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ABSTRACT

This paper describes a teacher-researcher's efforts to reshape students' attitudes and assumptions about mathematics in concert with the national mathematics standards. Changes as well as the ways students clung to traditional classroom roles are described. Mathematics formats and types of activities that were successful--and unsuccessful--for engaging students in mathematics inquiry are presented. Initiatives are proposed to move students further toward inquiry, and the supports teachers need to make these changes are identified. (Contains 13 references.) (MM)

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by Cornelia Tierney

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Preface

TERC is a nonprofit education research and development organization founded in 1965 and committed to improving science and mathematics learning and teaching. Our work includes research from both cognitive and sociocultural perspectives, creation of curriculum, technology innovation, and teacher development. Through our research we strive to deepen knowledge of how students and teachers construct their understanding of science and mathematics.

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Encouraging Inquiry in a Seventh-Grade Mathematics Class

by Cornelia Tierney

In this paper I describe my efforts to redesign the attitudes and assumptions of my "low-track," seventh-grade mathematics class in concert with the National Council of Teachers of Mathematics Standards (1989, 1991). I describe ways in which the students and I changed and ways in which we stayed stuck in traditional classroom roles, as well as the various formats and types of activities that were the most and the least successful to engage these students in doing mathematics.

Background

Like the "new mathematics" of the 1960s that was too different from current practice for the average teacher to implement (Sarason, 1971), the new directives in the National Council of Teachers of Mathematics Standards (1989, 1991) appear overwhelming to many teachers. The Standards promote methods of teaching in which students create their own problem solutions — in data analysis and geometry as well as in number — rather than memorize a set body of knowledge. To allow for such major changes in students' roles, the teacher's role and the whole structure of the mathematics class must change. Teachers who are motivated to make such changes are confused about how to proceed (Biggs, 1987; Darling-Hammond, 1990; Parker, 1993; Shifter & Fosnot, 1993). What should teachers do, and in what order?

After teaching mathematics in middle school for many years, I had been out of the classroom doing related work — developing mathematics curriculum, working with teachers, and doing research on how students learn. I felt somewhat irresponsible about encouraging teachers to take on nontraditional roles recommended by the Standards without trying to do so myself; therefore, I returned to the classroom part time.

In the tradition of others who have researched their own teaching practices (Ball, 1990; Burns, 1994; Lampert, 1990), I chose to teach one mathematics class — a seventh grade — and to use it as a laboratory for my own learning about teaching. I intended to make sense of and support what the students could do, not what they could not do, and to ask questions, not to find out if they were doing something "correctly," but to encourage them to communicate their own ideas and extend their thinking. Marilyn Burns, in consulting on teaching elementary mathematics (personal communication, 1992), described this process simply: *The children's job is to think and reason. The teacher's job is to delight in their thinking and reasoning.*

This paper is a description and discussion of my efforts to develop an inquiry-based (Postman & Weingartner, 1971) mathematics classroom that would be in line with the Standards.

The Setting

The class described in this paper was not typical; it was a small group identified as the lowest-achieving students in a highly competitive independent school. Five of the eleven students were new to the school; three entered from public schools and two from parochial schools. The students referred to themselves as "the dumb class." Halfway through the year they told a substitute that they were doing easier work than the other classes. This was not true. Except for replacing some work in sets with extra work on decimal, fraction, and percent relationships, they did the same work and sometimes more challenging work than the other classes.

A few weeks into teaching this class, I decided that one goal for the year would be to abolish low-track math classes and to prepare these students to succeed in the regular mixed-ability classes starting algebra the next year in eighth grade. It was crucial to teach them more mathematics. However, that would not be enough. A dramatic change must occur for "lower-track" students to catch up and keep up with their classmates — changes in their attitudes and behaviors and, ultimately, in their roles as students. I wanted to prepare the students to continue to be active learners after they left my class, whether they were doing mathematics in a traditional or nontraditional class, in a peer study group, or by themselves.

