey: So it looks like a beak that got...*

<95> *Dina: Smashed. [Tracey: Yeah.] It doesn't really look like one.*

Dina focused her analysis on the graph as a shape. She intended to draw a beak, but then it became something different, momentarily unintended, for which she tried to fit a new figural resemblance (like a beak *that got smashed*). Dina's prevalent approach to planning and interpreting graphs was based on her use of the drawing experience.

At this point, Tracey and Dina started to graph simultaneously with two buttons, one creating a blue graph and the other creating a yellow graph. Dina expressed a keen interest.

<96> *Dina: [the graph starts] Oh, it's started. Am I blue or yellow? Yellow. That looks like an interesting doodle. [Looking at the screen as if admiring a work of art]. It looks nice with the colors mixed.*

<97> *Tracey: It does.*

<98> *Dina: It looks like water with the blue. [Tracey: Yeah, I was trying to follow you.] Well, I'm not saying I want to or anything, but maybe if you had all four [buttons (there were four buttons on the table)], you could use one at a time on the same graph and you could make a nice picture with all the colors. That would be fun.*

<99> *Tracey: That would be beautiful. (...) What could we do with the two [buttons]?*

Dina assessed the aesthetic value of her graphical productions. Part of the drawing experience for her means to produce "nice" or "beautiful" images. It was natural to Dina to comment on how colorful the four-button graph would be, because this is the kind of judgment that we practice in appreciating a drawing.

<100> Dina: Do you know anything with two lines? Electricity poles. We can make straight down for the poles, and then go back up and make it straight and then another pole and so on. But I don't know. I'm not sure. [Tracey: Oh, OK. How would we start?] (...) Once it's about at the second square [on the screen] we can start the stick. We could go a little higher and then go straight down, and then go up. No, because that would be bumped and then straight. I'm not sure how that would make a stick.

<101> Tracey: It's hard to make a stick.

<102> Dina: And then go back up. Unless we traced the stick but I don't know how. That's not a too good idea.

Dina anticipated difficulties in graphing a vertical stick because the unavoidable displacement to the right would make it "bumped." This is another example — like the square that was so simple to draw and impossible to graph from Part I — of Dina's emerging sense of the difference between graphing and drawing; in this case it is the difficulty of graphing a vertical line by "going down" and then retracing it on its "way up." The impossibility of being twice on the same line led Dina to suggest in the next excerpt that, perhaps, one could start out from the bottom and make the stick just by going "straight up."

<103> Tracey: We could just do the wires somehow. What do the wires look like?

<104> Dina: They're just straight lines. [Tracey: Just straight lines.] We could make like...if we started out from the bottom and then went straight up, then made the wires. I don't know. I'm not sure.

Note how Dina used the verb "to go" to indicate motions in the graphical space; "go a little higher," "go straight down," "go up," "went straight up" meant in all cases a movement of a line within the graph. Such a "going" did not describe the motion of the hand in space but the genesis of the graph as a gradual movement on the computer screen. Tracey then turned the conversation toward the graph as a response to actions, motivating a different type of description.

<105> Tracey: Oh, so if we start at the bottom then how would we move to get it to go straight up?

<106> Dina: Get far, real fast.

<107> Tracey: OK, and then...to do the wires?

<108> Dina: You just keep it straight.

In the former exchange the motion on the graphical space and the body motion get intermingled. "Start at the bottom" indicates location and movement on the graph, and "how would we move?" is a question about their body motion. But "we," that is, Tracey and Dina, are at once in both realms: "*we* start at the bottom..." and "how would *we*

move..." Dina, responding to the body motion "side" of the question, says "Get far, real fast." Regarding the production of the wires Dina says "just keep it straight." Note that "just keep it straight" alludes to both, the action with the button, that is, the stillness of the hand holding the button and the straightness (horizontality) of the line going steadily to the right.

<109> Tracey: And do you want the wires? How do you want...?

<110> Dina: (...) usually they're [a pair of wires] kind of separate, so...One [of us] could be closer [to the tower] than the other.

<111> Tracey: Are you going to be the closer one or the further one? [Dina: I don't care.] OK, we'll just try it. (...)

For Dina the graph as a shape is a way of talking about it, a way of sharing plans and pointing at the different parts of the graph. Her image of the two parallel wires gets transformed in a plan: to do the wires they should do similar body motions but one closer to the tower than the other. Using the imagery of drawings enriched Dina's resourcefulness.

As they created Figure 18, Dina, at the 10th second, said:

<112> Dina: That looks...an elephant. (...)

After Figure 18 was complete, Dina made the following point:

<113> Dina: When we were still about here [she marks around the 10th second], I was thinking we could have made an elephant if we went straight down. So...an elephant drinking water?

"When we were still about here," Dina said. Where is "about here"? When in their past had they been "there"? Dina's utterance expresses a discourse in which "being somewhere," or "going from here or there," means at once body motion and displacement on the graph. Note that as she says "When we were still about here" Dina knows perfectly well that they have never been physically on the computer screen, and yet there is a web of relationships that are manifest *because* she points at their location on the screen; in other words, if she had said "about here," pointing at a place on the floor, nothing would have suggested to them why her idea of the elephant came up to her imagination on that particular spot; what counted was that Dina's "about here" on the screen made obvious because of the pictorial image that their whole body motion up to that point could be thought of as a half-elephant.

Dina's experience of fusion not only merged body actions and shapes on the computer screen but also past and present. Dina was talking about a past event: "When we were..." The genesis of the graph

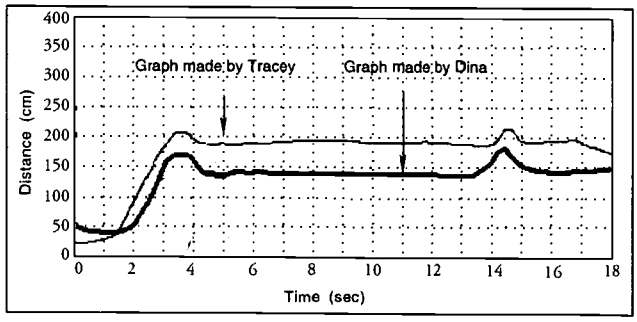

Figure 18. Dina's graph of an "elephant" reflects her resourcefulness in planning and discussing a graph.

is a record that endures the passage of time. The graphical plane has not only regions that can be distinguished by their being above or below but also by showing events that happen before or after. Dina pointed at Figure 18 telling Tracey something that she had noticed after creating the half-elephant and before spoiling it. The common space that Tracey and Dina experienced was populated by waves, smashed beaks, electricity poles, and elephants; it was structured by a field of locations which were at once on the floor and on the computer screen, in which the past and the future could be pointed at.

Tracey and Dina planned the next graph to produce an elephant drinking water.

<114> Tracey: Some water from our last picture, of the waves. [Dina: Yeah.] So you want us to go straight down there [at 10 seconds, Figure 18]?

<115> Dina: Well, that's an idea, but then it would have no front legs. And if it had front legs it would have no back. So it's kind of hard to make. Unless one [of us] did the legs while the other did the back or something. I don't know.

<116> Tracey: (...) So if you did the legs and I did the back, then what would I do?

<117> Dina: Well, you would have to go around up to here [gesturing the middle of the computer screen] and then go down and straight. I think.

<118> Tracey: Oh, OK, I see. OK. I'll try that.

<119> Dina: I'm not sure how it's going to come out, but it's [it will be] something. (...)

As Tracey and Dina were getting ready to start Figure 19, Dina asked the following question, as if in a rush:

<120> Dina: To get higher is to get further, right? [Tracey: Right.]

Dina responded to Tracey's question on how to do the elephant's back by describing a trip or trajectory on the graph: "you would have to go around up to here and then go down and straight." Dina felt that that was informative enough, that Tracey would know how to move her body appropriately to, for example, "go down and straight." Dina's trajectory on the graph tacitly implied body motions (e.g., walk far back, stay there for a few seconds). Therefore, even though the trajectory is described as if only on the graphical plane, it actually takes place at once in the room and on the graph, that is, in the graphical space.

Dina's question ("to get higher is to get further, right?"), urged by her immediate need to figure out what to do, shows that the phenomenon of fusion — the experience of a common space in which the graphical plane, the symbolized events, and the

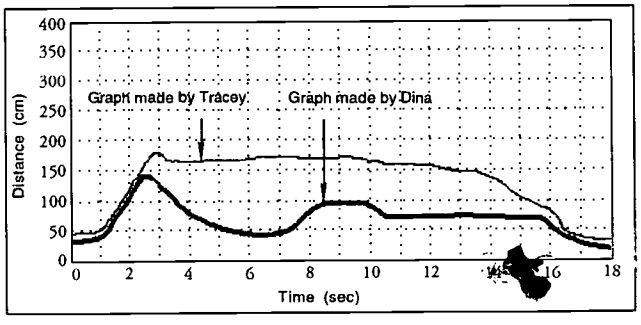

Figure 19. Dina's graph reflects the phenomenon of fusion — the experience of a common space in which the graphical plane, the symbolized events, and the graphers' interpretations coalesce.

graphers' interpretations coalesce — is not a "confusion" or a disordered "mixing up": the experience of fusion did not preclude Dina or Tracey from distinguishing between actions and symbols whenever they needed to make explicit such a distinction; Tracey understood Dina's question: "To get higher [on the graph] is to get further [with my body], right?" [Tracey: Right.]

<121> Dina: [at 4 seconds in the production of Figure 19] OK, I've got to go down and make a leg. Oh geez, that's not a leg. Oh, well, I don't know what I made. [After finishing Figure 19] now it looks like a bird! if you kind of cut the long tail off from there [around the 10th second], or something.

