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ABSTRACT

This report presents information from three special studies conducted as part of the National Assessment of Educational Progress (NAEP) 1996 mathematics assessment. It is intended primarily for mathematics educators and others concerned with mathematics education, such as curriculum specialists, teachers, and university faculty in schools of education. The three studies reported here were designed to provide greater detail on how students perform on particular types of mathematics questions. Studies include the Estimation Study, the Study of Mathematics-in-Context, and the Study of Students Taking Advanced Courses in Mathematics. The first study was designed to explore students' skills in estimation and was implemented at three grade levels. It concludes that although there has been significant improvement in mathematics performance overall since 1990 at all grade levels, the trend for student performance in estimation over the six years since the inception of the Estimation Study of 1990 is less clear. The second study was designed to assess problem-solving abilities within contexts that allow students to make connections across mathematics content areas. The Advanced study was administered at grades 8 and 12 and was designed to provide students who were taking or had taken advanced courses in mathematics an opportunity to demonstrate their full mathematical proficiency. (Contains 71 tables and figures.) (ASK)

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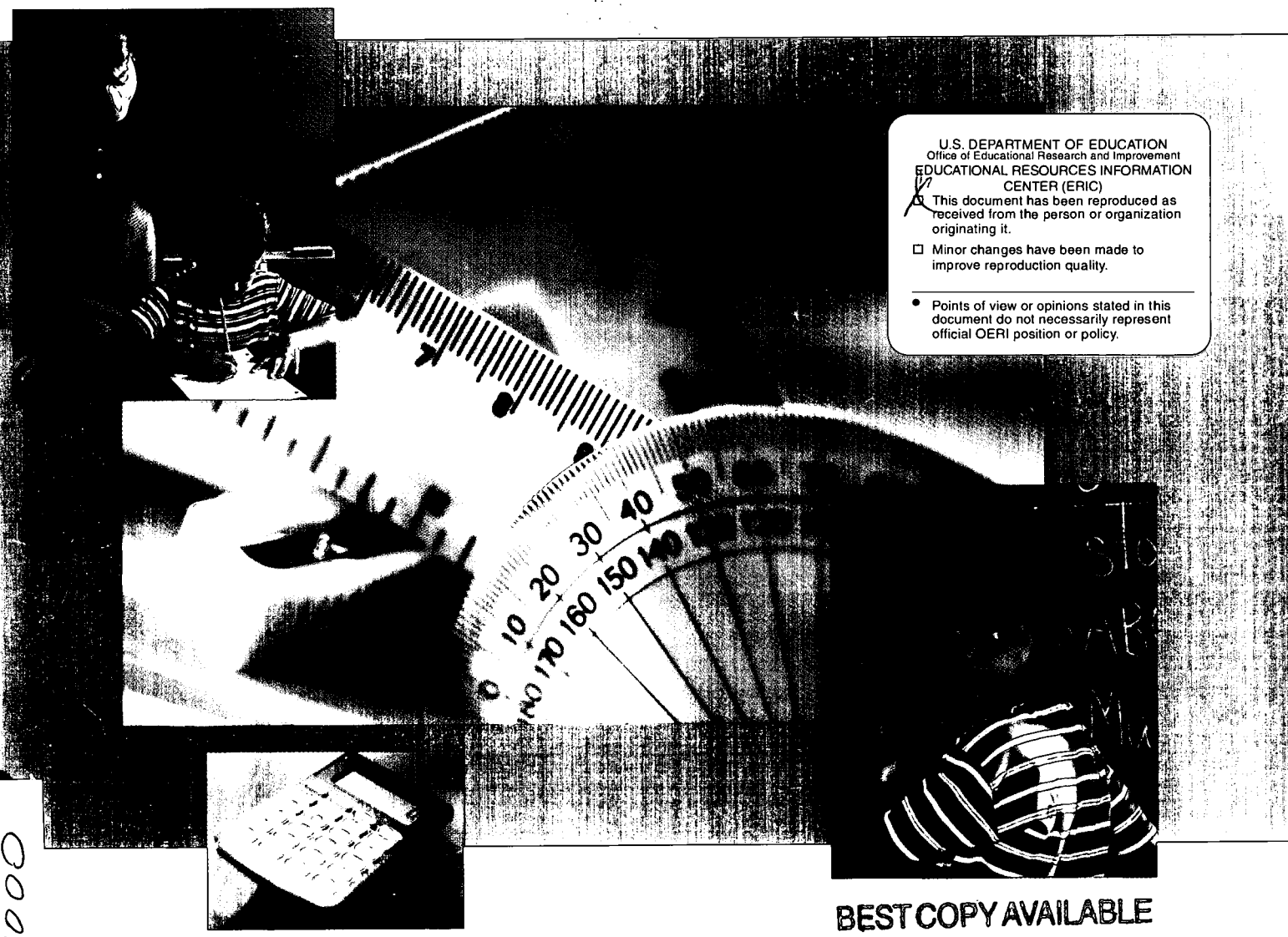
Estimation Skills, Mathematics-in-Context, and Advanced Skills in Mathematics

Results from Three Studies of the National Assessment of Educational Progress 1996 Mathematics Assessment



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The National Assessment Governing Board

NATIONAL CENTER FOR EDUCATION STATISTICS

Estimation Skills, Mathematics-in-Context, and Advanced Skills in Mathematics

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Due to the confidential nature of NAEP surveys, the photographs on the cover of this report do not portray actual students who participated in the NAEP mathematics assessment. The photographs used are from the PhotoDisc stock library.

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Chapter 1

Introduction

This report presents information from three special studies conducted as part of the National Assessment of Educational Progress (NAEP) 1996 mathematics assessment. For more than a quarter of a century, NAEP has been the only nationally representative and continuing assessment of what students in the United States know and can do in various academic subjects. Each NAEP assessment is guided by a framework that specifies important learning outcomes in that subject area; the 1996 mathematics framework was an enhancement of the one used for the mathematics assessments in 1990 and 1992. The goal of the new framework was to define a 1996 assessment that would: (1) more adequately reflect current curricular emphases and objectives, and yet (2) maintain a connection with the 1990 and 1992 assessments to measure trends in student performance.[1]

In addition to the main NAEP assessment, NAEP periodically conducts special studies focused on areas of interest to educators and others. Topics for some of these studies arise as a result of how students performed on NAEP; others are generated simply from research questions about teaching, learning, and assessment of student achievement. This report focuses on studies in mathematics; special studies have also been conducted in, for example, reading and writing.

Purpose and Audience for the Report

This report is intended primarily for mathematics educators and others concerned with mathematics education, such as curriculum specialists, teachers, and university faculty in schools of education. The three studies reported here were designed to provide greater detail on how students perform on particular types of mathematics questions. They include: the Estimation Study; the Study of Mathematics-in-Context, which will be referred to as the Theme Study; and the Study of Students Taking Advanced Courses in Mathematics, which will be referred to as the Advanced Study. The Theme Study and the Advanced Study were administered for the first time in 1996. The Estimation Study, on the other hand, had been administered twice before, in 1990 and 1992.

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[1] For a more in-depth description of the 1996 mathematics framework and how it guided the development of cognitive items, see the following: National Assessment Governing Board (1996). *Mathematics framework for the 1996 National Assessment of Educational Progress*. Washington, DC: Author; Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). *NAEP 1996 mathematics report card for the nation and the states*. Washington, DC: National Center for Education Statistics.

Major Findings of the Report

The first study was designed to explore students' skills in estimation. It was implemented at three grade levels and was the only one of the studies that provided trend information. Findings from the Estimation Study include the following:

- Although there has been significant improvement in mathematics performance overall since 1990 at all grade levels, the trend for student performance in estimation over the 6 years since the inception of the Estimation Study in 1990 is less clear.
- Student performance in Estimation at grades 4 and 12 was stronger in 1996 than in 1990.
- Student performance in Estimation at grade 8 appears to be level across the 3 years of the assessment.

The Theme Study was administered at three grade levels and was designed to assess problem-solving abilities within contexts that allow students to make connections across mathematics content areas. Findings from the Theme Study include:

- At the fourth-grade level, with the exception of the first problem, most students attempted to answer the questions posed, even though large percentages produced responses that were scored as "incorrect." Although not definitive, this may be evidence that the thematic context of the block of questions encouraged students' attention to the task of solving problems, even ones that proved to be difficult for most students.
- At grade 8, unlike grade 4, many students did not attempt to answer the more complex questions that required them to write explanations or apply concepts in problem settings.
- The response rate to the Theme questions at grade 12 was somewhere between the rates observed for grades 4 and 8, with most questions being attempted by at least 90 percent of the students.
- At all grade levels, students appear to have difficulty with complex multistep problems, even those that require only simple computational skills at each step of the problem.
- At all grade levels, many students seemed to lack the mathematical knowledge needed to solve problems. Other students, however, appeared to understand the underlying mathematics but provided incorrect or incomplete responses as a result of carelessness, inexperience in writing out solutions to problems, or confusion over the wording of the question.
- At all grade levels, no positive relationship was seen between the frequency with which students engaged in writing a few sentences about how to solve a mathematics problem, or writing reports or doing mathematics projects, and student performance on the Theme blocks.

The Advanced Study was administered at grades 8 and 12 and was designed to provide students who were taking or had taken advanced courses in mathematics an opportunity to demonstrate their full mathematical proficiency. Findings from the Advanced Study include:

- Students participating in the Advanced Study differed from those who did not qualify for the study in that they tended to come from homes providing a stronger educational context, both in materials and in level of parental education. In addition, based on their participation in Title I programs or qualification for the federal Free/Reduced-Price Lunch program, fewer Advanced Study students appeared to come from low-income homes.
- As would be expected, students at both grade levels who met the criterion for inclusion in the Advanced Study performed substantially better than other students on the main NAEP mathematics assessment.
- The results show that Advanced Study questions were quite difficult, even for students who were taking the more challenging mathematics courses that were prerequisite for participation in the study. Overall performance, measured by average percentage correct, was 36 percent at grade 8 and 30 percent at grade 12. At both grade levels, moreover, most of these students were unable to solve problems that required two or three successive steps to achieve the desired result.
- At grade 12, students who were currently taking mathematics or who were, or had been, enrolled in an Advanced Placement (AP) mathematics course outperformed students in the study who were not currently taking a mathematics course or who had not taken an AP course in mathematics.

The 1996 NAEP Mathematics Assessment

To provide a context for the special studies that are the focus of this report, the following sections give additional information about the NAEP 1996 mathematics assessment and about the manner in which the design and execution of the special studies relate to the main mathematics assessment.

NAEP Mathematics Framework

The NAEP mathematics framework encompasses three cross-cutting domains: a content domain, a domain of mathematical abilities, and a domain of mathematical power. The content domain has five strands: Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; and Algebra and Functions.[2] The domain of mathematical abilities describes the nature of the knowledge or processes that are involved in successfully handling mathematical tasks or problems; it includes Conceptual Understanding, Procedural Knowledge, and Problem Solving. The domain of mathematical power refers to students' ability to reason, to communicate, and to make connections of concepts and skills across mathematics strands, or from mathematics to other curricular areas. Figure 1.1 summarizes the structure of the framework for the NAEP 1996 mathematics assessment.

Figure 1.1 **Mathematics Framework for the 1996 Assessment**

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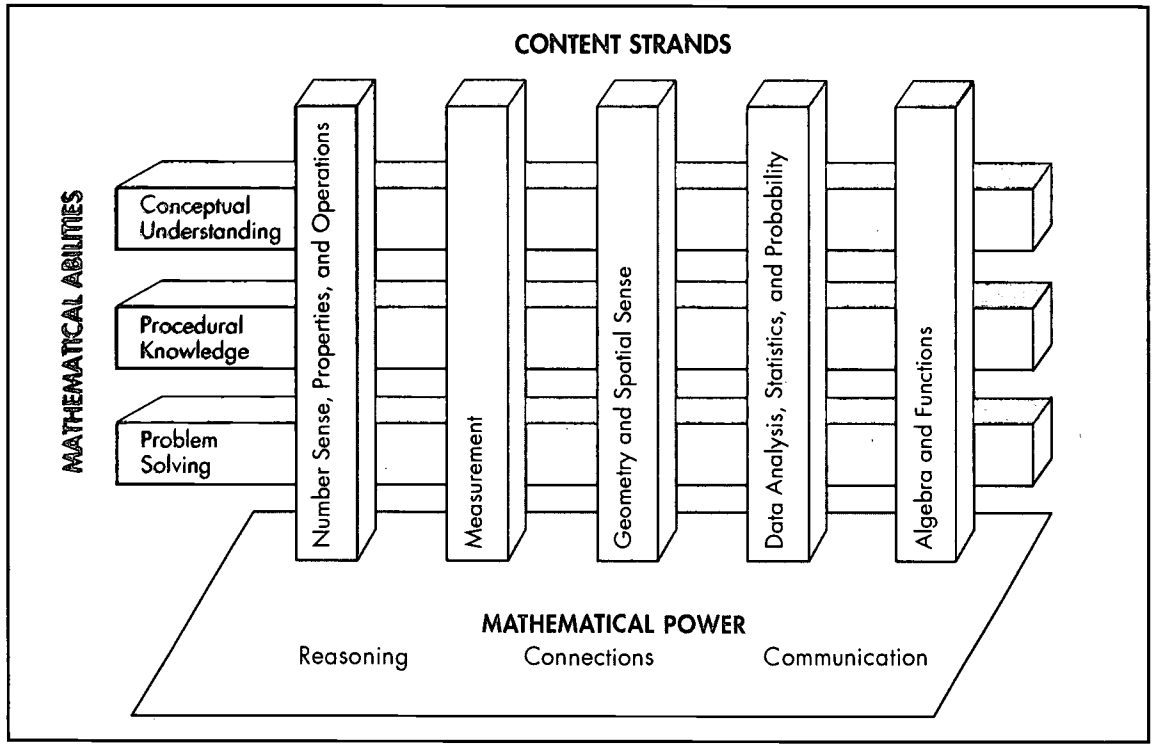

SOURCE: National Assessment Governing Board, *Mathematics Framework for the 1996 National Assessment of Educational Progress.*

[2] The content strand *Number Sense, Properties, and Operations* was called *Numbers and Operations* in the 1990 and 1992 assessments. The content strand *Geometry and Spatial Sense* was called *Geometry* in the 1990 and 1992 assessments.

The development of the questions for the special studies, although guided by the 1996 NAEP mathematics framework, naturally focused on the goal of each individual study. Questions for the Estimation Study cut across the five content strands, but the main intent was to assess estimation skills. Questions for the Theme Study tended to emphasize problem-solving abilities within the context of real-life types of experiences. Finally, questions for the Advanced Study tended to include more content in Algebra and Functions than did questions in the main NAEP assessment.

In addition to cognitive achievement questions, student assessment booklets for the special studies contained blocks of background questions. The background questions asked students to provide information about themselves, their classroom instruction, and their motivation to expend effort on the assessment. Teachers and school administrators of students participating in NAEP also responded to background questionnaires. Teachers provided information about their education, professional careers, curricular practices, and instructional approaches, as well as the resources available to them for teaching mathematics. School administrators answered questions about school policies and practices.

Samples

The NAEP 1996 mathematics assessment was conducted nationally at grades 4, 8, and 12. As mentioned earlier, both the Estimation Study and the Theme Study also were conducted at grades 4, 8, and 12, while the Advanced Study was conducted at grades 8 and 12 only. Students for the Estimation and Theme Studies were selected through the same sampling design as students for the main NAEP assessment and were representative of all U.S. public and nonpublic school students.[3] Students selected for the Advanced Study were representative of students who had taken, or were enrolled in, more advanced mathematics courses. Specifically, to qualify for the Advanced Study, eighth-grade students had to be currently enrolled in, or already have taken, first-year algebra or a more advanced course in mathematics; and twelfth-grade students had to be currently enrolled in, or already have taken, a pre-calculus or pre-calculus-equivalent course or a more advanced course such as calculus.

Following the model of the main NAEP data collection, school administrators of students participating in the special studies were surveyed at all grade levels, but mathematics teachers of participating students only were surveyed at grades 4 and 8. The exception was the Advanced Study, which included surveys of grade 12 mathematics teachers.

[3] See Appendix A for detailed information on sample selection.

Reporting NAEP Results

Student performance on NAEP assessments has been reported using a variety of measures. Results for the main NAEP mathematics assessment are reported using the NAEP composite mathematics scale, which summarizes performance across five separate subscales — one for each of the five content strands. In addition to the NAEP mathematics scale, results also are reported using the mathematics achievement levels as authorized by the NAEP legislation[4] and as adopted by the National Assessment Governing Board. The achievement levels are performance standards based on collective judgments about what students should be expected to know and to do. Viewing students' performance from this perspective provides some insight into the adequacy of students' knowledge and skills and the extent to which they achieved expected levels of performance. The Board reviewed and adopted the recommended achievement levels derived from the judgments of a broadly representative panel that included teachers, education specialists, and members of the general public.

For each grade tested, the Board has adopted three achievement levels: *Basic*, *Proficient*, and *Advanced*. For reporting purposes; the achievement level cut scores for each grade represent the boundaries between four ranges on the NAEP mathematics scale: below *Basic*, *Basic*, *Proficient*, and *Advanced*. The generic policy definitions of the achievement levels are shown below in Figure 1.2. The text of the descriptions of expected mathematics performance at each achievement level at each grade can be found in the *NAEP 1996 Mathematics Report Card*.[5]

Figure 1.2 — ***Policy Definitions of NAEP Achievement Levels*** — THE NATION'S REPORT CARD naep

Basic	This level denotes partial mastery of prerequisite knowledge and skills that are fundamental for proficient work at each grade.
Proficient	This level represents solid academic performance for each grade assessed. Students reaching this level have demonstrated competence in challenging subject matter, including subject-matter knowledge, application of such knowledge to real-world situations, and analytical skills appropriate to the subject matter.
Advanced	This level signifies superior performance.

SOURCE: *NAEP 1996 Mathematics Report Card for the Nation and the States.*

[4] The National Education Statistics Act of 1994 requires that the National Assessment Governing Board develop "appropriate student performance levels" for reporting NAEP results.

[5] Reese, et al., (1997). op. cit.

The NAEP legislation requires that the achievement levels be used on a developmental basis until the Commissioner of Education Statistics determines, as the result of a Congressionally mandated evaluation by one or more nationally recognized evaluation organizations that the achievement levels are "reasonable, valid, and informative to the public." Upon review of the available information, the Commissioner of Education Statistics agrees with the recent recommendation of the National Academy of Science that caution needs to be exercised in the use of the current achievement levels, since in the opinion of the Academy "... appropriate validity evidence for the cut scores is lacking; and the process has produced unreasonable results."[6] Therefore, the Commissioner concludes that these achievement levels should continue to be considered developmental and should continue to be interpreted and used with caution. The Commissioner and the Governing Board believe that the achievement levels are useful for reporting on trends in the educational achievement of students in the United States.

Reporting Results for the Special Studies

None of the special study assessment questions contributes to the NAEP composite mathematics scale. However, in 1990, the first year the composite scale and its component subscales were used, a separate scale was established that summarized performance on questions used in the Estimation Study. Each scale was constructed separately, and the metrics of the scales are arbitrary. Therefore, although each scale ranges from 0 to 500 across the three grade levels assessed, it is not possible to conclude that a student who performed at level 300 on the estimation scale, for example, and one who performed at level 300 on the main mathematics scale had both mastered the same proportion of their respective content domains.[7] The value of the estimation scale, like the value of the composite scale, is that it allows for trend analysis across years as well as making it possible to report the results using achievement levels.

The results from the Advanced Study and the Theme Study did not lend themselves to either the development of separate proficiency scales or equating to the main NAEP mathematics scales.[8] Consequently, the overall results from the Theme Study and the Advanced Study are reported simply in terms of the percentages of questions that students answered correctly. Student performance on individual items also is highlighted for each of these studies.

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[6] Pellegrino, J. W., Jones, L. R., & Mitchell, K. J. (Eds.). (1999). *Grading the nation's report card: Evaluating NAEP and transforming the assessment of educational progress*. Committee on the Evaluation of National and State Assessments of Educational Progress, Board on Testing and Assessment, Commission on Behavioral and Social Sciences and Education, National Research Council. (p. 182). Washington, DC: National Academy Press.

[7] In the initial year of use, each scale was set to have a mean of 250 and a standard deviation of 50.

[8] See Appendix A for more detail.

Organization of the Report

Each special study is presented in a separate chapter. The second chapter of this report describes the Estimation Study, the third chapter depicts the Theme Study, and the fourth chapter characterizes the Advanced Study. This report also includes two appendices. The first provides additional information on the procedural and technical aspects of these special studies, and the second includes standard error tables for the data presented in the body of the report.

Chapter 2

Estimation Study

Estimation is a process whereby one approximates, through rough calculations, the worth, size, or amount of an object or quantity that is present in a given situation. The approximation, or estimate, is a value that is deemed close enough to the exact value or measurement to answer the question being posed. Beginning with recommendations issued in 1975, estimation has been seen as an important, even necessary, daily living skill that all students should have, whether they are working with paper and pencil, doing mental mathematics, or using technology.[1] Estimation is now viewed as a necessary component of the school mathematics curriculum.

Estimation in the school curriculum can range from making quick ad hoc estimates to applying "rules of thumb." "Rule of thumb" operations include the application of mental checkmarks to a situation; for example, knowing that the circumference of a circle is slightly larger than 3 times the diameter. Other examples of estimation strategies include:

- adding only the leading digits to approximate the sum of a long column of numbers that all have the same number of digits (e.g., approximating the sum of 23 + 74 + 81 + 19 + 37 as 2 + 7 + 8 + 2 + 4 or about 23 tens, or 230);

- using compatible numbers for proportional problems (e.g., approximating the number of cups of oats that are needed to make 12 cups of granola, given that 1¾ cups of oats are needed to make 5 cups, by reasoning that it takes about 2 cups of oats for 6 cups of granola, so it will take about 4 cups for 12); and

- breaking or separating an uncommon shape into parts with known attributes to approximate a quality of the less common shape (e.g., approximating the area of an irregularly shaped region by separating it into 8 squares, each of which has an area of 4 square centimeters, and computing the area of the region as 8 × 4, or 32 square centimeters).[2]

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[1] National Advisory Committee on Mathematical Education. (1975). *Overview and analysis of school mathematics: Grades K-12.* Washington, DC: Conference Board of the Mathematical Sciences; National Council of Supervisors of Mathematics. (1978). Position paper on basic mathematical skills. *The Mathematics Teacher, 71*, pp. 147–152; National Council of Teachers of Mathematics. (1980). *An agenda for action.* Reston, VA: Author; National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

[2] Reys, B. (1988). Estimation. In Thomas R. Post (Ed.), *Teaching mathematics in grades K-8.* (pp. 262–284). Newton, MA: Allyn & Bacon, Inc.

The importance of estimation in the school curriculum was acknowledged in 1986 when the National Council of Teachers of Mathematics (NCTM) devoted its annual yearbook to the topic.[3] The NCTM also featured estimation as a central area within the *Curriculum and Evaluation Standards* in 1989.[4] Estimation is seen as a central mathematical process applicable across a wide range of mathematical content areas, including all five of the NAEP content strands: Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; and Algebra and Functions.

Design of the Estimation Study

It is because of the acknowledged importance of estimation that, since 1990, the national NAEP mathematics assessment has included blocks of questions focused on this topic. State data, however, were only collected in the 1992 Estimation Study.[5] The Estimation blocks include questions that require students to make estimates in a wide range of mathematical settings, from problems involving basic numeric operations to questions assessing students' knowledge of chance and growth. Some questions also require students to make use of spatial estimation skills, such as looking at pictures of dots in a region or marbles in a jar and making numerical estimates of the total.

Unlike the blocks in the main mathematics assessment or either of the other special studies, the Estimation blocks are administered using a paced audio tape. That is, students are paced through the Estimation Study questions by an audio tape that moves students along as they read and respond to the questions. This mode of administration is intended to minimize the time students have available to make actual computations or to give undue consideration to one question over another within the block. The test developers hope thus to increase the probability that students will actually use their estimation skills to respond to the questions. Furthermore, the paced audio tape reads the questions to the students, facilitating access to the problems for students with limited reading skills.

Because of these differences in administration procedures, the 1996 Estimation blocks were presented to students in separate test booklets rather than being randomly spiraled with the blocks from the main assessment. There was a separate Estimation booklet for each grade level, each containing three blocks of cognitive questions. The first block consisted of non-estimation questions taken from the main assessment. The second block (also referred to as the trend block) was based on the Estimation block used for that grade level in 1990 and 1992. Although some questions in the trend blocks were modified to improve their measurement characteristics, care was taken to maintain a sufficient core of unmodified questions from previous assessments to ensure the integrity of the trend data.

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[3] Schoen, H., & Zweng, M. (Eds.). (1986). *Estimation and mental calculations: The 1986 Yearbook of the NCTM.* Reston, VA: National Council of Teachers of Mathematics.

[4] National Council of Teachers of Mathematics. (1989). op. cit.

[5] Results from the state administration of the 1992 Estimation Study can be found in the following publications: Mullis, I. V. S., Dossey, J. A., Owen, E. H., & Phillips, G. W. (1993). *NAEP 1992 mathematics report card for the nation and the states.* Washington, DC: National Center for Education Statistics; National Center for Education Statistics (1993). *Data compendium for the NAEP 1992 mathematics assessment of the nation and the states.* Washington, DC: Author.

The third block in each booklet contained new estimation questions written for the 1996 assessment. In contrast to the trend blocks, the new Estimation blocks utilized constructed-response as well as multiple-choice questions, and they also included questions that were designed to assess students' ability to distinguish between situations in which it is more appropriate to over- or underestimate. In addition, the new blocks were written specifically for each grade level, whereas the trend blocks included a great deal of overlap across grades. As shown in Table 2.1, nearly half of the trend questions were used across all three grades, and, among the remaining questions, only those for grade 4 were unique. All of the questions in the new blocks, on the other hand, were unique to the grade level for which they were written.

Table 2.1 — *1996 Estimation Questions by Grade Level and Block*

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	Grade Level					
	Only 4	Only 8	Only 12	4-8	4-8-12	8-12
Trend	10	0	0	0	10	12
New	13	15	16	0	0	0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Content of the Estimation Blocks

Unfortunately, unlike the Theme Study and Advanced Study questions, which are discussed in later chapters of this report, none of the Estimation Study questions have been released to the public. This is because the number of Estimation Study questions is limited, and NAEP expects to readminister them in future NAEP assessments in order to continue to track trends over time. A few mock questions, based on actual assessment questions, were written for this report to provide the reader with a sense of the kinds of questions included in the Estimation blocks. The following paragraphs and mock questions provide a description of the content of the Estimation Study.

As shown in Table 2.2, multiple-choice questions predominated at each grade level, despite the introduction of some constructed-response questions in the new blocks written for the 1996 assessment.

Table 2.2 — *1996 Estimation Questions, by Grade Level and Question Type*

THE NATION'S REPORT CARD naep

	Multiple Choice	Constructed Response	Total
Grade 4	30	3	33
Grade 8	31	6	37
Grade 12	38	0	38

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The Estimation Study questions drew upon content from each of the five strands identified in the NAEP mathematics framework. However, as can be seen in Table 2.3, content from the Number Sense, Properties, and Operations strand and the Measurement strand predominated.

Table 2.3 — *1996 Estimation Questions, by Grade Level and Content Strand*

THE NATION'S REPORT CARD naep

	Number Sense, Properties, and Operations	Measurement	Geometry and Spatial Sense	Data Analysis, Statistics, and Probability	Algebra and Functions
Grade 4	21	9	0	3	0
Grade 8	20	9	2	4	2
Grade 12	15	13	1	3	6

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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In addition to questions in which the correct response was an actual estimate of the quantity posed, some questions asked students to indicate an appropriate *method* for arriving at an estimate, to identify the one response that could *not* satisfy the specified conditions, or to label possible responses as being either "reasonable" or "unreasonable" estimates. Some questions emphasized visual, or "sight" estimation, and a few required knowledge of whether over- or underestimation would be more prudent under a range of specified circumstances. In general, the questions designed for the higher grade levels exhibited increased complexity; for example, they required the combination of more facts or factors, involved more complex concepts or operations, or were presented in more general or abstract settings.

Number Sense, Properties, and Operations

Questions involving estimation that were classified in the Number Sense, Properties, and Operations strand required students to estimate quantities such as the following:

- the result of a single mathematical computation;
- the largest or smallest among the results of different arithmetic operations based on the same numbers;
- a number rounded to a specified decimal place value;
- the approximate total cost of several purchases;
- the total monetary value of several piles of coins;
- the amount of material needed for several articles, given the amount of material needed for one;
- the number of offspring after several cycles of reproduction;
- the number of individuals in a subgroup, given information about the size of the total group and the percentage represented by the subgroup;
- the mean score on a group of tests, given information about the number of tests and the minimum and maximum scores;
- the number of persons present at the end of a sequence of events in which people both come and go; and
- the number of words on a typewritten page, given an enlarged image of a portion of the page.

In addition to whole numbers, questions in this content strand involved fractions, decimals, money (including groups of coins), and time. At grade 12, exponents and roots also were included. A mock Number Sense, Properties, and Operations question similar to those that were asked of eighth-grade students is presented below. (As noted earlier, no Estimation Study items were released to the public, so the items shown in this chapter are not actual NAEP items.)

> As they were riding along a highway, Chris and Lee noticed that all of the cars in the opposite lane were stopped due to road construction. Driving at an average speed of 48 miles per hour, it took Chris and Lee 11 minutes to get from the beginning of the stopped cars to the end. If the average length of a car is 20 feet, show how you could round the numbers and compute in order to estimate how many cars were stopped in the opposite lane. (1 mile = 5,280 feet)
>
> *Not an operational NAEP question*

This question is relatively difficult because it requires more than one operation or comparison to reach a solution, whether or not the computational burden is reduced through appropriate rounding. Students also need to understand the relationship between time, distance, and rate in order to arrive at a correct solution.

One possible solution to this problem follows. However, because different students might approximate or round the values in different ways, credit must be based on appropriate identification and application of a solution strategy rather than a specific numeric outcome. For example, some students might approximate 11 minutes as 1/6 of an hour rather than 1/5 of an hour. Similarly, since the number of place values was not specified, the number of feet per mile might well be rounded up to 5,300 rather than down to 5,000.

One possible solution

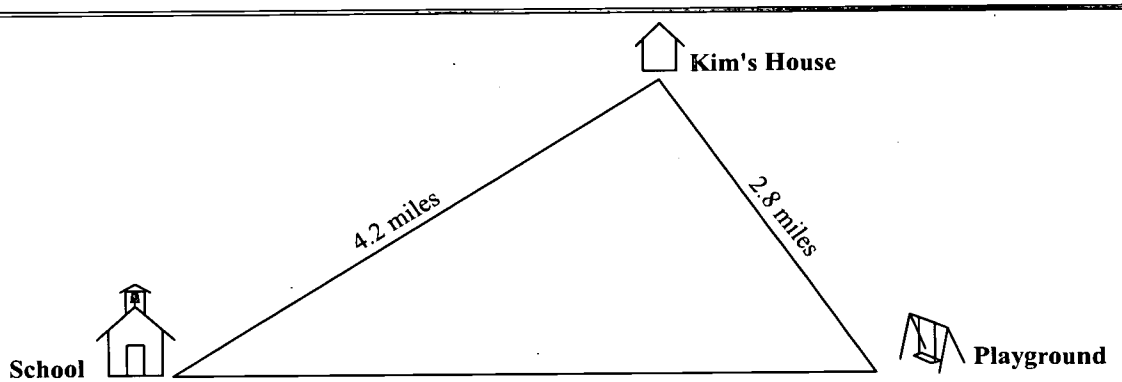
> 48 mph is almost 50 mph and 11 minutes is about $\frac{1}{5}$ hour, so the length of the stopped traffic is about $50 \times \frac{1}{5}$ or 10 miles. Since 5,280 feet rounds to 5,000, the number of cars stopped is approximately $\frac{10(5000)}{20}$, which is 2,500.

Measurement

Questions about estimation that were classified in the Measurement strand involved estimates of quantities such as the following:

- the length, in centimeters or inches, of an object shown to actual size;
- the relationship between readings of numbers on two different vertical scales;
- the number of objects that can be held in a one-quart container, given the number that fit into a one-cup container;
- the number of smaller objects that could be drawn into the interior of a larger object of the same shape;
- the length of a specified path between two points, given information about the shape of the path and the straight-line distance between the two points;
- the time difference between travel on two different routes, given rates and graphs drawn to scale for each route;
- the area of an irregularly shaped object, given a basic square unit of measure; and
- the area of an irregularly shaped object, given information about the dimensions of two rectangles, one larger and one smaller than the object.

A mock Measurement question similar to those that might have been asked of fourth-grade students is presented below.



Kim rides her bicycle from her house to school, then from school to the playground, and finally back home. She travels a total of 12.1 miles. About how many miles apart are the playground and the school?

(A) 4 miles

(B) 5 miles

(C) 7 miles

25

Not an operational NAEP question

The correct response is Option B, 5 miles. To answer the question, a student might read the graphic representation of Kim's journey, subtract the mileage for each of the two labeled segments from the total mileage for the journey, and round the answer to whole miles. If a student rounded all of the mileage values before doing the computations, the same numeric answer would be obtained. Alternatively, a student simply might reason from the picture that the third side is slightly longer than the side measuring 4.2 miles; hence, the answer must be 5 miles. (As noted earlier, all of the Estimation Study questions were presented via audio tape in order to establish the pace for the assessment and discourage students from undertaking precise calculations.)

Geometry and Spatial Sense

Only a few of the questions in the Estimation blocks tapped content classified as Geometry and Spatial Sense. Those that did required students to estimate the size of angles, given either a visual representation or information about the size of other angles in the same figure.

Data Analysis, Statistics, and Probability

Questions involving estimation that were classified in the Data Analysis, Statistics, and Probability strand all required the students to make estimates based on data presented in graphic or tabular format. Examples of the types of values they were asked to estimate include the following:

- the approximate value of a data point on a graph;
- the median of several values presented in a bar graph;
- which of two politicians performed best overall, given a tabular display of votes earned in each of several election districts; and
- the slope of the line that best fits data in a scatter plot.

An example of a Data Analysis, Statistics, and Probability question similar to those that might have been asked of twelfth-grade students follows.

MONTHLY COSTS FOR TELEPHONE CALLING OPTIONS
AT PHONEBELL TELEPHONE COMPANY

Calling Option	Cost per Month
Call Blocking	$4.04
Call Forwarding	$2.30
Call Waiting	$4.59
Caller ID	$6.55
Caller ID Deluxe	$7.50

The table above shows the monthly costs for certain calling options offered by Phonebell Telephone Company. Stephan chooses the following options: call blocking, call waiting, and caller ID. Approximately how much would Stephan pay over a 6-month period for these options?

(A) $15

(B) $35

(C) $90

Not an operational NAEP question

The correct answer is Option C, $90. Students would need to use the tabular data display to locate the monthly costs of the three specified options, sum them, and then multiply by six to obtain the 6-month total. These computations would be facilitated by using leading digits to sum the costs or by rounding the costs of the options before beginning. However, given the magnitude of the differences between response options, adept students would not have to perform any computations. They could see very quickly that only Option C was large enough to represent a plausible 6-month cost for three options, the cheapest of which costs more than $4.00 per month.

Algebra and Functions

Questions about estimation that were classified in the Algebra and Functions strand occurred only at grades 8 and 12. They required students to estimate quantities such as the following:

- the cube root of a four-digit number;
- the solution to a linear equation with decimal coefficients; and
- the value of a trigonometric function of a fractional number of radians, given a graph of the function.

Student Performance on Estimation

Overall performance on the NAEP mathematics scale

The tables that follow contain information on student performance on the NAEP Estimation Study over the past three assessments. The first table, Table 2.4, also includes performance data from the main NAEP mathematics assessment. These data show that, for all three grades on the main mathematics assessment, there was a pattern of continuous improvement between 1990 and 1992 and between 1992 and 1996 (although the improvement at grade 12 between 1992 and 1996 was not statistically significant). The trend for student performance on the Estimation Study, however, is not so clear. In part, this may reflect the smaller number of estimation items in the assessment, and the correspondingly greater error of measurement. In any event, student performance in Estimation at grade 8 appears to be level across the three years of the assessment, while performance at grades 4 and 12 was stronger in 1996 than in 1990.

In reviewing these data, it is important to remember that, because the Estimation Study was scaled separately from the main assessment, it is not appropriate to make direct comparisons of the average scale values obtained in a given year across the two scales. On the other hand, the Estimation Study scale, like the scale used for the main assessment, is a cross-grade scale. This means that all three grades are portrayed on a single scale, and it is possible to obtain rough estimates of the performance differences across grade levels.

Table 2.4 **Average Scale Scores for National NAEP and Estimation Studies, Grades 4, 8, and 12**

THE NATION'S REPORT CARD naep

	Assessment Year	Average Overall Scale Score in Mathematics NAEP	Average Estimation Scale Score
Grade 4	1996	224*†	206*
	1992	220*	208*
	1990	213	200
Grade 8	1996	272*†	270
	1992	268*	271
	1990	263	269
Grade 12	1996	304*	297*
	1992	299*	294
	1990	294	292

* Significant difference from 1990.

† Significant difference from 1992.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Other findings related to Estimation proficiency and student characteristics

Table 2.5 contains information from analyses made of the relationship between students' 1996 performance on the Estimation Study and their background characteristics. At grades 4 and 8, males and females performed about the same, while twelfth-grade boys outperformed twelfth-grade girls. At all three grades, White students and Asian/Pacific Islander students outperformed Black and Hispanic students. On average, students whose parents had at least some education after high school had higher scale scores than those whose parents did not finish high school, while at grades 4 and 12, students whose parents had a high school education also outperformed those who reported that their parents did not finish high school. Finally, students who did not participate in Title 1 and were not eligible for the federal Free/Reduced-Price Lunch program performed better than those who did participate or were eligible for these programs.

Data from earlier years are not included in Table 2.5 because there were few significant changes in subgroup performance in Estimation over the 6-year span from 1990 to 1996. The two significant changes that were observed were in the performance of males at grade 12 (where performance increased from 296 in 1990 to 303 in 1996) and in the performance of fourth-grade White students (where performance increased from 208 in 1990 to 216 in 1996).[6]

[6] The sources of these data are the 1990 and 1996 NAEP mathematics assessments.

Table 2.5

Scale Scores in Estimation by Background Variables, Grades 4, 8, and 12, 1996

THE NATION'S REPORT CARD

	Average Scale Score		
	Grade 4	Grade 8	Grade 12
Gender			
Males	209	272	303*†
Females	203	268	292
Students who Indicated Their Race/Ethnicity as...			
White	216*	278	303
Black	181	246	276
Hispanic	183	253	279
Asian/Pacific Islander	213!	271	318
American Indian	***	***	***
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	179	257	275
Graduated From High School	203	262	289
Some Education After High School	213	275	291
Graduated from College	217	278	306
I Don't Know	197	253	***
Students who Attend...			
Public Schools	204	269	295
Nonpublic Schools	221!	278!	308!
Title I Participation...			
Participated	178	248	271!
Did Not Participate	217	273	298
Free/Reduced-Price Lunch Program Eligibility...			
Eligible	183	254	273
Not Eligible	217	276	299
Information Unavailable	218	275!	304

* Significant difference from 1990.

† Significant difference from 1992.

*** Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Table 2.6 presents data on students' average Estimation scores at different percentile levels for 1990, 1992, and 1996. Here also, little change has occurred since 1990. At grade 4, students at the median and above have made progress since 1990. No significant changes have taken place at grade 8. At grade 12, there has been some improvement for students at the low end of the distribution — the 10th percentile. These data indicate that the pattern of limited or no improvement noted earlier for performance on the Estimation Study questions is fairly widespread across the full range of student performance, with the possible exception of students at the upper end of the grade 4 distribution.

Table 2.6

Average Scale Scores in Estimation at Different Percentile Levels, Grades 4, 8, and 12

THE NATION'S REPORT CARD naep

	Assessment Year	10th Percentile	25th Percentile	50th Percentile	75th Percentile	90th Percentile
Grade 4						
	1996	153	180	208	235*	256*
	1992	160	184	210*	234*	253
	1990	158	178	200	223	243
Grade 8						
	1996	232	251	272	290	304
	1992	233	251	272	291	306
	1990	232	249	269	288	305
Grade 12						
	1996	259*	278	298	317	333
	1992	258*	276	295	314	328
	1990	252	273	295	313	327

* Significant difference from 1990.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Performance on NAEP achievement levels

Along with the development of the NAEP Estimation Study score scale, *Advanced*, *Proficient*, and *Basic* achievement levels also were established for the estimation questions. These achievement levels were first reported in 1990 and were used again, though in slightly modified form, in 1992 and 1996.[7] This section includes information on the percentages of students scoring at or above the achievement levels set on the Estimation Study questions.

The data, presented in Table 2.7, reflect much the same picture as was seen in the earlier proficiency scale results — that is, there has been little change since 1990. The only area where a significant increase in student achievement appears to have taken place is at the fourth-grade level, where there was a significant increase from 1990 to 1996 in the percentage of students reaching at least the *Proficient* level of performance.

Table 2.7 — *National Percentages Attaining Achievement Levels in Estimation, Grades 4, 8, and 12*

THE NATION'S REPORT CARD naep

	Percentage of Students				
	Assessment Year	Advanced	At or Above Proficient	At or Above Basic	Below Basic
Grade 4					
	1996	2	30*	88	12
	1992	1	30*	91	9
	1990	0!	20	90	10
Grade 8					
	1996	1	19	68	32
	1992	1	20	67	33
	1990	1	18	64	36
Grade 12					
	1996	6	38	83	17
	1992	4	34	82	18
	1990	4	33	79	21

* Significant difference from 1990.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

[7] Mullis, I. V. S., Dossey, J. A., Owen, E. H., & Phillips, G. W. (1991). *The state of mathematics achievement: NAEP's 1990 assessment of the nation and the trial assessment of the states.* Washington, DC: National Center for Education Statistics; Mullis, et al., (1993). op. cit.

An overall analysis of the percentages of students reaching the various achievement levels in 1996 reflects a differentiated pattern across the grades. Grade 12 had the greatest percentage of students reaching the *Advanced* level (6%), while only one or two percent reached this level at the lower grades. The *Proficient* level was reached by almost one-third of the students at grades 4 and 12, while one-fifth of the eighth-grade students reached this level. At the same time, over 80 percent of the students at grades 4 and 12 reached at least the *Basic* level, compared to two-thirds of the grade 8 students.

The data in Table 2.8 provide information parallel to that provided in Table 2.5. Here one can see the percentages of students in different subgroups reaching at least the *Proficient* level on the Estimation Study for each of the three grade levels in 1996.

The data for 1990 and 1992 were not reported in the table due to the small number of significant differences in performance on estimation over the time period of the three assessments. However, at grade 4, male students, female students, White students, and students attending public schools all exhibited significant increases since 1990 in percentages reaching at least the *Proficient* level, indicating that this increase in proficiency occurred broadly across the largest subgroups of students.[8]

[8] The sources of these data are the 1990 and 1996 NAEP mathematics assessments.

Table 2.8

Percent of Students Reaching at Least Proficient Level in Estimation by Background Variables, Grades 4, 8, and 12, 1996

THE NATION'S REPORT CARD naep

	Percentage of Students Achieving Proficient or Better		
	Grade 4	Grade 8	Grade 12
Gender			
Males	32*	22	47
Females	28*	16	31
Students who Indicated Their Race/Ethnicity as...			
White	38*	25	45
Black	9	3	13
Hispanic	11	6	18
Asian/Pacific Islander	38!	16	70
American Indian	16!	6!	18!
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	7	6	14
Graduated From High School	27	10	25
Some Education After High School	36	19	27
Graduated From College	40	28	51
I Don't Know	21	6	18!
Students who Attend...			
Public Schools	28*	18	36
Nonpublic Schools	43!	29!	53!
Title I Participation...			
Participated	8	2!	7!
Did Not Participate	38	21	40
Free/Reduced-Price Lunch Program Eligibility...			
Not Eligible	38	24	39
Eligible	12	6	12
Information Unavailable	40	24!	49

* Significant difference from 1990.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Summary

The overall results presented in this chapter, indicate a lack of significant progress over the 6 years since the inception of the Estimation Study in 1990. In terms of the achievement expectations set for this area of mathematics, the greatest change appears to have taken place at grade 4, although smaller score increases were also found at grade 12. The differences in grade 4 performance may reflect action on the recommendation by the National Council of Teachers of Mathematics that emphasis be placed on estimation in the primary grades.[9] The lack of equivalent increases at the upper two grades may be an indication of the lack of focused instruction and opportunity to practice estimation concepts and skills in either classroom or assessment settings.

[9] National Council of Teachers of Mathematics. (1989). op. cit.

Chapter 3

Study of Mathematics-in-Context

Overview

The Study of Mathematics-in-Context (also referred to as the Theme Study) was implemented to address specific reform recommendations regarding mathematics instruction and assessment. These recommendations assert that students benefit from mathematics instruction and assessments that enable them to make connections across mathematics content areas (e.g., Number Sense, Properties, and Operations with Geometry and Spatial Sense, or Measurement with both Geometry and Spatial Sense and Algebra and Functions) and to real-world contexts.[1] These instructional and assessment strategies also reflect reform efforts to include more integrated curriculum and project-based instruction in our mathematics classrooms.

NAEP Administration

Students in grades 4, 8, and 12 who participated in the Theme Study were selected through the same procedures used in selecting students for the national NAEP mathematics assessment (also referred to as the main NAEP assessment).[2] Two 30-minute Theme blocks based on sustained real-life scenarios were constructed for each grade level. The use of a single real-world problem contexts for each Theme block was expected to engage students' interest and sustain motivation throughout the assessment. Additionally, the 30-minute Theme blocks provided an opportunity to explore student responses to questions that were more detailed and complex than questions in the main NAEP assessment, in which all blocks were 15 minutes in length.

Students participating in the Theme Study were each given an assessment booklet containing one of the Theme blocks for their grade level and a 15-minute, grade-appropriate block from the main NAEP assessment. In addition, the Theme Study booklets included blocks of background questions that also were administered to students in the main NAEP assessment.

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[1] National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author; Brutlag, D. & Maples, C. (1992). Making connections: Beyond the surface. *Arithmetic Teacher, 85*(3), 230–235.

[2] For information on sampling for the national NAEP mathematics assessment, see Appendix A and Allen, N. J., Jenkins, F., Kulick, E., & Zelenak, C. A. (1997). *Technical report of the NAEP 1996 state assessment program in mathematics.* Washington, DC: National Center for Education Statistics.

As with the main assessment, teachers of participating fourth- and eighth-grade students and principals from schools in which the fourth-, eighth-, and twelfth-grade students were enrolled also were surveyed. In addition, students were provided with calculators that they could use without restriction, and they were asked to indicate whether or not they had used the calculator when answering each question.

Although all of the questions in the Theme blocks were centered around a single theme, a correct response to one question did not depend on correct responses to other questions in the block, and each question was scored independently. However, some questions did include two or more interdependent parts that were scored as a single unit.

Unlike the Estimation Study or the main NAEP assessment, no achievement scale was developed for the Theme Study. Because the Theme Study was not intended to measure a unique aspect of mathematics achievement, a separate achievement scale was not appropriate. Moreover, the Theme Study booklets did not have enough questions per content strand (subscale) in common with the main assessment to allow for valid linking to the scale used in the main assessment.

Organization of Chapter

The performance data presented in this chapter include block-level average percentage correct scores as well as average percentage correct scores for individual questions: for all students, and by gender and race/ethnicity subgroups where sufficient sample sizes allow. One Theme block from each grade was released for public use, and these released questions are discussed in detail, although student performance on the unreleased blocks also is referenced.

Theme-block questions included mathematics content from more than one content strand and assessed a variety of mathematical abilities as defined by the 1996 NAEP Mathematics Framework.[3] Although assessment developers attempted to provide students with questions from an appropriate range of grade-level difficulty, there was no explicit attempt to make the two Theme blocks at each grade level (or across grade levels) equivalent in terms of overall difficulty level. Therefore, the reader should not make any formal comparisons of performance at different grade levels or across the two Theme blocks at any of the grade levels.

The remainder of this chapter is divided into four sections, one for each grade level and one that summarizes the findings across grade levels. Each grade-level section begins with background information on student demographics and the prevalence of two specific instructional practices thought to be related to skills required for answering the Theme block questions. The instructional practices data are based on teacher responses to the following questions: (1) How often are students asked to write a few sentences about how to solve a mathematics problem? and (2) How often are students asked to write reports or do mathematics projects? It was hypothesized that answering a series of thematically-related mathematics questions might be a more familiar task for students who had carried out sustained mathematics projects, and that students who had experience writing about mathematics might perform better

[3] For information about the NAEP 1996 Mathematics Framework, see National Assessment Governing Board. (1996). *Mathematics framework for the 1996 National Assessment of Educational Progress*. Washington, DC: Author.

on questions asking students to explain how they arrived at their answers. The background and instructional information is followed by a presentation of overall student performance on each Theme block administered to that grade and, finally, by a discussion of each of the questions students encountered in the released Theme block.

