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ABSTRACT

The purpose of professional development that supports changes in thinking is ultimately to develop practice that will lead to deeper and more robust learning for students. There is a complex relationship between teachers' beliefs and teaching practices. To a large extent, mathematics teacher professional development programs in recent years have attempted to support teachers to change their thinking in directions suggested by the Standards vision. This paper examines the goals and practices of several exemplary, reform-oriented mathematics teacher professional development programs to better understand the changes in teachers' thinking that they call for. In reviewing this literature, it begins by asking the following questions: What kinds of changes in teachers' thinking are exemplary mathematics teacher professional development programs seeking? How do they attempt to create these changes? and How do they seek to document their effectiveness in this regard? The information gleaned from this reading is organized and categorized into an interconnected set of thematic sketches of the goals and practices of these programs. Finally, the paper examines the level of cognitive complexity and perspective-taking required of teachers to meet these several goals. It suggests the utility of such a lens for understanding the experiences of teachers in mathematics teacher professional development programs. (Contains 111 references.) (ASK)

Exploring Development in Exemplary Mathematics Teacher Professional Development Programs: "Re-forming" Teachers' Thinking [1]

By James K. Hammerman

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Introduction

Ongoing learning and professional development has long been an element of teachers' work (Lieberman & Miller, 1990). The character of this ubiquitous teacher professional development, however, is dramatically shaped by what educators think teaching is and ought to be. A technical, "industrial" view of teaching leads to professional development focused on skills and empirically-based "best practices" as exemplified by process-product models (Sprinthall, Reiman, & Thies-Sprinthall, 1996). As reform visions shift our perception of teaching to be more of a complex practice, situated in time and place, requiring careful, ongoing judgment, and the ability to balance and manage several, often divergent, perspectives and sets of values (Ball, 1993; Lampert, 1985; McDonald, 1992; Sassi & Goldsmith, 1996), professional development must change as well (Little, 1993).

While many of those who would change teaching focus on features of the environment—new structures for schools and classrooms, new curricula or new materials, and so forth—others see changes in teachers' thinking as essential. "Changing practice is primarily a problem of learning, not a problem of organization. Teachers who see themselves as learners work continuously to develop new understandings and improve their practice" (Peterson, McCarthey, & Elmore, 1996, p. 148). Professional development designed to facilitate changes in teachers' thinking is a key element in this ongoing learning.

Upon the release of the National Council of Teachers of Mathematics (NCTM) *Standards* documents (1989; 1991; 1995), the mathematics education community formally embraced these changed views of teacher professional development.[2] Building on constructivist principles and describing teaching as "a complex practice...not reducible to recipes or prescriptions," (1991, p. 22) NCTM suggests professional development will need to foster inter-related shifts in how teachers view content, learning, teaching, and evaluation. Though teachers may need to develop expertise in new skills and techniques to be effective within this reform vision, technical changes and supports are not enough (Cohen, 1990; Grouws & Schultz, 1996; Little, 1993).

Instead, what may also need to change is teachers' *thinking*—their knowledge, beliefs, values, and assumptions—about themselves, their students, the content, and the context, as well as their thinking about interactions among these (Kennedy, 1991; Pajares, 1992; Prawat, 1992; Richardson, 1996). Such changes in teachers' thinking are necessary to facilitate deeper and longer-term changes in practice and the ability to generate teaching consistent with mathematics education reform visions (Ball, 1997; Loucks-Horsley, 1997).

Yet, generating such practices may require not only a shift in ideas, but thinking that is *qualitatively* different from what was previously required of teachers. Managing and balancing diverse perspectives and values, generating practices from underlying principles without "recipes" to guide the way, and negotiating ever-changing contexts, all demand not only that teachers think *differently*, but also that the *way* they think be highly complex and nuanced. How can professional development programs support teachers to deal with and develop this kind of complexity of thinking?

A focus on changing teachers' thinking is well within the broader traditions of teacher professional development (Sprinthall et al., 1996; Zeichner, 1983). Within the framework of professional development paradigms he describes, Zeichner might characterize this focus as primarily within the "personalistic" or "inquiry-oriented" paradigms: the former emphasizing reflection on self and personal growth and development; the latter on the effectiveness and implications of current practices for children's learning and broader societal goals. Such a focus contrasts with "behavioristic" and "traditional-craft" paradigms which emphasize received knowledge and acquisition of particular practices.

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[2]Recent political controversies have somewhat loosened the embrace, however (see, e.g., California Department of Education, 1999).

The purpose of professional development that supports changes in thinking is ultimately changes in practice that will lead to deeper and more robust learning for students. Yet there is a complex relationship between teachers' beliefs and teaching practices. Not only do changes in belief typically lead to new practices, but sometimes changes in practice can lead to changes in belief (Guskey, 1986). This happens when teachers discover the effectiveness of new practices after trying them, or come to see new things in their classrooms generated by changes in practice. Professional development programs sometimes make use of these connections between thinking and practice as they develop methods for stimulating change.

To a large extent, mathematics teacher professional development programs in recent years have attempted to support teachers to change their thinking in directions suggested by the *Standards* vision. However, systematic analysis of these programs is lacking. In fact, Grouws and Schultz (1996) in their review of the mathematics teacher education literature after the NCTM *Standards* lament a lack of both empirical research (p. 443) and of unifying theory (p. 453).

To address this concern, this Qualifying Paper will examine the goals and practices of several exemplary, reform-oriented, mathematics teacher professional development programs to better understand the changes in teachers' thinking they call for. In reviewing this literature, I begin by asking the following questions:

- What kinds of changes in teachers' thinking are exemplary mathematics teacher professional development programs seeking?
- How do they attempt to create these changes? and
- How do they seek to document their effectiveness in this regard?

I then organize and categorize the information gleaned from this reading into an interconnected set of thematic sketches of the goals and practices of these programs. Finally, I step back from this description to examine the level of cognitive complexity and perspective-taking required of teachers to meet these several goals. In doing so, I hope to suggest the utility of such a lens for understanding the experiences of teachers in mathematics teacher professional development programs.

Methods

Program and literature selection

The depth and breadth of mathematics education reform work in the decade since the release of the *Standards* in 1989 makes it impractical to review the entire corpus of professional development programs, so a sampling method is required. I sought a set of diverse, exemplary, in-service programs because these are more likely to represent a range of cutting-edge thinking and to address issues raised as currently practicing teachers "reinvent" their practice to meet the demands of the reforms.

I have chosen as my data set the 14 programs in K–6 Teacher Enhancement funded by the National Science Foundation (NSF) which were represented at an invitational conference in November, 1994, and whose projects are described briefly in *Reflecting on our Work* (Friel & Bright, 1997). The NSF sought to use the conference to summarize and synthesize "what we know about teacher enhancement programs K–6 in mathematics that can inform the design of large-scale teacher enhancement programs with optimal impact" (p. 2). These "pilot" and "experimental" programs (p. 2) were considered exemplary by the NSF and they "were selected to exemplify diversity in terms of professional development models" (p. 3).

For each program, I sought descriptions of their goals and methods in books, book chapters, and articles in both research and practitioner-focused journals, as well as in papers presented at research-based conferences. The extent of literature reviewed for each program varies because of differences in how much each published. However, this is not problematic since my focus, as described in more detail below, was on developing a descriptive typology representing goals for changes in teachers' thinking across the field rather than on case studies of the programs themselves.

Analytic method

Throughout this review I looked for explicit descriptions or implicit indicators of 1) the programs' goals especially with respect to the kinds of changes in teachers' thinking they hope to achieve; 2) their practices for promoting these changes; and 3) their methods for documenting the effectiveness of those practices in reaching their goals. I coded these descriptions primarily by allowing themes to emerge inductively from the texts but also, at times, grounded in ideas from the broader literature (Patton, 1990; Rossman & Rallis, 1998).

These themes then became the focus of my analysis as I sought to gather together the several ways programs approached the themes and to present the subtle variations and distinctions among them in brief thematic sketches. These thematic sketches begin with an idea and seek to use text-based descriptions of the programs' several approaches to that idea to make it come alive. I decided to focus on the themes rather than case studies of the programs to highlight similarities and differences in approach to apparently shared goals, and to illuminate underlying patterns that might otherwise remain obscured. When possible, I tried to sketch a single coherent picture from the seeming diversity of ways programs addressed a theme. When variation was overarching, I tried to paint the nuanced shades and hues of that.

My role as researcher in this analysis is also somewhat complex. As a mathematics teacher educator, I have close professional ties with some of the programs that I am reviewing. Specifically, I was one of four teacher educators for the Mathematics For Tomorrow (MFT) project which was housed at Education Development Center (EDC) along with Teaching to the Big Ideas (TBI). I also worked with Deborah Schifter and Virginia Bastable, co-project directors for TBI, at SummerMath for Teachers from 1986-89. In addition, Rebecca Corwin, one of my readers, was a project director for Talking Math.

There are also other connections among the projects—TBI, Talking Math, and TERC *Investigations* (TERC) were all directed, in part, by Susan Jo Russell; George Bright was involved in both FIRST and TeachStat. My connections to the projects I'm researching, and their connections to one another raise two main issues: First, these 14 programs are not entirely independent, reducing the potential for overall variation among them, although common goals and methods could also have developed through everyday channels of communication and intellectual discourse. Second, my integral role in the mathematics teacher education community gives me both access to insights as a practitioner that might not be available to a more distanced researcher, and also the obligation to be especially careful about backing up my claims. While my perspective is shaped by my experiences, my conclusions are grounded in the data of the published literature.

Overview of the programs and themes

In this section I begin by reviewing details of the goals, methods, and ways of documenting effectiveness of the 14 programs, primarily in chart form. I then briefly describe the several themes which I culled from my review of these program goals, before turning in the next section to the thematic sketches which form the bulk of my analysis and this paper.

Program features

The 14 programs which form the focus of this study vary in a number of ways. Some programs focused on individual teachers as the unit of change; others looked at the school- or state-wide level. Programs spent between 1 and 6 years working with participants with most in the 2 to 4 year range. Programs offered staff development workshops, new curriculum materials, and opportunities to learn subject content or new pedagogy. They helped teachers and schools write and analyze written or video cases, develop school improvement plans, build an inquiry culture, and more. Programs occasionally documented their success by attending to student achievement, but most focused primarily on the experiences of teachers through interviews, observations, survey data, and teacher-produced materials. Substantive characteristics of the 14 programs—their foci, activities, desired outcomes, and methods for documenting effectiveness—are summarized in chart form as Figure 1. This chart represents the first set of results for this study—rough answers to the three questions noted above that guided my reading of the literature on these programs. These features along with other program details—their location, number and type of participants, and time frame—are described again, program by program in Appendix A.

Summary of change themes

In reviewing the change goals of these 14 programs I have culled several categories or themes that describe the majority of changes in knowledge or beliefs sought by these programs. While other themes are sometimes present—for example, building a collaborative community among teachers and/ or university faculty; or teachers coming to see themselves as classroom researchers—these either don't focus substantially on changes in teachers' thinking or they are limited to just one or two programs. In addition, though these categories overlap to some extent, in part because the ideas could be divided in different ways, these seven themes seem to capture importantly different elements of the foci of these programs. The seven themes are:

- **Teaching practice**: Promoting reflection on and change in thinking about teaching practice;
- **Inquiry**: Fostering inquiry and a culture of inquiry;
- **Mathematics**: Increasing or changing the nature of teachers' knowledge of mathematics and beliefs about doing mathematics;
- **Children's mathematical thinking** : Increasing teachers' knowledge of and attention to children's mathematical thinking;
- **Learning and constructivism** : Supporting teachers to re-examine their theories of learning and encouraging the adoption of a constructivist perspective;
- **Equity**: Changing teachers' beliefs to support increasing educational equity;
- **Leadership**: Supporting teachers and administrators to develop knowledge and beliefs that will foster leadership and an understanding of the change process.

In some ways, this list is not at allsuprising. Its broad categories include attention to issues that have been raised in the literature before: knowledge of subject matter and how it is learned by children; attention to the process of learning itself and how it can be supported and facilitated by different approaches to teaching; attention to a variety of contextual variables including culture, community, and characteristics of students; and for leaders who are taking on some of the role of teacher educator, ameta-level version of all of these in understanding special issues that arise when the content is teaching as well as mathematics and when the students are teacher-peers rather than elementary and middle school students. Figure 2 charts which programs focus on which of these several goals.

Elements of this summary may be interesting—notice, for example, that all programs focusing on Learning/ constructivism also focus on Inquiry, but that Equity and Leadership concerns seem independent of an Inquiry focus. Yet, detailing some of the variation and nuances within this broad typology is essential to understanding the nature of change in thinking sought by these teacher professional development programs, which in turn may help us better understand their effectiveness at reaching their goals. I turn next to the sketches of each theme which will describe this variation and which forms the bulk of my analysis.