Note in Figure 20 the possible bird that Dina saw in Figure 19.

Discussion

We have examined the former episodes paying attention to all the aspects of Eleanor's and Dina's graphing that seemed potentially useful to characterize the specific forms of fusion that they experienced. This discussion will center on three traits that are woven throughout their explorations: the interplay between the graph as a shape and the graph as a response to actions, the interplay between graphing and drawing, and traveling along trajectories.

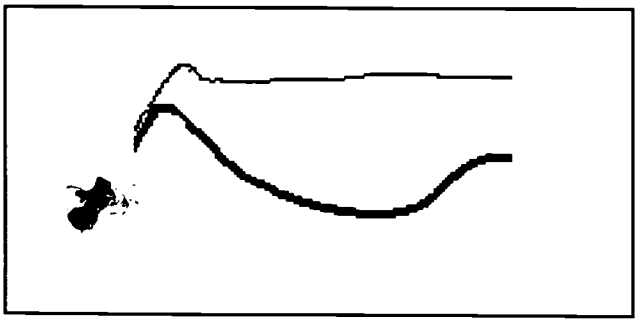

Figure 20. The segment of Dina's graph that she described as a bird.

The interplay between the graph as a shape and the graph as a response to actions

One of the main qualities of the form of fusion used by Eleanor and Dina was merging aspects of the graph as a shape and of the graph as a response to actions. The following examples illustrate these shifts and how Eleanor and Dina were always able to separate them whenever the interaction required it. Then we exemplify an utterance that fuses both types of attributes.

During their first use of the motion detector described in Part I, Eleanor and Dina gained an initial sense of what type of body motions the tool was sensitive to, and a view on what aspects of its responsiveness seemed significant to play and plan with. Their emergent perception that they could control the tool moved Eleanor and Dina to pursue new goals in their use of the motion detector.

Eleanor: OK, I'm going to try and make a pattern. (<67>)

Dina: I wonder if you could draw something? (<54>)

They articulated their new aims by imagining shapes to be displayed on the computer screen (e.g., a regular wave for Eleanor, a box for Dina). This

shift in purposes made salient the analysis of the graph as a shape.

> *Eleanor: It's all like zigzags...they look kind of mountains. (<73>)*

> *Dina: Well, here it kind of came out messy, but I was trying to make an "N." (<54>)*

However, in planning how to create their intended shapes or in assessing their actual graphical production, they often looked at the graph as a response to actions.

> *Eleanor: Because I didn't always...go in the same place. (<79>)*

> *Dina: Get far, real fast. (<106>)*

These two perspectives for the planning and interpretation of graphs — graph as a shape and as a response to actions — are not independent; on the contrary, they influence and permeate each other. The shifts back and forth between the two perspectives were often embedded in the language of intentions and causes, as in the following vignette (starting at <75>):

> *Eleanor: At first I was going [my intention was...] to have it stay on this line [tracing her finger on a horizontal line from the top of the highest zigzags], but they got. They kept on getting smaller, so.*

> *Tracey: Why did they get smaller? [What caused that?]*

> *Eleanor: Because I. I didn't walk as far.*

First Eleanor expressed how the resulting graph as a shape differed from the regular pattern that she intended ("They kept on getting smaller"). Tracey's question ("Why?") led Eleanor to explain it by resorting to the graph as a response to actions ("I didn't walk as far").

Consequently, a conspicuous trait of Eleanor's and Dina's fusion experience throughout their conversation with Tracey was the merging of qualities of the graph as a shape and of the graph as a response to actions. One example is Eleanor's utterance, "...and the line forward was that way" (<71>), by which she blended a quality of her body motion (forward) while gesturally emphasized a quality of the graphical shape (downward). What Eleanor was pointing at was not on the computer screen or in the room but in both at once; it was not in her present or her past but simultaneously in both. For these reasons we talk about an emerging graphical space shared by Tracey and Eleanor or Dina in which the absent is made present and in which pointing acts integrate shapes on the graphical plane, body motions performed in the room, and intentions held by the graphers.

The interplay between graphing and drawing

A resource that Eleanor and Dina brought to their graphing activities was their drawing experience. Interpreting the graph as a shape is, in a way, looking at the graph as if it were a drawing.

> *Tracey: The whole thing has a shape too, doesn't it? Eleanor: Yeah, it's all like zigzags. (<72>)*

Tracey: So it looks like a beak that got...
Dina: smashed. (<94>)

Their drawing experience was a resource for Eleanor and Dina because it was a source of ideas (Dina: "Do you know anything with two lines? Electricity poles!" <100>), ways of talking (Eleanor: "That one [the last peak] went up a little too high." <79>), and styles of appreciation (Dina: "You could make a nice picture with all the colors." <98>). They expressed the relevance of drawing by invoking imaginary objects whose silhouettes resembled graphical shapes. This aspect was particularly important to Dina: "When we were still about here, I was thinking we could have made an elephant if we went straight down" (<113>). In Dina's conversation with Tracey the elephant became a useful means for the mutual recognition of the different parts of the graph (Dina: "But then it would have no *front legs*. And if it had front legs it would have no *back*."), for planning their actions (Tracey: "so if you did the legs and I did the back, then *what would I do*?"), and for assessing results (Dina: "Oh geez, *that's not a leg*.").

The drawing experience was also important because it enabled Eleanor and Dina to notice ways in which graphing is *different* from drawing, such as when Dina concluded that making a vertical stick is a problem: "I'm not sure how that would make a stick. [Tracey: It's hard to make a stick] And then go back up" (<101>), or when Eleanor realized that a mark on the floor would help her to level off all the peaks. As Dina and Eleanor realized that something may be easy to draw but impossible or difficult to graph, they broadened and refined their drawing-based expectations. A domain in which they expressed this enrichment was in gesturing. Drawing nurtures a type of symbiosis between shapes and gestures. The act of drawing a certain shape is a way of gesturing it and is part of what the gesture comes to mean in a situation (e.g., a smooth wavelike shape and a gradual hand motion that traces it may evoke each other and elicit common connotations, such as a calm and slow rhythm). However, in graphing with the motion detector one's gestures are not always analogous to the developing graph. Eleanor and Dina incorporated the drawing-based continuity between shapes and gestures into their language, such as when Eleanor said "the line forward was that way" while tracing the downward segment on the computer screen, but they also extended a new dimension of gesturing based on the tools' graphical responses to body motion. For example, Eleanor explained to Tracey her production of Figure 15 by this gesture: "I was really going like this" [Eleanor moves her right arm in a back and forth motion], or Dina, looking at the pattern of crossing waves on the computer screen gestured: "I kind of...just thought of skiing" [Dina gestures her arms swinging back and forth].

Traveling along trajectories

Often ideas articulated by Eleanor and Dina included descriptions of trajectories on the graphical plane, using verbs such as "to go" and "to stay" that suggest forms of traveling across it.

Eleanor: I wonder if you could get it to go straight up*? (<10>)*

Eleanor: At first I was going to have it stay on this line *[tracing her finger on a horizontal line from left to right]. (<75>)*

Dina: We could go a little higher *and then* go straight down, *and then* go up. *(<100>)*

Dina: You would have to go around up to here *and then* go down and straight. *(<117>)*

The graphical plane serves as a pointable record on which trips can be planned or reviewed. How the graph is generated over time defines the temporal sequence of the trip: the trajectories always unfold from left to right and they never "go back." Even though the completed graph shows a trajectory that is all present at once, Eleanor and Dina learned to project on it a sense of time rooted in their experience of the genesis of the graph across time. The envisioning of trips across the graphical space seems to be a crucial aspect of the experience of fusion; they enable the graphers to create narratives (Nemirovsky, 1996) that express simultaneously the shape of the trajectory and the events that, graphers imagine, would take place along the way.

We would like to end this discussion describing a short "last minute" activity that took place at the end of the session with Dina because it introduces the theme of the ensuing Part III. Tracey asked Dina to create a graph using two buttons simultaneously and consequently generating two line graphs at the same time. Dina moved her arms back and forth generating crossing waves of small, then larger, and then smaller amplitude. She made the following comment:

Dina: I thought of getting bigger, bigger and then smaller and smaller. (...) I kind of... just thought of skiing [Dina gestures her arms swinging back and forth] or something.

Holding one button in each hand prompted Dina to experiment with a new kinesthetic behavior: swinging her arms back and forth, keeping them in opposite phase (one arm is behind her when the other one is in front, and vice versa). In this mode of body motion what was salient to her — the feature to play with — was the amplitude of the wavelike graph. We have seen many examples of Dina's multiple figurative interpretations of the graph ("Now it looks like a bird, if you kind of cut the long tail off from there"), but in the excerpt above we also find an example of an interpretation based on a kinesthetic pattern: "I kind of...just thought of skiing [Dina gestures her arms motion]." Skiing was not an image shown by the graph, but by the regular body movement with which Dina had created the graph. Kinesthetic patterns are significant elements of the graphical space which became particularly noticeable for us when the use of the tool changed in conspicuous ways, such as from holding one to holding two buttons at once. We will return to this theme in Part III.

Part III: Graphical Spaces

The graph on the screen is not merely something to be looked at, but instead an open gateway to a world where the human body can move and act within new frameworks of meaning. Like a playing field that builds a landscape within which certain moves, such as a "goal," become both possible and visible, the graph on the computer screen creates an arena for the perception and constitution of relevant action.