Grade 4

Student characteristics

The data in Tables 3.1 and 3.2 show that the fourth-grade students who participated in the Theme Study were similar to the students who participated in the main NAEP assessment on a variety of demographic variables and on the two classroom practice variables. Nearly equal numbers of male and female students took each of the Theme blocks and the main NAEP assessment. The majority of students in each of the samples were White; very small percentages were Asian/Pacific Islander or American Indian. When asked about the highest level of education attained by their parents, students most often indicated that their mothers and fathers had "graduated from college." However, more than a third of the students were unable to provide any information about their parents' education. Nearly 90 percent of students attended public as opposed to nonpublic schools. About one-fourth of the students participated in the Title I program, and about a third were identified as being eligible for the federal Free/Reduced-Price Lunch program.

The data in Table 3.2 show that fewer than 10 percent of the students in each of the samples had teachers who reported that students in their mathematics classes wrote a few sentences about solving a mathematics problem "nearly every day." Teacher responses for the remainder of the students were distributed fairly evenly across the other response categories: "once or twice a week," "once or twice a month," and "never or hardly ever." Teachers of these fourth-grade students reported asking students to write reports and do projects even less frequently than asking them to write a few sentences about solving a mathematics problem. For all samples, fewer than five percent had teachers who reported asking their students to write reports or do projects "once or twice a week" or more, and two-thirds or more had teachers who reported "never or hardly ever" asking their students to engage in such practices. However, the reader should keep in mind that reports and projects are typically long-term activities, and this factor may limit the frequency with which teachers could ask students to do them.

Given these similarities between students in the Theme Study samples and those participating in the main NAEP assessment, it is reasonable to expect that fourth-grade students across the nation would have performed similarly on the Theme blocks to the students who actually participated in the Theme Study.

Table 3.1

Student Demographic Distributions by Assessment, Grade 4, 1996

THE NATION'S REPORT CARD naep

Grade 4	Percentage of Students		
	Main Assessment	Theme Block 1 The Butterfly Booth	Theme Block 2 Recycling
Gender			
Males	51	51	53
Females	49	49	47
Students who Indicated Their Race/Ethnicity as...			
White	68	68	69
Black	15	15	14
Hispanic	13	12	13
Asian/Pacific Islander	3	3	3
American Indian	2	2	1
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	4	4	6
Graduated From High School	13	12	13
Some Education After High School	7	7	8
Graduated From College	40	42	38
I Don't Know	36	36	35
Students who Attend...			
Public Schools	89	87	88
Nonpublic Schools	11	13	12
Title I Participation...			
Participated	22	24	23
Did Not Participate	78	76	77
Free/Reduced-Price Lunch Program Eligibility...			
Eligible	31	33	35
Not Eligible	53	55	54
Information Not Available	16	12	11

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 3.2	Percentage of Students by Teachers' Reports on Classroom Practices, Grade 4, 1996	THE NATION'S REPORT CARD naep

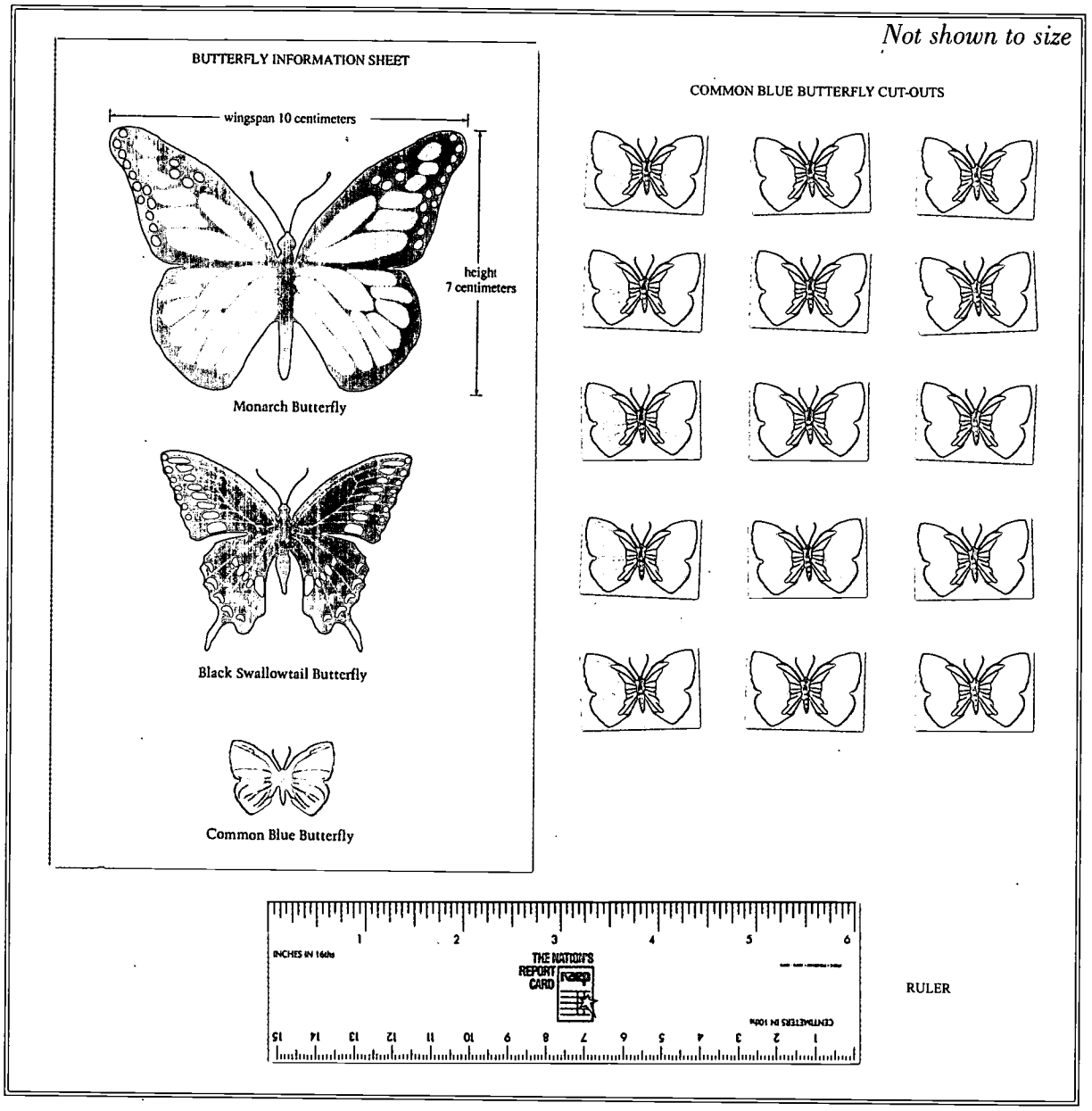
	Percentage of Students		
	Main Assessment	Theme Block 1 The Butterfly Booth	Theme Block 2 Recycling
Grade 4			
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem...			
Nearly Every Day	9	7	6
Once or Twice a Week	26	28	28
Once or Twice a Month	36	35	33
Never or Hardly Ever	29	30	33
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project...			
Nearly Every Day	1	0	0
Once or Twice a Week	4	2	2
Once or Twice a Month	29	27	25
Never or Hardly Ever	66	70	72

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Content of the Theme blocks

The developers of the assessment questions for the two fourth-grade Theme blocks selected real-life contexts that should have been familiar to most, if not all, fourth-grade students in the United States. The released block involved setting up a Butterfly Booth at a school science fair; the unreleased block was about recycling. With each of these blocks, students were provided with a packet of materials to be used in answering some of the questions. The packet that accompanied the Butterfly Booth block contained a Butterfly Information Sheet, which featured beautifully colored illustrations of a Monarch butterfly, a Black Swallowtail butterfly, and a Common Blue butterfly; a 6-inch ruler marked with both inches and centimeters; and 15 cut-outs of the Common Blue butterfly. The Butterfly Information Sheet also included information about the wingspan and height of the Monarch butterfly. Providing students with pictures of butterflies in such vivid, realistic colors may have made the task more interesting for many of the students.

Packet Contents for "The Butterfly Booth"



The questions in both blocks were similar in format. The six questions in the Butterfly Booth block were either short or extended constructed-response questions. Of the eight questions in the Recycling block, one was a multiple-choice question and the remainder were short or extended constructed-response questions. In both blocks, there were three or four questions in which students were asked to provide an explanation for how they arrived at their answer.

Overall student performance

Students' overall performance, as measured by average percentage correct scores, is presented in Table 3.3. For both blocks of questions, students' average percentage correct score was 30 percent. Male and female students performed similarly; White, Hispanic, and Asian/Pacific

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Islander students outperformed Black students, and White and Asian/Pacific Islander students also outperformed Hispanic students. The sample of American Indian students for both Theme blocks was too small to permit reliable estimates of their performance on the block as a whole or on individual questions. Therefore, the remainder of the discussion will not include information about the performance of American Indian students.

Classroom practices, as measured by the frequency of writing a few sentences about how to solve a mathematics problem, or writing reports or doing mathematics projects, were not found to be related to student performance on either of the Theme blocks.

Table 3.3 **Average Percentage Correct Scores by Theme Block, Grade 4, 1996**

THE NATION'S REPORT CARD naep

Grade 4	The Butterfly Booth	Recycling
All Students	30	30
Gender		
Males	29	30
Females	31	29
Race/Ethnicity		
White	34	33
Black	17	20
Hispanic	21	22
Asian/Pacific Islander	35	35
American Indian	***	***
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem...		
Nearly Every Day	35	34
Once or Twice a Week	30	32
Once or Twice a Month	29	27
Never or Hardly Ever	32	30
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project...		
Nearly Every Day	***	***
Once or Twice a Week	***	***
Once or Twice a Month	30	30
Never or Hardly Ever	30	30

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Planning a Butterfly Booth

The introduction to the block of questions about planning a Butterfly Booth for a school science fair is shown in Figure 3.1. The instructions included the following information about the expectations for acceptable responses: "In those questions where you must write an answer, it is important that your answer be clear and complete and that you show all of your work since partial credit may be awarded." The instructions also informed students that "[s]ome questions may each require 5 minutes or more to think about and answer." Thus, students were expected to take time to think about how to solve some of these problems. Finally, the manner in which the task of planning the booth was presented provided students the opportunity to take ownership of the task. For example, the introduction says, "Your class is planning to have a Butterfly Booth," and "You need to make decorations for the booth... " The hope was that students would feel that they were no longer just answering questions on another test but solving problems that they might encounter in their own lives at school, making the task of answering the questions more interesting and meaningful.

Figure 3.1 — ***Introduction to "Planning a Butterfly Booth" Theme Block, Grade 4, 1996*** — THE NATION'S REPORT CARD naep

This part has 6 questions. Mark your answers in your booklet. You will have to fill in an oval or write your answer as directed. In those questions where you must write an answer, it is important that your answer be clear and complete and that you show all of your work since partial credit may be awarded. Some questions may each require 5 minutes or more to think about and answer. After each question, fill in the oval to indicate whether you used the calculator.

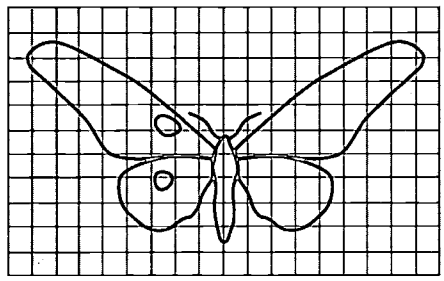
Use the packet you have been given to help you answer the questions in this section.

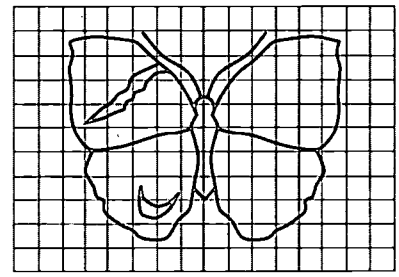
Each class in Oakville School will have a booth at the Science Fair. Your class is planning to have a Butterfly Booth.

Your class has a lot to do to get ready for the Science Fair. You need to make decorations for the booth, plan activities, and order materials.

Question 1. Draw symmetrical figure. The first question that students encountered in the block was classified as a Geometry and Spatial Sense question and was designed to assess Problem Solving ability. The question provided students with a context for the task they were asked to complete. That is, students could imagine rendering butterfly drawings that would be used to decorate the booth.

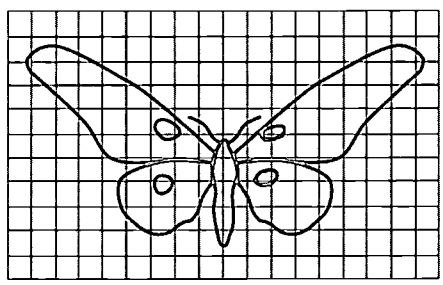
1. **The butterfly booth will be decorated with butterfly drawings. Draw only the missing markings on each picture to make each butterfly symmetrical.**

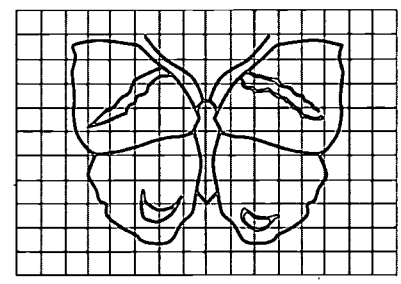




To respond successfully to this question, students needed to know what "symmetrical" meant, be familiar with using grids to complete drawings, and understand that the grids were important for getting a correct response. This question was scored with a 4-point rubric: "satisfactory," "partial," "minimal," and "incorrect."[4] To be scored "satisfactory," responses had to have all four markings correctly drawn as shown in the sample student response. Students were given full credit if their drawings appeared to show that they understood the concept of symmetry, even if the symmetrical markings were not perfectly drawn or not placed exactly on the grid.

Sample "satisfactory" response

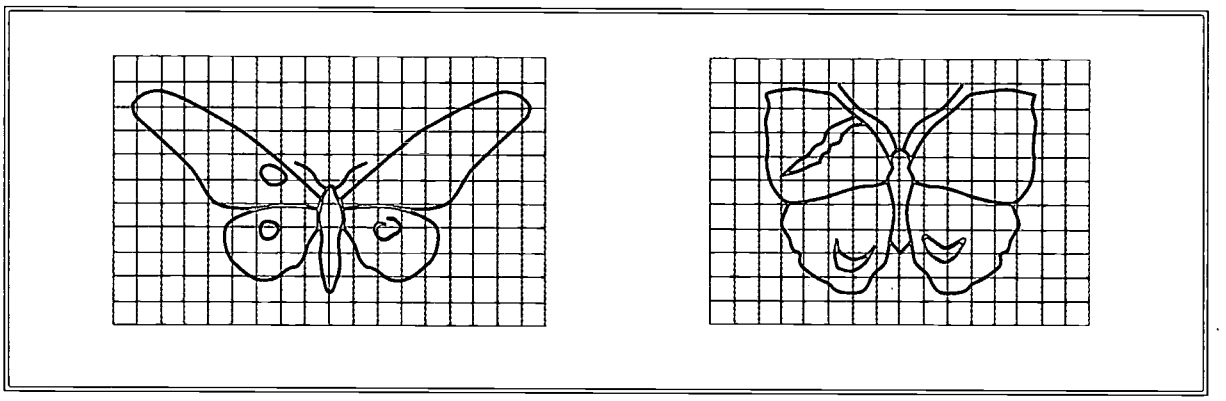




[4] Student responses for this and all other constructed-response questions also were scored as "off task" if the student provided a response that was deemed not related in content to the question asked. There are many examples of this type of response, but a simple one would be "I don't like this test." In contrast, responses scored as "incorrect" were valid attempts to answer the question that simply were wrong.

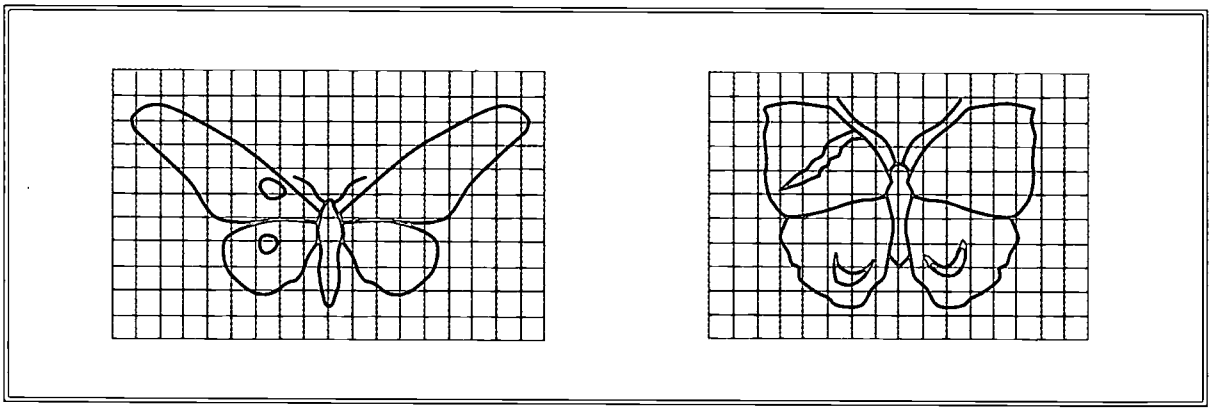
A "partial" response was one in which three of the four markings were correctly drawn; the fourth marking was either omitted or incorrectly drawn. In the "partial" response that follows, the student appears to have understood the concept of symmetry but did not duplicate the fourth marking. The student may have decided that, because this marking had a less uniform shape, it was too difficult to attempt to draw. Another possibility is that the student did not have enough time to finish the task.

Sample "partial" response

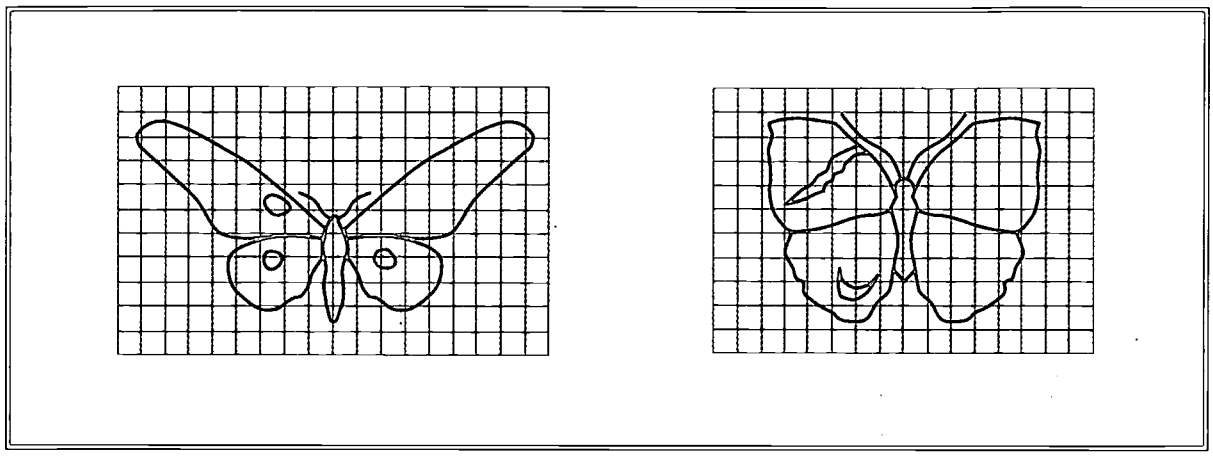


"Minimal" responses included those in which one or two of the markings were drawn correctly, and the remaining markings were either omitted or incorrectly drawn. The two "minimal" responses shown each have one marking correctly drawn. It is not clear why the students chose to draw only one marking. It is possible they did not fully understand the meaning of symmetrical. "Incorrect" responses had none of the parts correctly drawn.

Sample "minimal" response I



Sample "minimal" response 2



Information on students' performance on this question is presented in Table 3.4. The majority of students were able to draw at least some of the parts correctly; however, only 28 percent of the students drew all four parts correctly. Furthermore, a relatively large proportion of students (33%) did not attempt to draw symmetrical butterflies at all; that is, they skipped over this problem entirely. It is not clear why so many students omitted this question; possibly they did not know the word "symmetrical" or did not know how to use the grid paper.

Table 3.4 — *Score Percentages for "Draw Symmetrical Figure," Grade 4*

THE NATION'S REPORT CARD naep

Grade 4	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	28	17	11	11	33
Males	30	14	8	11	36
Females	27	20	13	10	29
White	34	19	10	10	26
Black	8	14	13	15	50
Hispanic	18	12	11	10	48
Asian/Pacific Islander	38	21	8	6	27
American Indian	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 2. Measure length using ruler. In the second question, students were asked to use the ruler that was provided in their packets to measure the wingspans of the Black Swallowtail and the Common Blue butterflies and report their measurements to the nearest centimeter. This question was classified as a Measurement question and was designed to assess Procedural Knowledge. Students needed to know how to measure using the ruler provided, what centimeters are, and what it means to measure to the nearest centimeter. Students also needed to know what the term "wingspan" means or be able to discern its meaning from the indication of the Monarch butterfly's wingspan on the Butterfly Information Sheet. Furthermore, although centimeters are clearly indicated on the ruler provided, knowledge of and experience in using centimeter measurements may have been helpful.

> 2. **Take the Butterfly Information Sheet from your packet.**
>
> On the Butterfly Information Sheet the wingspan of the Monarch butterfly is shown.
>
> Use your ruler to measure the wingspans of the other two butterflies on the sheet, the Black Swallowtail butterfly and the Common Blue butterfly, to the nearest centimeter.
>
> | | | |
> |---|---|---|
> | Black Swallowtail | Wingspan: _____ | centimeters |
> | Common Blue | Wingspan: _____ | centimeters |

The correct answer to the question is seven centimeters for the wingspan of the Black Swallowtail butterfly and three centimeters for the wingspan of the Common Blue butterfly. Students' responses were scored on a 5-point rubric: "extended," "satisfactory," "partial," "minimal," and "incorrect." An "extended" response was one in which both measurements were correct. Responses were scored "satisfactory" if one of the measurements was correct, if the two measurements were correct but reversed on the answer sheet, or if students did not round to the nearest centimeter but provided answers that would round to seven (6.5 to 7.5) and three (2.5 to 3.5). "Partial" responses were those in which students appeared to have measured the heights of the butterflies rather than their wingspans; that is, "partial" responses included answers for both measurements, with the wingspan ranging between 5 and 5.5 centimeters for the Black Swallowtail and between 2 and 2.5 centimeters for the Common Blue butterfly. Responses were scored "minimal" if they were in the correct range for inches, not centimeters. That is, if the measurements were 2.5 to 3 inches for the Black Swallowtail and 1 to 1.5 inches for the Common Blue. Responses that met none of the criteria already mentioned were scored as "incorrect."

Information on response scores for this question is presented in Table 3.5. Forty percent of fourth-grade students were able to provide the correct measurements for the wingspans of both butterflies. From student responses, it appears that many fourth-graders are familiar with metric measurements such as centimeters or are at least able to translate what they know about

using a ruler with inch markings into using a ruler with centimeter markings. Twenty-six percent of the responses were scored as "satisfactory" or "partial," suggesting that these students may not have been careful in their measurements or in their reading of the question, may not have understood the term "wingspan," or may not have understood what it means to measure to "the nearest centimeter."

Perhaps as evidence that fourth-grade students were more familiar with the task involved in this question compared to the first question, a much higher percentage of students chose to respond to this question (96%) than chose to respond to the first question (67%). Furthermore, the percentage of "extended" responses (40%) for this question was higher than the percentage of "satisfactory" responses (28%) for Question 1.

Table 3.5 — *Score Percentages for "Measure Length Using Ruler," Grade 4* — THE NATION'S REPORT CARD naep

Grade 4	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	40	14	12	3	27	4
Males	41	12	12	2	28	5
Females	40	16	12	4	26	3
White	47	14	12	3	23	2
Black	20	18	11	3	41	8
Hispanic	26	13	15	3	31	12
Asian/Pacific Islander	54	7	5	2	27	5
American Indian	***	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 3. Solve packing problem. Students were asked to use the 15 rectangular cut-outs of the Common Blue butterflies that were provided in their packets to answer this two-part question. The question included a two-dimensional representation of a storage case, drawn to size. Part 1 of the question asked students to determine the greatest number of Common Blue butterflies (of the size of the cut-outs) that could be stored in the case and to show how those butterflies would be arranged in the case; butterflies could not overlap each other or be stacked on top of each other. In the second part of the question, students were asked to determine how many storage cases of the same size they would need to store 28 Common Blue butterflies. Students also were asked to explain with drawings, words, or numbers how they arrived at their answers. Question 3 was classified as a Measurement question and was designed to measure Problem-Solving ability.

In responding to this question, students were not required to do any actual measuring with a ruler. To determine the number of butterflies that would fit, they simply needed to place the Common Blue butterfly cut-outs onto the picture of the storage case. To indicate how the butterflies would lie in the case, students could trace the outline of the cut-outs or they could draw butterflies of the approximate size of the Common Blue cut-outs. Having answered Part 1, there were numerous strategies students could use to solve Part 2. Whatever strategy students used, they were required to describe it in order to answer the question completely.

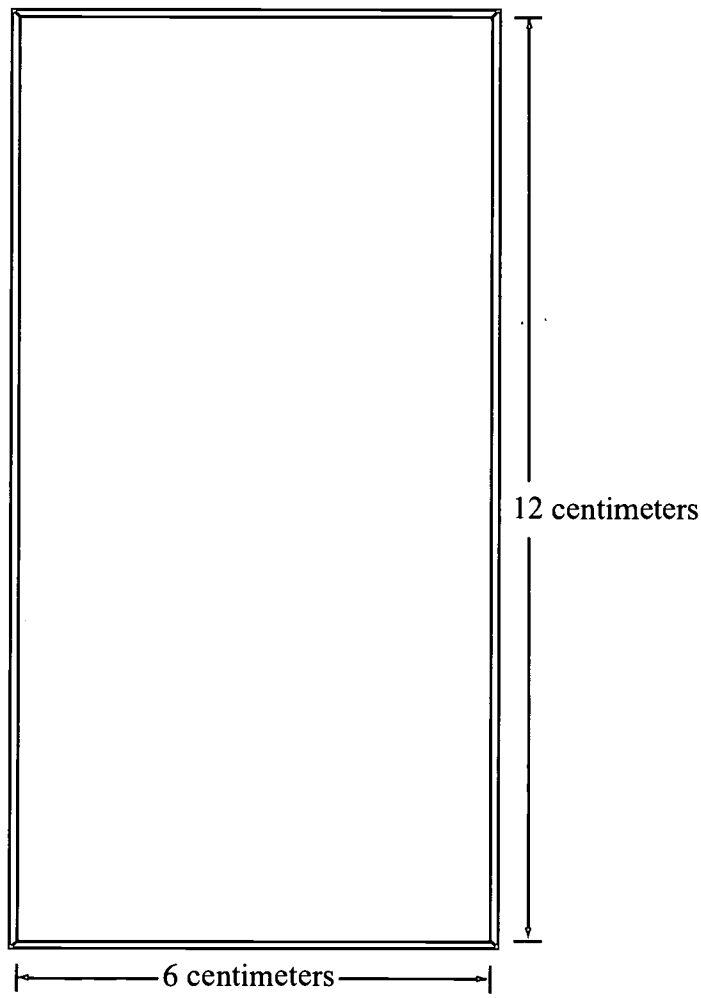
3. **Take the butterfly cutouts from your packet.**

What is the greatest number of Common Blue butterflies that can be stored in the case below? (When you put butterflies in the case, you can't stack them. The butterflies can touch, but they can't overlap at all.)

Answer: _____

Show how the butterflies fit in the case.

Storage Case



How many storage cases would you need to store 28 Common Blue butterflies?

Answer: _____

Use drawings, words, or numbers to explain how you got your answer.

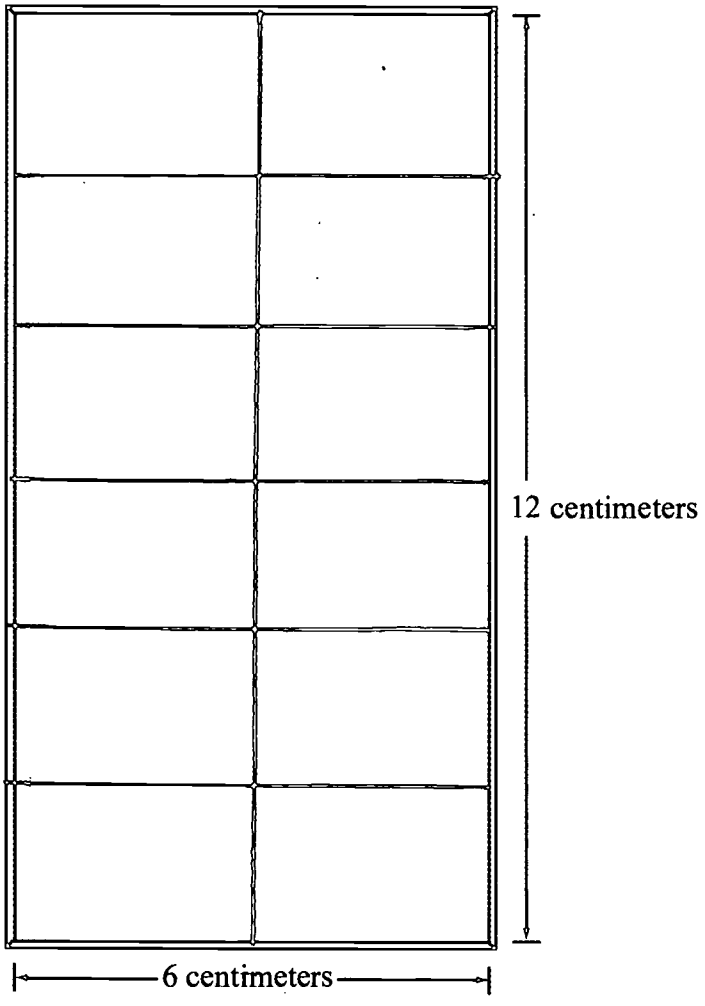
This question was scored on a 5-point rubric: "extended," "satisfactory," "partial," "minimal," and "incorrect." Responses scored as "extended" provided the answer 12 for the number of butterflies that would fit in the case, with a drawing of how the 12 would lie in the case, and the answer 3 for the number of cases needed to accommodate 28 butterflies, with an appropriate explanation for arriving at 3 cases. In the sample "extended" response that follows, the student subdivided the picture of the storage case into 12 appropriately sized rectangles to show the placement of the 12 non-overlapping butterflies. For the second part of the question, the student showed, through addition, how she or he arrived at the answer of 3 cases needed to accommodate the 28 butterflies.

Sample "extended" response

Answer: 12 butterflies

Show how the butterflies fit in the case.

Storage Case



How many storage cases would you need to store 28 Common Blue butterflies?

Answer: 3

Use drawings, words, or numbers to explain how you got your answer.

12 butterflies = 1 case
+12
24 butterflies = 2 cases
+12
36 butterflies = 3 cases

A response could be scored as "satisfactory" if it met any of the following criteria:

- Part 1, 12 butterflies with a correct drawing; and Part 2, an incorrect answer but a correct explanation;
- Part 1, 12 butterflies with a correct drawing; and Part 2, a correct answer but an inadequate or missing explanation;
- Part 1, 12 butterflies with drawing missing; and Part 2, a correct answer with a correct explanation; or
- Part 1, 10 or 11 butterflies with a correct drawing; and Part 2, a correct answer with a correct explanation.

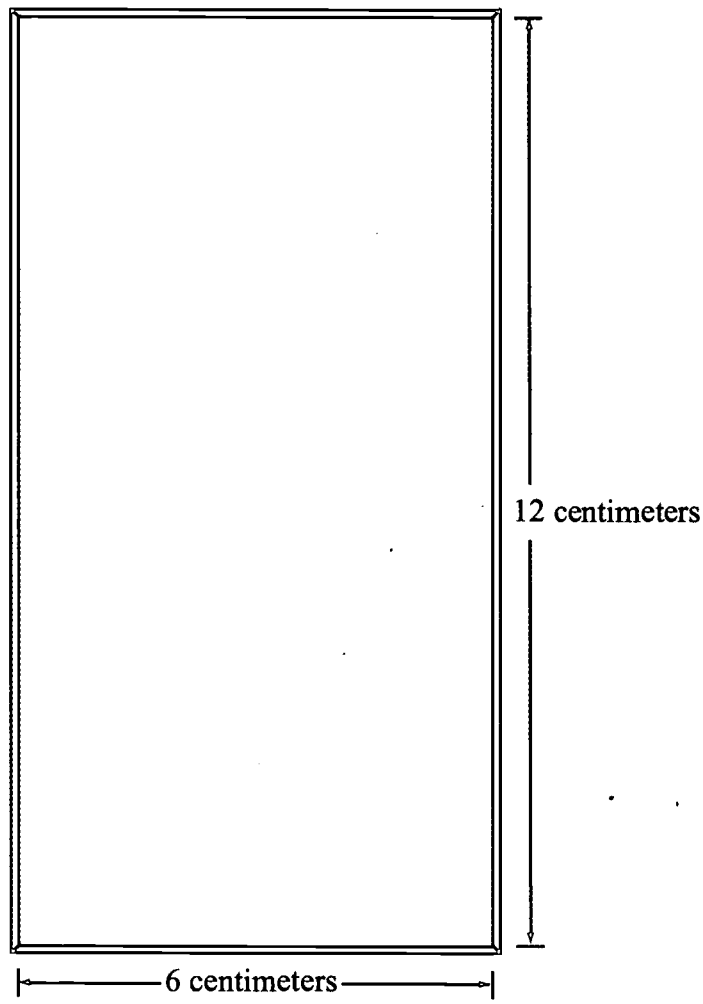
The following sample "satisfactory" response includes the correct answers for Parts 1 and 2 and an acceptable explanation for Part 2, but the drawing is missing for Part 1. It is not evident why the student did not complete the drawing. It is possible that she or he simply did not read the directions carefully and therefore did not realize that it was necessary to show how the 12 butterflies would fit in the case.

Sample "satisfactory" response

Answer: 12 butterflies

Show how the butterflies fit in the case.

Storage Case



53

How many storage cases would you need to store 28 Common Blue butterflies?

Answer: 3

Use drawings, words, or numbers to explain how you got your answer.

I new 12 could fit in a case. Then I multiplied 12 times and got 24. Then I new I would have to get another case because I couldnt any more in that case

A "partial" response would meet one of the following criteria:

- Part 1, 10 or 11 butterflies with or without a drawing; and Part 2, an incorrect answer with a correct explanation of the strategy used to determine that number;

- Part 1, 10 or 11 butterflies without a drawing; and Part 2, a correct answer with an acceptable explanation;

- Part 1, 10 or 11 butterflies with a correct drawing; and Part 2, a correct answer but a missing or inadequate explanation;

- Part 1, 12 butterflies without a drawing; and Part 2, an incorrect answer but an acceptable explanation; or

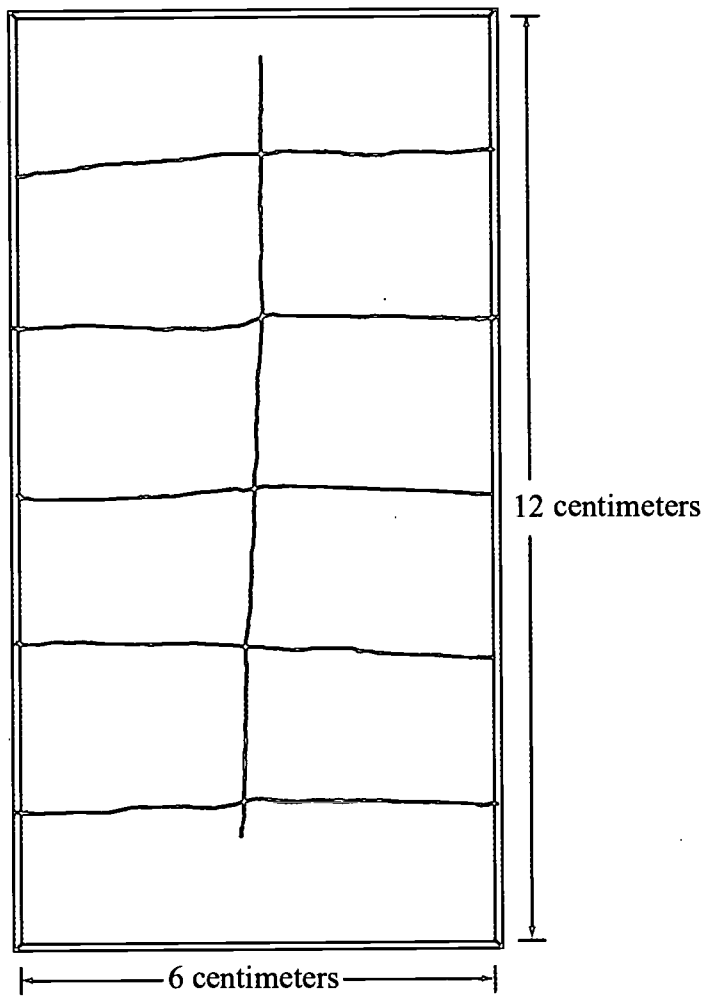
- Part 1, 12 butterflies without a drawing; and Part 2, a correct answer but an inadequate or missing explanation.

The sample "partial" response that follows indicated that 10 butterflies would fit in the case and provided a correct drawing of the 10 in the case. For Part 2, the student offered the correct answer of 3 cases but an inappropriate explanation of the solution process. The explanation in Part 2 is actually an explanation of deriving the answer in Part 1.

Answer: 10

Show how the butterflies fit in the case.

Storage Case



How many storage cases would you need to store 28 Common Blue butterflies?

Answer: 3

Use drawings, words, or numbers to explain how you got your answer.

I mesured the Blue Butterfly's wing span and hight then I took my ruler Put it on the box and mesured 5 dow and 2 across and I multiplied and got the answer 10

The different responses that could be scored as "minimal" include the following:

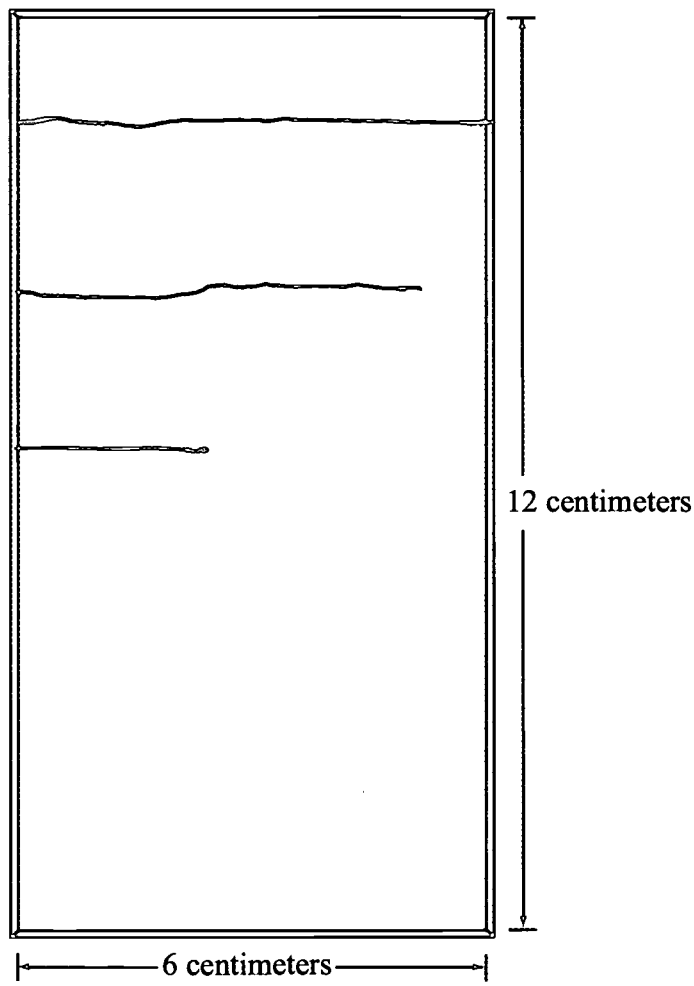
- Part 1, 10 or 11 butterflies with or without a correct drawing; and Part 2, an incorrect number of cases and explanation;
- Part 1, 10 or 11 butterflies with an incorrect drawing; and Part 2, a correct answer but an explanation that is missing or inadequate; or
- Part 1, 12 butterflies with or without a correct drawing; and Part 2, an incorrect answer and explanation.

The sample "minimal" response that follows has an acceptable number of 10 butterflies but an incomplete drawing for Part 1 and the correct number of 3 cases but an inadequate explanation for Part 2.

Answer: 10

Show how the butterflies fit in the case.

Storage Case



How many storage cases would you need to store 28 Common Blue butterflies?

Answer: 3

Use drawings, words, or numbers to explain how you got your answer.

I put the Butterflys in the storage Case.

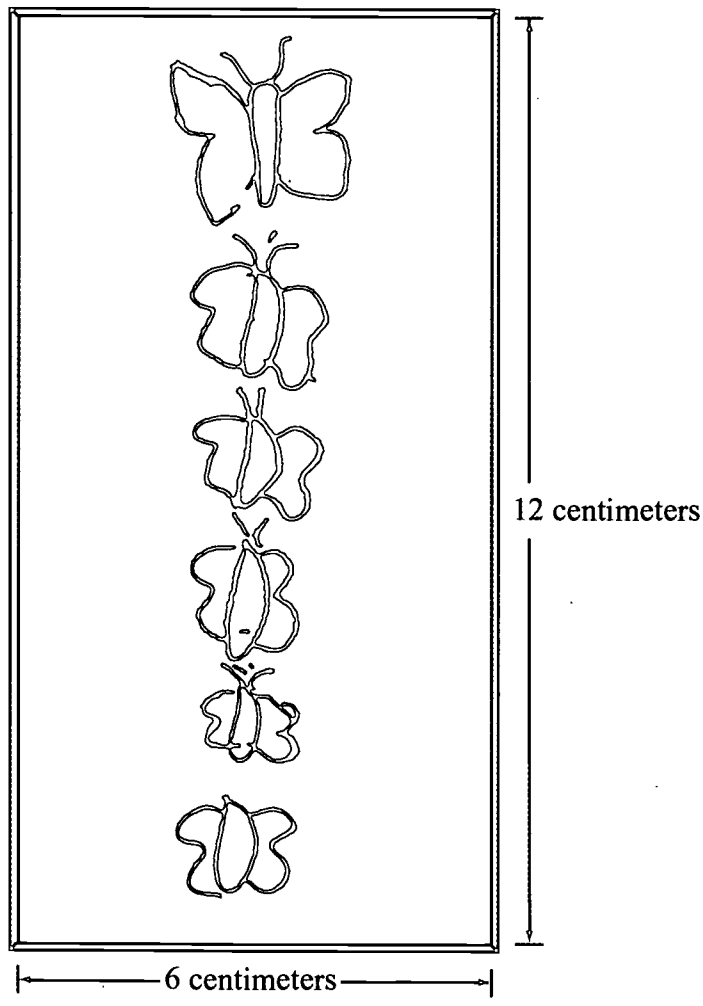
An "incorrect" response is one that meets none of the criteria previously mentioned. In particular, it provides no answer for the number of butterflies or an answer other than 10, 11, or 12. The following "incorrect" response indicates that the greatest number of butterflies that would fit in the case is 6, and that 34 storage cases would be necessary to fit 28 butterflies. Although the answer 6 is incorrect, the drawing for Part 1 shows some understanding of the task of explaining how the butterflies would lie in the case. The answers to the second part of the question appear to indicate that the student did not understand at all what the question was asking.

Sample "incorrect" response

Answer: 6

Show how the butterflies fit in the case.

Storage Case



Sample "incorrect" response (continued)

How many storage cases would you need to store 28 Common Blue butterflies?

Answer: 34

Use drawings, words, or numbers to explain how you got your answer.

IF There are 6 butterflies in one storage case add and 28 to get 34

Student performance information for this question is presented in Table 3.6. Students appear to have found this question difficult compared to the previous measurement question. Only a third of the responses were scored "partial" or higher, about a third were scored "minimal," and just under a third of the responses were "incorrect." Despite the apparent difficulty students encountered with this question, only one percent of the students chose not to attempt to answer it. It is perhaps both the multistep feature and the need to provide an explanation of the process of getting to an answer that accounted for much of the difficulty students had with this question.

Table 3.6 — *Score Percentages for "Solve Packing Problems," Grade 4*

THE NATION'S REPORT CARD naep

Grade 4	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	4	13	16	35	30	1
Males	4	12	14	36	34	1
Females	5	14	18	36	27	0
White	6	16	18	34	25	1
Black	2	4	8	34	50	2
Hispanic	1	6	12	42	39	0!
Asian/Pacific Islander	2	18	20	36	22	2!
American Indian	***	***	***	***	***	***

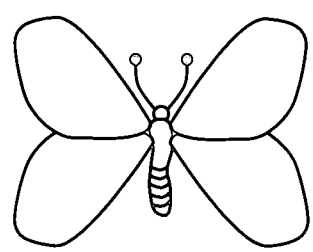
NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 4. Determine number of models. The fourth question explained that students who visit the booth would be building models of butterflies. Students were told how many of each part of the butterfly — 4 wings, 1 body, and 2 antennae — were needed to build one butterfly. They were asked to determine how many complete butterflies could be built with the supply of 29 wings, 8 bodies, and 13 antennae that the class had. Students also were asked to explain how they arrived at their answer using drawings, words, or numbers. This question was designed to assess Problem-Solving ability and content from the area of Number Sense, Properties, and Operations.

To answer this question correctly, students needed to understand that, after building all possible complete butterflies, they might have remaining wings, bodies, and/or antennae. Their task, actually, was to determine for each part how many models of butterflies they could accommodate. That is, they had enough wings for 7 butterflies, enough bodies for 8 butterflies, and enough antennae for 6 butterflies. So, students had to know that the answer they were seeking was the smallest of those three numbers, namely, 6.

4. The children who visit your booth are going to build models of butterflies. For each model, they will need the following:

When the model is put together it looks like this:



If the class has a supply of 29 wings, 8 bodies, and 13 antennae, how many complete butterfly models can be made?

Answer: _____

Use drawings, words, or numbers to explain how you got your answer.

This question was scored using a 4-point rubric: "satisfactory," "partial," "minimal," and "incorrect." "Satisfactory" responses had the correct number of butterflies, 6, and a correct explanation for how that number was determined. The sample "satisfactory" response shown had a correct answer and a complete explanation.

Sample "satisfactory" response

Answer: 6

Use drawings, words, or numbers to explain how you got your answer.

4 wings for each model.
4)29 with one leftover, There
is eight bodies so you'll have
1 left over but there is only
13 antennae, so you can only make
6

Responses scored as "partial" either had the correct answer but an incomplete or erroneous explanation or had a correct explanation with the six-and-a-half pairs of antennae rounded to 7 butterfly models. The "partial" answer that follows had the correct answer 6 but is lacking an explanation.

Sample "partial" response

Answer: 6

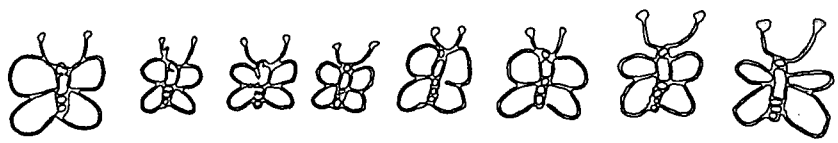
Use drawings, words, or numbers to explain how you got your answer.

Responses scored as "minimal" had an incorrect number of butterfly models but provided some evidence that the student understood that the number of parts (wings, bodies, and antennae) available determined the number of complete butterfly models that could be built. The following sample "minimal" response has the wrong answer, 8, but the drawing of the butterflies shows an understanding of how the different parts are needed to build a complete butterfly model.

Sample "minimal" response

Answer: 8

Use drawings, words, or numbers to explain how you got your answer.



An "incorrect" response showed no understanding of how to solve the problem. For example, the following "incorrect" response shows that the student simply added the various available butterfly parts and came up with the sum, 50, which is the wrong number of complete models possible.

Sample "incorrect" response

Answer: 50

Use drawings, words, or numbers to explain how you got your answer.

$$\begin{array}{r} 2 \\ 29 \\ +\ 8 \\ 13 \\ \hline 50 \end{array}$$

Student performance information on this question is presented in Table 3.7. Eighteen percent of the students were able to provide a response that received at least a "partial" score. Such responses appear to indicate that the student had some idea of how to solve the problem correctly. However, the majority of responses were scored as "incorrect," indicating that many students found this question very difficult.