Sketches of change themes

Teaching practice

Types of sought-after changes in practice

All the programs reviewed focus in some way on changes in teaching or beliefs about teaching. Sometimes the focus is on specific, technical changes in practice or on beliefs about the importance of integrating into practice new techniques, materials, or methods for

Figure 1: Program features

Project Name	Foci	Structures/ activities	Desired outcomes	Evaluation methods
California Mathematics Leadership Project (CMLP)	Mathematics & attitudes towards math; Constructivism; Leadership	Inservice workshops; Ongoing support for change in teaching and for leadership; Parent education	School-wide restructuring and change in culture; Change in teaching practice; Increased leadership capacity	Informal assessment of attitudes
Cognitively Guided Instruction (CGI)	Children's math understanding; Constructivist teaching	Two-week summer inservice; Academic year (AY) workshops; Various structures for several programs	Change in understanding of mathematics & children's math thinking; Constructivist teaching practices	CGI beliefs scale survey; Classroom observations; Teacher interviews & informal focus groups; Measures of children's learning
Elementary & Middle School Math & Technology Project (EM-MAT)	Math content; Change in teaching methods	Two-week summer math course; One release day/ month; Money for materials, computer, phone, modem; Classroom support; Support network	Develop math teacher leaders	Pre- & post-program surveys re: beliefs & behaviors; Case studies of teaching; Student math belief survey
FIRST Project (FIRST)	School improvement plans; Reform oriented change in teaching	Develop school improvement plans (SIPs); Year 1: Three-week summer workshop; Year 2: One-week summer workshop; Building level workshops to implement SIPs; Leadership training	Increase positive attitudes to math/ science; Increase math/ science teaching competency; Schoolwide improvement plans	Needs assessment instrument re: content, instruction, assessment & class climate; Monitor student performance, instructional practice, visions of reform
Project IMPACT: Increasing the Mathematical Power of All Children & Teachers (IMPACT)	Mathematics; Constructivism; Equity; Change in teaching; Children's math understanding	Summer inservice; School based math specialist for ongoing support	Increase math understanding of all; Change in predominantly minority schools; Constructivist teaching	Quantitative & qualitative data re: teacher change; Modified CGI beliefs scale; Phase I: Matched school comparisons

Figure 1: Program features, continued

Project Name	Foci	Structures/ activities	Desired outcomes	Evaluation methods
Kentucky K–4 Mathematics Specialist Program (Kentucky)	Change in attitudes and views of mathematics; Change in teaching strategies; Increased political voice for teachers	45 hour summer seminar; Specialists support teachers in classrooms; Specialists meet together monthly	Create network for change in teaching	Change in beliefs assessed through writing prompts & surveys; Specialist feedback; Interviews & observations of teachers & kids
Mathematics & Science Enhancement (MASE)	Leadership; Restructuring; Capacity for teacher decision-making; Math & science content; Constructivism; Children's thinking	Leadership development; Workshops, seminars, classroom demonstrations; Administrator workshops	Develop leaders/ change agents; Build capacity for knowledgeable decisions; Constructivist teaching; Change in math beliefs	Internal staff evaluation via written response, learning logs, classroom visits, student/ teacher portfolios, & teacher self-assessment
Mathematics for Tomorrow (MFT)	Mathematics; Constructivism; Change in teaching practice; Inquiry culture; Teacher leadership	Three-week summer institutes in two successive years; Biweekly district-based inquiry groups; Four day-long workshops / year; Classroom consultations; Administrator inquiry group; Inquiry group Sourcebook	Inquiry culture around math and teaching; Deepen math knowledge & change beliefs; Constructivist teaching; Administrator learning	Case studies of teacher change—observation & interviews; Teacher reflective journals
Talking Math (Talking Math)	Mathematical discourse; Children's thinking; Mathematics	Three-week summer seminar; Biweekly AY seminar; Resource package	Inquiry community; Increase math knowledge & confidence; Pedagogical development	Interviews; Videos of classrooms; Teacher writings
Teaching to the Big Ideas (TBI)	Mathematics; Children's mathematical thinking; Teaching; Leadership	Summer institutes; Biweekly AY seminars on math content & instructional implications; Biweekly classroom visits; Year 4: Teachers run peer workshops & seminars; Create professional development materials	Deeper math understanding; Constructivist teaching; Inquiry culture around math, kids' thinking, & teaching	Regular teacher writing; Classroom observation

Figure 1: Program features, continued

Project Name	Foci	Structures/ activities	Desired outcomes	Evaluation methods
Teach- Stat (Teach- Stat)	Statistics content; Change in teaching practice; Leadership	Year 1: Three-week summer pilot at one site for leaders; Years 2 & 3: Three-week summer at nine sites for teachers; Apprenticeship / coaching; Materials development for replication	Cadre of leaders for consistent state-wide professional development; Integrating statistics content into elementary curriculum	Phone interviews; Statistics content test; Classroom visits; Pedagogy & impact surveys; Informal reports; Student achievement
Teaching Excellence And Mathematics (TEAM)	Leadership; Math content	Three-week summer institutes; Three AY 1.5-2.5 day workshops; Years 3 & 4: Teachers prepare staff development materials; Teacher-run workshops for peers, administrators, parents, school boards	Leadership development; Increase math content knowledge; Develop school improvement plans	Count number & types of workshops; Teacher portfolios; Math post-test; Workshop evaluations
TERC *Investigations* Curriculum (TERC)	Math content; Children's thinking; Teaching practice	Formative & pilot testing; Materials development and distribution; Coordination with professional development programs	Curriculum that also supports ongoing math & pedagogy learning for teachers; Development of teachers' professional expertise	Observations & interviews in pilot testing; Observation of use of curriculum in professional development
Lead Teacher Program of the Virginia Quality Education in Science & Technology (V-QUEST)	Leadership support; Math content; Teaching issues	Two-week & one-week summer institutes; Two AY conferences; Monthly contact & support; Involve principals	Statewide K–14 reform; Leadership development & support	Collect systemic indicators; Ongoing workshop evaluation data

Figure 2: Thematic Goals of the 14 Programs

	Teaching practice	Inquiry	Math	Children's math thinking	Learning/ Construc -tivism	Equity	Leadership
CGI	X	X	X	X	X	X	
CMLP	X	X	X		X		X
EM-MAT	X		X			X	X
FIRST	X		?				X
IMPACT	X	X	X	X	X	X	
Kentucky	X	X	X			X	X
MASE	X	X	X	X	X	X	X
MFT	X	X	X	X	X		X
Talking Math	X	X	X	?	X		
TBI	X	X	X	X	X		X
Teach-Stat	X	X	X		?		X
TEAM	?	X	X				X
TERC	X	X	X	X			
V-QUEST	X		X			?	X

Note: X = explicitly present; ? = hinted at or uncertain

classroom organization such as manipulative materials, computers, calculators, collaborative or cooperative learning structures, alternative assessment methods, or problem-solving approaches (Kentucky K–4 Math Specialist Program (Kentucky): Bush, 1997; Elementary and Middle School Math and Technology project (EM-MAT): Grady, 1997; Lead Teacher Program of the Virginia Quality Education in Science & Technology (V-QUEST): Underhill, Abdi, & Peters, 1994).

Sometimes programs are also interested in deeper changes in beliefs and instructional practices, focusing on changing discourse and authority patterns, promoting new kinds of questions to deepen thinking, and developing an inquiry culture in the classroom, among others. These programs want to help teachers "reshape their own philosophy and classroom practices" towards a view of teaching as facilitating inquiry rather than telling (Mathematics And Science Enhancement (MASE): Gregg, 1997, p. 220); or to "offer experiences that challenge dominant instructional paradigms" and that "facilitate construction of a new pedagogical theory and practice" (TBI: Schifter, Bastable, & Russell, 1997, p. 256). While these programs may also be interested in specific technical changes in instruction such as increased use of manipulatives, problem-solving, or alternative assessment, such changes are considered to be tools in the service of the deeper changes in instructional paradigms which are their true focus.

This distinction between technical changes in practice and the deeper changes in thinking underlying them also appears in the broader mathematics education literature. Spillane and Zeuli (1999) describe such a difference in distinguishing between "behavioral" and "epistemological regularities" of teaching and their respective roles in implementing mathematics reform, seeing epistemological regularities as harder to change than behavioral regularities. The reforms, they say, are looking for a shift in tasks focused on principled rather than procedural mathematical knowledge, and in discourse patterns focused on students' reasoning and justification rather than on teachers' explanations and a search for single right answers or solution strategies. As part of a larger, long-term policy study, Spillane and Zeuli carefully observed 25 reform-oriented elementary and middle school teachers in six diverse districts in Michigan to "better understand the nature of reformed practice in classrooms where it was more likely to be consistent with reformers' ideals and the challenges teachers faced in enacting these ideals" (p. 4).

Analyzing their observations, they find three basic patterns of implementing the reform (Spillane & Zeuli, 1999, pp. 8-18): One (seen in 11 teachers of 25), in which teachers may talk about reform and in which specific teaching behaviors may have changed, but in which "the intellectual core of their practice, as captured in task and discourse norms, remained unchanged, firmly grounded in procedural knowledge and computational skills"; a second (ten teachers of 25) which focuses on problem-solving around tasks that *could* embody principled mathematical knowledge, but whose discourse patterns don't support reasoning about these ideas and therefore lose this potential; and a third (seen in four teachers of the 25) in which tasks and discourse patterns have changed to allow student inquiry and discussion around principled mathematical ideas (p. 18).

Simon and Schifter (Schifter & Simon, 1992; Simon & Schifter, 1991) make a similar distinction between changes in reform-oriented "strategies"—specific teaching practices including use of non-routine problems, manipulatives, diagrams, and alternative representations; exploring alternative solutions; employing wait time; encouraging student paraphrase of ideas expressed in class; etc.—as measured by the Levels of Use (LoU) half of their assessment instrument; and constructivist "epistemology"—teaching which "strives to maximize opportunties for students to construct concepts"—as measured by the Assessment of Constructivism in Mathematics Instruction (ACMI) half of their instrument (pp. 324-5). They, too, see changes in epistemology and the deeper changes in teaching practice associated with them as more difficult to achieve.

These patterns of change in teaching are also observed within the projects themselves. Distinguishing between externally observable changes in behavior and deeper changes in thinking that affect practice, Gregg hints that the former may not really constitute instructional change.

> Use of hands-on experiences and thematic units in mathematics and science can mask weak programs and create an illusion of reform and student learning. It is much easier to incorporate manipulatives, science kits, and thematic units into the classroom than actually change instructional practice. (MASE: Gregg, 1997, p. 218)

Project IMPACT describes the effect of the program on teaching practice (Campbell, 1996), saying "10%-15% of teachers have made no real change in their teaching"; and that "15% have changed their instructional style...but do not yet reflect on the understandings of their students." These teachers use manipulatives and group work, offer wait time, ask for alternative solution strategies and explanations of them but focus on correct answers without probing for understanding, and may or may not attend to issues of equity.

Another "40% have changed their instruction to reflect a constructivist perspective" and focus on "what his or her class understands...[while] keeping their mathematical goal or objective in mind" though "they may sometimes generalize the responses of a few children to be the understanding of the entire class." These teachers attend to issues of equitable participation, and use changes in teaching practice to support children's construction of deeper mathematical understanding.

Finally, about 30% of IMPACT teachers "know their curriculum and make links between mathematical topics" as they "focus children on thegeneralizability of mathematical ideas." They revise and redefine activities as they "reflect on the task at hand and the needs or strengths of her or his students" and they "use questioning to focus children and to address mathematical reasoning and connections.... These teachers constantly ask themselves what individual children understand, reflecting on the meaning of children's explanations and strategies" (Campbell, 1996, pp. 466-8). Notice that these levels of increasing success with IMPACT require greater and greater coordination of perspectives and viewpoints—from a focus merely on tasks, to a focus on children's thinking broadly defined, to a coordination of that with the mathematical ideas of the curriculum, to attention to the specific thinking of several children at once.

In their study of teachers four years after participation in Cognitively Guided Instruction (CGI) workshops, Knapp and Peterson (1995) describe three groups: Users of CGI (Group 1), Non-users (Group 2), and Declining users (Group 3). Non-users were those who interpreted the program as about specific practices rather than conceptual change—Knapp and Peterson define real change as requiring a change in beliefs about using children's thinking to inform instruction.

> [Users] defined CGI in conceptual, almost philosophical, terms like "building on children's already previously accumulated knowledge." [Non-users], however, saw CGI as a set of procedures, "using manipulatives" and teaching "strategies to solve word problems better." (p. 55)

Declining users espoused support for CGI beliefs but their "development ofCGI often seemed blocked by their interpretation of it as attached to specific classroom procedures" (p. 62). Though Group 3 teachers never "reported deciding to useCGI less because they valued other ways of teaching mathematics more" (p. 62), like Group 2 teachers, they saw the program as about implementation of procedures. This emphasis on behavioral rather than psychological features of the program kept them from succeeding with it. Knapp and Peterson also highlight the location of mathematical authority in distinguishing Users from Non-users:

> Perhaps the most clear-cut evidence of this difference came from teachers' reports of how knowledge was justified in their mathematics classes. Every Group 1 teacher described encouraging her students to justify their answers themselves...whereas Group 2 teachers all reported being the primary arbiter of correct and incorrect answers in their classrooms. (p. 53)

Thus, in both theory and practice, there seems to be a continuum of how reformed mathematics teaching is enacted with changes in behaviors tending to be easier thandeeper and more generative changes in epistemology.

Methods for facilitating change

What do programs that are seeking instructional changes based in these deeper epistemological shifts do to accomplish their goals? Typically they provide some mechanism for bringing in data about teaching and its impact on student learning—videotape excerpts, teacher-written cases or episodes, teachers' own experiences learning mathematics, readings about practice, audiotapes or transcripts of classroom dialogue, and so forth, that can serve to generate "shared images of instruction" (Talking Math: Corwin, 1997, p. 190) to be held in common within the group. These data become the basis for reflection and discussion—for analyzing the impact of particular practices on student learning, for identifying dilemmas of practice, for generating alternatives and possibilities and playing out their implications, and for uncovering hidden assumptions about learning and the role of a teacher in facilitating learning. Teachers might grapple with how to set up discussion of a problem to get at deep mathematical ideas, with ways to draw out students' thinking, with decisions about sharing mathematical strategies, affirming particular solutions, choosing which ideas to pursue or not, and so forth (see, also, Ball & Cohen, 1999).

In large part, these professional development discussions provide a vehicle for promoting more robust teacher decision-making—an explicit goal of several programs (IMPACT: Campbell & Robles, 1997; MASE: Gregg, 1997; TERC: Russell, 1997). This is consistent with the *Standards* which, when describing the teacher's role in discourse, for example, ask teachers to decide "what to pursue in depth from among the ideas that students bring up during a discussion; when and how to attach mathematical notation and language to students' ideas; when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty" (NCTM, 1991, p. 35). These decisions are not at all easy to make, especially since this new kind of teaching "confronts—and embraces—the uncertainties of learning and teaching" (Ball, 1997, p. 80).