— C. Goodwin (1995, p. 257)

Introduction

As we have seen in Parts I and II, Eleanor and Dina learned to create graphs by getting closer to or farther from the tower with the handheld button. The episodes that we will analyze in this Part III were at first puzzling to us. A change in the way of using the motion detector prompted Eleanor and Dina to explore the tool anew, apparently shifting them back to the approaches with which they had first investigated the tool, as described in Part I. The change for Eleanor was to use two buttons instead of one, so that she was able to generate two graphs at once, a yellow one and a blue one, corresponding to the respective colors of the buttons. For Dina, instead, the new feature was to attach the button to an electric train so that the graph would show the motion of the train. They treated the motion detector in the new contexts as if it were a different tool from the one they had used in Parts I and II. Eleanor started to investigate "what matters"; that is, what kind of motion elicits graphical responsiveness, whereas Dina went back to her original idea (see Part I) that the graph showed the speed of motion, as in "low is slow" (see <23>). We shall see that as they progressed in their analyses, Eleanor and Dina went through moments of "remembering" which brought back their experience with the motion detector in the original context of one handheld button.

Why did Eleanor and Dina go through such a process? Why didn't they directly see that using one or two buttons or that attaching the button to the train are variations which preserve what the graph is responding to, namely, the distance to the tower? Some readers may be tempted to explain Eleanor's and Dina's temporary "regression" by invoking matters of ignorance, such as "they had not *really* understood the motion detector and the graph of distance vs. time"; or "they had grasped only the *superficial* or situation-specific behavior of the computer graph." But these kinds of explanations are futile: What do we gain by saying that someone did not do x because she did not know x? What counts — what may help *us* to learn something new — is to develop a deeper understanding of the positive (present or existing) qualities of their new experience that made sensible Eleanor's and Dina's actions. Our research has something to contribute only to the extent that it nurtures in the reader a richer view of how the situation looked from the shoes of Eleanor or Dina.

Our analysis will make extensive use of a centuries-old distinction, that is, between what has often been called "subjective space and time" and "objective space and time." Everyone understands that the same minute, as measured by a clock, can be felt as an eternity by someone and as a fleeting instant by someone else. Subjective space and time are sometimes called intuitive, experi-

ential, or lived, whereas objective space and time are called with terms like metric, Newtonian, or absolute. We refer to them using the words "lived-in" and "metric" respectively. Ultimately, metric space is the notion of an infinite, regular, and quantifiable space. The lived-in space, instead, is forged on an ongoing basis, through our actions, expectations, and life history.

An example of the common experience with lived-in spaces will help us point out some of their attributes. From time to time it happens to many of us that changes are made in a very familiar place; for instance, removing a prominent table from a room that we frequently use in our house. Then, when we enter the room we immediately notice that it is not the same room we are accustomed to. Sometimes one does not easily recognize what has specifically been changed; but even if one identifies the missing table, the room as a whole looks different, possibly bigger and emptier. Other pieces of furniture present a new appearance, as if they changed their size, location, or appeal. The room may also seem more or less inviting as we perceive a demand to modify the ways of using and being in the room. As time goes by, as we start to live in this altered room, the sense of the customary begins to develop. The room becomes just what it is, not bigger or smaller, emptier or fuller, but a "normal" one. This gradual transformation of the room becoming ordinary is not a change in the metric properties of the physical configuration of the room, but in our experience of the room, in how we walk in it, in what activities we develop there, and in our daily reencounter with what used to be and has remained in the room.

From this example we want to highlight three properties of lived-in spaces, namely, that they are relational, intentional, and creative. By relational we mean that changes, even if they are physically circumscribed to a particular aspect, affect the lived-in space as a whole. The absence of the table is not a mere clearing of a portion of the space, but it makes the entire room different. This is evident from the fact that it is often difficult to guess what has specifically been changed.[4] Lived-in spaces are intentional in the sense that they are places to do things and to accomplish purposes. A room without a table may not be a good place to have dinner anymore, although it might have become very appropriate for doing a somersault; the kinds of activities that we perform in the room are an essential aspect of what it becomes as a lived-in space. Finally, by saying that lived-in spaces are creative we emphasize that they are not set and fixed but always subject to and constituted by the ongoing drift of the life experience. Even if we do not use a room for a long period, we continue to envision it as a lived-in space, so that if, much later, we go back to the unchanged room we may find ourselves surprised by how small or empty it looks.

We argue, in our analysis of the subsequent episodes with Eleanor and Dina, that for them the graphical space was a lived-in space and that Eleanor experienced the addition of a second button and

[4] This phenomenon of seeing changes holistically happens all the time. One of us was recently surprised to meet a colleague who looked so different because he had changed "something" in his face. Asked to guess what had happened, the hesitant response was that he had grown a mustache. It turns out that he had shaved his beard.

Dina the attachment of the button to the train, in ways roughly analogous to the former example with the missing table. These changes involved a modification in the type of actions that they performed with the motion detector as well as in the expected responsiveness of the tool. It was not immediately obvious to them what had changed and what had not changed. As they explored the new setups, they reencountered previous experiences with the single handheld button — events that they both described with the words "I remember."

Central questions opened by this analysis are about the role of metric spaces and the relationship between lived-in and metric spaces. Some may argue that the graph of distance vs. time represents the distance between the tower and the button *as a metric space*, and that only when this is conceptualized is one truly understanding the graph. So, what is the nature of the relationship between lived-in and metric spaces? Do they constitute a fundamental dichotomy? Are the metric spaces the ones for mathematics and the lived-in ones for poetry? At this point the reader may have little difficulty guessing that our answer is no; that far from a dichotomy, metric spaces become meaningful by participating in the more general experience of lived-in spaces. To clarify this idea the following example may be helpful.

Every person who comes to the United States from a country in which the temperature is normally given in Celsius degrees encounters the problem of making sense of the Fahrenheit scale. At first he or she "translates" it into the Celsius scale (some rules of thumb are widely used for this purpose, such as subtract 32 and divide by 2 to approximate the solution). Being taught the definition of the Fahrenheit scale is not necessarily helpful, even if one is well aware of its history and its two critical points. Still the information that the temperature will be, say, "between 65 and 70" is meaningless by how the weather would feel, what to wear, and so on.[5] Equally useless is being told that a person has a fever of "101 degrees" to know the seriousness of the health problem. By translating these numbers into the Celsius scale a whole background of expectations comes to the fore. However, as time passes and this person continues to live in an environment in which Fahrenheit degrees are customarily used, the Fahrenheit scale starts to have a meaning by its own and gradually the need to translate vanishes. We say that the person's experience with the weather is gradually *indexed* by Fahrenheit degrees.

Through the former example we want to suggest that metric spaces are not free-floating ideas held together exclusively by formal definitions. Indexing our experience with a metric space is a thoroughly situated process. H. Sacks (1995, p. 435) developed an analysis of this situatedness for the case of speed. He analyzed the expression "John likes to drive fast" and pointed out that this expression is not made more accurate by specifying the number range for

[5] Examples abound. People who are concerned about dieting often know a lot about calories intake, amount of calories for different types of food, and so forth, without necessarily knowing how a calorie is defined or what a calorie is a unit of. This does not imply that definitions are irrelevant, of course, but that they are only a piece of a much broader context that makes metric scales meaningful.

the speed at which John likes to drive. What is being said is that John drives fast in relation to the traffic situation, the shape of the road, the kind of car he drives and so forth; in some circumstances of heavy traffic, driving at 30 miles an hour is dangerously fast and in other circumstances, such as an empty highway, it may be banned for being too slow.[6]

Note that the view that metric spaces become meaningful by indexing lived-in spaces is different from the traditional notion that metric spaces are *abstractions*, in the sense of expressing the isolation of a specific aspect or property from a complex object. A classical example is the analysis of a statement such as "this table is 2 meters long" as indicating that, among all the infinite potential qualities of the table, the speaker has chosen to highlight its extension leaving out color, weight, volume, etc. By seeing the use of metric spaces as indexing, rather than abstracting, we want to emphasize the boundless and situated intuitions, expectations, and purposes that the subject brings to the meaningful practice with metric spaces. Saying that the statement "today is 65 degrees Fahrenheit" abstracts a particular property of the weather does not help in understanding why the newcomer to the United States finds it useless, even though he knows that it defines the temperature.[7]

Our analysis of the next episodes with Eleanor and Dina highlights how they began to develop a sense of metric space by indexing their experience with the motion detector. This process started at the very beginning of their use of the motion detector (described in Part I). Eleanor quickly noticed relations like "the closer I go, the lower it goes" developing a sense that her closeness to the tower is indexed by the height of the line on the graph. Later she used this qualitative relationship to mark exact positions on the floor and to help herself remain within a certain range on the graph. Dina, on the other hand, started expecting that what was being indexed by the graphical height was her speed ("low is slow" see <23>). In this context she used the numbers on the vertical scale (starting in <25>):

> *Dina: And here it also shows, it says 50 down below.*

[6] On August 22, 1995, the *Boston Globe* reported on speed laws. The following excerpt refers to opinions held by drivers from the state of Montana (p. 8): "...local drivers pose a variation of an oft-asked hypothetical question: If a car speeds across a state and no one is around to see it—not any police officers, not even any other drivers — is there really such a thing as traveling too fast?"

[7] On October 1, 1995, the *New York Times* reported on new steps taken in England to impose the use of the metric system. The following excerpt illustrates the difference between a metric space as an abstraction or as an index (p. 6): "But shopkeepers themselves are distraught. 'I can feel this cloth, I know it's 13 ounces,' said John Davis, a partner in Tobias Tailors on Savile Row, rubbing a Prince of Wales check from a patterns book between his fingers. 'How can I do this in grams? I can't (...).' Greta Hutsheson, owner of Lucy's delicatessen in Mayfair, said 'I felt resentment at the change.' She found it hard to visualize servings of pasta, beans, salads, and fish. 'I can do the conversion in my mind but I don't get a mental picture of how big it is,' she said."

Tracey: So you think if you moved it really fast then...

