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ABSTRACT

Evidence is presented from an ethnographic study of an eighth grade pre-algebra teacher's classroom in support of the idea that teachers' beliefs about the ontology and epistemology of math profoundly influence how they teach and thus what students learn. Specifically, beliefs based on a traditional view of mathematics as procedurally-based are explored. Classroom observations and teacher interviews provide evidence that expert, rule-based instruction may be beneficial in providing strong foundations for teacher and student efficacy, a clear belief system about the nature of mathematics, unambiguous guidelines for instruction, including areas such as scaffolding and assessment, and specific methods for classroom management. On the other hand, procedural instruction in this classroom was related to student misconceptions, unremarkable mathematics achievement, excessive student dependence on the instructor, and extrinsic student motivation. The implications of this analysis for implementing mathematics reforms based on the differing metaphors of learning are explored. Contains 42 references. (Author/WRM)

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Paradoxes and Paradigms in an Eighth Grade Pre-Algebra Class:

A Case Study of a "Good" Math Teacher

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Abstract

Evidence is presented from an ethnographic study of an eighth grade Pre-Algebra teacher's classroom in support of the idea that teachers' beliefs about the ontology and epistemology of math profoundly influence how they teach and thus what students learn Specifically, beliefs based on a traditional view of mathematics as procedurally-based are explored. Classroom observations and teacher interviews provide evidence that expert, rule-based instruction may be beneficial in providing strong foundations for teacher and student efficacy, a clear belief system about the nature of mathematics, unambiguous guidelines for instruction, including areas such as scaffolding and assessment, and specific methods for classroom management. On the other hand, procedural instruction in this classroom was related to student misconceptions, unremarkable mathematics achievement, excessive student dependence on the instructor, and extrinsic student motivation. The implications of this analysis for implementing mathematics reforms based on the differing metaphors of learning are explored.

> The issue then, is not, what is the best way to teach, but what is mathematics all about? (Hersch, as cited in Nickson, 1992, p.103)

What is mathematics? Is it a body of knowledge consisting of concepts and skills that can be learned and used to help one solve problems involving counting, numbers, measurement, or other concrete problems? Is it something invented by a community of scholars in response to the changing needs of the world (Cobb, Wood, Yackel, & McNeal, 1992)? Is it fixed? Is it invented? Mathematics teaching, so important to the students' success in math classes, is structured around how mathematics is conceived. Similarly, math pedagogy and curriculum are structured according to beliefs about how children learn. In this paper, I present evidence from an ethnographic study of a math teacher's classroom that supports the emerging understanding that one's beliefs about the ontology and epistemology of math profoundly influence how teachers teach and thus what students learn (Cohen, 1990; Ivey, 1996). Using Sfard's (1998) conceptualization of different metaphors for learning, I distinguish between three different kinds of mathematics beliefs and practices that are prevalent in the current literature. Finally, I suggest reasons for the tenacity of certain metaphors or beliefs, despite some of the negative outcomes they have for learning.

This study began in an effort to sort out the contradictions and discrepancies concerning the folk idea of what constitutes a good math teacher. On one hand, math reforms such as the NCTM Standards (1989) describe a way of actively engaging students in mathematics content in order to promote deeper understanding, complex problem solving, and powerful math communication. Similarly, other reformers promote deeper understanding by encouraging questioning and argument as powerful forms of math discourse (Lampert, Rittenhouse, & Crumbaugh, 1996). These are both representative of what is currently called inquiry mathematics (Cobb et al., 1992), and they both give primacy to conceptual knowledge over procedural knowledge (Hiebert & Carpenter, 1992). Although many examples of such reforms are described in the literature (Burns, 1992; Cohen & Ball, 1990; National Council of Teachers of Mathematics, 1989), their successful implementation into actual classrooms has been more problematic (Ball, 1990; Gregg, 1995; Wiemars, 1990). Our present math curriculum and pedagogy in the United States still seems rooted in what Cobb et al. call school mathematics, or what I call procedural instruction[1].

