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ABSTRACT

This paper describes how 8th grade students are using CBR, a motion detector linked to a graphing calculator, as a way of generating mathematical ideas regarding the motions concepts that surround their action. Students were previously introduced to the calculators in the classroom and teaching experiments were then carried out afterwards with a few pairs of students as a means of studying students' narratives as they faced the designed tasks. Students connected their body expressions to the Cartesian graphs generated by the motion detector. Discussion related to geometry, kinesthetic action, and functions emerged in student narratives. Data are presented based on the video-taping conducted throughout the teaching experiments and analysis developed with the help of GPIMEM, the research group. Results suggest that the use of the sensor can expand what has been labeled the epistemology of multiple representations. A theoretical view based on the notion of humans-with-media is sketched. (Contains 20 references.) (Author/ASK)

The Mathematics of Motion, Sensors, and the Introduction of Function to Eight Graders in Brazil

by
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THE MATHEMATICS OF MOTION, SENSORS, AND THE INTRODUCTION OF FUNCTION TO EIGHT GRADERS IN BRAZIL [1]

A paper presented at AERA-2001, April, 10-14, 2001, Seattle, U.S.A

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Abstract

This paper describes how 8th graders are using CBR, a motion dectector linked to a graphing calculator, as a way of generating mathematical ideas regarding the motions concepts which surround their action. Students, had previously been introduced to the calculators in their classroom and teaching experiments were then carried out by the second author afterwards with a few pairs of students , as a means of studying students' narratives as they face the tasks designed by the authors. Students connected their body expression to the Cartesian graphs generated by the motion detector. Discussion related to geometry, kinesthetic action and functions emerged in students narratives. Data is presented based on the video-taping conducted throughout the teaching experiments and the analysis developed by the authors with the help of GPIMEM, the research group to which they belong. Results suggest that the use of the sensor can expand what has been labeled the epistemology of multiple representations. A theoretical view based on the notion of humans-with-media is sketched.

INTRODUCTION

In the last decade there has been a trend in mathematics education to work with multiple representations (Tabach, 1999). As a result of greater acessibility to graphing calculators and computers, the use of multiple representations has been discussed intensively, especially for mathematics topics such as functions which seem to be suitable for such an approach. Multiple representational software have been developed for computers and calculators at a speed which is hard to keep up with (Elliot, Hudson & O'Reilly, 1999; Manrique et. al. 1998; Doerr & Zagor, 1999; Berger, 1998). Authors (e.g. Borba, 1994) have stressed the importance of such an approach as it facilitates students coordination of established mathematical representations such as tables, Cartesian graphs and algebraic expressions. With the development of interfaces for calculators, such as the CBRs with motion detector sensors, new possibilities exist for curriculum development

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and expansion of the epistemology of multiple representation (Borba & Confrey, 1996). As sensors make possible the connection of body motion to the Cartesian graph, the awareness of body movement can play a paramount role as conflicts between body movement and traditional mathematical representations may arise.

In this paper the we will present a study with 8th graders who worked with such technology in Brazil. A model relating body movement to the epistemology of multiple representations will be sketched. First we will outline the way the research was conducted, next we will present a research protocol with a small piece of the data collected, and finally we will discuss how we analyze the results from a theoretical perspective.

THE RESEARCH

GPIMEM, our research group, has developed an articulated study in which we have looked at different aspects of the use of technology in schools. Students, teachers and parents have been studied in settings such as schools, homes and universities using qualitative methods (Denzin & Lincoln, 2000) in which interviews, teaching experiments and participant observation are used as procedures.

One of these studies was developed in a school with a lower income population close to UNESP University, where GPIMEM is based. The second author of this paper developed exploratory studies with teachers in 1998 in which teachers and the researcher got acquainted and developed a pilot study with students. Both in the pilot study and in the main study, which was carried out in 1999, the teacher used graphing calculators in the classroom in collaboration with the researcher, and the latter met periodically with pairs of students to develop teaching experiments. The role of the classroom activity, for research purposes, was to acquaint the students with the graphing calculator TI-83, so they would know how to deal with it.

Teaching experiments (Cobb & Steffe, 1983, Villarreal, 1999) can be seen as a sequence of meetings in which a researcher tries to model how students think about tasks posed to them or which they posed to themselves. Besides the pair of students and the researcher, a technical assistant was present in the meetings to video-tape the activities. Video-tapes were transcribed, and both authors of this paper and some members of the research group analyzed selected episodes.

One of the tasks presented to the students was an open-ended one in which they were asked to move the sensor and think about what graph *distance x time* would result. It was made clear that the students understood that the sensor, the TI-CBR, would measure the distance from itself to an obstacle in front of it. The sensor has an internal clock which runs for 15 seconds. Once the students started the device the graphing calculator would graph the distance to an obstacle during 15 seconds. Besides the calculator, the two students also have available pencil and paper and chalk and two blackboards. The particular episode which will be presented in this paper took place in a small classroom (4.5 meters x 6.5 meters).