Dina: It would be at about 350 [on the vertical scale].

In Part III we trace how the shifts in Eleanor's and Dina's ways of using the motion detector led them to grapple with new lived-in spaces, that is, new kinds of actions and graphical responses, and how, as part of this development, they broadened their perception of what is being indexed by the height on a distance vs. time graph.

Eleanor

Introduction

The episode presented in this section spans from minute 14 to 21.

Between minutes 8 and 10, Eleanor explained that vertical lines are impossible, because time goes on "no matter what" (see <10>). She then produced a graph showing that one can get closer to the vertical if one goes fast. Her graph included very steep segments corresponding to moments in which she was walking fast toward the tower.

Let us briefly summarize the activities that took place between minutes 10 and 14. Eleanor raised a question for herself: "But you probably couldn't get it as fast going far back" implying, perhaps, that it is difficult to run backward fast. She then tried it out. As she ran backward, she accidentally turned the button away from the tower, producing an "anomaly" in the form of a sharp peak. This irregularity prompted her to explore again the close and far extremes for the sensitivity of the motion detector and the meaning of "pointiness" on the graph. Eleanor found that a way to create pointy graphs is by staying in one place and moving the arm back and forth very quickly. Eleanor says that the point is where you change directions and creates a few pointy graphs. She then played with the combination of walking and arm swinging to create the graph shown in Figure 21.

At the beginning of Segment B of Figure 21 she made the following comments:

<122> Eleanor: It went a little bit higher. Yeah [she looks at the complete graph], so it can kind of get to go in a straight line. [Tracey: Mmm-hmm.] Like, well, here [Segment A] I was trying to get it to go in between these squares like there. [that is, keeping the zigzags in between the two height lines on the grid of Figure 21].

<123> Tracey: What happened here [points at the transition between Segments B and C, Figure 21]?

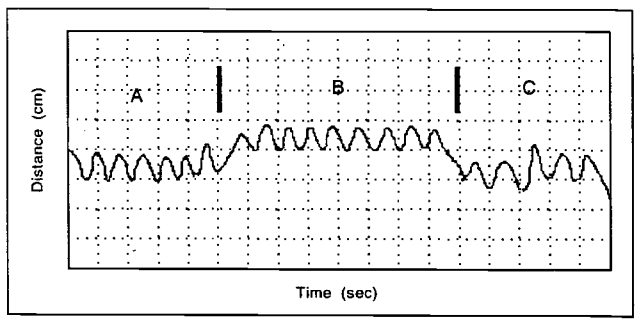

Figure 21. Eleanor's graph shows the result of her combining walking and arm swinging.

<124> *Eleanor: Well, maybe, I don't know. I just kind of went — I went like this [she walks toward the tower] a little closer to it [the tower].*

In creating Figure 21, Eleanor was playing with kinesthetic patterns that combined walking with arm waving. Figure 21 displayed an average height that shows how far she was from the tower — a dimension altered by the action of walking — and a finer up-and-down that expresses her continuous arm motion. The oscillation created by the swinging of her arm modulated her walking motion such that the kinesthetic composition of her body motions (walking and arm swinging) became expressed in the composite visual qualities of the graph (overall height and zigzags). Then Tracey introduce the second button. This is the point in time that we have chosen to begin the episode.

One or Two Buttons

Tracey showed a second button to Eleanor and suggested that they could use both, one held by Eleanor and another by Tracey to create a new type of graph. First Eleanor and Tracey each held one button and did a sort of improvisational dance together with Eleanor taking the lead and Tracey responding by moving her button sometimes alongside Eleanor's and sometimes in the opposite direction. Eleanor moved the button back and forth as well as up and down, in a diagonal trajectory. She ended by moving her hand quickly back and forth (the corresponding graph is not shown here). They commented on their graph. Eleanor said that the color of her graph was yellow and Tracey's blue, as indicated by the color of the buttons that each of them was holding. In the following graphs we show the yellow graph with a thick line and the blue one with a thin one. Then Eleanor asked to have both buttons.

<125> *Eleanor: (...) Wait, I want to try something with both buttons to see if I can kind of make them go right next to each other. (...) Maybe if you [I] hold both of them. [Eleanor takes Tracey's button from her so she is holding both, one in each hand.] Like you put — I wonder, which one would be above [in the graph]? OK. I'm just going to do something just like that. [She walks back and forth holding the two buttons close beside each other in front of her, equidistant from the sensor (beginning Segment A, see Figure 22). Then she holds them far apart one above the other and then out to the sides, still equidistant from the sensor as she moves back and forth (still Figure 22, Segment A). Then she stands still (Figure 22, Segment B) near the tower and moves the buttons alternately in and out, back and forth toward and away*

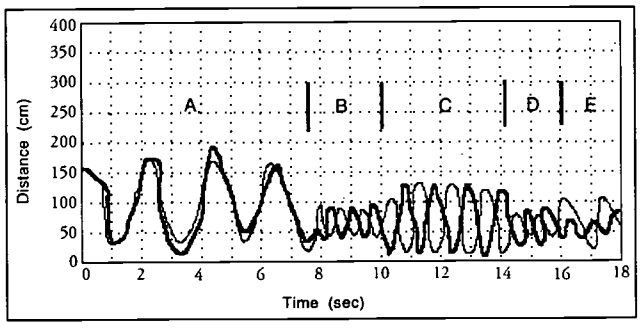

Figure 22. Eleanor's graph along with Tracey reflects her experimentation with two buttons.

from the tower, making an alternate zigzag.] Wow, that looks cool. [Eleanor does larger swings in and out (Figure 22, Segment C). Still standing near the tower, she circles the buttons around one another as if pushing peddles on a bicycle (Figure 22, Segment D). Finally, she swings the buttons out to the side and in until they touch (Figure 22, Segment E). The graph stops.]

As soon as Eleanor held the two buttons, she began to experiment with relational motion framed by a new question: "I wonder, which one would be above [in the graph]?" The actions that she had enacted formerly, with one button, were different ways of getting closer to or farther from the tower (walking, running, etc.). Her new movements and questions, instead, focused on different ways to separate the buttons (e.g., side by side, far apart to the sides, up and down), and the response that she intended to achieve was a relative separation between the two graphs on the computer screen. First Eleanor held the buttons together, side by side; then, after noticing the overlap between the two lines, she experimented with different ways of pulling the buttons apart, first vertically and then to the sides (Figure 22, Segment A) in attempts to separate the two lines on the computer screen. But she did not achieve a conspicuous response in the computer screen; that is, the lines were still together. Then she tried separating the buttons by moving one closer to the tower (Figure 22, Segment B), and immediately discovered a significant reaction when the two graphical lines moved apart ("Wow, that looks cool!"). She emphasized the responsiveness of the computer by enlarging the swings of her arms (Figure 22, Segment C) and playing with different ways of circling her hands around each other, toward and away from the tower (Figure 22, Segment D). The two line graphs on the computer screen expressed to Eleanor different kinesthetic patterns of two-arm motion; this is the theme of the ensuing dialogue:

<126> Eleanor: That's neat. I like this one if you go like this. [She points to the screen Figure 22, Segment B)]. Wait, I forget what I was doing. I think I was going like this [she moves the buttons alternately back and forth]. That looks neat.

<127> Tracey: Yeah, that was that part [points on the screen to Figure 22, Segment B].

<128> Eleanor: Yeah, and this part's kind of weird. [Figure 22, Segment E] I think I was going like this [stretching her hands out to the sides and then in together]. [Tracey: Right.] Wait, I was trying to figure out, which [way of pulling the buttons apart] makes them [she points on the screen to a vertical gap between the lines] go, like, farther apart? Like this [holding buttons apart to the side] or like that [holding one up and one down] or...

<129> Tracey: Oh, OK, right. And at first [Figure 22, Segment A] you were holding it like that [holding one hand up and one hand down] and you were also wondering which one would be on top [pointing to Graph 7, Segment A]? Which line would be on top?

<130> Eleanor: Yeah, except here [pointing to the screen where lines are close together in Figure 22, Segment A] I was holding them like this [holding them together].

<131> Tracey: Right. And what happened [pointing to first part of Figure 22, Segment A]?

<132> Eleanor: They were just — you can't tell the difference.

<133> Tracey: And then here, [second peak in Figure 22, Segment A] I was noticing you held this one [touching the yellow button]up high. But look, [at second peak in Figure 22, Segment A] the blue one is up higher than the yellow one.

<134> Eleanor: Well, let's. I'm going to try it and see if it, like, matters of, like, height or, like, if, like, it the yellow one's higher [holding yellow slightly above blue vertically] and just, like, have it [the graph] go straight across [horizontally]. [Tracey: Okay.]

Eleanor noticed Segment B, in Figure 22 ("That's neat"), where she had succeeded for the first time in separating the two graphs on the computer screen. She commented that that happened when she was "going like this," moving her two hands alternatively back and forth. Eleanor was explicit in her intended goal: "Wait, I was trying to figure out, which [way of pulling the buttons apart] makes them [the line graphs] go like farther apart?" Notice how she started to explore the motion detector anew. Suddenly, the distance to the tower was absent in her use of the tool: "Like this [holding buttons apart to the side] or like that [holding one up and one down] or..." A new range of actions and goals came to occupy Eleanor's concerns and curiosity, all of them centered on the relative distance between buttons and between the two line graphs. The kinesthetic patterns expressed in Segment A, Figure 22 "failed" because "you can't tell the difference [between the two graph lines]." Although Eleanor had found one style of arm motion that separated the two lines (in Segment B, Figure 22) she still felt that she did not know "what matters." Eleanor's movement back and forth in Segment B, Figure 22, was for her a particular case of arm motion that "worked," but she wondered if vertical separation of the buttons could also elicit the same response: "I'm going to try it and see if it, like, matters of, like, height." corresponding color on the screen is higher became her next question.