> Three sources of uncertainty stand out as endemic to this kind of teaching: the inherently incomplete nature of knowledge in teaching, the multiple commitments with which teachers work, and trying to teach in ways that are responsive to students. (Ball, 1997, p. 80)

Given these uncertainties, regular and ongoing discussion of dilemmas of practice is more likely to promote the development of teachers' capacities to make good decisions than will specific prescriptions for practice. Such discussions may lead to changes in teaching behaviors, but also to teachers' increased capacity to think about when and how to use these behaviors as they surface and learn to coordinate their multiple commitments. Such discussions also help teachers identify their assumptions, hear multiple perspectives, and reflect on the nature of learning and teaching itself.

Differing perspectives on the importance of specifying teaching behaviors are also played out among programs in their approach to providing lessons to be used directly in classrooms. Some more technically-oriented programs seem to pride themselves on offering teachers "classroom-tested" lessons or activities designed by more experienced teachers (e.g., EM-MAT, Teach-Stat, Teaching Excellence And Mathematics (TEAM)). These activity ideas, often shared in a "traditional-craft" model style (Zeichner, 1983), are intended to "work" but may not change teachers' thinking either through engagement with the activities themselves or by ensuring that something different happens in the classroom upon which teachers can reflect.

Other programs explicitly reject this approach, arguing that teachers don't have the time and experience to design really good curricula (e.g., California Mathematics Leadership Program (CMLP), IMPACT, MASE, TERC). These programs prefer offering materials designed by experts who have the time and knowledge to create activities and entire curricula that will engage students, and therefore their teachers, in different ways with mathematical ideas.

> The common practice of bringing together good teachers and providing time for them to write curriculum that can be shared with other teachers is often counter-productive. Writing good mathematics curriculum that can be used effectively by others is extremely difficult. (CMLP: Parker, 1997, p. 240)

The inclusion of the TERC *Investigations* curriculum in this set of programs offers a fascinating opportunity to explore the assumptions surrounding how curriculum can support changes in classroom practice. The TERC curriculum is innovative in that it integrates professional development support into the written material surrounding the activities themselves. This includes descriptions of underlying mathematical ideas and what's difficult about these for students; classroom dialogues to illustrate how children might talk about the ideas; and suggestions about different ways to approach using the materials.

Clearly TERC feels it is important to offer teachers ideas for what to do in their classrooms and to generate these ideas collaboratively among experts and classroom teachers throughout the curriculum development process. However, Susan Jo Russell, the project director, argues against several common views of curriculum in describing the purpose of the TERC curriculum. She says it is neither the "'teacher-proof'...Right Way" which will inevitably lead to student learning; nor the "necessary evil" required only until teachers have the knowledge and experience to continually design their own curricula in response to student needs; nor even the "reference material" to stimulate new thinking because teachers can't do all the curriculum design work themselves (Russell, 1997, p. 247). Instead of any of these, TERC sees "teachers and curriculum as partners" seeking to create the conditions for "intelligent teachers using intelligent curriculum intelligently" (p. 254) and so, offers support for teacher development as an essential component of the curriculum itself. Other programs also support such uses of curriculum materialsTBI, for example, sees the use of exemplary materials lik*Investigations* as a vehicle for stimulating changes in practice that might then change beliefs.

CGI chose yet another approach: not providing instructional activities for teachers at all. Instead, they asked teachers to use their focus on children's mathematical thinking to decide about any materials, believing both that teachers were better situated than program staff to make decisions about materials, and that the process of deciding would increase teachers' sense of ownership of the ideas.

> We did not include prescriptions of instructional activities teachers were to implement because we did not believe that we could specify instructional activities and materials that would take into consideration what we knew about teachers' thinking (Fennema, Carpenter, & Peterson, 1989). We anticipated that not only would teachers be better translators of knowledge about children's thinking into practice than we would, but having the opportunity to make the translation would enable them to assume ownership ofCGI ideas. (CGI: Fennema, Franke, Carpenter, & Carey, 1993, pp. 580-1)

This is not to say that CGI and other like-minded programs ignore teachers' need for good materials—many of these programs help teachers develop criteria for choosing and modifying commercially available materials to better meet reform goals. The experience of the IMPACT Project is instructive in this regard. They experimented with the idea of providing teachers with "ready to use materials" but discovered that "this was a mistake" (Campbell & White, 1997, p. 324).

> Teachers seemed to rely on these sample activities without fully addressing their mathematical potential. These teachers communicated the intent of the tasks, fostered the children's completion of the activities, and had some children share their strategies. At the same time, however, there seemed to be inadequate discussion of the mathematics underlying the tasks. (Campbell & White, 1997, p. 324)

To address these problems, the project developed three different ways to engage teachers with both the mathematical concepts and how the curriculum can elicit those in order to become more informed users of materials. These included review of commercial materials for their mathematical potential, review of a task explicitly focusing on pedagogical issues such as developing questions around it to ascertain student understanding, or modification of a mathematical exploration engaged in by teachers to make it effective with children. These methods yielded activities that teachers might use in their classrooms, but grounded the activities in a deeper understanding of the mathematics and how children might connect the tasks with important mathematical ideas. Teachers could experience themselves*deciding* to use materials based on principles of learning rather than merely because they were offered by the program.

The key difference, I think, among all these approaches to offering classroom materials to teachers is whether the curriculum is seen as an end in itself—tasks to be done as such—or as a vehicle for ongoing professional development by stimulating reflection on issues of mathematics, learning and teaching.

While reform-oriented changes in teaching practice clearly include specific new behaviors, they may also involve deeper changes in epistemology which have an impact on patterns of authority and discourse in the classroom. Programs often attempt to facilitate reflection on these deeper issues by generating and discussing "shared images" of practice via curricula, videotapes, and other case materials. This reflection is intended to support teacher decision-making informed by a broader and deeper set of understandings about mathematics, how children learn it, and teaching that can facilitate it. For many programs, this type of reflection is best accomplished through a process of inquiry into teaching practice that is comparable to what they often desire and implement for mathematics learning.

Inquiry

All but a few of the programs (EM-MAT, FIRST and V-QUEST) hope to build inquiry attitudes and cultures among teachers and sometimes in classrooms. They describe this hope as helping teachers develop an inquiry attitude (TBI), an exploratory mindset (Talking Math), a stance of inquiry (TERC), a "spirit of inquiry and continuous learning...so teachers can reshape their own philosophy and classroom practice" (MASE: Gregg, 1997, p. 219), or as creating "an environment of inquiry and reflection" grounded in sufficient mathematical and pedagogical knowledge (Kentucky: Bush, 1997, p. 176).

Some programs are primarily interested in supporting the development of inquiry among students in classrooms—a goal which requires shifts in teachers' beliefs about classroom learning, often along constructivist lines. Most programs, however, are also interested in developing inquiry attitudes and habits among teachers within professional development contexts. Programs may ask teachers to bring a variety of data from their practice to support this inquiry—for example, CGI asks teachers to engage in "practical inquiry" around students' mathematical thinking observed in their classrooms; Talking Math promotes the careful examination of classroom mathematical discourse; and TBI requires teachers to regularly write classroom episodes focusing on children's mathematical thinking.

Some programs describe what goes on for teachers in reflective discussions of these data. By creating "common experiences of carefully crafted model lessons and selected readings" MASE creates opportunities for dialogue around shared reference points so that "conflicting definitions and interpretations can be explored" and teachers can "re-examine their own beliefs, assumptions and practices" (MASE: Gregg, 1997, p. 218). Talking Math describes teachers "working together to generate their own theories of how mathematical discourse can best be supported in their classrooms" (Talking Math: Russell & Corwin, 1993, p. 558). MFT describes their inquiry groups as focusing on "the delicate middle ground" between theory and practice in order to "create real intellectual discourse and investigation tied to the particulars of teaching practice" (MFT: Hammerman, 1995a, p. 268). TBI's episodes provide "images which offer interpretations of a reform agenda which is still foreign to many teachers, [and which] supports both the development of visions of possibility as well as more fine-grained inquiry into specific aspects of the new pedagogy" (TBI: Schifter et al., 1997, p. 258). All of these methods ask teachers to step back from the data at hand to examine their own assumptions about what it means, to generate multiple possibilities and hypotheses to explore, and to collaboratively develop their own theories of practice—demands which may require a high level of cognitive complexity and perspective taking.

These kinds of inquiry require a substantial shift in the norms and values of teaching, towards those "where risk-taking is valued and change is expected" (CMLP: Parker, 1997, p. 239), and which support "ongoing intellectual curiosity for all" in which teachers can "explore and express doubts and uncertainties" about students' mathematical thinking and about pedagogy which might best help it grow and develop (MFT: Nelson, 1997c, pp. 231-2). New norms within teacher discussion groups might include:

> ...deep respect for teachers as learners and for the effort required to learn dramatically new ways of looking at and being in the world, a focus on judging ideas rather than individuals, a focus on intellectual rather than technical content, a respect for novel and diverse ideas, and clear expectations that [teachers] will grapple with new ideas even if these are difficult. (MFT: Hammerman, 1995b, p. 53)

By promoting values that support collaborative inquiry, programs create an environment that can challenge traditional norms of isolation and certainty in schooling. Several programs want teachers to become comfortable seeing knowledge about teaching as uncertain and constructable by the community of teachers (see, also, Ball, 1997), but they also recognize the cognitive and emotional difficulties associated with this change.

> Teachers will need to learn how to pose their own questions and will need reassurance that new cultural norms now allow them to freely discuss their own problems. And the school community will need to learn how to cope with the anxiety of leaving questions open long enough to examine them from a variety of perspectives. (TBI: Schifter et al., 1997, p. 259)

> How can the group support teachers to become more comfortable *being* confused and dealing with the inherent ambiguities of this new view of mathematics teaching, while they work ideas out? Can groups develop images of *uncertainty* that enable teachers to maintain a sense of control? (MFT: Hammerman, 1995b, p. 51)

Creating a community of discourse about practice among teachers is an important component of this shift in norms. Working together can nurture and sustain teachers as they explore these often uncomfortable new ways of being and thinking. While inquiry can and does happen in solitary reflection, a community can help teachers identify and challenge assumptions, generate a broader range of possibilities, and encourage ongoing attention to the intellectually and emotionally hard task of reflection and change (MFT: Hammerman, 1995b; Talking Math: Russell & Corwin, 1993; TBI: Schifter, Russell, & Bastable, 1999c).

Finally, it is interesting to note that inquiry goals frequently extend beyond programs' own participants—it seems that projects often want inquiry to be a larger part of the broader context of educational change and so it becomes a central part of their dissemination efforts. For example, in some of its articles for practitioners, the IMPACT Project suggests "Action Research Ideas" as ways to investigate aspects of equity and pedagogy (Campbell, 1997). MFT published an Inquiry Group Sourcebook (Davenport et al., 1998) to support the creation of and running of inquiry groups in the wider community focusing on mathematics, learning, and pedagogy. TBI has created the Developing Mathematical Ideas (DMI) professional development curriculum (Schifter et al., 1999a; Schifter et al., 1999b) to engage other teachers in exploration of the ideas embedded in episodes written by TBI teachers. Talking Math has published a multi-media resource package to support investigation of discourse (Corwin, Storeygard, Price, & Smith, 1996a). And the TERC *Investigations* curriculum (TERC, 1994) is designed to promote reflection on and continued growth stemming from the activities offered through the "collaboration of teachers and the curriculum" described above.

By focusing on inquiry, these programs attempt to sustain teachers' growth and development by embedding new norms and values in the culture of teaching. Next, I examine the topics towards which inquiry attitudes are usually focused.

Mathematics

Though one might expect teachers of elementary grades to understand the *mathematics* of what they teach, this is not necessarily the case (Ball, 1991; Schifter, 1993). Because reform visions of mathematics education call for a radical shift in the conception of what mathematics is and what it means to do mathematics—from a set of facts and procedures to be acquired towards a set of concepts to be constructed and understood (NCTM, 1989; 1991)—it is incumbent upon teacher professional development programs to help teachers develop these new ideas. And they *do* take on this responsibility. In fact, all of the programs reviewed except the FIRST Project seek changes of some kind in teachers' personal mathematics knowledge. Many, including FIRST, also seek changes in teachers' beliefs about the nature of mathematics or in their attitudes towards mathematics. Yet what they intend by these goals and their methods for accomplishing them varies in important ways.

Most programs hope to give teachers a deeper understanding of mathematical ideas and processes through experiences doing mathematics, though the level of mathematical activity, the method of engaging mathematical ideas, and the purpose in doing so varies. Teacher educators hope this deeper understanding of mathematical concepts and methods will yield a richer mathematical discourse in teachers' classrooms as teachers use their new understandings in the service of pedagogical decision-making. As Ruth Parker writes about the CMLP program, "Teachers cannot teach mathematics differently until they have experienced mathematics differently" (1997, p. 238).

Mathematical topics

The mathematical content of the programs varies. Some, like TeachStat and CGI, focus on specific mathematical areas, while most take a more eclectic approach. TeachStat focuses on "statistical content and statistical investigation" seeing it as an "interdisciplinary process" that can draw together different content areas (Friel & Danielson, 1997, p. 198). CGI's focus is on arithmetic operations—addition and subtraction, multiplication and division—and especially on communicating a research-based model of how *students* understand these operations and their connections to real-world situations in increasingly abstract ways (Peterson, Fennema, & Carpenter, 1988). In the process of exploring the ways that students' ideas can grow in each domain, teachers, too, come to a deeper view of the mathematics itself (Franke, Fennema, & Carpenter, 1997).

