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ABSTRACT

This project description is designed to show how graphing calculators and calculator-based laboratories (CBL) can be used to explore topics in the physics of sound. The activities address topics such as sound waves, musical notes, and chords. Teaching notes, calculator instructions, and blackline masters are included. (MM)

Analyzing Sound Waves Produced by Musical Notes & Chords

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Use trigonometric functions to analyze the nature of notes, chords, dissonance, consonance, resonance, and beats

Teachers Teaching with Technology
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Analyzing a Sound Wave

Situation

1. A tuning fork is struck (or some other musical instrument sounded) to produce a single note.
2. The graph of the corresponding sound wave is displayed on the graphing calculator using appropriate data collection devices.
3. The period, frequency, amplitude and phase shift are discovered by analyzing the graph.
4. These values are used to determine the particular form of $Y = A \sin(Bx + C)$ that best represents the sound wave.

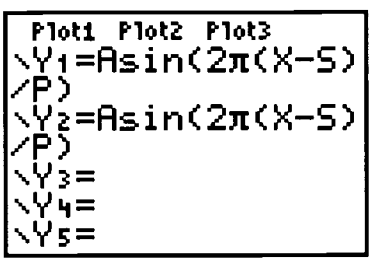
Equipment: TI-83 Plus), CBL 2, Microphone sensor, tuning fork.
(The tuning fork used for the data below was middle C – about 256 cycles per second).

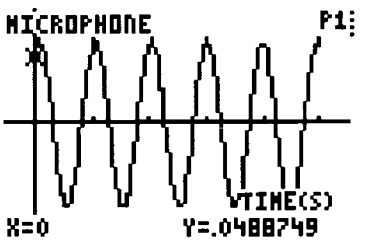
Discussion:

The usual form of the sine equation is $y = A \sin(Bx + C)$ where amplitude = A, period (P) = $2\pi/B$ and phase shift (S) = $-C/B$.
By substitution, we can determine an equivalent form of the usual equation as: $y = A\sin\left(\frac{2\pi(x-S)}{P}\right)$ where the amplitude, period and phase shift are clearly presented.

Steps to Follow:

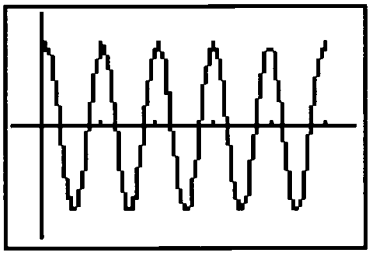
1. Enter the latter form of the sine function into Y1 andY2 of your graphing calculator. Strike the tuning fork and use the TI-83 Plus, CBL 2, microphone sensor to produce a sinusoidal graph of the note. It may take several attempts before you produce an acceptable graph.





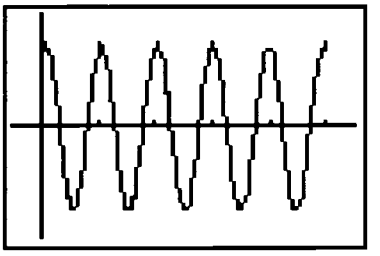
The data that produced the above graph is stored in L_1 and L_2.

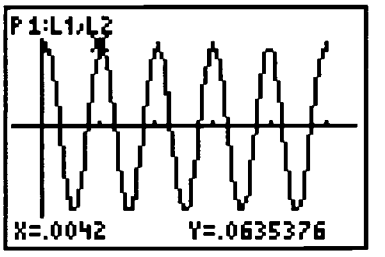
2. We will display the data again so that the words "Microphone" and "Time(s)" don't appear on the screen. (Go to the DRAW menu and select #1 – ClrDraw).

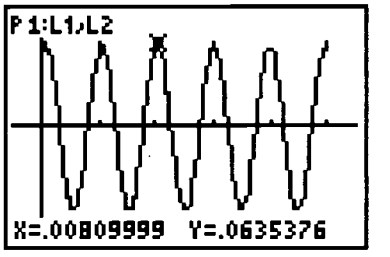


```
WINDOW
 Xmin=-.002
 Xmax=.022
 Xscl=.004
 Ymin=-.0851428...
 Ymax=.085140741
 Yscl=1
 Xres=1
```

3. One method to discover the period is to use the coordinates of two consecutive maxima. The difference between the x-values is the period (P) of the function and the y-value is the amplitude (A). We let the calculator do the recording and calculating for us.







