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ABSTRACT
        This paper is designed to show how graphics display
calculators can be used to analyze data. The paper describes a classroom
activity, where students calculate a linear regression on a data set and
answer questions related to the regression. Teaching notes, a full solution,
and calculator instructions are included as are blackline masters. (MM)

A Data Analysis Problem That Brews

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Keith Bobay*

Paper presented at the Annual International Teachers Teaching with Technology Conference
(13th, Columbus, OH, March 16-18, 2001)

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13th Annual International T[3] Conference

Columbus, OH

March 16-18, 2001

A Data Analysis Problem that Brews

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The Waste Management Problem

The United States is the largest waste producer in the world with most of the waste being dumped into the ground. As with most states, Indiana's landfills are filling at an alarming rate. To make adequate and financially feasible plans for the future, the County Solid Waste Management needs a way to predict the amount of waste that will need placement. Listed are the tons of waste generated <u>EACH YEAR</u> for the years 1990-1997. These numbers include solid waste and material that may be recycled.

Year	Tons of Waste in Monroe County
1990	132,754
1991	135,686
1992	136,102
1993	---------
1994	138,208
1995	142,147
1996	143,256
1997	143,430

1. Using the data above, create a linear function that describes as accurately as possible the data included in the table. Use t=0 for the year 1990.

2. State the slope, y-intercept, and x-intercept of your function and explain the meaning of each in the context of the problem situation.

3. What would your function predict for the amount of waste in 1993? **IF** the trend continues, what would you predict for the amount of waste in the year 2010? in 2025?

4. What is the difference in the actual tons of waste and the amount predicted by your model for the years 1991 and 1995 respectively?

5. At the end of 1994, the Monroe County Landfill still had room for three million tons of waste. If the current trend continues, in what year will the landfill be full? Create a function that approximates the <u>cumulative</u> waste in the Monroe County Landfill beginning at the end of 1994.

6. In 1994, 46,736 tons of the total waste were recycled and not put into the landfill. If that recycling pattern remains the same, then how long will it take for the landfill to be at its capacity?

7. At what rate must the people of Monroe County recycle in order for the current landfill to be usable until the year 2050?

8. Rework problem number one using t=1990 for the first year instead of t=0. How is your model different? Why? How is it the same? Why? What would this function predict for the waste in Monroe County in 1993? 2010? How do these answers compare to your answers in problem #3? Why?

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Solutions and Comments for Teacher

1. There are several ways that students may create a linear function. The way that we will create it here is by using the regression capabilities of the TI-83.

   [2ND][MEM][ClrAllLists][ENTER]

   [STAT][EDIT](enter Years in L1 and Waste in L2) and set the window similar to:

L1	L2	L3	3
0	132754		
1	135686		
2	136102		
4	138208		
5	142147		
6	143256		
7	143430		

   L3(1)=

   WINDOW
   Xmin=-1
   Xmax=10
   Xscl=1
   Ymin=130000
   Ymax=150000
   Yscl=0
   Xres=1

Set up the ScatterPlot

   Delete or Turn Off all of the equations in [Y=], then
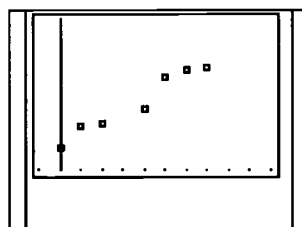   [2ND] [STAT PLOT] [PLOTS OFF] [ENTER]
   [2ND] [STAT PLOT] [ENTER] (to work with Plot 1)
   Set up Plot 1 as shown in the figure below (be sure to turn the plot on).
   Hit the [GRAPH] key.

   Plot1 Plot2 Plot3
   On Off
   Type: ▦ ⬚ ⬚
         ⬚ ⬚ ⬚
   Xlist:L1
   Ylist:L2
   Mark: ■ + ·

In order to have the calculator find and graph the regression equation, do the following:
   [2nd] [CATALOG] [D] [DiagnosticOn] [ENTER]
   [STAT] [CALC] [4] [2ND] [L1] [,] [2ND] [L2] [,] [VARS] [Y-VARS] [1] [1]

   What you should see on the screen is:

   LinReg(ax+b) L1,
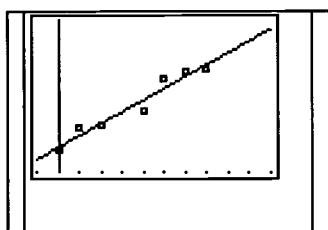   L2,Y1■

   Hit the [ENTER] key and you will
   see the regression equation:

   LinReg
   y=ax+b
   a=1559.746575
   b=133227.0479
   r²=.9540020406
   r=.9767302804

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If you look at [Y=], you will see that the calculator placed the equation there for you, too.
If you push the [GRAPH] key, you should see the regression equation graphed with the data as shown on the top of the next page.
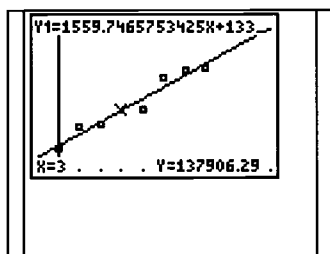

So, the function that is found using this technique is approximately: W(t) = 1559.75 t + 133227

2. We took the idea for asking these questions from Jerry Murdock. We now try to ask them for any real-world linear problem that we solve.

The slope is 1559.75 tons per year. It is the amount of increase in waste each year. The y-intercept is 133227 tons. It is the waste in tons for year 0 (1990). The x-intercept is approximately -85.4 years which is, according to the model, how many years prior to 1990 that no waste was produced. Interesting discussions can develop about the meaning of the non-integer part of the answer as well as the meaning of the part of the graph that is below the x-axis.

3. [2ND] [CALC] [VALUE] and enter "3" for x= and the calculator will return 137906.29.

NOTE: You need to change the "X-MAX" in the window to include the values 20 and 35, so change X-MAX to a number greater than 35. If this was done prior to calculating the value for x=3, then you can immediately type "20" in order to obtain the value for the year 2010 without typing anything else. Then, you can type "35" in order to obtain the value for the year 2025.

A more efficient way of obtaining values of a function, especially if finding several of them, is to use the table.
[2nd] [TBLSET] change "Indpnt" to [ASK] then [2nd] [TABLE] and input your "x" values

4.

Year	Actual	Predicted	Difference
1991	135,686	134,786.79	The model under-predicted by 899.21 tons

4

1995	142,147	141,025.78	The model under-predicted by 1121.22 tons

5. This problem caused difficulty for our students. In the problem that we gave to our students, we did not include the last sentence telling them to create the cumulative waste function. Accordingly, many students treated their answer to number 1 as a cumulative function. So, our revised problem includes the last sentence in problem 5 as well as an emphasis on the meaning of the data in the next to last sentence prior to the table of data in the original problem.

We believe that one characteristic of a good problem is that it "grows" beyond its original intent. This part (#5) of the problem "grew". We did not originally intend for it to go where it goes in the following paragraphs. The approach presented is a result of the direction established by student ideas.

NOTE: One of the difficulties that students experienced in this problem related to the different ways of referencing the years.

We can quickly generate the projected waste amounts for each year (after 1994) by doing the following:
   [STAT] [EDIT] and then move the cursor to highlight L3.
   [2ND] [LIST] [OPS] [5] [X] [,] [X] [,] [5] [,] [55] [,] [1] [ )] [ENTER]
This fills L3 with the integers from 5 through 55. We want to start with 5 because that represents the first year after 1994. (Note: The Catalog Help Apps that can be downloaded from the TI website provides the syntax for commands in the catalog.)

L1	L2	L3	3
0	132754	------	
1	135686		
2	136102		
4	138208		
5	142147		
6	143256		
7	143430		

L3 =seq(X,X,5,55

L1	L2	L3	3
0	132754	5	
1	135686	6	
2	136102	7	
4	138208	8	
5	142147	9	
6	143256	10	
7	143430	11	

L3 ={5,6,7,8,9,1...

Our next step is to generate the data for the predicted amount of waste EACH YEAR from 1995 through 2050. We will use our regression equation to generate that data in List 4.
   [STAT] [EDIT] and then move the cursor to highlight L4.
   [VARS] [Y-VARS] [FUNCTION] [Y1] [( ] [2ND] [L3] [ )]

5

L2	L3	**L4**	4
132754	5	------	
135686	6		
136102	7		
138208	8		
142147	9		
143256	10		
143430	11		

L4 =Y₁(L₃)

L2	L3	L4	4
132754	5	**141026**	
135686	6	142586	
136102	7	144145	
138208	8	145705	
142147	9	147265	
143256	10	148825	
143430	11	150384	

L4(1)=141025.7808...