An important goal of the Standards and of inquiry learning is for students to feel confident about their ability to do mathematics. However, students who are picked out as needing extra help tend to be particularly self-conscious. They fear that what they say will be "wrong," so they direct energy to figuring out what the teacher wants. Part of my job was to find and implement class structures that would support these students to feel safe enough to pay attention to the mathematics instead of to how they were doing. I began by looking closely at the students to see what behaviors, attitudes, and learning styles kept them from learning as effectively as their peers. After a few weeks I made a list of the behaviors I thought were getting in the way of their learning and the changes I wanted them to make in their approaches to learning mathematics. The new behaviors were ones that I thought successful students would exhibit in an inquiry classroom (see Table 1).

Thus I knew what I wanted the students to do; I needed to learn what I might do. As Postman and Weingartner suggest teachers do, I set out to treat these students as though they were capable of learning mathematics if only the setting were different. I would assume they had available the desired behaviors and could work independently and cooperatively to use their knowledge to find solutions to new sorts of problems.

This study became an exercise in problem solving for me as the teacher. It was highly subjective. I was both a participant and an observer in the class. In the manner of action research, I went through repeated cycles of questioning, planning, acting, observing, reflecting, and replanning classroom practice (Adler, 1992; Evans, 1991). My job was to find out about the culture of the class before, during, and after implementation of any changes. My questions were as follows:

Table 1

Current Behavior	Desired Behavior
Say they "understand" when they are able to imitate a method demonstrated by a teacher or parent.	*Construct their own problem solutions and justify them*: Delve into the structure of the problems to construct alternative solutions and defend their methods.
Speak up in class only when they are reasonably certain they have the "answers."	*Make conjectures in class*: Speak up in discussions to make and explain conjectures, to express their confusions, and to revise their own opinions.
Write down the first answer someone suggests; stop listening to alternative strategies once they have an answer.	*Challenge classmates' arguments*: Listen critically to their classmates to build on their solutions or to disagree with them.
Ask the teacher if their answers are correct.	*Evaluate their own work*: Figure out by themselves and with other students whether their answers make sense and be able to explain why their answers make sense.
Hurry to be the first with the answer; believe they can't do a problem unless they can see how to do it within 30 seconds.	*Take time*: Begin a problem by jotting down what they know before they see the whole solution. Enjoy spending a long time on a problem, returning to it after some time off, even doing a problem over several days.

How do students' behaviors and my curriculum choices affect each other?

What gets in the way of moving toward an inquiry class?

And at one step removed, what support is needed for students and teachers to make the sorts of changes the Standards suggest?

This project was planned week to week and sometimes day to day in response to my observations and those of a staff developer who observed the class every two or three weeks and discussed the students with me. Additional data included students' written responses about their experience, students' papers, and three videotapes of the class, one during each term.

What Happened

In this section, I describe some of the concerns I had, the strategies I used in response to these concerns, some changes I observed in the students' behaviors, and implications for mathematics teaching. I have chosen significant actions I took in adjusting formats and roles to show the evolution of the tone of the class and the students' engagement with the mathematics.

1. Solving Problems

I started the year assigning uncommon problems based on familiar information so that students could work out their own procedures. For example, I assigned this problem as part of the work on place value: Imagine that the telephone company needs to label 1000 telephone poles with numbers 1 to 1000 using a metal numeral for each digit. Find how many of each digit from 0 to 9 are needed to write all the numbers 1 to 1000. Only one student in the class, Ben,* was able to complete the problem on first try without help. Maria almost succeeded; like Ben, she had organized her work so that she could recall and explain her procedures. However, she counted groups of 100s incorrectly — 100, 200, 300,...900 — getting nine 100s in all. Other students counted incorrectly even within the 100s. The class debated whether there are 99 or 100 twos in the 100's place of the 200s. All agreed that there is one 2 in the number 200, but there was disagreement whether from 201 to 299 there are 98 or 99 numbers. Ben and Maria believed there were 99 but only Ben spoke up. He said the others were saying how many you had to add to go from 1 to 99, but Kay quite angrily said, "No you are wrong, we are subtracting," and missed Ben's concern that they needed to include both 201 and 299. Eventually, we counted in unison 201, 202, 203,...while I pantomimed moving objects, one for each count, onto a table, stopping every few counts to ask how many were on the table. By the time we had counted to 220, most of the students decided we would count 99 numbers if we counted from 201 to 299. This was one of many times that the students had difficulty applying what they knew to a new situation; they could count to 1000, but they had a problem organizing the partial computations.