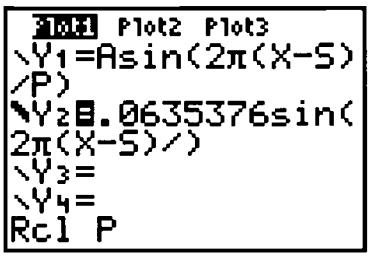

```
X→L3(1)
              .0042
X→L3(2)
          .00809999
L3(2)-L3(1)→L3(3
)
          .00389999
```

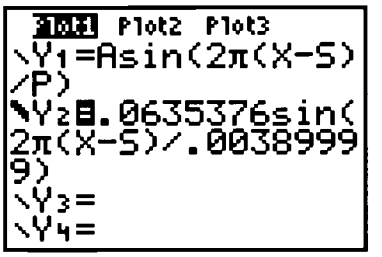
```
Y→L3(4)
          .0635376
L3(3)→P
         .00389999
L3(4)→A
          .0635376
```

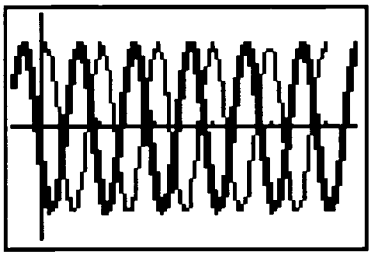
L1	L2	L3 1
0	.04887	.0042
1E-4	.05865	.0081
2E-4	.05865	.0039
3E-4	.06354	.06354
4E-4	.06354	------
5E-4	.05376	
6E-4	.05865	

L1(1)=0

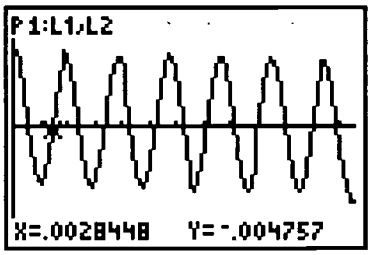
4. We now have P = $L_3(3)$ and A = $L_3(4)$. Substitute these values into Y2 and see what the graph looks like.

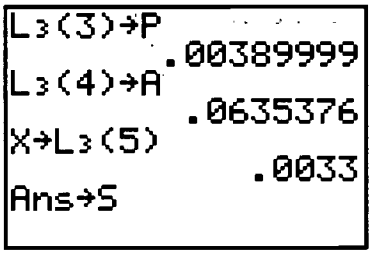


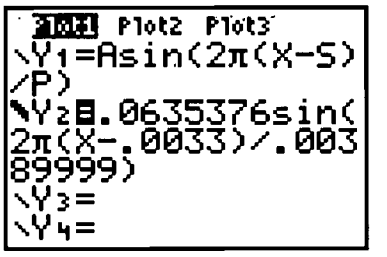




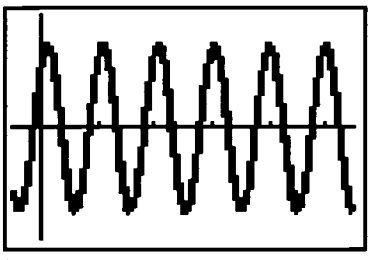
5. We are close. We just have to find the phase shift. We'll do this by determining the x-coordinate of the first full cycle of the sound wave and storing this value in $L_3(5)$ and S.



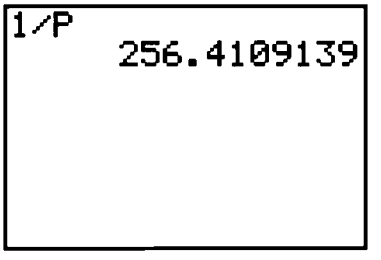


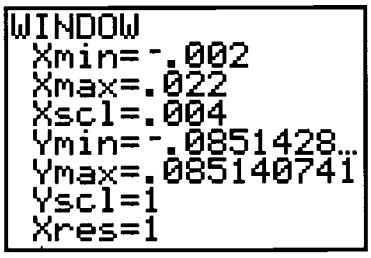


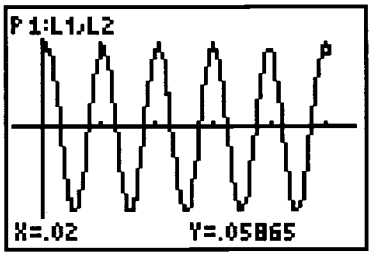
6 When we display the graphs of the original sound wave and the equation Y2 that we constructed, they very nearly coincide.