The topics addressed by more eclectic programs (when they're described explicitly) cover a wide range. They include number systems and place value, number sense, and whole number operations; rational numbers, fractions and ratios, and operations with these; measurement; graph construction and interpretation; geometry; statistics and data analysis; combinatorics and probability; variables, functions, patterns and algebraic thinking (Kentucky: Bush, 1997; IMPACT: Campbell, 1996; 1997; MFT: Davenport et al., 1998; MFT: Nelson & Hammerman, 1996; TBI: Schifter et al., 1999c; TERC, 1994). A few programs (esp. MASE and V-QUEST) hope to help teachers integrate mathematics and science topics.

Depth of mathematical knowledge

Yet this listing of topics tells us little about what these programs are really trying to do or how they do it. There are important and somewhat subtle differences to explore here. Some programs see mathematics learning for teachers as the sharing of expertise and the extension of teachers' knowledge into the next in a sequence of domains, or into the use of new techniques. They talk about the importance of "mathematics content presentations" conducted by university level experts (EM-MAT: Grady, 1997, p. 207; TEAM: Joyner, 1997, p. 224) or about "broadening teachers' view of mathematics" to include new topics (Kentucky: Bush, 1997, p. 174). Programs that take this approach often focus on the technical changes in teaching suggested in the *Standards*, describing the importance of using "manipulatives, computers, calculators, and problem-solving approaches" (Grady, 1997, p. 207) but not particularly in the service of deeper understanding—they seem to see these changes as ends in themselves rather than as methods for reaching a more conceptual goal.

Interestingly, the Kentucky program both used this approach and acknowledged its limitations. They state that while teachers "learned some mathematics in the program...these activities and assignments were not sufficient to provide them the background to understand many mathematical concepts or procedures" and their classroom mathematics lessons "were often filled with mathematical misconceptions and errors, and often lacked mathematical depth" (Bush, 1997, p. 175). At the same time, there were positive affective outcomes of this work, as "teachers began to enjoy doing mathematics and became confident in their ability to teach mathematics" (p. 174).

Programs that ascribe to this kind of additive approach to mathematics learning may fall into some of the "common, problematic interpretations" of the mathematics education reform movement described by Schifter (1997): 1) seeing it as about particular techniques and strategies (such as cooperative groups, or use of manipulatives or calculators) which "leave basic instructional goals...essentially unchanged"; 2) seeing it as about "problem solving and student engagement" without "[pursuing] substantial mathematical-conceptual objectives" leading it to "become a mere collection of activities without coherence"; and 3) seeing it as "the introduction of 'hot' new topics (e.g., fractals, discrete mathematics, probability, algebra in early grades) [which] may also get caught in either of the first two interpretations of reform" (pp. 259-60).

The majority of programs reviewed, however, seek deeper changes in teachers' thinking, asking them to engage in individual and collaborative mathematical investigations in order to enrich their own understanding of the relevant mathematics. They talk about doing "adult level mathematics," which most often means exploring and understanding the concepts behind the topics teachers teach through problems that are challenging at an adult level. For example, programs may ask teachers to invent a number system using base-5 materials to explore the meaning, power, and abstraction of place value, as well as to investigate the use of arithmetic operations within these several systems to understand how algorithms combine the essence of an arithmetic operation with characteristics of the number system to create a procedure that always works (MFT: Nelson & Hammerman, 1996, pp. 13-15). Or, they may focus on the importance of keeping track of the whole when operating on fractions (TBI: Schifter, 1997). Or they may ask teachers to seek to maximize the area for a figure with fixed perimeter to investigate the surprising independent relationship between area and perimeter (IMPACT: Campbell & White, 1997, p. 322). These explorations of the ideas behind topics that are superficially "simple" but in fact, have a lot of depth, contrast with an additive model that seeks to extend teachers' knowledge by teaching them more and more advanced topics.

Most programs do have these more conceptual goals, focusing on explorations of mathematics which will "enhance teachers' content understanding" (MASE: Gregg, 1997). They hope that by "explor[ing] adult mathematics, [teachers will] strengthen their appreciation of mathematical processes, and develop their abilities to understand and pose mathematical questions" (Talking Math: Corwin, 1997, p. 189). In doing so, they seek to help teachers become "knowledgeable, responsible, and reflective decision-makers, able to use mathematics in powerful ways to interpret information and to make sense of complex situations" (CMLP: Parker, 1997, p. 237).

Seeking this depth of mathematical understanding is substantially different from a more technical approach. Programs that do so typically engage teachers in mathematical tasks and discussions, emphasizing mathematical reasoning and justification over right answers. Teachers may work collaboratively or alone, using manipulative materials and other methods of representing mathematical situations, generating data and looking for patterns therein, conjecturing, predicting, and testing those conjectures, moving towards more and more abstract representations and general descriptions of situations, and communicating their findings with one another. Several programs talk about this as building a community of mathematical exploration (e.g., IMPACT: Campbell, 1996; 1997; Talking Math: Corwin, 1997; MFT: Hammerman, 1995b; CMLP: Parker, 1997; TERC: Russell, 1997).

These programs' moves towards promoting mathematical understanding are rich and deep. Yet, some programs go further still. In addition to asking teachers to explore the ideas underlying any piece of mathematics in order to develop real understanding, a few programs seek to build bridges and connections among topics or their underlying ideas, exploring the structure of mathematics in a more systematic way. The CGI program uses a research-based framework to connect and distinguish different ways of understanding addition and subtraction problems (Fennema et al., 1993; Peterson et al., 1988)[3]. This framework describes four types of real-world situations that can be characterized as involving addition or subtraction, shows structural similarities and differences among them, and details how different types of situations and problem structures affect children's abilities to understand and solve such problems.

The search for underlying mathematical structure is also the centerpiece of TBI's goals. By exploring mathematical ideas themselves, and bringing in to their regular group discussions "vignettes" of mathematical issues encountered in the classroom, teachers begin to discover deep mathematical ideas that arise over and over again in a variety of contexts. Ideas like "coordinating multiple units" may arise initially as children grapple with a place value system that treats the notion of "a ten" as both a single thing and as a collection of "ten ones." Children must be able to see numbers in both these ways and to make transformations between them in order to manipulate numbers effectively and understand the process. Later, the same underlying idea arises again as students ponder the remainder in the results of a fraction division problem. Here, students must be able to see that a fraction of the original whole—say 2/8 of a pizza—can simultaneously be a different fraction of the divisor—say 2/3 of a serving if servings are 3/8 of a pizza. Thus, the idea of coordinating multiple units recurs in different forms throughout the many years' process of learning mathematics—a characteristic of the "big ideas" that represent mathematical structure.

As the TBI project developed the DMI professional development curriculum for teachers (Schifter et al., 1999a;Schifter et al., 1999b), the big ideas that formed its focus are: "developing conceptions of number as students expand the domains they work with; coordinating multiple units in the contexts of whole numbers, rational numbers, and geometry; early algebraic thinking; studying attributes and constructing arguments in geometric contexts; and analyzing data" (Schifter et al., 1999c, p. 43). Exploring these deeper underlying issues in mathematics leads to yet another level of mathematical understanding for teachers.

Change in perspectives about mathematics

However, depth of mathematical knowledge is not the only hoped for outcome of mathematical explorations in any of these programs. Perhaps just as important is how teachers' experiences with mathematics change their view of the subject itself and their sense of themselves as doers and knowers of mathematics. Some programs want teachers to "broaden their view of math to include [new topics]" (Kentucky: Bush, 1997, p. 174). Others want teachers to "think about mathematics in terms of ideas rather than just facts, procedures, and strategies" (TBI: Schifter et al., 1997, p. 255); or to develop "a view of mathematics as a science that involves conjectures, observations, investigations and experiments" (CMLP: Parker, 1997, p. 238). When programs seek these larger shifts in teachers' beliefs about the nature of mathematics as a subject, they also hint at changes in the nature of learning which, in turn, have implications for teaching.

> Views of mathematics and science need to be reconstructed through experience and dialogue. Most educators, students, and parents have experienced science and mathematics as static bodies of knowledge and procedures to be memorized and transmitted. When they experience both mathematics and science as making conjectures, testing and verifying ideas, and communicating their ideas to others, they begin to view mathematics as a science of pattern and inquiry that reveals and describes order in our world. It is through meaningful experiences over time that learning becomes a process of inquiry, a search for meaning and sense-making that is enhanced by social interaction. (MASE: Gregg, 1997, p. 217)

[3]More recently, CGI workshops have expanded to include multiplication, division, fractions, the numeration system, and geometry (Carey, Fennema, Carpenter, & Franke, 1995, p. 123).

Schifter (1995) concurs, characterizing four "enacted conceptions of mathematics in the classroom" that integrate changing beliefs about the nature of mathematics with classroom practice. These include mathematics as:

1) An ad hoc accumulation of facts, definitions, and computational routines;
2) Student-centered activity, but with little or no systematic inquiry into issues of mathematical structure and validity;
3) Student-centered activity directed toward systematic inquiry into issues of mathematical structure and validity; and
4) Systematic mathematical inquiry organized around investigation of 'big' mathematical ideas. (Schifter, 1995, p. 18)

As teachers move through these schemas, they come to see themselves as active inquirers into mathematical structure in the context of their classrooms. This in turn, can lead to the development of even deeper mathematical knowledge as teachers continue to learn by engaging with students' ideas in their own classrooms (TBI: Russell et al., 1995).

Change in attitudes

Finally, programs hope that by engaging teachers in mathematical investigations wherein they are making sense of mathematical ideas, teachers will develop increased competence and a new sense of themselves as people who can do mathematics—even that they "will be captured by the pleasure of deep involvement in mathematics" (Talking Math: Russell & Corwin, 1993, p. 556). If math is no longer seen as a set of algorithms and formulas to be memorized to come up with single right answers, then those who were lost within this more traditional approach may be able to re-engage. This is especially important for many elementary teachers whose "prior experiences have left them feeling inadequate, fearful, and uninterested in mathematics" (Ball, 1996, p. 36; see, also, Goldsmith & Sassi, 1996; Yaffee, 1996). Helping teachers develop a sense of themselves as confident inquirers into mathematical topics and collaborative constructors of mathematical knowledge is seen as an essential shift in identity.

> We are now confident that doing mathematics and reflecting on it make a major contribution to a paradigm shift for many teachers in a long-term staff development program. Shifting the focus from their teaching helps some teachers pursue their own mathematical identities. Subsequently they develop more mathematical confidence. (Talking Math: Corwin, 1997, p. 188)

Other programs also report success in these efforts to shift attitudes. In CMLP, though teachers at first resisted opportunities to learn mathematics in their workshops, preferring that the program "provide new activities for their classrooms and tell them how to implement the activities," by the third year "teachers consistently requested more mathematics for themselves" (Parker, 1997, p. 238).

The mathematical focus of these programs is thus quite complex and intertwined. Attending well to it requires not only a description of topics covered, but also of depth of engagement with concepts and mathematical structure. It requires understanding how the nature of mathematics and mathematical knowing itself is characterized, as well as the attitudes about mathematics which programs hope to foster.

The majority of programs seek to help teachers develop deeper and more robust mathematical understandings including, sometimes, engagement with the underlying "big ideas" that constitute mathematical structure. They also hope that teachers will come to see mathematics as something to be investigated and constructed by teachers themselves, and come to see themselves as doers of mathematics.

<u>Children's mathematical thinking</u>

Related to program goals of extending teachers' own mathematical knowledge and changing their view of the nature of mathematics is a focus on how children understand mathematical content and come to learn it. This theme can be seen in 7 of the 14 programs—CGI, IMPACT, MASE, MFT, Talking Math, TBI, and TERC *Investigations*. Programs that address this theme typically do so in a variety of ways: through live or videotaped interviews of students conducted by staff and/or teachers themselves; through analysis of student work or assessment results; through videotapes of classroom discussions of mathematical ideas; through reflection on teachers' own experiences with students in classrooms either informally through some form of journalling, or more formally through writing, sharing, and/or analyzing vignettes of classroom practice; or through presentation of research-driven frameworks or data.

This theme is at the core of the CGI program, which focuses on communicating to teachers a set of research findings about elementary children's learning of arithmetic operations—both the mathematical framework described above, and information about how children move from more concrete representations of mathematical situations towards more abstract ones. This framework is intended to serve as the basis for attending carefully to students' thinking in the classroom which, in turn, leads to changing instruction to promote deeper student understanding (Chambers & Hankes, 1994; Peterson et al., 1988). For CGI, this knowledge of children's thinking is more powerful if it is "organized into a coherent network relating the different types of word problems to their difficulty and to children's cognitions for solving them" (Peterson et al., 1988, p. 44). At the same time, merely attending carefully to students' thinking also changes teachers' expectations. One teacher in CGI said,

> I didn't give them enough credit for what they knew. They certainly know more and have more sophisticated strategies than I ever dreamed of. First graders are capable of learning so much more than I thought they could. (Chambers & Hankes, 1994, p. 291)

Though CGI intends for all teachers to use their knowledge of student thinking in the service of instructional decision-making, teachers differ in the degree to which they believe that attending to students' thinking is important and in how they do so. CGI characterizes four levels of Cognitively Guided Beliefs and Instruction focusing on variations in teachers' beliefs about children's ability to solve problems without direct instruction and thus, the opportunities teachers provide children for doing so; in the degree to which the teacher elicits and students share their problem solving thinking with peers and the teacher; and in how well the teacher understands and uses that thinking in making instructional decisions.

