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## ABSTRACT

Noting that current views of mathematical learning and teaching focus on the child as a responsible student who attends to instruction and who constructs what is taught in a personal and meaningful way, this paper examines scaffolding and mediation strategies and describes the learning and teaching of elementary school level mathematics in Singapore. The paper first discusses mathematical knowledge and skill development in young children. The paper then describes elementary mathematics curriculum in Singapore, noting that its central focus is problem solving, and suggesting that teaching mathematics creatively is a challenge for mathematics teachers. The paper also describes the impact of educational policy changes on the school learning climate that have led to increased project work, multiple assessment modes, creative modes of learning, and focus on basic numeracy skills. Next, the paper outlines Vygotsky's views on thinking as a social process, focusing on the strategy of scaffolding and noting that Singapore teachers are highly interactive in the classroom as they try to induce mathematical abilities and skills through social experiences. The paper then describes Feuerstein's mediated learning experience approach, linking differences in learning propensities to an individual's cultural experiences. The paper maintains that many classroom learning problems are the result of insufficient or inadequate mediated learning experience. Finally, the paper describes two examples of mathematical concepts and activities that have inherent opportunities for elaboration and transfer. (Contains 11 references.) (KB)

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## Teaching and Learning Primary Mathematics in Singapore

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### Abstract

The purpose of this study is to examine the learning and teaching of mathematics in Singapore. It is hypothesised that Vygotsky's scaffolding and Feuerstein's mediation strategies are central to children's understanding of mathematical concepts. The child-teacher and teacher-child interactions are examined and critically analysed in terms of the scaffolding and mediation strategies to help the child reach his/her potential. Mathematical concepts and activities which have inherent opportunities for elaboration and transfer will be described in two examples.

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Current views of mathematical learning and teaching focus on the child as a responsible student who attends to instruction and who constructs what is taught in a personal and meaningful way. The learner is an active agent in the teaching-learning process and the role of the teacher has moved from one who imparts knowledge to one who helps students learn about learning. Contemporary mathematics educators not only want students to acquire considerable knowledge and skill, they also want students to think like mathematicians.

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Most children have a natural ability to think mathematically. By age three or four they start to use a number system and learn how to count which is an easy but important skill. They can also rank and arrange objects. According to Leeb-Lundburg (1985) logical thinking in children develops along with their mathematical discoveries eg at the age of two or three they know that one block on top of another equals two blocks. Children learn with their senses and their whole bodies and the majority of these experiences happen at home and their neighborhoods. In their elementary school, children acquire a range of strategies solving addition and subtraction problems and even apply these strategies in other contexts.

### Mathematical Knowledge and Skills

Brynes (1996) believed that children who are good at mathematics have a lot of mathematical knowledge and they can also engage in effective mathematical thinking. Mathematical knowledge can be split into conceptual and procedural knowledge. In the elementary school, some examples of conceptual knowledge include forming categories of mathematical entities (eg fractions, ratios, acute angles) and comprehending relative numerosity of groups of objects. On the other hand, procedural knowledge refers to the steps needed to attain specific mathematical outcomes (eg to count, add, subtract, divide, multiply or to find the area of a rectangle).

Mathematical ability consists of these two forms of knowledge and being good at mathematics does not bank on knowledge alone. The student must know how to use mathematical knowledge to solve problems. At different age groups Brynes (1996) noted that different mathematical skills are being mastered and it takes a long time for a child to think like a mathematician. Even after many years of mathematics at school, some students have a very shaky grasp of simple arithmetic.

In his book “The Mathematical Brain” Butterworth (1999) emphasises unravelling the nature of mathematical skill and its relationship with brain function. He argues that mathematics teaching would fail unless facts and procedures are integrated with understanding and concludes that “understanding means being able to transform the core of the problem into collections and numerosities, and the various solution strategies implied by these representations.”

Most of the studies that have examined what preschoolers know have focussed on either conservation of number or counting. According to Leeb-Lundberg, mathematics is more than counting. Real understanding of mathematics can only develop if children have plenty of opportunity to play in ways that will give them the foundation for later abstract thinking. Many children never develop this good sense of numbers because they are rushed into using the symbols and terms that signify an understanding of mathematics to many educators and parents. (Checkley, 1999). Many teachers in preschool have little knowledge of how to introduce the language of mathematical processes in the classroom. They must try to build and expand on the mathematical ideas that children are involved in through their daily activities.