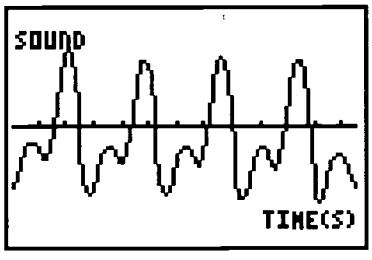
7. One final question: what is the frequency of the tuning fork that we sounded? Our period is 0.0038999 seconds. Therefore the frequency (in cycles per second) is the reciprocal of the period i. e. 1/P (not $2\pi/P$), about 256 cycles/s.



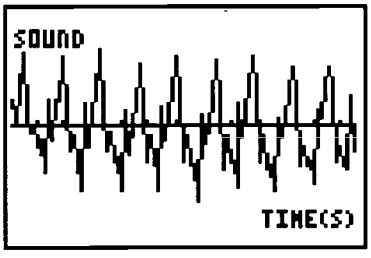




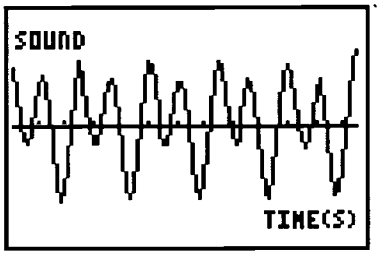
8. One final "note": a quick estimate of the frequency can be made by noting the number of cycles that the wave makes in our WINDOW of approximately 0.02 or 1/50 of a second.
We can count about 5 cycles and 5 x 50 = 250.



Piano



Pitch pipe



Voice

The last 3 graphs above represent attempts to produce a note of F below middle C (about 175 cycles/sec.). [The interval above is about 0.025 sec or 1/40 sec.]

The **piano** came close at 181 cycles/sec.

The **pitch pipe** seems to produce a graph which represents a note of 361 cycles/sec., which is F *above* middle C - a whole octave above where it should be! Why? What might be contributing to form this graph?

The **human voice** produced a note of 201 cycles per second (which is approximately G below middle C - a tone above the target sound of F).

2. Making Waves

Situation: With a set of tuning forks based on the 8-note C-Major octave, let's discover why some pairs of notes sound good together (consonance) and other pairs don't (dissonance).

Equipment : the same, with the addition of a set of tuning forks producing the following notes with their corresponding frequencies:

Tone	Frequency (cycles/s)
1 - C	256
2 - D	288
3 - E	320
4 - F	341.3
5 - G	384
6 - A	426.7
7 - B	480
8 - C '	512

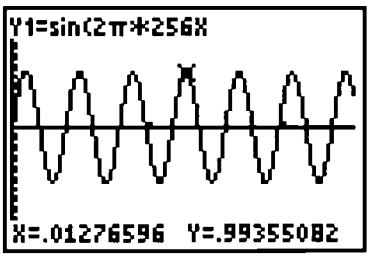
Represent these notes by equations Y1, Y2, Y3, etc. on the TI-83+ (as shown).

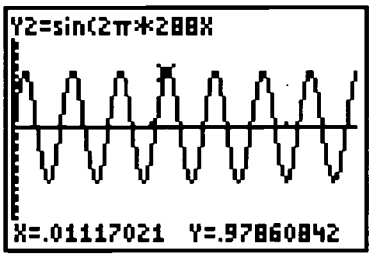
```
 Plot1 Plot2 Plot3
\Y1=sin(2π*256X
\Y2=sin(2π*288X
\Y3=sin(2π*320X
\Y4=sin(2π*341X
\Y5=sin(2π*384X
\Y6=Y1+Y3
\Y7=sin(2π*480X
```