According to Sfard (1998), these differing practices are rooted in two metaphors of learning: an acquisition metaphor (AM) and a participation metaphor (PM). The former, although dominant in current practice, is associated with all kinds of negative outcomes for students, from a lack of conceptual understanding to the inability to solve real world problems, to compliance and overreliance on the teacher as authority figure (Darling-Hammond, 1990; Gregg, 1995; Ivey, 1996; Schoenfeld, 1988). The latter, although embraced by researchers in math education, is not often successfully implemented in practice (Ball, 1990; Gregg, 1995; Smith III, 1996; Williams & Baxter, 1996). The original purpose of this study was to examine a good teacher's classroom to see if these metaphors could be analyzed with empirical data grounded in an ethnographic analysis. As often happens in good ethnographic research, the question changed as I became familiar with the culture of Ms. Bryans' classroom. The purpose of this paper is to explore this further question: if the acquisition metaphor of learning is so unpopular with researchers, and has empirically been shown to have some negative consequences for students, then why is a rule-based, AM teacher like Ms. Bryans still considered a good teacher? And more, why was her classroom so compelling to me, a former middle school math teacher committed to math reform as exemplified by the NCTM Standards? It is only by unraveling such questions in fine detail that a deeper understanding of the constraints to math reform and teacher change is possible.

Researchers in math education are interested in why teachers are not changing their practices even though good research is out there exhorting them to change (Ball, 1990; Darling-Hammond, 1990; Wiemars, 1990; Williams & Baxter, 1996). Many are examining how experienced teachers fail to correctly implement new reforms (Ball, 1990), and how novice teachers, with reform-oriented ideals, get inducted into the <u>school mathematics tradition</u> (Gregg, 1995). What has not been well-researched is what are the advantages of the school mathematics tradition, as compared to the inquiry oriented tradition, that keeps teachers yoked to its practice, even though they think they have significantly reformed their teaching [Ball, 1990; Cohen, 1990]. One notable exception is the study by Smith III (1996) that explained this discrepancy by examining teacher's sense of self-efficacy. Smith proposed that teaching-by-<u>telling</u> allows teachers to feel that they are accomplishing something in their classroom. He concluded by stating that for innovative math reforms to succeed, they must provide teachers with ways in which to experience new foundations for efficacy in their work. Although Smith's conclusions are persuasive, it is important to understand how such efficacy beliefs play out in the context of the classroom through an examination of empirical data. In subsequent sections, I will examine the relationship of efficacy beliefs to classroom practice to help inform an understanding of why procedural-based teaching, when it is done well, is considered good teaching and what are the implications of this for math reform based on teaching for understanding.

Method

In order to understand the complex, systemic nature of math teaching and its effects at the middle school level, I chose to do a case study of a particular good teacher's classroom using the ethnographic research techniques of participant observation and interviewing as outlined by Spradley (1980). I chose an experienced teacher because I wanted to see patterns that had consolidated into coherent practice. I chose a teacher who was considered "good" by those "in the know" because I wanted to explore what "good" teaching meant in the current context. In order to ensure that the teacher I would be observing was a "good" teacher, I questioned two experts in the field who knew of teachers in their local area that would be good candidates for study. Both were unanimous in recommending Ms. Bryans[2] as perfect for this purpose. I set up an informal interview with her in December, 1997. Upon meeting me, Sylvia Bryans told me it would be fine for me to observe her classroom as often as I wanted and interviews would be fine as well. At that point, I began the process of seeking human subjects committee approval that was accorded to me several weeks later.

Participant

Although my study of Ms. Bryans cannot be generalized to all math teachers, my analysis does allow me to investigate a complex phenomenon and create hypotheses and generate theory for further empirical testing. Still, it is important to know some background information about Ms. Bryans and the school setting so that the ensuing analysis can be situated in context. This is Sylvia Bryans 20th year of teaching math. She is certified to teach grades 5th through 12th, and this is her first year at Latham Middle School. She transferred from another middle school because they wanted her to teach a prescribed curriculum that she believed was opposed to her more activity-oriented curriculum.