Table 3.7 — *Score Percentages for "Determine Number of Models," Grade 4*

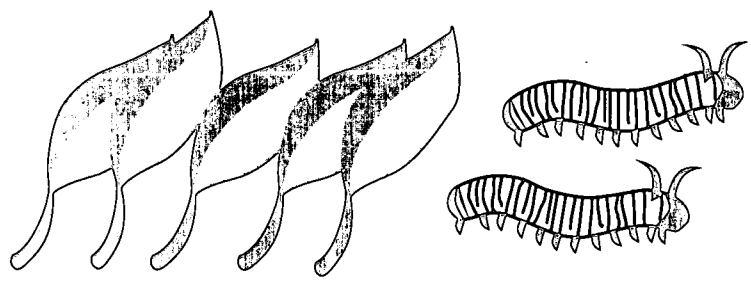
THE NATION'S REPORT CARD naep

Grade 4	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	3	15	18	61	2
Males	2	15	15	64	3
Females	3	15	21	59	2
White	3	19	21	55	2
Black	1	5	9	81	3
Hispanic	1	8	11	75	3
Asian/Pacific Islander	6	22	17	51	4
American Indian	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 5. Determine number of leaves. The fifth question used the context of the Butterfly Booth indirectly; that is, the question is about feeding caterpillars. Students could have answered this question correctly even if they did not know that butterflies are transformed caterpillars, but having that knowledge might have made the problem more interesting. In the question, students were told that a class has two caterpillars and needs five leaves a day to feed them. Students were asked to determine how many leaves the class would need each day if it had 12 caterpillars to feed. This question was classified as a Number Sense, Properties, and Operations question and was designed to assess Problem-Solving ability. As with other questions, in addition to specifying the number of leaves needed, students were asked to explain how they determined their answer.

There were a number of strategies that students might have used to answer this question. For example, students may have reasoned that 12 is 6 groups of 2, so 6 times 5 leaves, or 30 leaves are needed.



5. A fourth-grade class needs 5 leaves each day to feed its 2 caterpillars. How many leaves would they need each day for 12 caterpillars?

   Answer:_____

   Use drawings, words, or numbers to show how you got your answer.

This question was scored on a 3-point rubric: "complete," "partial," and "incorrect." Responses were scored "complete" if they had the right number of leaves, 30, and a correct explanation. The "complete" response that follows has the correct answer and an adequate computational explanation. Through the computations, the student appears to show an understanding that the number of caterpillars and the number of leaves needed to be multiplied by the same number, 6.

Sample "complete" response

Answer: 30 leaves

Use drawings, words, or numbers to show how you got your answer.

$$\begin{array}{r} 2 \\ \times 6 \\ \hline 12 \end{array} \qquad \begin{array}{r} 6 \\ \times 5 \\ \hline 30 \end{array}$$

"Partial" responses either had the correct number of leaves with an incomplete, erroneous, or no explanation or a correct explanation with a wrong number of leaves because of a computational error. In the sample response below, the student has the correct number of leaves, 30, but the explanation is incomplete. The student started with an acceptable process of determining the number of leaves one caterpillar needs a day, that is, two-and-a-half, but then jumped to the conclusion of needing 30 leaves without explaining that 30 leaves is the answer to two-and-a-half leaves per caterpillar multiplied by 12 caterpillars.

Sample "partial" response

Answer: 30

Use drawings, words, or numbers to show how you got your answer.

If each caterpillar gets 2½ leaves you need 30 leaves

"Incorrect" responses had the wrong number of leaves and either an erroneous or no explanation. In the sample "incorrect" response shown below, the student forgot that the five leaves were for two caterpillars instead of one. The process explained was an acceptable process. It would have resulted in the correct answer had the student remembered that the five leaves were for a pair of caterpillars and therefore only counted by fives 6 times rather than 12.

Sample "incorrect" response

Answer: 60

Use drawings, words, or numbers to show how you got your answer.

I counted by fives up to 12 and I got 60.

Student performance information on this question is presented in Table 3.8. It proved to be a very difficult question for students; 86 percent of their responses were scored as "incorrect." Despite the fact that students had difficulty determining the correct answer, most students attempted to answer the question.

Table 3.8 *Score Percentages for "Determine Number of Leaves," Grade 4*

THE NATION'S REPORT CARD naep

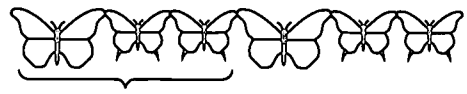
Grade 4	Complete	Partial	Incorrect	Omit
All Students	6	7	86	2
Males	6	7	85	2
Females	5	6	87	1
White	7	8	84	1
Black	2	3	92	3
Hispanic	3	2	93	2
Asian/Pacific Islander	11	8	75	6
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 6. Interpret pattern of figures. The last question in the Butterfly Booth block was about making a banner for the booth. Students were told that they should use the Butterfly Information Sheet to help them solve this problem. They were told the length of the banner, 130 centimeters, and the design on the banner; that is, a repeating pattern of one Monarch butterfly followed by two Black Swallowtail butterflies with wings touching but not overlapping. Students were asked to determine how many Monarch and Black Swallowtail butterflies would be needed to fill the banner. This question was classified as an Algebra and Functions question and was designed to assess Problem-Solving ability. As with other questions, students were asked to explain how they got their answer.

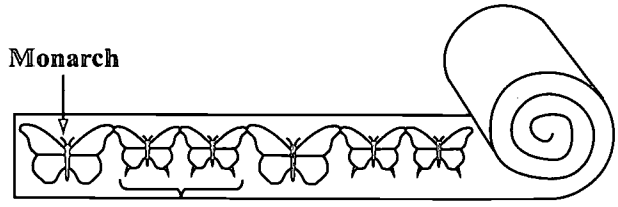
6. **Use the Butterfly Information Sheet and your answer from question 2 to solve this question.**

Your class has decided to have a banner that will be 130 centimeters long. This banner will have a repeating pattern of one Monarch butterfly followed by two Black Swallowtail butterflies, as shown here.



This part keeps repeating across the banner.

The butterflies will just touch but will not overlap.



How many of each type of butterfly are needed for the banner?

Monarch _____

Black Swallowtail _____

Show how you got your answers.

This was a relatively complex question. Students first had to determine which of all the information available was needed to solve the problem and, second, to determine a multistep process for arriving at the number of Monarchs and Black Swallowtails they needed to complete the banner. As with Question 2, students who previously had conducted measurements using centimeters may have had an advantage in solving this problem. The correct response was 6 Monarch and 10 Black Swallowtail butterflies and an adequate explanation of the process for obtaining that response.

Although there are other strategies that students could have used to solve the problem, the following is one possible strategy:

1. Measure in centimeters the wingspan of the Monarch butterfly and the Black Swallowtail butterfly — 10 centimeters and 7 centimeters.
2. Add the wingspan measurements of one Monarch and two Black Swallowtails — 24 centimeters.
3. Divide 24 centimeters into 130 centimeters, the length of the banner, to get the number of patterns needed to cover the banner — five patterns.
4. Realize that there was 10 centimeters of banner remaining — enough to fit Monarch butterfly, but no additional Black Swallowtails.

The responses to this question were scored on a 4-point rubric: "satisfactory," "partial," "minimal," and "incorrect." A "satisfactory" response had the correct number of Monarchs and Black Swallowtails and an adequate explanation of how the student arrived at those answers. The sample "satisfactory" response provides the correct answer and provides, as the explanation, a drawing of the how the butterflies (indicated with an "M" for Monarchs and a "W" for Black Swallowtails) would be positioned on the banner with the correct number of centimeters (for one Monarch and two Black Swallowtails) indicated below these letters.

Sample "satisfactory" response

How many of each type of butterfly are needed for the banner?

Monarch 6

Black Swallowtail 10

Show how you got your answers.

M WW M WW M WW M WW M WW M
10 14 10 14 10 14 10 14 10 14 10

"Partial" responses either had the correct number of Monarchs and Black Swallowtails but no explanation or an incomplete explanation, or had the correct strategy explained with the correct number of patterns but the wrong number of Monarchs and Black Swallowtails. The "partial" response shown had the wrong number of butterflies, but provided an explanation that showed an appropriate strategy for solving the problem. In the explanation, the student showed knowledge of the number of centimeters needed for the pattern, 24; had a counting strategy to get to the number of patterns; and understood that the banner could have an incomplete pattern at the end. Although it is not completely clear, it appears that the student made a mathematical calculation error while counting by 24 and came up with four repeating patterns with centimeters left over for the Monarch but no other Black Swallowtails.

Sample "partial" response

How many of each type of butterfly are needed for the banner?

Monarch 5

Black Swallowtail 8

Show how you got your answers.

Both of the wingspans on one set was 24 and I just kept adding and adding it together and counting as I go to get the answer of 5 Monarchs and 8 Black Swallowtail.

Responses that had any of the following pairs of numbers for Monarchs and Black Swallowtails with no or an inadequate explanation were scored as "minimal": 4 and 8, 5 and 8, 5 and 10, 6 and 12, or 7 and 12. These responses, which were classified as minimally acceptable, indicated measurement or computational error but showed that the student had some understanding of how to solve the problem. A sample "minimal" response follows. The student had a minimally acceptable number of repeating patterns, but did not provide any explanation and did not appear to take into consideration the remaining centimeters that could accommodate an additional Monarch butterfly.

Sample "minimal" response

How many of each type of butterfly are needed for the banner?

Monarch 4

Black Swallowtail 8

Show how you got your answers.

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"Incorrect" responses involved pairs of numbers other than those mentioned above or had missing numbers. The following "incorrect" response shows an understanding of some aspects of the problem, but the student clearly did not understand that it was necessary to take into account the wingspans of all three butterflies in the pattern. Although it is not possible to know with certainty from the response how the student went about solving the problem, it appears the student took the wingspan measurement of the Monarch butterfly, 10 centimeters, and determined that 13 such butterflies could fit on a 130-centimeter banner by dividing 130 by 10. Then, while not attending to the fact that the Black Swallowtails also have wingspans that take up space, the student laid out the repeating patterns with the 13 Monarchs. It is interesting, however, that the student did not add the two Black Swallowtails after the thirteenth Monarch butterfly. Therefore, the total number of Black Swallowtails summed to 24.

Sample "incorrect" response

How many of each type of butterfly are needed for the banner?

Monarch 13

Black Swallowtail 24

Show how you got your answers.

M BS BS M BS BS M BSBS M BS BS M BS BS
M BS BS M BS BS M BSBS M BS BS M BS BS
M BS BS M BSBSM

Student performance data on this last question in the Butterfly Booth block are presented in Table 3.9. Students apparently found this question very difficult to solve: 90 percent of the responses were scored as "incorrect."

Table 3.9 *Score Percentages for "Interpret Pattern of Figures," Grade 4*

THE NATION'S REPORT CARD naep

Grade 4	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	1	3	4	90	1
Males	1	2	5	90	1
Females	1	4	4	89	1
White	2	4	4	89	1
Black	0!	1	5	93	1
Hispanic	0!	1	4	91	1
Asian/Pacific Islander	0!	1!	9	87	3
American Indian	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Grade 8

Student characteristics

Eighth-grade students who participated in the Theme Study were similar to eighth-grade students in the main NAEP assessment in terms of a variety of demographic characteristics. (See Table 3.10.) Students in grade 8 also were similar to students in grade 4. For example, there were similar percentages of male and female students; White students were in the majority; the modal response regarding parents' highest level of education was "graduated from college." In addition, about 90 percent of the students were from public schools; the large majority were not Title I students; and just over a quarter of the students were eligible for the federal Free/Reduced-Price Lunch program.

Table 3.10

Student Demographic Distributions by Assessment, Grade 8, 1996

THE NATION'S REPORT CARD naep

	Percentage of Students		
	Main Assessment	Theme Block 1 Building a Doghouse	Theme Block 2 Flooding
Grade 8			
Gender			
Males	52	53	52
Females	48	47	48
Race/Ethnicity			
White	69	70	70
Black	14	14	14
Hispanic	12	12	12
Asian/Pacific Islander	3	2	2
American Indian	1	2	1
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	7	8	8
Graduated From High School	22	23	25
Some Education After High School	19	18	18
Graduated From College	42	42	39
I Don't Know	11	10	9
Students who Attend...			
Public Schools	89	90	91
Nonpublic Schools	11	10	9
Title I Participation...			
Participated	12	11	10
Did Not Participate	88	89	90
Free/Reduced-Price Lunch Program Eligibility...			
Eligible	27	25	26
Not Eligible	55	54	54
Information Not Available	17	21	21

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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The data in Table 3.11 show similar levels of exposure across the three samples of eighth-grade students to the classroom practice of writing a few sentences about how to solve a mathematics problem. Similarly, exposure to writing reports or doing mathematics projects did not vary significantly across the three samples of eighth-grade students.

Table 3.11 — ***Percentage of Students by Teachers' Reports on Classroom Practices, Grade 8, 1996***

THE NATION'S REPORT CARD naep

	Percentage of Students		
	Main Assessment	Theme Block 1 Building a Doghouse	Theme Block 2 Flooding
Grade 8			
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem...			
Nearly Every Day	5	5	4
Once or Twice a Week	25	19	21
Once or Twice a Month	37	40	36
Never or Hardly Ever	33	37	38
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project...			
Nearly Every Day	0!	0!	0!
Once or Twice a Week	3	4	4
Once or Twice a Month	33	34	31
Never or Hardly Ever	64	62	65

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

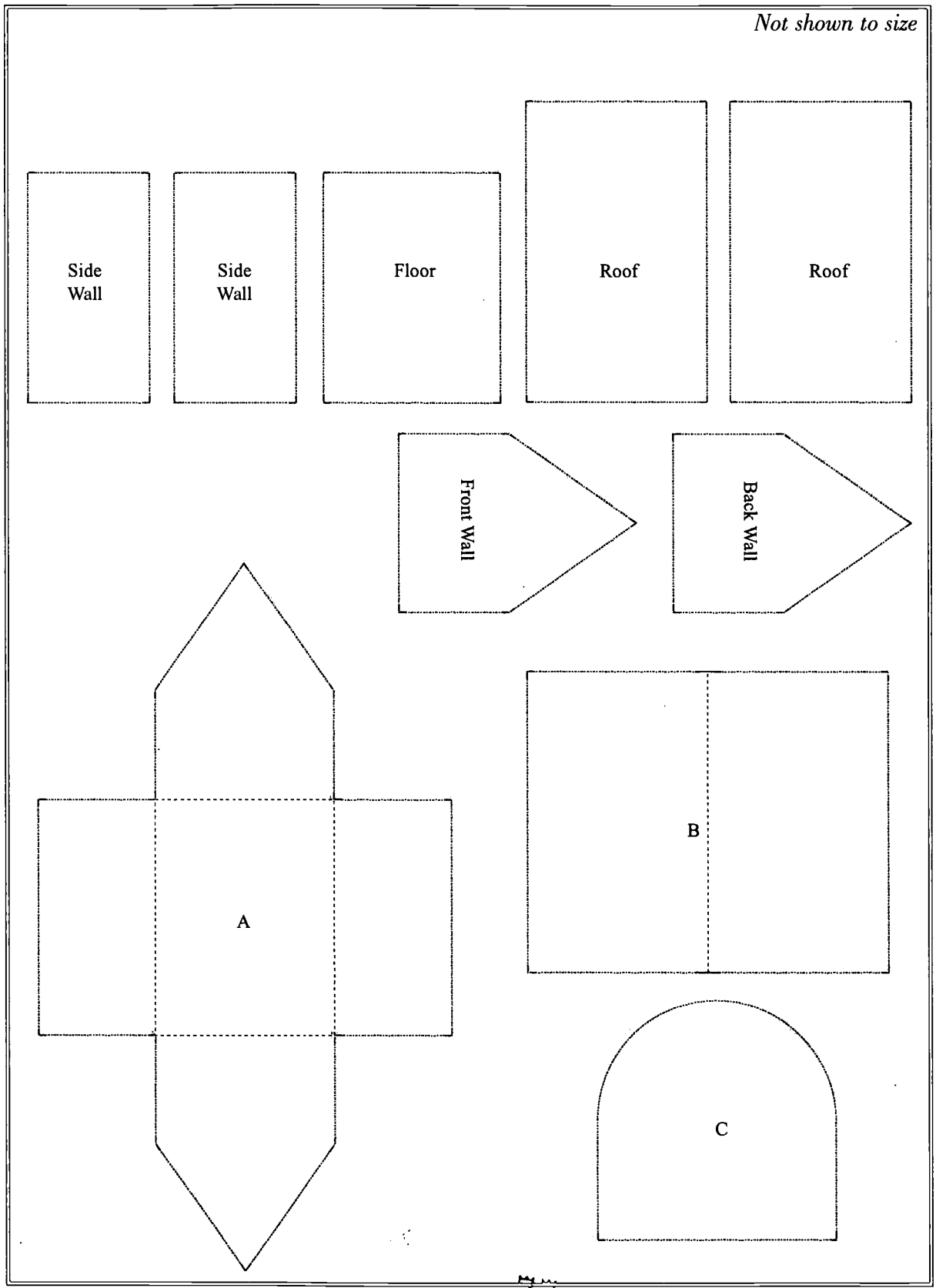
Content of the Theme blocks

The two blocks of questions for the eighth-grade Theme Study also had interesting and relevant contexts. The released block involved building a doghouse; the unreleased block was based on the flooding of the Mississippi River in the summer of 1993. Some eighth-grade students may have pets for which they have considered building a house. With regard to the Theme block on the flooding in the summer of 1993, most eighth-grade students had just completed their fifth-grade year at that time and may have been exposed to media attention about the floods. The flooding of the Mississippi River also was the context of the unreleased Theme block for grade 12. Seven of the 11 questions in that block were given to both eighth- and twelfth-grade students.

Students taking the Doghouse block of questions were given a sheet of push-outs representing parts of a doghouse. A picture of the sheet follows. The sheet included a set of seven push-outs that were parts of the doghouse — two roof pieces, two side walls, a front wall, a back wall, and a floor; a set of two push-outs (indicated with an "A" and a "B") that when folded together formed a model of the doghouse; and a separate push-out (indicated with a "C") that represented the door opening of the doghouse. Students were also provided with a ruler/protractor and a calculator.

Sheet of Push-Outs

Not shown to size



Both Theme blocks included multiple-choice and constructed-response questions. The Doghouse block included four multiple-choice and six constructed-response questions, whereas the Flooding-of-the-Mississippi block included four multiple-choice and seven constructed-response questions. In two of the questions in the Doghouse block, students were asked to show work that supported their answers. In the Flooding block, students were asked to explain their answers in five of the questions.

Overall student performance

Students' overall performance on the two blocks is presented in Table 3.12. The average percentage correct score was 41 percent for the Doghouse block and 30 percent for the Flooding block. In the Doghouse block, female students had a significantly higher percentage correct score than male students. White, Hispanic, and Asian/Pacific Islander students outperformed Black students, and White and Asian/Pacific Islander students also outperformed Hispanic students. For the Flooding block, male and female students performed similarly, while the pattern of performance by racial/ethnic groups was the same as noted on the Doghouse block. The frequency with which students engaged in the two classroom practices highlighted in this chapter was not found to be related to student performance. The sample of American Indian students for both Theme blocks was too small to permit reliable estimates of their performance on either the blocks as a whole or on individual questions. Therefore, the performance of American Indian students is not discussed.

Table 3.12

Average Percentage Correct Scores by Theme Block, Grade 8, 1996

THE NATION'S REPORT CARD naep

Grade 8	Build a Doghouse	Flooding
All Students	41	30
Gender		
Males	39	31
Females	43	30
Race/Ethnicity		
White	45	34
Black	27	18
Hispanic	33	22
Asian/Pacific Islander	43	35
American Indian	***	***
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem...		
Nearly Every Day	42	29
Once or Twice a Week	44	32
Once or Twice a Month	40	30
Never or Hardly Ever	42	31
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project...		
Nearly Every Day	***	***
Once or Twice a Week	39	30
Once or Twice a Month	42	32
Never or Hardly Ever	42	30

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Building a doghouse

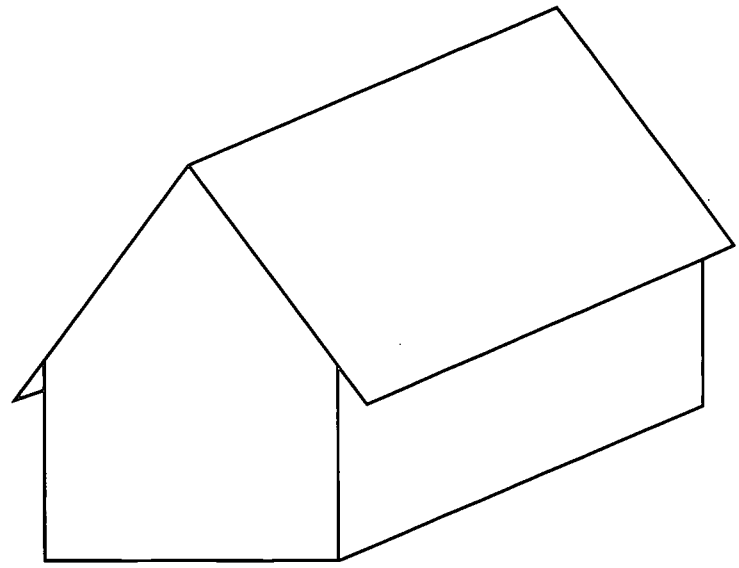
The introduction to the Doghouse block is shown in Figure 3.2. The instructions clarified what was expected of students with regard to responses. The description of the context for the block of questions brings the students directly into the task by telling them that Julie would like their help in building a doghouse. Students were asked first to put together a model of the doghouse using pieces "A" and "B" and following the directions given. The results of these efforts were not, however, collected or scored.

Figure 3.2

Introduction to the "Building a Doghouse" Theme Block, Grade 8, 1996

THE NATION'S REPORT CARD

This part has 10 questions. Mark your answers in your booklet. You will have to fill in an oval or write your answer as directed. In those questions where you must write an answer, it is important that your answer be clear and complete and that you show all of your work since partial credit may be awarded. The last two questions may each require 5 minutes or more to think about and answer. After each question, fill in the oval to indicate whether you used the calculator. If you are asked to round your answer, do not round any numbers except your final answer.



Julie wants to build a doghouse like the one shown in the picture above. She has asked you to help her build the doghouse.

The kit you have been given contains a model of a doghouse like the one Julie wants to build. Please put the model together now by following these instructions.

1. Separate pieces A and B from the paper. **Do not separate any other pieces from this paper until you are told to do so.**

2. Fold up the four walls on piece A so that they form right angles with the rectangular floor.

3. Fold the roof (piece B) in half, and set it on top of the house. The edges of the roof will extend slightly beyond the walls.

   Note: When Julie builds the house, the roof will be made up of two identical pieces of wood, since wood cannot be folded the way you folded the piece of paper just now to make the roof.

You may also use your calculator and ruler/protractor to help answer the questions in this part.

Question 1. Identifying needed information. The first actual question the students encountered asked them to consider whether each of five different measurements would help Julie determine whether the finished doghouse will be large enough to accommodate her dog. The question was designed to assess content from the Geometry and Spatial Sense content strand and the mathematical ability Conceptual Understanding. Students were asked to reply "yes" or "no" to the utility of each of the measurements, and each response was scored "correct" or "incorrect." Only measurement "d" contained specific mathematical language (i.e., "rectangular").

1. Consider each of the following measurements. Will knowing the measurement help Julie to determine whether the doghouse she plans to build will be large enough for her dog to sleep in and to go in and out of comfortably? (Answer "Yes" or "No" for each part.)

a. The length of the floor	◯ Yes	◯ No
b. The height of the house	◯ Yes	◯ No
c. The weight of the house	◯ Yes	◯ No
d. The width of the rectangular floor	◯ Yes	◯ No
e. The width and height of the door's opening	◯ Yes	◯ No

Student performance information for this question is presented in Tables 3.13 and 3.14. The data in Table 3.13 summarizes the percentage of students by the number of measurements correctly identified as useful in helping Julie determine whether the doghouse would accommodate her dog; Table 3.14 details the percentage correct scores for individual measurements. Students appear to have done relatively well on this question. Fifty-five percent of the students correctly evaluated the utility of each of the five measurements, and 23 percent evaluated four of the five measurements correctly.

Table 3.13

Score Percentages for "Identifying Needed Information," Grade 8

THE NATION'S REPORT CARD naep

Grade 8	5 Correct	4 Correct	3 Correct	2 Correct	1 Correct	0 Correct	Omit
All Students	55	23	9	6	6	1	0
Males	51	24	10	6	7	1	0
Females	60	22	7	6	4	0	0
White	62	20	7	5	5	1	0!
Black	34	30	14	10	10	1	1
Hispanic	35	37	13	7	6	1!	0
Asian/Pacific Islander	73	16	5!	4	1!	1!	0!
American Indian	***	***	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 3.14

Percentages Correct for "Identifying Needed Information," Grade 8

THE NATION'S REPORT CARD naep

Grade 8	1a — Yes	1b — Yes	1c — No	1d — Yes	1e — Yes
All Students	90	86	72	82	83
Males	87	85	69	79	81
Females	92	87	76	86	85
White	91	88	77	85	86
Black	82	77	58	70	74
Hispanic	86	83	61	79	74
Asian/Pacific Islander	98!	94	84	86	90
American Indian	***	***	***	***	***

***Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The data in Table 3.14 show that, in general, students appear quite knowledgeable about the use of each of these measurements: that is, for each measurement more than 70 percent of students were able to assess correctly whether the measurement would help Julie. Students had more difficulty correctly assessing what appeared to be more complicated or more formal measurements. For example, they were better at assessing correctly whether the "length of the floor" or the "height of the house" would be helpful than they were at assessing the usefulness of the "width of the rectangular floor" or the "width and height of the door's opening." Interestingly, students had most difficulty determining whether the "weight of the house" would be a helpful measurement, or they may have been inclined to answer "yes" to measurement "c" because "yes" was the correct answer to each of the other measurements presented.

Question 2. Determine minimum measuring needed. In the second question, students were first given information about the pieces that made up the doghouse — four walls, two roof pieces, and the floor. They were then told that some of the pieces were exactly the same size and shape, and that Julie did not want to measure all of the pieces, if not necessary. Students were presented with the problem of determining the smallest number of individual pieces Julie would need to actually measure in order to have the information she needed to cut out all of the pieces for the doghouse. The question was classified as a Geometry and Spatial Sense question and was designed to assess Conceptual Understanding.

In order to answer this question correctly, students could use either the model of the doghouse or the push-out pieces to determine that there were three matched pairs among the seven pieces that Julie needed to cut; therefore, she only needed to make four unique measurements. The problem was simplified if students realized that they did not need to actually measure any of the pieces themselves.

2. Seven pieces — four walls, two roof pieces, and the floor — make up the doghouse. Since some of the pieces are exactly the same in size and shape, Julie does not need to measure every piece. She can measure and cut a piece and then make identical pieces <u>without measuring</u> by tracing an outline of the cut piece onto the wood and then cutting out the traced shape.

   How many of the seven pieces does Julie need to <u>measure</u> before she cuts?

   (A) Two

   (B) Three

   (C) Four

   (D) Five

   (E) Seven

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Information on student performance on this multiple-choice question is presented in Table 3.15. Forty-two percent of the students were able to correctly identify four pieces as the correct response. The second highest percentage, 24 percent, chose Option B, three pieces. There are a number of reasons that students could have chosen three pieces. This is the answer students would arrive at if they assumed that all the walls were the same size, the two roof pieces were of the same size, and the floor was of a size different from the walls or roof pieces. This would also be the correct response if students assumed that they could use the two adjacent walls to trace out the floor and therefore did not need to measure the floor piece itself.

Table 3.15 — *Percentage Correct for "Determine Minimum Measuring Needed," Grade 8*

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
All Students	42
Males	40
Females	43
White	46
Black	28
Hispanic	32
Asian/Pacific Islander	37
American Indian	***

***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 3. Measure lengths using ruler. The third question asked students to actually measure, in inches, the pieces of the model doghouse that they put together initially. For example, students were told to measure "the longer side of the rectangular floor." This meant that the student had to know what a "rectangular floor" was, determine which was the "longer side," and use the ruler correctly to measure. This question was designed to assess Measurement content and Procedural Knowledge.

3. The model of the doghouse that you put together is a smaller version of the actual house. Measure the following lengths, in inches, on your model and record your results in the spaces below.

Longer side of rectangular floor _____ inches

Shorter side of rectangular floor _____ inches

Height from floor to highest point of roof _____ inches

The responses to this question were scored on a 3-point rubric: "complete," "partial," and "incorrect." As shown below, a "complete" response gave three correct measurements: two inches for the longer side of the rectangular floor, one-and-one-half inches for the shorter side of the rectangular floor, and two inches for the height from the floor to the highest point of the roof.

Sample "complete" response

3. The model of the doghouse that you put together is a smaller version of the actual house. Measure the following lengths, in inches, on your model and record your results in the spaces below.

Longer side of rectangular floor	2	inches
Shorter side of rectangular floor	1.5	inches
Height from floor to highest point of roof	2	inches

A "partial" response provided only one or two correct measurements in inches. In the sample "partial" response shown, it appears that the student may have been careless in conducting the measurements and used the centimeter side of the ruler to measure the longer side of the rectangular floor, arriving at 5 (centimeters rather than inches). The remaining dimensions were measured correctly in inches.

Sample "partial" response

3. The model of the doghouse that you put together is a smaller version of the actual house. Measure the following lengths, in inches, on your model and record your results in the spaces below.

Longer side of rectangular floor	5	inches
Shorter side of rectangular floor	1½	inches
Height from floor to highest point of roof	2	inches

Finally, an "incorrect" response gave no correct measurements in inches. The measurements in the sample "incorrect" response appear to have been measured in centimeters rather than inches. It is not clear if the student did not understand the difference between centimeters and inches or if the student simply was careless and used the wrong side of the ruler to conduct the measurements.

Sample "incorrect" response

3. The model of the doghouse that you put together is a smaller version of the actual house. Measure the following lengths, in inches, on your model and record your results in the spaces below.

Longer side of rectangular floor	5	inches
Shorter side of rectangular floor	4	inches
Height from floor to highest point of roof	5	inches

Student performance information on this question is presented in Table 3.16. The modal score for the responses was "complete" (46%) with similar percentages of responses scored "partial" and "incorrect" (23% and 22%, respectively).

Table 3.16 — **Score Percentages for "Measure Lengths Using Ruler," Grade 8** — THE NATION'S REPORT CARD naep

Grade 8	Complete	Partial	Incorrect	Omit
All Students	46	23	22	8
Males	46	22	23	8
Females	47	24	21	8
White	54	21	18	6
Black	17	31	34	17
Hispanic	36	25	31	8
Asian/Pacific Islander	48	24	25	2!
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Question 4. Apply concept of ratio. The fourth question was prefaced by a scale that showed an inch representing 1½ feet (18 inches). Students were asked to explain how they would use the scale to determine in feet the measurements they obtained in inches in Question 3. Students were also told that they were not required to find the actual measurements; that is, students were simply being asked to provide an explanation of the procedure they would follow to convert measurements in inches to measurements in feet when given the conversion scale. This question was classified as a Number Sense, Properties, and Operations question and was designed to assess Conceptual Understanding, specifically, of the use of ratios.

1 inch represents $1\frac{1}{2}$ feet (18 inches)

4. Explain what you would need to do to each of your measurements in question 3 to find the measurements in feet of the actual house.

   (You do not have to find each of the actual measurements.)

Responses to the question were scored using a 3-point rubric: "complete," "partial," and "incorrect." A "complete" response provided an explanation of the correct procedure for translating the inches into feet using the scale provided. The sample response shown below describes that procedure in one sentence. Although not necessary, it also provides the measurements in feet.

Sample "complete" response

You must multiply the measurments (previous question) by 1.5 to get the # of feet.

1. 3 ft.
2. 2.25 ft.
3. 3 ft.

Responses were scored as "partial" if they met any of the following criteria:

- the correct procedure for finding the measurements in inches rather than feet;
- the correct procedure using one of the measurements from Question 3, but not generalizing to the other measurements;
- the three correct measurements in feet, but no explanation of the procedure; or
- an example of the correct procedure using one of the measurements in Question 3 multiplied by 18, 1.5, or 18/12.

The "partial" response that follows provides a procedure for finding the measurement in inches rather than feet.

Sample "partial" response

> If 1 inch represents 1½ feet then when you measure
> Just change 1 inch to 18 inches.

An "incorrect" response met none of the previous criteria. The following "incorrect" response begins with an attempt to explain how to solve the problem, "To find the actual measurements...," and ends the sentence with the general procedure to convert inches into feet with no reference to use of the scale provided.

Sample "incorrect" response

> To find the actual measurements, you would have to divide the numbers by 12 because there are 12 inches in a foot. My model would be

Student performance data for this question are presented in Table 3.17. Fourteen percent of the students were able to provide "complete" responses; however, 45 percent of students were not able to provide even a partially correct answer. It is not clear whether eighth-grade students do not understand how to use a scale, or whether they cannot explain in words a general mathematical procedure, or both. Nevertheless, there is certainly hesitancy on the part of many students to attempt to respond to a question like this because a fifth of the students chose not to attempt to answer this question at all.

Table 3.17	Score Percentages for "Apply Concept of Ratio," Grade 8	THE NATION'S REPORT CARD NAEP

Grade 8	Complete	Partial	Incorrect	Omit
All Students	14	17	45	21
Males	14	16	44	24
Females	15	19	46	18
White	17	21	44	16
Black	3	6	53	36
Hispanic	8	10	49	31
Asian/Pacific Islander	22	21	40	16
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 5. Understand concept of ratio (I). The fifth question students encountered was a multiple-choice question classified in the Number Sense, Properties, and Operations content strand that was designed to assess Conceptual Understanding, specifically of the use of scales and ratios. The question asked students to select, from among five options, the scale that would produce the largest doghouse. There are a number of different strategies students could use to solve the problem and thereby select the correct option. One strategy, for example, would involve referencing all of the scales to the same number of inches; that is, convert all of the scales so that each indicates the number of feet represented by one inch. The scale with the largest number of feet is then the correct response. Some students also could have been able immediately to see ways to eliminate certain options before doing any computations.
For example, some students may have seen that Option D would produce a larger doghouse than Option E, and that Option B would produce a larger doghouse than Option C, and therefore eliminated Options E and C at the outset.

5. Of the following scales, which one would produce the largest doghouse?

Ⓐ 2 inches on model represents 5 feet on actual house.

Ⓑ 1 inch on model represents 3 feet on actual house.

Ⓒ 1 inch on model represents $1\frac{1}{2}$ feet on actual house.

Ⓓ $\frac{1}{2}$ inch on model represents 1 foot on actual house.

Ⓔ $\frac{1}{2}$ inch on model represents $\frac{3}{4}$ foot on actual house.

Student performance data on this question are presented in Table 3.18. Just over a third of the students were able to select the correct scale that would produce the largest doghouse, Option B. Forty-seven percent of students selected Option A. From this high percentage, it appears that many students did not know how to use scales, and consequently, used other reasoning in selecting their answer. Option A has both the largest number of inches and the largest number of feet, so it appears that many students may have simply selected that option because it had the largest numbers. It is not clear, however, from students' choices, whether they have little or no understanding of scales or simply do not know how to compare differing ratios.

Table 3.18 — ***Percentage Correct for "Understand Concept of Ratio (I)," Grade 8*** — THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
All Students	35
Males	39
Females	31
White	38
Black	21
Hispanic	27
Asian/Pacific Islander	42
American Indian	***

***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 6. Understand concept of ratio (II). In the sixth question, students were provided with a scale they were to use in answering the question and were asked to indicate, based on the scale provided, how much taller the actual doghouse would be compared to the model. This question also was designed to assess Conceptual Understanding, specifically the use of scales and ratios, and content from the area of Number Sense, Properties, and Operations.

1 inch represents $1\frac{1}{2}$ feet (18 inches)

6. The height of the actual house will be how many times as tall as the height of the model?

   (A) $1\frac{1}{2}$

   (B) 9

   (C) 18

   (D) 24

   (E) 27

If, in answering the question, students understood the use of scales and also realized that they should be comparing inches to inches rather than converting the height of the actual house to feet, the correct response, Option C, would be straightforward. That is, they would see that an inch on the model represents 18 inches of the actual doghouse, and that therefore the actual doghouse would be 18 times as tall as the height of the model. Student performance data on this question are presented in Table 3.19. Thirty-five percent of the students were able to get the correct answer. The next highest percentage of students, 26 percent, selected Option A, which specified that the actual doghouse would be one-and-a-half times as tall as the model. It may be that this option attracted students who did not think about the metric in which the height was being measured, or they may have been drawn to it because of the repeated number (1½).

Table 3.19	Percentage Correct for "Understand Concept of Ratio (II)," Grade 8	THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
All Students	35
Males	34
Females	35
White	38
Black	24
Hispanic	30
Asian/Pacific Islander	39
American Indian	***

***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

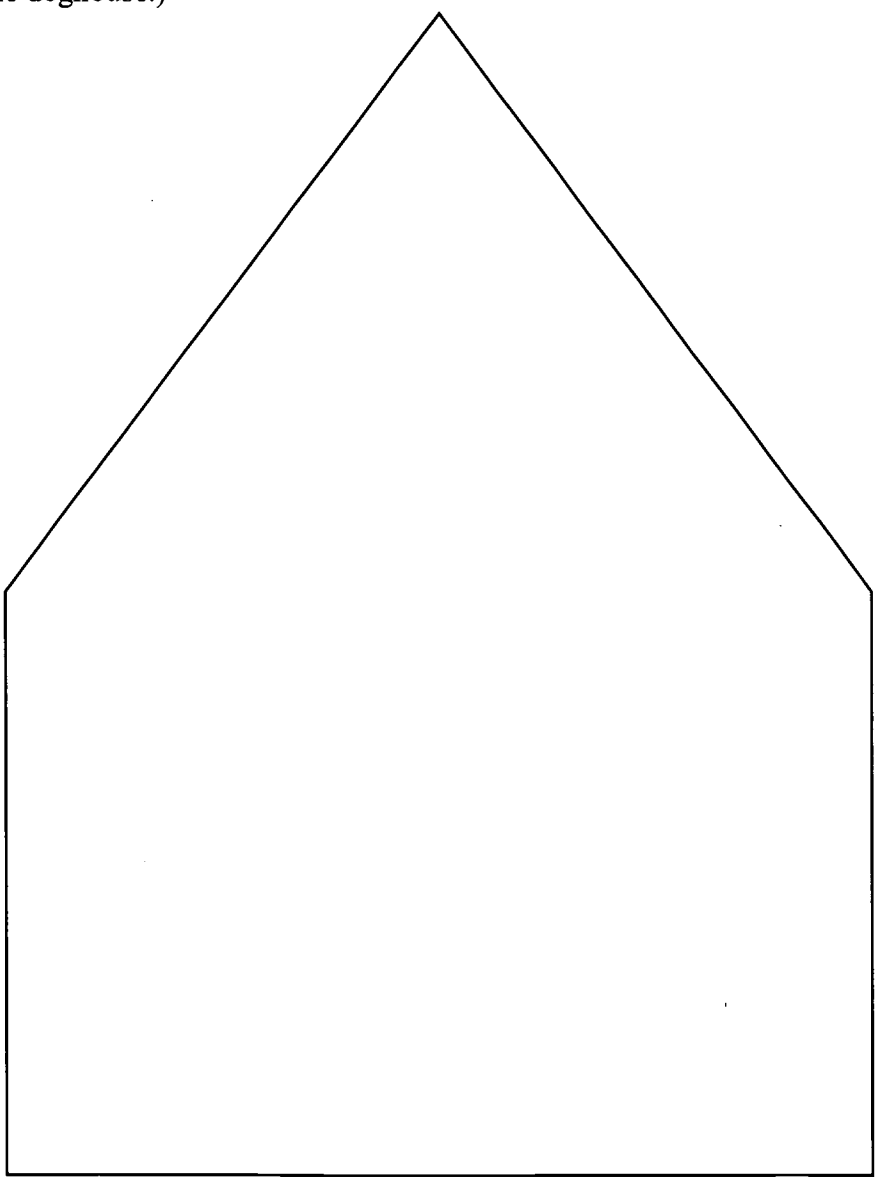
Question 7. Correctly position door. The seventh question required students to trace the correct position of the door opening onto a drawing of the front wall of the doghouse, using push-out piece "C" as a template. Students were told how to position the door, using the measurements of the actual doghouse, and given a conversion scale for the model pieces. In addition, they were cautioned that this conversion scale was not the same as the one used in the previous question. This question was classified as a Geometry and Spatial Sense question and was designed to assess Problem-Solving ability.

7. **You will now need piece C to answer this question. Separate piece C from the paper.**

2 inches represents 1 foot (12 inches)

Piece C represents a scale model of the door for the doghouse. The front wall of the doghouse, shown below, as well as piece C has been drawn to a different scale than the one used in the previous question. The scale is 2 inches represents 1 foot.

On the drawing below, use piece C to locate the door on the wall so that it will be $\frac{1}{2}$ foot above the floor level of the doghouse (to keep the water out) and centered exactly between the vertical edges of the wall. When you have correctly positioned the door, trace its location on the drawing. (Disregard the thickness of the wood that will be used to build the doghouse.)

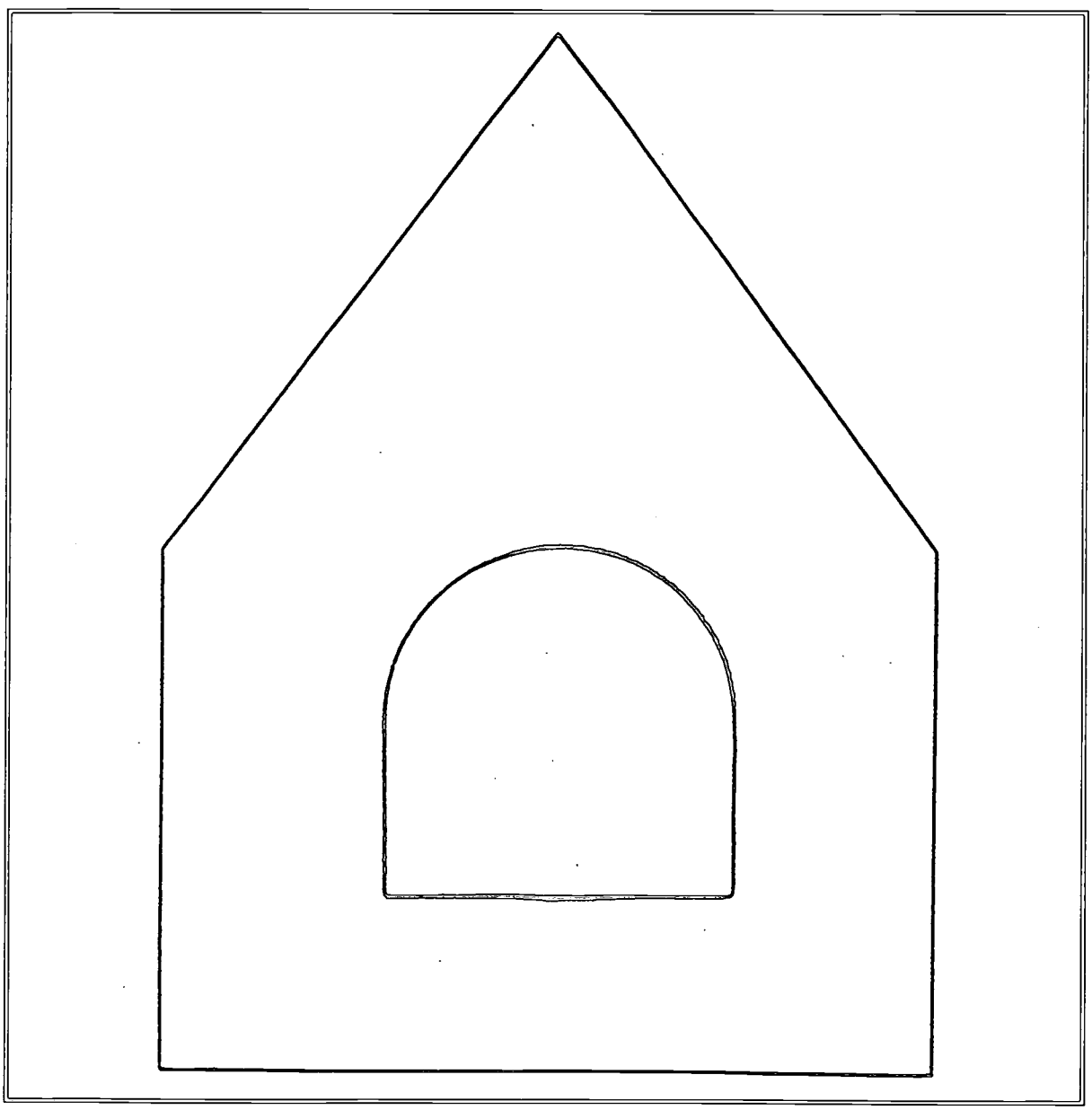


Floor

In order to solve this problem correctly, students had to convert half-a-foot above the floor level to one inch above the floor line on the drawing of the front wall. There were several strategies students could use to center the door opening. One of the simplest is to measure both the width of the front wall and the width of the door opening, find the centers of both, and align the centers. Finally, the student had to be able to trace the door opening in the correct position.

The responses to this question were scored on a 3-point rubric: "complete," "partial," and "incorrect." "Complete" responses had the bottom of the door opening positioned between and including 0.906 to 1.064 inches above the floor and the sides of the door opening positioned between and including 1.142 to 1.314 inches from the side edges of the front wall. A "complete" response follows.

Sample "complete" response



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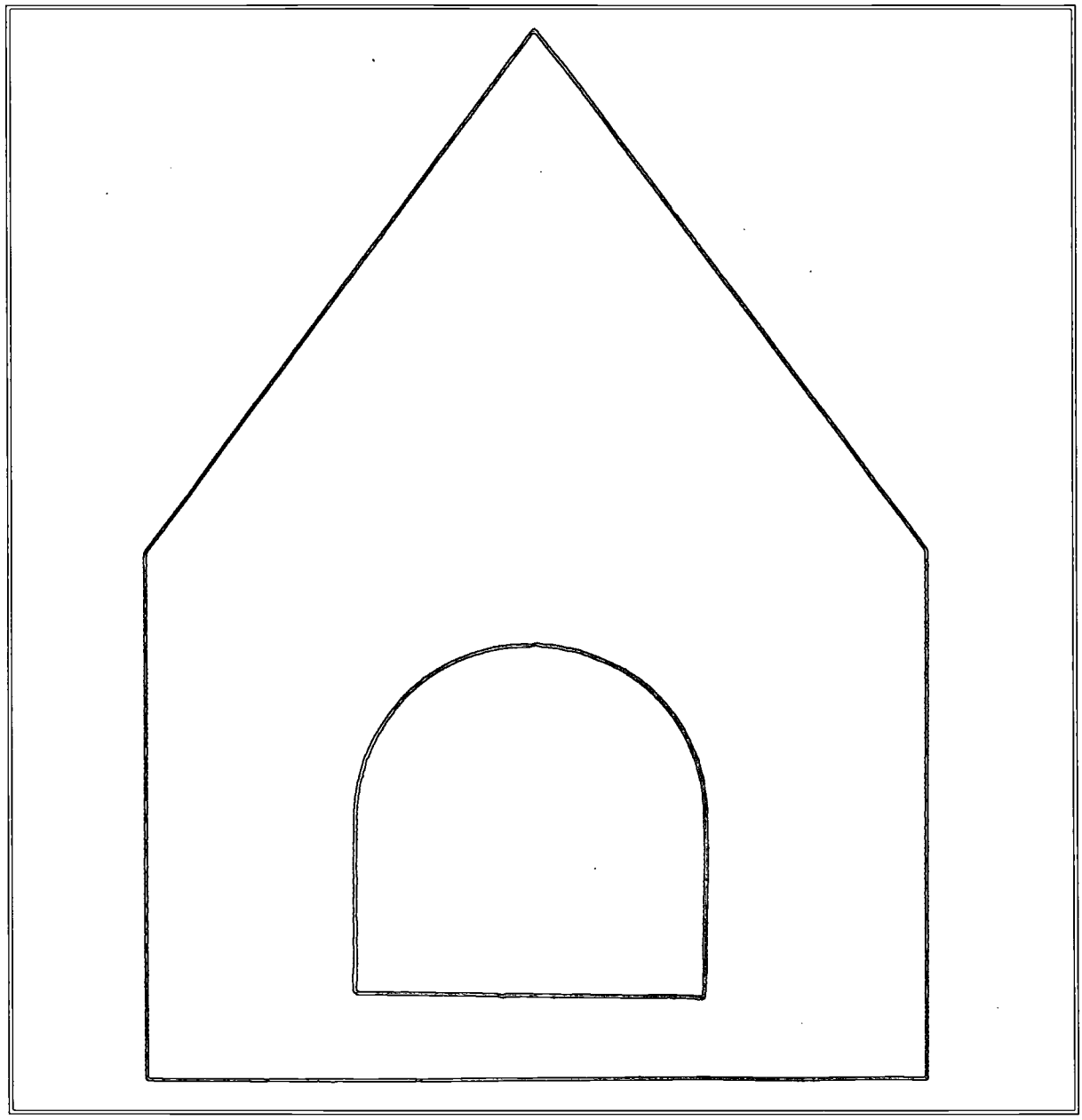
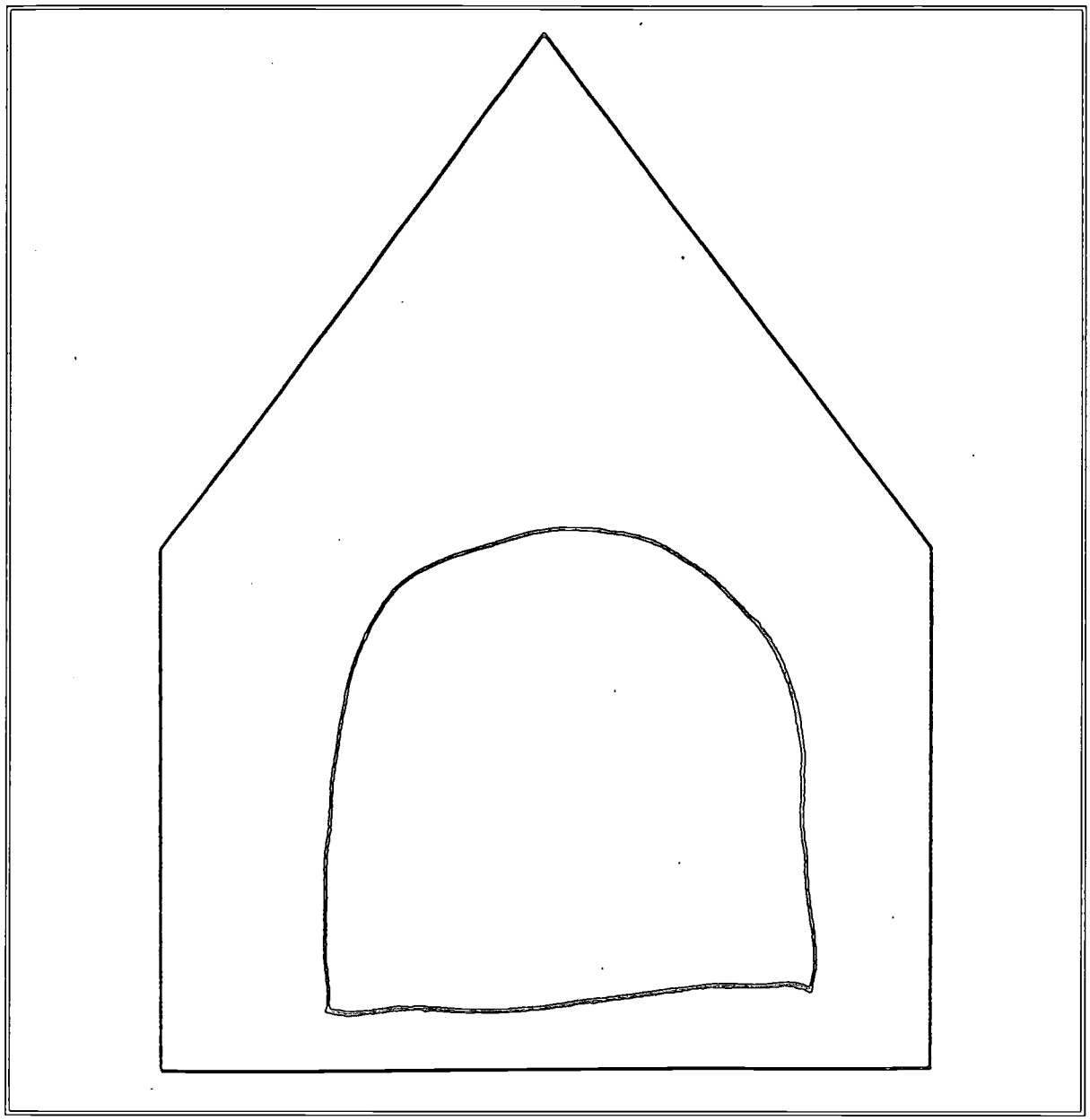
A "partial" response either had the door centered correctly but not located an inch above floor level, or located an inch above the floor level but not centered correctly. The door opening in the "partial" response shown below is correctly centered but only half an inch above the floor of the doghouse. Although the exact reason for this mistake is not clear, the student appears to have forgotten about the scale and instead erroneously translated the half-a-foot instruction into half an inch on the drawing.