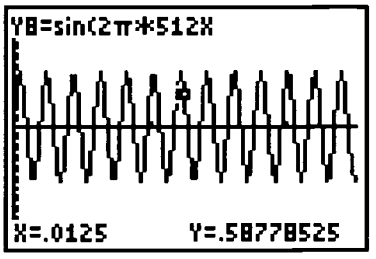
```
 Plot1 Plot2 Plot3
\Y4=sin(2π*341X
\Y5=sin(2π*384X
\Y6=Y1+Y3
\Y7=sin(2π*480X
\Y8=sin(2π*512X
\Y9=Y1+Y8
\Y0=Y1+Y5
```

```
WINDOW
 Xmin=0
 Xmax=.025
 Xscl=.01
 Ymin=-2
 Ymax=2
 Yscl=.1
 Xres=1
```

Here are the graphical representations of C, D and C '.

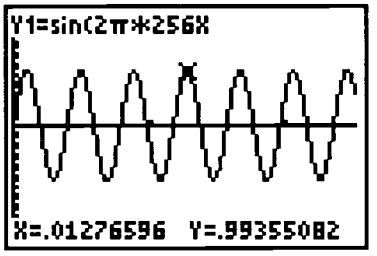


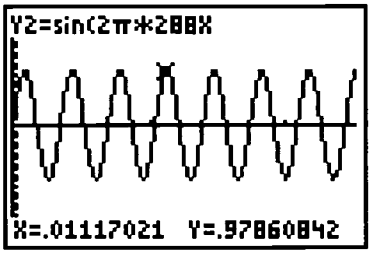


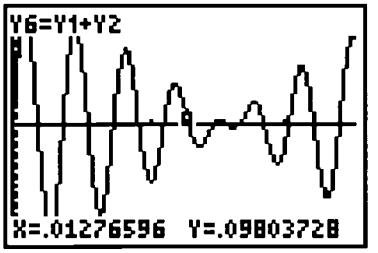


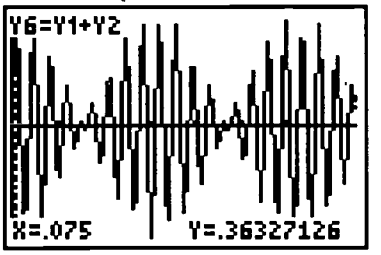
Dissonance and Consonance

Let's theoretically play a couple of notes at the same time by adding their mathematical functions. We'll start with C and D.







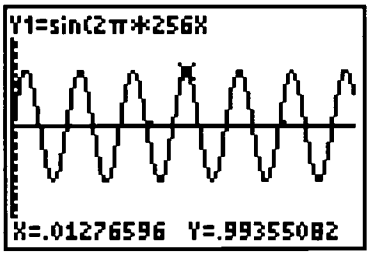


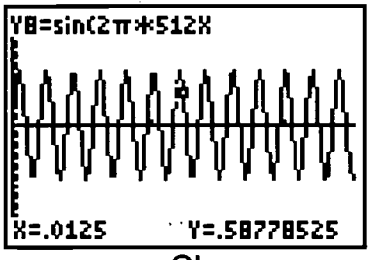
D C + D C + D

When two notes of slightly different frequencies (pitches) are sounded, an audible pulse (or **beat**) is produced at regular intervals. This is known as **dissonance** and is evident in the graph above where the combination of the first and second tones (C and D) produced a sound punctuated with periodic silences. Dissonant sounds are usually unpleasant to the ear.

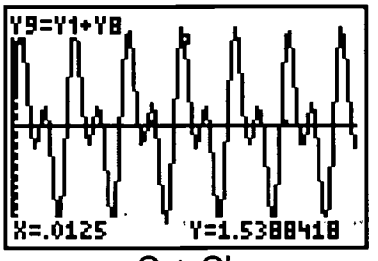
However, dissonance is fundamental to tuning instruments by ear. In stringed instruments, for example, the adjustment of the tension of a string produces changes in the frequency with which it vibrates when played. Two strings of slightly different frequencies will produce a certain number of beats per second and this number depends on, and is therefore an indication of the difference in pitch between the two notes. Chorus directors also use this phenomenon when they strive to achieve a unit sound within each of the harmony sections.

Now, let's combine C and its octave C'.