Sylvia is the teacher sponsor of Mu Alpha Theta, a competitive math club, and she is the person on the eighth grade math team who is responsible for designing the Pre-Algebra curriculum, which was the class I observed. She also teaches Algebra 1 Honors and Algebra 1 to eighth graders. The other teachers have similarly distributed math classes, except that Ms. Zaccaro also teaches Math 8, the lowest tracked math class. In addition to teaching full-time and sponsoring math clubs (in which she takes students to math competitions), Ms. Bryans works at her in-laws' family fabric store on weekends and holidays.

Ms. Bryans believes that her teaching changed dramatically fifteen years ago when she attended a week-long problem-solving workshop by a university math educator. Before the workshop, her classroom was "pretty traditional" (I2, p.1). After the workshop, she began to "use these problem solving activities now for mental math and front-end activities....Because of him [the university problem-solving expert], I changed my style of teaching" (I1, p.2).

Setting

Latham Middle School provides the setting and context for Ms. Bryans' teaching. LMS is located in a suburban area of a mid-sized city in Florida, and has a teacher-student ratio of about 1:19. It serves sixth through eighth graders and 28% of its student population are minority students. Many of the policies and practices implemented by LMS are those that characterize exemplary middle schools (George & Alexander, 1993). For instance, they have block scheduling of four periods per day that last for 80 minutes each, with the addition of a half hour for lunch. Each grade is housed in its own building, creating a school-within-a-school atmosphere. There are no bells to signal the end or beginning of classes; students are trusted to get to class on time (although tardy policies exist for students who are consistently late to class). Furthermore, there is an extensive network of parent volunteers that supports LMS in a variety of ways; in fact, I've often encountered these volunteers when signing in for my observation or when calling LMS. The administrators keep in close contact with parents via the PTA, informal principal "coffees" on several Saturdays throughout the year, conferences, and an annual school climate survey that parents are asked to fill out. Other LMS practices that are representative of good middle schools include a teacher advisor program, exploratory programs, and teacher teams.

Research Schedule

I began my first observation on January 12, 1998 and completed the data-gathering portion of this study on April 8, 1998. During this time period, I formally observed Ms. Bryans' classroom 16 times, usually visiting twice a week for approximately two hours each time. In addition, I interviewed Ms. Bryans four times (two were audiotaped), the principal, Dr. Cannell, two times, and one of the assistant principals, Mr. Previce, once. Total hours of observation equaled 27 hours and 21 minutes. Total interview time equaled three hours and 35 minutes. From this, I generated 103 pages of expanded field notes, 28 pages of transcribed interviews, and 172 analyzed domains. In addition, artifacts such as teacher tests, school newsletters, test scores and homework grades were also collected.

There were several limitations of this study. First, even though I was primarily interested in Ms. Bryans' method of teaching, interviewing students would have provided specific information on how students perceived her teaching. Without this data, student responses must be inferred from classroom observations or from the perceptions of others, such as Ms. Bryans, Dr. Cannell, or myself. This is a significant weakness of the present study that was due to the lack of time to collect the necessary approvals once I had realized that such data would be useful. Another weakness of the study has to do with the lack of a comparison class with which to contrast Ms. Bryans' teaching instruction.

Metaphors for Learning

Teacher beliefs influence instructional practice (Nickson, 1992). As the quote at the start of the paper stated, the ontological inquiry into what constitutes mathematics must be decided before we can make a judgment about whether teaching is effective or not. The question is, effective for what purpose? Even though student understanding is an important outcome supported by researchers (Campione, Shapiro, & Brown, 1995; Hiebert & Carpenter, 1992; Ivey, 1996; Lave, 1990; Perkins, Jay, & Tishman, 1993), understanding is an elusive concept and difficult to assess. Math performance, such as problem solving or solving equations, is much easier to quantify. In addition, understanding is conceptualized differently according to the different metaphors of learning. On one hand, those who subscribe to the AM metaphor would see understanding more along the lines of cognitive science, as "modeling internal representations;" (Hiebert & Carpenter, 1992, p. 65) while a PM approach frames understanding as much more contextual and situationally determined.