RELEVANT RESULTS

The main actors in the episode which will be described are André and Naíta. André was 13 years old and Naíta was 14 at the time of the teaching experiment. André is white and Naíta black, and they are both from low income families. The episode was extracted from the third meeting with the pair of eight graders.

As André was asked to make any movement he wanted with the sensor, he chose to position himself in the middle of the room and turned his own body around with the CBR pointing against the walls as he moved (Fig. 1). The dialogue, which was originally in Portuguese, has been translated into English by the authors. The protocol which

Fig. 1

follows starts with Naíta asking if what André is doing is legitimate and the assurance of the researcher, Nilce, that it is.

Naíta: *Can it keep going around?*

Nilce: *Yes, it can.*

André: *At the time when I was there* [pointing to the center of the room] *and I started to go around, I took the radar [*that is how André referred to the CBR*] a little bit foward*

Nilce: *In reality how is this movement?*

André: *It is a circular one.*

Nilce: *Circular one, hum. . .* [the researcher gave him a piece of chalk and asked him a question] *What do you think the graph of this movement would be like, going around like you did?* [he does the circle as shown in Fig.2].

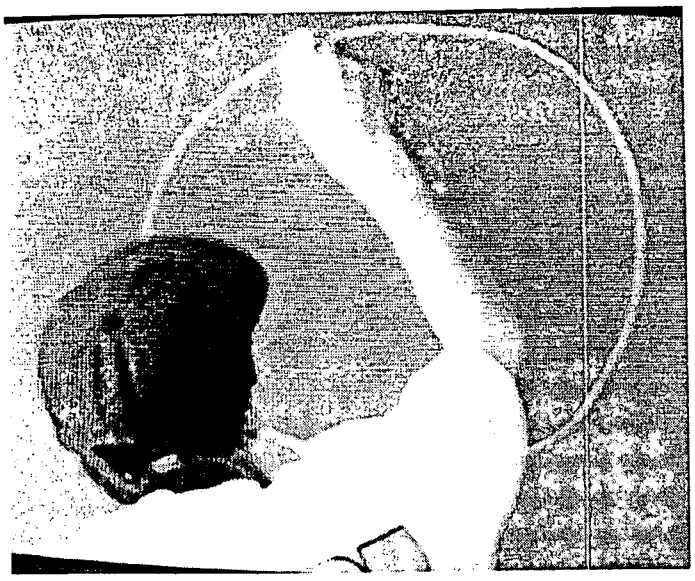

Fig. 2

Then the researcher asked him:

Nilce: *Why like that?*

André: *Because I was going around, wasn't I?*

Nilce: *Where were you?*

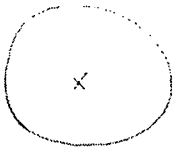

Fig.3

André: *In the center* [he went and marked an x in the center of the circle previously drawn in the blackboard as shown in Fig. 3]

Nilce*: So do you think the graph would be like this?*

André: *Yes.*

Nilce: *So, André, how do you explain to us the graph that the calculator shows?* [the researcher pointed to the calculator screen. See Fig. 4].

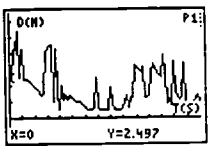

Fig.4

Nilce: *What happened there* (Fig. 4)?

André seems to be thinking hard, staring at both graphs. Naíta looks mostly to the graph that André drew and says:

The graph is not a "pizza graph"; it is made by straight lines.

Nilce: *Hmm.*

Naíta: *It [*the sensor] *gets the distance from the point where it was to the blackboard , or to the window, or to the door* [Naíta points to the different targets that the CBR may have reached when André made his circle-like movement].

Nilce*: Hmm. It got these distances, you think?*

Naíta: *Then, if were in another place, the graph would be neater, for instance, . . then it would be four positions, it would be the blackboard, the door, the window and these other two blackboards* [pointing to the different targets]

Nilce*: Hmm!*

Naíta: *They* [the calculator and the CBR] *only got the distance.*

.....

Naíta: *I think that he* [pointing to André] *imagined it that way* [pointing to the drawing in the blackboard, Fig. 2] *because he, I think that he did not consider the four walls. He wanted to make a "pizza graph" but he forgot to take into account the four walls, he wanted a pizza graph, right? But when he went around* [pointing to both walls] *the distance was not the same. He was closer to both walls, then there was the difference. Did you get it? Here, he was closer to the wall, that is the reason for the difference* [we believe she refers to the bumps in the graph of Fig. 4].