These levels of increasingly complex use of children's thinking begin with non-use, in which instruction is centered neither on problem solving nor on sharing of thinking (Level 1), to a limited focus on these through more open-ended problems (Level 2), to a focus on problem solving and sharing of multiple solution strategies but without drawing these ideas together or using them as the basis of instruction (Level 3), to the use of student thinking to drive instruction (Levels 4A and 4B). The distinction between these final two sublevels is based on whether the knowledge of student thinking that teachers use is general knowledge primarily known from external research, or knowledge of the thinking of specific children in their classrooms (Fennema et al., 1996, p. 412-3).[4] Franke et al. (1998) claim that teachers whose knowledge of children's thinking is of this last type, Level 4B, are "generative" and, in another article, that such teachers are engaging in a form of "practical inquiry" (Richardson, 1994) in their classrooms which will enable them to continue to grow and develop on their own and collaboratively far into the future (Franke et al., 1997).

This kind of classroom inquiry into students' thinking is also a key component of the TBI program. There, teachers are regularly asked to write two- to five-page "episodes" describing the mathematical thinking of a student or group of students as a springboard for explorations of mathematical "big ideas." "It is by listening to students, remarking on

[4]Notice that this schema has similarities to Schifter's (1995) framework of enacted conceptions of mathematics and to Simon and Schifter's Assessment of Constructivist Mathematics Instruction (ACMI) scale (1991), both described above.

common areas of confusion or persistently intriguing questions, and then analyzing underlying issues that these big ideas are identified" (Schifter et al., 1997, p. 255). In this way, a focus on student thinking can serve, in part, as a vehicle for deepening teachers' own mathematical knowledge.

Ball (1993) cautions, however, that it isn't easy to integrate children's ways of thinking about mathematics with disciplinary ways of thinking. At the same time, she sees the teaching dilemmas raised in trying to do so as fascinating and productive vehicles for teacher learning.

TERC *Investigations* uses episodes created in TBI, along with data from their own formative research, to build into the curriculum materials descriptions of how students might think about the ideas embedded in the activities and how these might change over time, ways to attend to and elicit these different conceptions, and ideas about how to promote focused and productive thinking about them (Russell, 1997).

Some programs seem to focus more on helping teachers come to value student thinking generally than on the specifics of that thinking. In some ways, for example, Talking Math's emphasis on developing communities of mathematical discourse seeks to promote *process* for eliciting, engaging with, and communicating mathematical thinking as a vehicle for deeper learning. The program makes several suggestions for the kinds of tasks, materials, questions, instructional approaches, and teacher roles and attitudes that can promote good mathematical thinking and discourse. Encouraging teachers to get students to talk about their ideas can begin the process of deeper use of children's mathematical thinking. This may serve as an important step along the path of teachers' development; perhaps implicitly oriented to encourage movement toward CGI's Level 3, for example, which can then serve as a platform for further development. As Franke et al. (1998) state:

> We speculate that once teachers reach a Level 3, participating with other teachers engaged in practical inquiry would provide them an opportunity to develop further. As they observe other teachers adapt, create and challenge their own thinking they could begin to see that they could learn from their students in their classrooms and their colleagues. They could also become aware that knowledge was their own to continually recreate. (pp. 33-34)

At the same time, though teachers can prepare for the creation of mathematical community, there are elements of this work that are necessarily improvisational, and this improvisational view may require a much more sophisticated level of cognitive complexity as teachers are asked to take students' perspectives in the moment while also considering a deeper and more nuanced understanding of mathematics itself.

> If you're ready, you can pick up on subtle questions or less obvious ideas. If you aren't ready to listen to students, you will be less likely to hear what they're saying. Furthermore, if you aren't listening to students, they are not likely to listen to each other... [But] good mathematics discussions cannot always be planned; they tend to resist orchestration, may even seem spur-of-the-moment. We recommend that you seize opportunities to follow children's unexpected comments or questions. This often leads to excellent discussions. (Talking Math: Corwin, with Storeygard, & Price, 1996b, p. 17)

This kind of improvisation requires designing instruction based on carefully listening to the particulars of students' ideas in the moment—an approach characteristic of CGI's Level 4B. Thus, different kinds of structures and goals within the very same program may be simultaneously supporting teachers at very different levels of pedagogical development. This notion, that teacher education programs may support teachers at different stages or levels in their growth and development will be an important one to keep in mind as we continue our review of the change themes derived from these programs.

Whether programs provide a framework for understanding students' thinking or hope that teachers will develop that from their own classroom-based observations; whether they focus on the specifics of the mathematics explored or more on the development of a culture which will support talk about mathematics, programs that focus on children's mathematical thinking expect teachers to come to see students *as* thinkers and knowers of mathematics. This view is closely connected to beliefs about learning itself, which will be explored next.

Learning and constructivism

As discussed above, for some programs a focus on mathematics and children's understanding of mathematics can quickly become intermingled with reflections on how people come to learn mathematics, though these foci are not the same. Programs that focus explicitly on learning typically seek to help teachers develop a new, constructivist view of learning in place of what has typically been a model based on transmission of information. This is consistent with the principles underlying the mathematics education reform movement (NCTM 1989; 1991; 1995) and has been described by some as a shift in paradigm (Goldsmith & Schifter, 1997; Nelson, 1997a; Schifter et al., 1997; Schifter & Fosnot, 1993) because it not only changes the lens through which teachers see the events of their classrooms, but also the questions they ask about those events (Kuhn, 1970).

In fact, ten of the 14 programs reviewed—CGI, CMLP, IMPACT, MASE, MFT, Talking Math, TBI, Teach-Stat, TEAM, and TERC *Investigations*—focus in some way on changes in teachers' beliefs about learning. For example, MFT talks about the need for teachers "to examine long-standing beliefs about the nature of knowledge and learning, deepen their mathematics knowledge, and reinvent their classroom practice from within a new conceptual frame" (Nelson, 1997c, p. 229), and TBI wants to "facilitate construction of a new pedagogical theory and practice" (Schifter et al., 1997, p. 256). CMLP, MASE and IMPACT also describe themselves as "based on the tenets of constructivism" (see, also, IMPACT: Campbell, 1996; Campbell & Robles, 1997; MASE: Gregg, 1997, p. 215; CMLP: Parker, 1997).

CGI, too, supports constructivist views of learning, views which are reflected in the CGI Beliefs Scale—a 48-item, 5-point Likert scale style questionnaire designed to assess teachers' agreement with "four interrelated constructs representing fundamental assumptions about children's learning." These assumptions are: 1) Children construct their own mathematical knowledge; 2) Mathematics instruction should be organized to facilitate children's construction of knowledge; 3) Children's development of mathematical ideas should provide the basis for sequencing topics for instruction; and 4) Mathematical skills should be taught in relation to understanding and problem solving (Peterson, Fennema, Carpenter, & Loef, 1989, pp. 4-5).

For many programs, these changes in belief come in part from the new perspectives gleaned by teachers from listening carefully to children as they grapple with novel problems in creative and sophisticated ways. The sense of surprise at children's thinking cited above (p. 37), is an example of this, and leads teachers to see that students can solve problems without direct instruction (CGI: Fennema et al., 1996). Similarly, Teach-Stat hopes that their emphasis on statistical inquiry—a focus on "opinions supported by evidence" rather than right answers—in a novel content area will open teachers' eyes to more constructivist ways of teaching by eliciting "surprising" student responses, especially because "teachers don't have preconceived notions" of students' capabilities in statistics (Friel & Danielson, 1997, pp. 198-9).

For many of these programs, learning about the nature of learning also occurs as teachers step back from their own mathematical explorations to reflect on the learning process. For this strategy to be successful, programs often try to model the methods of mathematics instruction that they hope teachers will develop in their classrooms. For example, MFT describes "Starfish Math" as a chance for teachers to collaboratively invent different number systems that meet specific criteria and experience taking "the standpoint of someone learning them for the first time" (Nelson & Hammerman, 1996, pp. 13-15). Talking Math describes some of the features of the learning community they're trying to model, explaining,

> Keeping an exploratory frame of mind alive in staff development experiences is essential if we want teachers to replicate that mindset in their classrooms. A challenge for large programs, it is still necessary to find ways of retaining tentativeness, serendipity, and spontaneity.(Corwin, 1997, p. 189)

The TEAM project claims, "Participants need to construct new understandings just as their students do. The instruction must model the philosophy being espoused" (Joyner, 1997, p. 226). When programs do model a constructivist approach, then teachers' reflections on their own learning can promote the development of constructivist views.

Programs use a variety of methods for engaging teachers with issues of learning and construction of knowledge. Some common methods include reflective journal writing, analysis of children's learning, reflection on readings, and explicit discussions of the learning process. These discussions can focus on teachers' own mathematics learning experiences, on analysis of students' learning in the classroom, or on the results of small teaching experiments.

By providing contexts in which students and teachers can generate new mathematical understandings, and opportunities for teachers to reflect on the implications of those experiences, programs support teachers in the development of new, constructivist beliefs about learning itself. Yet changing deep beliefs like this—a paradigm shift—often involves stepping back from one's assumptions to see them afresh, and getting this kind of perspective on oneself can be a difficult process.

Equity

Mathematics for All is an essential theme of the mathematics reform movement—mathematics education should serve learners independent of their race and ethnicity, their gender, their prior access to educational opportunities, or their prior success in mathematics (NCTM, 1989; 1991, p. 4). While equity may be an implicit goal in the work of many teacher professional development programs, there are just a few (CGI, EM-MAT, IMPACT, Kentucky, MASE, and V-QUEST) that focus explicitly in their writings on issues of equity in mathematics learning.

The kinds of diversity that these projects deal with varies—some are situated primarily in urban districts (EM-MAT, IMPACT and some CGI work); others mix urban and rural districts (Kentucky, MASE and V-QUEST). Diversity can refer to the gender, race, class, ethnicity, primary language, and special learning needs of students as well as of teachers in the target schools or districts.

Sometimes a focus on equity primarily takes the form of declarations that all students can learn—along with attempts to encourage teachers to believe that—and a choice to enact this declaration by situating the work in schools with diverse populations which, in turn, leads to engagement with the special issues that arise in those contexts. The two statewide programs, Kentucky and Virginia's V-QUEST, deal with equity issues in this way. Specifically, Kentucky distributes participation across the varied regions of Kentucky to reach 80% of districts that employ 90% of the state's teachers (Bush, 1994) while V-QUEST uses a "selection process [that] has special requirements and opportunities for traditional[ly] underserved and underrepresented populations, especially related to ethnicity and income" (Underhill, 1997, p. 265). It is not clear in these programs what is done explicitly to influence teachers' beliefs about the learning potential of diverse learners, though participating alongside teachers who work with a range of learners and sharing common struggles and concerns may have an indirect impact.

The Boston-based program EM-MAT, while also recruiting teachers from varied backgrounds and those who serve varied constituencies, feels that the pedagogical approaches it espouses work well for many kinds of learners and wants teachers to believe that as well. "EM-MAT has consciously worked to develop successful models for teaching mathematics to all students.... [through] hands-on, project based, technology-infused [activities]" (Grady, 1997, p. 210).

Several projects built on constructivist principles—MASE, CGI, and Project IMPACT—also seek equity goals by offering their regular program to teachers of students from under-served communities. They argue that their principled emphasis on careful attention to the thinking of *every* learner may equip them to better meet these equity goals. For example, the MASE project selects "teacher leaders representative of the diverse teacher population" in Clark County, Nevada, and calls for curricular and pedagogical "change that provides all students access to quality, inquiry-based, developmentally appropriate mathematics and science education" (Gregg, 1997, pp. 220-1).

CGI has done some of its work in the Prince George's County, Maryland, schools—an urban district bordering Washington, DC, which has 70% African-American students with 7 of the 11 schools receiving Chapter I supports (Carey et al., 1995, p. 104). In this context, because the program doesn't prescribe particular practices but emphasizes careful listening and responsiveness to all students, teachers using the CGI approach can tailor their instruction to the needs of diverse individuals and communities of learners.

> What happens in a CGI class is that as the teacher comes to understand each child's thinking, he or she designs a mathematics program on the basis of what each child knows and can do. Because the child has to be able to make connections between intuitive, informal knowledge and school-based knowledge, the curriculum includes components the child understands, which are derived from the child's experiences and culture. (Carey et al., 1995, p. 101)

Project IMPACT, which is situated entirely within the predominantly minority and low-income portions of the Montgomery County, Maryland, public schools, also uses this approach to meeting equity goals.

> The premise in IMPACT is to teach students mathematics by building on their existing knowledge, by initially framing instruction within meaningful contexts, by focusing on problem solving and concept building, and by expecting and demanding that every student participate in mathematical inquiry (Campbell & Langrall, 1993). (Campbell, 1996, p. 456)

By engaging all children in mathematical explorations, these approaches value children's thinking at all levels of mathematical sophistication and from all cultural backgrounds, help children make meaning of mathematical ideas based on their own experiences, and build from their informal knowledge towards more abstract and formal understandings. "Teaching for understanding yields growth for children at all ability levels" (IMPACT: Campbell, 1997, p. 107). As teachers come to see that constructivist practices work for varied learners, their expectations about who can succeed in mathematics may change.

An important additional component of IMPACT's philosophy is the focus on school-wide efforts for change. This strategy was adopted to address 1) the resistance to change in urban settings; 2) the limitations of isolated teachers effecting real change; 3) the failure of traditional teaching practices to meet the needs of urban students; and 4) the need for programmatic consistency across the grades (Campbell, 1996, p. 452). By working with all the teachers in a school, it is hoped that teachers can form a community for promoting change, and that students who had not been well-served by traditional approaches would have a better chance at success through continuity of approach. Because its goals relate both to teacher beliefs and student learning, IMPACT evaluated its success in part through statistical measures of student achievement in relation to demographically comparable districts, and in part through measures of changes in teachers' attitudes and beliefs. The latter consisted of a modification of the CGI Beliefs Scale with additional questions that probed for teachers' beliefs about equity.

Thus, programs that espouse equity goals do so primarily through the process of selecting and working with teachers and schools that represent diverse populations, but also by promoting pedagogical strategies intended to engage all students in thinking deeply about mathematics.

