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ABSTRACT

This study describes the shifts in thinking and practice made by teachers in primary grades in their first year of implementing an elementary school mathematics reform curriculum. All teachers were observed during professional development sessions and submitted written reflections. A sample of those teachers was interviewed and observed teaching to provide additional insight into how they were interacting with the curriculum materials. The analysis of the data focuses on changes in teachers' goals for what their students should know about numbers and the resulting impact on instructional practice. While teachers reported major shifts in goals from an emphasis on skills to understanding, enacting those shifts in the classroom proved to be more challenging. As teachers attempted to teach for understanding, they mainly struggled with knowing how to elicit and respond to students' ideas. (Contains 23 references.) (Author/SM)

Understanding Teachers' Changing Beliefs & Practice While Implementing a Reform Curriculum

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Understanding Teachers' Changing Beliefs & Practice While Implementing a Reform Curriculum

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This study describes the shifts in thinking and practice made by teachers in primary grades in their first year implementing an elementary school mathematics reform curriculum. All teachers were observed during professional development sessions and submitted written reflections. A sample of those teachers were interviewed and observed teaching to provide additional insight into how they were interacting with the curriculum materials. The analysis of the data focuses on changes in teachers' goals for what their students should know about number and the resulting impact on instructional practice. While teachers reported major shifts in goals from an emphasis on skills to understanding, enacting those shifts in the classroom proved to be more challenging. As teachers attempted to teach for understanding, they mainly struggled with knowing how to elicit and respond to students' ideas.

The current reform movement has been shaped by successes and failures of past reforms. A major obstacle identified by analysts of the last major reform movement in mathematics ("New Math") is that the professional development opportunities were of limited availability, costly, and focused primarily on new content rather than pedagogical issues (Cohen & Barnes, 1993). As a result, many teachers had only the textbook to guide them, within which there were "little to no philosophical or theoretical guidelines" (Bossé , 1995, p.186).

Since that time, we have learned a great deal about the complexities of changing teachers' beliefs and practice (e.g., Lloyd, 1999; Simon & Schifter, 1991; Wood, Cobb, & Yackel, 1991). This work has identified some of the inhibitors of change, which often relate to the organization of schooling and/or the teachers' personal characteristics, such as biography, beliefs, and knowledge (Richardson, 1990). While inhibitors have been identified, we are just beginning to understand how to best enable change (Ball, 1996). Promising results have been found when teachers are encouraged to look carefully at student thinking and use the information gained to make instructional decisions (Franke, Carpenter, Fennema, Ansell, & Behrend, 1998). Further analysis of the factors that enable change is necessary to inform the design and implementation of successful professional development programs.

The current reform movement in mathematics has the advantage of hindsight and a greater understanding of the demands that such a major reform places on teachers, students and parents. This reform movement began with a vision, described in the widely publicized *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). Further specificity of how such a vision might be implemented came later: in the next two Standards documents on teaching (NCTM, 1991) and assessment (NCTM, 1995); in the Addenda series; and most recently, in the form of complete curricular materials funded by NSF. Unlike their predecessors, these new curricula were designed with the intention to be learning tools for students and teachers alike.

The curriculum used in this study, *Investigations in Number, Data and Space* (*Investigations*), has a primary goal to communicate mathematics content and pedagogy to teachers through a variety of components which draw on the research on student thinking and the authors "collaborations with many teachers and students over many years" (TERC, 1998, p.17). It requires a major rethinking of the traditional role of teacher and student consistent with that which is recommended in the three Standards documents mentioned above. The teacher must emphasize thinking rather than be a dispenser of information that is memorized by students and must view the construction of knowledge as the major vehicle for learning.

This study is designed to better understand the shifts in the thinking and practice made by teachers as they interact with such a curriculum as both a guide for day-to-day teaching and a site for reflection in professional development sessions.

Methodology

This study takes place in a suburban/rural school district during the first year of a three-year phased adoption plan for implementing the reform elementary mathematics curriculum, *Investigations*. In the first year, grades K-2 implemented the curriculum; in the second year, grades 3-4 will begin; and in the third year, grade 5. All teachers began, or will begin, with a selection of modules from their grade level that focus on number development and add one to two additional units in subsequent years on such topics as measurement and data analysis. Five

professional development release days were provided for the K-2 teachers during year one, and all teachers were required to attend.

Professional Development. All professional development sessions were facilitated by the authors. The main goals for these sessions were to develop teachers' understanding of the mathematics content in the program, effective teaching practices, and the ways in which students learn. The curriculum materials themselves were the main "text" for these professional development sessions as each unit included: (a) an overview detailing the mathematical goals for the unit; and (b) specific sections that provide examples of student work on assessment items and sample conversations one may have with students to elicit explanations of reasoning. Each professional development day was structured the same and involved discussing questions/sharing stories from the previous unit and preparing for the new unit by doing many of the activities together, discussing the mathematics, and discussing pedagogical implications.

Beyond the resources in the curriculum, written and video cases from *Developing Mathematical Ideas* (Schifter, Bastable, & Russell, 1999) were discussed as well, and samples of student work from previous units that were submitted by the teachers were analyzed. These cases provided illustrations of real teachers in real classrooms using *Investigations*. Discussions centered around mathematical issues as well as pedagogical issues and the moves the teachers made to support their students' understanding.

Data. The data collected from all the grades 1-2 teachers included reflective writings and student work samples submitted at the professional development sessions. In addition, a subset of the teachers volunteered to participate in classroom observations and interviews. These three teachers, one in grade 1 and two in grade 2, were observed at least three times during the year by both authors. Interviews typically took place after an observation and involved general questions on how effective they thought the lessons were as well as specific questions on interchanges with their students. In addition, end-of-the-year interviews with the teachers were conducted to probe their personal reflections on how they changed throughout the year. In one case, this included having the teacher view videotapes of her instruction throughout the year and comment on the

changes she witnessed. All observations and interviews were audiotaped and/or videotaped. The authors also conducted audio-taped debriefings after most of these events, including the professional development sessions described previously, and all tapes were transcribed.