It is important to notice that Naíta was oscillating between a perspective in which if they were in a particular room she was imagining, a pizza graph would be generated: and another in which she explained the bumps in the graph displayed by the calculator. As we will see later, she solves this conflict even though we have no data to infer any model about how she did so. On the other hand, André gets back into the debate proposing that, if the room were in a circular shape, then the graph he drew in the black board would be right:

André: *In that circle-like movement* [pointing to the figure he drew on the blackboard], *being in a round room, right? and right in the center of it, the distance would be just one.*

Nilce: *And so what would the graph be like?*

André: *It would be circular.*

Nilce: *Would it be circular?*

Naíta: *Since the calculator represents it with straight lines, it would be just one line.*

Nilce: *Hmm. Interesting. Then if we had in a round room, as André says, this graph* [pointing to to the graph in Fig. 4] *would be like what?*

Naíta: *For example, if the movement of André's arm was not shaky, if he did not move the arm and just turned his body around, it would be a straight line because it would be just one distance* [she indicates that she means what we would call a constant function].

DISCUSSION OF THE RESULTS AND THEORETICAL PERSPECTIVES

The episode presented may be analyzed from many different perspectives. It may be seen from a mathematical perspective in which the student generated a problem which can have different degrees of complexity depending on whether the speed of the movement is constant, or whether the room is a square, rectangle and so on. It could also be discussed from a pespective of how appropriate the interventions of the teacher/researcher were. However we would like to focus in this paper, on the role that episodes like this could play in the introduction of the notion of function. Coordination of multiple representations shoud also include coordinating mathematical representations with your body movements.

In analyzing the episode, we felt the need to study the role of the body in knowing, as we have not emphasized its role in our previous study regarding technology (Borba, 1994, 1999, 2000). This led us to study Merlau-Ponty (1994) who stresses, among other points, the indivisibility of body movement and awareness of it. This author views the

mind and body as a whole - and not the mind in opposition to the body - that perceives, that knows. As André, in the episode above, starts the movement around himself with the CBR pointing against the walls which bound the room he was in, he was first inclined, to believe that the graph that the calculator would show would be a circle (fig. 2). The researcher's probe confirms that, as he affirms that he was in the center of the circle (fig. 3). It is not new in the literature that some students think the graph resembles the motion. What adds to other studies is that André was engaged in the task (as his concentrated look suggests in figure 1) and he was very aware of his attempt since his own body was involved in it. The other point that differs from other studies is the fact that the calculator provides feedback, as shown in figure 4, and André has to coordinate his awareness of his own body movement with the representation of the Cartesian graphs, which reduces the totality of his movement to a two-dimensional graph.

Then a third factor emerges: his interaction with the researcher and his colleague, Naíta. Naíta suggests that maybe the unexpected graph shown in the calculator is due to the chairs, windows and so on, that were within the reach of the sensor. Although this explanation helps them to understand what is happening, Naíta needs to demonstrate that the distance from the sensor to the blackboard is less than the distance to the corner of the room, for example. That is how she makes sense of the bumps in the graph shown on the calculator.

André insists that if the room were in the form of a circle, then the graph would be in the form of the circle, and Naíta again refutes him proposing that the graph would be a straight line.

CONCLUSION

The mathematical narratives - understood by Nemirovsky (1996) as a sequential description intertwined with mathematical symbols (Nemirovsky, 1994) - developed by the two students were enacted with the full use of gestures and body language. The awareness of body movement, as previously discussed, was central to the episode presented. It should be emphasized, though, that Merlau-Ponty (1994) did not think of computers as technology of intelligence (Levy, 1993), so we have to expand the ideas we are borrowing from him to think about this episode. In this sense, based on Levy (1993) and Tikhomirov (1981) we propose that the role of technology is central to the way one knows because the possibilities of the body are transformed due to the feedback provided by artifacts like the calculators. Also, the ways one knows are always influenced by other humans, like Naíta or the designers of the calculator. Thus, we believe that this episode substantiates the theoretical work behind the notion of humans-with-media (Borba, 1999, 2000) as the main unit of the collective subject, since it suggests the relevance of humans and non-humans in the process of knowing. According to Levy (1993), humans have always known in union with technologies of intelligence such as orality, writing and informatics. The episode presented highlights, among other things, the relevance of a new non-human interface in the process of knowing.

For the discussion regarding function, we believe that the above example extends the notion that understanding function is not only coordinating graphs with tables and algebra, but must also include motions, such as the one reported above. In this sense, coordination of multiple representations should also be in harmony with motions experienced by collectives like the one formed by the Naíta, André, Nilce , the graphing calculator and the CBR. We believe that such a theoretical perspective about knowing and about functions can open new perspectives for the development of curricula. The mathematics of motion in which technological actors take part could link concerns about introducing calculus concepts in grades like the 8th grade with new possibilities for introducing functions.

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