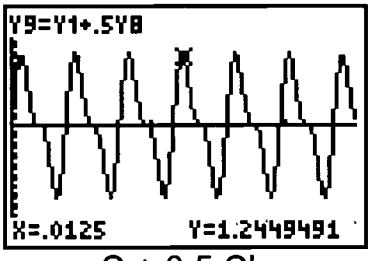




C'



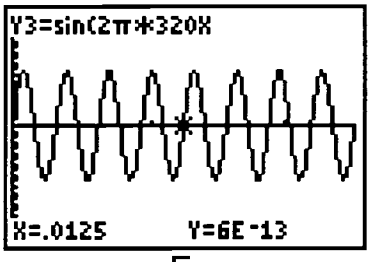
C + C'



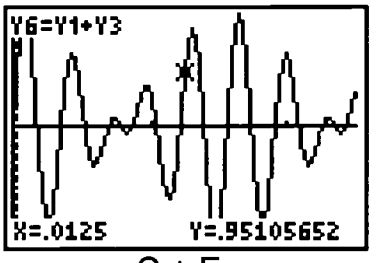
C + 0.5 C'.

The combination of these notes produces a regular pleasant sound and is an example of **consonance**. Notice the change in the graphs above when the "volume" of C' is diminished by half. This is a representation of the manner in which a chorus director will try to achieve a more pleasant, balanced sound by adjusting the volume of the various harmony sections.

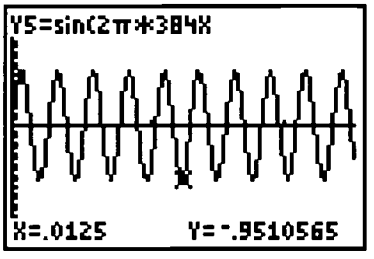
Thirds and fifths are also commonly used to achieve consonant sounds.



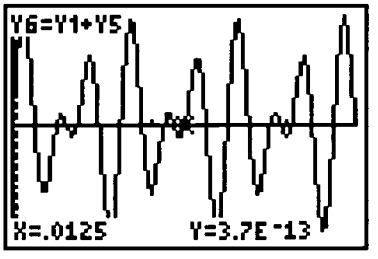
E



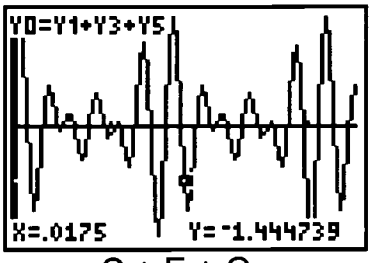
C + E



G



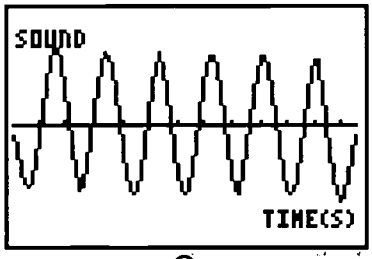
C + G



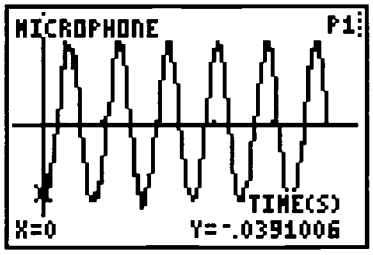
C + E + G
[C-Major chord]

Let's Get Wet!

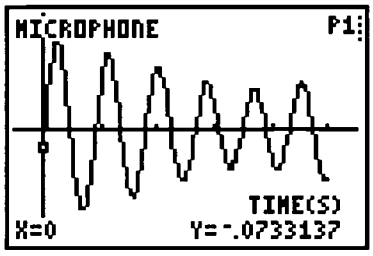
Our theoretical music above produces some nice, acceptable graphs. Let's use our CBL 2, tuning forks and other instruments to produce some real waves.
(We may get splashed in the process.)