Analysis. Reflective writings from the professional development sessions on goals for teaching number were analyzed and sorted into categories. In most cases the categories were self-evident. If the description was unclear, and no explanation was provided, the item was taken at face value. Field notes of professional development sessions and samples of student work were used to provide further explanations for these findings. Transcripts of classroom episodes were analyzed for common themes within and between the three teachers. In particular, we looked for classroom exchanges that highlighted how teachers were eliciting and reacting to their students' thinking. We used a coding scheme that identified teacher questions as high level or low level in that they either elicited explanations of thinking when solutions were correct or incorrect, or they elicited answers without explanations of thinking. These low level questions included fill-in-the-blank type questions, those requiring a yes or no as a response, and rhetorical questions.

Results

In this section, we will first describe the changes in beliefs of all of the teachers based upon their written reflections, discussions during professional development sessions, and submitted samples of student work. Next, the changes in practice of three of those teachers will be described based upon observations of teaching and reflective interviews. To capture the different struggles these teachers experienced in this first year, the change in practice data will be reported in case study form.

Change in Teachers as a Group

All grades 1 and 2 teachers were asked to identify what they thought was important for their students to know about number by the end of the year. They wrote about their ideas at the beginning of the year and after teaching *Investigations* for one full year. A summary of the results from the writing reflections is shown in Table 1.

Insert Table 1 here.

In the beginning of the year, teachers' comments were more skill-oriented, focusing on the typical goals of traditional instruction in mathematics. The major goal, adding and subtracting, was often described by teachers as 'regrouping,' suggesting a procedural interpretation of subtraction. In addition, their descriptions of other items on the list suggested a narrow interpretation of those items. For example, number characteristics in the broad sense could include substantial ideas about the ways numbers behave. However, the teachers that identified number characteristics as important mainly described it as "identifying odd and even numbers."

At the end of the year, the majority of teachers identified many of the "big ideas" about whole numbers, focusing on number relationships. They described number relationships as involving landmark numbers, such as 10, 25, 50, etc., knowing how to use these landmarks to move around the number system, the importance of ten and multiples of ten, and knowing what happens when you operate on numbers. All but two teachers recognized the importance of their students' ability to solve problems in a variety of ways and/or explain their solution strategies. It was obvious that a major shift had occurred in their thinking about number toward an emphasis on understanding and away from skill development alone. Many of the teachers recognized this dramatic shift in their thinking when asked to look at their writing from the beginning of the year and compare it to what they wrote at the end of the year. They commented:

> "I see that I seemed more concerned with computation than understanding. I did not look toward the underlying need to see relationships and patterns that is necessary to total understanding when working math problems." (Grade 2 Teacher)

> "I now expect more from my students. Before I expected more rote learning but now I expect the children to think about what they are doing and then to explain what they are doing." (Grade 1)

Such striking changes in reported goals raises two important questions: how did these shifts occur, and what changes occurred in actual classroom practice. The conversations that took

place during the professional development sessions and those with the teachers who allowed us to observe their teaching provide some insight into these questions.

These teachers, as evidenced by their goals at the beginning of the year, were concerned with basic skills: fact memorization and proficiency with standard algorithms. During the year their struggle to move to a more balanced perspective was evident. An eruption about computation occurred during a mid-year professional development session. One of the second grade teachers, Kathy, mumbled "I don't think we're supposed to teach anymore. Do we ever introduce the standard algorithm? Do we *teach?*" Kathy's comments seemed to express the underlying concerns of all of the teachers (as was evident in both the conversation and the written reflections from that session). As the conversation continued, we worked to redirect their attention on what their students were learning about number by looking at student work submitted by one of the teachers. Many teachers recognized the understanding that these students were displaying but were skeptical about how all of their students would look at the end of the year. At that point, they had only completed the first number unit, which focused primarily on the meaning of the operations rather than computational skill. With this in mind, we reminded teachers that computational skill would be addressed more heavily in the second number unit and that we would continue the conversation then.

By the time we met for the end of the year professional development session, most teachers had completed the second number unit and brought examples of their students' "reasoning" procedures for computation (more commonly referred to as "invented" procedures). There was a marked difference in the kind of work they brought to this meeting as compared to previous meetings. The samples at the end of the year displayed a wide variety of approaches to multi-digit computation, many of which were quite complex. The teachers seemed more comfortable dealing with these reasoning procedures and were beginning to recognize the value of encouraging their students to develop and discuss them (as evidenced by the shift in goals discussed above). While they recognized the value of this approach to computation, many remained skeptical about the feasibility of using these reasoning procedures with larger numbers in later grades.

Change at the Individual Level

The three teachers who volunteered to have their teaching videotaped provide three different pictures of the process of change in practice. Two major issues emerged as we analyzed what these teachers believed to be important in teaching mathematics and how these beliefs were enacted in their practice. These issues were their ability to elicit and engage with students' ideas and the extent to which and the way in which they modified the curriculum.

The Case of Mary

Mary is an experienced teacher who has been teaching at the early elementary level for 21 years. She has a background in special education but enjoys teaching first grade the most. As she explains, "Oh I love the kids and it's just so fun to watch them grow over the year, seeing them start to read, things that they start to do that they couldn't do when they came in." She describes herself as "not real strong" in mathematics and much "stronger in the language areas." Before the adoption of *Investigations*, she had been using *Mathematics Their Way*, a K-2 curriculum published in 1975, as her main source for teaching mathematics. She liked the program for several reasons:

> When we first got it, I loved it, because what we had done was dittos and books like this where you rip out the pages and you didn't spend a lot of time on a concept, you know, so you expected them to get it and move on. And they couldn't. I mean, they weren't ready. And so when we went to *Math Their Way*, it was just, like, wonderful, because now they had manipulatives and they were able to do a lot more and spend more time on one concept.

However, she also expressed a feeling that it was time to change. At first, she did not point to any inadequacies in *Math Their Way*, but rather talked about changing simply for the sake of change. "I know when I've been in one place for a long time, I've always found that I like to do change, change my curriculum, change the way I do things." When probed further, she did explain that she did not feel that *Math Their Way* supported reasoning sufficiently. She explained:

> I think the reasoning part was missing…The kids would come up with answers, and even though they were saying things to me, I guess I really didn't—maybe I wasn't hearing what they were saying or…we really didn't talk about the way they were thinking. You know, it was like, yep, you came up with 2 and 5, and you came up with 4 and 3, yep, that's 7.