This matter is complicated by the nature of many reforms that do not make clear the distinction in the underlying metaphors of math learning. For instance, The California State Department of Education's new math framework distinguishes between teaching procedures and teaching for understanding (Wiemars, 1990). Understanding in this case is vaguely conceptualized as transferable knowing that is "difficult to teach" and "difficult to test" (CSDE as cited in Wiemars, 1990, p.284). Both types of pedagogy however, are rooted in the AM metaphor; consequently, we have another distinction to consider in this paper: the AM metaphor as operationalized by cognitive scientists and the AM metaphor as subscribed to by traditional, rule-based teachers. Although the cognitive scientists and situational learning theorists have their debates (Bereiter, 1997; Lampert et al., 1996; Lave, 1990; Salomon, 1993), neither camp subscribes to the traditional pedagogy of direct procedural instruction. Why then is such teaching still occurring, and more importantly, why is it considered good?

One possible answer to this question is that teachers think they have radically changed their pedagogy when all they have changed is surface features of instruction. This phenomenon is documented in Cohen's case study (1990) of a teacher who thought she had aligned her pedagogy with reform principles, but in fact, had changed only surface features while the underlying framework was based on traditional conceptions of math. Ms. Bryans demonstrated similar beliefs.

> G: Could you give some examples of how you teach differently, like how you used to teach and how you teach now?

> T: [In the past my teaching] was, umm, pretty traditional. You go over the homework, then you [have] the lesson, then you give them work to do. Umm, then I changed it to doing something different at the beginning of class, like problem solving. And eventually that led to doing mental math and logic puzzles… and with block scheduling that makes it really easy to do everything where in regular schedule I was really pushing <pause> to get it all done. (I2, p.1)

For Ms. Bryans, her teaching was no longer traditional once she added problem solving to her curriculum. In her opinion, the main things that good math teachers do were to get students "involved in their own learning" (I2, p.1). She accomplished this through

> Hands-on activities, making the problems pertain to them. Like we'll change them and make holiday problems out of them or you know things that maybe have to do with the (local college football team) or the Giants or you know things that they would understand better….(I2, p.1)

Such views originated in Ms. Bryans' beliefs about mathematics. For her, math was something concrete, not creative (in an interview she distinguished between creative writing, which did not enjoy, and math, which she did). Since problem solving was the area that Ms. Bryans believed distinguished her from other teachers (that, plus the fact that she's the "activity guru" because everyone comes to her for activities, I1, p.3) it is worth examining her specific beliefs about this domain.

> G: You're really focused on problem solving; I noticed that. How do you think kids become good problem solvers? I mean, they don't start out

being good problem solvers, so how do they become good problem solvers?

T: They have to practice it. Like, umm, the, oh you missed it yesterday. We're doing Venn Diagrams (she laughs). And it's awful. They can't, even my Honors kids couldn't get the answer.

G: Really?

T: Yeah. And I changed the problem for Pre-Algebra, but I don't know what I am going to do today. And, they have to be taught, once I taught them how to do it yesterday, then today, when I gave the problem, they could do it. It's just a matter of being taught how to do it.

G: That's really hard, I mean…word problems for example, were the hardest thing to teach.

T: But see, but word problems and problem solving aren't the same thing.

G: Ok, what's the difference?

T: They're close but they're not the same. Problem solving, is like, there are strategies to solve it, like drawing a picture. Word problems can be that way, but they're questions like, a frog goes up the well, 7 ft and drops down four feet, how many days, If the well is 10 ft high, how many days will it take him to get out of the well? As opposed to, there's three apples over here and four apples there, how many apples are there all together? …It's much more involved.

G: And you focus on that (problem solving) more than word problems?

