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ABSTRACT
        This workbook introduces curriculum mapping, a process used
by schools and districts to reform and improve curricula. The school's or
district's elementary and middle level science curriculum can be mapped out
by indicating what topics are taught at what grade levels. This workbook is
part of the North Central Regional Educational Laboratory's (NCREL's) ongoing
effort to develop accessible and user-friendly, data-driven decision making
tools. The workbook allows for the analysis and comparison of individual
mathematics curriculum to the United States as a whole and to top achieving
nations around the globe. The curriculum is first mapped out on a tally sheet
by indicating what topics are taught at which grades, then a plot is made of
tally sheet totals on a variety of templates showing comparable data from the
United States and top achieving nations. (SAH)

# Mathematics
# Curriculum
# Mapping
## *Workbook*

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# NCREL
North Central Regional Educational Laboratory
*"Applying Research and Technology to Learning"*

## Curriculum Mapping Workbook

This workbook introduces "curriculum mapping," a process used by schools and districts to reform and improve curricula. With this workbook, you can "map out" your school or district's elementary and middle-level mathematics curriculum (Grades 1-8) by indicating what topics are taught at which grade level.

A Web-based version of this tool is available for more advanced analysis in both mathematics and science. With the Web-based version, rich international mathematics and science curriculum maps from top-achieving nations are available for comparison to your local curricula.

## Background

Today, the U.S. educational system faces a myriad of challenges to initiate and implement curriculum reform. Results from the Third International Mathematics and Science Study (TIMSS), the most extensive study of international achievement to date, provide overwhelming evidence that topics in U.S. curricula are too numerous and retained too long. Unfortunately, district administrators lack user-friendly methods to make informed decisions about curricula.

The North Central Regional Educational Laboratory (NCREL), in partnership with the U.S. TIMSS National Research Center at Michigan State University, has responded by developing tools to facilitate more comprehensive mapping and analysis of curricula.[1] This introductory workbook and the accompanying Web site are designed to help schools and districts reform their mathematics and science curricula. These products are part of NCREL's ongoing effort to develop accessible and user-friendly data-driven decision-making tools.

## How It Works

This workbook allows you to analyze and compare your mathematics curriculum to the U.S. as a whole and to top-achieving nations around the globe. You begin by "mapping out" your curriculum on your tally sheet by indicating which topics are taught at which grade levels (K-8). Then you plot totals from your tally sheet on a variety of templates—or displays—showing comparable data from the U.S. and top-achieving nations. These comparisons will help you make informed decisions about curriculum reform strategies.

For more sophisticated curriculum analysis, use our Web site (www.ncrel.org/currmap) to map your mathematics curriculum. Web users are guided through a series of analyses, complete with graphic displays that superimpose their curricula against that of top-achieving TIMSS nations.

## Before You Start

Quality curriculum analysis and evaluation is a time-consuming task. We recommend that you allow 1-2 hours for the mapping process. Effective curriculum mapping also requires the right personnel. We suggest that you bring together the following personnel and documentation:

* District-level staff knowledgeable of your standardized elementary mathematics curriculum (The standardized curriculum appears in your district textbooks and/or your district curriculum guide.)

* Mathematics curriculum guides developed for your district

* Other personnel and written material that influence what is taught at what grade level(s)

### Refer to the Pullout Section for Instructions on Completing Your Tally Sheet

[1]The TIMSS curriculum frameworks used in these products were developed by the Survey of Mathematics and Science Opportunities (SMSO) research project, funded by the U.S. Department of Education and the National Science Foundation. References include:

Survey of Mathematics and Science Opportunities. (1992). *Science curriculum framework* (Research Report Series 37). East Lansing, MI: Michigan State University.

Survey of Mathematics and Science Opportunities. (1992). *Mathematics curriculum framework* (Research Report Series 38). East Lansing, MI: Michigan State University.

Robitaille, D. F., Schmidt, W. H., Raizen, S., McKnight, C., Britton, E., & Nicol, C. (1993). *Curriculum frameworks for mathematics and science* (TIMSS Monograph No. 1). Vancouver: Pacific Educational Press.