Leadership and the change process

Finally, many programs (CMLP, EM-MAT, FIRST, Kentucky, MASE, MFT, Teach-Stat, TBI, TEAM, and V-QUEST) seek to support teachers and administrators in acquiring new beliefs and knowledge as they adopt new roles as facilitators of other teachers' learning in schools. Goals for changes in the thinking of administrators and teacher-leaders overlap in some areas but also differ. Programs hope that both groups will come to understand aspects of the process of teacher learning including theories of adult change and growth and how to facilitate it, affective issues in the change process and how to manage them, and development of a new conception of staff development. Teacher-leaders need to come to see themselves in the new role of facilitator for their peers' learning—"to transform their self-perception from good teachers to building-level staff developers" (V-QUEST: Underhill, 1997, p. 264)—while administrators need to understand enough about new views of mathematics and pedagogy to support teachers politically and logistically in developing their practices, and "to develop supervision strategies that are compatible with child-centered, inquiry-based programs" (MASE: Gregg, 1997, p. 219).

Programs expect teacher-leaders to begin by developing a deep understanding of mathematics, learning, and teaching, as each program defines them. Some programs also expect this kind of content understanding from administrators, hoping administrators "can develop a deep understanding of mathematics instruction and can develop the attitudes, orientations, and skills which will permit them to support and sustain progressive mathematics instruction in their school and districts over the long-term" (MFT: Nelson, 1997b; Nelson, 1997c, pp. 231-2). Such programs worry that "shallow [administrator] understanding can be eroded by political pressure" and therefore they seek "principals who understand constructivism and reform goals" (MASE: Gregg, 1997, p. 219).

Some programs offer specific supports to teachers in becoming leaders. The nature of this support depends in large part on the kinds of changes they envision—some programs, like FIRST, Kentucky, TEAM and V-QUEST, offer more technical, "how-to" supports. V-QUEST, for example, reports, "about 25% of our training focuses on adult learners; how to plan, conduct and deliver workshops; how to work with administrators; possible activities and roles for Lead Teachers, etc." (Underhill, 1997, p. 266). Other programs, however, focus on more transformative elements of the change process itself—context, content, affect, and facilitative roles, among other things.

> The complexities of preparing such teacher leaders must not be underestimated. Leaders facilitating school-wide efforts must understand fully what it takes to implement a restructured mathematics program in their own classrooms. They also need to understand the change process and be prepared to challenge current practices and support teachers through times of discomfort and discontent. (CMLP: Parker, 1997, p. 244)

Programs seeking deeper changes in thinking and practice make a point of calibrating expectations about the length of time that these sorts of changes take—years rather than weeks or months. Many programs also emphasize the importance of attending to and managing the discomfort, frustration, and confusion—as well as the curiosity, excitement, and satisfaction—that teachers are bound to experience in programs that ask them to "reconstruct" their beliefs and practices from within a new conceptual frame. CMLP, MASE, MFT, TBI, TEAM and Teach-Stat each work with leaders to address affective issues in change to some extent (as does IMPACT, though only in its work with participants, not leaders). These affective components of change become relevant to the work of teacher-leaders especially when the professional development learning they are asking of colleagues is a transformative process of reconceptualizing thinking about content and teaching rather than just an additive process of acquisition of new knowledge, skills, or techniques (Goldsmith & Davenport, 1995).

> Change of this magnitude will be messy, and will involve levels of discomfort, frustration, and even anger. A natural part of the change process is a long period

> of time when teachers are dissatisfied with their old practices before they feel competent with their newly developing understandings and methods. During this time of struggle there will be a tendency for many to retreat to a safer place. (CMLP: Parker, 1997, p. 238)

Programs often explicate their theories of change for teacher-participants through their work with teacher-leaders. Some programs emphasize issues of "ownership" of change, for example, inFIRST's involvement of teachers in the development of school improvement plans (Bright, Miller, Nesbit, & Wallace, 1997) or inIMPACT's commitment to asking teachers how they will "attempt change" and helping them "create a setting that supports that growth" (Campbell, 1996, p. 474).

Many programs talk about the need to support teachers to integrate new ideas into their everyday practice and use such techniques as classroom consultations, demonstration lessons, and observation of small group clinical interviews, to do so. In part, they see these practices as grounding theoretical ideas in classroom realities—"the classroom is an incomparably rich environment for continued learning about learning" (TBI: Schifter et al., 1999c, p. 30)—and providing a context for continued collaborative investigation of new practices. It is not entirely clear from the literature how programs support teachers to take on these new roles in others' classrooms. Though constructivist images of teacher learning dramatically shift teachers' expectations of the help they might get from peers through these activities—facilitation of inquiry rather than offering ideas and answers—leaders may still have to fight against more traditional images of staff development both among their colleagues and perhaps, even, within themselves.

Programs also talk about the power of teachers' stories of success and struggle in motivating and informing the change efforts of other teachers. Such stories have greater credibility than comments coming from teacher educators, they capture some of the complexity of the relationship between cognitive and affective elements of the change process, and they can encourage teachers to struggle through difficulties because these can be seen as surmountable. Whether through teachers' stories or otherwise, programs often find it essential to help teachers develop new images, possibilities, perspectives, interpretations, and options for practice (MFT: Nelson, 1997c; CMLP: Parker, 1997).

Finally, some programs help teacher-leaders learn to develop and sustain a culture of inquiry among their peers by asking them to replicate the experiences they themselves participated in and by providing explicit material support for curricula and pedagogy that will generate such a culture (MFT: Davenport et al., 1998; TBI: Schifter et al., 1999c).

Programs provide a variety of supports to teachers and administrators taking on what is often a difficult new leadership role. Being a facilitator of change rather than a more experienced expert evokes special challenges. In order for teacher-leaders to understand their own role in facilitating cognitive and affective aspects of their peers' change processes, they may need to put their own stories and experiences in relationship to others', come to see themselves through others' eyes in the context of larger change goals, and develop a larger institutional vision. These are not at all easy tasks.

Discussion and analysis

These sketches of themes offer rich descriptions of the range of goals for changes in teachers' thinking sought by mathematics teacher professional development programs. Such descriptions are useful in their own right, helping us understand the features and themes of mathematics teacher professional development. But can we draw further insights by stepping back from the individual thematic sketches to consider common threads or elements underlying several or all of the themes?

One clear thread within several of the themes is the variation in degree or depth of changes that programs seek. Sometimes they seem to seek more surface changes in knowledge, behavior and skills; sometimes they seek deeper transformations of perception, meaning, and conceptual structure. We see this in Mathematics, for example, where some programs seek to extend teachers' mathematical knowledge to new or more advanced topics; others seek a deeper, more connected understanding of the topics teachers already teach; and others push deeper still, asking teachers to grapple with the structure of big ideas underlying various mathematical topics. Similar patterns of variation in depth can be seen in the different ways programs approach the Teaching, Inquiry, Equity, and Leadership themes.

In addition, the several schemes for understanding the development of reformed mathematics teaching presented above (Campbell, 1996; Fennema et al., 1996; Franke et al., 1998; Schifter, 1995; Simon & Schifter, 1991; Spillane & Zeuli, 1999)—schemes used especially to describe teacher growth in the Teaching, Mathematics, Children's mathematical thinking, and Learning/ constructivism themes—can be seen as measuring a deepening of understanding. Some distinguish explicitly between more surface changes in behavior and more cognitive, epistemological changes; others do so implicitly by describing more advanced levels as requiring an integration of deeper understanding of mathematics or children's thinking into teaching itself.

But what kind of a deepening is this? I propose that the deepening of understanding sought by programs may not only require teachers to add new information and knowledge to their existing bag of tricks, but also to learn to take and coordinate new and different perspectives, to identify and use one or more underlying principles to generate and then evaluate the effectiveness of new practices, and to deal with a higher level of complexity than has been true with traditional textbook-driven practice. These issues of complexity, authority, and perspective taking are the very ones addressed by broader, theories of adult meaning-making (e.g., Belenky, Clinchy, Goldberger, & Tarule, 1986; Kegan, 1982; 1994; King & Kitchener, 1994; Kitchener, 1986; Loevinger & Blasi, 1976; Perry, 1970). In this section, I will use such a frame to draw together some seemingly disparate threads from the several themes, to suggest possible insights about programs and their level of success in meeting their goals, and to raise questions for future research.

Clearly the goals and methods of these programs can be understood in several other ways as well—for example, as a straightforward attempt to increase teachers' breadth of formal and practical knowledge; as elements of a multi-prong attempt to change the resistant institution of schooling; or as part of the reculturing that goes along with a shift in the intellectual paradigm around teaching and learning. All these frames may be informative in their way. However, a constructive-developmental lens focusing on changes in adult meaning-making will enable us to ponder how teachers themselves might make sense of their experiences in these programs.

Others, too, have hinted at the importance of using a developmental framework to examine teacher education and teacher professional development. Sprinthall, Reiman and Thies-Sprinthall (1996) in their review of teacher professional development argue strongly for the value of a broader cognitive-developmental perspective in researching and understanding teacher professional development. A few teacher professional development programs both use cognitive-developmental theories to understand their work and describe themselves as explicitly seeking cognitive-developmental changes (e.g., Bell & Gilbert, 1994; Glassberg & Oja, 1981; Oja & Ham, 1984). Berger (1999) discusses how a constructive-developmental lens can shed light on issues and difficulties in changing beliefs in the context of preservice teacher education. I have recently raised developmental issues specifically in the domain of mathematics teacher professional development (Hammerman, 1999b), as have Cooney & Shealey (1997) who focus on issues of reflection and authority.

In this analysis I am interested in the qualitatively different ways that adults construct their understanding of themselves, their roles, knowledge, authority, and learning and teaching. There are many theories of adult development that could help us understand these issues (e.g., Belenky et al., 1986; Kegan, 1982; 1994; King & Kitchener, 1994; Kitchener, 1986; Loevinger & Blasi, 1976; Perry, 1970). All posit qualitative changes in the complexity of a person's meaning-making system, focusing less on the *specific content* of thought and more on its *form or structure*. Many of these theories can be easily correlated with one another. Yet some of these theories were developed with specific populations (Belenky et al., 1986; Perry, 1970) or with specific issues (King & Kitchener, 1994; Kitchener, 1986) in mind and so, are less useful to me in this analysis. I choose to use Robert Kegan's theory (1982; 1994) as a tool for this constructive-developmental analysis as it addresses multiple domains of meaning making (cognitive, interpersonal, and intrapersonal) and has been developed and used with a general population of adults.

Kegan's theory of adult development[5]

Kegan's constructive-developmental theory (1982; 1994) describes changes in meaning-making structures across the lifespan but focuses especially on changes which typically occur during adolescence and adulthood. It distinguishes between learning that adds more knowledge or ideas to an existing structure of mind—learning that "informs" our thinking—and learning that changes the structure of mind itself—learning that "transforms" or "re-forms" our perspective (Kegan, 1994, pp. 163-4).

Kegan posits five Orders of Mind that hold across the lifespan, with several transitional positions between each of the five Orders (Lahey, Souvaine, Kegan, Goodman, & Felix, 1988). Because Kegan's is a subject-object relations theory, a key issue at any of the five Orders of Mind is the distinction between which aspects of ourselves and the world we can work with, relate to, and have some distance on and control over (aspects that Kegan calls "object"); and which aspects we are made up by, provide the lens or frame through which we see, and shape our perspective because they constitute what we *are* (aspects that Kegan calls "subject"). For Kegan, it is the shifting in these "subject-object" balances that constitutes development. The larger the realm of what we consider "object," the more we can take responsibility for, the more perspective we have, and the more complex our understanding of ourselves in the world and of the world itself.

The first two meaning-making Orders of Mind typically describe younger and older children respectively, though some adults (roughly 10% to 25%) continue to make meaning in whole or in part in Second Order ways.[6] At the Second Order, people are oriented towards concrete consequences of actions and enduring characteristics of self, others, and the world. They develop a point of view and care about how others perceive them because those perceptions may have concrete consequences for them.

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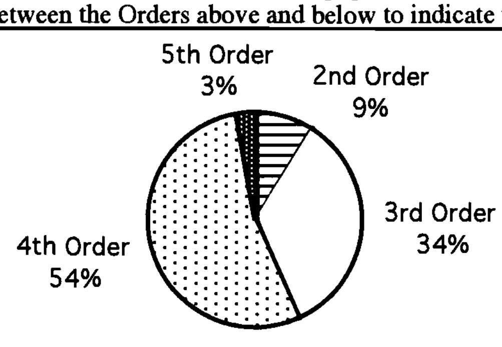
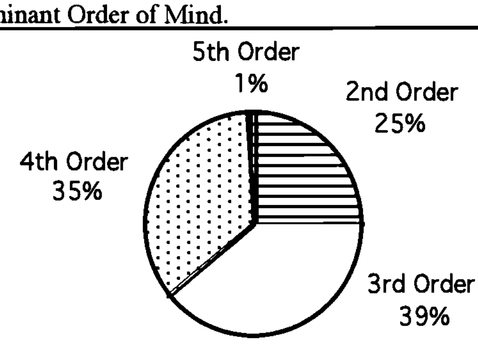
[5]The description of Kegan's theory in this section borrows heavily from a previous paper I presented at AERA (Hammerman, 1999b).

[6]The statistics I report here are compiled from Kegan's description of the distribution of Subject-Object Interview scores for 282 people drawn from 12 dissertation studies (1994, pp. 188-197). Proportions of people at each Order are correlated, in part, with age, education, and socio-economic status; thus the rough ranges presented here. The full dissertation sample reported by Kegan is biased towards a professional and well-educated population, though he also describes a subset of three studies (N=75) that represents a more complete SES range. Below I chart distributions of Orders of Mind for both of these samples—it is not clear which better describes the population of teachers. I have split the reports of transitional stages equally between the Orders above and below to indicate the dominant Order of Mind.