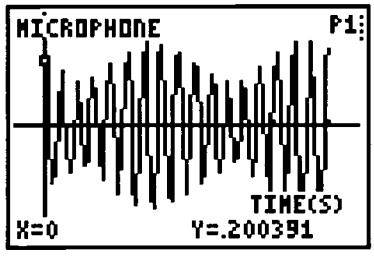
C



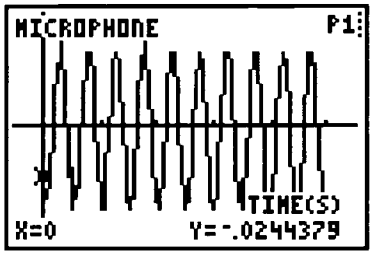
D



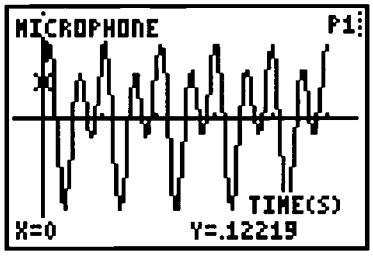
C + D



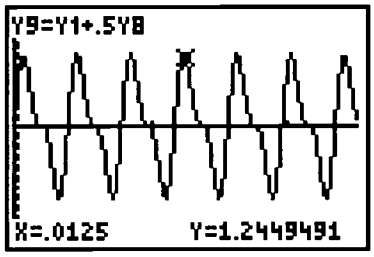
C + D



C'



C + aC'



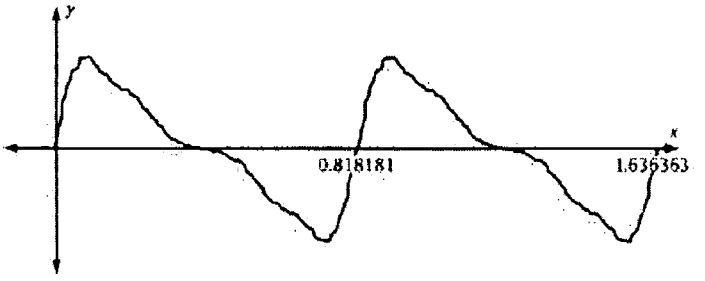
C + 0.5 C'

Hmmm… not too bad! It's interesting to compare the final two graphs above of C and its octave C' in the real and theoretical settings respectively.

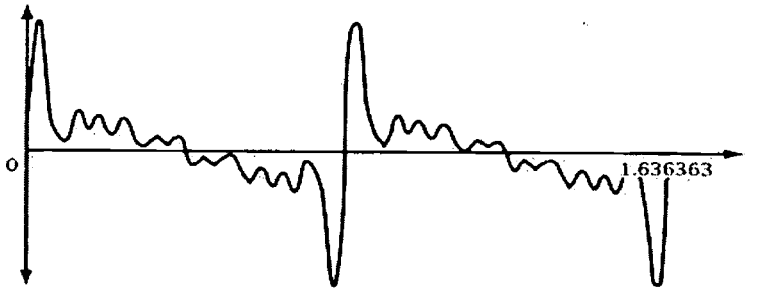
3. Tuning Forks vs Musical Instruments

While a tuning fork produces a classical sinusoidal wave as the graph of its note, the same cannot be said for most musical instruments. Because of its material components, the sound produced by a musical instrument is actually a combination of several different sounds consisting of the original note and several of its overtones (higher octave notes). The overtones are present to varying degrees of loudness which affects the resultant sound.

One reason flutes and violins sound different even if they play the same note is the presence of the overtones. The graphs shown below are those of a flute and a violin playing the same A note of 440 cycles per second. The flute produces a 'pure' sound, influenced very little by its 4 overtones. By contrast, the interesting, complex sound of the violin consists of the original A note and 14 of its overtones, many of which have relatively large coefficients.



Flute: $y = 16 \sin 440x + 9 \sin 880x + 3 \sin 1320x + 2.5 \sin 1760x + 1.0 \sin 2200x$



Violin: $y = 19 \sin 440x + 9 \sin 880x + 8 \sin 1320x$
$+ 9 \sin 1760x + 12.5 \sin 2200x + 10.5 \sin 2640x$
$+ 14 \sin 3080x + 11 \sin 3520x + 8 \sin 3960x$
$+ 7 \sin 4400x + 5.5 \sin 4840x + 1.0 \sin 5280x$
$+ 4.5 \sin 5720x + 4.0 \sin 6160x + 3 \sin 6600x$

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