<135> Eleanor: Hmm. [Tracey starts the computer. Eleanor holds the buttons vertically, in opposite positions again, one up one down, keeping them still for a few seconds each time. Then, keeping one up and one down, she stretches very far, one to the floor and one up as high as she can reach. At 5 seconds (Figure 23) she says:] I wonder if I hold it like that [holding each one out to the side]. Hmm. [She holds the buttons out to opposite sides] I don't know. [She goes back to holding them apart vertically,

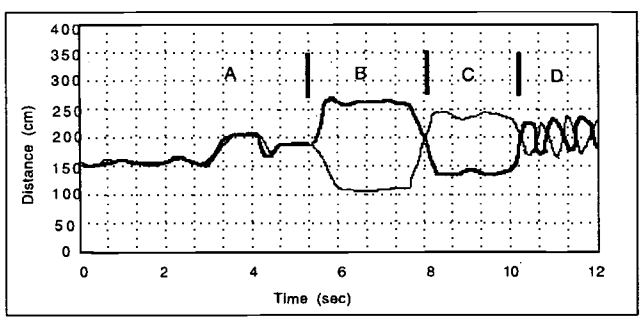

Figure 23. Eleanor's graph reflects her experimentation with ways of separating the buttons and their corresponding graphs.

one up one down, still at Segment A; Figure 23]. They [the two line graphs] kind of go the same. [Tracey: Yeah.]

<136> Eleanor: [holding left hand, blue button forward and right hand yellow button back, on Figure 23, Segment B] There, kind of found kind of a combination. [She sounds pleased, swings her arms slowly, Figure 23, Segment C, and then quickly Figure 23, Segment D. Graph stops.] Hmm...that's neat, so I was kind of holding them like this and like that. [Tracey: Right] [Repeating previous motion from Segments B and C, holding left hand in front of the right and swinging arms so that right is in front, all on same height]. Like one's [button] back and higher [gesturing with left hand] and one's forward and lower [gesturing with right hand].

Eleanor created Figure 23 by experimenting again with ways of separating the buttons and their corresponding graphs. When she finally achieved a conspicuous separation between the graphs she remarked: "There, kind of found a combination." Note how she described her arm motion by highlighting that one was back *and higher*. Eleanor found again a successful kinesthetic action but her recollection also included a component of vertical displacement. Does the height of the button matter or not? Her first action putting the buttons up and down in Segment A, Figure 23 suggested that button's height does not matter, and yet the "successful" Segments B and C appeared to be related to vertical displacement of the buttons.

<137> Tracey: Right. And which part was that on the screen?

<138> Eleanor: [She interprets the question as what she had been doing in Figure 23, Segment C]. I think when the yellow one [button] was back, it [the yellow graph] was higher, I think. [Tracey: Yeah?] Yeah, I remember the farther back [gesturing back from the tower] you hold it [pulling her right arm and body dramatically backward] the higher it is [gesturing upward with right hand].

<139> Tracey: Right, from the beginning [of working with the motion detector].

Right after Eleanor said "I think that when the yellow one was back it was higher," she recalled memories of her former experience with one button, in which being back away from the tower meant being higher on the graph. This is a moment of re-encountering previous ways of acting and interpreting the graph. She experienced a sudden "remembering" that cast light on her question of "What does matter [to separate the two graphs]?" Eleanor immediately stopped looking for different ways to achieve such a separation because it ceased to be a question for her.

<140> Eleanor: I'm going to try to make a pattern like that kind of [Figure 23, Segment D].

Feeling a new sense of control, Eleanor reacted by challenging herself to generate visual patterns. The parallel with Part II is striking. At that time she strove to find out what moves the graph up and down. Let us recall that former episode: after Eleanor

developed the conclusion that "the closer to the tower it gets, the lower" — which enabled her to control the vertical displacement of the graph by walking back and forth — she said, "OK, I'm going to try and make a pattern." The use of two buttons prompted Eleanor to start an investigation very similar to her initial one with the motion detector.

<141> *Tracey: So in the beginning part [pointing to Figure 23, Segment A] you were experimenting with it (...) [gesturing at the same time as Eleanor].*

<142> *Eleanor: I was holding it [the buttons] like this [moving one button up and the other down] and like that [out to the sides], but that didn't really work. But then I remembered that, like, when you held... when you went back farther [pulling her arm back as she steps back] it went up higher [pointing up with her front hand as if drawing in the air]. OK, I'm going to try to do, like, patterns with them.*

It seems she had temporarily "forgotten" that the distance to the tower is what counts. Why? Did she deem that by using two buttons the tool might work in entirely new ways? Was it a matter of putting her former conclusions "on hold"? We think not; this type of interpretation would amount to a very deliberate attitude that is inconsistent with Eleanor's ongoing surprise in meeting unexpected results as well as with her sudden remembering that seemed to settle the question as to what counts. In our view, what Eleanor experienced with the use of the two buttons was a new realm of possible actions and responses. The notion of a lived-in space implies that the kind of actions that one performs and perceives constitute the space itself; working with two buttons shifted Eleanor into a *new* space, so that her movements, gestures, and graphical productions came to be permeated with new kinesthetic and visual qualities. As she had done at the time of her initial encounter with the motion detector, Eleanor proceeded to enact the universe of possible actions, to test its limits, and to assess its responsiveness. This does not mean that her previous experiences in Parts I and II disappeared. They were available to her through a remembering act: "But then I remembered that, like, when you held..."

Using two buttons was for Eleanor different from using a single one. The kinesthetic experience of the relative motion of one's arms focuses on issues of how spread apart are one's hands, to what extent they are back or ahead of each other, or the synchrony between arm movements. All of these are aspects that counted for Eleanor; they were an integral part of her experience with the tool and the graphs. This episode shows that a lived-in space is an ongoing creation; that far from being an inert structure, it demands a constant re-creation as we practice new possibilities and ways of acting in it.

Dina

Introduction

Parts I and II described the first 20 minutes during which Dina started using the motion detector, then the session ended. During the next session, Dina and Tracey continued to develop activities with the motion detector. At the very beginning Tracey explained the plan for the session:

<143> Tracey: We're going to use a sensor and the tower like last time, the button. (...) we can attach the button to the train [she gestures how the button could be attached to the front side of the electric train]. And I think we'll start with you holding the button [with your hand]. And the first thing that I'm wondering is do you think you could find a way to make a straight [horizontal] line?

<144> Dina: Yes.

<145> Tracey: What's your plan?

<146> Dina: To keep it still.

During the first 9 minutes they reviewed ideas from the previous session, such as how to make horizontal lines or a broken graph as shown in Figure 24.

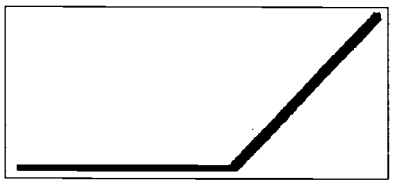

Figure 24. Dina and Tracey use this example to discuss how to make a line graph.

This next section includes the episodes that took place during the 12 subsequent minutes (from minutes 9 to 21) as well as three short vignettes (each taking one minute or less) that occurred later, at minutes 34, 46, and 49.

Hand or Train Motion

Tracey finished drawing the figure on a paper sheet resting on an easel (see Figure 25).

<147> Dina: OK. Are these two supposed to be the same size [the two arms of the "V" in Figure 25], or this one's [second one] longer?

<148> Tracey: Um, they're really supposed to be the same size.

<149 Dina: OK. Should I do it?

<150> Tracey: Do you have a plan? [Dina: I think.] What's your plan?

<151> Dina: To get closer [she walks closer to the tower with the button] and then to get further (...) [she walks away from the tower with the button, then she starts to produce Figure 26 by moving forward and then backward. When the right hand arm of the "V" has reached the height where the left hand arm started, at Point A in Figure 26, she says:] Oh, I don't think it's right.

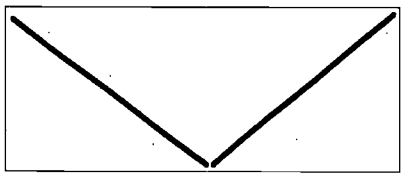

Figure 25. The graph model for Dina's experimentation in graphical spaces.

One is longer than the other. One is going to be longer than the other. I have to get even further [next time make the starting point higher on the graph].

Dina planned to produce the graph in Figure 25 by enacting the body motions with which she had learned to control the down movement and up of the graph: "get closer and then get further." Note that what was important to her was not just the "V" shape, but also the lengths of the arms and how they fit the screen as a whole (e.g., the vertex in the middle, the overall symmetry). She intended to create a graph resembling Tracey's drawing with respect to the borders of the paper (see Figure 25), which were apparently taken by Dina as corresponding to the borders of the computer screen. Because Figure 26 is an asymmetrical "V," Dina reckons that the starting point of the graph should be higher to compensate: "I have to get even further." In producing Figure 26 Dina was clearly focusing on her distance to the tower. This is a significant remark, given the shift that she will soon express.

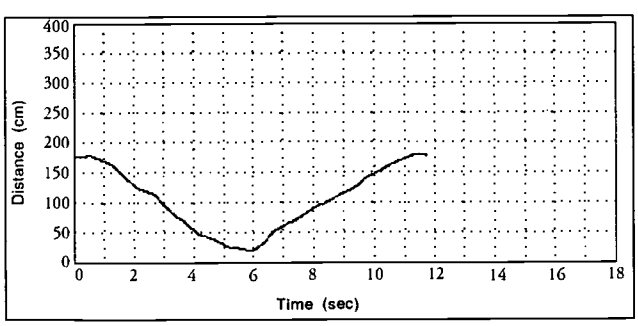

Figure 26. Dina's graph reflects her initial attempt to reproduce the model graph with hand motion.