When asked to describe her impressions of the new curriculum, *Investigations*, Mary responded favorably for the most part. She liked the conceptual development and the focus on an understanding of numbers, but seemed particularly impressed with the support for language development as children learn to express their mathematical ideas. She recognized the value in having children investigate and explore ideas without always being told how to do things. "That's what I think I like about this program. It makes kids think. Or it makes the teachers think!" Mary did express some difficulty maneuvering through the teacher's guides. She explains:

> I think it's a great program from what I see so far. There's a lot of reading....And the thing I think I find frustrating about the manual right now, it's like I start reading and it goes, "Go to page 53." Then you go there and it goes, "Go to page 49." Then you read a little bit further and I feel like you're jumping back and forth, back and forth. But yet it's much more organized than *Math Their Way* was, so I do like that.

Although every lesson appears on consecutive pages in the teacher's guide, there are additional pages designed to support teachers as they learn new mathematics and ways to encourage student understanding. These additional pages, called Teacher Notes and Dialogue Boxes, are often found interspersed throughout the teacher's guide, but are integral to teaching *Investigations* effectively.

Mary's beliefs about mathematics teaching and learning seemed to be aligned with the intent of *Investigations*, but some of her comments suggest that issues other than the focus on thinking may have been more salient to her. Her teaching during the first year certainly embodied the difficulty of teaching mathematics for understanding. Several reoccurring issues emerged throughout the year in Mary's teaching. She altered the intent of the curriculum, because she had difficulty focusing on the mathematical intent of lessons. She also had difficulty shifting her focus to understanding, because she continued to rely predominantly on questions that asked for solutions without checking for understanding.

Modifying the Curriculum. A problem type that is revisited throughout the year in first grade is the "How Many of Each?" problem. Students are presented with a total number and asked to find combinations of two (or three) items that make that total. They are always presented in the context of a word problem or story problem. The mathematical intent of lessons involving these problems is to help students learn that there is more than one solution to some problems and to

search for patterns in their list of combinations and use one found combination to figure out another. In one lesson, students were working on the following problem:

> *I have seven things on my plate. Some of them are peas, and some of them are carrots. What could I have? How many peas? How many carrots? Remember, I have seven things in all.*

As Mary mingled among students who were working in pairs, she mainly helped them with their representations on paper, getting them to label which number represented the peas and which number represented the carrots. One may argue that this is certainly part of the "mathematics" that the students are working on—to represent their mathematical ideas on paper so that others are able to understand them. However, what she neglects to do is ask any questions about the combinations themselves. She does not ask students how they know any particular combination makes seven, if they see any relationships between combinations, or if they can use one they have found to figure out another.

This is counter to what is suggested in the teacher's guide: "It's important to regularly ask students to explain their thinking behind an answer, whether it is correct or incorrect, and to encourage them to find ways to double-check their solutions on their own" (Kliman, Russell, Wright & Mokros, 1998, p. 47). In a separate Teacher Note on "Exploring Multiple Solutions" (Kliman et al., 1998, p. 49), it suggests that having students explore such questions as--"Have I found all the solutions?; Is there another way of thinking about the problems that would yield more solutions?; and Are there any relationships among the solutions I found?--is appropriate for this investigation.

On another occasion, later in the year, students were working on the following "How Many of Each?" problem:

> *Suppose I have 12 pets. Some of them are cats and some of them are dogs. How many of each could I have? How many cats? How many dogs? Remember, I have 12 things in all.*

When Mary introduced the lesson, she referred to 12 as the "magic" number and that they were looking for ways to come up with combinations for 12. There was little reference to the context of the problem in the introduction or in her discussions with students while they were working. Not

surprisingly, some students found combinations of three numbers to make 12. When they were offered as solutions at the end of the lesson, Mary accepted them whole-heartedly and did not question the students as to whether this solution fit the problem. There was even a subtraction equation (13 – 1) presented, which Mary added to the running list of solutions without question.

When asked afterwards if these equations made sense for this lesson, she commented, "They were looking for combinations, is what I said to them to do—find numbers that would equal 12." She had lost sight of the fact that this mathematical activity was embedded in a context that students should have had to think about. The section on facilitating a concluding discussion in the teacher's guide says explicitly to "encourage students to think whether the [solution] is appropriate to the problem at hand" (Kliman & Russell, 1998, p. 41). This suggests that Mary is either not attending carefully to the teacher's guide or is having difficulty focusing on key ideas.

Eliciting and Responding to Student Explanations. Almost all of Mary's questions revolve around asking for answers rather than asking for explanations of reasoning. In the "12 Cats and Dogs" lesson, there was only one time in the entire whole group discussion when she asked for an explanation of how a solution was arrived at. This happened to be on the occasion when a combination using three addends was offered as a solution, and the student was one that she considered "very bright."

T: Now, Hayley and Megan had some other interesting answers that I would like them to share with us.
S: 5 + 5 + 2.
T: They had 5 + 5 + 2 = 12. What's another one Hayley?
S: 6 + 4 + 2 = 12.
T: Now would you tell the boys and girls how you came up with that Hayley? Come on up here and tell us.
S: Um, we um, I thought it would work and told Megan about it.
T: *Okay, so how did you come up with those two numbers*? [Points to the two 5's.]
S: I knew that 5 and 5 is 10, and 2 more is 12.
T: Good. She knew that 5 and 5 would make 10, and 10 + 2 would make 12. *Now how did you come up with those two numbers*? [Points to the 6 and 4.]
S: I put 1 more to the 5 to make 6 and I took away 1 from the other 5.
T: Good. So you added 1 here to make 6, and she took 1 away to make 4. And she knew that 6 and 4 made how many?
S: 10.
T: And 2 more made?
S: 12
T: Good.

Even after having Hayley share her reasoning, she did not pursue it or encourage discussion about the logic of the explanation with the rest of the class. Her reaction to Haley's quite sophisticated strategy of changing two addends in opposite directions to keep the equation in tact (changing 5 + 5 to 6 + 4) was simply to restate it.

Mary's questioning techniques showed some change as time went on. During our third observation, students worked on a game where they rolled two dice and covered the sum with a marker on a game board. While she was describing how to play the game, Mary did several sample rolls with the class and asked them what the sum would be. After each response, she asked either "How did you know?" or "How did you figure that out?" However, she did not probe further if their explanation was correct. In her exchanges with students while they worked, she still predominantly accepted correct answers without questioning or told students with incorrect solutions to do the problem again.