Sample "partial" response



Responses scored as "incorrect" met none of the criteria mentioned above. In the following "incorrect" response, the door opening appears to be a free-hand drawing that is larger than the push-out, less than an inch above the floor, and not centered properly. If the drawing shown was actually traced, as required by the question, the tracing was rather imprecise.

Sample "incorrect" response



Information on how students performed on this question is presented in Table 3.20. Almost a fifth of the responses were scored as "complete," and 44 percent were scored as "partial."

Table 3.20 — ***Score Percentages for "Correctly Position Door," Grade 8***

THE NATION'S REPORT CARD naep

Grade 8	Complete	Partial	Incorrect	Omit
All Students	19	44	26	10
Males	16	42	29	13
Females	21	46	24	8
White	23	45	25	7
Black	5	40	31	24
Hispanic	11	43	30	16
Asian/Pacific Islander	14	40	32	15
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 8. Visualize cut-outs on grid. The eighth question that students encountered was classified as a Problem-Solving question in the Measurement content strand. Students were instructed to use the seven pieces of the doghouse — the four walls, two roof pieces, and the floor — to help answer the question. They were presented with a scale, told that the plywood sheets from which doghouse pieces would be cut were four feet wide by eight feet long, and provided with drawings of three plywood sheets. (Only one piece is shown here.) The students were further told that the pieces of the doghouse and the representations of the plywood sheet were all drawn to the same scale. Students were asked to trace the doghouse pieces onto the drawings of the plywood sheet in order to demonstrate the fewest number of plywood sheets needed to cut out all seven pieces of the doghouse.

Not shown to size

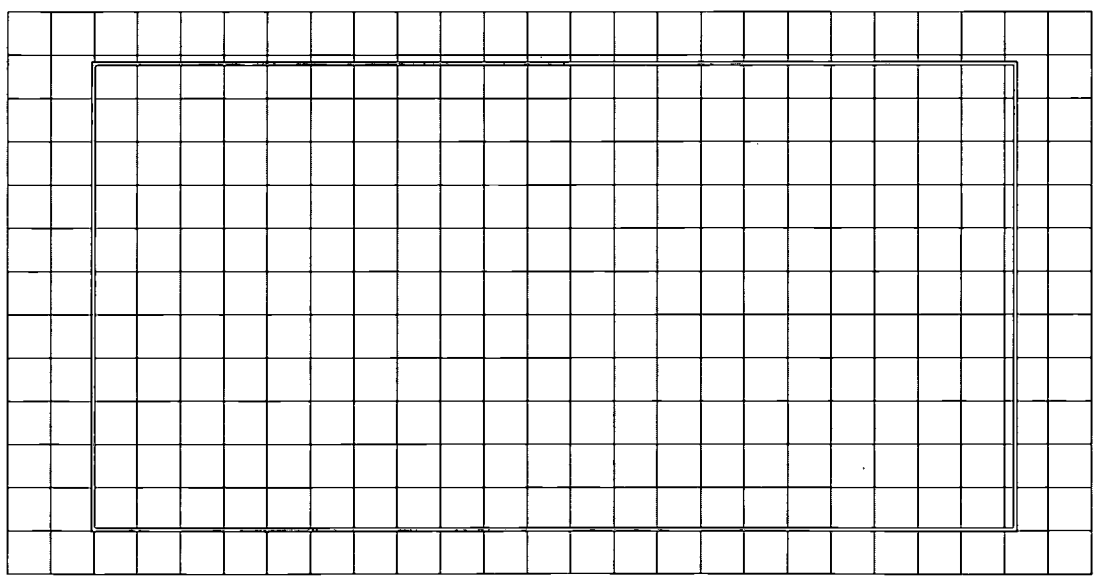
8. Separate the <u>remaining</u> seven pieces from the paper, and use <u>only those seven pieces</u> to help you answer the following question.

1 inch represents $1\frac{1}{2}$ feet (18 inches)

Julie plans to use plywood to build her doghouse, using the scale above. The plywood is sold in rectangular sheets that are each 4 feet wide and 8 feet long. She wants to determine the fewest number of sheets that she will need.

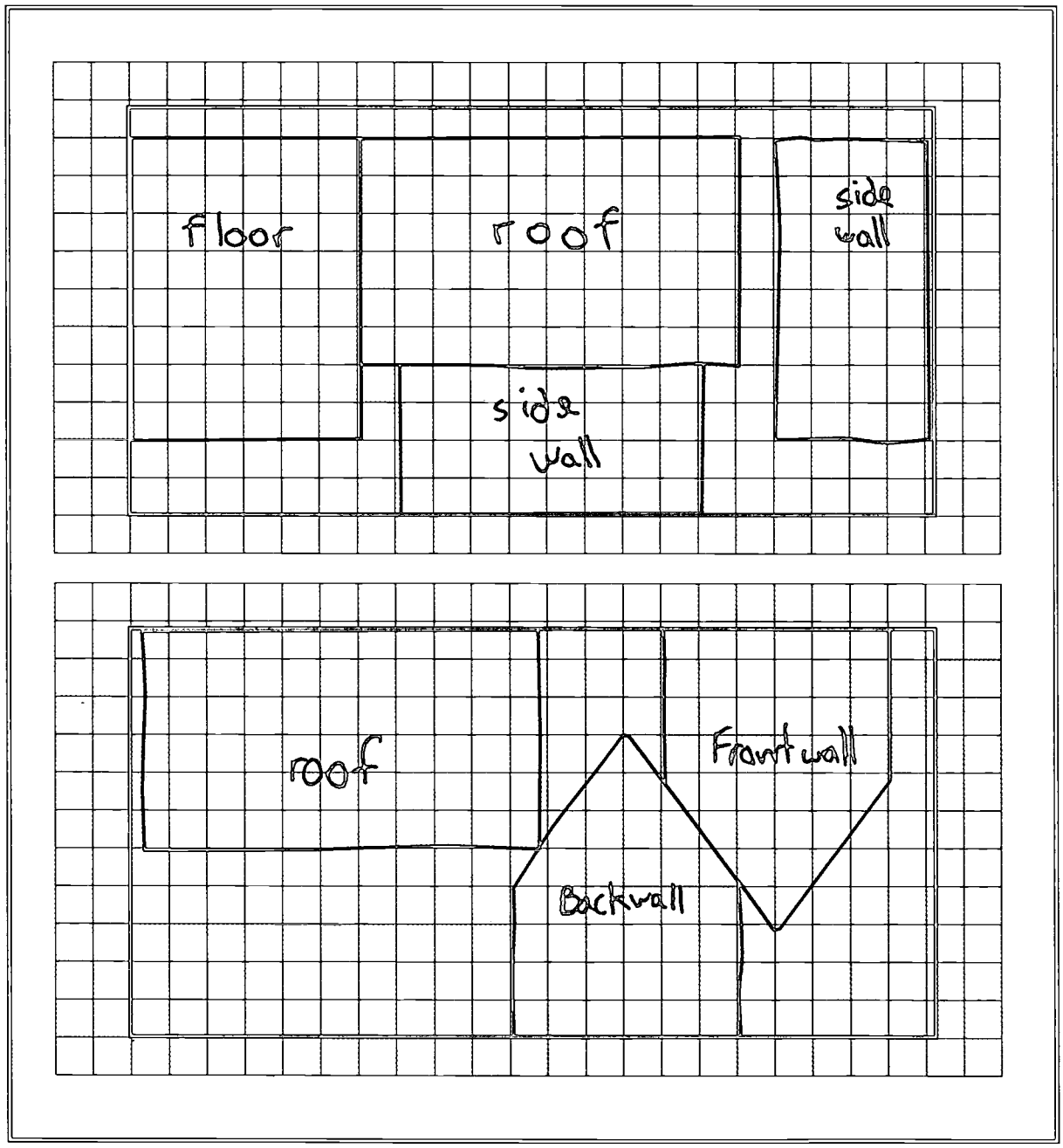
On the grids below and on the next page, the plywood sheets have been drawn to the same scale as the seven pieces. Show how the seven pieces (four walls, two roof pieces, and the floor) could be cut from the plywood sheets so that the <u>fewest</u> number of sheets are used. This should be done by tracing the pieces on the sheets.

(Note: There may be more sheets shown than you will need to use.)



The responses to this question were scored with a 3-point rubric: "complete," "partial," and "incorrect." The fewest number of plywood sheets needed for the seven pieces was two. A "complete" response was one in which the seven pieces were correctly drawn onto two plywood sheets. Responses that showed two side pieces, two front or back wall pieces, or two roof pieces drawn as one piece but correctly labeled were considered correctly drawn. Although students were not required to label the pieces, if they did so incorrectly, their responses were not scored as "complete." The following "complete" response is just one of several configurations that would accommodate the seven pieces on two plywood sheets.

Sample "complete" response



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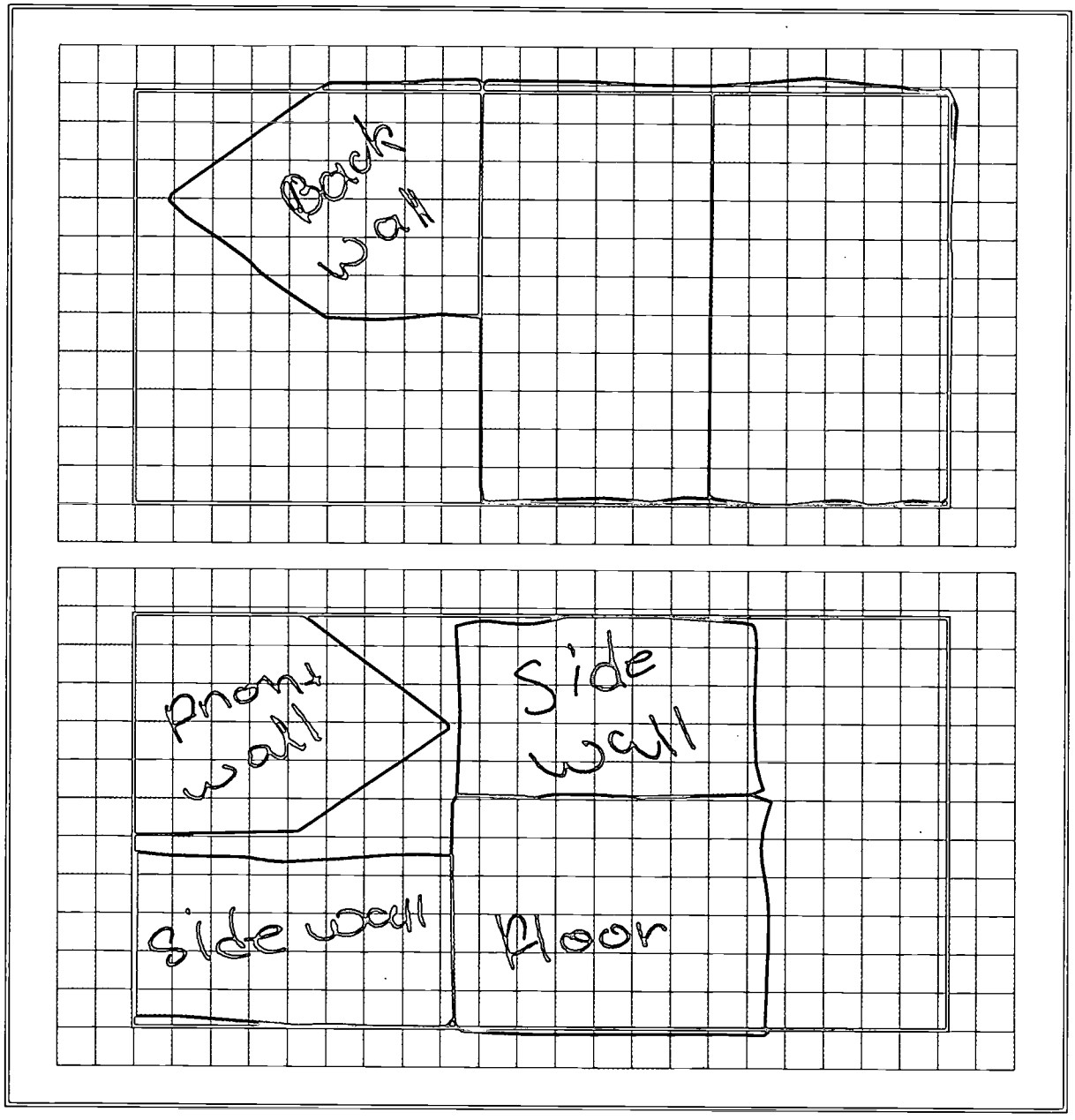
Responses meeting one of the following criteria were scored as "partial":

- six of the seven pieces correctly traced on two sheets with the seventh piece missing;
- six of the seven pieces correctly traced on two sheets with the seventh piece drawn incorrectly, but the seventh piece is not the door opening (piece "C") or a duplicate piece already accounted for; or
- seven pieces correctly traced on to two sheets, but also piece "C" drawn on one of the two sheets or on a third sheet.

If pieces were labeled incorrectly, responses were still scored as "partial." In the "partial" response shown below, the student appears to have drawn one of the side walls larger than the push-out piece provided.

Sample "partial" response



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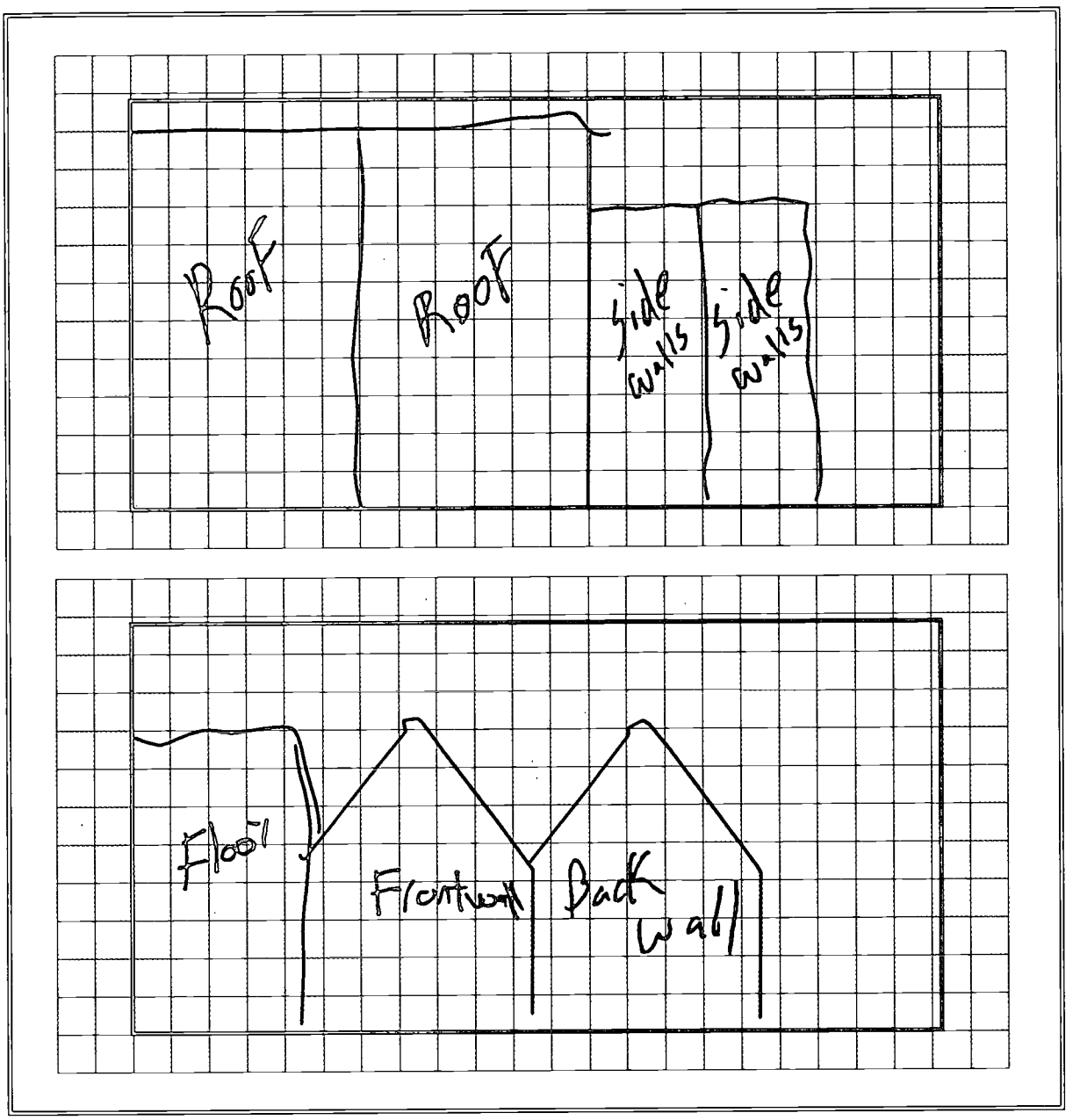
An "incorrect" response is one that used three sheets, had drawings of pieces that appeared to be freehand or not drawn to scale, or included more than one of any of the seven requisite pieces. In the "incorrect" response shown below, it is apparent that although the roof pieces, front wall, and back wall were traced, the side walls and floor were drawn freehand and were not of the correct size.

Sample "incorrect" response



As the data in Table 3.21 show, 48 percent of the students understood and correctly solved the problem. This was a problem where few students provided responses that were scored as "partial." That is, if students were able to correctly trace and position six of the seven pieces, it was uncommon for them not to succeed with the seventh piece as well. A substantial percentage of the responses, 36 percent, were scored as "incorrect." In addition, 10 percent of the students chose not to attempt this question.

Table 3.21 — *Score Percentages for "Visualize Cut-Outs on Grid, Grade 8"*

THE NATION'S REPORT CARD naep

Grade 8	Complete	Partial	Incorrect	Omit
All Students	48	7	36	10
Males	43	8	37	12
Females	54	6	34	7
White	56	7	30	7
Black	20	2	58	20
Hispanic	36	6	45	13
Asian/Pacific Islander	45	11	34	10
American Indian	***	***	***	***

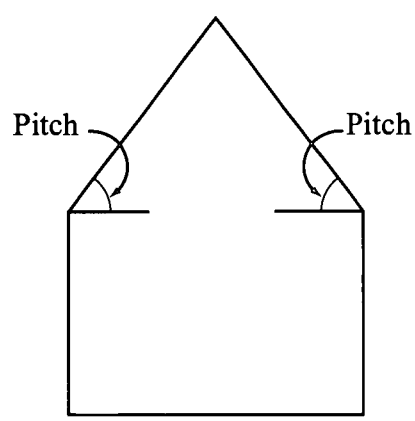
NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 9. Apply geometry in model. The ninth question was a multistep question classified as Problem-Solving and designed to assess content from both the Algebra and Functions and the Geometry and Spatial Sense content strands. In the introduction to the problem, students were told that they were required to show how they solved the problem and to explain their reasoning process. A scale that students had seen in previous problems was provided. The question began with a definition of the pitch of a roof. Students were told that Julie had read that the optimal pitch for best air flow was 30°, and that the pitch of the model doghouse was 53°. They then were asked to determine the number of feet by which the height of the actual doghouse would decrease if Julie decreased the pitch from 53° to 30°.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all of your work.

1 inch represents $1\frac{1}{2}$ feet (18 inches)

9. The drawing below shows a wall of Julie's doghouse. The **pitch** is defined as the slope of the roof; it can also be described as the angle formed between the roof and a horizontal line, as shown in the drawing.



The pitch of the roof in the drawing is slightly more than 53 degrees, which is the same as the roof pitch on your model.

Julie read in a book that the best air flow inside a doghouse occurs when the roof pitch is 30°. If the height of the doghouse is measured from the floor to the highest point on the roof, by about how many feet is the actual height of the doghouse decreased when the pitch is decreased from 53° to 30°?

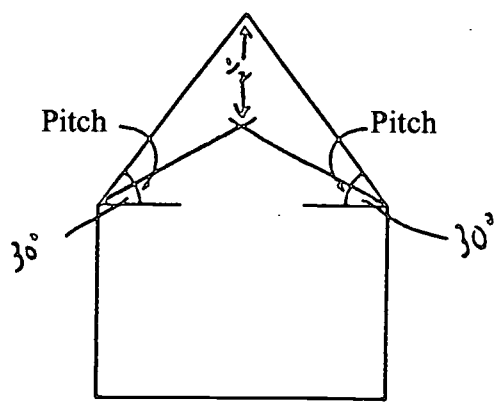
Show how you got your answer. (You may find it helpful to mark on the drawing.)

To solve the problem, students needed to understand the definition of pitch and be able to draw a new roof slope based on a pitch of a different degree. That is, they needed to know what an angle is, what a horizontal line is, and how to use a protractor to measure an angle. Finally, students needed to know how to use the scale provided to convert measures on the drawing of the model to measures of the actual doghouse. Responses to this question were scored using a 4-point rubric: "satisfactory," "partial," "minimal," and "incorrect." The responses were scored on the presence of three attributes:

1. correct indication of the 30° angle on the drawing;
2. correct measurement of the height of the old doghouse, the height of the new doghouse, or the sides opposite the 30° angle in the right triangle; or correct subtraction or measurement to obtain the difference between the heights of the old and new doghouses; and
3. correct conversion of the decrease in height from inches to feet using the scale provided.

A "satisfactory" response included all three of the attributes detailed above. The sample "satisfactory" response shown has the correct indication of the 30° angle on the drawing, the correct measurement (with explanation) of the decrease in the height of the roof from the old to the new, and proper conversion of inches to feet using the scale provided, resulting in the correct answer of ¾ of a foot.

Sample "satisfactory" response

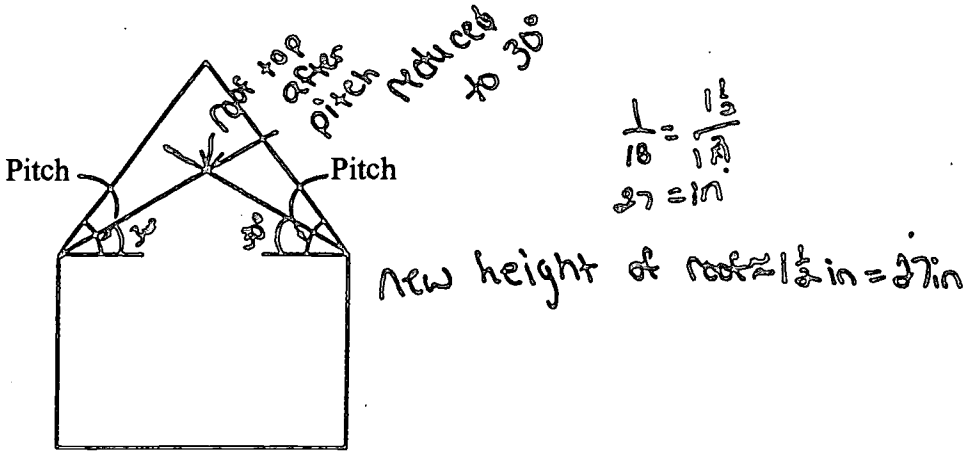


First you mark a 30° angle on the drawing. Then make the roof to fit that. Measure the length of the top of the original roof to the new one. The length is half of an inch.

$\left(\frac{1}{2}'\right)\left(\frac{1\frac{1}{2}\text{ feet}}{1\text{ in.}}\right) = \frac{3}{4}$ of a foot

A "partial" response had two of the attributes detailed above. The following "partial" response has the 30° angle correctly measured and drawn and has the correct measurements of the old and new heights for the full-scale doghouse shown in inches. The response also includes the decrease in the height but leaves it in inches and does not carry out the conversion to feet.

Sample "partial" response



18 in · 2 = 36 in ⇒ first found actual height of doghouse

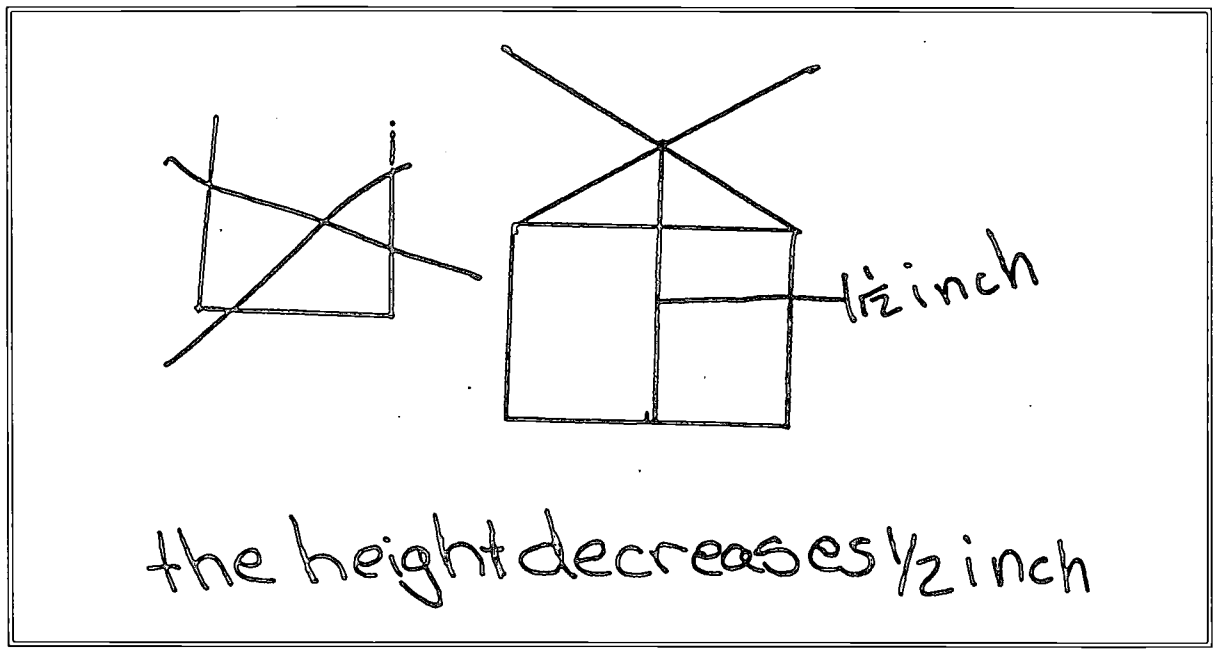
next measure pitch as 30° and drew lines to show roof top

new height would be 27 in

36 − 27 = 9 inches lost from 36 in to 27 in

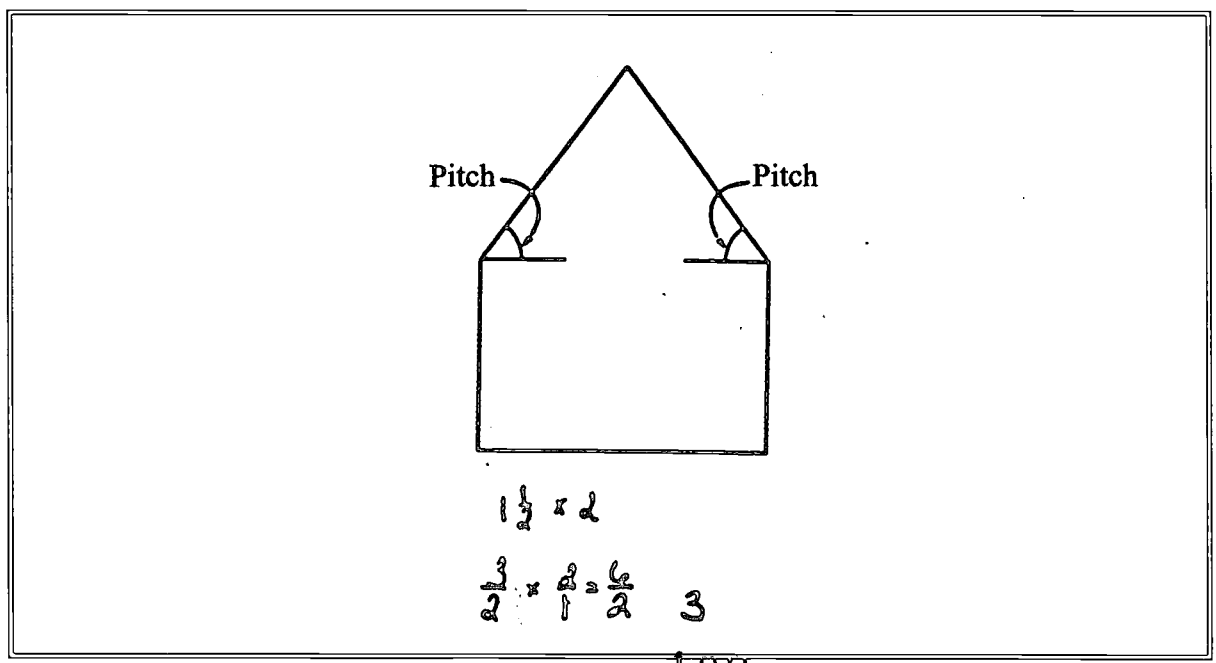
A "minimal" response has one of the correct attributes. The sample "minimal" response shows a correct measure of the new height of the model doghouse. It also includes the decrease in heights of the old and new models but does not show or mention the use of the 30° angle and does not convert inches into feet using the scale provided.

Sample "minimal" response



An "incorrect" response had none of the correct attributes. The sample "incorrect" response shows little understanding of how to approach solving the problem.

Sample "incorrect" response



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The data in Table 3.22 show that nearly half of the responses from students were scored as "incorrect." Furthermore, only 22 percent of the responses received any credit, and 24 percent of the students chose not to attempt to solve this problem at all.

Table 3.22 — **Score Percentages for "Apply Geometry in Model," Grade 8**

THE NATION'S REPORT CARD naep

Grade 8	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	2	6	14	49	24
Males	1	6	13	47	28
Females	2	7	14	52	19
White	2	8	16	47	21
Black	0!	0!	6	60	32
Hispanic	0!	4	8	53	32
Asian/Pacific Islander	***	***	***	***	***
American Indian	***	***	***	***	***

NOTE: Raw percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 10. Find maximum area when perimeter is fixed. The final question was classified as a Measurement question and was designed to assess Problem-Solving ability. Students were introduced to this question with information about what was required of their responses. Then they were presented with the problem. They were told that Julie wanted to build a fence around a section of the yard for her dog and that she was able to purchase 36 feet of fencing. They also were told what the enclosed area should look like (i.e., a rectangle with whole number lengths). Students were asked to determine the largest area that Julie could enclose with her fencing and to show work that would convince Julie that their solution had indeed yielded the largest area.

> This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all of your work.
>
> 10. Julie wants to fence in an area in her yard for her dog. After paying for the materials to build her doghouse, she can afford to buy only 36 feet of fencing.
>
> She is considering various different shapes for the enclosed area. However, she wants all of her shapes to have 4 sides that are whole number lengths and contain 4 right angles. All 4 sides are to have fencing.
>
> What is the largest area that Julie can enclose with 36 feet of fencing?
>
> Support your answer by showing work that would convince Julie that your area is the largest.

To answer this question students needed to know what a rectangle is; have a conceptual understanding of the fact that rectangles with the same perimeter could have different areas; and be able to calculate the area from the length and width of the sides of the rectangle. This question was scored on a 5-point scale: "extended," "satisfactory," "partial," "minimal," and "incorrect." An "extended" response needed to provide the correct answer of 81 square feet along with evidence that all length/width combinations had been considered in determining the largest area enclosed. The sample "extended" response clearly explained that the largest rectangular area of perimeter 36 feet would be a 9-foot square, which provides an area of 81 square feet. It also included a table that listed all the possible combinations of widths and lengths that would result in a perimeter of 36 feet, thus providing evidence that could "convince Julie" and also showing that the student clearly understood the relationships between the lengths of the sides of the rectangle, perimeter, and area.

Sample "extended" response

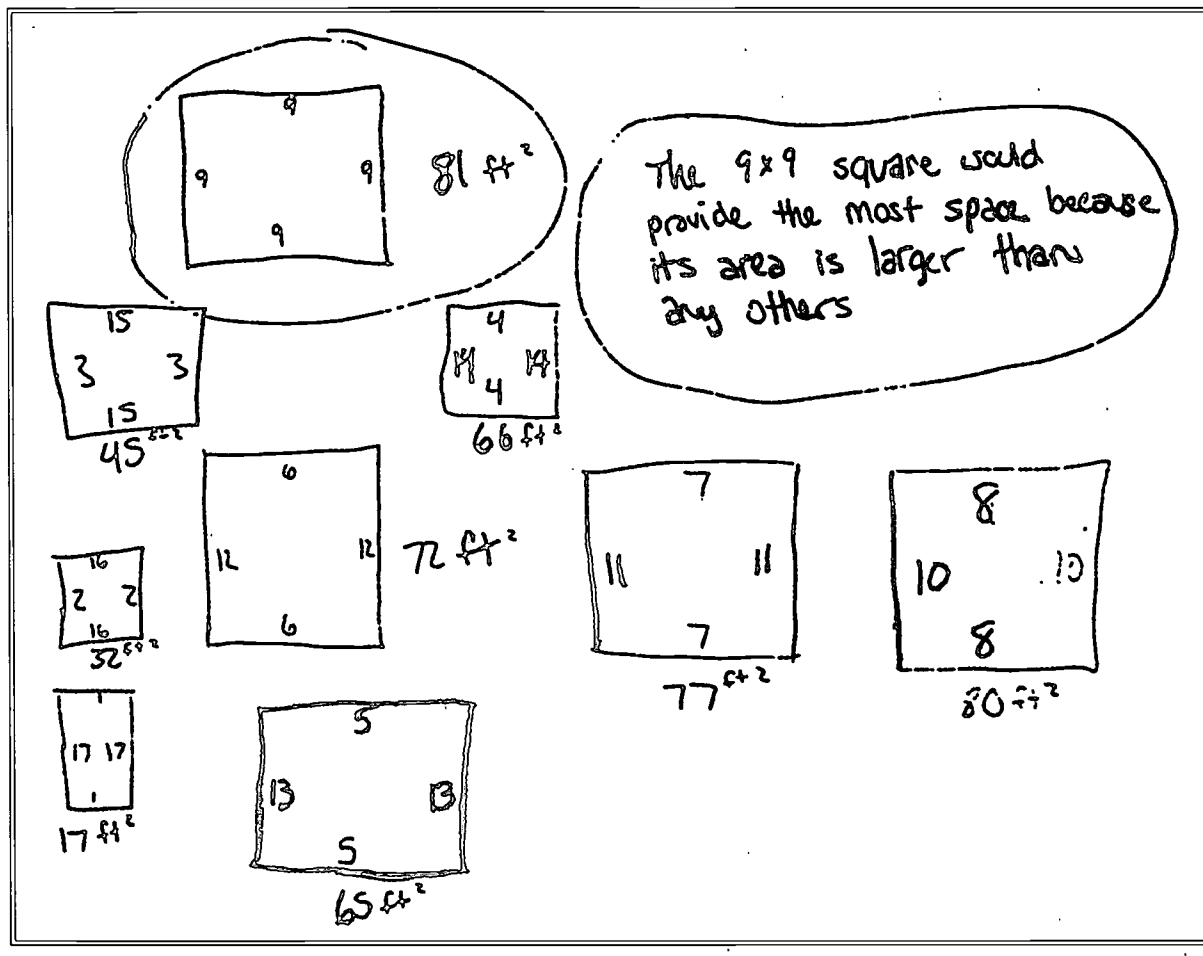
The largest area would be a square with sides of 9 ft. which would allow for 81 ft.^{2} of area.

This is because of the following:

Width	Length	Area	Perimeter
9	9	81	36
10	8	80	36
11	7	77	36
12	6	72	36
13	5	65	36
14	4	56	36
15	3	45	36
16	2	32	36
17	1	17	36

"Satisfactory" responses indicated that the 9-by-9 square has the maximum area of 81 square feet; indicated that another rectangle has the maximum area, due to a mathematical error in the accompanying work; or contained calculations for all nine rectangles (i.e., widths of 1 though 9) but did not specify the maximum area. The sample "satisfactory" response shown correctly designates the rectangle with largest area; however there is a computational error in the supporting work.

Sample "satisfactory" response



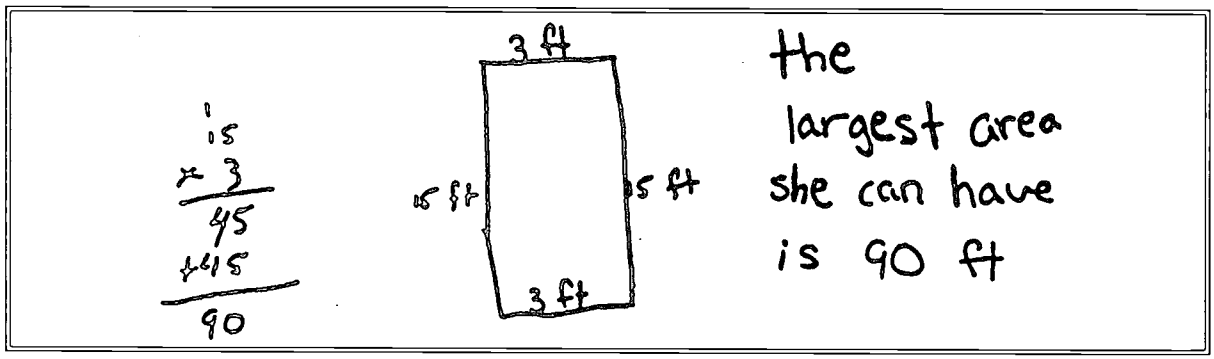
A "partial" response was one that a) showed at least three different rectangles with their dimensions and areas and might have included an indication that one of those rectangles yielded the maximum area, or b) indicated that the 9-by-9 square had the maximum area of 81 square feet, but showed no accompanying work. The "partial" response shown below specifies the correct rectangle and area (although the area units are incorrectly labeled as "feet" rather than "square feet"), but the work shows no consideration of other possible rectangles.

Sample "partial" response

Area = base x height
x = 9 x 9
81 = area
She can enclose 81 feet

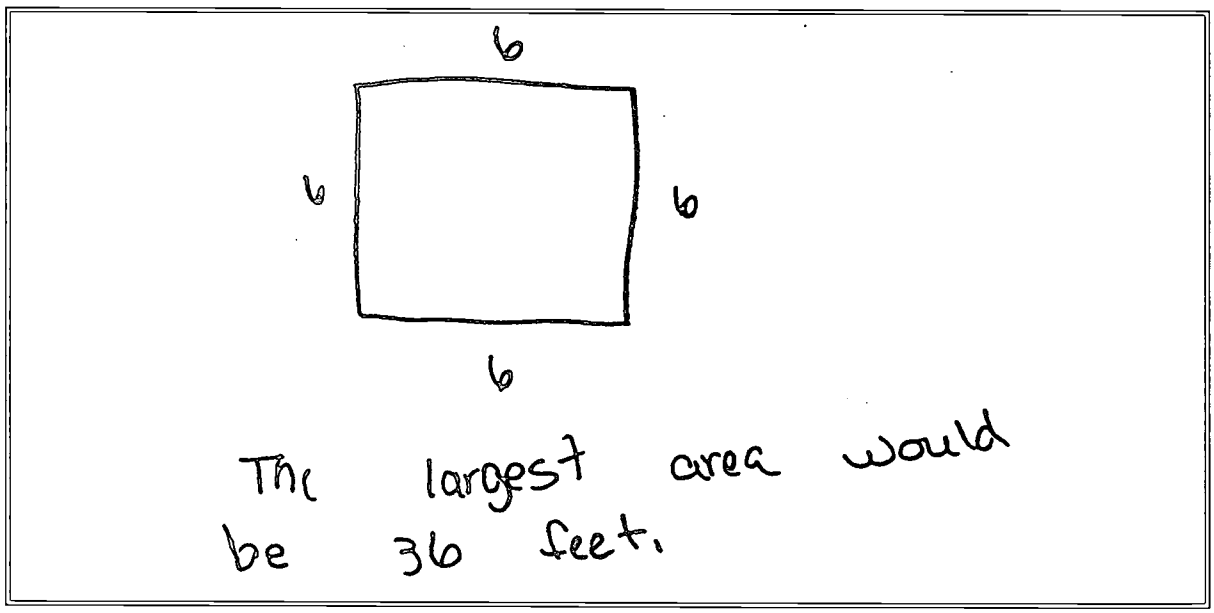
A "minimal" response must have included some evidence of understanding that area and perimeter formulas for rectangles were needed to solve the problem and might have included an attempt to organize the data. The sample "minimal" response shows that the student had an understanding of using the 36-foot-long fencing to form a rectangle, but was unclear as to the exact formula for calculating the area of that rectangle.

Sample "minimal" response



An "incorrect" response had none of the preceding criteria. Although it is not definitive from the "incorrect" response shown, it appears that the student knew how to calculate the area of a rectangle, but did not fully understand the task explained in the problem, since the perimeter of the student's rectangle does not equal 36 feet.

Sample "incorrect" response



Student performance data are presented in Table 3.23. Less than one-half of one percent of the responses were scored "extended" or "satisfactory." A third of the responses received at least minimal credit, while 38 percent were scored as "incorrect." A large percentage of students, 27 percent, chose not to attempt this question.

Table 3.23 — *Score Percentages for "Find Maximum Area When Perimeter is Fixed," Grade 8*

THE NATION'S REPORT CARD naep

Grade 8	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	0	0	29	4	38	27
Males	0	0!	28	4	35	30
Females	0	0	29	3	40	23
White	0	0!	34	4	38	21
Black	0!	0!	10	2	40	45
Hispanic	0!	0!	13	2	37	44
Asian/Pacific Islander	***	***	***	***	***	***
American Indian	***	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Grade 12

Student characteristics

Twelfth-grade students who participated in the Theme Study were similar demographically to students who participated in the national NAEP assessment in mathematics. The data in Table 3.24 show that similar percentages of male and female students participated in each of the samples; the majority of students were White students; and the modal response from students regarding their parents' highest level of education was "graduated from college." In addition, the large majority of twelfth-grade students were from public schools, very few students were part of the Title I program, and about 10 percent of students were eligible for the federal Free/Reduced-Price Lunch program.

Teachers of twelfth-grade students were not surveyed either in the main NAEP assessment or in the Theme Study. Therefore, the information presented on classroom practices in Table 3.25 is based on responses from the twelfth-grade students themselves. Students in each of the different samples indicated similar levels of frequency of writing about solving a mathematics problem and of writing reports or doing a mathematics project. Notice that only small percentages of students reported engaging in either of these practices with any frequency, while 60–70 percent of students reported "never or hardly ever" doing so. In interpreting these data, however, one must keep in mind that the "never or hardly ever" category also includes responses from students not currently enrolled in mathematics. In the main NAEP assessment, only 62 percent of grade 12 students indicated being enrolled in mathematics.

Table 3.24

Student Demographic Distributions by Assessment, Grade 12, 1996

THE NATION'S REPORT CARD naep

Grade 12	Percentage of Students		
	Main Assessment	Theme Block 1 Buying a Car	Theme Block 2 Flooding
Gender			
Males	48	50	49
Females	52	50	51
Race/Ethnicity			
White	70	69	69
Black	14	14	14
Hispanic	11	12	12
Asian/Pacific Islander	4	4	4
American Indian	1	0	0
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	6	7	7
Graduated From High School	19	18	20
Some Education After High School	25	26	27
Graduated From College	47	46	43
I Don't Know	3	3	3
Students who Attend...			
Public Schools	88	89	88
Nonpublic Schools	12	11	12
Title I Participation...			
Participated	2	2	2
Did Not Participate	98	98	98
Free/Reduced-Price Lunch Program Eligibility...			
Eligible	13	10	10
Not Eligible	60	64	65
Information Not Availabile	27	26	25

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 3.25	Percentage of Students by Reports on Classroom Practices, Grade 12, 1996*	THE NATION'S REPORT CARD naep

Grade 12	Percentage of Students		
	Main Assessment	Buying a Car	Flooding
Students who Report Writing a Few Sentences About How to Solve a Mathematics Problem...			
Nearly Every Day	6	6	7
Once or Twice a Week	12	13	13
Once or Twice a Month	18	19	18
Never or Hardly Ever	64	63	63
Students who Report Writing Reports or Doing a Mathematics Project...			
Nearly Every Day	2	1	2
Once or Twice a Week	4	4	5
Once or Twice a Month	24	22	25
Never or Hardly Ever	70	72	69

* Teachers of twelfth-grade students were not surveyed in these assessments.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Content of the Theme blocks

As with grades 4 and 8, the two blocks of Theme Study questions at the twelfth-grade level also were structured around engaging contexts. The released block involved buying or leasing a car, a topic that is likely to be of interest to twelfth-grade students. The unreleased block had the same context and many of the same questions as the unreleased block at the eighth-grade level: It concerned the flooding of the Mississippi River during the summer of 1993.

Both of the Theme blocks at grade 12 included multiple-choice and constructed-response questions. In the Buying-a-Car block, 2 of the 7 questions were multiple-choice. Four of the remaining five constructed-response questions required the students to show how they solved the problem. In the Flooding block, 4 of the 11 questions were multiple-choice, and in 4 of the 7 constructed-response questions, students were asked to show how they arrived at their answers.

Overall student performance

Information on students' performance on the two Theme blocks is presented in Table 3.26. The average percentage correct score is 41 percent for the Buying-a-Car block and 38 percent for the Flooding block. On both of the Theme blocks, male and female students performed similarly. However, on the Buying-a-Car block, White, Hispanic, and Asian/Pacific Islander students outperformed Black students, and White students outperformed Hispanic students, whereas on the Flooding block, White students outperformed Black, Hispanic, and Asian/Pacific Islander students, and Asian/Pacific Islander students outperformed Black students. The sample

of American Indian students for both Theme blocks was too small to permit reliable estimates if their performance on either the blocks as a whole or on individual questions. Therefore, the performance of American Indian students is not discussed.