## Instructions for Mapping Your Mathematics Curriculum

To complete your Tally Sheet for Mathematics Curriculum, read the description for each topic (in this instruction booklet) and decide for each grade level whether the topic is:

- **Not Taught (leave the box on the tally sheet blank).** A topic is not taught if **ANY** of the following statements are true:

    - The topic is not taught at that grade in your school or district.

    - The grade level is not included in your school or district.

    - The curriculum includes ONLY incidental coverage of the topic.

    **Note:** Avoid using "Not Taught" as a substitute for "I don't know."

- **Taught (write a "1" in the box on the tally sheet).** A topic is taught if at least one lesson addresses the topic. This category does not include incidental coverage of a topic.

- **Emphasized (write a "2" in the box on the tally sheet).** A topic is emphasized if several lessons are devoted to teaching the topic.

In the example below, the topic "Whole numbers: Meaning" is taught (1) in first grade, and it is emphasized (2) in Grades 2, 3, and 4. The topic is not taught in Grades 5-8.

| Topic | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total Grades per Topic |
|---|---|---|---|---|---|---|---|---|---|
| Whole numbers: Meaning | 1 | 2 | 2 | 2 | | | | | 4 |

## Special Instructions for Mapping Middle School Curricula

In instances where a district has specialized tracks of study in middle school, separate curriculum maps should be completed to describe each intended mathematics curriculum.

## Mathematics Topic Descriptions

### Numbers

Whole numbers

- ✓ Meaning—including the uses of numbers; place value and numeration; and ordering and comparing numbers

- ✓ Operations—including addition, subtraction, multiplication, division, and mixed operations

- ✓ Properties of operations—including commutative property, distributive property, etc.

Fractions and decimals

- ✓ Common fractions—including the meaning and representation of common fractions, and computations with common fractions and mixed numbers

- ✓ Decimal fractions—including the meaning and representation of decimals, and computations with decimals

**NOTE:** This form is a modified version of the GTTM developed for the Third International Mathematics and Science Study (TIMSS) of 1995.

✓ Relationships of common and decimal fractions—including conversion to equivalent forms and ordering of fractions and decimals

✓ Percentages—including all work with percent computations and various types of percent problems

✓ Properties of common and decimal fractions—including commutative, distributive, etc.

Integer, rational, and real numbers

✓ Negative numbers, integers, and their properties

✓ Rational numbers and their properties—including terminating and recurring decimals

✓ Real numbers, their subsets, and their properties

Other numbers and number concepts

✓ Binary arithmetic and/or other number bases

✓ Exponents, roots, and radicals—including integer, rational, and real exponents

✓ Complex numbers and their properties

✓ Number theory—including primes and factorization, elementary number theory, etc.

✓ Counting—including permutations, combinations, etc.

Estimation and number sense

✓ Estimating quantity and size

✓ Rounding and significant figures

✓ Estimating computations—including mental arithmetic and reasonableness of results

✓ Exponents and orders of magnitude

## Measurement

✓ Units—concept of measure and standard units (including metric system); use of appropriate instruments (precision and accuracy); common measures (length, area, volume, capacity, time and the calendar, money, temperature, mass and weighing, angles, quotients and products of units [km/h, m/s, etc.]); dimensional analysis

✓ Perimeter, area, and volume—concepts of perimeter, area, surface area, and volume; formulas for perimeters, areas, surface areas, and volumes

✓ Estimation and errors—estimation of measurements and errors of measurement; precision and accuracy of measurements

## Geometry: position, visualization, and shape

✓ Two-dimensional geometry: coordinate geometry—line and coordinate graphs, equation of line n the plane, and conic sections and their equations

✓ Two-dimensional geometry: basic—points, lines, segments, rays, angles, parallelism, and perpendicularity

✓ Two-dimensional geometry: polygons and circles—triangles; classification and properties of quadrilaterals; Pythagorean Theorem and applications; other polygons, circles, and their properties

✓ Three-dimensional geometry—three-dimensional shapes and surfaces and their properties; planes and lines in space; spatial perception and visualization; coordinate systems in three dimensions; equations of lines, planes, and surfaces in space

✓ Vectors

## Geometry: symmetry, congruence, and similarity

✓ Transformations—patterns, tessellations, friezes, stencils, etc.; symmetry (line and rotational symmetry, symmetry in three dimensions, symmetry in algebra and number patterns); transformations: symmetries and congruence, enlargements (dilations), combinations of geometric transformations, group structure of transformations, and matrix representation of transformations