In contrast with the other students at that time, Ben was what Postman and Weingartner (1971) would call an effective learner. He was confident that if he worked on problems he would eventually solve them. He had been placed in the lowest track because he had a reputation for not doing homework. In our class he didn't do all the assigned work, but he worked on challenging problems that he could look forward to discussing in class. Indeed, without Ben in our class at the beginning of the school year, there might not have been any discussion among the students. The other students were silent or they called out answers for me to confirm or deny. Ben talked about mathematics in ways that made sense to him, and often his way made more sense to the other students than my way did.

I kept Ben in our class as long as I could convince myself that some of the others, perhaps

*All the students' names in this paper are pseudonyms. The students in order of mention are Ben, Maria, Kay, Ari, June, Ellen, Joe, Mark, Eva, Shanta, Raphael.

Maria or perhaps Ari who was new and conscientious, would gain a voice and provide Ben with some challenge. But Ben needed other reflective and vocal students with whom to discuss ideas. I promoted him to regular math class at the end of October. When Ben left our class, we had lost a voice of reason. This left me with the challenge of how to get students into debates about problems.

2. Polling the Whole Class

I wanted to involve other students, as Ben had been, in working out ways to solve problems, in keeping track of their work, and in explaining their solutions. To push students to take a stand, I began to poll them all for their answers and then to ask volunteers to explain the various answers. Sometimes a student managed to convince others of an unpopular opinion. For example, when I polled students for their answers to the problem of finding the smallest number divisible by all the integers 1 to 10, most suggested 3,628,800 which is the product of all these integers; only June voted for her smaller and correct answer — 2520 — which contains only the necessary factors. Her explanation convinced most of the others.

However, Kay still wanted me to tell which was the right answer, and she believed that if a procedure was done correctly the answer couldn't be wrong. She couldn't understand how 3,628,800 could be wrong because she felt sure her computation was correct; she had used a calculator to multiply all the numbers from 1 to 10. In choosing a procedure, she didn't consider the part of the question that asked for the *smallest* answer.

Pressured by Ari and Kay, I demonstrated some computation to the whole class — multiplication of mixed numbers. Although I didn't believe in teaching "one right method" to do computation, I was willing to show a number of methods to do one problem. I worked out some problems on the board in several different ways. I drew diagrams and explained as I went along, enlisting advice from the students about next steps, as in a typical high school lecture. I left the work on the board and gave the students similar problems to do. Only a few of the students were able to use these methods to do other problems, and Kay was not among them.

Ari and Maria, who did the problems correctly, were able to discuss the values of the numbers and their roles in the operation, and Ellen was able to relate the solution methods that were new to her to others that she already knew. Ellen had been absent much of the time and had missed introductions to new material. Thus she couldn't make sense of homework assignments. She often hid by sitting silently in the back of the room, but she became involved in this class when the work was familiar to her.

In working with decimals and fractions, understanding the values of the numbers involves knowing which changes in notation maintain the value (e.g., $3.50 = 3.5$) and which do not (e.g., $3.05 \neq 3.50$). Students like Kay just tried to mimic actions on the numbers. They did not have the basic number knowledge necessary to make sense of the problems. When I asked them to double $^{3}/_{4}$, Maria said $^{6}/_{8}$, and most of the others agreed. I became convinced that teaching algorithms is useful only after the learner already has a way to solve the problem that is based on the relationships among the numbers and operations in the problems. To do these mixed number problems, the students needed to build knowledge of fraction and whole number relationships.

3. Seeing Problems Embedded in Visual Contexts

Ari, Kay, and Maria needed fraction knowledge on which to base the computation they wanted to learn, whereas some of the others refused to try routine computation at all. Therefore, after a failed attempt at fraction computation, the class looked at relationships among fractions.