In terms of the highlighted classroom practices, the levels of frequency of writing a few sentences and of writing reports or doing a mathematics project were not found to be related to student performance on the questions in the Buying-a-Car block. Performance on the Flooding block, however, was found to have some relationship to the frequency of those classroom practices, although not necessarily in the direction that one might predict. The data show that students who "nearly every day" are asked to write a few sentences about how to solve a mathematics problem were outperformed by students who indicated a lower frequency, and students who indicated writing reports or doing mathematics projects "once or twice a week" were outperformed by students who indicated a lower frequency. One can only speculate about the reasons for these unexpected relationships. It is possible that they reflect ambiguities in the meaning of phrases such as "mathematics projects" or other sources of inaccuracies in the self-reported answers to the survey questions. Another possibility is that activities such as mathematics projects and writing about how to solve mathematics problems are being used less frequently in classes that enroll more high achieving students. Finally, it is possible, that contrary to expectations, introduction of these types of teaching practices actually decrease mathematics learning, perhaps by taking time away from more effective instructional activities.

Table 3.26

Average Percentage Correct Scores by Theme Block, Grade 12, 1996

THE NATION'S REPORT CARD naep

Grade 12	Buying a Car	Flooding
All Students	41	38
Gender		
Males	40	39
Females	42	37
Race/Ethnicity		
White	46	44
Black	24	23
Hispanic	30	26
Asian/Pacific Islander	45	34
American Indian	***	***
Students Whose Teachers Ask Them to Write a Few Sentences About How to Solve a Mathematics Problem...		
Nearly Every Day	33	31
Once or Twice a Week	43	38
Once or Twice a Month	43	40
Never or Hardly Ever	41	39
Students Whose Teachers Ask Them to Write Reports or Do a Mathematics Project...		
Nearly Every Day	***	***
Once or Twice a Week	37	31
Once or Twice a Month	42	40
Never or Hardly Ever	42	39

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Buying a car

The released Theme block at the twelfth-grade level is about buying versus leasing a car. Unlike the released Theme blocks in grades 4 and 8, this block was not accompanied by any supplemental materials other than a calculator. All of the materials or information needed to solve the problems were provided in the questions themselves. The introduction to the block of items is shown in Figure 3.3. In this introduction, students were told about the expectation for completeness of responses, given information that Question 5 in the block might require more time for thinking and answering, told that they should indicate whether they used a calculator on each question, and instructed to round only at the final answer when calculating. Then, students were introduced to the context for the block of questions: A family is trying to decide whether to buy a new car with a loan or to lease the car. Although familiarity with buying or leasing a car might have some influence on students' comfort level with the questions in this block, prior knowledge about buying and leasing cars was not essential to solving the problems.

Figure 3.3 — **Introduction to "Buying a Car" Theme Block, Grade 12, 1996**

THE NATION'S REPORT CARD naep

This part has 7 questions. Mark your answers in your booklet. You will have to fill in an oval or write your answer as directed. In those questions where you must write an answer, it is important that your answer be clear and complete and that you show all of your work since partial credit may be awarded. Question 5 may require 5 minutes or more to think about and answer. After each question, fill in the oval to indicate whether you used the calculator. If you are asked to round your answer, do not round any numbers except your final answer.

A family is trying to decide how to pay for the new car they want. Should they buy the car with a loan or should they lease it? If they lease the car, then they must return the car to the dealer at the end of the lease period. If they buy the car, then at the end of the loan period the car belongs to them. The monthly payments for the car, whether they buy it or lease it, will depend upon the size of their down payment. If they make a big down payment now, they will have smaller monthly payments, but they might like to use the down payment money to buy something else. How will the interest rate and the period of time for the loan or lease affect the total amount they will pay?

All the problems on this test are related to financial questions similar to those asked above that involve buying or leasing a car.

Question 1. Find amount of down payment. In the first question, students were introduced to Donna, who wants to buy a car, and were provided with the selling price of the car she wants to buy, $16,500. They were asked to determine the required down payment in dollars, given a down payment requirement of 20 percent of the selling price. This is a relatively straightforward computational question that was designed to assess content from the Number Sense, Properties, and Operations content strand and Procedural Knowledge. One way to answer the question is to translate the 20 percent into the decimal 0.20 and then to multiply the selling price by the decimal fraction. However, students could have used a number of other strategies. For example, some students may have known that 20 percent is equivalent to 1/5 and multiplied the selling price by 1/5 or divided it by 5. Others may simply have used the percent key on their calculators.

1. Donna decides to buy a new car that is selling for $16,500. If she is required to pay 20 percent of the selling price as a down payment, what is the number of dollars required for the down payment?

   (A) $330

   (B) $1,650

   (C) $3,300

   (D) $4,125

   (E) $13,200

The correct response to this multiple-choice question is $3,300, Option C. Student performance information is presented in Table 3.27. A large majority of the students, 82 percent, were able to answer this question correctly.

Table 3.27	Percentage Correct for "Find Amount of Down Payment," Grade 12	THE NATION'S REPORT CARD

	Percentage Correct
Grade 12	
All Students	82
Males	82
Females	83
White	86
Black	73
Hispanic	70
Asian/Pacific Islander	83
American Indian	***

***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 2. Find total amount paid for car. In the introduction to the second question, students were told that the information given was to be used in Questions 2 and 3. The information included the $19,200 selling price of Bill's car; his down payment of $4,800; and the duration of his loan, 36 months. In Question 2, students were asked to determine how much Bill paid in total for his car, given the down payment and monthly payments of $451. This question was classified as a Number Sense, Properties, and Operations question and was designed to assess Procedural Knowledge. Like the first question, this was a relatively straightforward computational question. The one difficulty that students might have had was in realizing that the selling price of the car, $19,200, was extraneous to answering the question.

Questions 2-3 refer to the following information.

Bill purchased a new car that was selling for a price of $19,200. He paid $4,800 as a down payment and obtained a 36-month car loan to finance the remainder of the selling price.

2. If his monthly payments were $451, what was the total amount, including the down payment, that Bill paid for the car?

Ⓐ $11,436

Ⓑ $16,236

Ⓒ $19,200

Ⓓ $21,036

Ⓔ $24,000

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The data on students' performance presented in Table 3.28 show that a large percentage (83%) of students were able to select the correct option, D. The second highest percentage of students, seven percent, selected Option B, which is the number of dollars for the 36 monthly payments of $451 without the down payment. Five percent of the students selected Option E, which is the sum of the selling price of the car and the down payment.

Table 3.28 — *Percentage Correct for "Find Total Amount Paid For Car," Grade 12*

THE NATION'S REPORT CARD

	Percentage Correct
Grade 12	
All Students	83
Males	83
Females	82
White	89
Black	62
Hispanic	74
Asian/Pacific Islander	77
American Indian	***

***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 3. Find difference between total amount paid and price. The third question builds on the results of the second question: students were asked to indicate how much more Bill paid for his car, that is, with the down payment and the loan, than the selling price of the car. This question, like Questions 1 and 2, was designed to assess content from the Number Sense, Properties, and Operations strand and Procedural Knowledge.

> 3. By how much did the total amount Bill paid for the car exceed the selling price for the car?

In order to answer this short constructed-response question correctly, students needed to determine which numbers they needed for solving the problem and perform the subtraction accurately. The question was scored simply "correct" or "incorrect"; that is, students were not given partial credit for their work. To be scored "correct," the response must have been $1,836 (that is, $21,036 − $19,200). Any other response was scored as "incorrect." Information on student performance on this dichotomously-scored question is presented in Table 3.29; 80 percent of the students were able to determine the correct answer.

Table 3.29 — ***Percentage Correct for "Find Difference Between Total Amount Paid and Price," Grade 12***

THE NATION'S REPORT CARD naep

Grade 12	Percentage Correct
All Students	80
Males	79
Females	80
White	86
Black	60
Hispanic	69
Asian/Pacific Islander	75
American Indian	***

***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Question 4. Find amount to be financed. The fourth question introduced the term "residual value." Students were asked to determine the number of dollars that would be needed to finance a two-year lease on a car, if the amount to be financed is the selling price minus the sum of the down payment and the residual value of that car at the end of the lease period. They were given the down payment, $1,500; the selling price of the car, $18,700; and were told that the residual value of that car after a two-year lease was 53 percent of the selling price. This question also was classified as a Number Sense, Properties, and Operations question and was designed to assess Procedural Knowledge. Students were asked to show their work.

> 4. When leasing a new car, the amount to be financed is the amount left when the down payment and residual value are subtracted from the selling price. (The residual value is the value of the car at the end of the lease period.) The residual value for a two-year-old car, whose original selling price was $18,700, is estimated to be 53 percent of the selling price. If the amount of the down payment is $1,500, what is the number of dollars to be financed for a two-year lease? Show the work that led to your answer.

In essence, students were asked to use knowledge and skills similar to those that they needed for the first three questions, only this time in combination, to solve a multistep problem. The computational skills required to solve the problem were relatively simple; the difficult part of the problem was in setting up the problem and determining the steps involved. One way to solve for the amount to be financed is first to calculate the residual value, 53 percent of $18,700 or $9,911; add that amount to the down payment to get $11,411; and, finally, subtract that sum from the selling price to get $7,289.

The responses to this question were scored on a 3-point rubric: "complete," "partial," and "incorrect." A "complete" response had the correct answer and showed an acceptable process for getting to that answer. The sample "complete" response shows the process through mathematical calculations.

Sample "complete" response

$.53(18,700) = 9,911$

$+ 1,500$

$11,411$

$18,700$

$-11,411$

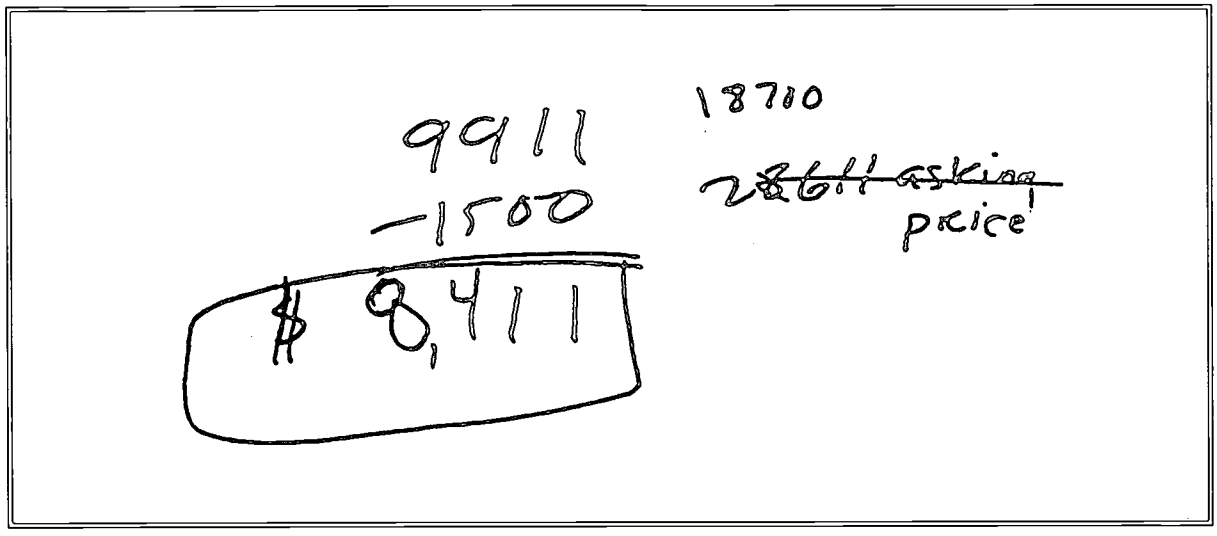
$\$7,289$

The following types of responses were scored as "partial" responses:

- the correct residual value — $9,911;
- the correct difference between selling price and residual value — $8,789;
- an incorrect residual value, but with all calculations performed correctly — for example, finds 53 percent of $17,200, which is the selling price minus the down payment, instead of 53 percent of the selling price;
- a correct procedure but work shown contains arithmetic errors;
- the correct sum of the residual value and the down payment — $11,411; and
- the correct difference between the residual value and the down payment — $8,411.

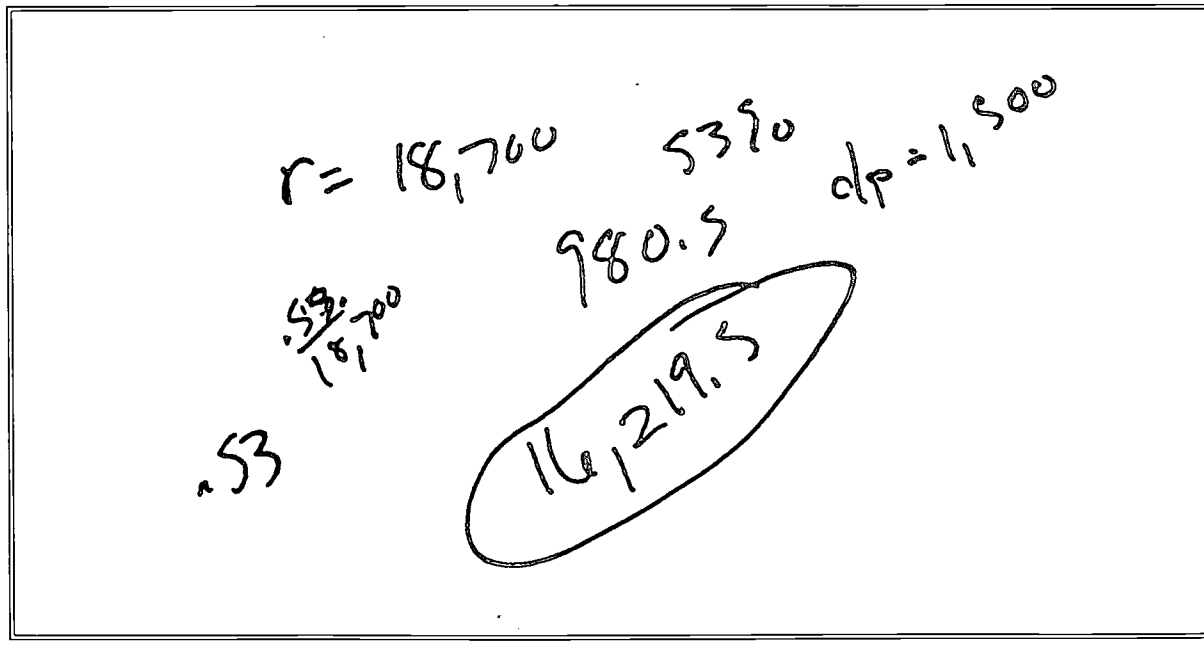
The sample "partial" response is an example of the correct difference between the residual value and the down payment.

Sample "partial" response



An "incorrect" response met none of the preceding criteria. The "incorrect" response shown below contains evidence of some understanding of the process for solving the problem. That is, it is not clear how the student arrived at 980.5, but if that were the residual value, when added to the down payment of 1,500 and subtracted from the selling price of 18,700, the answer obtained would be the 16,219.5 shown. The response nevertheless is scored as "incorrect."

Sample "incorrect" response



Information on students' performance on this question is presented in Table 3.30. Thirty-four percent of student responses were scored as "complete," and 31 percent were scored as "partial."

Table 3.30 — ***Score Percentages for "Find Amount To Be Financed," Grade 12***

THE NATION'S REPORT CARD naep

Grade 12	Complete	Partial	Incorrect	Omit
All Students	34	31	24	8
Males	33	28	26	10
Females	36	34	22	6
White	39	30	22	6
Black	17	30	34	16
Hispanic	27	36	26	7
Asian/Pacific Islander	42	30	18	7
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Question 5. Use formula to find total cost. The fifth question is prefaced with requirements for the students about showing their work and explaining their reasoning. In the question, students were asked to calculate the total cost of a car given the following information: a formula for calculating monthly car payments, the selling price of the car, the down payment, the amount to be financed, the annual percent rate of the amount to be financed, and the length of the loan. Students also were given an approximate value to use for one of the quantities used in the formula — $\left(1+\frac{.08}{12}\right)^{48}$. This question was classified as an Algebra and Functions question and was designed to assess Problem-Solving ability.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> of your work.

5. A formula used to calculate a monthly car payment is

$$MP = A\left[\frac{\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{n}}{\left(1+\frac{r}{12}\right)^{n}-1}\right]$$

where MP is the monthly payment,
A is the number of dollars to be financed,
r is the annual percent rate (e.g., if the annual percent rate is 7%, then r is 0.07), and
n is the length of the loan in months.

Use the formula shown above to help calculate the <u>total cost</u> of purchasing an $18,000 car if there is a down payment of $4,000 and the remaining $14,000 is to be financed at an annual percent rate of 8 percent for 48 months.

Show the work that led to your answer.

Note: In your calculations, use 1.3757 as the approximate value of

$$\left(1+\frac{0.08}{12}\right)^{48}$$

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This problem was difficult because students needed to determine how to organize and use the information provided in order to solve the problem. In other words, students needed to know how to use a formula with variables to compute the monthly payments and how to correctly substitute the approximate value provided in the "Note" into the formula. They also had to recognize that the formula is for a monthly payment; because the question asks for the total cost of purchasing the car, the monthly payment needs to be multiplied by 48 months and added to the down payment in order to arrive at the total cost.

The responses to this question were scored with a 5-point rubric: "extended," "satisfactory," "minimal," "partial," and "incorrect." The scoring rubrics took into consideration that the answers students gave may have differed because of rounding, although students were told at the beginning of the block that they should not round in their calculations until the final answer. Therefore, the following answers, if accompanied by evidence of correct work, were all scored as "extended" — $20,404.48; $20,404.41; $20,404.35; or $20,405.48. The sample "extended" response has the correct answer, $20,404.48 and shows the mathematical calculations done to arrive at the correct answer. The student first calculated the monthly payment, using the approximate value, 1.3757, in the calculations; then calculated the total payments by multiplying the monthly payment by 48 months; and, finally, calculated the total cost by adding the total of the monthly payments and the down payment.

Sample "extended" response

$$MP = 14000\left[\frac{\left(\frac{.08}{12}\right)(1.3757)}{(1.3757)-1}\right]$$

$$MP = 14000\left(\frac{.0091713}{.3757}\right)$$

MP = 341.76

TP = 341.76 x 48

TP = 16404.48

16404.48
+ 4000.00
$20,404.48

A "satisfactory" response was one that produced an acceptable monthly payment shown in dollars and cents or produced a total cost derived from rounded values (e.g., using 0.0092 for 0.0091713 in the numerator of the monthly payment formula). The sample "satisfactory" response provides an acceptable answer for the monthly payment, but fails to show the total cost of purchasing the car. If the student had completed the calculations, a correct answer would have been obtained.

Sample "satisfactory" response

$$14000\left[\frac{\left(\frac{.08}{12}\right)\overset{1.3757}{\left(1+\frac{.08}{12}\right)^{48}}}{\left(1+\frac{.08}{12}\right)^{48}-1}\right]$$

$$(.006667)($$

$$\frac{.0091713}{.3756661}$$

$$\$341.79$$

A "partial" response is one in which any of the following are correctly calculated and indicated: the monthly payment using rounded values; the numerator in the brackets, 0.0091713; and/or the amount to be financed. The following "partial" response shows a rounded monthly payment of $342. However, unlike the "satisfactory" response, this response would have led to an incorrect final answer if the student had completed the calculations, due to the intermediate rounding up to $342.

Sample "partial" response

$$MP = 14{,}000 \left[\frac{\frac{.08}{12}(1.3757)}{1.3757 - 1} \right]$$

$$MP = \$342$$

A "minimal" response is one that shows a correct substitution of the given values into the formula. In the "minimal" response shown below, the student rounded the monthly payment down incorrectly, made an error in multiplying that monthly payment by 48 months, and finally, added the selling price of the car to the monthly payments rather than the down payment, to arrive at a total cost for the car that was nearly double the $18,000 selling price.

Sample "minimal" response

0.0067

$$M.P = 14000 \left[\frac{\left(\frac{0.08}{12}\right)(1.3757)}{-0.3757} \right]$$

$$M.P = -341 \times 48 \text{ months} = 16404$$

$$+18000$$

$$34{,}404$$

negative just to indicate that your loosing money

An "incorrect" response met none of the criteria mentioned above. In the sample "incorrect" response shown below, the student relisted the information provided but did not attempt any calculations with that information.

Sample "incorrect" response

$18,000 total price
4,000 down payment
14,000
8%
48 months

The data on student performance on this question are presented in Table 3.31. Twenty-three percent of the responses were scored at least "satisfactory," but 44 percent of the responses were scored as "incorrect."

Table 3.31 — *Score Percentages for "Use Formula to Find Total Cost," Grade 12* — THE NATION'S REPORT CARD naep

Grade 12	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	9	14	7	12	44	9
Males	9	12	7	10	45	12
Female	8	16	8	14	43	7
White	10	16	8	12	40	8
Black	2	4	4	12	54	19
Hispanic	4	11	4	13	55	8
Asian/Pacific Islander	19	15	8	10	38	8
American Indian	***	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 6. Find amount saved if leased. As students encountered Question 6, they were provided with information that they were told would be relevant for Questions 6 and 7. This information included formulas for computing the cost of leasing or buying a particular car and for determining the amount of money for which the car could be resold after 4 years.

In the sixth question, students were told that Mary bought a car and sold it after 4 years. They were asked to determine the amount of money Mary could have saved if she had leased the car for 4 years instead of buying it. This question was designed to assess content from the Algebra and Functions strand and Procedural Knowledge. Students were instructed to show how they solved the problem.

> **Questions 6-7** refer to the following information.
>
> Mary is interested in leasing or buying a particular car model. She has determined that the cost of leasing this car for 4 years is $0.78p + 450$, where p is the selling price. Mary has also determined that the cost of buying this car, including finance charges, is $1.2p + 80$. At the end of 4 years, when the car is paid off, it can be sold for $0.4p$.
>
> **6.** Mary decided to buy the car at a selling price of $20,000. She kept the car for 4 years and then sold the car to recover some of the costs. If Mary had decided to lease the same car for 4 years, what amount of money would she have saved? Show the work that led to your answer.

To solve the problem, students needed to understand how to use the formulas provided, correctly substituting in the selling price of $20,000. Computing the cost of leasing is relatively straightforward, 0.78(20,000) + 450. However, computing the cost of buying involved adding the finance charges, 1.2(20,000) + 80, and also subtracting out the amount for which the car can be sold at the end of 4 years, 0.4(20,000). Finally, students had to subtract the cost of leasing from the cost of buying and reselling to get the answer of $30, the amount saved by leasing.

The responses to this question were scored on a 4-point rubric: "satisfactory," "partial," "minimal," and "incorrect." Responses scored as "satisfactory" had the correct number of dollars saved, $30, and showed an acceptable procedure for getting to that answer. The sample "satisfactory" response shows the correct procedure for calculating the cost of leasing, then the cost of buying and reselling, and, finally, the correct savings of $30.

Sample "satisfactory" response

Leasing = .78(20,000) + 450
= $16,050

buying = 1.2(20,000) + 80
$ 24080

Selling : $8000
Buying : resell $16,080

She would have saved $30

"Partial" responses satisfied one of the following criteria:

- both costs were correctly calculated, but the amount saved with leasing was not specified;
- the cost to buy the car was calculated without subtracting out the amount gained from reselling; or
- the correct amount saved, $30, was indicated, but the work involved in calculating that number was not shown.

The sample "partial" response shown below includes the correctly calculated cost of leasing and buying the car but fails to take into account the amount gained from reselling the car; it therefore gives $8,030 as the amount saved by leasing.

Sample "partial" response

$$.8(20{,}000)+450=$$
$$16{,}050$$

$$1.2(20{,}000)+80=$$
$$24{,}080$$

$$\begin{array}{r} 24{,}080 \\ -16{,}050 \\ \hline \boxed{\$8{,}030} \end{array}$$

A "minimal" response is one in which the cost of either leasing or buying is correctly calculated, or the cost to buy is correctly calculated but the gain from reselling is not taken into consideration. The sample "minimal" response correctly calculates the cost of buying, and although it has the correct formula for leasing, incorrectly calculates the cost of leasing. Furthermore, in determining the dollars saved by leasing, the student failed to take into account the amount gained from reselling.

Sample "minimal" response

$1.78p + 450 = 15680 =$ leased

$((1.2p + 90) = 24080 =$ buying

24080
-15680
$8400 saved

An "incorrect" response satisfied none of the criteria mentioned above. The following "incorrect" response correctly calculates a part of the formula for the cost of leasing, but goes no further.

Sample "incorrect" response

20,000 × .78

15,600

The information presented in Table 3.32 shows that the students did relatively well on this question. Twenty-seven percent of the responses were scored as "satisfactory," and 23 percent were scored as "partial." This means that students at least were able to calculate the cost of leasing and buying; in some cases the failure to include the gain from the resale was probably an oversight rather than inability to understand how to solve the problem.

Table 3.32 — *Score Percentages for "Find Amount Saved if Leased," Grade 12*

THE NATION'S REPORT CARD naep

Grade 12	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	27	23	20	21	5
Males	27	19	18	24	7
Females	27	27	21	18	3
White	33	24	20	16	4
Black	6	24	19	36	11
Hispanic	12	23	22	30	7
Asian/Pacific Islander	29	16	20	32	2
American Indian	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Question 7. Price lease vs. buy. In the seventh question, students were asked to find the selling price of a car for which the leasing cost for 4 years and the buying cost with a resale after 4 years are the same. As in the other questions in the question block, students also were asked to show how they got their answer. This question was classified as an Algebra and Functions question designed to assess Procedural Knowledge.

> 7. For the particular car model that Mary is interested in, determine the selling price for which the car costs the same to buy and resell at the end of 4 years as it costs to lease the car for 4 years. Show the work that led to your answer.

To answer this question most efficiently, students had to know how to set up an equation that equated the cost of leasing with the cost of buying and reselling, and to solve for the selling price of the car that would satisfy that equation. The responses to this question were scored using a 4-point rubric: "satisfactory," "partial," "minimal," and "incorrect." A "satisfactory" answer is one in which the students find the correct selling price of $18,500 either through setting up and solving the correct equation or through trial and error. In the sample "satisfactory" response shown below, the student wrote out the correct equation and calculated the selling price correctly. The student also included computations to show that, with the selling price of $18,500, both buying and leasing the car would cost $14,880.

Sample "satisfactory" response

Selling price 18500 $.78p + 450 = (1.2p\ 180) - (0.4p)$

$.78p + 450 = 0.8p + 80$

$.78p - 0.8p = 80 - 450$

$-0.02p = -370$

$p = 18500$

$.78(18500)\ 1450$ 14880

$1.2(18500)180 = 22280$

$.4(18500) - 7400$

14880

Responses scored as "partial" were those in which the correct equation was written out, but not solved, or the correct selling price was obtained, but the work contained a computational error. In the "partial" response shown, the correct answer is obtained, but there appears to be some carelessness in writing out the computations. For example, in the original equation, the student shows the resale price as being added to the selling price rather than subtracted from it. However, in the next line of the solution, the student has correctly subtracted the resale price after all.

Sample "partial" response

$$.78p + 450 = (1.2p + 80) + .4p$$

$$.78p + 450 = .8p + 80$$

$$.02p = 370$$

$$p = \$18{,}500$$

"Minimal" responses included an equation without the resale price and therefore arrival at a wrong answer; or use of the resale price, $0.4p$, incorrectly in the algebraic relationship. In the following "minimal" response, the student confused the equations for leasing and buying the car, although the resale proceeds were correctly subtracted out from what the student labeled as the cost of buying the car. The calculations were carried out correctly but, of course, the wrong answer was obtained.

Sample "minimal" response

buy & resell cost = lease cost

$$.78p + 450 - .4p = 1.2p + 80$$

$$.38p + 450 = 1.2p + 80$$

$$\frac{370}{.82} = \frac{.82p}{.82}$$

$$\boxed{\$451.22 = p}$$

An "incorrect" response is one in which none of the above criteria were satisfied. The sample "incorrect" response used the selling price of the car mentioned in Question 6 and therefore appeared to show that the student did not understand how to approach solving this problem.

Sample "incorrect" response

WP of $\frac{20000}{20000} = \frac{8030}{20000}$

$= .4015p$

$\boxed{\$8030}$

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Information on student performance on this item is presented in Table 3.33. Sixteen percent of the responses were scored as "satisfactory," while 57 percent were scored "incorrect." Student responses were more likely to be scored "satisfactory" than "partial" or "minimal," suggesting that, if they understood the question at all, they were likely to carry through to the final solution correctly.

Table 3.33 — *Score Percentages for "Price Lease vs. Buy," Grade 12*

THE NATION'S REPORT CARD naep

Grade 12	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	16	2	7	57	8
Males	17	2	7	52	11
Females	16	1	7	62	6
White	19	2	8	55	7
Black	4	1!	4	59	19
Hispanic	7	0!	3	66	9
Asian/Pacific Islander	18	3	14	57	2
American Indian	***	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
***Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Summary

The fourth-grade released Theme block was about planning a Butterfly Booth for the school's science fair. Of the six questions posed, one was classified as measuring Procedural Knowledge, the remaining five were classified as measuring Problem-Solving abilities. Questions were designed to assess content in four content strands: Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; and Algebra and Functions. In addition to solving the problems, students were generally asked to provide explanations of their answers, using mathematical computations, drawings, or words.

The following are findings of what students appear to know and are able to do based on their performance on the questions in the Butterfly Booth block:

- A majority of fourth-grade students appeared to be familiar with centimeter measurements and were able to measure using a ruler marked off in centimeters. A substantial proportion of students, however, appeared not to understand what it means to measure to the nearest centimeter.

- In most of the questions, students were required to solve a multistep problem. Students appeared to have difficulty with multistep problems, even those that required straightforward calculations at each step of the problem.

- Students appeared either to not understand what is meant by the word "symmetrical" or not understand how to use grid paper to show symmetry in a drawing.

- Students appeared to be familiar with using proportions in reasoning but, in many cases, they had difficulty carrying out proportional operations with acceptable precision.

- With the exception of the first question, most students attempted to answer the questions posed, even though large percentages produced responses that were scored as "incorrect."

- Although students' explanations were often incomplete, most students attempted to provide explanations for their answers.

- In many cases, students seemed to lack the mathematical knowledge needed to solve the problems. In other cases, students appeared to understand the underlying mathematics but provided incorrect or incomplete responses as a result of carelessness, inexperience in writing out solutions to problems, or confusion over the wording of the question.

The eighth-grade released Theme block was about building a doghouse. Of the 10 questions that students encountered in this block, 4 were designed to assess Conceptual Understanding, 1 to assess Procedural Knowledge, and 5 to assess Problem-Solving ability. The questions were classified in the content strands of Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; and Algebra and Functions. In three of the

questions, students were asked to write an explanation of how they solved a problem or to describe a mathematical operation. The following are findings of what students appear to know and are able to do based on their performance on the questions in the Doghouse block:

- Many eighth-grade students appeared to understand how to interpret and use linear dimensions.

- Some students showed understanding of the use of scales and ratios; however, for many students this understanding was not fully developed.

- Some students appeared to understand the relationship between perimeters and areas but had difficulty applying these concepts in a relatively complex problem.

- Many students appeared to lack the depth of mathematical understanding required to plan and execute solutions to the problems. Others appeared to have the requisite conceptual and procedural knowledge, but were not careful in reading the problems or precise in carrying out the operations necessary for solving them.

- Omit rates for the constructed-response questions were much higher than those observed at grade 4, particularly for the two questions at the end of the block, which also happened to be among the most difficult.

In the twelfth-grade released block, students were told that a family was considering whether to buy or lease a car. Most of the questions involved evaluating the costs of one of these options in relation to the other. Of the seven questions included in the block, six were designed to assess Procedural Knowledge and one to assess Problem-Solving ability. Four of the questions were classified as Number Sense, Properties, and Operations, and the other three were classified as Algebra and Functions. The following are findings based on student performance on the Buying-a-Car Theme block questions:

- Most students were able to solve problems involving simple computational skills. However, some of these students seemed to have difficulty with more complicated multistep problems, even those that required only simple computational skills at each step of the problem.

- Many, but not nearly the majority, of twelfth-grade students appeared able to solve an algebraic problem if they were given the equation and values for variables in the equation. However, many students appeared to have trouble reading through a relatively complicated word problem in order to isolate the information needed to solve the problem completely.

At *all* grade levels, the frequency with which students engaged in writing a few sentences about how to solve a mathematics problem, or engaged in writing reports or doing mathematics projects, was examined in relationship to student performance on the Theme blocks. Given the many recommendations to increase attention to extended-response problems in the classroom, it is perhaps significant that no positive relationship was seen between these two instructional practice variables and performance on the Theme blocks.

Chapter 4

Assessment of Performance of Students Taking Advanced Courses in Mathematics

Overview

The NAEP 1996 assessment of mathematics included a special study to examine the performance of students at the eighth- and twelfth-grade levels who were taking or had taken advanced courses in mathematics (hereafter referred to as the Advanced Study). The motivation for the Advanced Study was an observation that the main NAEP assessment did not include enough advanced mathematics questions to allow students with appropriate preparation to demonstrate the full extent of their proficiency.

For example, the main eighth-grade NAEP assessment covers the standard content of the basic eighth-grade curriculum. This means that most topics in the Algebra and Functions content strand of the NAEP framework currently are assessed at the pre-algebra level — that is, with limited use of literal terms to represent variables and limitations on the types of applications and formulas on which students can be assessed. However, a substantial percentage of eighth-grade students are currently enrolled in first-year algebra. The main NAEP assessment does not provide these students with sufficient opportunity to display what they know and are able to do in algebraic situations. A similar situation exists at the twelfth-grade level, where many students have taken pre-calculus or calculus, but are limited in what mathematical proficiency they can display on the main NAEP assessment.

As a result, the National Assessment Governing Board (NAGB) and the National Center for Educational Statistics (NCES) decided to undertake a special study to assess what these "advanced" students could do as part of the NAEP 1996 mathematics assessment. At the eighth-grade level, the questions in the Advanced Study focused on algebra, with a special emphasis on examining what students know about various representations — graphical, numerical, symbolic, and written — for algebraic concepts and relationships, and the transformations among the various representations. At grade 12, the Advanced Study included a more even distribution of questions from all of the content strands of the NAEP mathematics framework.

Administration of the Advanced Study

To qualify for the Advanced Study, eighth-grade students had to be currently enrolled in or already have taken first-year algebra or a more advanced course such as geometry. Twelfth-grade students had to be enrolled in or have already taken a pre-calculus course, a course equivalent to pre-calculus, or a more advanced course such as calculus. These qualifying students represented 21 percent of the student population at grade 8 and 24 percent of the student population at grade 12. Students at participating schools who qualified but were not selected for the Advanced Study were eligible for the main NAEP assessment, where their performance could be directly compared to that of students with less rigorous preparation. (Appendix A provides more detail on the sampling for the Advanced Study.)

At both grade levels, each student in the Advanced Study completed a special NAEP assessment booklet that consisted of three blocks of mathematics questions: two Advanced blocks and a block comprising questions selected from the main NAEP assessment. At the eighth-grade level, the questions from the main NAEP were mostly from the Algebra and Functions content strand, along with some questions from the Data Analysis, Statistics, and Probability strand. At the twelfth-grade level, the questions from the main assessment covered the more advanced content from the content strands of Algebra and Functions; Data Analysis, Statistics, and Probability; and Geometry and Spatial Sense. The block of questions from the main NAEP assessment was 15 minutes in length for both grade levels and was the first cognitive block in the Advanced Study assessment booklet. At the eighth-grade level, the two special blocks containing Advanced Study questions were each 20 minutes in length, whereas at the twelfth-grade level, the Advanced blocks were each 30 minutes long. All students participating in the Advanced Study were told they could bring their own calculators; students who did not bring a calculator were provided with a scientific calculator.

Grade Eight Advanced Study

Student background characteristics

Table 4.1 compares the background characteristics of the grade 8 students in the Advanced Study with the characteristics of the students who took the national NAEP assessment but were not eligible for the Advanced Study. Table 4.2 includes additional data about students' school types and their eligibility for income-related, school-based intervention programs.

The data in Table 4.1 reflect some differences between students who qualified for the Advanced Study and those who did not. The percentage of female students was higher in the Advanced Study than among the students who were not eligible for the Advanced Study. In terms of racial/ethnic origins, greater percentages of White students and smaller percentages of Hispanic students were in the Advanced Study. In addition, Advanced Study students were more likely to have parents who had graduated from college.

Table 4.1	Student Demographic Distributions, Grade 8, 1996	THE NATION'S REPORT CARD

	Percentage of Students	
	Advanced Study	Not Eligible for Advanced Study
Grade 8		
Gender		
Males	48	53
Females	52	47
Students who Indicated Their Race/Ethnicity as...		
White	71	62
Black	14	18
Hispanic	6	13
Asian/Pacific Islander	6	6
American Indian	2	2
Students who Reported Their Parents' Highest Level of Education as...		
Did Not Finish High School	3	8
Graduated From High School	14	23
Some Education After High School	20	19
Graduated From College	58	38
I Don't Know	5	12
Home Environment Contains...		
0–2 Types of Educational Materials	12	23
3 Types of Educational Materials	26	32
4 Types of Educational Materials	62	45
Students From...		
Northeast	27	18
Southeast	17	22
Central	30	28
West	26	31
Students Live in...		
Center City	36	34
Urban Fringe/Small City	39	35
Rural	25	31

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Paralleling the findings for parental education, the data on types of educational materials in the home (newspapers, books, encyclopedias, magazines, etc.) show that students included in the Advanced Study were more likely to have 4 such materials in their homes, while students not eligible for the Advanced Study were more likely to have 0–2 or 3 of these materials in their homes.

There were no differences in the proportions of Advanced Study or not eligible students based on region or type of community.

Student/school demographics

The data in Table 4.2 describe the types of schools students attended and present categorical information about special programs within schools for which the students may have been eligible. The data for type of school do not show any differences in the proportions of students participating in the Advanced Study relative to those eighth-grade students who were not eligible. However, the data for special programs do show differences; specifically, fewer students in the Advanced Study participated in Title I programs or were eligible for the federal Free/Reduced-Price Lunch program.

Table 4.2 — **Student/School Demographic Distributions, Grade 8, 1996**

THE NATION'S REPORT CARD naep

	Percentage of Students	
	Advanced Study	Not Eligible for Advanced Study
Grade 8		
Students who Attend...		
Public Schools	84	90
Nonpublic Schools	16	10
Title I Participation...		
Participated	3	14
Did Not Participate	97	86
Free/Reduced-Price Lunch Program Eligibility...		
Eligible	16	32
Not Eligible	63	52
Information Not Available	22	16

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Classroom content emphases

NAEP also collected information from students' teachers concerning the instructional emphases they placed on content areas in the curriculum and on developing students' mathematical process abilities. Questionnaires were used to collect this information, as well as information on the use of calculators during mathematics instruction. Data from the teachers' responses are shown in Tables 4.3 through 4.5. The percentages in the tables reflect the percentages of students whose teachers responded as shown.

The data in Table 4.3 indicate that eighth-grade students in the Advanced Study were more likely to receive "some" emphasis on the area of Numbers and Operations and less likely to receive "a lot" of emphasis on this area than students who were not eligible for the Advanced Study. In addition, the Advanced Study students were more likely to receive "a little" emphasis on Measurement in their programs and less likely to receive "a lot" of emphasis on Geometry and Spatial Sense. There were no differences in student exposure to the area of Data Analysis, Statistics, and Probability.

Grade 8 students in the Advanced Study also were reported as receiving more emphasis in the area of Algebra and Functions than students not qualifying for the Advanced Study. This is consistent with the curricular prerequisites on which study selection was based.

Table 4.3

Content Emphases in Mathematics Classes, Grade 8, 1996

THE NATION'S REPORT CARD naep

Grade 8	Percentage of Students Whose Teachers Responded			
	None	A Little	Some	A Lot
In This Mathematics Class How Often Do You Address...				
Numbers and Operations				
Advanced Study	0!	6	21	73
Non-Eligibles	0!	1	9	90
Measurement				
Advanced Study	4	39	45	13
Non-Eligibles	1	19	59	21
Geometry				
Advanced Study	6	26	54	14
Non-Eligibles	2	20	52	26
Data Analysis, Statistics, and Probability				
Advanced Study	9	33	46	12
Non-Eligibles	6	30	48	16
Algebra and Functions				
Advanced Study	0!	1	7	92
Non-Eligibles	2	9	40	49

NOTE: Row percentages may not total 100 due to rounding.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Classroom process emphases

The data in Table 4.4 relate to four general mathematical processes: problem solving, reasoning, connections, and communication. The data indicate that the students in the Advanced Study received much the same instructional emphases as other grade 8 students on learning mathematical facts and solving routine problems. However, the students qualifying for the Advanced Study were given more emphasis on solving unique problems and communicating ideas in mathematics. This finding substantiates the opportunity-to-learn differences that arise from the curricular tracking of students in the upper middle grades/junior high schools.[1]

[1] Kifer, E. (1993). Opportunities, talents, & participation. In L. Burstein (Ed.), *The IEA Study of Mathematics III: Student growth and classroom processes* (pp. 279–308). New York: Pergamon Press; Schmidt, W. H., et al. (1998). *Facing the consequences: Using TIMSS for a closer look at United States mathematics and science education*. Dordrecht, Netherlands: Kluwer Academic Publishers.

Table 4.4

Process Emphases in Mathematics Classes, Grade 8, 1996

THE NATION'S REPORT CARD naep

Grade 8	Percentage of Students Whose Teachers Responded			
	None	A Little	Some	A Lot
In This Mathematics Class How Often Do You Address...				
Learning Mathematical Facts and Concepts				
Advanced Study	2	7	13	78
Non-Eligibles	0	5	16	79
Learning Skills and Procedures Needed to Solve Routine Problems				
Advanced Study	0!	4	13	82
Non-Eligibles	1!	1	18	80
Developing Reasoning and Analytical Ability to Solve Unique Problems				
Advanced Study	0!	5	28	68
Non-Eligibles	1	8	44	46
Learning How to Communicate Ideas in Mathematics Effectively				
Advanced Study	0!	10	34	55
Non-Eligibles	1	17	43	39

NOTE: Row percentages may not total 100 due to rounding.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Calculator access and usage

NAEP also collected information on the extent to which students use calculators as part of their normal mathematics experience in schools. Table 4.5 contains the data reported by grade 8 teachers relative to student access to and use of calculators as part of their regular instructional programs. The data reflect that students in the Advanced Study had fewer restrictions on the use of calculators in their regular school mathematics programs than students not eligible for the Advanced Study. Similar results were found regarding the use of calculators on classroom tests; more than 80 percent of the Advanced Study students, compared to about two-thirds of the non-eligible students, had teachers who allowed calculators to be used in these circumstances. Responses to questions dealing with access to school-owned calculators and with instruction on the use of calculators, however, showed no differences between eighth-grade students in the Advanced Study and those who were not eligible for the study.

Table 4.5 — **Calculator Emphases in Mathematics Classes, Grade 8, 1996** — THE NATION'S REPORT CARD naep

	Percentage of Students Whose Teachers Responded	
	Yes	No
Grade 8		
Do You Permit Students in This Class Unrestricted Use of Calculators...		
Advanced Study	61	39
Non-Eligibles	42	58
Do You Permit Students in This Class to Use Calculators for Tests...		
Advanced Study	82	18
Non-Eligibles	64	36
Do the Students in This Class Have Access to Calculators Owned by the School...		
Advanced Study	81	19
Non-Eligibles	81	19
Do You Provide Instruction to Students in This Class in the Use of Calculators...		
Advanced Study	80	20
Non-Eligibles	84	16

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Performance on the Main Assessment

The results from the Advanced Study did not lend themselves to either the development of a separate proficiency scale or equating to the main NAEP mathematics scale (see Appendix A). However, as noted above, some of the students who were eligible for the Advanced Study actually participated in the main NAEP assessment. This affords the opportunity to compare directly the performance of students who were eligible for the Advanced Study and those who were not eligible, using the composite NAEP scale. The data in Table 4.6 show that students in the eighth grade who were eligible for the Advanced Study had an average NAEP score of 300, while non-eligible students had an average score of 264.

Table 4.6 — *Average Mathematics Scale Scores by Eligibility for Advanced Study, Grade 8, 1996*

THE NATION'S REPORT CARD naep

	Average Scale Score
Grade 8	
All Students	272
Eligible	300
Non-Eligible	264

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

General description of Advanced Study questions

The two blocks of Advanced Study questions at the eighth-grade level consisted of 10 and 12 questions, respectively. The questions were classified primarily in the Algebra and Functions content strand of the NAEP mathematics framework, although a few questions designed to assess concepts and procedures from the Measurement and Data Analysis, Statistics, and Probability strands also were included among the 22 questions. Furthermore, some of the questions classified as Algebra and Functions were open to solutions via Number Sense, Properties, and Operations approaches.

Six of the algebra questions required students to make an interpretation or complete an operation based on understanding symbolic representations of algebraic relationships. Nine of the questions required students to begin by considering written descriptions of algebraic relationships, possibly translate these relationships into numerical or algebraic representations, and then find solutions for the situations described. Another question was designed to measure students' understanding of and ability to use the Pythagorean theorem in an applied setting.

Table 4.7 displays the breakdown of questions in the Grade 8 Advanced Study by content strand and response structure. As noted previously, the Advanced Study at grade 8 differed from the main NAEP assessment in the distribution across content strands.[2] In addition, the Advanced Study contained a far greater percentage of questions that required students to show or explain their work than were found in the main NAEP.

Table 4.7 — ***Distribution of Questions by Content Strand and Response Format, Grade 8, 1996***

THE NATION'S REPORT CARD naep

	Number of Items
Content Strand Coverage	
Number Sense, Properties, and Operations	1
Measurement	0
Geometry and Spatial Sense	1
Data Analysis, Statistics, and Probability	2
Algebra and Functions	18
Response Formats for Items	
Multiple-Choice	9
Short Constructed-Response	11
Extended Constructed-Response	2

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

[2] Reese. C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). *NAEP 1996 mathematics report card for the nation and the states.* Washington, DC: National Center for Education Statistics.

Performance on the Advanced Study

Students' overall performance on the Advanced Study, measured by average percentage correct scores, is presented in Table 4.8. Students' average percentage correct score was 36 percent, and male students outperformed females. White, Hispanic, and Asian/Pacific Islander students outperformed Black students, and White and Asian/Pacific Islander students also outperformed Hispanic students. The sample of American Indian students was too small to permit reliable estimates of the performance on the Advanced Study blocks or on individual questions.

Table 4.8 — ***Average Percentage Correct Scores, Advanced Study, Grade 8, 1996***

THE NATION'S REPORT CARD naep

	Percentage Correct
Grade 8	
All Students	36
Males	38
Females	34
White	40
Black	19
Hispanic	28
Asian/Pacific Islander	50
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

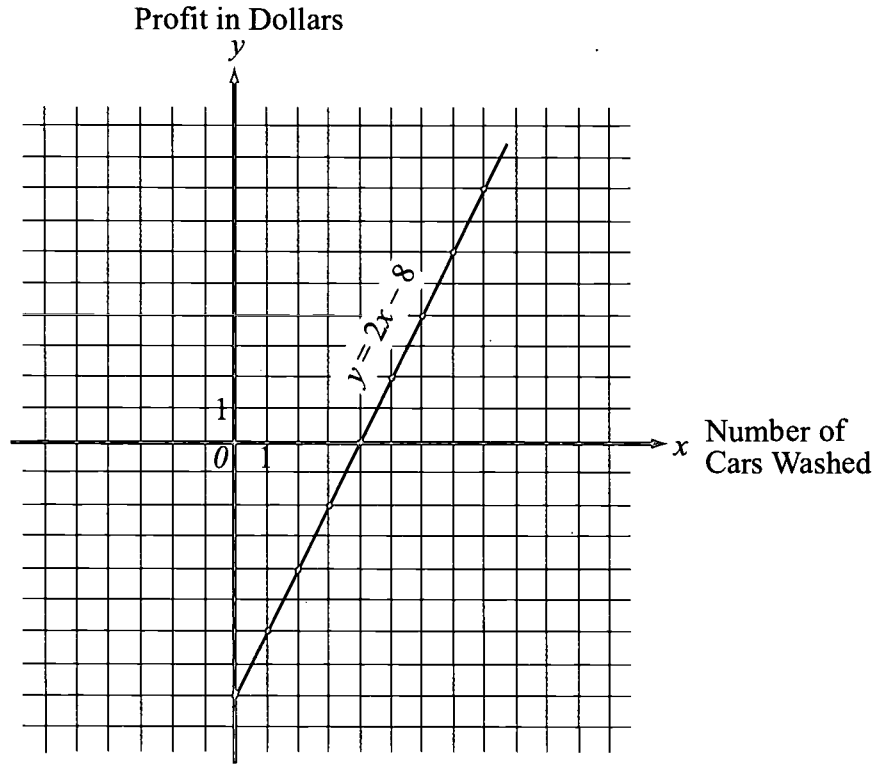
Examples of Advanced Study questions and student performance

Some sample questions from the Grade 8 Advanced Study are presented below to show how well the Advanced Study students could perform on specific tasks and to explicate the types of knowledge and skills students at the eighth-grade level needed to have to solve these problems. Information on how students performed on these questions is presented in terms of percentages correct on the individual questions. Performance information is provided for all students and by gender and race/ethnicity subgroups.

The first three sample questions constitute a family of questions based on a car wash being conducted by an eighth-grade class. The problem stimulus provided students with important data in various forms: written, graphical, and symbolic. Based on this information, students were asked to complete three different short constructed-response questions.