✓ Congruence and similarity—congruences (congruent triangles and their properties [SSS, SAS, etc.], congruent quadrilaterals and polygons and their properties); similarities (similar triangles and their properties)

✓ Constructions using straight edge and compass

## Proportionality

✓ Proportionality concepts—meaning of ratio and proportion; direct and inverse proportion

✓ Proportionality problems—solving proportional equations; solving practical problems with proportionality; scales (maps and plans); proportions based on similarity

✓ Slope and trigonometry—slope and gradient in straight-line graphs; trigonometry of right-angled triangles

✓ Linear interpolation and extrapolation

## Functions, relations, and equations

✓ Patterns, relations, and functions—number patterns; relations and their properties; functions and their properties; representation of relations and functions; families of functions (graphs and properties); operations on functions; related functions (inverse, derivative, etc.); relationship of functions and equations (e.g., zeros of functions as roots of equations); interpretation of function graphs; functions of several variables; recursion

✓ Equations and formulas—representation of numerical situations; informal solution of simple equations; operations and expressions; equivalent expressions (factorization and simplification); linear equations and their formal (closed) solutions; quadratic equations and their formal (closed) solutions; polynomial equations and their solutions; trigonometrical equations and identities; logarithmic and exponential equations and their solutions; solution of equations reducing to quadratics, radical equations, absolute value equations, etc.; other solution methods for equations (e.g., successive approximation); inequalities and their graphical representation; systems of equations and their solutions (including matrix solutions); systems of inequalities; substituting into or rearranging formulas; the general equation of the second degree

## Data representation, probability, and statistics

✓ Data representation and analysis—collecting data from experiments and simple surveys; representing data; interpreting tables, charts, plots, and graphs; kinds of scales (nominal, ordinal, interval, ration); measures of central tendency; measures of dispersion; sampling, randomness, and bias; prediction and inferences from data; fitting lines and curves to data; correlations and other measures of relations; use and misuse of statistics

✓ Uncertainty and probability—informal likelihoods and the vocabulary of likelihoods; numerical probability and probability models; counting principles; mutually exclusive events; conditional probability and independent events; Bayes' Theorem; contingency tables; probability distributions for discrete random variables; probability distributions for continuous random variables; expectation; sampling; estimation of population parameters; hypothesis testing; confidence intervals; bivariate distributions; Markov processes; Monte Carlo methods and computer simulations

## Elementary analysis

✓ Infinite processes—arithmetic and geometric sequences; arithmetic and geometric series; Binomial Theorem; other sequences and series; limits and convergence of series; limits and convergence of functions; continuity

✓ Change—growth and decay; differentiation; integration; differential equations; partial differentiation

## Validation and structure

✓ Validation and justification—logical connectives; quantifiers ("for all," "there exists"); Boolean algebra and truth tables; conditional statements; equivalence of statements (including converse, contrapositive, and inverse); inference schemes (e.g., modus ponens, modus tollens); direct deductive proofs; indirect proofs and proof by contradiction; proof by induction; consistency and independence of axiom systems

✓ Structuring and abstracting—sets, set notation and set combinations; equivalence relations, partitions, and classes; groups; fields; linear (vector) spaces; subgroups, subspaces, etc.; other axiomatic systems (e.g., finite geometries)

## Other content

✓ Informatics—operation of computers, flow charts, learning a programming language, programs, algorithms with applications to the computer, complexity; history and nature of mathematics; special applications of mathematics (kinematics, Newtonian mechanics, population growth [discrete or continuous models], networks [applications of graph theory], linear programming, critical path analysis, examples from economics); problem-solving heuristics; non-mathematical science content; non-mathematical content other than science

## Breadth of Curriculum

Breadth of curriculum is defined as the number of topics taught or intended at each grade level. Analysis of breadth gives you a first look at your curriculum. It does not tell you what, when, or how long topics are taught, but rather it provides a general sense of the number of topics at each grade. Analysis of breadth is important because of the implications it can have for depth of instruction. Given the limited number of instructional hours in a school year, greater breadth implies less depth of instruction. In the case of breadth, *more is not better.*

You might pose the following questions to your school or district improvement team to generate discussion about curriculum breadth:

- Are we comfortable with the total number of topics intended at each grade level? Do we think there are too many? not enough?