Mary did not recognize the opportunity provided by incorrect solutions to allow the class to wrestle with some key mathematical ideas. And because she struggled with asking questions about reasoning, she was unable to allow students to explain incorrect solutions and perhaps catch their mistakes along the way. When incorrect solutions were shared, she typically told students to check their solution, with manipulatives in most cases. For example, in the beginning of the year, a pair of students offered 5 and 3 as a solution to the "Seven Peas and Carrots" problem.

T: Okay. Caleb, you had one you wanna share with us.
S: Yeah. 3 and 5. 5 carrots and 3 peas.
T: You had 5 carrots and 3 peas? Okay. Do you know what Caleb and Vince? I want you guys to go get 5 carrots and 3 peas and bring them up here for a minute with us.
T: Okay. Would you boys count that for us please?
S: 1, 2, 3, 4, 5, 6, 7, 8.
T: Now what was our magic number today?
S: 7.
T: 7. So, what could we do to make 7?
S: Take one off.
T: Take one off. Now, come here Caleb. Bring that up here. Now Caleb and Vince, how many carrots do you have?
S: 1, 2, 3, 4, 5.
T: How many carrots?
S: 1, 2, 3, 4, 5.
T: 5. Okay. So you have 5 carrots, and how many peas?
S: 2.

T: And 2. Look at those boys. They came up with a different group of numbers. 5 carrots and 2 peas.

Notice the student's suggestion to take one off and the fact that Mary did not pursue this with the rest of the students. She missed the opportunity to have them think about what they might do or if it mattered if you take away a carrot or a pea.

Later in the year, Mary's reaction to an incorrect response of 3 + 6 as a solution to the "12 Cats and Dogs" problem was much the same.

T: Give me another one Kayley.
S: Um, 3 + 6.
T: 3 + 6? Okay. You know what I'm gonna ask you to do? I'm gonna ask you to go get the cubes over there and get 3 in a group and get 6 in a group, and then I want you to come back and tell us - and tell us what it is. Go work it out honey. I want you to work it out right now.
[Mary takes a few additional solutions from students that she writes on the board.]
T: Okay. Kayley - did you figure that out? What honey?
S: [inaudible]
T: Okay, so if you had 3 and 6, what did that make?
S: 9.
T: 9. How many more would you need to make 12? Now go figure that one out. Okay.
[Mary takes one more solution from a student that she writes on the board.]
T: Okay, Kayley, how many more?
S: 3.
T: 3. So we can actually have 3 + 3 + 6 = 12. [She writes this on the board.] Okay? Good job.

Reflections on the Year. Unfortunately, we were unable to have an end of the year interview with Mary. At the final professional development session, Mary informed us that she did not want to continue her participation in the study. In fact, as early as our second observation we began to sense Mary's discomfort with our presence in her classroom. In a phone discussion shortly after this observation, Mary expressed her feelings of discomfort with our interviews. She felt that we were "throwing questions back at her" as we attempted to get her thoughts on her teaching. We discussed the purpose of the study again, reminding her that we did not have to videotape her if she did not want us to, and offered to do a demonstration lesson for her. Although Mary did let us observe on one more occasion, she did not take us up on the offer to do a demonstration lesson, nor did she return phone calls for several months after this final observation. We did not speak with Mary again until the final professional development session in May. At that time she apologized for not returning our calls and cited problems in her personal life that were

removing her from the classroom more than usual, but one wonders if this was also indicative of the extreme struggle she was undergoing as she attempted to teach for understanding a subject she did not understand well herself.

The Case of Ed

After completing high school, Ed joined the service and began taking college courses near his base. When he left the service, Ed finished his degree and got a job teaching in the same building that he teaches today. Ed has been teaching for 29 years in that building: he taught sixth grade math and science for nine years; third grade for five years; and finally switched to second grade 15 years ago. When asked about moving to lower grades, Ed informed us that he did his student teaching in second grade and wanted to teach second grade, but, "twenty nine years ago nobody would hire a man in second grade."

The aspect of Ed's background that seems most indicative of his reaction to teaching with *Investigations* is his matter-of-fact response to being asked how his teaching of mathematics had changed over his career of 29 years: "It hasn't changed a whole lot." Ed was equally clear about his goals for second graders' understanding of number: basic facts; carry and borrow procedures; knowing numbers to hundreds; using symbols appropriately; and knowing how to make change. Given Ed's emphasis on facts and procedures, we were not surprised to hear that he had "some reservations about the program," though he did not elaborate on what exactly those reservations were:

> I was a little concerned because they [the math committee] didn't seem too positive about it [the curriculum]. There were concerns about not having enough mechanical things, dittos, and that kind of thing. That didn't seem to bother me because I like math. And there are all kinds of ways to teach math, so you know, I had some reservations about the program, yet. But I haven't seen the other four books yet, either. So I don't know what the full program is yet.

Eliciting and Responding to Student Explanations. During our first observation it was evident that Ed's questioning was dominated by a few different types of "low-level" questions: rhetorical questions; "fill-in-the-blank" questions, predominantly with *row* or *column* being the appropriate response that day; yes/no questions; and questions requiring students to count the

number of blocks being used. Also evident were the ways that he consistently dealt with correct and incorrect responses to his questions. Ed had a few standard ways of dealing with incorrect answers; he would either repeat the question, tell the student the correct answer, or ask the student to perform a specific task (during our first observation, it was counting) to demonstrate that they were wrong. On the other hand, correct answers were often silently verified by Ed moving on to the next question. Finally, there was no evidence that he attempted to elicit student explanations for their answers, whether correct or incorrect. In these and other ways, it was clear that Ed's conceptions of teaching and of the goals for mathematics learning were at odds with the current reform movement in general, and the *Investigations* program in particular.

As we observed Ed during the year, we began to see some changes in the ways he orchestrated the sharing of student solutions. This can best be illustrated by considering his teaching of one of the second grade routines, "Today's Number." In this routine, which is to be done almost daily, students are asked to come up with equations that equal the number of days they have been in school thus far. We observed Ed doing this routine on three separate occasions, in November, February and May. During the first and second observations of this routine, days 59 and 98, Ed kept the whole class sharing of equations under tight control. A typical exchange on day 59 looked like this:

T:	Which group has not shared one yet?	S:	20 plus 10.
T:	20+10. [T records on the blackboard]	S:	Plus 10.
	... [T continues until this equation is on the board: 20+10+10+9+2+8]		
T:	Is that it?	S:	Um-hmm.
T:	Let's see how they did. 20 plus 10?	S:	30.
T:	Plus 10?	S:	40.
T:	Plus 10? [T circles the 2 and 8, skipping over the 9]	S:	50.
T:	Is 50. Plus ?	S:	9.
T:	Fantastic. Fantastic.		