<152> Tracey: OK. [Tracey clears the screen and Dina backs up to start a new graph.]

After making Figure 27, Dina is concerned that the "V" is not quite symmetrical.

<153> Dina: Well I think the second one [right arm of the "V" in Figure 27] should have been a little straighter instead of curved.

<154> Tracey: It did curve, didn't it? What made it curve? [Dina: I don't know.] Do you have any ideas of why it might have curved?

<155> Dina: Well, the way I was moving my arm.

<156> Tracey: How were you moving your arm? (...)

<157> Dina: Well, I think I was kind of slow on moving it. I wanted to go straight. (...) I was just kind of trying to slow down because I thought it [second arm of "V", Figure 27] would be too long again.

In trying to avoid the same "mistake" of an excessively long line for the second arm of the "V," Dina made a curved arm for the "V." Note that

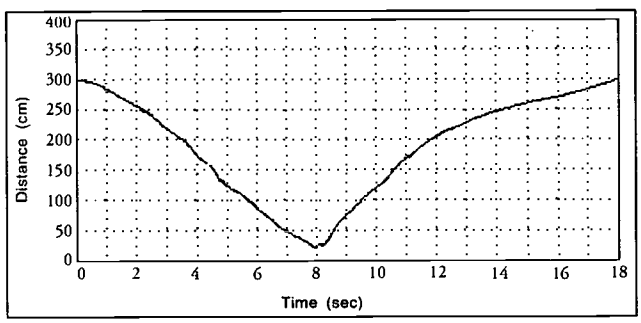

Figure 27. Dina's graph reflects her second attempt to reproduce the model graph with hand motion using an adjustment of speed.

she included considerations of speed ("I was kind of slow on moving it"), based on her recollection of how she tried to "stop" the line from elongating too much. Dina saw her arm's slowing down as affecting the straightness of the graph.

<158> Tracey: (...) The other thing that we can do to try to make some of these lines is you can choose whether you think it would work better to use the train or with your hands. Like we could try this one [Figure 25] with the train.

<159> Dina: I think it's easier with the train.

<160> Tracey: How come?

<161> Dina: Because all you do is start at a high speed, get slower to one way [to produce the downhill piece of the "V"], and get faster on the other [to produce the uphill piece of the "V"]. But, um, on that one it's kind of hard to control your arm, because you can't tell what's going to happen.

<162> Tracey: Mmm-hmm. OK, so if you wanted to have more control, you'd choose to use the train?

<163> Dina: Yeah.

As soon as Tracey suggested Dina produce the graph in Figure 25 by moving the train, Dina immediately reinterpreted the graph as telling a completely different story of events: "All you do is start at a high speed, get slower to one way [to produce the downhill piece of the 'V'], and get faster on the other [to produce the uphill piece of the 'V']." Through this striking shift, Dina appears to make the same type of interpretation that she had expressed at the very beginning of her use of the motion detector, in which graphical height meant speed ("Because I think that low is slow" [see <23>]). It is as if she was suddenly dealing with a different tool and graphical meanings. Dina had played with the electric train, inventing different ways to represent its motion by drawings on paper, but without the motion detector. In her previous use of the electric train Dina had controlled its speed with Speed Controller A (see Figure 28), with which the speed and direction are set independently with a knob and a switch respectively. Because the two independent controls had been cumbersome to Dina, Tracey decided this session to use Speed Controller B (see Figure 28), in which the same knob controls speed and direction simultaneously, so that 0 is stop and negative/positive speed indicate direction.

<164> Tracey: And we, actually we changed to a different speed control. This was the one from last time [see Speed Controller A]. And this one [see Speed Controller B] works a little bit differently, and actually I think you might like it [Speed Controller B] better. (...) So you can

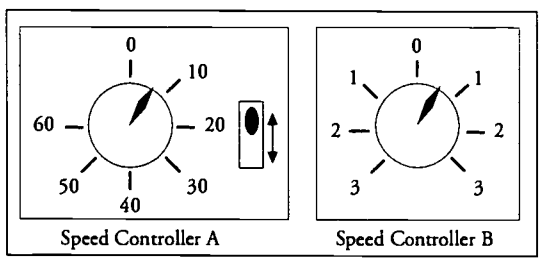

Figure 28. Speed controllers used to control the speed and direction of an electric train: Speed Controller A regulates speed and direction independently with a knob and a switch; Speed Controller B regulates speed and direction simultaneously.

just kind of play around with that and see how you like the speed control [B].

<165> *Dina: [after trying it out] Number 1 [on the number scale of Speed Controller B] goes much faster than the other [Number 1 on Speed Controller A]. Because Number 1 didn't even move on that one [on the number scale of Speed Controller A].*

Dina experimented with moving the train. She said that the train tended to go too fast. Speed Controller B is more powerful; with it, it was easy to get the train going significantly faster. Tracey asked her opinion about the speed controllers.

<166> *Dina: Well, I don't know. Because the other one [Speed Controller A] I got all mixed up with the [change of] directions. But with this one [Speed Controller B], it kind of...I don't know, it goes too fast, but then this one [Speed Controller B] I can stop it much easier.*

<167> *Tracey: What are some ways you could stop it?*

<168> *Dina: Well, just get it back up there to the zero.*

Dina's experimentation with Speed Controller B raised one of her more central concerns about the electric train: how to avoid going too fast and hitting the bumpers on either end of the track that prevent the train from falling off the table. She had mixed feelings regarding this new speed controller. On the one hand, it reached high speed too soon; on the other hand, it was easier to stop. Note how she used the numbers on the scales: "Number 1 goes much faster than the other [Speed Controller A]. Because Number 1 didn't even move on that one." In other words, "1" was not a quantified speed in the conventional sense of one unit of distance over one unit of time; rather, "1" indexed her experience with the speed controllers: it meant fast with Speed Controller B and no motion with Speed Controller A. Dina's attitudes toward the Speed Controller suggest that its use involved issues relevant to her (e.g., how to avoid hitting the bumpers at the ends of the track) that were related to handling the train's speed; this is likely to have influenced her subsequent shift toward a focus on speed to interpret graphs.

Tracey asked Dina to make the same "V" she did with the handheld button. This was the spatial setting of the devices (see Figure 29).

<169> *Tracey: Shall we put the button on the train and you can try this picture [Figure 25, using the train]?*

Before they turn on the motion detector, Dina thinks aloud about how the train should move to make the "V," and she practices the motion with the train and the speed control.

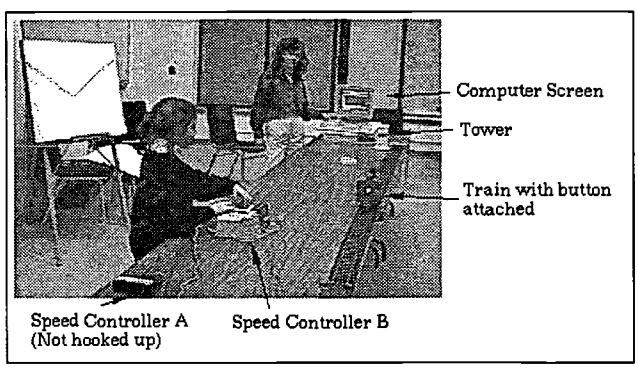

Figure 29. Dina uses Speed Controller B to regulate the speed and direction of the electric train.

<170> Dina: OK. Hold on [to herself quietly] to get slower. So I have to start. I have to [moving dial of speed controller] [now to Tracey] I kind of have to get faster, cause to start it. So that we're going to have to get a little faster. [The train is at the end of the track near the tower. She turns the controller knob sharply counterclockwise turning it on. The train spins its wheels going backward.] Oh, woops, wrong way. [She turns the controller clockwise. The train moves away from the tower at a moderate speed. As it approaches the 2/3 point toward the end, she turns the speed controller sharply to the neutral position. The train stops and then reverses direction. It goes about 3 feet very slowly until Dina stops it.] Oh, oh, oops. [Pause.] Can I just get it back over there? [She wants to move the train back to the tower.] [Tracey: Um-hum.]

As Dina began to plan how to produce the "V" of Figure 25, first she interpreted the first arm as "get slower," but then she faced the issue of the starting velocity ("So I have to start...."). She decided that she had to start the graph by first moving the train "faster," so that the line could begin higher up. Note that to get ready to begin the graph, Dina positioned the train close to the tower. She would keep this feature through all her attempts. This tacit and self-imposed constraint prevented her from noticing that the graph could start high without going fast, because by setting the train close to the tower in all cases, her graphs started always uphill from zero.

<171> Dina: OK. [She runs train away from the tower at a fairly constant speed. Then, as it reaches a point close the end, she speeds it up. Then she stops it.] I did it wrong again [she wanted to speed up the train more in the middle of the track].

<172> Tracey: Oh — what happened?

<173> Dina: In the middle [of the track or of the trip] I have to change it and I kind of thought to do the whole track. Unless I do the whole track, so there are two ways I can do it.

<174> Tracey: Two ways, so — what would the first way be?

<175> Dina: Well, to use the whole track going at lower speed, and then use the whole track getting higher. Or you could use half getting lower and half getting higher. [Tracey: Mmm-hmm. OK.]

Dina's "two ways" had to do with whether the whole track should be used to make the first line segment of the graph, with the whole track being used to make the second segment when the train returns, or whether half the track should be used to make the first segment and the next half to be used to make the second segment (see Figure 30).