- Is the breadth of the curriculum so great that it is compromising depth of content coverage?

- Can we eliminate topics at any grade level?

- Is the breadth of our curriculum similar to that of the United States? of top-achieving TIMSS nations?

- Does the breadth of the curriculum increase from the elementary to middle grades? Are topics accumulating?

Results from TIMSS revealed substantial breadth in the U.S. curriculum when compared to the 40 nations participating in the study. One way to determine whether your school or district has excess breadth in its curriculum is to hand-plot the curriculum against the international distribution. On the display below, plot the "Total Topics per Grade" from your tally sheet with an "X" for each grade level. If your "X" is to the right of the gray bar for any grade, you are including more topics than 75 percent of the TIMSS nations for that grade. Therefore, you may want to evaluate the scope and sequence of your curriculum. Always refer to your state standards when considering reform.

This display offers only a cursory look at the breadth of your curriculum. More advanced comparative displays can be generated from the Web site (www.ncrel.org/currmap). These advanced displays provide a wealth of information to guide data-driven curriculum reform.

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### Display 1: Number of Intended Mathematics Topics

The U.S. composite for each grade is indicated with a circle. A composite for the top-achieving nations for each grade is indicated with a square. The gray bar shows how many mathematics topics were intended at each grade level for all TIMSS countries, from the 25th percentile to the 75th percentile. The blue line indicates the median number of topics at each grade level.

**Directions:** With an "X," plot the number of intended topics for your curriculum using the "Total Topics per Grade" at the bottom of your tally sheet.
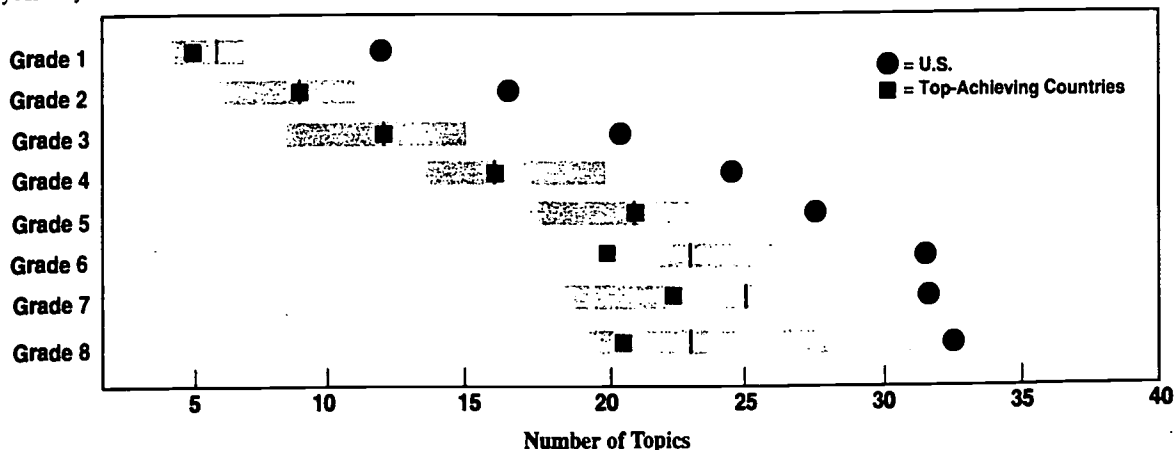


Source: Schmidt, W. H., McKnight, C. C., & Raizen, S. A. (1997). *A splintered vision.* The Netherlands: Kluwer.