During the second observation, day 98, Ed allowed the students to come to the board and write down their equations. However, he continued to control the process by which the number strings were calculated, rather than allowing the child to explain how s/he did this:

T: Who thinks they have a subtraction one for us that hasn't been up there. Okay.
S1: [goes to board and writes 112 - 14 = 98]
T: Okay. [to class] We have 112. That's what she's starting out with. If she subtracts 10, people, how much will she have? ...
[T writes 112 - 10 on the board in vertical format.]
Class: [mumbling only]
T: If you subtract 10 from 112, how much does she have?
S2: one hundred and [inaudible]
T: one hundred and ... two. [T records 102, and then writes - 4 underneath (in standard vertical format).]
And now, if we subtract 4 more, she would be at ...
Class: [some clearly say 98; some are saying other things, like one hundred and (inaudible)]
T: She would be at ...?
Class: 98.
T: 98. So, she did a nice job, didn't she? There you go. [Student returns to seat.]

Not only does Ed fail to elicit this student's thinking as to how she did the subtraction, he also rewrites the equation from the horizontal format the student used, to the traditional vertical format. When asked about this particular student solution during the interview, Ed explained that some of his brighter kids were being taught regrouping at home and that he was assuming that this student used regrouping to do the subtraction.

In contrast, during our last observation Ed relinquished more control by allowing the students to come to the board and prove that their equations were correct. This was quite a dramatic change from what we had witnessed in the past in the sense that he stood off to the side of the room and attempted to allow the students to take center stage. On this day, Ed combined the sharing of Today's Number (170) with sharing solutions for story problems and added a competitive component to the routine by creating teams and offering points for correct solutions. One of the problems the students were working on was:

The pet store had a large tank filled with 100 goldfish. Kyra bought 23 and Jake bought 19. How many goldfish were left at the pet store?

The whole class sharing of all solutions that day were handled similarly. Below is an example of a typical interchange around the problem stated above.

S: [Student writes on the board: 100 - 23 - 19 = □]
T: That's your number sentence?
S: Um-hmm.
T: So you've got 100 - 23 -19. Okay, can you explain how you're gonna get your answer now?
S: ?

T: Excuse me? Can you try it one more time? You talk so softly, I didn't quite hear you.
S: I'm starting with 100.
T: Yep.
S: (student reads off of paper) I subtracted three 10's, then twelve 1's, ... it equals 49.
T: But, you just kinda said some numbers that you subtracted. That doesn't really explain it to us. Can you use the number chart, or can you write some numbers that will show us what you're talking about. We'd like to kinda see it.
S: [Student goes over to the 100 chart]
T: Okay, you started at 100. Now you're going back 3 10's. Do that please. Now you're gonna go back---
S: Twelve 1's.
T: Twelve 1's. Would you show us that?
S: [Student counts back by ones starting at 70, pauses at 60, and then continues to 58.]
T: And where did you end up at?
S: 58.
T: So what do you think the answer is?
S: 58.
T: Ah, the answer is 58. The answer is 58. All right. See? You had it wrong, but ... how did you come up with the right answer? ... What should you have done the next time before you try it again? What would you do the next time?
S: Um. . .
T: What did you just do for us?
S: Proved it?
T: Proved it, didn't you? See, proving answers, double checking. That's what we're always talking about, prove answers, double check. She proved it. And by proving it and double checking it, you found out the . . . ?
S: Right answer.
T: Right answer. Give yourself a point. Give yourself a point. Good job. Good job.

This interchange was typical of the interchanges that day in the following ways. First, all students were required to "prove" their solutions. Second, when a student was incorrect, Ed responded primarily in one of the following ways: (1) ask the student, "Do you want to double check that?"; (2) ask the class, "How many agree with ___?"; or, as in the above passage, (3) ask the student to perform a specific task to illustrate his/her error. As in the beginning of the year, students offering correct answers and explanations did not have such questions posed to them.

Modifying the Curriculum. Throughout the year there were indications that Ed was altering the curriculum in seemingly small, but substantial ways. In general, the ways Ed dealt with whole class sharing of solutions and strategies, as discussed above, often served to circumvent the kinds of rich discussions intended in the curriculum. The intent of the curriculum in this area is made clear in the teacher's edition through sections called Dialogue Boxes, which offer sample dialogues between students and students, and students and teachers, based on classroom testing of the curriculum. One striking difference between the dialogues in the text and the above excerpts from

Ed's class is the dominance of the student in the text, and the dominance of the teacher in Ed's class. For example, in the Dialogue Box "Counting On and Counting Back" (Economopolous & Russell, 1998, pp.110-111) two students offer different solutions to a story problem for 40 – 26. Although both students used counting backwards, they did it in different ways and arrived at different answers. The teacher probes to understand the strategies of both children, and then allows other students to question and comment on the strategies. As it becomes clear that several students are confused about the issue, the teacher poses a question to the entire class to force them to think about this issue. Thus the teacher's role is one of helping the students come to consensus on a particular mathematical issue, rather than being the sole mathematical authority in the classroom.

Related to this was the way Ed utilized his time with small groups of students. During every class we observed there was a period of time when students were working in small groups on some mathematics activity. In each of these instances, Ed's interchanges with the students in those groups were dominated by either making sure directions were being followed or by responding to student work with praise for correct answers, identification of incorrect answers, or "hints" that a better solution existed other than the one they had found (e.g., "Is that the best you can do?"). In contrast, the teacher's guide frequently suggests that these times are opportunities to better understand student thinking, and often provide suggestions for what mathematical ideas to focus on when talking with individuals or small groups.

Finally, there was an instance in which Ed altered the directions for an activity, and the order in which a series of activities took place. When we discussed these alterations in a phone interview later that day, Ed indicated that this was a deliberate move on his part. Furthermore, when it was proposed that one of the difficulties that he was having might be solved by reverting to the order suggested in the text, Ed's response was half-hearted, saying, "Well, that might work."