Dissertation Composite, N=282



Full SES Composite, N=75

The work of adolescence is typically to gain some perspective on the Second Order's concrete orientation to construct a Third Order understanding that can integrate different views within larger cognitive and social principles. People at the Third Order can coordinate several points of view within a sense of their own role within a social structure. They care about and can consider others' opinions of them as such, no longer seeing these strictly in terms of the consequences of others' actions towards them. People at this Order can use abstractions and inference to coordinate concrete data, and can develop hypotheses and respond to abstract ideals and values generated elsewhere. The idea of doing things "because it's the right thing to do" even if it's not in your own self-interest makes sense at this Order. Kegan describes this meaning-making structure as "Traditionalist" or "Socializing" in that it includes an internalized sense of mutual reciprocity and cultural expectations, and therefore enables people at this Order to be responsible for their own role within a larger social structure. People at this Order, however, are unable to generate these roles or principles underlying them on their own, so may need appropriate external models for belief and action to guide their thinking and behavior. Between 1/3 and 2/5 of adults make meaning in this way (see Note 5).

Unfortunately, Kegan argues that our society doesn't always provide the supports and models needed by people making meaning at the Third Order. Instead, society often demands something more from adults (Kegan, 1994), demands which are not met until people can construct meaning at the Fourth Order he posits. In a Fourth Order view, adults come to coordinate their multiple roles and the different expectations others hold for them within their own self-generated, relationship-regulating framework. While someone at the Third Order might be torn apart by competing roles or expectations from important external others, at the Fourth Order people aren't "made up by" others' expectations (responding either by cooperating or rebelling) because they have a larger frame from which to judge and make sense of those expectations. People at this Order internally mediate between abstractions through abstract systems and ideologies—e.g., a teacher coordinating expectations for promoting high achievement with expectations for equity can create a classroom environment that she then judges with both criteria at once. People at this Order take responsibility for their own inner states and take a perspective on culturally or socially mediated definitions of reality. Kegan calls this Order "Self-authoring" in that those constructing reality in this way can distinguish their own role in shaping their understanding of the world and are not determined by the cultural milieu in which they find themselves. This way of constructing meaning is the primary mode for another 1/3 to 1/2 of adults (see Note 5).

Finally, Kegan claims that a small percentage of adults primarily in mid-life or beyond move towards the Fifth Order, where they come to see the Fourth Order's personally constructed ideologies themselves as constructed objects from a "dialectical" or "self-transformational" perspective. This Order of Mind is quite rare and won't be a substantial focus of this paper.

Our concern for this discussion will be primarily on differences between Third and Fourth Order ways of making meaning, since the vast majority of adults—including teachers—construct their world in one of these two ways or in transitional places that emphasize these ways of making meaning.

Meaning-making demands of the programs

How can a constructive developmental framework help us understand the goals of mathematics teacher professional development programs? I begin by describing how several goals described in the thematic sketches may require capacities that are representative of Fourth Order thinking or beyond. I then briefly consider several other issues through a constructive-developmental lens—how particular professional development methods might provide supports for teachers at other Orders of Mind; how program activities might actually encourage developmental change; and how teachers at different Orders of Mind might experience the same methods differently. Finally, I pose some questions stemming from these observations that might be fruitfully pursued with further research.

Some programs' goals may require a level of cognitive complexity that constructive-developmental theory can help us understand. For example, CGI and other programs that ask teachers to consider children's mathematical thinking in the design of their instructional practice require several things of teachers: 1) They must be able to take the perspective of the children to whom they are listening, to be able to see mathematical ideas as the children see them; 2) They must keep in mind a set of goals for children's mathematical thinking, imagining how they want children to think about the mathematics; and 3) They must have an image of how to help children move from one place to the other. Each of these goals by itself requires a Third Order capacity which allows teachers to take another's perspective on the world or to have an internal image of what another's understanding might look like—a capacity that teachers at the Second Order would not have. Coordinating these components, however, requires an additional ability that is more characteristic of Fourth Order thinking—being able to imagine another's cognitive state *while also* comparing it to a desired one as well as how one's own pedagogical actions might influence that thinking. While imagining any one of these components is possible at the Third Order, putting them together to build a teaching practice that considers several at once requires Fourth Order thinking.

Constructivist teaching—which offers learners experiences to help them build from their current understandings towards new knowledge—may require coordinating just this set of demands. If so, then the developmental considerations described above could explain in part the difficulties that programs experience helping teachers develop constructivist practices.

> The knowledge and instructional expertise demanded of teachers who are using constructivist methods to foster mathematical power in students is exponentially greater than the expertise required to follow a textbook page by page or to drill students in low-level skills and procedures. (IMPACT: Campbell, 1996, p. 465)

When teacher-leaders are asked to consider their colleagues' constructions of mathematics and pedagogy along with an ideal and an image of the change process as part of supporting their professional development, then these same developmental considerations may again apply.

The levels of the Cognitively Guided Beliefs and Instruction scale (CGI: Fennema et al., 1996) are intended to describe how teachers learn to teach in cognitively guided ways, but they also hint at a broader constructive-developmental framework. Teachers at Kegan's Third Order who buy into the program would see CGI as a respected authority, internalizing its demands for particular practices in stereotyped ways. These teachers could be at several, but perhaps not all, of the CGI Levels. Thus, Third Order teachers at CGI Level 2 would see CGI asking them to use more open-ended problems; those at CGI Level 3 would see CGI also demanding they make more space for students' voices and multiple solution strategies. Even teachers at CGI Level 4A, who see the knowledge about student thinking they are using to design instruction as necessarily generated by expert researchers, may be operating from a Third Order view embedded in the more complex Fourth Order coordination of perspectives needed to enact such a design, perhaps indicating a transitional constructive-developmental place between the Third and Fourth Orders.

> Teachers at [CGI] Level 3 and Level 4A thought that the knowledge about children's thinking was critical and central to their teaching, but they saw knowledge as something passed on to them. Ms. Mason [a Level 3 teacher] ... wanted to learn how the researchers thought she should be using children's mathematical thinking, essentially, how she could do it right. (Franke et al., 1998, p. 19)

This focus on outside authorities contrasts with another teacher's succinct statement of CGI as a principled approach: "CGI is a philosophy versus a recipe" (Fennema et al., 1993, p. 580). Teachers at CGI Levels 2 and 3 can seem to be following formulaic prescriptions for practice, as described above. At Levels 4A and 4B teachers find ways to actually use students' thinking to interactively design instruction—still based primarily on general knowledge of students from outside research at Level 4A, but grounded in the specific thinking of children in their classrooms at Level 4B. At these levels, "the analysis of the mathematics of children's thinking involves more than lists of problems and strategies; rather it involves an integrated perspective based on relationships among problems and strategies" (Franke et al., 1998, p. 13). Would teachers who don't have the Fourth Order capacity for cognitive complexity required to grapple with this integrated perspective be able to progress to these highest levels of CGI practice?

Several programs (CGI, MFT, Talking Math, TBI) hope that teachers will become generators of new knowledge about children's mathematical thinking or about classroom practice, typically through collaborative inquiry. They ask teachers to "work together to generate their own theories of how mathematical discourse can best be supported in their classrooms" (Talking Math: Russell & Corwin, 1993, p. 558); to become aware that "knowledge of children's mathematical thinking is their own to create, adapt, and investigate" (CGI: Franke et al., 1998, p. 31); or to discover the "big ideas" of mathematics by writing and exploring vignettes of their own classroom practice (TBI: Schifter et al., 1999c). In this, programs ask teachers to assume that knowledge and authority can come from within themselves rather than necessarily coming from the outside, an assumption that requires a Fourth Order view.

Of course, not all programs ask teachers to claim such authority. Some (e.g., EM-MAT, Kentucky, Teach-Stat, TEAM) are clear about offering "expert" courses and providing activities for teachers to try directly in their classrooms. Perhaps these programs are implicitly catering to the needs of teachers with Third Order capacities. Yet, other programs either refuse to provide classroom activities or, like IMPACT, ask teachers to engage with materials to reclaim decision-making authority grounded in underlying principles. These programs may be expecting a more Fourth Order perspective.

TERC's insistence that "the curriculum itself must assume that what it suggests won't always work" (Russell, 1997, p. 251) directly contradicts a Third Order view that curriculum is a prescription for practice. Describing itself as a "partnership with teachers" (p. 248), the curriculum clearly asks teachers to be responsible for considering how the activities suggested would be experienced by real students in their classrooms. "The teacher's role is to connect the particulars of her classroom and students to investigations in the curriculum" (p. 251). This requires a Fourth Order coordination of perspectives and consideration of the unique features of classroom contexts.

When programs see teaching as a context-dependent, dilemma-filled practice (Ball, 1993; Lampert, 1985), they may also be pointing towards the need for Fourth Order capacities or more. Such views suggest the need for ongoing generation of new knowledge and best practices, responding to the particulars of the moment and several perspectives at once, rather than following externally-derived prescriptions for teaching. This requires an internal sense of authority which is characteristic of the Fourth Order.

> It appears that the new mathematical understandings teachers must develop and the teaching situations they must negotiate are too varied, complex, and context-dependent to be anticipated in one or even several courses. Thus teachers must become learners in their own classrooms. (TBI: Russell et al., 1995, p. 10)

> Mathematics teaching is a complex domain that combines thought and action. It involves a coordination of internal, psychological constructions of mathematics, epistemology, and pedagogy that guide a multitude of on-the-spot practical decisions and external actions made in the context of the specific classroom conditions that prevail on a given day and time. (TBI & MFT: Goldsmith & Schifter, 1997, p. 38)

If negotiating dilemmas of practice also requires making choices while holding competing values—for example, simultaneously considering concerns for promoting students' creative mathematical thinking, equitable classroom participation, and obligations to promoting mathematical knowledge recognizable in the broader community—then teachers may need an ability to coordinate self-generated principles that is suggestive of a move towards Fifth Order thinking.

The nature of the inquiry process in these programs may also have constructive-developmental implications. Programs that ask teachers to step back from their own experiences learning mathematics in new ways to reflect on what that tells them about learning more generally require teachers not only to see themselves and their own thinking as an object for reflection (a Third Order demand), but to use that perspective to generate for themselves broader principles about mathematics learning (a Fourth Order demand). Sometimes the constructivist principles underlying their experiences as a learner are substantially different from those underlying their teaching if it's done in a transmission mode, and this conflict in principles also requires Fourth Order capacities to negotiate.

Remember, it is not reflection per se that is Fourth Order—a call for reflection can support developmental change at any level by asking people to see themselves, however they construct that, as Object. In fact, teachers at the Third Order might see the conflict they discover through reflection as about competing authorities and expectations—"The text says I should teach one way and the program says I should teach another. What do I do?" Yet, the focus on unearthing potentially competing internally-held theories and principles, rather than on competing authorities and expectations (or even competing consequences) is what makes these demands Fourth Order.

Thus, programs ask teachers to do several things that seem to require Fourth Order capacities: 1) to coordinate students' perspectives with images of disciplinary knowledge to design instruction; 2) to see themselves as generators of new knowledge from within their classroom practice rather than relying on external authorities; 3) to negotiate a context-dependent practice filled with principled dilemmas; and 4) to step back from their own learning to discover new guiding principles for teaching. Yet not all teachers have the level of cognitive complexity and perspective taking abilities that will enable them to meet these demands. Fortunately, as I discuss below, even programs that seem to make Fourth Order demands also provide supports for people making meaning at the Third Order, supports which may serve to scaffold a developmental change, as well.

Other constructive-developmental observations

Though some programs seem to make Fourth Order demands on teachers, aspects of their goals may be more appropriate for teachers who make meaning at the Third Order and, in fact, quite challenging for those whose thinking is primarily Second Order. Some of these Third Order elements have been described above—for example, taking children's perspectives on mathematics, imagining ideal mathematical thinking, and teachers reflecting on their own thinking. These practices can provide vehicles for deeper learning—about children's thinking or mathematics, for example—even if they can't be coordinated and integrated into a larger picture.

Often, too, programs serve as authorities for new knowledge or new practices, and this fits easily into a Third Order way of thinking. By offering expert knowledge about mathematics or children's mathematical learning, by providing activities to try in classrooms, by bringing math specialists or other leaders into classrooms to do demonstration lessons, by creating groups that can explore and then seem to warrant best practices—in all these ways and more, programs serve as authorities to teachers with a Third Order worldview and this support can help teachers sustain new practices even in the face of more traditional images of mathematics teaching. In fact, more technical programs that make fewer Fourth Order demands may be implicitly designed for teachers at the Third Order by providing such clear, authoritative images of reformed practice. Thus, Third Order teaching may change in desired directions based on teachers "doing what the program wants" even if the principles underlying these new practices are not internalized (see, also, Berger & Hammerman, In press).

Program elements can also seem to support developmental change itself. The collaborative inquiry process, especially, can support developmental change by creating an environment which offers the combination of support and challenge that Kegan (1982; 1994) claims serves as a bridge from one Order of Mind to another. By providing a vehicle for sharing images or data, for generating alternatives through "the development of visions of possibility" (Schifter et al., 1997, p. 258), and for surfacing assumptions about mathematics, learning, and teaching, these groups help teachers step back from their assumptions either explicitly, or by putting the implications of assumptions in perspective by generating and sharing alternative courses of action. By meeting over the course of an academic year or more and grounding new images in ongoing changes in practice, groups hold new ideas long enough so that teachers must grapple with them and their implications over time, and they support teachers to reintegrate practices into a new conceptual frame. Kegan would describe this process as supporting a Subject-Object transformation in that teachers come to see their assumptions, which were formerly implicit and unquestioned, as something they can examine, work with, and understand. For exampleMFT asks:

> How can teachers learn to support one another to delve deeply into the mathematical and pedagogical issues at hand...posing questions that challenge assumptions but are seen, nonetheless, as supportive rather than evaluative? (Hammerman, 1995b, p. 52)

Finally, even the same support vehicle can be experienced differently by teachers at different Orders of Mind. For example,Franke et al. (1998) also see a group engaging in inquiry as a potential support for teachers, but they feel the nature of that support may vary depending on teachers' levels ofCGI beliefs.