# Tally Sheet for Mathematics Curriculum

| Topic | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total Grades per Topic* |
|---|---|---|---|---|---|---|---|---|---|
| **Numbers** | | | | | | | | | |
| Whole numbers: Meaning | | | | | | | | | |
| Whole numbers: Operations | | | | | | | | | |
| Whole numbers: Properties of operations | | | | | | | | | |
| Common fractions | | | | | | | | | |
| Decimal fractions | | | | | | | | | |
| Relationships of common and decimal fractions | | | | | | | | | |
| Percentages | | | | | | | | | |
| Properties of common and decimal fractions | | | | | | | | | |
| Negative numbers, integers, and their properties | | | | | | | | | |
| Rational numbers and their properties | | | | | | | | | |
| Real numbers, their subsets, and their properties | | | | | | | | | |
| Binary arithmetic and/or other number bases | | | | | | | | | |
| Exponents, roots, and radicals | | | | | | | | | |
| Complex numbers and their properties | | | | | | | | | |
| Number theory | | | | | | | | | |
| Counting | | | | | | | | | |
| Estimating quantity and size | | | | | | | | | |
| Rounding and significant figures | | | | | | | | | |
| Estimating computations | | | | | | | | | |
| Exponents and orders of magnitude | | | | | | | | | |
| **Measurements** | | | | | | | | | |
| Units | | | | | | | | | |
| Perimeter, area, and volume | | | | | | | | | |
| Estimation and errors | | | | | | | | | |
| **Geometry: position, visualization, and shape** | | | | | | | | | |
| Two-dimensional geometry: coordinate geometry | | | | | | | | | |
| Two-dimensional geometry: basic | | | | | | | | | |
| Two-dimensional geometry: polygons and circles | | | | | | | | | |
| Three-dimensional geometry | | | | | | | | | |
| Vectors | | | | | | | | | |
| **Geometry: symmetry, congruence, and similarity** | | | | | | | | | |
| Transformations | | | | | | | | | |
| Congruence and similarity | | | | | | | | | |
| Constructions using straight edge and compass | | | | | | | | | |
| **Proportionality** | | | | | | | | | |
| Proportionality concepts | | | | | | | | | |
| Proportionality problems | | | | | | | | | |
| Slope and trigonometry | | | | | | | | | |
| Linear interpolation and extrapolation | | | | | | | | | |
| **Functions, relations, and equations** | | | | | | | | | |
| Patterns, relations, and functions | | | | | | | | | |
| Equations and formulas | | | | | | | | | |
| **Data representation, probability, and statistics** | | | | | | | | | |
| Data representation and analysis | | | | | | | | | |
| Uncertainty and probability | | | | | | | | | |
| **Elementary analysis** | | | | | | | | | |
| Infinite processes | | | | | | | | | |
| Change (growth and decay, differentiation) | | | | | | | | | |
| **Validation and structure** | | | | | | | | | |
| Validation and justification | | | | | | | | | |
| Structuring and abstracting | | | | | | | | | |
| **Other content** | | | | | | | | | |
| Informatics | | | | | | | | | |
| Total Topics per Grade (1s and 2s) | | | | | | | | | = A** |
| Total Topics in Curriculum | | | | | | | | | B*** |

*Indicate total number of grades a topic appears in your curriculum. If a topic is not in your curriculum, put an "X" in the box.

**Sum total of all topics in Grades 1-8

***Number of boxes in "Total Grade" column *without* an "X"

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## Topic Duration

Duration of topic is the length of time (number of grades) a topic is retained in a curriculum. Duration of topic has a direct relationship to breadth of curriculum. For example, curricular material is often recycled, reinforced, or re-introduced rather than dropped from the curriculum, increasing the breadth of the curriculum at each successive grade level. As a result, more advanced topics may be crowded out or not covered in depth, reducing the rigor of the curriculum.

You might pose the following questions to your school or district improvement team to generate discussion about duration of topics:

- Are topics retained in the curriculum too long?

- How does the duration of topics reflect teachers' expectations of students' ability to master material at any grade level?

- Are we retaining topics in the curriculum after they are taught? after they are mastered?

- Do state standards require a high or low duration of certain topics? Or is topic duration a direct result of the way we designed the curriculum?

Use the Duration Worksheet below to compare your school or district's average topic duration to that of top-achieving TIMSS nations.