> T: Yeah, although with this book (referring to the text), they do a lot of word problems, so, kids have to be able to read 'em. And they can fall back on some of their problem solving strategies. To help them. If they get past the reading. (I2, p.6)

Problem solving for Ms. Bryans was distinct from solving word problems in that the problems were more complex, but both are based on specific strategies, (e.g. "drawing a picture") and could be taught and learned, with practice. This was a different view of problem solving than one advocated by inquiry-oriented math researchers in which specific heuristic strategies are not explicitly conveyed to students, and where students participate in groups to solve a complex problem that might take more than a day to complete (Schoenfeld, 1992).

Even though Ms. Bryans' emphasis on problem solving reflected the spirit of current math reforms (National Council of Teachers of Mathematics, 1989), it was still not something that she assessed, although it is something that the Florida state assessment evaluates. When asked what were the minimal skills she expected her students to master, problem solving was not one of them. Instead the list included adding, subtracting, multiplying, and dividing the following:

> whole numbers, fractions, and decimals...(and) integers. And I would like them to be able to solve one step equations, and two-step equations, but it has become apparent that they cannot solve equations with variables on two sides. They cannot do that. We did that, and it did not work. So, we decided that this group couldn't do that. That maybe they hadn't reached

> Piaget's level of <pause> thinking yet, that they couldn't do that. Um, and we would like them to be able to graph lines. (I5, p.4)

So not only was problem solving not something that Ms. Bryans expected all of her students to achieve at the end of class, deeper understanding was not emphasized either. Instead, the focus of her Pre-Algebra curriculum was to get students to be able to perform those math procedures that were presented in the Pre-Algebra textbook as appropriate for eighth grade. Curriculum items may be deleted, however, if they turned out to be beyond the students' developmental capabilities, according to the judgment of the eighth grade math team.

It is apparent that Mrs. Bryans' beliefs led her to have certain views about math that were not only a part of the AM metaphor, but which also fit the framework of traditional or "rule-based" teaching. These beliefs, along with the structure and context of schooling, support a certain pedagogy that has particular consequences for the students and the instructors themselves. Although I previously claimed that these consequences are almost universally denounced by math researchers subscribing to either the AM metaphor of learning within a cognitive science point of view or within the PM metaphor of shared discourse, I have likewise argued that such traditional practices must provide some positive outcomes for all those affected by the classroom context (teachers, students, parents, and administrators); otherwise, they would have been easier to surrender once the new math reforms became widely espoused. In the next section, I examine a particular problem solving teaching episode in order to frame the ensuing discussion about possible benefits of such an approach.

Problem Solving Episode

It was a typical day in Ms. Bryans's Pre-Algebra class. The problem of the day was displayed on the screen, handwritten in bright green marker. Most of the 31 students were in their seats, but some were still getting settled. Walking in the room from her duty as hall monitor, Ms. Bryans headed right to the overhead projector and started talking amidst the noise.

> All right, here we go....Shhh....One more time. This is problem solving. The name of the problem is "The Gambler." (P10, p1)

Some students were still socializing, talking, and getting their books out. The electric sharpener loudly grinded Tom's pencil to a suitable point. Ms. Bryans, in a loud voice, said,

> That's ONE! When I am talking that means you're not supposed to be talking. One of the problems is you're trying to pass your homework forward, 'so go ahead and pass your homework forward quietly, then I'll go over this.'

Students settled down at the warning, and there was now only moderate talking. Around 30 seconds later, Ms. Bryans told the class that she would read the overhead problem to them as they passed their papers forward.

> A gambler has 2 kinds of chips. The red ones are worth \$5 and the blue ones are worth \$8. What is the largest bet he CANNOT make?

As soon as she finished reading the problem, Ms. Bryans began to break it down into more manageable steps.

> Let's see. Let's make a chart of the ones he cannot make and the ones he can make and maybe if we do this in some organized fashion, we can determine how many coins he has....How many total coins do we have?

A few students said they don't know. She echoed this:

> T: We don't know. I'm going to tell you he has an infinite number of coins.
>
> S: What does that mean?