## Teaching and Learning Mathematics in Singapore

Mathematics is a compulsory component of the elementary school curriculum in Singapore. Key mathematical topics and concepts are given in a series of teachers' guides and worksheets prepared by the Curriculum and Development Unit in the Ministry of Education. Teachers are advised to select the appropriate topics and concepts in their instruction. There is a high element of freedom for this choice and it is usually done by a group of mathematics teachers under the guidance of the mathematics head of department within a school.

One of the challenges facing the mathematics teacher is to teach mathematics creatively. Tan (1998) singled out at least six components in creative mathematics teaching. The first component comprises basic pedagogical skills such as lesson planning, classroom management, communication and evaluation. The second component refers to the content mathematics, creative techniques and knowledge of developmental processes. The third component is related to the competence in selecting appropriate assessment modes. The fourth component refers to teachers' and pupils' motivation. The fifth and sixth components are related to the learning climate and environment, educational policies and the school culture.

In Singapore, educational policies influence the school learning climate. Since the early nineties, the mathematics curriculum has been officially developed and refined and has been used as a guide for teachers to plan their maths program. Teachers are not bound by the choice and sequence of topics presented but are encouraged to exercise flexibility and creativity and to ensure that the linkage within the curriculum are maintained. Recommended big changes have taken place at the pre and primary school levels and these include project work, multiple modes of assessment, creative modes of learning and focus on basic numeracy skills. Critical thinking for both explicit and infusion maths lessons is emphasised and schools are encouraged to carry out school-initiated projects and activities to develop creativity.

Problem solving has been the central focus of Singapore school mathematics curriculum since 1992. The attainment of mathematical problem solving skills is dependent on five interrelated components – Concepts Skills Processes Attitudes Metacognition. The key features in the program include the development of concepts through meaningful activities, competence in basic skills, mathematical communication through oral work, group discussion and presentation, investigative work and mathematical thinking. In a study (Yeap 1999) done on a small sample of Singapore secondary school students on the role of metacognition in mathematical problem solving, it was found that these students solve problems differently in the classroom. Five types of metacognitive behaviours were identified. These are stating a plan of action: clarifying task requirements: reviewing progress: recognizing errors and detecting new development. Very few students were able to assess the difficulty of their tasks. At the primary school, metacognitive awareness is exhibited in a totally different way.

Current research indicates that traditional views on teaching mathematics based on Piagetian theory are being challenged. The concept of conservation has had a great impact on the learning of mathematics for many years in the 60s and 70s. The teaching of numbers in the early childhood classroom was heavily dominated by activities such as matching, sorting and classifying. According to Piaget, as children develop they build up their knowledge personally and independently explore their physical and social worlds. This constructive process is located within and governed by the individual child. However if this way of understanding is similar across all children, they probably have the same cognitive process to interpret their experiences.

### Vygotsky and Scaffolding

Vygotsky viewed thinking as a profoundly social process. Social experiences shape the child's interpretation of the world and language plays an important role as a primary means of communication. Anthropological studies from various cultures have pointed out that humans are inherently social and communicative beings. For example, many children become remarkably skilled conversationalists by 2 to 3 years of age. Today developmental psychologists and educators believe that the social and cognitive are essential aspects of one another. In the classrooms, teachers are trying to adjust learning experiences so as to acknowledge the productive use of the social experiences of the child. Questioning, play activities and collaborative work are embedded in intellectual tasks.

In Singapore teachers are highly interactive in the classrooms and concentrate a lot on question-asking since questions are a particularly important communication tool. Many of them try to induce key mathematical abilities and skills through social experience. It was discovered that difficult mathematical tasks could be performed effectively by having peer groups and agemates with differing opinions to exchange ideas. What was most helpful come from the teacher's instructions and questions, providing correct explanations and pointing out errors. According to Vygotsky (1978), through cooperative dialogues with more knowledgeable classmates during challenging tasks, children learn to think and behave in ways that reflect their community's culture. Vygotsky believed that as more mature partners- both adults and peers – offer guidance to children mastering culturally meaningful activities, the communication with these partners becomes part of children's thinking. (P 19, Berk & Winsler, 1995).