Joe, June, and Mark enjoyed the kinds of problems they could visualize, but they had language difficulties that hindered their ability both to understand what was being asked and sometimes to explain their thinking. Joe would work with numbers only if it involved relationships among shapes. He spent hours making an accurate partitioning of a single rectangle into many different-size fractional parts — $^1/_4$, $^1/_8$, $^1/_{16}$, $^1/_{32}$, $^1/_{64}$, $^1/_{128}$, 2 of $^1/_{256}$, $^1/_3$, $^1/_{12}$, $^1/_{24}$, $^1/_{48}$, 2 of $^1/_{96}$ — which he displayed and explained to the class. Eva, Joe, and Mark enthralled some of the other students with their ability to put assorted fractions — $^4/_9$, $^7/_6$, $^1/_2$, $^6/_5$, $^3/_5$ — in order quickly. Mark claimed that he "just knew," but the others would explain their thinking. From the way Joe would talk about what he "saw," he was apparently "seeing" how to organize information and what to look for. For example, seeing that $^6/_5$ is larger than $^7/_6$ requires that one knows both these fractions are one piece more than a whole and that one compares the size of that extra piece. Eva described alternative methods that were not based on seeing; for example, "$^4/_9$ is smaller than $^1/_2$ because four and a half ninths is one half" or "if I double $^4/_9$, I get $^8/_9$, which is smaller than a whole." These kinds of methods are not appreciated by students like Kay who want a rule (e.g., converting to common denominator) that they can apply to a large group of problems.

Students who develop their own ways of seeing a problem need access to the board or overhead projector to show rather than just talk about their strategies, and they need tools such as blocks, geometric shapes, rulers, and calculators available to use in solving problems and in explaining their solutions to others. Had we kept at computation all year, as "special education" classes so often do, I might never have seen the abilities of this group of students. Joe's score in a seventh-grade problem-solving contest placed him, along with Ben, in the middle of the grade, above anyone else currently in our class.

4. Requiring Two Procedures for Each Problem

If I wanted all the students to consider the value of the numbers, I needed to find a way to motivate them to do so. This began to happen when I required that they show more than one solution for each fraction problem or that they use a solution they hadn't been taught.

Kay drew little icons that she split into fractional parts by circling groups of them. Doing this forced her to interpret the operations instead of only using a procedure that she associated with a particular sign. The class, led by Eva and Joe, had figured out percent equivalents for fractions based on what they knew; for example, $^1/_4 = 25\%$, so $^1/_8$ must be half of that, or 12.5%, and $^3/_8$ is $^1/_4 + {}^1/_8$, or 37.5%). Now Ari and Maria used this knowledge to do the problems in decimal form on a calculator; they would do $1^7/_8 \times 1^1/_5$ as $1.875 * 1.2 = 2.25$ and write the answer as $2^1/_4$, or they would do $^2/_3 \times 6$ as $^2/_3 * 6$ and get 3.9999996 on the calculator and write it as 4. Besides getting the problems done, Ari and Maria became adept at fraction-decimal equivalents which Ari called "tools." When the students wrote about the work they had learned the most from, Ari wrote that he had learned the most from challenging problems in which he could use all the "math tools" he knew, and Kay wrote about doing "fractions and decimal problems a few ways and talking about numbers, fractions, decimals and what they do."

Requiring two methods allowed students to use the procedures they had learned while they stretched to invent another method. Requiring a particular different method might have given that method the status of another algorithm to be memorized without understanding — just what I didn't want. Searching for new methods allowed the students who loved computation to stay in this domain but pushed them to consider the value of the numbers and the effect of the operations.

5. Doing Mental Arithmetic and Estimation

I introduced multiple-choice estimation "quizzes" to increase the students' attention to the numbers in the problem. The students were to pick the closest answer; for example, 5210 + 298 is the closest to 5400, 5500, 7000, or 8000? Only enough time was given for students to write round number approximations for the numbers in the problems before computing mentally.

Ellen came alive again when we did these estimation quizzes or any kind of mental arithmetic. She invented many tricks for herself; for example, to multiply by 25, she multiplied by 100 and divided by 4. As they had showed in the fractions work, Eva and Joe were far better at this work, which required thinking about the quantities, than they were at written computation, with which they believed they were to use a set method. Mark also did reasonably well but he would sometimes apply the wrong sign, apparently reading it incorrectly. Ari, like Ellen, also excelled at these problems and was able to move from learned algorithms for computation to inventing his own. Ari chose a page of the estimate problems as the paper he was most proud of in the second half of the year. For homework, he had written out all the procedures he used and made up additional problems. For example, on one of the quiz problems, estimating 26% of 79, he wrote, "The closest would be 20 because I round 26% to 25%, or $^1/_4$, and 79 to 80, and $^1/_4$ of 80 is 20." For one problem that Ari made up, $120.041 \div 4 =$, he wrote, "It's just like saying $12 \div 4$ which equals 3, and so

120 ÷ 4 equals 30." Kay did badly. She wrote that the quizzes were the hardest thing we had done all year: "Estimate quizzes are supposed to be done in your head. I need to write things down to figure them out."