Press [ENTER] and L4 will be filled with the data.

Remember that L4 is the data for each year, and we want the cumulative data. The calculator will place that in L5 for us.

[STAT] [EDIT] and then move the cursor to highlight L5.
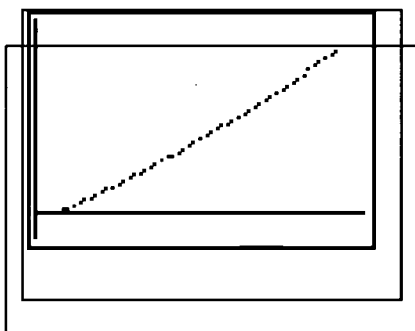[2ND] [LIST] [OPS] [6] [2ND] [L4] [ )]

L3	L4	**L5**	5
5	141026	------	
6	142586		
7	144145		
8	145705		
9	147265		
10	148825		
11	150384		

L5 =cumSum(L4)

L3	L4	L5	5
5	141026	**141026**	
6	142586	283611	
7	144145	427757	
8	145705	573462	
9	147265	720726	
10	148825	869551	
11	150384	1.02E6	

L5(1)=141025.7808...

Press [ENTER] and L5 will be filled with the cumulative data.

By scrolling down L5, we can see that the landfill would be full (that is, used up the space for three millions tons of waste) by the year 2014.
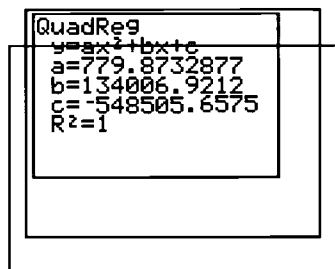
Now for the cumulative waste function. That, too, can be done by regression. We can do a scatterplot of L3 and L5 using the window x ∈ [0, 60] and y ∈ [-1400000, 11000000]

Most of our students thought that this was linear,(it does look that way, doesn't it?), and performed a linear regression:

```
LinReg
y=ax+b
a=180799.3185
b=-1081419.071
r²=.9967890138
r=.998393216
```

6

This could possibly be used as an example that even with a calculator we need to know mathematics. Although our students probably don't know this yet, as teachers, we know that the sequence in L4 that we summed is linear and the cumulative sum of a linear sequence is quadratic. Yet, the r-value of .996789 seems to support the linearity of the data. However, the data was generated by a mathematical formula, so an r-value of 1 should be expected. If we do a quadratic fit, then we get:

```
QuadReg
y=ax²+bx+c
 a=779.8732877
 b=134006.9212
 c=-548505.6575
 R²=1
```

Note: Tom Reardon suggested in his T³ presentation in 1998 that regression methods could be used to introduce sum formula that students are often given as problems for math induction proofs. That sounds like a good suggestion. If we have time today, we will revisit it.

Let's check the fit both graphically and via the lists.

We could now use the equation to solve the problem analytically: $779.8732877x^2 + 134006.9212x - 548505.6575 = 3,000,000$. We can use the calculator to solve this via the quadratic formula or by using [MATH] [SOLVER]. Once again, the Catalog Help Apps from the website may help with SOLVER.

6. One way of interpreting this problem is to determine that 46,736 is 33.8% of 138,208 and assume that the problem means to continue to recycle 33.8% of the waste. We can use L6 to be 66.2% of L5.
   [STAT] [EDIT] and then move the cursor to highlight L6.
   .662 times L5.
   Let's move L4 and L5 out of the way, so that we can match the data with the years.
   [STAT] [EDIT] and move the cursor to highlight L4 and [DEL]
   [STAT] [EDIT] and move the cursor to highlight L5 and [DEL]
   We can recall these lists, they are off the screen but still in the calculator.
   Scrolling down L6, the first value in excess of 3 million is in the year 2022 (that is, 32 years after 1990).

   This problem, too, can be solved analytically by solving the following equation as we previously solved a quadratic: $.662(.779.8732877x^2 + 134006.9212x - 548505.6575) = 3,000,000$

7. Most of our students (that is, most of those who got this far) used a trial-and-error method by changing the percentage in #6 above until they found the correct answer. This, too, can be solved symbolically: $r(779.8732877(60)^2 + 134006.9212(60) - 548505.6575) = 3,000,000$

8. We were surprised at the difficulty that our students had with this. Obviously (to us) the answers for specific years should be the same, but the input must change appropriately.

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