## Duration Worksheet

*From your tally sheet...*

1. Enter the sum total of all topics in Grades 1-8 (box A): _____

2. Enter the number of boxes in the "Total Grade" column without an "X" (box B): _____

*Calculation 1: Average Topic Duration?*

Box A ÷ Box B = Average Duration

$$\boxed{\phantom{xx}} \div \boxed{\phantom{xx}} = \boxed{\phantom{xxxx}}$$

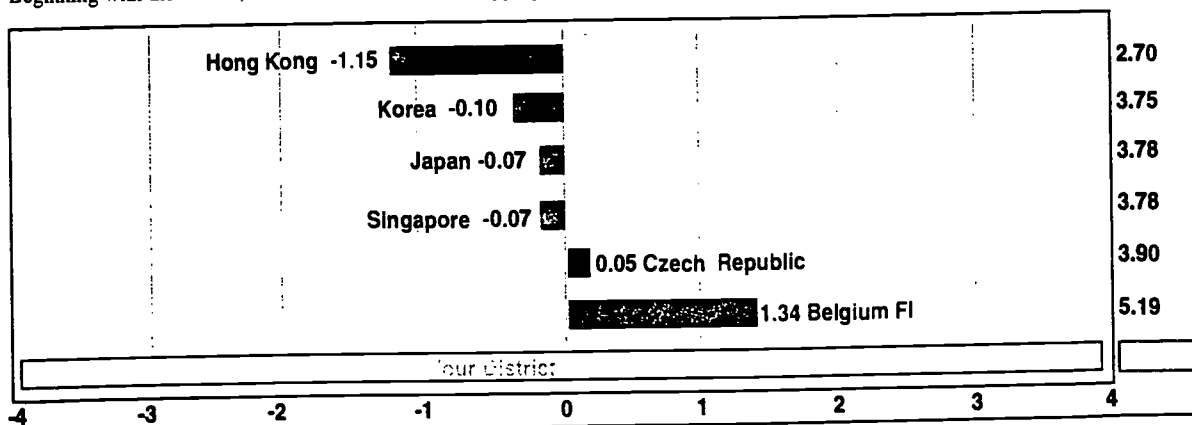*Calculation 2: How Do You Compare With Top-Achieving Nations?*

Your Average Duration (calculation 1) – Average Duration for Top-Achieving Nations = Length of Your Bar

$$\boxed{\phantom{xx}} - \boxed{3.85} = \boxed{\phantom{xxxx}}$$

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### Display 2: Average Duration of Mathematics Topics Relative to Top-Achieving Nations

The anchor, or zero mark. for this display is the average duration (3.85 grades) for six TIMSS nations that significantly outperformed most other nations around the world in Grade 8 mathematics achievement. The individual average duration for each of the six nations is provided to the right of the display. The length of the bars indicates how much more or less topics are retained in a nation's mathematics curriculum compared to the composite average. For example, on average, topics are retained in Japan's curriculum 3.78 grades, which is .07 grades less than the composite average of 3.85. Therefore, Japan's bar is -.07.

**Directions:** Write the average duration of topics in your curriculum (calculation 1) in the box in the lower right corner of the display. Beginning with the anchor, or zero mark, shade in the appropriate length of your bar (calculation 2).

| | |
|---|---|
| Hong Kong -1.15 | 2.70 |
| Korea -0.10 | 3.75 |
| Japan -0.07 | 3.78 |
| Singapore -0.07 | 3.78 |
| 0.05 Czech Republic | 3.90 |
| 1.34 Belgium Fl | 5.19 |
| Your District | |

-4   -3   -2   -1   0   1   2   3   4

## About NCREL

The North Central Regional Educational Laboratory (NCREL) is a not-for-profit organization dedicated to helping schools—and the children they serve—reach their full potential. One of ten regional educational laboratories, NCREL serves a seven-state region of the Midwest: Illinois, Indiana, Iowa, Michigan, Minnesota, Ohio, and Wisconsin.

NCREL's vision is for effective educational systems in which all students are successful learners. Our mission is to improve the performance of all parts and levels of the K-12 educational system from the classroom to the state by:

- Making research-based knowledge useful to and usable by educators, policymakers, and the wider community.

- Integrating research, policy, and practice around issues of transforming schools into high-performing learning communities.

- Providing regional and national leadership in applying technology to improve student learning.

NCREL also operates the North Central Mathematics and Science Consortium (NCMSC) and the North Central Regional Technology in Education Consortium (NCRTEC).

## NCREL

**North Central Regional Educational Laboratory**

*"Applying Research and Technology to Learning"*