In classrooms where most of the "mathematics" is teaching of traditional algorithms for computation, some students, like Joe and June and Mark, become averse to doing written work because they fear they will get the wrong answers. Others, like Ari and Kay and Maria, do their best to memorize what the teacher shows them. However, many people, both those who learned to use routine procedures for written computation and those who didn't, are inventive when thinking about how to do a problem mentally. In this class only Kay and one other student, Shanta, whom I will describe next, were unable to make this shift. Perhaps they could have estimated and invented their own solutions if they hadn't already become so sure that doing math was mimicking the procedures they had been shown.

6. Making Observations and Solving Open-ended Problems

The student I was most concerned about was Shanta. Typically, Shanta collected answers from other students or procedures from her mother then memorized them for tests. She was bewildered when she failed a test on angles and polygons that she had taken home to prepare: "I had all the angles that Eva and I found and I memorized them and my mother tested me." On a quiz about square numbers, she got 100% on the facts but could do none of the problems that required thinking, such as investigating what happens to the area of a rectangle if its dimensions are doubled.

Only when Shanta was asked to write or tell everything *she* noticed did she have a chance to excel. This happened when she dealt with problems that didn't have only one answer, as when she brainstormed with the class about patterns in the multiplication tables. She listened to other students to add to her written lists of patterns and to find out if the observations she wanted to make had already been made. Shanta pointed out to her table mates that the nines table was contained in the threes table. Then, when another student found the twelves table in the threes table, Shanta generalized from this and told the whole class that all the tables of multiples of three could be found in the threes table. This was the first time I had noticed that she went beyond just memorizing.

Eva and Maria thrived on some more difficult investigations with many answers. One of these was finding similarities and differences between base 10 and another base. They stayed after class to show me what they had found out. Maria noticed that although the place values in whole numbers are larger for larger bases, the fractional place values are smaller for larger bases ($.1_{(5)}$ is one-fifth, but $.1_{(10)}$ is only one-tenth). Eva pointed out that to turn base 10 numerals of multiples of 5 into base 5 numerals with the same value, one needs to multiply by powers of 2; for example, $50_{(10)}$ is multiplied by 4 to get the same value in base 5, that is, $200_{(5)}$, and $2{,}500_{(10)}$ is multiplied by 16 to get the same value, $40{,}000_{(5)}$. Joe also did well on the bases work and explained his reasoning using diagrams; he said he could do it "because it could be put into pictures."

Eva, Maria, and I discussed real mathematics. I asked them to explain what they had found because it interested me; I wanted to understand. I never had a chance to discuss math with Shanta. She usually wanted me just to tell her she had done well. If I asked her a question she took it as a criticism. Shanta and I needed to have conversations about what she observed, perhaps outside the class and away from other students so that she would be less likely to feel the need to compete. When the questions are about what the students think and observe, all students have something to say.

7. Working on Problems Over Several Days

When our class studied number bases, I wanted the students to look at the whole subject instead of only playing the games and solving little problems provided on worksheets. Besides assigning the comparison of two bases that had caught Eva's and Maria's interests, I made up a test of difficult number base problems for students to prepare at home. All the problems asked students to provide a description of their procedures and justification for their answers. The students took the test home to work on and get help with. Then, without any notes, they wrote in class independently on two successive days, getting additional help between the two class sessions if they wanted it. This was a real test; it counted toward their grade for the unit of work on number bases.

The group of students that benefited from this preparation cut across the class; it included all of those who could get help at home, because it let them know exactly what was expected. This prepared test worked for Rafael who seldom understood what was being asked and depended on following other students' lead. I was delighted to see that for him to do well did not mean the work needed to be routine. Ellen failed the test. She had not been able to figure out what some of the problems asked, and she had been too scared to ask for help from me or from her father who had offered. Ellen didn't need help of the typical kind in which the tutor shows the student how to do the problem. She needed more support to get into a problem and understand what was being asked and what were the criteria for a successful solution. She also needed to be sure that no particular procedure was expected.