Ms. Bryans explained what infinite means. She then drew two columns on the overhead with the headings **cannot** and **can**. She placed an asterisk next to **cannot** to highlight that this was the column of interest. She then began calling out the counting numbers, starting with one. After each number, she asked the class, "can I make a bet of ____?" The class answered yes or no to each. She repeated each answer and wrote the number in the appropriate column. The overhead looked like this:

*cannot	can
1, 2,3,4,6,7,9,11,12	5,8,10,13

Ms. Bryans then told the class:

> I want you to continue in the same fashion until you find one he cannot make.... It is a number, and it's a lot less than a 100. So continue on with this chart and find the one he cannot make. Spend a couple of minutes and see if you can find the one....

She then returned to her desk to grade homework assignments while students worked on completing the chart. Students occasionally called out remarks to Ms. Bryans as they worked, such as "I have to ask you a question," or "Is there a pattern here?" To the first, Ms. Bryans told the student to come up to her desk. To the second, she said "Sort of." Ms. Bryans occasionally called out reminders for students to keep on task like "Randy, get busy please" or "Settle down. Shhh!" The class was working quietly. Most students were working on the assignment, except for Tony who was trying to fix the pencil sharpener that had jammed. When he succeeded at repairing it, he announced to no one in particular, "I work miracles." At his seat, he turned around to the student next to him and says, "Was that glorious?"

After about five minutes of individual problem solving, Ms. Bryans walked around the room, handing back the homework. Classroom noise has been steadily increasing as students either solved the problem or gave up on trying to solve it. She told the class, "All right, when I come around, show me your work." Students were chatting socially among themselves while Ms. Bryans walked up and down the orderly rows, checking their answers. She told the class "Shhh" a few times, and the noise never became too loud or chaotic. After checking each student's paper, Ms. Bryans walked to the overhead projector.

> All eyes should be up here; all mouths closed. (Turning to the overhead chart, marker in hand) Can we make the number 14?

Some students called out "no. She continued like this, with students (about five or six voices at a time—mostly the same voices) chorally calling out yes/no as appropriate to fill in the chart.

T: 28?

Class: Yes

T: Yes, cause I can make 18 and 10.

T: 29?

Class: Yes.

T: All right, 13 and 16 good....30, can I make 30?

Class: Yes.

Yes. 32?

Class: Yes. No.

T: Yes or no? Someone tell me why 32 will work?

S: Four eights.

T: Four eights, thank you. (Ms. Bryans stops writing numbers on the chart and asks the class, pointing to the list under "**can**:") Would you think if I could make 30-40, I could make all the rest of the numbers? ...because all I would have to do is add a ten to them?

Class: Yes.

T: So, what is the largest number I can make?

Class: '27'

T: Mr. Johnson is the only student who got this right so 'be proud of it.' Sam blushed as a few of his classmates acknowledge this accomplishment by calling out "woah." Ms. Bryans told students how to record this in their notebooks. She asked them what strategy this problem used, then repeated their answer. "Make a table is correct."

This episode was representative of the usual sequence of the day in Ms. Bryans' classroom. Often the day began with a problem like the one just described, and these were organized according to the strategy used to solve them: make a table, logical reasoning, estimation, guess and check, or draw a picture. If there was no problem of the day, then the warm up activity was usually either a mental math exercise, five minute check of previously learned skills, or review for a standardized test, at least during the months I was observing. The rest of the class period usually followed a similar structure. Ms. Bryans read the answers to the homework, students asked questions on problems they wanted her to go over, and then the main lesson of the day was presented. Students went to lunch halfway through the period, then returned to continue the main lesson and perhaps begin their homework.