In Dina's interpretation of Figure 25 the direction of the train is irrelevant; what counts is whether the speed increases or decreases in either direction. As we shall see, Dina preferred the first option because speeding up the whole way would get the train going too fast. We want to emphasize that in producing graphs with the electric train — as opposed to with her hand motion — Dina enacted a completely

different kind of actions and concerns. Rather than getting closer or farther from the tower, what was salient for her was getting faster or slower, stopping, and avoiding high speeds. Even the idea of "stopping," which in the case of hand motion expressed just stillness or absence of motion, had a different connotation with the train because it required an active and timely turning of the knob.

<176> Dina: But to me, I think using half for each is easier with this one, because it has, it goes faster [Speed Controller B is faster than A was]. So I don't really want to get so fast.

<177> Tracey: OK. So you're going to try it [Figure 25] with me turning this [motion detector] on?

<178> Dina: OK, hold on...[she runs the train back to the tower and she ponders something] I kinda. OK. Now hold on now...[Thinking to herself. She stares at the graph, while moving her fingers on the speed controller, planning how to turn the knob.] OK. [Tracey starts the motion detector. Dina runs the train away from the tower at a fairly fast but unchanging speed. As the train moves, she takes a quick glance at the graph. As it nears the end of the track, she quickly reverses its direction and sends it back to the tower. When the train is almost at the tower, she says "Whoopsy" and stops it for a second.] Oh well, I'll just do wig-wags for now. [She relaxes her body, but continues to move the train, moving it away from the tower. When it gets near the end of the track, she reverses direction and makes it come closer to the tower again. She has produced Figure 31.]

<179> Dina: OK, I did it completely wrong [shrugs her shoulders]. I messed up. 'Cause when, I have to start off when it's fast. So it [the graph] has to go uphill first.

<180> Tracey: [pause] It has to go uphill first?

<181> Dina: Yeah, cause I have to start it — cause it [first half of the "V"] goes from fast to slow. So I have to start it at fast, and to get it there the train moved.

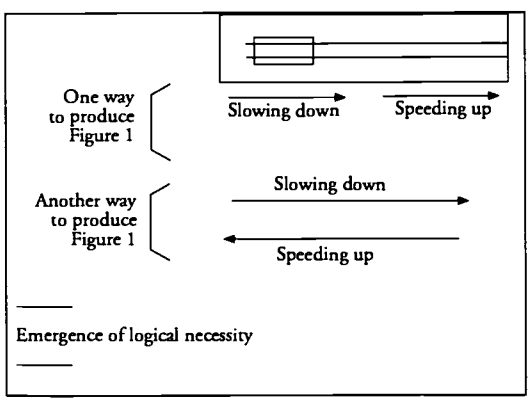

Figure 30. Dina's two ways to reproduce the model graph.

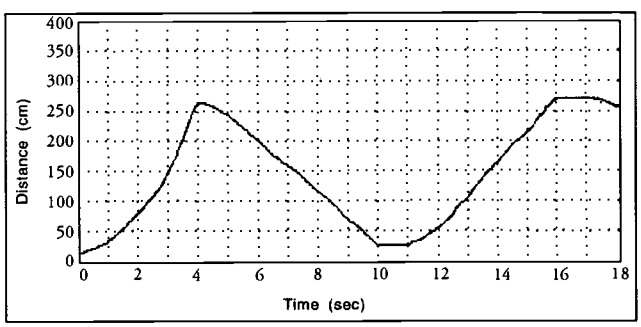

Figure 31. Dina's graph reflects her first attempt to reproduce the model graph with train motion.

<182> Tracey: I see what you're saying. OK. Right. You can't just start really fast.

Dina saw in the first half of the "V" the train going "from fast to slow." The initial "uphill" in Figure 31 reflected for her the necessary increasing to a fast speed. Note how Dina used "fast" as a location: "So I have to start it *at* fast, and to get it *there* the train moved." This sense of "fast" as a location is part of Dina's interpretation of the graphical space when using the train; fast is being at a certain region, higher up, on the graph. Figure 31 did not present to Dina evidence contradicting her assumptions; because she kept starting the train close to the tower she could made sense of the undesired uphill as an unavoidable initial speeding up because she always started moving the train from the tower. How did she reconcile this conclusion with the fact that she had been able to create the "V" with hand motion? This is the theme of the next dialogue.

<183> Dina: You can do it [the "V"] without the train, but not with...

<184> Tracey: You could do it without the train?

<185> Dina: Well, you could do it [the "V"] with the button by itself, but... 'Cause the train has to get faster first.

<186> Tracey: Uh-huh. And you don't. Your body — doesn't? with the button? You wouldn't have to get fast first?

<187> Dina: No, because you'd start out from behind [far from the tower] and you already start at the top [of the graph]. [Tracey: Uh-huh.] On this [the train] you have to get the speed up.

<188> Tracey: Uh-huh. Do you think this [Figure 25] is possible with the train, to do?

<189> Dina: Unless I do an uphill first. But I can't think of another way.

<190> Tracey: You think you can do [with the train] this [gesturing an upside down "V"].

<191> Dina: Yeah.

Puzzling Tracey, Dina seemed to describe *two* graphical spaces. In the one sensitive to body motion, the graph shows one's getting closer or farther to the tower and the graphical upness or downness distinguish directionality of motion. In the one sensitive to train motion, what is being portrayed is how fast or slow the train goes regardless of direction, as well as how quickly one turns the knob on the speed controller. A property of the latter is that graphs must start from zero because the train has to start from rest; therefore, a "V" is impossible. These two graphical spaces were for Dina lived-in spaces, that is, spaces populated by distinctive modes of actions, expectations, and examples. Dina did not reach this interpretive duality by ignoring the evidence, on the contrary, so far she has developed coherent accounts for all her graphical productions. But 13 minutes later, Tracey brought up the case of a "flat line" that would make Dina question her approach and trigger a sudden remembering act. The following vignettes encompass the three moments in which Dina reflected on the issue of producing a horizontal line with the electric train.

Vignette 1, minute 34 (34.30 to 35.47)

<192> Tracey: We're going to do this one [pointing to a horizontal line drawn on paper].

<193> Dina: [telling her plan to graph with the electric train] Um, start them at the speed of one and just do the whole thing on that speed. [Tracey: OK.] Because if I wanted to start it on 2. Well, if it wasn't so fast, and I wanted to start it on 2, it...[pause] oh!, I also remembered another thing on the computer. If I wanted to make a straight line on that [computer screen], I wouldn't be able to move the train. So on that [computer screen] it's not going to be a straight line. So...it's going to probably be a downhill or something. So...

<194> Tracey: So if you want to put...to make a flat line with the train...

<195> Dina: I wouldn't be able to move it.

Vignette 2, minute 46 (46.22 to 46.50)

<196> Tracey: What if you wanted to use the train and make a flat line?

<197> Dina: A straight line?

<198> Tracey: Yeah [gesturing a horizontal line].

<199> Dina: I wouldn't move it at all, I don't think.

<200> Tracey: Where would you put it [the train].

<201> Dina: It depends where I want to — how high I wanted it [the line on the graph].

Dina's initial plan to create a flat line was to keep the speed of the train constant. She wondered whether to set the speed at 1 or 2, expressing her recurrent concern that the train might go too fast. However, as she was staring at the computer screen, she remembered that flat lines meant stillness. Her past experience with graphical flatness brought to her the certainty that she "wouldn't be able to move the train." It is suggestive that this meaning of flatness re-presented itself to Dina *before* she tried it out with the motion detector and saw unexpected results. She insisted two times, "on *that* [the computer screen] it's not going to be a straight line [if the train moves]." Then, in Vignette 2, Dina expressed her clear sense that the height of the horizontal line would show the position of the train. Let us highlight that at this time the flat line was for Dina a special case. What she said in Vignette 2 did not necessarily imply that in general —for any graphical shape — what is being shown is the position and not the speed of the train. This remark is suggested by the next vignette.

Vignette 3, minute 49 (49.07 to 50.03)

<202> Tracey: And now what's your plan for making that picture [flat then uphill, see Figure 24]?

<203> Dina: Well, starting out with a straight line and then get faster....Where should I start [moves train to the extreme of the track closer to the tower]? OK. [Tracey starts the computer measurement and Dina moves the train.] Oh [she says when Figure 32 is about to reach 4 seconds] wait. Um, kind of...

<204> Tracey: What did you make?

<205> *Dina: I kind of messed up. Because I had to leave the train straight.*

<206> *Tracey: You had to what?*

<207> *Dina: Leave this train without moving it.*

<208> *Tracey: To make?*

<209> *Dina: That [the horizontal piece of Figure 24], yeah.... Should I do it again? [Tracey: Yeah.] OK, I'm going to just get it a little far [she places the train a few feet from the tower and leaves it still while the computer begins to generate a horizontal piece].*

Then Dina produced a graph that resembled Figure 24 although she deemed that the uphill piece was too steep.

Although Dina's description of her plan was ambiguous ("starting out with a straight line" does not tell how to do it), her actions in her first graphical production suggest that she was interpreting Figure 24 as a sequence of variations of speed (constant and then speeding up). This initial interpretive stance reflects a central quality of lived-in spaces: their horizons of possibilities present themselves to us. What we initially come to see and expect is not the result of deliberate and consistent inferences; rather, one finds oneself noticing patterns and expecting events. Even though in Vignettes 1 and 2 Dina had become keenly aware that graphical flatness must prevent the train from moving, Figure 24 was another case. She *saw* in Figure 24 "starting out with a straight line and then get faster." However, in this last vignette, the uphill segment appearing on the computer screen did *not* elicit to Dina, this time, the idea that Figure 24 is impossible to do with the train; instead, it led her to conclude that "I kind of messed up." Dina explained her mistake to Tracey by using a remarkable expression of fusion: "Because I had to leave the train straight"; moreover, demonstrating the difference between fusion and confusion, she clarified: "leave this train without moving it."