By March the students were working well in small groups that were self-selected. They generally worked together to compare solutions and to help one another informally. When our school had a visiting day for grandparents and special older friends, I didn't want to put anyone on the spot, so I assigned difficult geometry problems for the students to work out in small groups. I went to the students who had a visitor and asked which problem each would like to prepare to present. Rafael, who had done so well on the prepared number bases test but never volunteered to explain problems in class, gave a clear explanation of a solution in a confident voice. His group had helped him prepare, but he made sure he could explain the problem clearly by himself.

The lesson for me in these examples — one that I had gotten hints of many times before — was that even with a teacher or other adult willing to explain, it *takes time just to understand what the* *problem is and what is being asked.* Students need time to think about a problem and they need more than one try to solve it. This does not mean just having a longer time; most insecure students don't do any more work if the assignment is due in a week than if it is due in a day. They need to make a first try, check it with other people, and get feedback. The typical expectation that students are able to do work successfully the first time they are asked is like maintaining the pressure of an examination on them all the time. This allows little chance for them to engage with the problem and generate ideas.

How Successful Were the Changes?

How successful was this attempt to structure a mathematics class to support the growth of the students as learners of mathematics? Did the students come to enjoy solving problems, and did they gain confidence in their ability as problem solvers? Did they exhibit the behaviors I described as typical of good math students at the beginning of this paper; namely, take time to construct their own problem solutions, listen critically, and communicate with classmates? In Table 2, I list the months in which I first observed the students becoming active learners in the style described by Postman and Weingartner and by the Standards. Once the students began exhibiting these behaviors they usually maintained them throughout the year.

More than half the students, seven out of the original eleven — Ari, Ben, Ellen, Eva, Joe, Maria, and Mark — made real gains in their engagement with the math work and showed confidence that they could work things out for themselves. Ari, Ben, and Maria worked consistently and with pleasure at homework and in class. Although Ellen, Eva, Joe, and Mark remained erratic in doing homework, they became engaged in class with problem solving and discussing their ideas. June and Rafael also became more engaged in class. June, however, needed to check her computation with classmates because she could never be sure of her own accuracy; and Rafael depended on his mother or other students for help to understand what was asked and

Table 2

Student	Construct and Justify Problem Solutions	Make Conjectures in Class	Challenge Classmates' Arguments	Evaluate Own Work	Take Time to Complete Problems
Ben	September	September	September		September
Maria	September	November	November	December	November
Eva	November	November	November		November
Ari	December			December	November
Joe	December	November	December		November
Mark	March	November			
Ellen	October	(November)*			February
June	(October)	(October)			
Rafael	May	May			November
Kay			(October)		
Shanta		(November)			

*I use parentheses to denote behaviors I observed for only brief periods.

for assurance that he was on the right track. With this dependence on others, they didn't gain the confidence that comes with ownership of ideas.

By these measures, Kay and Shanta gained the least in this class. They fought against the way the class was run. They knew only one inroad — work hard to learn computation methods or memorize facts — and they felt that I was attempting to take that away. Because of Kay's thoroughness and promptness with homework and generally good work habits, she earned a good grade in the class. I believe that in a traditional class where procedures are usually demonstrated, Kay would have done equally well and felt more secure. Shanta's situation was different. She wanted to do well at memorization because that was the way she had previously been asked to prove her competence. In fact, I think that she might have done well making sense of the numbers and using visual tools if she had invested her time in those activities.

What Would I Do Next Time?

Observing Postman and Weingartner's suggestions, I tried to treat this class the way I would treat a class of students known to be successful math students: to expect that they all could learn math in their own way. The course was to be based on the students' construction and discussion of problem solutions. However, I did some things that did not fit with trusting that they could find their own solutions. For a brief period I lectured on procedures for computation. More important, for a better part of the year, I evaluated most of their work, including their homework, instead of leaving it to them to talk through and check. I believe that to help engage students with the mathematics, teachers need to avoid even positive evaluation and instead show an enjoyment of students' work by asking about *their ideas*.