Reflections on the Year. During our final interview with Ed, we asked that he reflect on his teaching during this past year: "Well, I've had to adjust my style a bit. ... [I need to] back off more. Get it set up and then back off more. Let the kids kind of investigate on their own a little

more. That's probably the hardest thing to do, but that's what needs to be done, I guess [emphasis ours]." This type of comment was somewhat typical for the interview. Ed would make a statement indicating that his conceptions of teaching and his goals for mathematics were moving toward the philosophy of *Investigations*, and then he would follow up with something noncommittal (e.g., the "I guess" above) or potentially conflicting (as in the next statement). Ed's reflections on what was, and was not, accomplished that year were peppered with comments about valuing the students thinking and their understanding of number on the one hand, and his discomfort with letting go of some of his traditional goals like knowing the borrowing and carrying procedures and knowing numbers to 1000. When asked directly why he thought his kids were "thinking better" about number, Ed attributed this difference to the students, not his teaching nor the activities in the program. Finally, he expressed an unhappiness with being tied down to a specific curriculum: "I just can't follow a book like a recipe. It's just not me. I'm just more like, do my own type of thing. ... Because there are lots of ways to teach math. Even the experts would agree with that."

The Case of Pamela

Pamela has been teaching for about 15 years at various levels, kindergarten through eighth grade, in three different locations in the country. She spent several of those years teaching middle school language arts and home economics. Although she considers herself a language arts person, Pamela also likes mathematics and feels confident teaching mathematics. She is in her third year of teaching second grade in her current position and describes second grade as being the lowest grade she would like to teach. When she was asked to describe the differences among her teaching positions and how she has changed along the way, she explains:

> Well actually, I think it's more of a challenge to teach, because children are so diverse or they -- they've always been diverse, but it seems that they come from such different backgrounds and I really feel that I have to use every trick that's in my bag to keep their attention and work with them. But yet, they're also much greater thinkers. So it's really a challenge to me to see how I can get them to think. And they have so much stimuli from, uh, outside the classroom that they bring, so in a lot of ways it's more interesting.

Pamela described some dissatisfaction with the traditional textbook they were using before the adoption of *Investigations*. She had to supplement with activities that she felt were more engaging to students, to motivate them to work with numbers more frequently. She expressed great enthusiasm when she heard the district's decision to adopt a new program that focused on mathematical reasoning and helping children understand the mathematics they were learning. In response to reading *Beyond Arithmetic*, a companion book to the *Investigations* curriculum which describes the philosophical underpinnings of the program, Pamela explained:

> This agrees with my philosophy of teaching math. Uh, one thing I think I had been doing too much of, I think my eyes have really been opened, is that we really do need to do a lot more investigation with the models and things before we memorize facts. And even though children have had these models in kindergarten and first, they still weren't really putting them together in the ways that we are here. And so I'm anxious to see now once we have done this all first semester, I'll be anxious to see if they really know those math facts better than the other way that I taught it which was more by -- not that I didn't use the models, I did--but I also used a lot of rote memory.

Pamela believed in encouraging children to think about mathematics as a way to help them understand it better and she worked hard to enact these beliefs in her teaching. Of the three teachers, she was the one who clearly established an environment in her classroom right from the beginning of the year that valued thinking and explanations of answers. However, she still struggled with honestly listening to students' ideas and handling incorrect answers.

Eliciting and Responding to Student Explanations. From the beginning of the year, Pamela was very good at focusing on the ways students were thinking rather than simply asking for solutions. She recognized the value in focusing on thinking and knew that this was the intent of the curriculum. She commented that "I really think the curriculum makes it very clear about that—that what we want here is thinking." However, she sometimes neglected appropriately probing students' ideas when answers were correct or when explanations were not totally clear. For example, in the beginning of the year, students were providing estimates for how far away they thought 124 was from 90. One student responded, "About 14 numbers away, 'cause, um, well if it's 90, then there's 9 and I putted, um, 4 more." Another student gave 35 as her estimate and when asked how she got that, responded, "'Cause I thought about 24--so I kept that number in

my head, and then I counted on." Pamela questioned students about their thinking, but had difficulty carrying through to make sure the strategies and explanations made sense.

By the end of the year, she continued to struggle with probing ideas to make sure they made sense. In one example, a student offered 80 + 10 + 9 = 99 as a way to make 99. After she wrote her number sentence on the board, Pamela asked her "How did you know to put 80?" She waited a full 15 seconds before she rephrased the question to "What do you know about 80?", and waited again before the student explained that 80 has eight tens. On the one hand, Pamela can be congratulated again for probing the students' thinking and for her patience and long wait time in order to get that explanation. However, this was another example of incomplete probing. Some of the students may have been wondering what the fact that 80 having 8 tens had to do with thinking about a way to make 99. Perhaps a more complete discussion should have included the idea that 99 has 9 tens and that 8 tens might be a good place to start, because it is close.

Pamela also struggled with appropriate ways of responding to incorrect answers. In early observations, she tended to react to them by suggesting a strategy and telling the student how to use the strategy to find the correct solution. In the exchange below, students were working with multilink cubes on "Today's Number" to find as many ways as they could to make 22. They were instructed to write their findings on a recording sheet. Pamela suggested that there were a multitude of ways to make 22 but asked students to also think in particular about how they could make 22 with only two parts. She came upon a pair of students who had 15 + 4 = 22 among many other correct equations written on their paper and the following exchange occurred.

S1: Is this right? [points to 15 + 4 = 22] Are these right?
T: *Well, I want you to check them and make sure they're right.* Okay. Use your cubes and count them and see. Take 15 and add 4 to it. How else could you figure out if that was right instead of taking all the cubes? Well, if you had 15 and you wanted to figure out how much 4 more would be, what could you do? What's that strategy we talked about?
S1: Count.....
T: Yeah, counting.....[motions upwards with her hands].
S1: Numbers?
T: Counting.....
S1: Umm.....
T: Counting up.
S1: Up.
T: So, if I already have 15, I only have to count 4, don't I? So what would it be? 15, 16, 17, 18, 19. So, is that correct?