Questions 4–6 refer to the following information and graph.

The eighth-grade class at Carter School is going to hold a car wash to raise money for a class trip. The class determined that it had purchased enough supplies to wash at most 50 cars. The graph below shows the relationship between the number of cars washed and the profit earned in dollars. The line that can be drawn through these points is represented by the equation $y = 2x - 8$.



Profit is defined as the amount of money collected from the people whose cars are washed minus the amount of money that was spent on car wash supplies.

The first question asked students to interpret the information in the graph to find the fixed costs, or amount spent on supplies, before the first car was washed. In order to answer the question correctly, students needed to consider the y-intercept on the graph (i.e., the profit value associated with the situation before the first car was washed). They could do this either by inspecting the graph and finding the y-intercept at $y = -8$, or by substituting $x = 0$ into the equation shown in the stimulus and solving to find $y = -8$.

4. According to the graph, how much money did the class spend on car wash supplies?

_____

Explain how you found your answer.

Student responses were rated "correct," "partial," or "incorrect." The scoring rubric for the question required that students' responses indicate an understanding that initially the number of cars washed was zero (i.e., that the y-intercept must be used). That is, to receive a rating of "correct," students were required to give both the correct response of 8 and an explanation that showed a correct understanding of the y-intercept in the graph or in the equation. Two sample "correct" responses are shown below. These responses clearly indicate both a correct answer and a rationale for why that answer is correct. The first response indicates recognition of the values associated with zero cars washed. The second response contains evidence of understanding the meaning of the y-intercept in the symbolic equation presented in the stimulus of the problem.

Sample "correct" response 1

4. According to the graph, how much money did the class spend on car wash supplies?

8 DOLLARS

Explain how you found your answer. (0, -8)

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Sample "correct" response 2

4. According to the graph, how much money did the class spend on car wash supplies?

$8.00

Explain how you found your answer.

because profit equals money collected minus money spent.
The line is $y = 2x - 8$

Students responding with the value 8 and no explanation or an incorrect explanation or students responding without a value or with an incorrect value but with an explanation that indicated that they knew that the value of the y-intercept answered the question received "partial" ratings. The sample "partial" response shown below indicates a correct value, but lacks an acceptable rationale for the answer.

Sample "partial" response

4. According to the graph, how much money did the class spend on car wash supplies?

8

Explain how you found your answer.

The last point on the graph is 8.

160

Students simply responding – 8 for the amount spent on car wash supplies received a rating of "incorrect." Student responses that contained an incorrect value and an explanation that indicated a lack of understanding that the answer should be found in the value of the y-intercept also received an "incorrect" rating. The "incorrect" response shown demonstrates such a lack of understanding.

Sample "incorrect" response

4. According to the graph, how much money did the class spend on car wash supplies?

$ 9.75

Explain how you found your answer.

I counted up the y axis, there were 9 boxes and about 3/4 left over.

Table 4.9 presents the breakdown of student performance according to gender and race/ethnicity. Overall, 13 percent of the student responses were rated "correct," 26 percent were rated "partial," and 36 percent were rated "incorrect." Twenty-one percent of the students did not respond to the question, and an additional four percent, not shown in the table, only wrote off-task remarks on their papers.[3]

[3] Student responses for this and all other constructed-response questions were scored as "off task" if the student provided a response that was deemed not related in content to the question asked. There are many examples of these types of responses, but a simple one would be "I don't like this test." In contrast, responses scored as "incorrect" were valid attempts to answer the question that were simply wrong.

Table 4.9

Score Percentages for "Car Wash Supplies," Grade 8

THE NATION'S REPORT CARD

Grade 8	Correct	Partial	Incorrect	Omit
All Students	13	26	36	21
Males	15	30	32	20
Females	10	23	41	21
White	15	30	34	19
Black	3!	14!	46!	32!
Hispanic	10	23	40	23
Asian/Pacific Islander	14	33	35	17
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The second question in this three-question family asked students to find the number of cars that must be washed before the class would start to earn a profit. In order to answer the question correctly, students needed to find the value of x for which $y = 0$. This value could be found by locating the x-intercept of the graph (i.e., $x = 4$), or by solving $y = 2x - 8$ for $y = 0$. Some students also responded with the value of 5 cars, as this is the first point at which there is a positive profit. Values of either 4 or 5 were accepted as correct numerical answers for the value portion of the question, and correct corresponding explanations were accepted for the explanation portion of the question.

5. How many cars do the students have to wash before the class would start to earn a profit?

_____

Explain your answer.

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Student responses were rated as "correct," "partial," or "incorrect" according to the following rubric, or rating scale. To receive a rating of "correct" a response had to contain both the correct answer and an explanation of why it was the right value. Responses received a rating of "partial" when they contained a numerical answer of 4 or 5 without a supporting explanation, or when they contained a number other than 4 or 5 but demonstrated an understanding that the x-intercept must be used. Responses were rated "incorrect" when students failed to give either the correct answer or an explanation of how to get a correct answer.

The "correct" response shown below reflects the work of a student who recognized that washing four cars would pay for the supplies and that washing a fifth car would be needed to achieve a positive profit.

Sample "correct" response

5. How many cars do the students have to wash before the class would start to earn a profit?

4 cars to break even 5 to have some profit

Explain your answer.

x = the amount of cars you washed
2 = how much they were paid for each car
To make up for the money they spent on supplies they would have to wash 4 cars at 2 dollars each (8 total) Then to gain a profit they would have to wash another car (5 cars total washed) and have a $2 profit.

The "partial" response that follows shows a correct numerical answer but has an explanation that contains errors.

Sample "partial" response

5. How many cars do the students have to wash before the class would start to earn a profit?

5

Explain your answer.

Because when the line on the graph comes above 0 they have made a profit and the line comes up to 0 they have washed 5 cars

Information on student performance is presented in Table 4.10. Overall, 21 percent of students submitted responses rated "correct," 19 percent submitted responses rated "partial," and 39 percent turned in responses rated "incorrect." Another 18 percent of the students omitted the question. Performance on this question was somewhat better than performance on the previous problem. In this case, compared to the preceding problem, students had to make less of a conceptual leap to interpret what the problem was asking them to do. The problem also was more open to being solved through numerical reasoning, $2 \times [\ ] = 8$, and therefore, was more straightforward than the fixed-costs problem. There was some ambiguity in the phrasing of the question, however, which required acceptance of either 4 or 5 as the correct answer.

Table 4.10	Score Percentages for "Begin to Earn Profit," Grade 8	THE NATION'S REPORT CARD naep

Grade 8	Correct	Partial	Incorrect	Omit
All Students	21	19	39	18
Males	23	36	16	
Females	19	16	42	20
White	24	22	36	15
Black	7!	6!	53!	32!
Hispanic	16	14	49	19
Asian/Pacific Islander	30	14	38	17
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The third question in the family, also rated on a three-point scale, asked students to find the greatest profit (in dollars) that the class could expect to make from the car wash. To solve this problem, students needed to find the profit associated with $x = 50$, the maximum number of cars that could be washed. By substituting into the equation $y = 2x - 8$, they could find that the maximum profit that could be expected was $92.

6. What is the greatest profit (in dollars) that the class can expect to earn?

_____

Show your work.

165

To receive a rating of "correct," a student only had to give the correct answer of $92. A supporting rationale was not required for the response to be rated "correct" even though the question asked students to show their work. The scoring rubric allowed raters to give partial credit for an incorrect value if the correct process was shown. While many of the students who received full credit simply wrote in $92 or showed something like 2(50) – 8 = 92, the "correct" response shown below is from a student who clearly communicated his or her approach as well as giving the correct solution.

Sample "correct" response

6. What is the greatest profit (in dollars) that the class can expect to earn?

$92.00

Show your work. equation: y = 2x - 8

They expect to wash at the most 50 cars. When they wash 50 cars they make $100. But it cost them $8.00 to buy supplies, so to find their profit you subtract the money they used for supplies from the money they collected from washing cars.

100 - 8 = $\boxed{92}$ → profit

Students who used the equation and made a correct substitution, but made an error in calculation, were awarded a rating of "partial." For example, some divided the 92 by 2 or incorrectly multiplied 2 × 50 to get 200 and then subtracted 8 to arrive at an answer of 192, as in the "partial" response shown.

Sample "partial" response

6. What is the greatest profit (in dollars) that the class can expect to earn?

192

Show your work.

$y = 2x - 8$

$y = 2(50) - 8$

$y = 200 - 8$

$y = 192$

Student responses were rated "incorrect" if they neither found the value nor gave a method for finding that value. Some students receiving "incorrect" ratings omitted the start-up costs and simply multiplied 50 × 2 to arrive at an answer of $100; others attempted to estimate the answer, as shown in the following "incorrect" response.

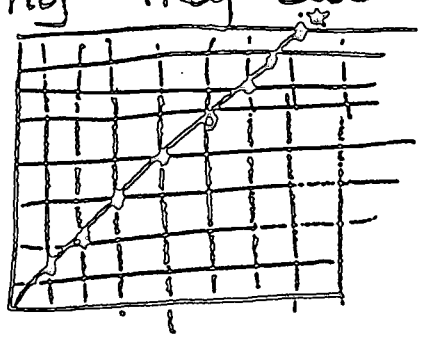
Sample "incorrect" response

6. What is the greatest profit (in dollars) that the class can expect to earn?

$80.00

Explain your answer.

The graph on the "y" axis is positive it goes up about 8 squares assuming they each represent 10.



8 · 10 = $80.00

Information on student performance is presented in Table 4.11. Overall, 27 percent of responses were rated "correct," 4 percent were rated "partial," and 40 percent were rated "incorrect." Twenty-six percent of the students omitted the problem. On this question, a higher percentage of students submitted responses that were rated "correct," but there was a slight decrease in overall performance ("correct" and "partial") compared with the two previous questions. The increase in responses receiving a "correct" rating could be due to the fact that the student could read the problem and then simply substitute into the equation to find the answer. While the question is open to solution by numerical patterns or an extension of the graph, these are less likely approaches. It also is possible that the score distribution reflects the decision to give full credit for a correct numerical answer without requiring that work be shown.

Table 4.11	Score Percentages for "Greatest Profit Expected," Grade 8	THE NATION'S REPORT CARD naep

	Correct	Partial	Incorrect	Omit
Grade 8				
All Students	27	4	40	26
Males	29	4	39	24
Females	25	4	41	27
White	30	4	40	22
Black	12!	2!	40!	44!
Hispanic	20	5	44	28
Asian/Pacific Islander	36	3	33	26
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The fourth sample question presented here for grade 8 is one that asked students to consider a situation in which one hot-air balloon is ascending and another balloon is descending. Students were given the rate of change in elevation and the initial starting point for each balloon and were asked to find the time required for the two balloons to reach the same altitude. The problem is most readily solved through the use of a single equation $3t = 1000 - 5t$ or the system of equations: $y = 3t$ and $y = 1000 - 5t$. However, a student could solve the problem through a table of values and estimation or through experimentation.

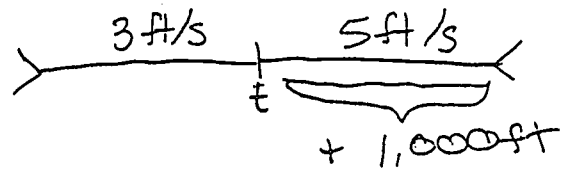
> **7.** A hot-air balloon begins rising at the rate of 3 feet per second. At the same time, a second hot-air balloon that is 1,000 feet above the first balloon begins to descend at the rate of 5 feet per second. In how many seconds will the balloons reach the same altitude?

Students' solutions to this problem were rated on a three-point scale. "Correct" responses gave the correct numeric response of 125 seconds with or without an explicit rationale. Responses that reflected work that did more than restate the problem but failed to achieve a correct answer for the time in seconds were rated "partial." The rating of "incorrect" was given to responses that contained neither a correct value nor a correct equation or formulation of the problem.

While many "correct" responses resulted from students simply solving $3t = 1000 - 5t$, the sample "correct" response shows both the modeling that led to the development of the equation and the equation-solving approach employed.

Sample "correct" response

7. A hot-air balloon begins rising at the rate of 3 feet per second. At the same time, a second hot-air balloon that is 1,000 feet above the first balloon begins to descend at the rate of 5 feet per second. In how many seconds will the balloons reach the same altitude?



	d =	r ×	t
A	$3t$	3	t
D	$-5t + 1{,}000$	-5	t

$$3t = -5t + 1{,}000$$

$$\frac{8t}{8} = \frac{1{,}000}{8}$$

$$t = 125 \text{ s.}$$

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The following "partial" response demonstrates some effort toward working out the problem but a failure to obtain the correct answer.

Sample "partial" response

7. A hot-air balloon begins rising at the rate of 3 feet per second. At the same time, a second hot-air balloon that is 1,000 feet above the first balloon begins to descend at the rate of 5 feet per second. In how many seconds will the balloons reach the same altitude?

1,000 feet above

ascend		descend	
3ft per	1 second	5 feet per	1 second
6ft "	2 "	10 "	" 2 "
9ft "	3 "	15 "	" 3 "
12 ft "	4 "	20 "	" 4 "
15 ft "	5 "	25 "	" 5 "
18ft "	6 "	30 "	" 6 "

The results, presented in Table 4.12, show that 19 percent of the responses received a "correct" rating, 7 percent received a "partial" rating, and 58 percent received an "incorrect" rating. In addition, 13 percent of the students omitted the question. Students used a number of approaches in answering this question. While the use of a system of equations or a single equation that equated the two ways of representing height at the time desired, $3t = 1000 - 5t$, were most common, a number of numerical approaches were noted among the Advanced Study students.

Table 4.12 — Score Percentages for "Hot Air Balloon," Grade 8

THE NATION'S REPORT CARD naep

	Correct	Partial	Incorrect	Omit
Grade 8				
All Students	19	7	58	13
Males	27	6	52	13
Females	12	8	64	13
White	23	8	57	10
Black	2!	4!	68!	25!
Hispanic	14	4	60	18
Asian/Pacific Islander	24	10	51	12
American Indian	***	***	***	***

NOTE: Raw percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Grade Twelve Advanced Study

Student background characteristics

The data in Table 4.13 describe the student background characteristics of the grade 12 students in the Advanced Study compared to those of students in the main NAEP assessment who had not taken advanced courses and were therefore not eligible for the Advanced Study. Compared to the non-eligible group, higher percentages of Asian/Pacific Islander and White students and a lower percentage of Black students participated in the Advanced Study. In addition, students in the Advanced Study were more likely to have parents who had graduated from college.

Similar to the findings at grade 8, the data on types of educational materials in the home (newspapers, books, encyclopedias, magazines, etc.) show that students included in the Advanced Study were more likely to have 4 such materials in their homes, while students not eligible for the Advanced Study were more likely to have 0–2 of these materials in their homes. There were no significant differences in Advanced Study participation based on region or type of community.

Table 4.13	*Student Demographic Distributions, Grade 12, 1996*	THE NATION'S REPORT CARD

	Percentage of Students	
	Advanced Study	Not Eligible for Advanced Study
Grade 12		
Gender		
Males	51	48
Females	49	52
Students who Indicated Their Race/Ethnicity as...		
White	74	68
Black	7	15
Hispanic	8	11
Asian/Pacific Islander	10	4
American Indian	0	2
Students who Reported Their Parents' Highest Level of Education as...		
Did Not Finish High School	4	7
Graduated From High School	13	22
Some Education After High School	23	26
Graduated From College	59	41
I Don't Know	2	3
Home Environment Contains...		
0–2 Types of Educational Materials	13	20
3 Types of Educational Materials	25	27
4 Types of Educational Materials	62	53
Students From...		
Northeast	25	22
Southeast	21	23
Central	28	24
West	26	30
Students Live in...		
Center City	32	31
Urban Fringe/Small City	38	38
Rural	29	31

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Student/school demographics

Information on type of school and student participation in categorical programs is shown in Table 4.14. When these data are considered, more differences between the groups begin to emerge. A larger proportion of grade 12 students participating in the Advanced Study were from nonpublic schools than were students not eligible for the Advanced Study. The two groups did not differ in terms of participation in Title I programs, but fewer Advanced Study students were eligible for the federal Free/Reduced-Price Lunch program.

Table 4.14 — *Student/School Demographic Distributions, Grade 12, 1996*

THE NATION'S REPORT CARD naep

	Percentage of Students	
	Advanced Study	Not Eligible for Advanced Study
Grade 12		
Students who Attend...		
Public Schools	82	91
Nonpublic Schools	18	9
Title I Participation...		
Participated	2	2
Did Not Participate	98	98
Free/Reduced-Price Lunch Program Eligibility...		
Eligible	7	15
Not Eligible	56	62
Information Not Available	37	23

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Classroom content emphases

Table 4.15 contains the data from teachers' responses to questions requesting an indication of the amount of emphasis they placed on each of a number of subject matter content areas within the grade 12 curriculum. Note that the data shown here are only from the teachers who were teaching students in the Advanced Study; teachers of twelfth-grade students who took the main NAEP assessment were not surveyed. The data shown in Table 4.15 indicate that, in classes taken by Advanced Study participants, there was more "heavy emphasis" placed on Functions than on any other topic. Algebra and Trigonometry ranked next and did not differ much from each other in frequency of "heavy emphasis." Geometry, at 23 percent, received "heavy emphasis" less often than Functions, Algebra, or Trigonometry, but more often than Statistics, Probability, or Discrete Mathematics.

Table 4.15 — **Content Emphases in Classes Taken by Advanced Study Students, Grade 12, 1996**

THE NATION'S REPORT CARD naep

	Percentage of Students Receiving		
	Little Emphasis	Moderate Emphasis	Heavy Emphasis
Grade 12			
How Much Emphasis on...			
Algebra	4	23	73
Geometry	14	63	23
Trigonometry	5	28	67
Functions	2	13	85
Statistics	65	25	10
Probability	66	25	9
Discrete Mathematics	60	34	6

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Classroom process emphases

In Table 4.16, a second set of data from classroom teachers of Grade 12 Advanced Study students describes the emphases teachers reported placing on process-related activities. Specifically, teachers were questioned about four common mathematical processes: problem solving, reasoning, connections, and communication. Most students had teachers who reported that they placed "heavy emphasis" on the first three areas: facts and concepts, skills and procedures for solving routine problems, and reasoning and analysis for solving nonroutine problems. However, fewer students had teachers who reported a "heavy emphasis" on

preparing students to communicate the results of their mathematical endeavors. This latter finding is consistent with classroom practices data reported by twelfth-grade students who participated in the main NAEP assessment.[4]

Table 4.16 — ***Process Emphases in Classes Taken by Advanced Study Students, Grade 12, 1996*** — THE NATION'S REPORT CARD naep

	Percentage of Students Receiving		
	Little Emphasis	Moderate Emphasis	Heavy Emphasis
Grade 12			
How Much Emphasis on...			
Learning Mathematical Facts and Concepts	2	20	78
Learning Skills and Procedures Needed to Solve Routine Problems	1	22	77
Developing Reasoning and Analytical Ability to Solve Unique Problems	2	25	73
Learning How to Communicate Ideas in Mathematics Effectively	10	47	43

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Calculator access and usage

Data on calculator access and usage for students in the Advanced Study also were collected from their teachers. Table 4.17 presents findings about calculator access. The data reflect an even level of access to both scientific and graphing calculators in the classroom, but greater access to scientific calculators outside the classroom. From a different perspective, the data indicate that 80 percent of Advanced Study students were permitted unlimited access to calculators in their mathematics classes. However, the percentage of students having access to calculators during assessment sessions was somewhat lower, with teachers of 71 percent of the students reporting that their students always could use calculators on tests; the remainder of the teachers reported that their students were sometimes permitted to use calculators on tests.

[4] Reese, et al., (1997). op. cit.

Table 4.17

Calculator Access in Classes Taken by Advanced Study Students, Grade 12, 1996

THE NATION'S REPORT CARD

Grade 12	Percentage of Students
Do You Permit Students in This Class Unrestricted Use of Calculators...	
Yes	80
No	20
Do Students in This Class Have Access to Scientific Calculators in Class...	
Yes	83
No	17
Do Students in This Class Have Access to Scientific Calculators Out of Class...	
Yes	82
No	18
Do Students in This Class Have Access to Graphing Calculators in Class...	
Yes	84
No	16
Do Students in This Class Have Access to Graphing Calculators Out of Class...	
Yes	56
No	44
Do You Permit Students in This Class to Use Calculators on Tests...	
Yes All	71
Yes Some	28
No	0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Calculator instruction

Table 4.18 contains more detail about calculator instruction and graphing calculators. The responses indicate that more students received instruction in how to use graphing calculators than scientific calculators. The larger extent of training being devoted to graphing calculators may be due partially to their special uses in these classes and partially to their newness in grade 12 classrooms.

Over 70 percent of the students in Advanced Study classrooms had full access to a graphing calculator for their study of mathematics, either from a complete set in the classroom or through a personally owned graphing calculator. Reportedly these calculators were primarily utilized for their unique capability — that is, graphing functions.

Table 4.18	Calculator Usage and Instruction in Classes Taken by Advanced Study Students, Grade 12, 1996

THE NATION'S REPORT CARD naep

Grade 12	Percentage of Students
In This Class Students are Provided Instruction in the Use of Scientific Calculators...	
Yes	65
No	35
In This Class Students are Provided Instruction in the Use of Graphing Calculators...	
Yes	87
No	13
Which of the Following Best Describes the Availability of Graphing Calculators in This Class...	
One	1
Less Than Six	3
Complete Set	48
Some Students Have One	8
Most Students Have One	9
All Students Have One	24
No Student Has One	6
If Graphing Calculators are Used in This Class, What is Their Primary Usage...	
Calculating	7
Graphing	76
Tables	0
Statistics	1
Symbolic Manipulation	3
Not Used	13

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Performance on the Main Assessment

Table 4.19 compares performance on the composite NAEP mathematics scale for grade 12 students in the main assessment who were, or were not, eligible for the Advanced Study. As can be seen, students who were eligible for the Advanced Study had an average scale score of 327, while non-eligible students had a lower average score of 297.

Table 4.19 — **Average Mathematics Scale Scores by Eligibility for Advanced Study, Grade 12, 1996**

THE NATION'S REPORT CARD naep

	Average Scale Score
Grade 12	
All Students	304
Eligible	327
Non-Eligible	297

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

General description of the Grade 12 Advanced Study questions

The two blocks of Advanced Study questions for grade 12 students each consisted of 11 questions. Although the questions were advanced in nature, no questions required calculus. Most of the questions could be solved by several methods.

The 22 questions were balanced among situations that were presented in symbolic, graphical, written, and visual formats. Many of the questions required students to make connections between two or more forms of representation in order to complete the requirements set by the question.

Table 4.20 displays the breakdown of questions by content strand and response structure. All five NAEP mathematics content strands were represented in the Advanced Study. Compared to the main NAEP assessment, however, the Advanced Study included considerably higher percentages of questions classified in the Algebra and Functions; Data Analysis, Statistics, and Probability; and Geometry and Spatial Sense strands; there also was a far greater percentage of questions that required students to show or explain their work.[5]

Table 4.20 **Distribution of Questions by Content Strand and Response Format, Grade 12, 1996**

THE NATION'S REPORT CARD naep

	Number of Items
Content Strand Coverage	
Number Sense, Properties, and Operations	1
Measurement	2
Geometry and Spatial Sense	7
Data Analysis, Statistics, and Probability	5
Algebra and Functions	7
Response Formats for Items	
Multiple-Choice	7
Short Constructed-Response	10
Extended Constructed-Response	5

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Performance on the Advanced Study

Students' overall performance on the Grade 12 Advanced Study, measured by average percentage correct scores, is presented in Table 4.21. For the group as a whole, the average percentage correct score was 30 percent. Male students outperformed female students, and White and Asian/Pacific Islander students outperformed Black and Hispanic students. The sample of American Indian students was too small to permit reliable estimates of the performance on the Advanced Study blocks or on individual questions. Students who were currently taking mathematics or who were taking, or had taken, an AP course in mathematics outperformed students who were not currently taking a mathematics course or who had not taken an AP course in this subject area.

[5] Ibid.

Table 4.21	Average Percentage Correct Scores, Advanced Study, Grade 12, 1996	THE NATION'S REPORT CARD naep

Grade 12	Percentage Correct
All Students	30
Males	32
Females	27
White	32
Black	14
Hispanic	19
Asian/Pacific Islander	32
American Indian	***
Are Students Presently Enrolled in Mathematics...	
Yes	30
No	18
Are Students Presently Enrolled in or Have Previously Taken an Advanced Placement (AP) Mathematics Course...	
Yes	37
No	26

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Examples of individual questions and student performance

Four sample questions from the Grade 12 Advanced Study are presented below to show the types of knowledge and skills students at the twelfth-grade level needed to solve the Advanced Study problems. As before, student performance information is presented in terms of percentages correct on the individual questions. The subgroup comparisons for grade 12 include gender, race/ethnicity, whether students are presently enrolled in mathematics, and whether students are enrolled in or have taken an AP mathematics course.

The first sample question asked students to find a third value for a linear function, f, given two other values for the function. Students were asked to show their work. In order to solve the problem, students first had to realize that they were being asked to find the third point on a straight line, given two other points on the line. Students then could have proceeded to find the slope of the line connecting the two given points graphically or by using the equation for a straight line. The value of the slope, along with the coordinates of the given points, one of which is the y-intercept, allows for an algebraic construction of an equation for the line. Solving this equation for the value $x = 3.8$ results in the correct answer of 1.58 for $f(3.8)$. Other students might have approached the problem graphically and more geometrically and used proportions such as $0.35/1 = h/3.8$ to find h, which they then added to the value of the y-intercept to get the required value of 1.58.

> 3. If f is a linear function such that $f(0) = 0.25$ and $f(1) = 0.6$, what is the value of $f(3.8)$? Show your work.

Student responses were rated "correct," "partial," or "incorrect." Responses of 1.58 or 1.6 with correct accompanying work were rated "correct." If no work was shown, the answer had to be 1.58 in order to be considered "correct." Responses were rated "partial" if they contained some form of the equation $y = 0.35x + 0.25$ *or* correctly found the values of $f(3) = 1.3$ and $f(4) = 1.65$ *or* correctly found the slope, $m = 0.35$. All other responses were rated "incorrect."

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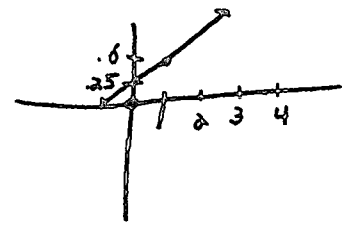
One example of a "correct" student response is shown below. Here the student correctly calculated the slope and then used the slope value and one of the given ordered pairs to find an equation of the line. Finally, the value of $f(3.8)$ was computed correctly.

Sample "correct" response

3. If f is a linear function such that $f(0) = 0.25$ and $f(1) = 0.6$, what is the value of $f(3.8)$? Show your work.

$(0, .25)$
$(1, .6)$

$$\frac{.35}{1}$$



$y - .6 = .35(x - 1)$

$y - .6 = .35x - .35$

$y = .35x + .25$

$y = .35(3.8) + .25$

1.58

One response that received a "partial" rating follows. The student correctly calculated the slope as 7/20 but did not complete the problem.

Sample "partial" response

3. If f is a linear function such that $f(0) = 0.25$ and $f(1) = 0.6$, what is the value of $f(3.8)$? Show your work.

$(0, \frac{1}{4})$ $(1, \frac{3}{5})$ $\frac{\frac{3}{5} - \frac{1}{4}}{1 - 0}$ $\frac{\frac{2}{20}}{1} = \frac{2}{20}$

$(3.8, y) = \frac{2}{20}$

The data in Table 4.22 reflect student performance on the linear functions question. Twenty percent overall submitted a response rated "correct," and another 11 percent received partial credit. This performance is somewhat disappointing, given that the question covers basic concepts of linear functions. Common errors included reversing values for the independent and dependent variables, perhaps indicating a fundamental misunderstanding of the functional notation, and reversing the definition of a slope. Some students incorrectly defined the slope as the change in x over the change in y.

Table 4.22 — **Score Percentages for "Use Linear Function," Grade 12**

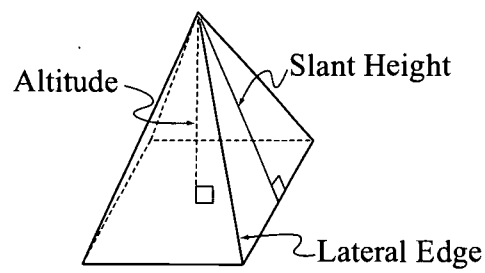
THE NATION'S REPORT CARD naep

	Correct	Partial	Incorrect	Omit
Grade 12				
All Students	20	11	43	23
Males	25	10	38	23
Females	16	11	48	23
White	23	11	42	21
Black	4	8	44	43
Hispanic	11!	6!	46!	32!
Asian/Pacific Islander	22	12	42	22
American Indian	***	***	***	***
Are Students Presently Enrolled in Mathematics...				
Yes	21	11	43	23
No	9!	4!	46!	37!
Are Students Presently Enrolled in or Have Previously Taken an Advanced Placement (AP) Mathematics Course...				
Yes	31	12	38	17
No	13	10	46	27

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The second sample question from the Grade 12 Advanced Study asked students to rank order the volumes of three square pyramids from smallest to largest and supply a rationale for their ranking. As data, students were given one of the measurements of altitude, slant height, or lateral edge for each pyramid and told that all pyramids resided on a square base with sides of 10 units. Students also were given the general formula for the volume of a pyramid.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all of your work.



10. The figure above shows the altitude, lateral edge, and slant height of a square pyramid. Each of three square pyramids P, Q, and R (not shown) has a base with side 10. In each pyramid the lateral edges are of equal length. Pyramids P, Q, and R have the following characteristics.

    a. The altitude of pyramid P is 10.

    b. The lateral edge of pyramid Q is 10.

    c. The slant height of pyramid R is 10.

    List the pyramids by order of volume from smallest volume to largest volume.

    Support your conclusion with mathematical evidence.

    [The formula for the volume of a pyramid with base area B and height (altitude) h is $V = \frac{1}{3}Bh$.]

Since the area of the base was the same for all three pyramids, the solution to the problem was dependent on the height, or altitude (h), of the pyramids. The height of pyramid P was given as 10 units. The height of pyramid R could be found by recognizing that the distance from the foot of the segment representing the slant height to the middle of the base was 5 units and then using the Pythagorean theorem to find the height of the pyramid, namely $5\sqrt{3}$, or approximately 8.66 units. The height of pyramid Q also could be found by using the Pythagorean theorem. In this case, however, students had first to solve for either the slant height or the length of the segment running from the base of the lateral edge to the foot of the altitude. Once either of these dimensions had been determined, it was possible to solve for the altitude, or height. The correct solution was $5\sqrt{2}$, or approximately 7.07 units.

Since all three pyramids had the same base area, and their volumes could be found by multiplying the area of the bases by one-third of the heights, the volumes were ordered from smallest to largest in the same order as the heights were ordered from shortest to longest. That is, $V_Q < V_R < V_P$, since $h_Q < h_R < h_P$.

A few students noted that there is a quicker way to solve the problem without doing any calculations. Because the hypotenuse of a right triangle is the side of greatest length, it can be argued that the lateral edge is greater than the slant height, which in turn is greater than the altitude. Consequently, the pyramid with a lateral edge of 10 is "shorter" than the pyramid with a slant height of 10, which is "shorter" than the pyramid with an altitude of 10. Since "shorter" relates to the height of the pyramids, and since all of the pyramids are on the same base, the correct ordering follows.

Responses were rated "extended," "satisfactory," "partial," "minimal," and "incorrect." The percentage of students achieving each rating is shown in Table 4.23, which follows the presentation of the sample responses.

Only responses that gave the correct ordering of the volumes of the three pyramids along with some supporting evidence received a rating of "extended." In addition, if the student did not indicate that the answer was only dependent on height, the numerical values of all three volumes were required to be correct. Only five percent of the students participating in the Advanced Study reached this level of performance. The following response is an example of an "extended" response.

Sample "extended" response

V of P = 1/3 Bh B = 100 h = 10

$V_P = \frac{1000}{3} \doteq 333$

V of Q = 1/3 Bh B = 100 Le = 10 so

$Sh = \sqrt{75}$, so $h = \sqrt{50}$

$\frac{100\sqrt{50}}{3} = 236$

$V_R = 1/3\ Bh$ B = 100 sh = 10 so h = √

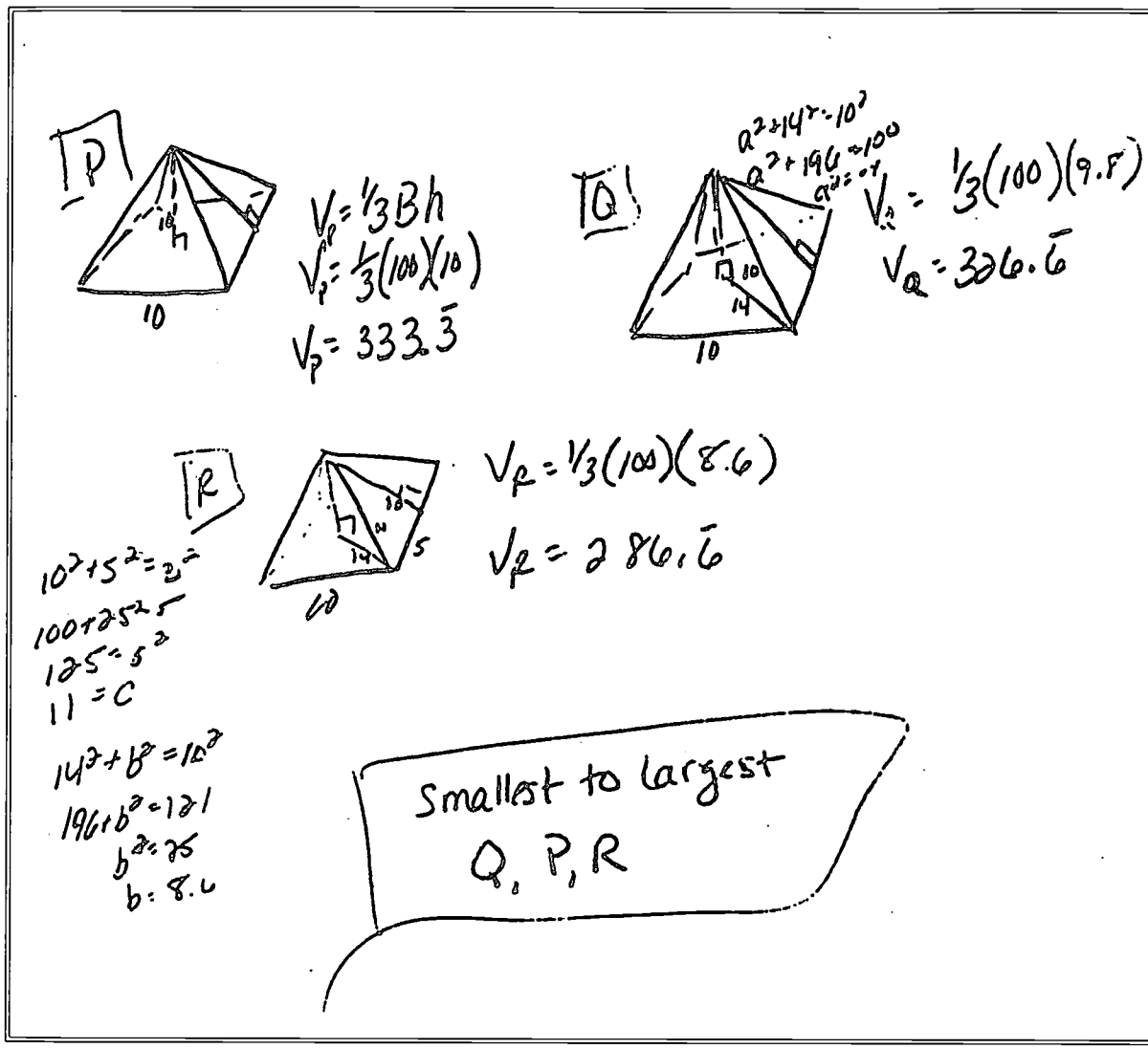
$V_R \doteq 289.$

$V_Q < V_R < V_P$

Because the area of the base is equal, it can be cancelled as can the 1/3. So, Therefore, the volume is directly proportional to the height.

Responses that failed to order the three pyramids correctly but that provided correct solutions for the volumes of two or three pyramids, or for the heights of Q and R, or that correctly compared the heights of Q, P, and R, were rated "satisfactory." The following response was rated "satisfactory" because the volumes of pyramids P and R are correct.

Sample "satisfactory" response



Responses were rated "partial" if they showed a correct solution for either the height or the volume of pyramid Q or R. The following response was rated "partial" because 288.7 is the volume of pyramid R.

Sample "partial" response

$V = \frac{1}{3}(100)(10)$

$V = 33\frac{1}{3} \Rightarrow P$

$V = \frac{1}{3}(100)(5\sqrt{3})$

$V = 288.7$

$x^2 + 25 = 50$

$x = 5$

$V = \frac{1}{3}100(5)$

$V = 166\frac{2}{3} \Rightarrow Q$

$5^2 + 5^2 = h^2$

$h = 5\sqrt{2}$

$100 + 25 = h^2$

$125 = h^2$

$h = 5\sqrt{5}$

$25 + x^2 = 100$

$x^2 = 75$

$x = 5\sqrt{3}$

smallest = P

Q

R

190

Responses that showed or stated that the three pyramids had the same base area or that $\frac{1}{3}$ (area of the base) was the same for all three pyramids were rated "minimal." Responses that contained a correct solution for the volume of pyramid P also received a "minimal" rating. The following response was rated "minimal," since it clearly states that the bases of the three pyramids are equal. The area of the bases, however, should have been shown as 100, not 10.

Sample "minimal" response

$V = \frac{1}{3}Bh$

$B = 10$

Base of pyramid Q = Base of pyramid P = Base of pyr. R
Lateral edge " " lateral " " = lateral " " "

$V = \frac{1}{3}BH$

Pyramid P: Base = 10
Alt = 10
lateral = 10

$V = \frac{1}{3} \cdot 10 \cdot 10$

$V = 33.\overline{3}$

Pyramid Q

$V = \frac{1}{3} \cdot 10 \cdot 10$

$= 33.3$

Pyramid R

$V = \frac{1}{3} \cdot 10 \cdot 10$

$= 33.3$

Responses that attempted to answer the question, but did not satisfy the conditions for at least a "minimal" response, were rated "incorrect."

Performance data on the square pyramid question are presented in Table 4.23. This question was very challenging for students because it involved a significant number of steps. Students showed a number of approaches to solving the problem. Most students whose responses were rated at least "satisfactory" approached the problem analytically, similar to the sample responses shown previously. The fact that only 10 percent of the students received "satisfactory" or above is somewhat disappointing but may be explained by the fact that many students might not have worked with geometric volume problems since they took geometry, possibly as long as 2 or 3 years before the assessment.

Table 4.23 — *Score Percentages for "Compare Volumes of Pyramids," Grade 12*

THE NATION'S REPORT CARD naep

Grade 12	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	5	5	5	11	52	20
Males	6	6	5	12	46	22
Females	4	3	5	9	59	18
White	5	5	5	12	52	18
Black	0!	1!	1	6	58	32
Hispanic	3!	2!	1!	5!	57!	30!
Asian/Pacific Islander	8	6	5	9	51	18
American Indian	***	***	***	***	***	***
Are Students Presently Enrolled in Mathematics...						
Yes	5	5	5	11	53	19
No	2!	2!	1!	9!	46!	35!
Are Students Presently Enrolled in or Have Previously Taken an Advanced Placement (AP) Mathematics Course...						
Yes	8	8	6	13	47	16
No	2	3	4	9	56	22

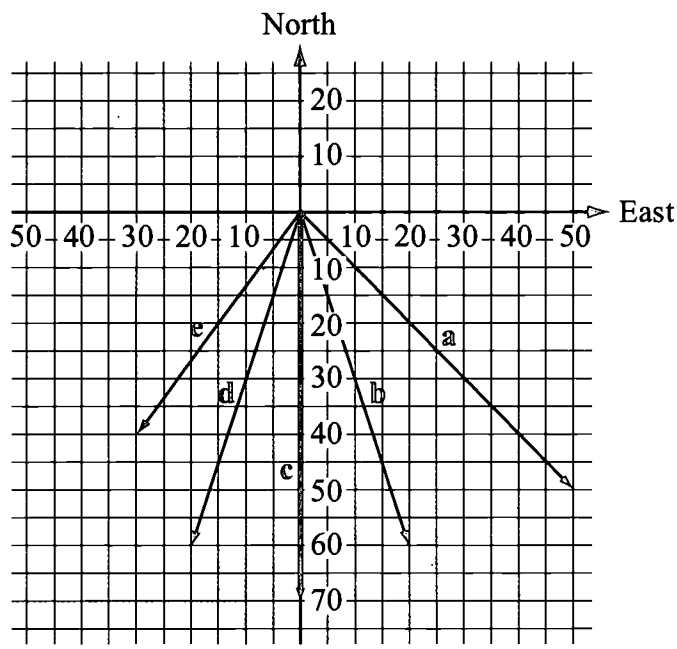
NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.

*** Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The third sample question from the Grade 12 Advanced Study is a multiple-choice question assessing students' ability to select the vector that correctly represented the result of a ship's movement from the origin of a coordinate grid to a position described in terms of the sum of two direction vectors. Students had to recognize that the resultant sum, or position, of a trip due south for 40 miles followed by a trip of 30 miles southwest could be represented by the vector associated with Option D. Sixty-three percent of the grade 12 students correctly selected Option D.



1. A ship travels due south for 40 miles and then southwest for 30 miles. Which of the vectors in the figure above best represents the result of the ship's movement from its starting point?

   (A) a

   (B) b

   (C) c

   (D) d

   (E) e

The only other option that was selected by more than five percent of the students was Option E. Twenty-seven percent of the students made this incorrect choice. These students selected a vector representing the sum of movement due south and due west rather than the sum of movement due south and southwest.

The problem could be solved in a variety of ways. Most students would not have had this type of question (representing and solving problems using vector methods) as part of their geometry classroom experience, but those who had studied physics or pre-calculus would have studied vectors. The result of 63 percent correct, shown in Table 4.24, is a measure of students' problem-solving skills related to interpreting a situation, representing it, and solving it. However, fewer students might have been successful if the task had been to draw the vector rather than simply to recognize it, or if they had been required to discriminate between two vectors with the same direction but different lengths.

Table 4.24 — *Percentage Correct for "Find Resultant Vector," Grade 12*

THE NATION'S REPORT CARD

Grade 12	Percentage Correct
All Students	63
Males	67
Females	58
White	66
Black	51
Hispanic	50
Asian/Pacific Islanders	55
American Indian	***
Are Students Presently Enrolled in Mathematics...	
Yes	63
No	52
Are Students Presently Enrolled in or Have Previously Taken an Advanced Placement (AP) Mathematics Course...	
Yes	67
No	60

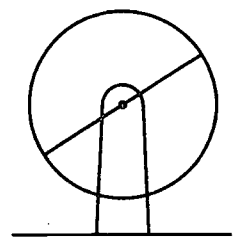
*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The final sample question from the Grade 12 Advanced Study was classified as an Algebra and Functions question. It presented students with a written description of the motion of a Ferris wheel. Using the given information, students were to draw a graph depicting the height, over 45 seconds of time, of an individual on a wheel that rotates once every 15 seconds, if the individual is at the bottom of the wheel at time 0. In order to answer the question correctly, students needed to attend to three conditions:

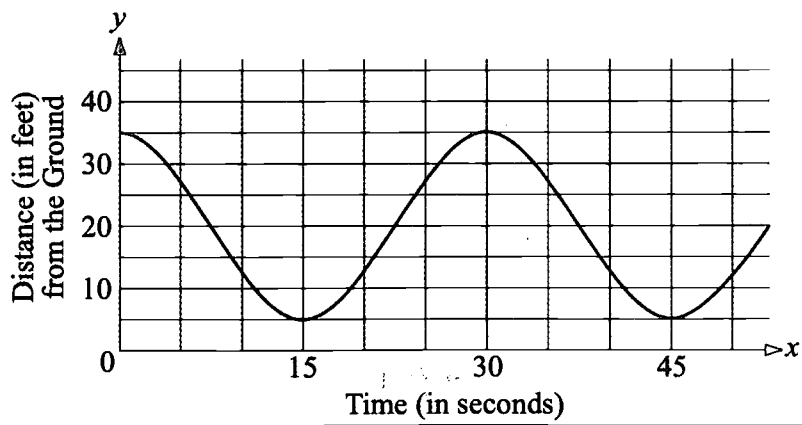
- at time $t = 0$, $h = 5$;
- the *height* can only vary from 5 feet to 35 feet throughout the graph; and
- the *period* in the solution graph is 15 seconds (i.e., the graph repeats itself every 15 seconds).

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all of your work.



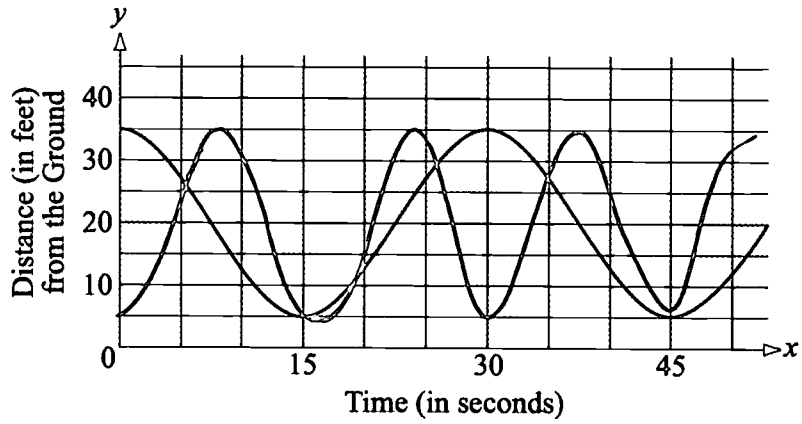
10. The Ferris wheel above is 30 feet in diameter and 5 feet above the ground. It turns at a steady rate of one revolution each 30 seconds. The graph below shows a person's distance from the ground as a function of time if the person is at the top of the Ferris wheel at time 0.

On the same graph, draw a second curve that shows a person's distance from the ground, as a function of time, if that person is at the bottom of the Ferris wheel at time 0 and if the Ferris wheel turns at a steady rate of one revolution each 15 seconds. Sketch the graph from time equals 0 to time equals 45 seconds.



Responses were rated "extended," "satisfactory," "partial," "minimal," or "incorrect." The rating of "extended" was awarded to responses that showed a complete and correct graph extending over the 45-second period, as shown below.

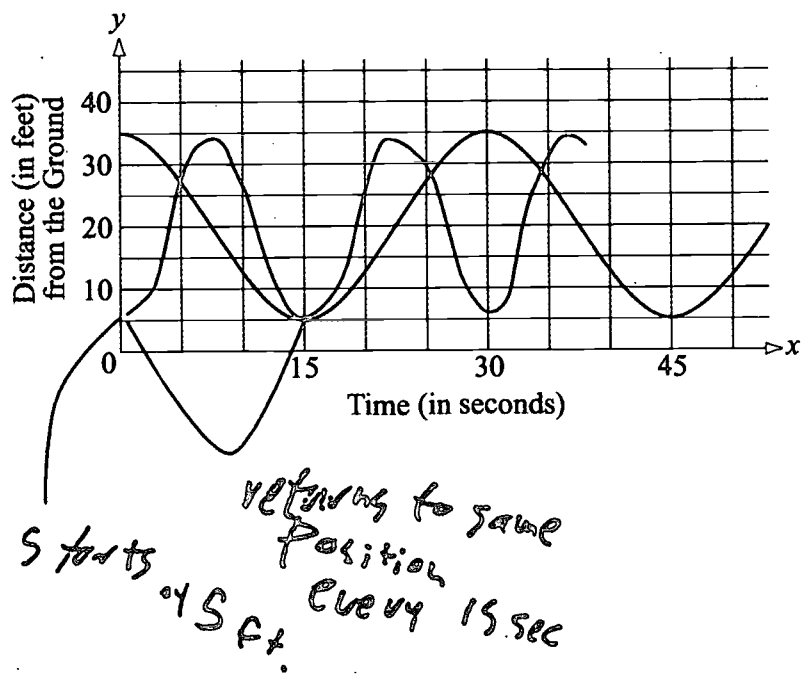
Sample "extended" response



This is simple — all you do is start the graph at 5 feet (the starting point for the wheel) and you draw the curve just like the one before but at a faster rate. (One period for the graph I drew will be half that of the original curve).