Vygotsky believed that it is not so much WHO participates in the social exchanges, adult-child or child-child, as HOW these children participate in their collaborative activity that is so significant. Rogoff (1990) has identified 'guided participation' a way of social experience as most effective in stimulating children's cognitive growth. Another way is scaffolding (Wood 1989) and this is a metaphor used to refer to a support system for children's efforts and which is sensitive to their needs. Adults prompt and monitor children's learning in this social engagement and encouragement. This is a region in which a transfer of ability from the shared

environment to the individual takes place and is called the zone of proximal development, a most well known concept in Vygotsky's work in cognitive development of the young.

Scaffolding refers to a special quality of adult-child interaction or collaboration. Berk and Winsler (1995) identify five key components and goals in effective scaffolding which can be applied to maths learning and teaching. These are

1. Joint problem solving
2. Intersubjectivity
3. Warmth and responsiveness
4. Keeping the child in the ZPD
5. Promoting self-regulation.

Briefly, the first component of scaffolding engages the child in an interesting and culturally meaningful, collaborative problem-solving activity. (adult-child or child-child groupings.) Intersubjectivity refers to the process whereby two participants who begin a task with a different understanding arrive at a shared understanding. Warmth and responsiveness concern the emotional tone of the interaction and it helps when the adult is pleasant and gives verbal praise. To keep the child in the ZPD zone means that the adult structures the task in such a way that it is appropriate and challenging. Another goal of scaffolding is to foster self-regulation and this includes letting the child to make decisions and to solve the problem himself.

What makes effective 'scaffolding' varies from culture to culture. Its characteristics can only be understood in terms of the values and requirements of the child's society as a whole. According to Berk and Winsler (1995) Vygotskian scaffolding is limited to Western culture children and some other societies may have different but socially appropriate ways of interacting with their young. However, ZPD is a specially useful framework for mathematical school learning. Vygotsky observed that effective teachers plan and carry out learning activities within children's ZPDs, through dialogue and scaffolding.

#### Feuerstein and Mediated Learning Experience (MLE)

How do young children make the jump from a natural level of mental organization to higher cognitive processes with the assistance of their caregivers or teachers ? It is found that if the child and the adult focus their attention together, the development of higher order thinking is mutual. Research shows that this kind of joint attentional focus eg between mothers and babies provides a communicative context whereby language and problem solving are enhanced. Mothers provide short, clear explanation of what the baby is looking at , long before the infant is able to verbalise what is seen.

The theory of mediated learning experience (MLE) dates back to the 1950s. Reuven Feuerstein (1980) developed it to explain individuals' different propensities for

learning referring to an example of young adults emigrating from different cultures to Israel showing different levels of learning propensity in adapting to Israel's technology-oriented society. Some of these variations are explained by the nature of the cultures from which these individuals came. What is more interesting, according to Reuven are the differences in the learning propensities among individuals belonging to the same culture. In this respect, the observed intragroup differences were often greater than the intergroup ones. Observations made by researchers attempting to define and explain the cognitive structure of culturally different groups could not pin point it to the culture the immigrants came from. (Skuy, 1996)

Feuerstein links the differences in learning propensity to an individual's exposure through MLE to his own culture, irrespective of its nature or level of conceptualisation, technology, or institutionalized education. Culturally different individuals have become "different" by learning their own culture. This learning experience, usually gained through an MLE process, turns individuals into efficient learners. They use their previously acquired learning experiences to confront a new culture. Culturally deprived individuals, on the other hand, have not been exposed to their own culture. They have not learned to learn and hence it is difficult for them to adapt to the new, more complex conditions of life. Therefore, according to Reuven cultural difference is not the same as cultural deprivation which is a universal phenomena and can be observed in a large variety of ethnic, socioeconomical and professional environments. Cultural deprivation or lack of MLE lowers the individual's 'flexibility and elasticity'.

Feuerstein has shown that individuals have the potential to change and are modifiable if provided with the opportunities to engage in the right kind of interaction. The quality of this interaction is paramount in determining and allowing the individual to develop efficient thinking skills that will enable him or her to become a self-regulated learner. Embedded in MLE is a process by which a mediator organises and interprets the world to the child. When an individual gives meaning to events, helps children select relevant from irrelevant variables, assists in abstracting rules for regularly occurring phenomena, and generally attempts to develop children's abilities to think, that individual is engaged in mediated learning.