T: Oh, now tell us what did you do?
S: I put 40 + 20 + 20 + 19 = 99, because 20 plus 20 equals 40 and 40 plus 40 equals 80 and 80 plus 10 equals 90 and then I added the 9, equals 99.
T: Thumbs up if you think that is a good idea. Great Jordan. Anyone have a question or anything about this one?

Modifying the Curriculum. Unlike the other two teachers, Pamela did not modify the curriculum in apparent ways. She understood the mathematical intent of the lessons she was teaching, recognized the value of and attempted to elicit student explanations of thinking, and followed the teacher's guide fairly closely. In professional development sessions, when her colleagues complained about the amount of time and effort to read the teacher's guide and prepare materials for lessons, Pamela often disagreed. She explained that she felt the teacher's guide was easier to use, because of all of the supporting materials provided for teachers, such as examples of student thinking and possible student misconceptions.

Reflections on the Year. When asked to reflect on how she changed throughout the year, Pamela explained that she is "more open to having the children make choices in how they want to solve the problems" and "more patient about give them time to work through it." She explained the importance of letting students work through their answers to see if they really had the right answer. She also explained how important it was to help her students be more willing to "take risks" and offer ideas for the rest of the class to think about. She lamented about how she "was never allowed to take a risk in math as a child" and that "you had to be correct or you certainly didn't answer."

A videotape of segments from three of Pamela's lessons throughout the year was compiled for use in the final professional development session. We watched the tape together in this final interview with Pamela to allow her to reflect on her changing practice before the videotape was shown to her colleagues. When she was shown the segment from the beginning of the year described previously where a student had 15 + 4 = 22 recorded on his sheet and she told him to count-up to check his answer, Pamela exclaimed, "I'm doing it for him. He should be doing it! I should have had him figure out more!" In addition, when asked what she thought was the greatest factor enabling her to change, she replied, "Oh, probably seeing how excited the children get about

S1: Yeah.
S2: I'm way ahead (of him).
T: (to S1) It is?
T: (to S2) Oh, we need to stay together. Is that right? Would that be right?
S2: [Nods her head.]
T: 15 + 4? Oh, if I already have 15 and I add 4? 15, 16, 17, 18, 19? Oh, how many more do I need to get 22?
[S1, looking confused, changes his paper to read 15 + 19 = 22.]

Notice that Pamela's initial reaction to the student's question about whether or not his equations were correct was to tell him that she wanted *him* to figure that out. She seemed to recognize the value in getting students to check their own answers and to figure out for themselves whether they were or were not correct. However, without hesitation she proceeded to walk him through a counting-up procedure to check the only incorrect equation on his paper. And based upon how he changed the equation to 15 + 19 =22, it was clear that he did not understand the exchange with Pamela. When she was asked to reflect on this exchange after the lesson (before looking at the recording sheets), it was interesting that she was sure he did understand why 15 + 4 = 22 was incorrect and knew how to change the equation appropriately.

There was a striking transformation by the end of the year in how Pamela reacted to incorrect responses. She responded the same way to incorrect answers as she did to correct ones; that is, to ask for explanations of reasoning. For example, in one exchange, students were working on "Today's Number," which was ways to make 99. Three students came to the board and wrote their equations (all correct), explained their thinking, and answered any questions from other students. A fourth student then came to the board and wrote an incorrect solution. Pamela responded in the same way as she had responded to the previous correct solutions as evidenced in the exchange below.

S: [Writes 40 + 20 + 20 + 9 = 99 on the board.]
T: All right, Jordan, tell us what you did.
S: Umm, I put 40 plus 20 plus 20 plus 9.
T: Okay, you want to break that down for us a little bit & tell us how you knew to put those numbers together.
S: Because I know that 20 plus 20 equals 40 and 40 plus 40 equals 80, and --[long pause]-- I got messed up.
T: Oh, well go ahead and fix it. Do whatever you need to do. That's all right. We are willing to take a risk, aren't we boys and girls? Sometimes we have to talk through it to make sure that it is the way we want it.
S: [Student writes on board.] 40 + 20 + 20 + 19 = 99

putting the larger numbers together, and putting the tens together. They have an understanding of mathematics far greater than my children did last year." Therefore, it was the positive change in her students that prodded her on to think about her teaching and what elements of her practice encouraged her students to think.

Discussion

It is a daunting undertaking for teachers to learn about a new curriculum whose approach may or may not align with their own philosophy, think about the changing nature of mathematics in society and resulting changes in emphasis and value in the elementary grades, and adjust their teaching in sometimes substantial ways to effectively implement the new curriculum. The most encouraging change occurred in the teachers' goals for number. Before the curriculum adoption, it was not surprising that most of the teachers viewed number as isolated bits of information that needed to be taught directly to students. Their shift in goals to that of understanding number and how numbers are related was promising and supports the notion that curriculum materials can support teacher learning. The activities and investigations in the number units emphasize relationships, and the support for teachers in such supplementary pages as "Encouraging Thinking and Reasoning," "Finding Relationships Among Solutions," and "The Relationship Between Addition and Subtraction," among others, clearly emphasize relationships and understanding operations.

Also persuasive was the way students responded to the activities. The teachers commented positively on their students' engagement in the investigations and expressed surprise at the complexities of their students' thinking. The routines contained in the curriculum, such as "Today's Number," also provided the teachers with more concrete evidence as to how their students were thinking and how that thinking developed over the year. They commented on how their students were much more advanced in the ways they looked at number, took them apart, and put them back together. Although most teachers were positively persuaded by their students' level of engagement and understanding, many teachers remained concerned, to varying degrees, about *all* students reaching acceptable competence in number and computation.

Some of the hesitancy and resistance seemed to be caused by teachers' weak understanding of the content and their uncertainty about whether or not to abandon having all students adopt the same computational algorithms. Many were frustrated with the reasoning procedures that students were developing to add and subtract, because they could not understand them. On several occasions, teachers shared examples of procedures that some of their "bright" students had offered. And even though they did not understand them, they accepted them saying, "These kids understand numbers better than I do!" We spent time in professional development sessions discussing these procedures and thinking about whether they were reasonable or erroneous. In some cases, they were erroneous, and if the teachers had asked more questions about the student's thinking, they would have figured that out. But again, because that practice was slow to develop, their ability to discriminate among procedures was slow to develop as well. This difficulty discriminating among reasoning procedures may have made them more hesitant to give up the traditional approach of teaching one specific algorithm.