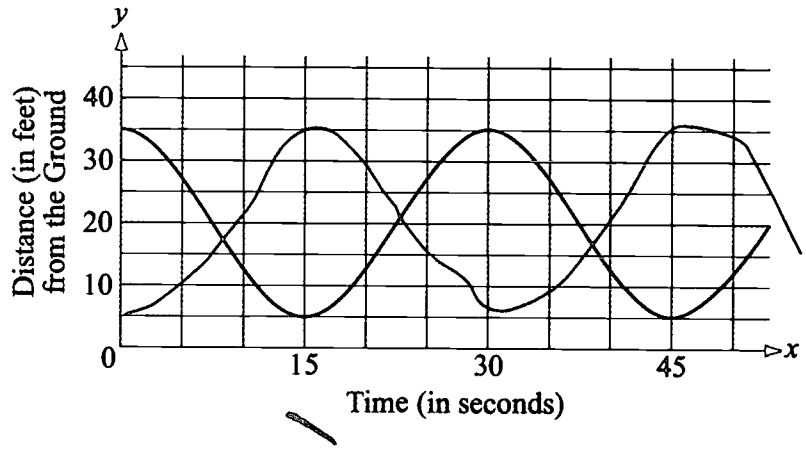
Responses containing a graph of *three* of the conditions but that failed to extend the graph from time 0 to 45 seconds were rated "satisfactory." The sample "satisfactory" response shown satisfied three conditions, but the sketch was not shown for the entire time from $t = 0$ to $t = 45$ seconds. Very few students received "satisfactory" scores. In the sample response shown, the response error suggests inattention to the full requirements of the problem rather than a lack of understanding.

Sample "satisfactory" response



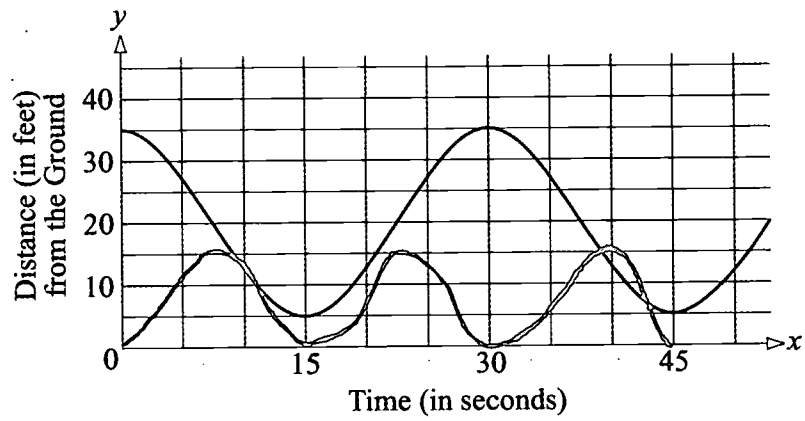
Ratings of "partial" were given to responses showing a graph of *two* of the conditions. The sample "partial" response shown satisfied the conditions that at time $t = 0$, $h = 5$ and that the height vary from 5 feet to 35 feet throughout the graph, but did not portray the correct period of 15 seconds per rotation.

Sample "partial" response



"Minimal" ratings were awarded to responses that correctly depicted a graph of *one* of the conditions. The sample "minimal" response shown has the 15-second period graphed correctly.

Sample "minimal" response



Responses that did not move beyond a simple restatement of the problem were rated "incorrect."

Table 4.25 presents student performance data on the Ferris wheel problem. The data show that students did relatively well on this question; 39 percent of the responses were rated as "extended." The percentage of students receiving the "extended" rating is somewhat surprising, considering student performance on the other problems. This higher level of performance may be a reflection of how recently students in the Grade 12 Advanced Study had dealt with similar content. The height and vertical variation in the graph indicate the amplitude of a periodic function. The compression of the graph horizontally represents the period of the graph. Both of these topics, amplitude and periodicity, receive a great deal of attention in the study of trigonometric, or circular, functions in pre-calculus and calculus.

Table 4.25 *Score Percentages for "Ferris Wheel," Grade 12*

THE NATION'S REPORT CARD

Grade 12	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	39	0	18	13	19	10
Males	44	1	18	11	14	10
Females	33	0!	18	16	23	9
White	44	0	19	13	16	7
Black	13	0!	13	12	34	25
Hispanic	22!	0!	12!	12!	30!	20!
Asian/Pacific Islander	33	0!	19	14	22	11
American Indian	***	***	***	***	***	***
Are Students Presently Enrolled in Mathematics...						
Yes	40	0	18	13	18	9
No	24!	0!	19!	17!	21!	14!
Are Students Presently Enrolled in or Have Previously Taken an Advanced Placement (AP) Mathematics Course...						
Yes	50	0!	17	12	14	8
No	32	0	20	14	22	10

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The Effects of Grade 12 Mathematics and Advanced Placement

An analysis of the data related to the performance of students in the Advanced Study who reported that they were currently taking mathematics or who were, or had been, enrolled in an AP mathematics course shows that in each case these students outperformed students in the study who were not currently taking a mathematics course or who had not taken such an AP course. Some of those not currently taking a course may have taken mathematics during the first semester of their senior year as part of a block scheduling arrangement at their schools. Others may simply have completed their schools' mathematics curriculum earlier and were "lying fallow" mathematically during the assessment semester. Finally, it also is possible that some of the students who, by self-report, were not currently enrolled in mathematics had been misclassified by the individuals responsible for the NAEP samples at their schools and were not really eligible for the Advanced Study.

Summary

The background and demographic data collected along with the achievement data indicate that both grade 8 and grade 12 students participating in the Advanced Study were different from those who did not qualify for the study. They tended to come from homes providing a stronger educational context, both in materials and in parental education. Further, these students appeared to have somewhat better financial means, as a smaller proportion qualified for the federal Free/Reduced-Price Lunch program. Similarly, fewer of them participated in Title I programs.

The Advanced Study was designed to provide students who were taking or had taken advanced courses in mathematics with an opportunity to demonstrate their full mathematical proficiency. Students at both grade levels who met the criterion for inclusion in the Advanced Study performed substantially better than other students on the main NAEP mathematics assessment. However, student performance on the Advanced Study itself shows that the study questions were quite difficult, even for students who were taking the more challenging mathematics courses that were prerequisite to qualify. Overall performance, measured by percentage correct, was 36 percent at grade 8 and 30 percent at grade 12, and, at both grade levels, most of these students were unable to solve problems that required two or three successive steps to achieve the desired result.

Appendix A

Procedures

The NAEP 1996 Mathematics Assessment

The 1996 assessment was the first update of the NAEP mathematics assessment framework since the release of the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics*.[1] This update reflected refinements in the specifications governing the development of the NAEP 1996 mathematics assessment while ensuring comparability of results across the 1990, 1992, and 1996 assessments.

Special Studies in the 1996 NAEP Mathematics Assessments

In addition to the main NAEP 1996 mathematics assessment, three special studies were conducted: an Estimation Study, a Theme Study, and an Advanced Study. In each of the studies, students were presented with an assessment booklet consisting of blocks of cognitive questions. The Estimation and Advanced Studies had a booklet for each grade level, while the Theme Study had two different booklets at each grade level.

The booklets consisted of blocks of cognitive questions and blocks of background questions. The cognitive block structure of the special study booklets is displayed in Table A.1. The Estimation and Theme booklets began with block M4, a Balanced Incomplete Block (BIB) linking block taken from the main assessment. In the Estimation booklets, the linking block was followed by two Estimation blocks: a trend block, M16, and a new block, M17. In the Theme booklets, the linking block was followed by a single Theme block, either M21 or M22. The Advanced booklet began with a nonadvanced block, M20, which was composed of questions from various BIB (main assessment) blocks; this was followed by two Advanced blocks, M18 and M19. In addition to the cognitive blocks, all of the special study booklets had three blocks in common with the main assessment: a general student background block, a mathematics background block, and a motivation block.

[1] National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

Table A.1	Block Structure of the Special Study Booklets	THE NATION'S REPORT CARD

Study	Booklet	Cognitive Blocks
Estimation	127	M4, M16, M17
Theme	128	M4, M21
Theme	129	M4, M22
Advanced Mathematics*	130	M20, M18, M19

* Grades 8 and 12 only

The cognitive portion of the Estimation booklet was administered in two sections: the first (BIB) block was self-paced; and the two Estimation blocks were administered by a paced audio tape. The Theme and Advanced booklets were entirely self-paced.

The special studies were not part of the main assessment. However, the Estimation and Theme samples were drawn from the same population as the main assessment; that is, they were chosen to be representative of all students at the target grade level. The grades 8 and 12 Advanced samples, by contrast, were drawn from populations of students who were considered high mathematics achievers based on their enrollment in advanced mathematics classes during the 1995–1996 school year.

Sampling Procedures for the Special Studies

The populations for the special studies were sampled as part of the complex multistage sample design used for the main national sample. The design involved sampling students from selected schools within 94 selected geographic areas, called primary sampling units (PSUs), across the United States. There were five steps in the selection process:

1. Selection of geographic PSUs (counties or groups of counties).
2. Selection of schools within PSUs.
3. Assignment of sample types to schools.
4. Assignment of session types to schools.
5. Selection of students for session types within schools.

The samples for the main and special studies were drawn for grades 4, 8, and 12. In addition, separate age-based samples were selected for the long-term trend studies in mathematics. Table A.2 shows the numbers of students and schools that were assigned to each condition. In addition to representing the respective populations as a whole, the main and special study samples involved oversampling of nonpublic schools, and of public schools with moderate or high enrollment of Black or Hispanic students. This oversampling was undertaken to increase the sample sizes of nonpublic school students and minority students, so as to increase the reliability of estimates for those groups of students. The assessment period for the main assessment and special studies was in the winter of 1996, between January 3 and March 29.

Table A.2

Number of Students Per School for Each Session Type

THE NATION'S REPORT CARD

Sample	Number of Assessed Students	Number of Schools	Mean Number of Students Per Assessment Per School	Mean Number of Students Per Item Per School
Age Class 9				
Long-Term Trend				
Print Booklets 51–56	5,019	215	23.3	3.9–7.8*
Tape Booklet 91	1,852	127	14.6	14.6
Tape Booklet 92	1,721	116	14.8	14.8
Tape Booklet 93	1,840	125	14.7	14.7
Grade 4 Main				
Print Mathematics	10,830	445	24.3	5.6
Print Science	11,578	421	27.5	4.5–7.4*
Tape Mathematics Estimation	2,115	120	17.6	17.6
Print Mathematics Theme	4,004	230	17.4	8.7
Age Class 13				
Long-Term Trend				
Print Booklets 51–56	5,493	221	24.9	4.1–8.3*
Tape Booklet 91	1,928	128	15.1	15.1
Tape Booklet 92	1,866	125	14.9	14.9
Tape Booklet 93	1,864	124	15.0	15.0
Grade 8 Main				
Print Mathematics	11,521	411	28.0	6.5
Print Science	11,971	346	34.6	5.6–9.4*
Tape Mathematics Estimation	2,244	104	21.6	21.6
Print Mathematics Theme	4,227	175	24.2	12.1
Print Advanced Mathematics	2,365	253	9.3	9.3
Age Class 17				
Long-Term Trend				
Print Booklets 51–56	4,669	186	25.1	4.2–8.4*
Tape Booklet 84	1,848	133	13.9	13.9
Tape Booklet 85	1,691	122	13.9	13.9
Grade 12 Main				
Print Mathematics	10,600	430	24.8	5.7
Print Science	11,481	401	28.6	4.6–7.7*
Tape Mathematics Estimation	1,889	96	19.7	19.7
Print Mathematics Theme	3,860	196	19.7	9.8
Print Advanced Mathematics	2,965	207	14.3	14.3
Print Advanced Science	2,431	222	11.0	11.0

* This number varied because some item blocks appeared more often than others in the set of booklets used for this sample.

Following is a brief summary of the general features of the sampling procedure. Many details and adjustments left out of this description can be found in Chapter 3 of *The NAEP 1996 Technical Report.* [2]

PSU Selection

Of the population of 1,000 PSUs in the country, the 22 largest PSUs were included in the sample with certainty. The remaining PSUs were partitioned into 72 strata (defined by region and various socioeconomic characteristics), and one PSU was chosen from each stratum with probability proportional to size (of the population of the PSU).

Selection of Schools

In the second-stage sampling a list was created, for each grade, of all public and nonpublic schools in the 94 selected PSUs. Schools were selected (without replacement) across all PSUs with probabilities proportional to assigned measures of size. Details of the assignment of school size can be found in Section 3.3 of *The NAEP 1996 Technical Report.*

Assignment of Sample Type to Schools

In order to determine the effect of using different guidelines for excluding or assessing students with disabilities or limited English proficiency, three different sample types were assigned to schools selected for the main or special study assessments. These sample types were:

- S1, in which students were subject to the exclusion criteria used in 1990 and 1992;
- S2, in which students were subject to new 1996 exclusion criteria; and
- S3, in which students were subject to new 1996 exclusion criteria and accommodations were offered to students with disabilities (SD) and limited English-proficient (LEP) students.[3]

[2] Allen, N. L., Carlson, J. E., & Zelenak, C. A. (1999). *The NAEP 1996 technical report.* Washington, DC: National Center for Education Statistics.

[3] See Chapter 5 of the *NAEP 1996 Technical Report* for a fuller description of sample types, exclusion criteria, and accommodations.

Sample type was assigned to schools separately for each grade. For schools that were not also selected for the State Assessment, sample type was assigned to schools according to the percentages shown in Table A.3.

Table A.3 **Percent of Schools Assigned to Sample Type in the 1996 Assessment**

THE NATION'S REPORT CARD naep

Grade	Sample Type S1	S2	S3
4	20	45	35
8	16	42	42
12	15	46	39

Sampling Students

The first step in sampling students was to compile a list of all grade-eligible students in a school. Along with this list, schools at grades 8 and 12 were asked to provide course enrollment information to be used in determining eligibility for the Advanced Study. Specifically, eligibility was based on reported enrollment in Algebra 1 or beyond at grade 8 and on reported enrollment in Algebra 3, Pre-Calculus, Calculus and Analytical Geometry, Calculus, or AP Calculus at grade 12.

Systematic selection of students from the grade-eligible list was made to obtain the target sample size for each school. Westat, Inc. district supervisors then assigned sampled students to one of the various session types according to specified procedures. Session types included the various subject areas and, within the subject area of mathematics, whether the students would be included in the main (BIB), long-term trend, Estimation, Theme, or Advanced assessments. Obviously, only students who were eligible for the Advanced Study could be assigned to Advanced sessions, but students eligible for the Advanced Study also were assigned proportionally to all other session types.

To ensure an adequate sample size of SD/LEP students, oversampling procedures were applied to these students in sample type 2 (S2) and sample type 3 (S3). In general, SD/LEP students were sampled (within schools) at twice the rate at which non-SD/LEP students were sampled. For grades 8 and 12, because of the way sessions were assigned, oversampling of SD/LEP students took place only among students who were not eligible for the Advanced assessment.

Collected and Reporting Samples

The exclusion and accommodation conditions that were applied in S3 are anticipated to be similar to those that will be used in future NAEP assessments. Consequently, the data from S3 constitute a statistical bridge that can be used in maintaining future trend lines. However, in order to allow timely reporting of 1996 results, data from S3 were generally not included in the reporting sample, but were set aside for further analysis of the impact of the availability of accommodations on group achievement estimates.

The following tables — A.4 through A.6 — illustrate how the special studies were distributed across the S1, S2, and S3 samples. In the tables, the notation "B" refers to that subset of the participating students who were identified as having either disabilities or limited English proficiency. "A" refers to all other participating students. As can be seen, the data reported for the Estimation Study were generally drawn from S1, but S2 students were added at grade 12 in order to obtain a sufficient sample size. However, to insure comparability across grades, results from S2 students with disabilities or limited English proficiency were not reported. The Theme Study sample was drawn from S2. The Estimation and Theme Studies also were administered to S3, as a statistical bridge to future administrations of these studies. The Advanced Study sample was taken from S3 at grade 12 and, in order to obtain a sufficient sample size, from both S2 and S3 at grade 8. Due to the nature of the Advanced Study selection criteria, accommodations were infrequent in this sample, particularly at grade 12. Consequently, it was judged acceptable to report Advanced Study results from S3. As a precaution, however, data from grade 8 students with disabilities or limited English proficiency (where some accommodations did occur) were held back from reporting.

Table A.4 — ***Collected and Reporting Samples for the Estimation Assessment, Grades 4, 8, and 12****

THE NATION'S REPORT CARD naep

	S1	S2	S3
Grade 4			
A	R	X	NR
B	R	X	NR
Grade 8			
A	R	X	NR
B	R	X	NR
Grade 12			
A	R	R	NR
B	R	NR	NR

* An "R" indicates that this cell was part of the reporting sample. An "NR" cell indicates that data were collected but not included in the reporting sample. An "X" indicates that no data were collected from that category.

Table A.5 — *Collected and Reporting Samples for the Theme Assessment Grades 4, 8, and 12**

THE NATION'S REPORT CARD naep

	S1	S2	S3
A	X	R	NR
B	X	R	NR

* An "R" indicates that this cell was part of the reporting sample. An "NR" cell indicates that data were collected but not included in the reporting sample. An "X" indicates that no data were collected from that category.

Table A.6 — *Collected and Reporting Samples for the Advanced Assessment, Grades 8 and 12 Only**

THE NATION'S REPORT CARD naep

	S1	S2	S3
Grade 8			
A	X	R	R
B	X	R	NR
Grade 12			
A	X	X	R
B	X	X	R

* An "R" indicates that this cell was part of the reporting sample. An "NR" cell indicates that data were collected but not included in the reporting sample. An "X" indicates that no data were collected from that category.

Participation Rates

In order for the data to be reported, NCES school and student participation rates must be met. School nonparticipation or student nonparticipation in the form of absenteeism creates a potential for bias to be introduced in the reporting of the data. The participation rates of schools and students included in the 1996 assessments were inspected for any differences. NCES standards regarding acceptable potentials for bias are expressed in terms of weighted participation rates. Table A.7 shows the weighted participation rates by grade and session type for the main reporting samples. For the main samples, the student participation rates are similar for different session types at grades 4 and 8, but the student participation rates at grade 12 and the school participation rates at all grades vary by session type. The differential school participation rates show that different session types include different schools. This is due to the assignment of schools to sample type, the fact that all session types were not assessed in all sample types, and the specific sample types included in the reporting populations for each session type.

Table A.7	Weighted Participation Rates (in %), by Grade and Session Type, 1996 Main NAEP Reporting Samples	THE NATION'S REPORT CARD naep

	Mathematics Print	Science Print	Mathematics Estimation Print	Mathematics Theme Print	Advanced Mathematics Print	Advanced Science Print
Grade 4						
School Participation	82.3	77.8	93.5	77.9	—	—
Student Participation	95.3	94.9	96.7	95.4	—	—
Overall Participation	78.4	73.8	90.4	74.4	—	—
Grade 8						
School Participation	81.5	79.7	85.3	86.8	77.0	—
Student Participation	92.9	93.1	93.8	92.7	95.6	—
Overall Participation	75.7	74.3	80.0	80.4	73.6	—
Grade 12						
School Participation	76.2	77.4	63.9	78.4	77.6	77.7
Student Participation	82.3	77.5	81.0	78.2	85.8	86.5
Overall Participation	62.7	60.0	51.7	61.3	66.6	67.2

Evaluation of Potential for Bias

Although school and student nonresponse adjustments are intended to reduce the potential for nonparticipation to bias the assessment results, they cannot completely eliminate this potential bias with certainty. The extent of bias remains unknown since there are no assessment data for the nonparticipating schools and students.

Some insight can be gained about the potential for residual nonresponse bias by examining the weighted school- and student-level distributions of characteristics known for both participants and nonparticipants, especially for those characteristics known or thought likely to be related to achievement on the assessment. If the distributions for the full sample of schools (or students) without the use of nonresponse adjustments are close to those for the participants with nonresponse adjustments applied, there is reason to be confident that the bias from nonparticipation is small.

A nonresponse bias analysis completed on the reporting population for the science assessment can be found in the *NAEP 1996 Technical Report*.[4] Science was chosen because it contained the largest number of students and could, therefore, provide the most precise estimates of student distributions across several demographic characteristics (i.e., age, race, gender, type of school and school size). Generally, the findings show that the student distributions before and after school and/or student nonresponse adjustments were similar, with a few exceptions. Most of these exceptions were at grade 12 due to its relatively high nonresponse rate (20.3% for grade 12 students). An additional nonresponse bias analysis was completed on the

[4] Allen, et al., (1999). op. cit.

reporting populations of Mathematics, Mathematics Theme, Mathematics Estimation and Advanced Mathematics. Each of these reporting populations have smaller numbers of schools and students in the sample. Thus, the variance of the estimated distributions will be larger for these subjects, than for science.

Within the NAEP data, there are several school-level characteristics available for both participating and nonparticipating schools. The tables that follow show the combined impact of nonresponse and of the nonresponse adjustments on the distributions of schools (weighted by the estimated number of eligible students enrolled) and students, by the type of school (public, Catholic, other nonpublic), the size of the school (measured by the estimated number of eligible students enrolled) and whether the school is located in an urban/rural place. Three size classes have been defined for each grade. The data are for the 1996 samples excluding the science assessment.

Several student-level characteristics are available for both absent and assessed students. The tables that follow show the impact of school nonresponse and nonresponse adjustments, and student nonresponse and nonresponse adjustments on the distributions of eligible students for each grade. The distributions are presented by age category (at or below modal age, and above modal age), race/ethnicity category (White, Black, Hispanic, and other), gender, SD and LEP.

Table A.8 shows the weighted marginal distributions of students for each of the three classification variables for grade 12, using weighted eligible schools. The distributions before school nonresponse adjustments are based on the full sample of in-scope schools for each assessment — those participating, plus those refusals for which no substitute participated. The distributions after school nonresponse adjustments are based only on participating schools for each assessment, with school nonresponse adjustments applied to them. The weighted school-level nonparticipation rates at grade 12 are as follows: Mathematics, 27.8 percent; Mathematics Theme, 21.6 percent; Mathematics Estimation, 36.1 percent; and Advanced Mathematics, 22.4 percent. For more detail, see the *NAEP 1996 Technical Report*.[5]

Table A.8 — **Distribution (in %) of Populations of Eligible Students Based on Full Weighted Sample of Eligible Schools, Before and After School Nonresponse Adjustments, 1996 Main NAEP Samples (Excluding Science), Grade 12**

THE NATION'S REPORT CARD

	Mathematics		Mathematics Theme		Mathematics Estimation		Advanced Mathematics		Advanced Science	
Population	Before	After	Before	After	Before	After	Before	After	Before	After
School Type										
Catholic	5.8	6.6	4.5	5.5	7.5	7.1	4.5	5.2	5.4	5.5
Other Nonpublic	3.6	3.1	4.5	3.8	3.9	4.8	4.1	3.4	3.8	3.1
Public	90.6	90.3	91.0	90.8	88.5	88.1	91.4	91.4	90.7	91.4
School Size[1]										
1 (1–49)	6.0	6.2	5.7	5.1	6.8	8.3	7.1	7.3	5.5	5.0
2 (50–399)	67.9	66.0	71.6	71.4	72.1	66.4	72.7	72.9	69.4	65.7
3 (400+)	26.1	27.7	22.7	23.5	21.1	25.3	20.2	19.9	25.1	29.2
School Location										
Large City	14.8	15.1	14.9	17.5	14.7	15.4	13.5	15.1	15.4	16.0
Midsize City	18.6	21.2	14.6	15.6	20.8	23.4	17.3	18.2	16.3	18.4
Urban Fringe/ Large City	22.4	19.1	27.5	23.5	18.5	19.5	25.0	22.6	23.8	22.5
Urban Fringe/ Midsize City	14.5	15.2	13.3	15.2	17.5	14.4	14.3	15.2	15.0	13.9
Large Town	1.1	1.2	0.7	0.9	0.7	1.1	0.5	0.7	1.5	0.9
Small Town	12.1	13.1	14.5	14.6	11.5	11.5	16.0	16.1	14.7	15.9
Rural MSA	3.8	3.0	6.0	5.7	4.5	2.1	5.6	4.5	4.1	4.1
Rural nonMSA	12.8	12.0	8.4	7.0	11.8	12.6	7.7	7.6	9.1	8.4

[1]Distributions by school size are not comparable to previous assessments, since students were eligible by grade only (instead of by grade or age) in 1996. School size refers to the number of eligible students enrolled.

[5] Allen, et al., (1999). op. cit.

It can be seen from Table A.8 that, overall, the distributions for school type, school size and school location remain similar. For Mathematics, even though the nonresponse rate is 27.8 percent, the only exceptions may be midsize cities and urban fringe of large cities; for Mathematics Theme, the exceptions may be large cities and their urban fringe; for Mathematics Estimation, exceptions may be at medium and large schools, midsize cities and their urban fringe, and rural MSAs (Metropolitan Statistical Areas). For Advanced Mathematics, even though the school nonparticipation rate is 22.4 percent, the only exception may be urban fringes of large cities. The potential for bias is greatest for Mathematics Estimation, which also has the highest nonparticipation rate for schools.

Table A.9 shows the distributions of two school-level characteristics — school type and school location, plus additional distributions of student-level characteristics, using weighted eligible students. The distributions before student nonresponse adjustments are based on assessed and absent students (with base weights adjusted for school nonparticipation).The distributions after student nonresponse adjustments are based on assessed students only, with the student nonresponse adjustments also applied to them. The rates of student nonparticipation for the five subjects at grade 12 are as follows: Mathematics, 17.7 percent; Mathematics Theme, 21.8 percent; Mathematics Estimation, 19.0 percent; Advanced Mathematics, 14.2 percent. For more detail, see the *NAEP 1996 Technical Report*.[6]

[6] Allen, et al., (1999). op. cit.

Table A.9

Distribution (in %) of Populations of Eligible Students Before and After Student Nonresponse Adjustments, 1996 Main NAEP Samples (Excluding Science), Grade 12

THE NATION'S REPORT CARD

Population	Mathematics		Mathematics Theme		Mathematics Estimation		Advanced Mathematics		Advanced Science	
	Before	After	Before	After	Before	After	Before	After	Before	After
School Type										
Catholic	7.2	8.0	5.9	7.5	7.9	9.2	9.1	10.6	10.6	11.7
Other Nonpublic	3.3	3.8	3.8	4.1	5.2	6.1	7.1	7.9	4.9	5.4
Public	89.5	88.1	90.3	88.3	86.9	84.7	83.7	81.5	84.5	82.9
School Location										
Large City	15.3	14.6	17.2	17.3	15.0	14.5	15.5	16.9	15.5	15.2
Midsize City	21.3	20.8	15.8	15.4	23.4	23.5	20.4	19.8	16.7	15.6
Urban Fringe/ Large City	19.2	18.8	23.8	23.6	19.2	19.4	19.5	18.8	27.1	27.6
Urban Fringe/ Midsize City	14.4	15.2	15.0	15.7	14.3	15.1	15.5	14.8	12.1	12.9
Large Town	1.3	1.2	0.9	0.9	1.3	1.2	0.7	0.7	0.8	0.7
Small Town	13.2	13.1	14.6	14.3	11.5	10.7	14.4	14.6	13.9	13.5
Rural MSA	2.9	3.1	5.9	5.8	2.0	2.2	6.0	6.6	5.6	5.9
Rural nonMSA	12.5	13.1	6.8	7.0	13.2	13.4	7.9	7.9	8.3	8.7
Age Category										
At Modal Age or Younger	65.9	66.0	65.4	65.7	67.0	66.6	74.0	73.7	72.5	72.3
Older than Modal Age	34.1	34.0	34.6	34.3	33.0	33.4	26.0	26.3	27.5	27.7
Race/Ethnicity										
White	70.0	70.0	69.1	68.6	70.5	70.9	74.8	74.1	74.8	74.1
Black	13.1	13.4	13.1	13.0	12.7	12.8	7.3	7.2	8.9	9.1
Hispanic	10.1	9.6	10.6	11.2	11.6	10.9	7.5	7.9	6.8	7.0
Other	6.8	7.0	7.1	7.2	5.1	5.4	10.4	10.8	9.5	9.7
Gender[1]										
Male	47.9	47.6	49.8	49.1	47.4	47.3	50.2	50.3	49.4	49.2
Female	52.1	52.4	50.2	50.8	52.6	52.7	49.1	49.0	50.5	50.8
SD										
Yes	2.7	2.7	2.5	2.2	2.7	2.4	0.6	0.7	0.4	0.3
No	97.3	97.3	97.5	97.8	97.3	97.6	99.4	99.3	99.6	99.7
LEP										
Yes	1.1	1.1	1.8	1.6	0.9	1.0	1.2	1.2	1.1	1.0
No	98.9	98.9	98.2	98.4	99.1	99.0	98.8	98.8	98.9	99.0
SD, LEP										
SD Yes; LEP Yes	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SD Yes; LEP No	2.7	2.7	2.4	2.2	2.7	2.4	0.6	0.7	0.4	0.3
SD No; LEP Yes	1.1	1.1	1.8	1.6	0.9	1.0	1.2	1.2	1.1	1.0
SD No; LEP No	96.2	96.2	95.7	96.2	96.3	96.5	98.2	98.1	98.5	98.6

[1] Gender is unknown for a small percentage of students.

Table A.9 shows that with two exceptions at grade 12, for the distributions of type of school attended and place where the school is located, the effect of the student nonparticipation adjustment has resulted in very little change in distribution. The changes in the distribution of school type for Mathematics Estimation and Advanced Mathematics reflect the relatively high nonresponse rate of grade 12 public school students (22.7% versus 8.5% for nonpublic school students; from the *NAEP 1996 Technical report*).[7]

Table A.10 shows the weighted distributions of eligible students in participating schools, using the base weights of assessed and absent students unadjusted for school-level nonresponse. Tables A.9 and A.10 show that both school and student-level nonresponse and nonresponse adjustments have little effect on the distributions of eligible students by age, race/ethnicity, gender, and SD and LEP status. All of the distributions in the tables are similar.

Table A.10 — **Distribution (in %) of Populations of Eligible Students Before School and Student Nonresponse Adjustment, 1996 Main NAEP Samples (Excluding Science), Grade 12**

THE NATION'S REPORT CARD naep

Population	Mathematics	Mathematics Theme	Mathematics Estimation	Advanced Mathematics	Advanced Science
Age Category					
At Modal Age or Younger	65.6	64.9	66.3	74.0	72.1
Older than Modal Age	34.4	35.1	33.7	26.0	27.9
Race/Ethnicity					
White	71.0	69.1	70.8	75.4	76.0
Black	12.8	13.4	12.6	7.5	8.8
Hispanic	9.5	10.3	11.6	7.3	6.3
Other	6.8	7.1	5.1	9.8	9.0
Gender[1]					
Male	47.9	49.9	47.8	49.8	49.6
Female	52.1	50.0	52.2	49.4	50.3
SD					
Yes	2.6	2.5	2.8	0.5	0.4
No	97.4	97.5	97.2	99.5	99.6
LEP					
Yes	1.0	1.8	1.0	1.1	1.0
No	99.0	98.2	99.0	98.9	99.0
SD, LEP					
SD Yes; LEP Yes	0.0	0.0	0.0	0.0	0.0
SD Yes; LEP No	2.6	2.5	2.8	0.5	0.4
SD No; LEP Yes	1.0	1.8	1.0	1.1	1.0
SD No; LEP No	96.3	95.7	96.2	98.4	98.6

[1] Gender is unknown for a small percentage of students.

[7] Allen, et al., (1999). op. cit.

When comparing the distributions in Table A.9 before and after student nonresponse adjustments, we expect the distributions by age category and race/ethnicity to be similar because these variables were used to determine student nonresponse adjustment classes. However, the distributions by gender and SD and LEP status are also similar. To the extent that nonrespondents would perform like respondents with the same characteristics (defined by the characteristics in the tables), the bias in the assessment data is small.

Further information about potential nonresponse bias can be gained by studying the absent students. NAEP proficiency estimates are biased to the extent that assessed and absent students within the same weighting class differ in their distribution of proficiency. It seems likely that the assumption that absent students are similar in proficiency to assessed students is reasonable for some absent students — namely, those whose absence can be characterized as random. Conversely, it seems likely that students with longer and more consistent patterns of absenteeism — such as truants, dropouts, near dropouts, and the chronically ill — are unlikely to be as proficient as their assessed counterparts.

In the 1996 assessments, schools were asked to classify each absent student into one of nine categories. The results of this classification for the assessments are shown in Table A.11. Table A.11 shows that, as anticipated, the majority of absenteeism from the assessment was the result of an absence from school of a temporary and unscheduled nature. The table shows that among the two Advanced sessions, the absenteeism rate is lower than among the 'non-Advanced' sessions. The proportion of absenteeism classified as temporary is similar across subjects, including science (63.6%).

Table A.11	Weighted Distribution (in %) of Absent Students, by Nature of Absenteeism, 1996 Assessments (Excluding Science), Grade 12	THE NATION'S REPORT CARD

Nature of Absenteeism	Mathematics	Mathematics Theme	Mathematics Estimation	Advanced Mathematics	Advanced Science
Temporay Absence[1]	65.2	59.9	58.9	60.9	64.5
Long-term Absence[2]	1.3	1.0	2.0	1.0	0.0
Chronic Truant	1.0	1.4	1.5	0.1	0.0
Suspended or Expelled	0.4	0.2	0.3	0.1	0.3
Parent Refusal	8.1	5.8	12.8	11.1	11.7
Student Refusal	10.8	14.7	6.9	14.3	12.9
In School, Did not Attend Session	8.2	10.6	9.3	9.3	5.7
In School, not Invited[3]	0.0	0.0	0.0	0.0	0.0
Other	4.9	6.3	8.3	3.2	4.8
Missing	0.0	0.0	0.0	0.0	0.0
Total Absentee Sample	2,598	1,112	449	460	379
Total Sample Size	13, 258	4,972	2,338	3,425	2,810
Overall Absentee Rate	19.6	22.4	19.2	13.4	13.5

[1] Absent less than two weeks due to illness, disability, or excused absence.
[2] Absent more than two weeks due to illness or disability.
[3] In school, but not invited to assessment session due to disruptive behavior.

For each subject in grade 12, a significant component of absenteeism is not temporary or due to parental refusal. Chronic truants, those suspended, and those in school but not attending, constitute the obvious candidates for potential bias. These groups comprise 6.0 to 9.5 percent of absent students in the Advanced sessions (or 0.8% to 1.3% of each total sample). Among the non-Advanced sessions, the groups comprise 9.6 to 12.2 percent of absent students (or 1.8% to 2.7% of each total sample). Thus their potential for introducing significant bias under the current procedures is minor.

As with all NAEP assessments, data collection was conducted by trained Westat field staff. Materials collected as part of the 1996 assessment were shipped to National Computer Systems, where trained staff evaluated the responses to the constructed-response questions using scoring rubrics or guides prepared by the Educational Testing Service (ETS).

Each constructed-response question had a unique scoring rubric that defined the criteria used to evaluate students' responses. The extended constructed-response questions were evaluated with four- or five-level rubrics (e.g., no evidence of understanding, evidence of minimal understanding, evidence of partial understanding, and evidence of satisfactory or extended understanding). Short constructed-response questions first appearing in the 1996 assessment were rated according to three-level rubrics that permitted partial credit (e.g., evidence of little or no understanding, evidence of partial understanding, and evidence of full

understanding). Other short constructed-response questions that were carried over from previous assessments were scored as either "correct" or "incorrect." For more information, see the *NAEP 1996 Technical Report*.[8]

Student responses for constructed responses also were scored as "off task" if the student provided a response that was deemed not related in content to the question asked. A simple example of this type of response is, "I don't like this test." In contrast, responses scored as "incorrect" were valid attempts to answer the question that were simply wrong.

Analysis Procedures

The results from the Estimation Study were analyzed and reported in terms of the NAEP proficiency scale metric, while the results from the Theme Study and the Advanced Study were reported simply in terms of block percent correct scores. It was possible to scale the results from the Estimation Study because all of the questions were presumed to measure a relatively unidimensional mathematical trait: the ability to estimate and work with estimated data. However, no such single trait was hypothesized to underlie the questions from either the Theme Study or the Advanced Study. Rather, Theme Study questions were characterized by being presented in a single practical context, and Advanced Study questions were characterized by their more challenging content, some of which might only be covered in advanced courses.

For the Theme and Advanced Studies, another alternative might have been to scale the items onto the mathematics scales used in the main NAEP assessment. However, this would have required a very large number of linking questions because the main reporting scale in mathematics is actually a composite of five separately scaled subscales representing the various mathematics content areas. Theme Study and Advanced Study questions therefore would have had to be disaggregated onto five subscales for analysis and scaling, and stable links would have been required for each subscale.

Estimation Study

The Estimation booklet at each grade was scaled with an IRT analysis, and proficiencies were calculated for students and put on the scale of the 1992 Mathematics Estimation assessment. The calculation of proficiencies for each grade involved a number of steps. First, the Estimation data were scaled, using the PARSCALE program to estimate item parameters. This scaling was entirely separate from the analysis of the main BIB data, and data from the non-estimation block, M4, were not used.

Next, conditioned proficiency scores were created. Here, the same background variables and the same resulting contrasts as in the main assessment were used. However, because the principal components of the contrasts are sample dependent, these principal components were calculated separately for the Estimation and the main data.

[5] Allen, et al., (1999). op. cit.

Conditioned scores are initially in an arbitrary metric, with a mean approximately zero and a standard deviation approximately one. Therefore, the conditioned Estimation scores were linearly transformed in order to place them onto the 1992 Estimation proficiency scale. That is, each score was multiplied by one constant and then another constant was added to it, in the form:

$$P_i = A*x_i + B,$$

where x_i is the score for student i in the arbitrary metric, P_i is the score for student i in the 1992 Estimation scale metric, and A and B are appropriate constants. These constants, called transformation constants, also are sample dependent and therefore specific to the Estimation samples.

Table A.12 shows the A and B constants used to transform the initial 1996 Estimation scores from an arbitrary metric to the 1992 metric.

Table A.12 — *Constants Used to Transform 1996 Estimation Scale Scores to the 1992 NAEP Estimation Proficiency Metric*

THE NATION'S REPORT CARD naep

Grade	A	B
Grade 4	37.9729	207.2608
Grade 8	28.0702	269.9908
Grade 12	27.9881	295.8688

For this report, the following Estimation Study performance data were presented for each grade and for selected subgroups within each grade:

- average proficiency means, with standard errors reflecting sampling and measurement error;
- average proficiency means at percentile levels — 5, 10, 25, 50, 75, 90, and 95; and
- the percentage of students at or above achievement levels of *Advanced*, *Proficient*, *Basic*, and below *Basic*.

Block Percent Correct Scores for Each Student. The linking block, M4, was dropped from the analysis, and a block percent correct score for each student was calculated using the Theme block (M21 or M22). The block percent correct score was calculated as a straight percentage correct with not-reached items counted as wrong. Dichotomous items were scored 0 (wrong) and 1 (correct), and polytomous items were scored from 0 to $m - 1$ (m being the number of categories in a given polytomous item), so 0 = 0, the first partial credit category = 1, the second = 2,..., and so on to $m - 1$.

The student percent correct scores were broken down by:

- gender;
- race/ethnicity;
- frequency of writing a few sentences about how to solve a mathematics problem; and
- frequency of writing reports or doing a mathematics project.

For the "frequency of writing" variables, teacher responses were used at grades 4 and 8. Because teachers of twelfth-grade students were not surveyed, student responses were used at grade 12.

Significance tests were run for families of tests defined by the contrasts among levels of each variable. This was accomplished by using standard almanac programs.

Item-by-Item Statistics for Questions in the Released Block. For the individual item statistics for questions in the released Theme blocks at each grade, the following procedures were used:

- For dichotomous items, percentages responding to each alternative were calculated, but generally only the percentages for the correct response were included in the report.
- For polytomous items, percentages responding to each partial credit category were calculated and generally reported.
- The percentages responding in categories were reported for the whole group and were also broken down by categories of four variables:

  1. gender;
  2. race/ethnicity;
  3. frequency of writing a few sentences about how to solve a mathematics problem; and
  4. frequency of writing reports or doing a mathematics project.

Advanced Study

Booklet percent correct scores were calculated for each student in a manner similar to the block percents created for the Theme analysis. The difference is that the two Advanced blocks were analyzed together to create a single booklet percent correct score for each student. The linking block, M20, was deleted from the analysis.

The booklet percent correct scores were reported for the whole group and for subgroups defined by gender and race/ethnicity. In addition, for grade 12, a breakdown of the booklet percents was done by two self-report variables: (1) whether the student was currently taking mathematics and (2) whether the student was taking or had taken an AP mathematics course.

Response category frequencies and percentages were reported on various background, course-taking, and classroom practices variables. These responses also were reported for a comparison sample of students from the main assessment who were not eligible for the Advanced Study.

For each group, information on the following demographic variables was reported:

- gender;
- race/ethnicity;
- parent education;
- home resources;
- region;
- location;
- type of school;
- Title 1 participation; and
- eligibility for the federal Free/Reduced-Price School Lunch program.

In addition, information was reported on the following classroom practices variables:

- emphasis on mathematics topics;
- emphasis on mathematics skills;
- unrestricted use of calculators;
- permitted use of calculators on tests;
- access to calculators;
- provision of instruction in use of calculators;

- availability of graphing calculators; and
- primary use of graphing calculators.

The classroom practices variables are based on teacher responses. At grade 12, these were only available for the Advanced sample because grade 12 teachers in the main sample were not surveyed.

In addition, the report provides information on the average proficiency of the Advanced-eligible and not-Advanced-eligible students in the main sample. Because the Advanced-eligible students in the main sample were selected by exactly the same criterion as the students who actually participated in the Advanced Study, this affords a direct comparison of the mathematics achievement of the two groups.

Item-by-item statistics for items in both Advanced blocks were reported for grades 8 and 12.

a) For dichotomous items this included percentages responding to each alternative, with the key (correct) alternative marked.

b) For polytomous items this included percentages responding to each partial credit category.

c) The percentages responding in categories were shown for the whole group and broken down by the variables:

- gender, and
- race/ethnicity.

d) In addition for grade 12, the breakdown variables also included:

- whether the student was currently taking mathematics; and
- whether student had taken an AP course in mathematics.

NAEP Reporting Groups

In this report, results are provided for groups of students defined by shared characteristics — gender, race/ethnicity, parental education, type of school, region of the country, participation in Title I programs, and eligibility for the federal Free/Reduced-Price School Lunch program. Results are reported for subpopulations only when sufficient numbers of students and adequate school representation are present. For public and nonpublic school students in the national assessment, the minimum requirement is at least 62 students in a particular subgroup from at least 5 primary sampling units (PSUs).[5] However, the data for all students, regardless of whether their subgroup was reported separately, were included in computing overall results. Definitions of the subgroups referred to in this report are presented below.

[5] For the national assessment, a PSU is a selected geographic region (a county, group of counties, or metropolitan statistical area).

Gender

Results are reported separately for males and females.

Race/ethnicity

The race/ethnicity variable is derived from school records and two questions asked of students from the set of general student background questions.

> If you are Hispanic, what is your Hispanic background?
>
> ○ I am not Hispanic
> ○ Mexican, Mexican American, or Chicano
> ○ Puerto Rican
> ○ Cuban
> ○ Other Spanish or Hispanic background

Students who responded to this question by filling in the second, third, fourth, or fifth oval were considered Hispanic. For students who filled in the first oval, did not respond to the question, or provided information that was illegible or could not be classified, responses to the following question were examined to determine their race/ethnicity.

> Which best describes you?
>
> ○ White (not Hispanic)
> ○ Black (not Hispanic)
> ○ Hispanic ("Hispanic" means someone who is Mexican, Mexican American, Chicano, Puerto Rican, Cuban, or of other Spanish or Hispanic background.)
> ○ Asian or Pacific Islander ("Asian or Pacific Islander" means someone who is from a Chinese, Japanese, Korean, Filipino, Vietnamese, or other Asian or Pacific Islander background.)
> ○ American Indian or Alaskan Native ("American Indian or Alaskan Native" means someone who is from one of the American Indian tribes or is one of the original people of Alaska.)
> ○ Other (specify) _____

Students' race/ethnicity was then assigned on the basis of their responses. For students who filled in the sixth oval ("Other"), provided illegible information or information that could not be classified, or who did not respond at all, race/ethnicity was assigned as determined by school records.[6]

Race/ethnicity could not be determined for students who did not respond to either of the demographic questions and whose schools did not provide information about race/ethnicity.

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[6] The procedure for assigning race/ethnicity was modified for Hawaii. See the *NAEP 1996 Technical Report* for details.

Details of how race/ethnicity classifications were derived are presented so that readers can determine how useful the results are for their particular purposes. Also, some students indicated that they were from a Hispanic background (e.g., Puerto Rican or Cuban) and that a racial/ethnic category other than Hispanic best describes them. These students were classified as Hispanic based on the rules described above. Furthermore, information from the schools did not always correspond to how students described themselves. Therefore, the racial/ethnic results presented in this report attempt to provide a clear picture based on several sources of information.

Parents' highest level of education

The variable representing the level of parental education is derived from responses to two questions from the set of general student background questions. Students were asked to indicate the extent of their mother's education and their father's education.

How far in school did your mother go?

- ○ She did not finish high school.
- ○ She graduated from high school.
- ○ She had some education after high school.
- ○ She graduated from college.
- ○ I don't know.

How far in school did your father go?

- ○ He did not finish high school.
- ○ He graduated from high school.
- ○ He had some education after high school.
- ○ He graduated from college.
- ○ I don't know.

The information was combined into one parental education reporting variable determined through the following process. If a student indicated the extent of education for only one parent, that level was included in the data. If a student indicated the extent of education for both parents, the higher of the two levels was included in the data. If a student did not know the level of education for both parents or did not know the level for one parent and did not respond to the other, the parental education level was classified as "I don't know." If the student did not respond for either parent, the student was recorded as having provided no response.

Type of school

Results are reported by the type of school that the student attended — public or nonpublic. Nonpublic schools include Catholic and other private schools. Although Bureau of Indian Affairs (BIA) schools and Department of Defense Domestic Dependent Elementary and Secondary Schools (DDESS) are not included in either the public or nonpublic categories, they are included in the overall national results.

Region

Results are reported for four regions of the nation: Northeast, Southeast, Central, and West. Figure A.1 shows how states are subdivided into these regions.

Figure A.1 — *Regions of the Country* — THE NATION'S REPORT CARD naep

Northeast	Southeast	Central	West
Connecticut	Alabama	Illinois	Alaska
Delaware	Arkansas	Indiana	Arizona
District of Columbia	Florida	Iowa	California
Maine	Georgia	Kansas	Colorado
Maryland	Kentucky	Michigan	Hawaii
Massachusetts	Louisiana	Minnesota	Idaho
New Hampshire	Mississippi	Missouri	Montana
New Jersey	North Carolina	Nebraska	Nevada
New York	South Carolina	North Dakota	New Mexico
Pennsylvania	Tennessee	Ohio	Oklahoma
Rhode Island	Virginia*	South Dakota	Oregon
Vermont	West Virginia	Wisconsin	Texas
Virginia*			Utah
			Washington
			Wyoming

* Note: The part of Virginia that is included in the Washington, DC metropolitan area is included in the Northeast region; the remainder of the state is included in the Southeast region.

Title I participation

Based on available school records, students were classified either as currently participating in a Title 1 program or receiving Title 1 services, or as not receiving such services. The classification applies only to the school year when the assessment was administered (i.e., the 1995–96 school year) and was not based on participation in previous years. If the school did not offer Title 1 programs or services, all students in that school would be classified as not participating.

Eligibility for the federal Free/Reduced-Price School Lunch program

Based on available school records, students were classified as either currently eligible for the Free/Reduced-Price School Lunch component of the Department of Agriculture's National School Lunch Program or not eligible. The classification applies only to the school year when the assessment was administered (i.e., the 1995–96 school year) and was not based on eligibility in previous years. If school records were not available, the student was classified as "Information not available." If the school did not participate in the program, all students in that school were classified as "Information not available."

Cautions in Interpretation

This report describes students', teachers', and principals' responses to background questions as well as mathematics performance for fourth-, eighth-, and twelfth-grade students. The report also compares the performance results for various groups of students within these populations (e.g., those who responded to a specific background question in a particular way or by individual course-taking groups as described above). However, it does not include an analysis of the relationships among combinations of these subpopulations or background questions. In interpreting these data, it is important to understand that a relationship that exists between achievement and another variable does not reveal its underlying cause, which may be influenced by a number of other variables. Similarly, the assessments do not capture the influence of unmeasured variables. The results are most useful when they are considered in combination with other knowledge about the student population and the educational system, such as trends in instruction, changes in the school-age population, and societal demands and expectations.

Estimating Variability

Because the statistics presented in this report are estimates of group and subgroup performance based on samples of students rather than the estimates that could be calculated if every student in the nation answered every question, the degree of uncertainty associated with the estimates should be taken into account. Two components of uncertainty are accounted for in the variability statistics based on student ability: (1) the uncertainty due to sampling only a relatively small number of students and (2) the uncertainty due to sampling only a relatively small number of cognitive questions. The first component accounts for the variability associated with the estimated percentages of students who had certain background characteristics or who answered a certain cognitive question correctly.

Because NAEP uses complex sampling procedures, conventional formulas for estimating sampling variability that assume simple random sampling are inappropriate. NAEP uses a jackknife replication procedure to estimate standard error. The jackknife standard error provides a reasonable measure of uncertainty for any student information that can be observed without error. However, because each student typically responds to only a few questions within any

content strand, the scale score for any single student would be imprecise. In this case, plausible values technology can be used to describe the performance of groups or subgroups of students, but the underlying imprecision involved in this step adds another component of variability to statistics based on NAEP scale scores.[7]

Typically, when the standard error is based on a small number of students or when the group of students is enrolled in a small number of schools, the amount of uncertainty associated with the standard error may be quite large. Throughout this report, estimates of standard errors subject to a large degree of uncertainty are designated.