MLE begins within the family context with parents and significant others passing on cultural norms, values and modes of thought from one generation to another. A lack of MLE leads to deficient cognitive functioning and low levels of modifiability. The child fails to adapt to and learn from interactions in his/her environment. Many classroom problems in learning are the result of insufficient or inadequate mediated learning experience.

## Mathematics in the Primary Schools: Examples of Scaffolding and MLE

The following two lessons based on Feuerstein's work with MLE are taken from Rodriguez and Bellanca's (1996) examples illustrating mathematical ideas and topics which work well with young children. The use of concept themes and problem solving are infused into the maths instruction.

### PRIMARY SCHOOL LESSON (Example 1)

#### COWS and CHICKENS: A MATHS LESSON

Problem: How to solve a math problem.

Focus Intelligence: Logical/Mathematical

Supporting Intelligences: Verbal/Linguistic, Visual/Spatial and Interpersonal

#### CHECKING PRIOR KNOWLEDGE

- 1 Ask the class to tell you what they know about cows and chickens.
- 2 Sketch each animal as they reply.

#### STRUCTURING THE TASK

- 1 Write on the board: There are four cows and three chickens. How many feet and tails are there all together?
- 2 Put students into pairs and give each pair one pencil and one piece of paper. Invite them to agree on one answer to the question. They can do work on the paper to figure out the answer. Invite them to make their notations large enough for others to see from the class circle.
- 3 Check for understanding of the task.
- 4 Circulate among the pairs and observe how they work together to solve the problem.

#### LOOKING BACK

- 1 Assemble the pairs in the class circle. Ask a number of the pairs to share what they did to solve the problem. Ask them to save their answers. Invite all to use their listening skills when they are not speaking.
- 2 Comment on each strategy with positive feedback.

#### BRIDGING FORWARD

- 1 Ask random pairs to tell what they learned about problem-solving
- 2 Identify the answers.

It is important not only for students to obtain information, especially when that information has connection to their prior knowledge and experience, but also for students to use that information. The more stimulating and creative the opportunity for use, the more likely it is that students will "make sense" of the raw information. By transforming print information into another medium, the students will have a richer opportunity to

develop a second or third intelligence, lock the information into short term-term memory, and build a sense of pride in their work. In essence , as Feuerstein points out, the teacher structures a meaningful task. In addition, the task provides multiple opportunities for the teacher-mediator to mediate for meaning as the project underfolds.

This primary-grade mathematics task can be adapted by the teacher to fit into the child's cultural experience.

## PRIMARY SCHOOL LESSON (Example 2)

### SHAPES

Problem: How to describe shapes in the world around us.

Focus Intelligence: Naturalist

Supporting Intelligences: Logical/Mathematical, Visual/Spatial, Interpersonal, and Bodily/Kinesthetic

### CHECKING PRIOR KNOWLEDGE

On the board, draw a circle, a square, a rectangle, and a triangle. Ask each student to think where he or she might have seen these shapes. Allow pairs to discuss the sightings before you ask individuals to share. Under each shape, list the appropriate responses.

### STRUCTURING THE TASK

- 1 Break the class into four to six groups. Invite each group to form the shape you specify.
- 2 Invite the class to label each shape as you point to each example on the board. Ask "What makes this shape special?"
- 3 Give each group a worksheet with the four shapes. The groups will search throughout the room to find objects in which each shape is found. Let them write the object's name or sketch it.
- 4 Conduct a round robin and invite explanations/reasons for each selection.

### LOOKING BACK

Invite each group to add an example to each list on the board. Conduct a round robin until all lists are complete.

### BRIDGING FORWARD

Instruct each child to take a shape worksheet home so that they can find at least three items which contain each shape. Use the reciprocal model to check for understanding.

### Assessing Student Performance

Each child can identify and name the shape within an object and can explain why each example is reflective of the definition.

### Variation

Give each child a page of shapes to cut out and make a simple picture. Ask the child to explain why the picture is a specific shape.

### Recognizing Patterns

For the primary grades, learning about mathematical shapes is an important topic. Many workbooks end the lesson by asking students to match a shape with a word. The above lesson designed to mediate transcendence goes beyond simple recognition of isolated shapes. It instead enables children to recognize each shape wherever it may be located. Notice how the lesson calls for explanations of the definition .

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