> Being part of a supportive relationship did not mean the same thing to each teacher within the different groups.... There was a noticeable difference in how the Level 2 teacher, Ms. Conti, viewed the grade-level collaboration and the way the other teachers involved viewed it. MsConti participated in the weekly planning meetings and considered her colleagues supportive. Even so, she rarely engaged with the teachers about children's thinking during their planning meetings. She would take the tasks that the group developed and use them in her classroom in a way that made sense to her, but she never really engaged with the community beyond sharing tasks. (CGI: Franke et al., 1998, p. 28)

Thus CGI Level 2 teachers use the group as a source for classroom tasks. By contrast, teachers at CGI Level 4 are "not simply sharing; [but] building principled knowledge on which to base their ongoing instructional decisions" (p. 33). As described above (pp. 39-40), CGI Level 3 teachers who might be at Kegan's Third Order, could use a peer group engaged in this kind of inquiry as a model of movement towards the use of principled knowledge in practice and a different—more Fourth Order—relationship to the knowledge itself. How teachers engage with issues in collaborative groups makes a difference—sharing of tasks is dramatically different from principled exploration. Yet, even when groups model engagement with principles that can drive practice, not all teachers can make good use of this modeling. For some, such groups still provide merely a source of tasks, for others, a warrant for experimentation based on the authority of the group itself.

Thus, constructive-developmental theory can help us understand aspects of what is going on in programs besides how they may be making Fourth Order demands on teachers. For example, we can come to see how teaching behaviors for teachers at different Orders of Mind might change in the same direction but built on different foundations—the authority of the program for Third Order teachers; underlying principles for those at Fourth Order. We can see how the same program components may be experienced differently by teachers at different Orders of Mind. And we can see how some program practices might even begin to support constructive-developmental transformation itself[7].

Implications

This analysis has implications both for the design of mathematics teacher professional development programs and for further research. This analysis indicates that many exemplary mathematics teacher professional development programs seem to have goals for changes in teachers' thinking that require particular developmentally-linked capacities. Because not all teachers will have these capacities, programs might carefully consider whether such expectations are necessary for them to reach their goals for changing practice. If not, then they might want to explicitly build in supports for teachers constructing meaning in a Second or Third Order way. If so, then they must consider whether and how to support developmental change itself (Berger & Hammerman, In press) or expect that teachers will not be able to meet their goals.

At the same time, this analysis leaves open several fascinating questions. First, it's clear that teachers participating in a mathematics teacher professional development program will make sense of their experiences in these programs from their current Order of Mind. Thus, even if programs seem to demand Fourth Order capacities, a teacher making meaning in a Third Order way can only interpret these demands through this lens. What is that experience like? In what ways can such teachers succeed in these types of programs?

Also, how do the structures built into programs that seem to offer supports for developmental change actually function? When and how does participation in such a program actually promote increasing perspective taking and complexity of mind? These and other questions might be fruitfully addressed using a constructive-developmental framework to further examine teachers' experiences in teacher professional development programs.

Conclusion

This study has been ambitious. I began by looking at the goals and methods of exemplary mathematics teacher professional development programs as they seek changes in teachers' thinking. In doing so, I found several themes that can characterize the majority of these change goals and sought to paintnuanced sketches of these themes and the several ways that programs approach these goals. I then stepped back from these themes using a constructive-developmental lens to describe both how these professional development goals may require cognitive capacities that are not universally held, and how teachers constructing meaning at different Orders of Mind might experience program demands and supports. Finally, I hinted at some possible implications of this perspective on professional development practice and for further research. I hope that the lens I propose and begin to use here can help us better understand how teacher professional development experiences can better serve teachers and thus, how we can move towards improved teaching in the service of deeper and more robust student learning.

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[7]Careful readers will notice that I have not addressed the Equity theme in this section. This is due to the fact that I have not been able to draw insights about constructive-developmental implications of how programs address this theme from the literature reviewed.

about the appropriate scope of this paper. Sara's voice, both in anticipation and reality, kept me searching hard to capture and communicate the essences and nuances of the programs and ideas I describe. My work with Bob in several contexts—as his advisee and student, as a teaching fellow for his course, as colleagues on a research project—has pushed me to refine and clarify the developmental ideas that I chose to pursue in this paper. His questions about both the paper's content and my process for producing it helped me write much more of the QP I really wanted to create. The smiles and laughter and encouragement from all three are also much appreciated.

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Appendix A: Program features detailed

CALIFORNIA MATHEMATICS LEADERSHIP PROJECT —(CMLP)

Participants : Elementary & middle school teacher leaders & principals; School & district focus
Locations : California districts
Time Frame : 4-5 years per participant; 7 years total
Foci : Mathematics & attitudes towards math; Constructivism; Leadership
Structures/ activities : Inservice workshops; Ongoing support for change in teaching and for leadership; Parent education
Desired outcomes : School-wide restructuring and change in culture; Change in teaching practice; Develop leadership capacity
Evaluation methods : Informal assessment of attitudes

COGNITIVELY GUIDED INSTRUCTION (CGI)

Participants : Primary grades teachers
Locations : Madison, WI; Prince George's County, MD; others
Time Frame : 1 month to several years; Since 1985
Foci : Children's math understanding; Constructivist teaching
Structures/ activities : 2 week summer inservice; Academic year workshops; Varied approaches
Desired outcomes : Change in understanding of mathematics & children's math thinking; Constructivist teaching practices
Evaluation methods : CGI beliefs scale survey; Classroom observations; Teacher interviews & informal focus groups; Measures of children's learning

ELEMENTARY & MIDDLE SCHOOL MATH & TECHNOLOGY PROJECT — (EM-MAT)

Participants : 220 Boston K–8 teachers from 100 schools
Locations : Boston, MA
Time Frame : 1 year per participant; 6 years total
Foci : Math content; Change in teaching methods
Structures/ activities : 2 week summer math course; 1 release day/ month; Money for materials, computer, phone, modem; Classroom support; Support network
Desired outcomes : Develop math teacher leaders
Evaluation methods : Pre- & post-program surveys re: beliefs & behaviors; Case studies of teaching; Student math belief survey

FIRST PROJECT —(FIRST)

Participants : 357 elementary teachers; 181 principals; Site coordinators; University partners
Locations : North Carolina State-wide
Time Frame : 1 year planning; 2 years program
Foci : School improvement plans; Reform oriented change in teaching
Structures/ activities : Develop school improvement plans (SIPs); Year 1: 3 week summer workshop; Year 2: 1 week summer workshop; Building level workshops to implement SIPs; Leadership training
Desired outcomes : Increase positive attitudes to math/ science; Increase math/ science teaching competency; Schoolwide improvement plans

FIRST, CONTINUED

Evaluation methods : Needs assessment instrument re: content, instruction, assessment & class climate; Monitor student performance, instructional practice, visions of reform.

PROJECT IMPACT: INCREASING THE MATHEMATICAL POWER OF ALL CHILDREN & TEACHERS —(IMPACT)

Participants : All K–3 teachers in 3 primary schools; Then grades 4–5 teachers in 5 elementary schools; Teachers of minority & at-risk children
Locations : Montgomery County, MD Public Schools
Time Frame : 1990-93 K–3 teachers; 1993-95 grades 4–5 teachers
Foci : Mathematics; Constructivism; Equity; Change in teaching; Children's math understanding
Structures/ activities : Summer inservice; School based math specialist for ongoing support
Desired outcomes : Increase math understanding of all; Change in predominantly minority schools; Constructivist teaching
Evaluation methods : Quantitative & qualitative data re: teacher change; ModifiedCGI beliefs scale; Phase I: Matched school comparisons

KENTUCKY K–4 MATHEMATICS SPECIALIST PROGRAM —(KENTUCKY)

Participants : 435 K–4 math specialists in 143 districts & 25 private schools; University faculty; School administrators; Parents
Locations : Kentucky State-wide
Time Frame : 1 year preparation; 1 year program; Several years follow-up; 4 years total
Foci : Change in attitudes and views of mathematics; Change in teaching strategies; Increase political voice
Structures/ activities : 45 hour summer seminar; Specialists support teachers in classrooms; Specialists meet together monthly
Desired outcomes : Create network for change in teaching
Evaluation methods : Change in beliefs assessed through writing prompts & surveys; Specialist feedback; Interviews & observations of teachers & kids

MATHEMATICS & SCIENCE ENHANCEMENT —(MASE)

Participants : 50 K–6 teacher leaders; 260 site liaisons; 150 administrators; 127 urban & rural elementary districts
Locations : Clark County, NV; Includes Las Vegas
Time Frame : 4 years
Foci : Leadership; Restructuring; Build capacity for teacher decision-making; Math & science content; Constructivism; Children's thinking
Structures/ activities : Leadership development; Workshops, seminars, classroom demonstraions; Administrator workshops
Desired outcomes : Develop leaders/ change agents; Build capacity for knowledgeable decisions; Constructivist teaching; Change in math beliefs
Evaluation methods : Internal staff evaluation via written response, learning logs, classroom visits, student/ teacher portfolios, & teacher self-assessment

MATHEMATICS FOR TOMORROW —(MFT)

Participants : Phase I: 26 K–8 teachers, 14 administrators; Phase II: 40 teachers, 20 administrators; School-based teams from 4 districts
Locations : Boston-area districts
Time Frame : Two groups, —2 years per participant; Leadership extension
Foci : Mathematics; Constructivism; Change in teaching practice; Inquiry culture; Teacher leadership
Structures/ activities : Two 3-week summer institutes; Biweekly district-based inquiry groups; 4 day-long workshops / year; Classroom consultations; Administrator inquiry group; Inquiry group Sourcebook

MFT, CONTINUED

Desired outcomes : Inquiry culture around math and teaching; Deepen math knowledge & change beliefs; Constructivist teaching; Administrator learning
Evaluation methods : Case studies of teacher change—observation & interviews; Teacher reflective journals

TALKING MATH—(TALKING MATH)

Participants : Group I: 12 elementary teachers; Group II: 25 teachers
Locations : 1/2 Boston; 1/2 suburban Boston
Time Frame : 2 or 3 year phases
Foci : Mathematical discourse; Children's thinking; Mathematics
Structures/ activities : 3-week summer seminar; Biweekly academic year seminar; Resource package
Desired outcomes : Inquiry community; Increase math knowledge & confidence; Pedagogical development
Evaluation methods : Interviews; Videos of classrooms; Teacher writings

TEACHING TO THE BIG IDEAS—(TBI)

Participants : 36 elementary teachers
Locations : Eastern & Western Mass.
Time Frame : 4 years per participant
Foci : Mathematics; Children's mathematical thinking; Teaching; Leadership
Structures/ activities : Summer institutes; Biweekly academic year seminars on math content & instructional implications; Biweekly classroom visits; Year 4: Teachers run peer workshops & seminars; Create professional development materials
Desired outcomes : Deeper math understanding; Constructivist teaching; Inquiry culture around math, kids' thinking, & teaching;
Evaluation methods : Regular teacher writing; Classroom observation

TEACH-STAT—(TEACH-STAT)

Participants : 80 "Statistics educators"; 480 grades 1–6 teachers; University faculty; U NC Math & Science Education Network
Locations : North Carolina, state-wide
Time Frame : 1 year per participant; 3 years total
Foci : Statistics content; Change in teaching practice; Leadership;
Structures/ activities : Year 1: 3 week summer pilot at one site for leaders; Years 2 & 3: 3 week summer at 9 sites for teachers; Apprenticeship/ coaching; Materials development for replication
Desired outcomes : Cadre of leaders for consistent state-wide professional development; Integrating statistics content into elementary curriculum
Evaluation methods : Phone interviews; Statistics content test; Classroom visits; Pedagogy & impact surveys; Informal reports; Student achievement

TEACHING EXCELLENCE AND MATHEMATICS—(TEAM)

Participants : 50 elementary teachers in pairs from same school systems
Locations : North Carolina, state-wide
Time Frame : 4 years
Foci : Leadership; Math content
Structures/ activities : 3 week summer institutes; 3 academic year 1.5-2.5 day workshops; Years 3 & 4: Prepare staff development materials; Teacher-run workshops for peers, administrators, parents, school boards
Desired outcomes : Leadership development; Increase math content knowledge; Develop school improvement plans
Evaluation methods : Count number & types of workshops; Teacher portfolios; Math post-test; Workshop evaluations

TERC *INVESTIGATIONS* CURRICULUM —(TERC)

Participants : Formative & pilot testing teachers; Users of the materials
Locations : Distributed nation-wide
Time Frame : Published 1994
Foci : Math content; Children's thinking; Teaching practice
Structures/ activities : Formative & pilot testing; Materials development and distribution; Coordination with professional development programs
Desired outcomes : Curriculum that also supports ongoing math & pedagogy learning for teachers; Development of teachers' professional expertise
Evaluation methods : Observations & interviews in pilot testing; Observation of use of curriculum in professional development

LEAD TEACHER PROGRAM OF THE VIRGINIA QUALITY EDUCATION IN SCIENCE & TECHNOLOGY —(V-QUEST)

Participants : 30 pilot lead teachers; 480 other lead teachers (240 each year) in building pairs; Adminstrators
Locations : Virginia, state-wide
Time Frame : 3 years
Foci : Leadership support; Math content; Teaching issues
Structures/ activities : 2 week & 1 week summer institutes; 2 academic year conferences; Monthly contact & support; Involve principals
Desired outcomes : Statewide K–14 reform; Leadership development & support;
Evaluation methods : Collect systemic indicators; Ongoing workshop evaluation data

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