The reader is reminded that, like findings from all surveys, NAEP results are subject to other kinds of error, including the effects of imperfect adjustments for student and school nonresponse and unknowable effects associated with the particular instrumentation and data collection methods. Nonsampling errors can be attributed to a number of sources — inability to obtain complete information about all selected schools in the sample (some students or schools refused to participate, or students participated but answered only certain questions); ambiguous definitions; differences in interpreting questions; inability or unwillingness to give correct information; mistakes in recording, coding, or scoring data; and other errors in collecting, processing, sampling, and estimating missing data. The extent of nonsampling error is difficult to estimate, and, because of their nature, the impact of such errors cannot be reflected in the data-based estimates of uncertainty provided in NAEP reports.

Drawing Inferences from the Results

As noted, because the percentages of students and their average scale scores are based on samples rather than on the entire population of fourth-, eighth-, or twelfth-graders in the nation or a jurisdiction, the numbers reported are estimates. As such, they are subject to a measure of uncertainty, reflected in the standard error of the estimate. When the percentages or average scale scores of certain groups are compared, the standard error should be taken into account, and observed similarities or differences should not be relied on solely. Therefore, the comparisons discussed in this report are based on statistical tests that consider the standard errors of those statistics and the magnitude of the difference among the averages or percentages.

The results from the sample, taking into account the uncertainty associated with all samples, are used to make inferences about the population. Using confidence intervals based on the standard errors provides a way to make inferences about the population averages and percentages in a manner that reflects the uncertainty associated with the sample estimates. An estimated sample average scale score ± 2 standard errors approximates a 95 percent confidence interval for the corresponding population quantity. This statement means that one can conclude with approximately a 5 percent level of confidence that the average performance of the entire population of interest (e.g., all fourth-grade students in public schools in a jurisdiction) is within ± 2 standard errors of the sample average.

[7] For more details, see Johnson, E. G. & Rust, K. F. (1992). Population inferences and variance estimation for NAEP data. *Journal of Educational Statistics*, *17*(2), 175–190.

As an example, suppose that the average mathematics scale score of the students in a particular group was 256, with a standard error of 1.2. A 95 percent confidence interval for the population quantity could be described in any of the following ways:

Average ± 2 standard errors
$256 \pm 2 \times 1.2$
256 ± 2.4
253.6, 258.4

Thus, one can conclude with a 5 percent level of confidence that the average scale score for the entire population of students in that group is between 253.6 and 258.4.

Similar confidence intervals can be constructed for percentages, if the percentages are not extremely large or extremely small. For extreme percentages, confidence intervals constructed in the above manner may not be appropriate, and accurate confidence intervals can be constructed only using procedures that are quite complicated.

Extreme percentages, defined by both the magnitude of the percentage and the size of the sample from which it was derived, should be interpreted with caution. The *NAEP 1996 Technical Report* contains a more complete discussion of extreme percentages.[8]

Analyzing Group Differences in Averages and Percentages

The statistical tests determine whether the evidence, based on the data from the groups in the sample, is strong enough to conclude that the averages or percentages are actually different for those groups in the population. If the evidence is strong (i.e., the difference is statistically significant), the report describes the group averages or percentages as being different (e.g., one group performed higher than or lower than another group), regardless of whether the sample averages or percentages appear to be approximately the same. If the evidence is not sufficiently strong (i.e., the difference is not statistically significant), the averages or percentages are described as being not significantly different, regardless of whether the sample averages or percentages appear to be approximately the same or widely discrepant.

The reader is cautioned to rely on the results of the statistical tests rather than on the apparent magnitude of the difference between sample averages or percentages when determining whether the sample differences are likely to represent actual differences among the groups in the population.

To determine whether a real difference exists between the average scale scores (or percentages of a certain attribute) for two groups in the population, one needs to obtain an estimate of the degree of uncertainty associated with the difference between the averages (or percentages) of these groups for the sample. This estimate of the degree of uncertainty, called the standard error of the difference between the groups, is obtained by taking the square of each group's standard error, summing the squared standard errors, and taking the square root of that sum.

[8] Allen, et al., (1999). op. cit.

Standard Error of the Difference = $SE_{A-B} = \sqrt{(SE_A^2 + SE_B^2)}$

Similar to how the standard error for an individual group average or percentage is used, the standard error of the difference can be used to help determine whether differences among groups in the population are real. The difference between the averages or percentages of the two groups ± 2 standard errors of the difference represents an approximate 95 percent confidence interval. If the resulting interval includes zero, there is insufficient evidence to claim that a real difference between the groups is statistically significant (different) at the 0.05 level. In this report, differences among groups that involve poorly defined variability estimates or extreme percentages are not discussed.

As an example, to determine whether the average mathematics scale score of Group A is higher than that of Group B, suppose that the sample estimates of the average scale score and standard errors were as follows:

Group	Average Scale Score	Standard Error
A	218	0.9
B	216	1.1

The difference between the estimates of the average scale scores of Groups A and B is 2 points (218–216). The standard error of this difference is:

$$\sqrt{(0.9^2 + 1.1^2)} = 1.4$$

Thus, an approximate 95 percent confidence interval for this difference is:

Difference ± 2 standard errors of the difference

$$2 \pm 2 \times 1.4$$
$$2 \pm 2.8$$
$$-0.8, 4.8$$

The value zero is within the confidence interval; therefore, there is insufficient evidence to claim that Group A outperformed Group B.

The procedures described in this section and the certainty ascribed to intervals (e.g., a 95 percent confidence interval) are based on statistical theory that assumes that only one confidence interval or test of statistical significance is being performed. However, in Chapters 2–4 of this report, many different groups are being compared (i.e., multiple sets of confidence intervals are being analyzed). In sets of confidence intervals, statistical theory indicates that the certainty associated with the entire set of intervals is less than that attributable to each individual comparison from the set. To hold the significance level for the set of comparisons at a particular level (e.g., 0.05), adjustments (called multiple comparison procedures) must be made to the methods described in the previous section. One such procedure, the Bonferroni method, was used in the analyses described in this report to obtain confidence intervals for the differences among groups when sets of comparisons were considered.[9] Thus, the confidence intervals for the sets of comparisons in the text are more conservative than those described on the previous pages.

[9] Miller, R. G. (1966). *Simultaneous statistical inference*. New York: Wiley.

Most of the multiple comparisons in this report pertain to relatively small sets or families of comparisons. For example, for discussions concerning comparisons of parents' level of education, six comparisons were conducted — all pairs of the four parental education levels. In these situations, Bonferroni procedures were appropriate.

Appendix B

Standard Error Tables

The comparisons presented in this report are based on statistical tests that consider the magnitude of the difference between group averages or percentages and the standard errors of those statistics. The following appendix contains the standard errors for the averages and percentages discussed in Chapters 2, 3, and 4. For ease of reference, the format and headings of each table in this appendix match the corresponding chapter table, although the numbers that appear are actually standard errors.

Table B2.4	Standard Errors for Average Scale Scores for National NAEP and Estimation Studies, Grades 4, 8, and 12	THE NATION'S REPORT CARD naep

	Assessment Year	Average Overall Scale Score in Mathematics NAEP	Average Estimation Scale Score
Grade 4			
	1996	0.9	2.1
	1992	0.7	1.5
	1990	0.9	1.5
Grade 8			
	1996	1.1	1.2
	1992	0.9	1.3
	1990	1.3	1.2
Grade 12			
	1996	1.0	1.2
	1992	0.9	1.2
	1990	1.1	1.2

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Table B2.5

Standard Errors for Scale Scores in Estimation by Background Variables, Grades 4, 8, and 12, 1996

THE NATION'S REPORT CARD

	Average Scale Score		
	Grade 4	Grade 8	Grade 12
Gender			
Males	2.3	1.7	1.6
Females	2.3	1.6	1.4
Students who Indicated Their Race/Ethnicity as...			
White	2.5	1.6	1.4
Black	4.2	2.0	3.0
Hispanic	2.8	2.8	3.1
Asian/Pacific Islander	9.2!	3.0	6.1
American Indian	***	***	***
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	5.1	2.6	4.2
Graduated From High School	3.6	1.7	1.7
Some Education After High School	5.6	1.5	1.7
Graduated From College	2.9	1.9	1.6
I Don't Know	2.7	2.0	***
Students who Attend...			
Public Schools	2.2	1.3	1.2
Nonpublic Schools	4.9!	3.6!	4.4!
Title I Participation...			
Participated	3.2	3.2	4.9!
Did Not Participate	2.3	1.4	1.3
Free/Reduced-Price Lunch Program Eligibility...			
Not Eligible	2.1	1.9	1.4
Eligible	2.8	1.7	3.1
Information Unavailable	5.8	2.8!	2.8

*** Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B2.6

Standard Errors for Average Scale Scores in Estimation at Different Percentile Levels, Grades 4, 8, and 12

THE NATION'S REPORT CARD

	Assessment Year	10th Percentile	25th Percentile	50th Percentile	75th Percentile	90th Percentile
Grade 4						
	1996	5.5	2.6	1.5	3.0	3.6
	1992	1.5	2.4	1.8	1.5	3.1
	1990	2.4	2.0	2.6	1.9	2.4
Grade 8						
	1996	1.2	1.4	1.7	1.1	2.5
	1992	2.1	2.0	1.6	1.9	2.5
	1990	1.3	1.8	2.3	2.3	1.2
Grade 12						
	1996	1.2	1.9	1.6	1.2	1.7
	1992	1.4	1.9	1.5	0.9	2.1
	1990	1.5	1.6	2.2	1.3	1.1

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Table B2.7	Standard Errors for National Percentages Attaining Achievement Levels in Estimation, Grades 4, 8, and 12	THE NATION'S REPORT CARD naep

	Percentage of Students				
	Assessment Year	Advanced	At or Above Proficient	At or Above Basic	Below Basic
Grade 4					
	1996	0.4	2.4	1.3	1.3
	1992	0.3	1.7	1.1	1.1
	1990	---	1.3	1.8	1.8
Grade 8					
	1996	0.3	1.6	1.9	1.9
	1992	0.5	1.8	2.0	2.0
	1990	0.5	1.6	2.3	2.3
Grade 12					
	1996	1.0	2.5	1.2	1.2
	1992	0.7	2.0	1.7	1.7
	1990	0.9	2.1	1.6	1.6

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Table B2.8	Standard Errors for Percent of Students Reaching at Least Proficient Level in Estimation by Background Variables, Grades 4, 8, and 12, 1996	THE NATION'S REPORT CARD naep

	Percentage of Students Achieving Proficient or Better		
	Grade 4	Grade 8	Grade 12
Gender			
Males	3.1	2.0	2.7
Females	2.7	2.1	3.2
Students who Indicated Their Race/Ethnicity as...			
White	3.1	2.1	3.1
Black	2.6	1.1	3.9
Hispanic	2.4	2.4	3.6
Asian/Pacific Islander	9.9!	3.5	8.2
American Indian	9.9!	---	---
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	4.2	3.1	4.6
Graduated From High School	4.0	2.3	3.1
Some Education After High School	5.9	2.9	3.5
Graduated From College	4.0	2.7	3.0
I Don't Know	2.9	2.1	---
Students who Attend...			
Public Schools	2.6	1.7	2.2
Nonpublic Schools	5.8!	5.2!	7.4!
Title I Participation...			
Participated	2.3	---	4.2!
Did Not Participate	2.6	1.7	2.6
Free/Reduced-Price Lunch Program Eligibility...			
Not Eligible	3.1	2.8	2.7
Eligible	2.4	1.9	3.4
Information Unavailable	6.1	2.7!	4.4

--- Standard errors could not be accurately determined.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.1

Standard Errors for Student Demographic Distributions by Assessment, Grade 4, 1996

THE NATION'S REPORT CARD naep

Grade 4	Percentage of Students		
	Main Assessment	Theme Block 1 The Butterfly Booth	Theme Block 2 Recycling
Gender			
Males	0.7	1.4	1.4
Females	0.7	1.4	1.4
Students who Indicated Their Race/Ethnicity as...			
White	0.4	0.9	0.9
Black	0.2	0.6	0.6
Hispanic	0.4	0.6	0.7
Asian/Pacific Islander	0.2	0.3	0.4
American Indian	0.2	0.3	0.3
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	0.3	0.5	0.8
Graduated From High School	0.6	1.0	1.0
Some Education After High School	0.4	0.7	0.7
Graduated From College	1.2	1.3	1.4
I Don't Know	0.9	1.1	1.2
Students who Attend...			
Public Schools	1.6	1.8	1.6
Nonpublic Schools	1.6	1.8	1.6
Title I Participation...			
Participated	1.4	1.9	1.9
Did Not Participate	1.4	1.9	1.9
Free/Reduced-Price Lunch Program Eligibility...			
Eligible	1.4	1.6	1.3
Not Eligible	2.5	2.7	2.6
Information Not Available	3.0	2.4	2.3

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.2	Standard Errors for Percentage of Students by Teachers' Reports on Classroom Practices, Grade 4, 1996	THE NATION'S REPORT CARD naep

Grade 4	Percentage of Students		
	Main Assessment	Theme Block 1 The Butterfly Booth	Theme Block 2 Recycling
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem...			
Nearly Every Day	1.4	1.2	0.9
Once or Twice a Week	2.1	2.1	2.4
Once or Twice a Month	2.6	2.3	2.7
Never or Hardly Ever	2.4	2.8	3.0
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project...			
Nearly Every Day	0.5	0.1	0.1
Once or Twice a Week	0.8	0.6	0.5
Once or Twice a Month	2.3	2.7	2.6
Never or Hardly Ever	2.4	2.7	2.7

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.3

Standard Errors for Average Percentage Correct Scores by Theme Block, Grade 4, 1996

THE NATION'S REPORT CARD naep

Grade 4	The Butterfly Booth	Recycling
All Students	0.7	0.6
Gender		
Males	0.9	0.8
Females	0.8	0.6
Race/Ethnicity		
White	0.8	0.7
Black	0.7	1.1
Hispanic	0.9	1.1
Asian/Pacific Islander	2.4	2.1
American Indian	***	***
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem...		
Nearly Every Day	3.3	1.9
Once or Twice a Week	1.2	1.3
Once or Twice a Month	1.2	1.4
Never or Hardly Ever	1.2	1.3
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project...		
Nearly Every Day	***	***
Once or Twice a Week	***	***
Once or Twice a Month	1.4	1.6
Never or Hardly Ever	0.8	0.7

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.4	Standard Errors for Score Percentages for "Draw Symmetrical Figure," Grade 4	THE NATION'S REPORT CARD naep

	Satisfactory	Partial	Minimal	Incorrect	Omit
Grade 4					
All Students	1.2	0.8	0.9	0.7	1.4
Males	1.6	1.1	1.3	1.1	1.8
Females	1.8	1.3	1.2	0.9	1.7
White	1.4	0.9	1.1	0.9	1.5
Black	1.7	1.8	2.1	1.9	2.6
Hispanic	1.7	2.0	1.5	1.5	2.9
Asian/Pacific Islander	7.6	4.1	3.1	2.8	5.5
American Indian	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.5	Standard Errors for Score Percentages for "Measure Length Using Ruler," Grade 4	THE NATION'S REPORT CARD naep

	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Grade 4						
All Students	1.4	0.8	1.0	0.5	1.3	0.5
Males	2.2	1.2	1.4	0.5	2.0	0.8
Females	1.7	1.4	1.1	0.9	1.4	0.6
White	1.7	1.0	1.2	0.7	1.6	0.4
Black	2.0	2.3	2.0	1.3	2.7	1.5
Hispanic	2.7	1.5	2.2	0.9	3.2	2.3
Asian/Pacific Islander	6.0	2.8	2.5	1.3	5.3	1.6
American Indian	***	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.6 — Standard Errors for Score Percentages for "Solve Packing Problems," Grade 4

THE NATION'S REPORT CARD naep

Grade 4	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	0.7	1.0	1.1	1.4	1.3	0.2
Males	1.2	1.3	1.2	1.8	1.7	0.4
Females	0.9	1.3	1.6	1.7	1.8	0.2
White	0.9	1.3	1.5	1.9	1.8	0.3
Black	1.0	0.9	1.4	2.6	2.4	0.6
Hispanic	0.4	1.6	1.8	2.9	3.2	---
Asian/Pacific Islander	1.4	6.4	4.3	5.8	4.4	---
American Indian	***	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.7 — Standard Errors for Score Percentages for "Determine Number of Models," Grade 4

THE NATION'S REPORT CARD naep

Grade 4	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	0.4	1.0	1.0	1.7	0.4
Males	0.5	1.3	1.3	2.0	0.7
Females	0.5	1.5	1.4	2.2	0.4
White	0.6	1.3	1.4	2.2	0.5
Black	0.6	1.2	1.7	2.5	1.0
Hispanic	0.4	1.5	2.0	2.3	1.1
Asian/Pacific Islander	2.6	4.4	5.0	6.0	1.9
American Indian	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.8

Standard Errors for Score Percentages for "Determine Number of Leaves," Grade 4

THE NATION'S REPORT CARD naep

	Complete	Partial	Incorrect	Omit
Grade 4				
All Students	0.6	0.8	0.9	0.3
Males	0.9	0.9	1.1	0.4
Females	0.7	1.1	1.4	0.4
White	0.7	1.1	1.3	0.3
Black	1.0	0.9	1.8	1.1
Hispanic	0.8	0.8	1.4	0.8
Asian/Pacific Islander	3.2	3.3	4.9	3.0
American Indian	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.9

Standard Errors for Score Percentages for "Interpret Pattern of Figures," Grade 4

THE NATION'S REPORT CARD naep

	Satisfactory	Partial	Minimal	Incorrect	Omit
Grade 4					
All Students	0.3	0.4	0.5	0.8	0.2
Males	0.4	0.6	0.7	1.2	0.3
Females	0.3	0.6	0.6	1.0	0.4
White	0.4	0.6	0.7	1.1	0.3
Black	---	0.7	1.2	1.6	0.6
Hispanic	---	0.5	1.1	2.1	0.6
Asian/Pacific Islander	---	---	3.2	3.7	2.2
American Indian	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.10

Standard Errors for Student Demographic Distributions by Assessment, Grade 8, 1996

THE NATION'S REPORT CARD naep

Grade 8	Percentage of Students		
	Main Assessment	Theme Block 1 Building a Doghouse	Theme Block 2 Flooding
Gender			
Males	0.8	1.0	1.2
Females	0.8	1.0	1.2
Race/Ethnicity			
White	0.2	0.7	0.6
Black	0.2	0.6	0.5
Hispanic	0.1	0.5	0.5
Asian/Pacific Islander	0.2	0.3	0.3
American Indian	0.2	0.3	0.3
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	0.4	0.7	1.1
Graduated From High School	0.8	1.2	1.1
Some Education After High School	0.7	1.0	1.0
Graduated From College	1.3	1.6	1.7
I Don't Know	0.6	0.7	0.7
Students who Attend...			
Public Schools	1.1	1.9	1.9
Nonpublic Schools	1.1	1.9	1.9
Title I Participation...			
Participated	1.6	1.7	1.7
Did Not Participate	1.6	1.7	1.7
Free/Reduced-Price Lunch Program Eligibility...			
Eligible	1.4	1.5	2.0
Not Eligible	2.4	2.5	2.8
Information Not Available	2.9	2.9	3.0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.11

Standard Errors for Percentage of Students by Teachers' Reports on Classroom Practices, Grade 8, 1996

THE NATION'S REPORT CARD

	Percentage of Students		
	Main Assessment	Theme Block 1 Building a Doghouse	Theme Block 2 Flooding
Grade 8			
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem...			
Nearly Every Day	1.1	1.1	1.2
Once or Twice a Week	2.8	3.2	3.7
Once or Twice a Month	2.8	3.2	3.6
Never or Hardly Ever	3.3	3.4	3.6
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project...			
Nearly Every Day	---	---	---
Once or Twice a Week	1.1	1.3	1.4
Once or Twice a Month	3.2	2.7	2.8
Never or Hardly Ever	3.3	2.5	2.8

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.12

Standard Errors for Average Percentage Correct Scores by Theme Block, Grade 8, 1996

THE NATION'S REPORT CARD naep

Grade 8	Build a Doghouse	Flooding
All Students	0.5	0.7
Gender		
Males	0.6	0.8
Females	0.7	0.9
Race/Ethnicity		
White	0.7	1.0
Black	0.8	0.5
Hispanic	0.8	0.8
Asian/Pacific Islander	2.5	2.8
American Indian	***	***
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem...		
Nearly Every Day	3.1	3.0
Once or Twice a Week	1.6	2.4
Once or Twice a Month	1.1	1.3
Never or Hardly Ever	1.0	1.4
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project...		
Nearly Every Day	***	***
Once or Twice a Week	2.6	2.1
Once or Twice a Month	1.1	1.4
Never or Hardly Ever	0.9	1.0

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.13 — Standard Errors for Score Percentages for "Identifying Needed Information," Grade 8

THE NATION'S REPORT CARD naep

Grade 8	5 Correct	4 Correct	3 Correct	2 Correct	1 Correct	0 Correct	Omit
All Students	1.5	1.0	0.7	0.7	0.7	0.3	0.1
Males	1.8	1.3	1.2	0.9	1.2	0.5	0.1
Females	2.1	1.5	0.9	0.9	0.8	0.2	0.1
White	1.9	1.1	0.8	0.7	0.7	0.4	---
Black	2.5	2.0	2.4	1.4	1.2	0.5	0.5
Hispanic	2.5	2.8	2.0	1.7	1.5	---	0.3
Asian/Pacific Islander	5.2	4.4	---	2.5	---	---	---
American Indian	***	***	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.14 — Standard Errors for Percentages Correct for "Identifying Needed Information," Grade 8

THE NATION'S REPORT CARD naep

Grade 8	1a — Yes	1b — Yes	1c — No	1d — Yes	1e — Yes
All Students	0.9	0.9	1.2	1.0	1.2
Males	1.6	1.3	1.5	1.6	1.6
Females	0.9	1.1	1.6	0.9	1.3
White	1.1	1.1	1.7	1.2	1.5
Black	2.2	1.9	1.9	2.4	2.5
Hispanic	1.8	1.6	3.0	2.6	2.7
Asian/Pacific Islander	---	3.1	4.1	4.5	4.1
American Indian	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.15	Standard Errors for Percentage Correct for "Determine Minimum Measuring Needed," Grade 8	THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
All Students	1.3
Males	1.3
Females	2.2
White	1.6
Black	2.6
Hispanic	2.4
Asian/Pacific Islander	7.4
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 0Mathematics Assessment.

Table B3.16	Standard Errors for Score Percentages for "Measure Lengths Using Ruler," Grade 8	THE NATION'S REPORT CARD naep

Grade 8	Complete	Partial	Incorrect	Omit
All Students	1.5	1.2	1.2	1.3
Males	1.8	1.4	1.5	1.2
Females	2.0	1.7	1.6	1.6
White	1.8	1.5	1.4	1.4
Black	2.4	3.1	4.1	2.7
Hispanic	3.0	3.0	2.9	1.5
Asian/Pacific Islander	7.3	5.8	8.5	---
American Indian	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.17	Standard Errors for Score Percentages for "Apply Concept of Ratio," Grade 8	THE NATION'S REPORT CARD naep

	Complete	Partial	Incorrect	Omit
Grade 8				
All Students	1.1	1.2	1.5	1.1
Males	1.4	1.5	1.7	1.1
Females	1.2	1.5	2.0	1.7
White	1.4	1.5	1.8	1.4
Black	0.9	1.1	2.8	3.3
Hispanic	1.9	1.7	3.1	2.5
Asian/Pacific Islander	4.8	6.6	7.2	4.5
American Indian	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.18	Standard Errors for Percentage Correct for "Understand Concept of Ratio (I)," Grade 8	THE NATION'S REPORT CARD naep

	Percentage Correct
Grade 8	
All Students	1.3
Males	1.9
Females	1.3
White	1.6
Black	2.3
Hispanic	3.1
Asian/Pacific Islander	9.0
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.19	Standard Errors for Percentage Correct for "Understand Concept of Ratio (II)," Grade 8	THE NATION'S REPORT CARD naep

	Percentage Correct
Grade 8	
All Students	1.2
Males	1.7
Females	1.7
White	1.4
Black	2.4
Hispanic	2.8
Asian/Pacific Islander	8.2
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.20	Standard Errors for Score Percentages for "Correctly Position Door," Grade 8	THE NATION'S REPORT CARD naep

	Complete	Partial	Incorrect	Omit
Grade 8				
All Students	1.3	1.2	1.3	0.8
Males	2.0	1.8	2.0	1.3
Females	1.7	1.4	1.6	0.9
White	1.8	1.4	1.7	0.9
Black	1.1	2.9	2.7	2.7
Hispanic	1.9	3.3	3.2	2.4
Asian/Pacific Islander	4.5	8.3	6.6	6.8
American Indian	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.21	Standard Errors for Score Percentages for "Visualize Cut-Outs on Grid," Grade 8	THE NATION'S REPORT CARD naep

	Complete	Partial	Incorrect	Omit
Grade 8				
All Students	1.5	0.6	1.4	0.8
Males	1.7	0.9	1.5	1.1
Females	2.1	1.2	2.1	1.1
White	1.9	0.8	1.7	0.8
Black	2.2	0.8	2.3	2.6
Hispanic	2.6	1.3	2.7	3.0
Asian/Pacific Islander	8.6	5.0	6.4	4.8
American Indian	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.22	Standard Errors for Score Percentages for "Apply Geometry in Model," Grade 8	THE NATION'S REPORT CARD naep

	Satisfactory	Partial	Minimal	Incorrect	Omit
Grade 8					
All Students	0.5	0.7	0.8	1.1	1.6
Males	0.4	1.0	1.3	2.1	2.3
Females	0.8	0.9	1.2	1.6	1.5
White	0.7	1.0	1.1	1.2	2.0
Black	---	---	1.4	3.4	3.1
Hispanic	---	1.1	2.0	3.8	3.5
Asian/Pacific Islander	***	***	***	***	***
American Indian	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.23	Standard Errors for Score Percentages for "Find Maximum Area When Perimeter is Fixed," Grade 8

THE NATION'S REPORT CARD naep

Grade 8	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	0.1	0.0	1.2	0.6	1.7	1.3
Males	0.1	---	1.8	1.0	1.7	1.7
Females	0.2	0.1	1.5	0.4	2.2	2.0
White	0.1	---	1.5	0.8	1.9	1.4
Black	---	---	1.7	0.9	3.3	3.1
Hispanic	---	---	2.6	1.0	3.2	4.0
Asian/Pacific Islander	***	***	***	***	***	***
American Indian	***	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.24

Standard Errors for Student Demographic Distributions by Assessment, Grade 12, 1996

THE NATION'S REPORT CARD naep

Grade 12	Percentage of Students		
	Main Assessment	Theme Block 1 Buying a Car	Theme Block 2 Flooding
Gender			
Males	0.9	1.4	1.3
Females	0.9	1.4	1.3
Race/Ethnicity			
White	0.5	0.8	0.8
Black	0.4	0.6	0.6
Hispanic	0.4	0.6	0.6
Asian/Pacific Islander	0.4	0.3	0.4
American Indian	0.6	0.2	0.2
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	0.5	0.8	0.6
Graduated From High School	0.8	1.6	1.3
Some Education After High School	0.8	1.1	1.3
Graduated From College	1.5	1.8	1.8
I Don't Know	0.2	0.5	0.5
Students who Attend...			
Public Schools	1.6	1.9	2.0
Nonpublic Schools	1.5	1.9	2.0
Title I Participation...			
Participated	0.6	0.8	0.8
Did Not Participate	0.6	0.8	0.8
Free/Reduced-Price Lunch Program Eligibility...			
Eligible	1.3	1.2	1.1
Not Eligible	3.7	3.5	3.7
Information Not Available	3.8	3.5	3.6

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.25

Standard Errors for Percentage of Students by Reports on Classroom Practices, Grade 12, 1996*

THE NATION'S REPORT CARD

	Percentage of Students		
	Main Assessment	Buying a Car	Flooding
Grade 12			
Students who Report Writing a Few Sentences About How to Solve a Mathematics Problem...			
Nearly Every Day	0.4	0.7	0.7
Once or Twice a Week	0.5	0.8	0.9
Once or Twice a Month	0.6	1.4	0.9
Never or Hardly Ever	0.8	1.6	1.6
Students who Report Writing Reports or Doing a Mathematics Project...			
Nearly Every Day	0.2	---	---
Once or Twice a Week	0.3	0.5	0.5
Once or Twice a Month	1.0	1.6	1.1
Never or Hardly Ever	1.0	1.8	1.3

* Teachers of twelfth-grade students were not surveyed in the Main Assessment.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.26

Standard Errors for Average Percentage Correct Scores by Theme Block, Grade 12, 1996

THE NATION'S REPORT CARD naep

Grade 12	Buying a Car	Flooding
All Students	1.1	0.8
Gender		
Males	1.4	1.1
Females	1.2	1.1
Race/Ethnicity		
White	1.3	1.0
Black	1.3	1.1
Hispanic	2.1	1.9
Asian/Pacific Islander	3.8	3.3
American Indian	***	***
Students Whose Teachers Ask Them to Write a Few Sentences About How to Solve a Mathematics Problem...		
Nearly Every Day	4.4	1.9
Once or Twice a Week	2.3	1.8
Once or Twice a Month	2.1	1.5
Never or Hardly Ever	1.1	1.1
Students Whose Teachers Ask Them to Write Reports or Do a Mathematics Project...		
Nearly Every Day	***	***
Once or Twice a Week	2.9	2.7
Once or Twice a Month	2.0	1.4
Never or Hardly Ever	1.4	1.0

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.27	Standard Errors for Percentage Correct for "Find Amount of Down Payment," Grade 12	THE NATION'S REPORT CARD naep

	Percentage Correct
Grade 12	
All Students	1.1
Males	1.6
Females	1.6
White	1.0
Black	3.1
Hispanic	3.3
Asian/Pacific Islander	4.1
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.28	Standard Errors for Percentage Correct for "Find Total Amount Paid for Car," Grade 12	THE NATION'S REPORT CARD naep

	Percentage Correct
Grade 12	
All Students	1.1
Males	1.5
Females	1.2
White	1.2
Black	3.4
Hispanic	3.8
Asian/Pacific Islander	4.5
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.29	Standard Errors for Percentage Correct for "Find Difference Between Total Amount Paid and Price," Grade 12	THE NATION'S REPORT CARD naep

	Percentage Correct
Grade 12	
All Students	1.2
Males	1.9
Females	1.4
White	1.3
Black	3.6
Hispanic	3.1
Asian/Pacific Islander	4.1
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.30	Standard Errors for Score Percentages for "Find Amount To Be Financed," Grade 12	THE NATION'S REPORT CARD naep

	Complete	Partial	Incorrect	Omit
Grade 12				
All Students	1.4	1.5	1.4	0.6
Males	2.0	1.8	1.7	1.2
Females	1.6	1.8	1.6	0.6
White	1.7	1.6	1.8	0.6
Black	2.5	3.2	2.9	2.2
Hispanic	3.8	3.5	3.3	2.0
Asian/Pacific Islander	5.9	5.1	3.5	3.2
American Indian	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.31	Standard Errors for Score Percentages for "Use Formula to Find Total Cost," Grade 12	THE NATION'S REPORT CARD naep

Grade 12	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	0.9	1.1	0.9	0.8	1.5	0.8
Males	1.0	1.1	1.0	1.2	1.9	1.2
Females	1.2	1.6	1.2	1.1	2.0	1.1
White	1.2	1.4	1.2	1.0	1.9	1.0
Black	0.8	0.9	1.3	1.9	2.9	2.0
Hispanic	1.6	2.7	1.2	2.8	4.0	1.9
Asian/Pacific Islander	3.9	4.1	2.5	2.3	5.8	2.9
American Indian	***	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.32	Standard Errors for Score Percentages for "Find Amount Saved if Leased," Grade 12	THE NATION'S REPORT CARD naep

Grade 12	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	1.6	1.0	1.3	1.3	0.5
Males	2.0	1.6	1.5	1.9	0.9
Females	1.9	1.6	1.9	1.7	0.7
White	1.8	1.2	1.5	1.3	0.6
Black	1.7	2.6	4.2	3.3	2.6
Hispanic	3.7	3.2	3.6	3.9	2.2
Asian/Pacific Islander	4.3	3.9	4.9	7.1	0.8
American Indian	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.33

Standard Errors for Score Percentages for "Price Lease vs. Buy," Grade 12

THE NATION'S REPORT CARD

Grade 12	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	1.5	0.4	0.9	1.7	0.9
Males	1.9	0.7	1.4	2.7	1.1
Females	1.7	0.4	1.4	1.9	1.2
White	1.8	0.6	1.1	1.9	0.9
Black	1.4	- - -	1.8	4.1	4.0
Hispanic	2.8	- - -	1.3	3.9	2.7
Asian/Pacific Islander	4.2	1.5	4.8	7.0	1.1
American Indian	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

- - - Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.1

Standard Errors for Student Demographic Distributions, Grade 8, 1996

THE NATION'S REPORT CARD naep

Grade 8	Percentage of Students: Advanced Study	Percentage of Students: Not Eligible for Advanced Study
Gender		
Males	1.2	0.9
Females	1.2	0.9
Students who Indicated Their Race/Ethnicity as...		
White	2.6	0.7
Black	2.7	0.5
Hispanic	0.9	0.4
Asian/Pacific Islander	0.8	0.5
American Indian	0.8	0.5
Students who Reported Their Parents' Highest Level of Education as...		
Did Not Finish High School	0.4	0.5
Graduated From High School	1.2	0.8
Some Education After High School	1.0	0.7
Graduated From College	1.6	1.2
I Don't Know	0.5	0.8
Home Environment Contains...		
0–2 types of educational materials	1.1	0.7
3 types of educational materials	1.2	1.0
4 types of educational materials	1.4	1.0
Students From...		
Northeast	3.8	1.4
Southeast	2.2	1.8
Central	4.0	1.7
West	2.7	1.7
Students Live in...		
Center City	4.2	2.8
Urban Fringe/Small City	4.6	3.1
Rural	3.8	3.2

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.2

Standard Errors for Student/School Demographic Distributions, Grade 8, 1996

THE NATION'S REPORT CARD naep

	Percentage of Students	
	Advanced Study	Not Eligible for Advanced Study
Grade 8		
Students who Attend...		
Public Schools	2.3	1.0
Nonpublic Schools	2.3	1.0
Title I Participation...		
Participated	0.8	1.9
Did Not Participate	0.8	1.9
Free/Reduced-Price Lunch Program Eligibility...		
Eligible	2.0	1.6
Not Eligible	3.3	2.3
Information Not Available	3.4	2.8

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.3

Standard Errors for Content Emphases in Mathematics, Grade 8, 1996

THE NATION'S REPORT CARD naep

Grade 8	Percentage of Students Whose Teachers Responded			
	None	A Little	Some	A Lot
In This Mathematics Class How Often Do You Address...				
Numbers and Operations				
Advanced Study	---	1.8	3.7	3.7
Non-Eligibles	---	0.5	1.8	1.9
Measurement				
Advanced Study	1.6	4.0	4.5	2.5
Non-Eligibles	0.3	2.7	3.7	3.1
Geometry				
Advanced Study	3.1	3.3	3.6	2.8
Non-Eligibles	0.7	3.2	3.4	3.0
Data Analysis, Statistics, and Probability				
Advanced Study	3.1	4.9	5.0	2.7
Non-Eligibles	1.4	3.4	3.5	2.5
Algebra and Functions				
Advanced Study	---	0.5	2.2	2.2
Non-Eligibles	0.4	1.4	3.5	3.8

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.4

Standard Errors for Process Emphases in Mathematics, Grade 8, 1996

THE NATION'S REPORT CARD

	Percentage of Students Whose Teachers Responded			
	None	A Little	Some	A Lot
Grade 8				
In This Mathematics Class How Often Do You Address...				
Learning Mathematical Facts and Concepts				
Advanced Study	1.4	2.5	2.3	3.8
Non-Eligibles	0.0	1.9	2.6	3.0
Learning Skills and Procedures Needed to Solve Routine Problems				
Advanced Study	---	1.6	2.2	2.7
Non-Eligibles	---	0.3	2.5	2.5
Developing Reasoning and Analytical Ability to Solve Unique Problems				
Advanced Study	---	1.7	3.8	4.4
Non-Eligibles	0.3	2.0	3.5	3.4
Learning How to Communicate Ideas in Mathematics Effectively				
Advanced Study	---	2.9	3.1	4.6
Non-Eligibles	0.2	2.3	3.3	3.2

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.5	Standard Errors for Calculator Emphases in Mathematics, Grade 8, 1996	THE NATION'S REPORT CARD naep

	Percentage of Students Whose Teachers Responded	
	Yes	No
Grade 8		
Do You Permit Students in This Class Unrestricted Use of Calculators...		
Advanced Study	5.2	5.2
Non-Eligibles	3.2	3.2
Do You Permit Students in This Class to Use Calculators for Tests...		
Advanced Study	3.3	3.3
Non-Eligibles	2.8	2.8
Do Students in This Class Have Access to Calculators Owned by the School...		
Advanced Study	3.6	3.6
Non-Eligibles	3.2	3.2
Do You Provide Instruction to Students in This Class in the Use of Calculators...		
Advanced Study	3.8	3.8
Non-Eligibles	2.7	2.7

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.6	Standard Errors for Average Mathematics Scale Scores by Eligibility for Advanced Study, Grade 8	THE NATION'S REPORT CARD naep

	Average Scale Score
Grade 8	
All Students	1.1
Eligible	2.0
Not Eligible	1.0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.8	Standard Errors for Average Percentage Correct Scores, Advanced Study, Grade 8, 1996	THE NATION'S REPORT CARD naep

	Percentage Correct
Grade 8	
All Students	1.1
Males	1.3
Females	1.1
White	0.9
Black	2.3
Hispanic	2.1
Asian/Pacific Islander	1.7
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.9

Standard Errors for Score Percentages for "Car Wash Supplies," Grade 8

THE NATION'S REPORT CARD

	Correct	Partial	Incorrect	Omit
Grade 8				
All Students	1.2	1.8	1.7	1.3
Males	1.6	2.1	1.9	1.4
Females	1.2	2.0	2.1	1.6
White	1.4	1.9	2.0	1.7
Black	1.2!	3.3!	3.9!	1.9!
Hispanic	2.7	3.6	4.0	3.4
Asian/Pacific Islander	3.2	4.4	3.9	3.7
American Indian	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.10

Standard Errors for Score Percentages for "Begin to Earn Profit," Grade 8

THE NATION'S REPORT CARD naep

	Correct	Partial	Incorrect	Omit
Grade 8				
All Students	1.6	1.3	1.8	1.5
Males	2.0	1.8	2.2	1.5
Females	1.6	1.2	2.1	1.9
White	1.8	1.5	2.1	1.8
Black	2.3!	0.9!	1.8!	2.3!
Hispanic	3.5	2.6	4.8	3.2
Asian/Pacific Islander	3.9	2.8	4.3	3.4
American Indian	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.11

Standard Errors for Score Percentages for "Greatest Profit Expected," Grade 8

THE NATION'S REPORT CARD naep

Grade 8	Correct	Partial	Incorrect	Omit
All Students	1.9	0.7	1.8	2.1
Males	2.2	0.9	2.4	2.4
Females	2.2	0.8	1.9	2.2
White	2.0	0.8	2.4	2.5
Black	3.1!	0.7!	2.1!	3.6!
Hispanic	3.2	1.3	3.4	4.0
Asian/Pacific Islander	5.9	1.4	6.1	4.2
American Indian	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.12

Standard Errors for Score Percentages for "Hot Air Balloon," Grade 8

THE NATION'S REPORT CARD naep

Grade 8	Correct	Partial	Incorrect	Omit
All Students	1.3	1.2	1.6	1.3
Males	1.7	1.1	2.0	1.6
Females	1.3	1.6	2.0	1.3
White	1.6	1.2	1.9	1.3
Black	0.9!	1.7!	2.8!	3.5!
Hispanic	3.2	1.4	2.9	3.2
Asian/Pacific Islander	5.0	4.2	5.6	3.6
American Indian	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.13

Standard Errors for Student Demographic Distributions, Grade 12, 1996

THE NATION'S REPORT CARD naep

Grade 12	Percentage of Students: Advanced Study	Percentage of Students: Not Eligible for Advanced Study
Gender		
Males	1.8	0.9
Females	1.8	0.9
Students who Indicated Their Race/Ethnicity as...		
White	2.2	0.9
Black	1.0	0.6
Hispanic	1.5	0.5
Asian/Pacific Islander	1.6	0.6
American Indian	0.1	1.0
Students who Reported Their Parents' Highest Level of Education as...		
Did Not Finish High School	0.7	0.6
Graduated From High School	1.0	0.9
Some Education After High School	1.1	0.9
Graduated From College	1.7	1.6
I Don't Know	0.3	0.3
Home Environment Contains...		
0–2 types of educational materials	1.0	0.9
3 types of educational materials	1.0	0.8
4 types of educational materials	1.3	1.2
Students From...		
Northeast	3.8	1.4
Southeast	3.4	2.3
Central	2.9	1.7
West	3.0	2.6
Students Live in...		
Center City	4.3	3.9
Urban Fringe/Small City	4.9	3.9
Rural	4.1	3.0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.14	Standard Errors for Student/School Demographic Distributions, Grade 12, 1996	THE NATION'S REPORT CARD naep

	Percentage of Students	
	Advanced Study	Not Eligible for Advanced Study
Grade 12		
Students who Attend...		
Public Schools	3.6	1.2
Nonpublic Schools	3.6	1.2
Title I Participation...		
Participated	1.0	0.7
Did Not Participate	1.0	0.7
Free/Reduced-Price Lunch Program Eligibility...		
Eligible	1.4	1.5
Not Eligible	4.5	3.6
Information Not Available	4.7	3.6

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.15

Standard Errors for Content Emphases in Classes Taken by Advanced Study Students, Grade 12, 1996

THE NATION'S REPORT CARD

Grade 12	Percentage of Students Receiving		
	Little Emphasis	Moderate Emphasis	Heavy Emphasis
How Much Emphasis on...			
Algebra	1.3	3.6	3.3
Geometry	2.1	4.1	3.6
Trigonometry	1.4	2.8	2.7
Functions	0.8	2.1	2.1
Statistics	2.9	2.7	1.9
Probability	2.4	2.6	2.1
Discrete Mathematics	3.2	3.1	1.4

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.16	Standard Errors for Process Emphases in Classes Taken by Advanced Study Students, Grade 12, 1996	THE NATION'S REPORT CARD

	Percentage of Students Receiving		
	Little Emphasis	Moderate Emphasis	Heavy Emphasis
Grade 12			
How Much Emphasis on...			
Learning Mathematical Facts and Concepts	0.9	3.3	3.3
Learning Skills and Procedures Needed to Solve Routine Problems	0.4	3.1	3.2
Developing Reasoning and Analytical Ability to Solve Unique Problems	1.0	2.9	3.1
Learning How to Communicate Ideas in Mathematics Effectively	1.9	3.7	3.5

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.17	Standard Errors for Calculator Access in Classes Taken by Advanced Study Students, Grade 12, 1996	THE NATION'S REPORT CARD naep

Grade 12	Percentage of Students
Do You Permit Students in This Class Unrestricted Use of Calculators...	
Yes	2.4
No	2.4
Do Students in This Class Have Access to Scientific Calculators in Class...	
Yes	2.8
No	2.8
Do Students in This Class Have Access to Scientific Calculators Out of Class...	
Yes	2.6
No	2.6
Do Students in This Class Have Access to Graphing Calculators in Class...	
Yes	3.0
No	3.0
Do Students in This Class Have Access to Graphing Calculators Out of Class...	
Yes	3.4
No	3.4
Do You Permit Students in This Class the Use of Calculators on Tests...	
Yes All	3.3
Yes Some	3.3
No	0.1

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.18	Standard Errors for Calculator Usage and Instruction Classes Taken by Advanced Study Students, Grade 12, 1996	THE NATION'S REPORT CARD naep

Grade 12	Percentage of Students
In This Class Students are Provided Instruction in the Use of Scientific Calculators...	
Yes	3.0
No	3.0
In This Class Students are Provided Instruction in the Use of Graphing Calculators...	
Yes	2.6
No	2.6
Which of the Following Best Describes the Availability of Graphing Calculators in This Class...	
One	0.8
Less than Six	1.0
Complete Set	4.0
Some Students Have One	1.9
Most Students Have One	2.3
All Students Have One	3.5
No Student Has One	1.7
If Graphing Calculators are Used in This Class, What is Their Primary Usage...	
Calculating	1.8
Graphing	3.0
Tables	0.2
Statistics	0.5
Symbolic Manipulation	1.4
Not Used	2.8

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.19	Standard Errors for Average Mathematics Scale Scores by Eligibility for Advanced Study, Grade 12	THE NATION'S REPORT CARD

Grade 12	Average Scale Score
All Students	1.0
Eligible	1.6
Not Eligible	0.9

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.21	Standard Errors for Average Percentage Correct Scores, Advanced Study, Grade 12, 1996	THE NATION'S REPORT CARD

Grade 12	Percentage Correct
All Students	0.4
Males	1.0
Females	0.9
White	0.8
Black	1.6
Hispanic	1.4
Asian/Pacific Islander	2.2
American Indian	***
Are Students Presently Enrolled in Mathematics...	
Yes	0.8
No	1.4
Are Students Presently Enrolled in or Have They Previously Taken an Advanced Placement (AP) Mathematics Course...	
Yes	1.1
No	0.9

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.22

Standard Errors for Score Percentages for "Use Linear Function," Grade 12

THE NATION'S REPORT CARD naep

	Correct	Partial	Incorrect	Omit
Grade 12				
All Students	1.1	0.6	1.2	1.3
Males	1.3	0.9	1.5	1.6
Females	1.3	0.9	1.7	1.4
White	1.2	0.7	1.4	1.3
Black	1.2	1.7	3.3	3.4
Hispanic	2.4!	1.7!	3.5!	3.2!
Asian/Pacific Islander	4.2	1.7	4.0	2.6
American Indian	***	***	***	***
Are Students Presently Enrolled in Mathematics...				
Yes	1.1	0.7	1.2	1.3
No	3.2!	---	5.6!	5.2!
Are Students Presently Enrolled in or Have They Previously Taken an Advanced Placement (AP) Mathematics Course...				
Yes	1.9	0.9	1.6	1.6
No	1.4	0.9	1.7	1.7

*** Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.23

Standard Errors for Score Percentages for "Compare Volumes of Pyramids," Grade 12

THE NATION'S REPORT CARD

Grade 12	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	0.5	0.5	0.7	0.9	1.2	1.2
Males	0.7	0.8	0.8	1.1	1.5	1.7
Females	0.6	0.7	0.7	1.1	1.8	1.3
White	0.5	0.6	0.8	1.2	1.3	1.2
Black	---	---	0.6	1.5	5.4	5.2
Hispanic	1.2!	0.7!	0.5!	1.5!	5.1!	4.5!
Asian/Pacific Islander	1.9	1.7	2.0	1.6	2.5	2.4
American Indian	***	***	***	***	***	***
Are Students Presently Enrolled in Mathematics...						
Yes	0.5	0.5	0.7	0.9	1.3	1.2
No	1.1!	---	0.4!	4.4!	5.9!	5.1!
Are Students Presently Enrolled in or Have They Previously Taken an Advanced Placement (AP) Mathematics Course...						
Yes	1.1	0.9	0.9	1.3	1.7	1.3
No	0.5	0.6	0.9	1.2	1.6	1.7

*** Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.24	Percentage Correct for "Find Resultant Vector," Grade 12	THE NATION'S REPORT CARD naep

Grade 12	Percentage of Students
All Students	1.0
Males	1.5
Females	1.3
White	1.1
Black	4.1
Hispanic	3.3!
Asian/Pacific Islander	3.1
American Indian	***
Are Students Presently Enrolled in Mathematics...	
Yes	1.0
No	4.2
Are Students Presently Enrolled in or Have They Previously Taken an Advanced Placement (AP) Mathematics Course...	
Yes	1.7
No	1.2

*** Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.25

Standard Errors for Score Percentages for "Ferris Wheel," Grade 12

THE NATION'S REPORT CARD naep

	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Grade 12						
All Students	1.5	0.1	0.9	0.8	1.2	0.9
Males	1.9	0.3	1.1	1.1	1.3	1.1
Females	1.7	---	1.3	1.1	1.6	1.0
White	1.6	0.2	1.1	0.9	1.4	0.8
Black	2.9	---	2.8	2.3	2.8	4.3
Hispanic	4.1!	---	1.7!	2.0!	2.7!	3.1!
Asian/Pacific Islander	3.2	---	2.4	2.2	2.7	2.3
American Indian	***	***	***	***	***	***
Are Students Presently Enrolled in Mathematics...						
Yes	1.5	0.1	0.9	0.8	1.2	1.0
No	4.2!	---	3.7!	4.3!	4.0!	3.3!
Are Students Presently Enrolled in or Have They Previously Taken an Advanced Placement (AP) Mathematics Course...						
Yes	1.7	---	1.4	1.0	1.1	1.1
No	2.1	0.2	1.1	1.2	1.6	1.1

*** Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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