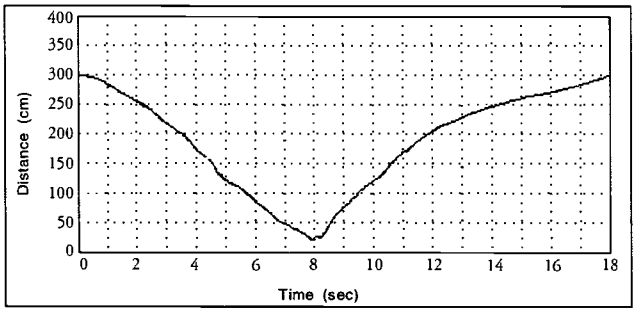

Figure 32. Dina's graph reflects her attempt to reproduce a flat-then-uphill picture with train motion.

Discussion

In this discussion we want to articulate two points: that the graphical spaces were, for Eleanor and Dina, lived-in spaces, and that the distance and time indicated by the graphical location were meaningful to Eleanor and Dina as indexes of their experience with the tool.

1. Graphical spaces as lived-in spaces.

Eleanor started out playing with two kinesthetic patterns: arm waving and walking, each one with a certain identifiable visual response (waves and overall height). As she used two buttons she explored new kinesthetic patterns focused on the relative distance

between her hands aiming at a particular graphical response (separation between line graphs). She perceived a holistic change in her use of the tool. She wondered anew on "what matters." In a particular configuration of circumstances (the button more to the back had generated a higher graph) she remembered that "the farther back the higher." Her insightful reencounter with her previous idea helped her to gain a sense of control over the tool that she expressed with exactly the same words that she had used in <67>: "OK, I'm going to try to make a pattern" (see <140>). In both circumstances she moved from trying to figure out how the tool worked to using the tool for the production of visual patterns.

Dina dealt with the issue of graphing the "V," first with the handheld button then with the train. In each context she found relevant different actions (e.g., getting closer vs. getting faster) and aspects that the graph is sensitive to (e.g., with the train it does not sense direction). She then seemed to envision two distinct graphical spaces. By attaching the button to the train she found that, instead of walking and arm waving, she needed to control the *speed* of the train and be concerned about other issues (e.g., not to get too fast, having to start from rest). The case of the flat line disrupted this dual perspective. She remembered that the flat line meant stillness.

For Eleanor and Dina the distance to the tower was the arena in which they created different lived-in spaces populated by specific intentions (e.g., keep the wave straight, separate the graphs, make the "V," avoid the uphill piece) and different types of actions (e.g., arm waving, hand separation, getting closer, stopping by turning the knob) eliciting diverse graphical responsiveness.[8]

2. Graphical locations indexing one's experience with the tool.

If we use "abstraction" in the sense of "the act of singling out" (James, 1983, p. 477) an element of reality in its relative isolation, we note that for Eleanor and Dina the distance from the tower was not primarily an abstraction. As they created new lived-in spaces different forms of indexing emerged (e.g., "low is slow," "1 does not move on this one"). At the beginning of Part III Eleanor and Dina saw the height of the graph indexing their distance to the tower with the handheld button. The use of two buttons prompted Eleanor to open up the issue of what is being indexed by the separation between the two lines. As she described her body actions in creating Figure 23 (see <138>), Eleanor recognized that the relative separation between the buttons that matters is the one that puts them more or less close to the tower: "Yeah, I remember the farther back you hold it [the button] the higher it [the graph] is." Dina, on the other hand, saw the graphical height as indexing either her distance to the tower or the speed with which she drove the train. The case of the horizontal line, however, led her to preserve

[8] This notion that the graph is part of a lived-in space is overlooked by those researchers and teachers who overemphasize the importance of the labels on the axis of a graph, as if noticing them would be enough to "fix" the meaning of the graph, or as if students' mistakes *must* be derived from their attaching wrong variables to the axes.

only the mode of indexing that announces the distance to the tower. Note how Dina extended this form of indexing to the train as she responded to Tracey's question on where to put the train to start the graph (see <202>): "It depends...how high I wanted it [the line on the graph]."

The idea of distance as indexing one's spatial experience has been highlighted by several philosophers. Merleau-Ponty (1946/1989) called distance the "dimension of inactuality" because nearness and farness emerge from the "hold" that one can have on the object; in other words, being near is primordially the possibility of touching, seeing "eye to eye," and all the other activities enabled by body proximity.[9] "Distance is not primarily a matter of meters, but concerns the impossibility of certain actions which would be possible were the object nearby." Similarly, distance to the tower was not for Eleanor and Dina primarily a matter of meters but concerned the possibility and impossibility of certain actions and their perception of how the tool responded graphically to them.

[9] O. Sacks (1995) noticed that: "It has been reported that if people who have lived their entire lives in dense rain forest, with a far point no more than a few feet away, are brought into a wide, empty landscape, they may reach out and try to touch the mountaintops with their hands; they have no concept of how far the mountains are" (p. 119). An instance of such a report is in Turnbull (1961) which includes the example of an Ituri forest pygmy looking at buffalos for the first time, and asking what "insects" they were. Heidegger (1927/1962) elaborated on how the experience of something being more or less far expresses one's interest in "making the remoteness of that something disappear, in bringing it close" (p. 139): "When, for instance, a man wears a pair of spectacles which are so close to him distantially that they are 'sitting on his nose,' they are environmentally more remote to him than the picture on the opposite wall" (p. 141).

Conclusions

You'd have to kind of stop the time.

Eleanor, from <12>.

Because I had to leave the train straight.

Dina, from <205>.

Throughout the paper we have come to three series of conclusions that characterize Eleanor's and Dina's emerging resources and patterns of significance:

1. On Tool Perspectives

Eleanor and Dina focused on grasping the *motion detector's responsiveness to their body motions*, striving to identify the aspects of body motion that the tool is sensitive to as well as how they become graphically noticeable. We emphasize Eleanor's and Dina's treatment of *the tool as a point of view* from which they discriminated between significant and idiosyncratic qualities, that is, between those attributes used to interpret and control the creation of graphs (e.g., getting closer to or farther away from the tower) and those that one has to comply with to achieve a good performance (e.g., keep the button oriented toward the tower). Finally, we notice that in the context of using the tool as a point of view, Eleanor and Dina expressed *the emergence of a sense of logical necessity* which enabled them to envision how the graph should look and to distinguish between possible and impossible graphs.

2. On Fusion

A conspicuous trait of Eleanor's and Dina's fusion experience was the interplay between the graph as a shape and the graph as a response to actions. This interplay integrated the language of intentions and causes (e.g., "At first I was going [my *intention* was...] to have it stay on this line...but they kept on getting smaller....Because [the *cause* was] I didn't walk as far" [see <75>]). It also involved the interplay between graphing and drawing. Treating graphs as drawings was a fruitful source of ideas and language for Eleanor and Dina. At the same time, as they attempted to use the motion detector to draw shapes, they encountered constraints that made "easy" drawings impossible to graph. Our analysis underscores instances in which Eleanor and Dina reflected on graphs imagining *travels along trajectories* embedded in narratives; trajectories that traversed not only the graphical plane but also the events and body motions encountered along the path.

3. On Graphical Spaces

We characterize the graphical spaces experienced by Eleanor and Dina as *lived-in spaces*; populated by kinesthetic and visual patterns, graphical responses, intentions to experiment with, as well as past and present conversations. We also emphasize that Eleanor and Dina perceived the graphical height not primarily as a detached variable but as *indexing* their experience with the tool.

We think that these conclusions are relevant in a number of areas. In relation to the ethnographic

studies on practitioners of science and technology (Ochs et al., 1994; Goodwin, 1995), they suggest that the difference between expert and lay graphers is not a difference in kind but part of a continuum. Our conclusions nourish views according to which learning graphing takes place in the broadening and enrichment of the students' lived-in spaces, in the growth of novel forms of fusion, and in the manifold development of tool perspectives.[10] Concerning the nature of symbolizing itself, these conclusions seem to invite the notions that symbolizing is a movement toward more refined forms of indexing human experience, that playfulness is a crucial aspect of the creation and interpretation of symbolic expressions, and that projecting anthropomorphic qualities on tools (e.g., seeing, distorting) nurtures the subtle dynamics weaving logical necessity and empirical evidence.

[10]Some readers might argue that while the relationship between Eleanor's and Dina's graphing and experts' graphing could be supported by the ethnographic studies on scientific practices, there is no reason to assume that their ideas would be relevant for the school teaching of graphing because they are based on short individual interviews that are so different from the context of the classroom and group work. Our belief that Eleanor's and Dina's ideas *are* relevant for school teaching, however, is mostly based on our own experience in classrooms. We have developed and tested a number of curricular units involving graphing (Tierney, Weinberg, & Nemirovsky, 1995; Tierney, Nemirovsky, & Weinberg, 1995; Tierney, Nemirovsky, & Noble, 1996; Wright, Nemirovsky, & Tierney, 1997) and conducted teaching experiments in elementary and high school classrooms. We have noticed frequent instances of students describing trips on a graphical plane as they unfold narratives that involve simultaneously events and visual attributes of the graph, students exploring tools anew when the context of use changed or high school students wondering, for example, why the "line keeps going to the right even when [they] don't move the button." We think that the traditional focus on both labeling axis and plotting points should *not* be dismissed. These are useful practices. Our point is this: These are small pieces of a much broader and richer domain of ideas and activities.

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Ricardo Nemirovsky, Cornelia Tierney, and Tracey Wright

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