Perhaps it is not surprising that the ability to elicit and engage with students' ideas and explanations of thinking was the slowest teaching practice to develop. It may be an indication of how little this is emphasized in education in general, let alone in mathematics, where a weak understanding of content can exacerbate the problem. One may wonder if we saw so little of this going on because of teachers' concerns about orchestrating discussions so that all students are engaged and so that appropriate and correct mathematical ideas (and not erroneous ones that might confuse other students) are being discussed. Although these issues were ones that the teachers struggled with, to varying degrees, it seemed that the overriding issue was their entrenchment in their former mode of instruction and subsequent difficulty in taking those first steps to elicit ideas and explanations rather than solutions alone.

Dealing with incorrect answers and viewing them as opportunities to learn proved to be a particular challenge. The inclination to respond to incorrect answers by telling students how to solve the problem correctly or by directing them through a particular strategy is perhaps "natural." But teachers can be pushed beyond "natural" tendencies when they have motivation and reason to

do so and when they enact that reason into behavior. One of the barriers to dealing with both correct and incorrect answers productively seemed to be the teachers' assumption that they knew what their students were thinking even though they had no specific evidence. In some of our interviews with Ed, for example, he would explain that he "assumed" that the student had done this or that, and therefore, did not see a need to ask.

A second reaction to incorrect answers—that of directing students to manipulatives to check their answer—is problematic as well. Perhaps this is a result of the emphasis placed on the use of manipulatives to help students make sense of abstract ideas, which has developed into an overreliance. The problem of treating manipulatives as a panacea has been addressed by many mathematics educators (e.g., Ball, 1992; Burrill, 1997). In any case, always directing students to manipulatives to check their incorrect answers before asking them how they arrived at their answers disavows their reasoning ability. With young children, it can be even more detrimental by encouraging them to count by ones to figure out numerical solutions rather than using reasoning and what they know about numbers to do so.

Despite the difficulties discussed above, the three teachers we observed all became better at engaging with students' ideas. Some researchers have characterized various aspects of teachers' conceptions of mathematics teaching through the use of a series of continua (e.g., Grant, Hiebert & Wearne, 1998; Knapp and Peterson, 1995). In our case, one can imagine a continuum illustrating the major focus of instruction. On one end, the focus would be on the correct answer and one correct strategy for arriving at that answer, and at the other end, the focus would be on students' thinking. Both Ed and Mary began the year on the "one answer, one strategy end," while Pamela seemed to be positioned further along this continuum. Although all three clearly moved in the same direction, it would be difficult, and seemingly unnecessary, to quantify the different distances that they moved.

It is important to note that the teachers "starting" position on this continuum was not the only feature that distinguished the teachers in this study. One can imagine a series of parallel continuums representing the host of factors influencing change. In the case of the three teachers in

this study, there were indications of additional differences that may have allowed Pamela to engage more effectively with students' ideas. For example, Pamela seemed more equipped than her peers for change in several ways: (1) she was more comfortable with mathematics than Mary; (2) she had good classroom management skills that Mary lacked, while at the same time, did not resort to total teacher dominance as Ed did; and (3) although she and Mary were both interested in changing, Pamela was the only one who took the initiative to read the book *Beyond Arithmetic* to better understand the curriculum they were about to adopt.

Conclusions

The changes required of these teachers in implementing such a radically different curriculum can be overwhelming. The philosophy is different, the goals are different, the mathematics is different, the instructional techniques are different, the assessment is different, and the teacher's guide looks different. Perhaps it would be more reasonable to address these issues in a step-by-step fashion before a new curriculum is adopted. In other words, having discussions about philosophy and what it means to teach mathematics for understanding may be more appropriate before a curriculum adoption. Or analyzing student work to support teachers as they learn how to interpret students' thinking and understanding of mathematics may be more effective if it occurred before new curricula were considered. While this may make the curriculum implementation phase less overwhelming, one must question whether such an approach is realistic and whether it would enable teachers to avoid the overwhelming sensation of teaching a reform curriculum for the first time.

The changes that did occur among the teachers in this study, albeit modest, suggest that change can occur as teachers negotiate through the myriad of ideas, beliefs, and attitudes that accompany an adoption of new curricula. Although this study reports on only the first year of change, it is heartening that we saw a substantial shift in teachers' goals regarding the mainstay of the elementary curriculum--number and computation. While it is impossible to isolate the effect of the curriculum materials alone on the teachers' beliefs and practice, it was evident in the

professional development sessions that the constructivist design of the lessons continually encouraged the teachers to reflect upon effective instructional techniques.

Of course, the extent to which individual teachers changed their thinking about the goals of mathematics instruction and their ability to implement the program as intended varied greatly. As earlier studies have reported, teachers' beliefs about mathematics teaching and learning affected their ability to move toward a more reform-minded view of teaching (e.g., Grant et al., 1998; Knapp & Peterson, 1995; Lloyd & Wilson, 1998). Teachers whose beliefs were more closely aligned with the philosophy of the curriculum more readily enacted the suggestions for instruction contained in the curriculum. But those who had conflicting beliefs or misgivings about the program's approach changed more slowly. This study extends earlier work by suggesting roles that curriculum support materials and professional development opportunities play in supporting the change process. Although the curriculum contains information on how to engage with students' ideas based on actual teachers' classrooms, some teachers needed examples from their peers to convince them, before attempting to change their own practice. Having the opportunity to interact with colleagues on a regular basis was a necessary factor in enabling these teachers to change.

Table 1: Beginning & End-of-Year Comparison of Goals for Number

Beginning of Year (n=17)		End of Year (n=18)	
Adds and subtracts	76 %	Number relationships	72%
Knows basic facts	65	Problem solving strategies	72
Recognizes coins	59	Higher level thinking	39
Number characteristics	59	Place value	39
Tells time	47	Explains strategies	33
Writes numerals	35	Counts by groups	28
Story problems	35	Understands addition/ subtraction	28
Patterns	35	Story problems	28
1-to-1 correspondence	29	Patterns	17
Knows how to count	29	Tells time	17
Place value	29	Adds and subtracts	11
Recognizes numerals	29	Knows how to count	11
Understands addition/ subtraction	24	Writes numerals	11
Problem solving strategies	18	Knows basic facts	6
Counts by groups	12	Recognizes coins	6
Number relationships	6	Number characteristics	6

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