Every one of the students in this low-track class demonstrated that he or she could work out some problem solutions without being taught. Those who had difficulty with computation were able at estimation and at problems requiring spatial reasoning. The similarities among this group of students were not in their knowledge gaps or in their learning style, but in needing time to thoroughly understand what was being asked; many of their "errors" came from answering a question different from the one asked. A predominant misinterpretation for many of the students is that problems call for certain learned procedures; for several students, this meant that they were hesitant to try at all if they couldn't think of such a procedure. I felt a tension in myself between defining problems or tasks clearly enough so that the students would have confidence they were on the right track and explaining so much that they would sense I had one best procedure in mind. In most schools in the United States, students have long been indoctrinated in the early grades to believe that applying computational methods efficiently is the measure of success as a math student. This approach to teaching appears to be the wrong way around. Students need to have their own methods of solving computation problems before they can analyze a procedure they are shown.

Having fewer problems that serve as the central work would allow for discussion, not just checking answers, in class. Instead of rushing through problems and topics for coverage, more time is needed for students to understand what each problem asks and to return to a problem between one discussion of it and the next. Students do not need to get everything clear in a single class, and I would not want to assign any more problems than students would have time to discuss with at least one other person.

Central to mathematics learning needs to be the expectation that students look for more than one way to do most problems and that they collect alternative strategies from one another. In the ideal class, I would help students evaluate and celebrate their own solutions to problems. Unfamiliar types of problems, including those that could be done from a visual as well as an analytic point of view, involve students more easily in creating their own solutions. Open investigations such as looking for patterns in multiples or comparing two number bases involve all students to some degree. These could be as simple as "Write everything you know about the number 60."

Because so many students can't manage to get homework done in their current circumstances and because these students need more time to understand the problems they are to do, I would start a homework club to meet daily after school so that students could work together talking quietly. One or more adults or high school students would be present doing their own work and available to consult with groups of students. One of the goals would be for students to complete their "homework" before they go home.

I would like to implement formats that support the Standards from the beginning of the year. Doing so would take time for students and me to practice the new formats. The course would need to become our course. A few weeks into the course, I would relate to students my overall goals for them and how I intended to help us move in directions that would support their growth and self-confidence. I would then enlist their help to monitor our implementation of the changes. I would need the students' input and feedback on the problems I assigned and on their role and my role in the classroom.

What Outside Supports Did We Need to Meet the Standards?

I would have liked support to eliminate tracking. I tended to correct the students more often in this class than I would have in a mixed class, more frequently even than I did at the beginning of the year when Ben was part of the class. I could depend on him to start a debate about a problem. Without him, conversation between the students was limited. The whole class could agree on a wrong answer. Only when other students became more confident and outspoken did I feel that I could back off to let them all voice disagreement or support one another.

I would have liked support to abolish course grading. Were I not grading in this class, I would have demanded more, not less, of the students. They could take a risk at trying problems that might require a long time or that they might fail at. No student would need to ask me, "Will this be on the test?" Without course grading, the students wouldn't have had to worry about their parents' response to low grades. I would have given assessments that were more like projects embedded in the everyday work. Grading the work of these insecure students was especially destructive in that doing so kept their attention on how they were doing instead of on the mathematics. This put me in the position of critic instead of coach. For all the students, except Ari and Ben and Maria who did well by the end of the year, having grades was discouraging.

Implementing the Standards requires change for most teachers. It means breaking habits practiced for the duration of our experience as students and teachers in mathematics classes. Without an overall plan, the changes I made in my behavior were smaller and less effective than I had hoped for. I began the year with goals for the students. At that time, I wasn't thinking in the same terms about myself. I made no list of improvements I would have liked to see in my teaching and no criteria for being an inquiry teacher. I saw the overall issue as one of empowering the students, but I hadn't specified how I would change my role to do that. I needed to undo my habits of directing and evaluating students' thinking. As the year progressed, I set specific goals for my behavior in the classroom. To change my role to that of researcher of students' understanding, I tried to implement the following actions:

asking questions to understand the students' thinking about a problem, not to push students toward the right answer, and asking students to explain correct answers as often as incorrect answers;

allowing plenty of wait time after asking a question for students to understand the question and to formulate their answers;

probing students' answers to learn more about their thinking and to get students to look further at their own procedures and evaluate their own answers.

Learning to teach to support students' learning is a never-ending process that is interwoven with teachers' own philosophy of learning. To make learning about students' thinking a priority requires believing that all students can think and giving up the idea that teaching is telling.

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