Thus far, Mrs. Bryans' room looks quite traditional. Her method of instruction for problem solving is based on a recitation-style question and choral response format. Students get some time to work on the problem alone, but this is after Ms. Bryans has already determined the strategy and broken down the problem into manageable parts. To ensure that this style of instruction wasn't being used solely with this class due to their lower level skills, I observed her Algebra 1 class one day, paying special attention to the problem-solving portion of the class. That day's problem asked students to figure out a three-digit number from a series of given clues: It is not divisible by four. It is a three-digit whole number. Its hundred's digit is odd. The sum of its hundred's digit and its tens digit is 13. For each clue, Ms. Bryans asked students "Who can set that up for us to help us solve it?" It is "harder" than the problems she gave to her Pre-Algebra students, according to Ms. Bryans in a short talk with me after class, but solving it required the use of similar heuristic strategies. Ms. Bryans read the problem to the class and helped them set it up so that they could solve it; however, she asked these higher-tracked students more questions at first, eliciting from them how to interpret the clues to the problem. After giving these students about five minutes to solve the problem on their own, she went to the overhead to explain how to solve it using a similar recitation-style of instruction. I will briefly describe this scenario to show that both episodes were, in fact, more similar than different.

> T: Did (clue) number 2 help you? (Tim says yes, and explains why it was helpful. Peter questions this explanation. Ms. Bryans ignores their conversation and moves on.)
>
> T: Who can tell me what works for clue #3 that I can use the tens and ones for? Tamika? (Tamika responds with a quick, brief answer.) Now how about four, who can tell me what the hundreds place is going to be...? (No one answers.) Boy, nobody knows what 8+1 is? If this is two it has to be…? If this is 0, then this has to be 13, since that doesn't work the zero is gone and so is the five….Which one of these digits is a square number? John?
>
> John: Nine.
>
> T: Right. So this is puzzle number 30, it's 943, and it took us how many clues?
>
> S: Five.
>
> T: Five, so record that on your logic sheet.

In both instances, Ms. Bryans guided students in how to begin solving a complex problem. She used recitation techniques to engage the class in beginning to break down the problem into smaller steps. Students worked individually for about five minutes, continuing where she left off, then Ms. Bryans asked specific questions designed to get students to give answers to each stage of the problem until the final answer was arrived at, a joint effort between the class and herself. I will argue that this rule-based, or procedural, pedagogy offers significant positive outcomes for both students and Ms. Bryans, and these advantages extend out into the larger school and family context in the areas of beliefs, efficacy, instruction, and classroom management.

Benefits of Procedural Teaching

Beliefs

The metaphors underlying a procedural pedagogy were previously discussed. They form the basis for how Ms. Bryans, her students, the school, and the community can judge whether or not mathematics learning is occurring in this Pre-Algebra class. Even though this study was limited by lack of specific parent and student interviews, there was still a sense that each subgroup subscribes to the same beliefs about schooling, or what Gregg (1995) called "the school mathematics tradition." For Ms. Bryans, if students were

> sitting in their seats, and I'm in control, they're usually quiet and doing what they're supposed to be doing, but…if they have an activity to do, they're gonna get a little louder because they're communicating to each other which is more than just one person communicating to me, so the class will be louder. And then there are kids who can't handle that, you

> know, group behavior, and there are kids that can handle it, so I would expect it to get louder. And some would be off-task, but how many kids are on task in the classroom when you're lecturing to them to begin with anyway? (I2, p.4)

This quote reflected Ms. Bryans' belief that group tasks weren't good for all students, but then again, neither was lecturing to them. This belief may at first seem in favor of reform-oriented practices, but for the classes I observed, cooperative groups comprised only about three lessons out of the sixteen that I observed, and those that I did observe were not true collaborative groups, but primarily independent hands-on activities in which students sat next to their peers and shared supplies. Furthermore, the products of the group work were assessed on the same basis as homework grades: 100 points for completion, 50 points for late work. For instance, at the end of the second quarter, Ms. Bryans presented a unit on creating tessellations. Students were responsible for producing at least two tessellations, one by sliding a shape and the other by rotating it. The student conversation I heard was primarily social. Heather told her group about a lady who "was delivering candles. She was like, 'you all have to stop." (P4, p2) Student dialogue two days later was similarly social:

> Tony: Shut up blondie!
>
> Hannah: Ooh, that really hurt. Tony is there any time you are not talking?
>
> Tony: Yes, when I'm sleeping.
>
> James (to Ms. Bryans): This is really cool!
>
> Tomas: Oh, James wants extra credit! (P5, p.5)

Not only was such work not assessed to the same degree as a test or quiz, it was used to supplement the regular curriculum or as a break for students. As Ms. Bryans told the class, "This week we're going to do stuff that's a little unusual because I know your brains are 'tapped.'" Consequently, although she may not lecture, Ms. Bryans did teach procedures and problem-solving heuristics through whole group instruction. Students were less passive than they would be in a lecture, but Ms. Bryans was still the classroom authority and math expert. By using techniques such as hands-on activities and groups, students got actively involved in learning, but that only went so far and then it was time to "get down to business" as this next passage shows. During an interview, in response to my question about her reaction to a constructivist point of view, Ms. Bryans replied

> T: Well, we do hands-on stuff with the equations and the chips and so on and so forth so we actually do demonstrations that way, so that they can maybe make their own (equations). See they've already been taught it before they reach 8th grade anyway, but we do, you know, a little bit of manipulations, going from one side of the equation to the other, and then there's only so much of that you can do until it's time to get down to business and solve equations. So it's a combination (of hands-on and didactic instruction).
>
> G: What if someone said that it's never time to get down to business. You should never ever teach them those equations? This is a very extreme view, but I just want to know if you think there's any…

> T (interrupting me): This person is obviously not a teacher. Because you wouldn't get anywhere. You would never get anywhere, and so you'd never get past that. (I2, p3)

These statements reflect Ms. Bryans' beliefs about the importance of getting to the "business" of actually solving problems, equations, or whatever was the task at hand. Math, ultimately, is a skill that everyone can do and something everyone needs to know how to do. Mrs. Bryans was adamant about this point, stating that math is like reading because

> you're going to use it the rest of your life, you know. Which is, it's kind of weird cause you would never admit that you can't read, but it's ok that parents go, 'I was never good at math,' you know...And they think that's a reason their kids can't do math. And I don't agree with that at all. But that's just a skill you're going to use your whole life.

Finally, Ms. Bryans' traditional beliefs about mathematics were reflected in her statements that "Kids have a right to fail" (I9, p.6) and that "competition (is a part of) the real world" (I5, p.7); both of which imply that math was something individual and achievable due to effort.

Although my evidence is weaker here, there is a sense that students, parents, and the administration of the school support this view of education as well. For instance, Ms. Bryans, worried about parents' possible reactions to the tessellation activity, told the class:

T: The only problem is you're going to go home and tell your parents, "Hey we did this really neat art thing in math." Who can tell me how does this relates to math?

S: 'It's geometry.'

T: There you (go), they're geometric figures.

The need to justify certain types of creative, hands-on activities to parents was felt by Ms. Bryans whereas there was no evidence that she felt she has to justify tests or other traditional activities to them. Along the same lines, Dr. Cannell, the principal of LMS, told me that Ms. Bryans was "exemplary" at teaching problem solving because she "presents material to kids clearly, logically, and sequentially" (I3, p2); thus demonstrating that she too, believed that math is something concrete, fixed, and serial.

Student beliefs may be inferred from some of the questions they asked. The majority of questions asked were procedural questions about the parameters of an assignment:

> What answer do we put down? (P7, p.3) Do we need to show our work? (Do) we need to use a calculator? (P10, p.6). Should we check it [their work, P2, p.4]?

There were daily requests asking Ms. Bryans to solve one of the homework problems (e.g. "Could you do 7? P14, p.3)." Other questions had to do with being able to use the algorithm itself: "How did you get that?" (P11, p.3) or "How do you find...?" (P2, p.4) or "How did you get 0.9?" (P7, p.5) or "Do you have to put the zero before the decimal point?" (P8, p.4) This overview suggests that students see math as something that has clearly defined parameters and procedures, is concrete, and has to be done in a certain way, with one right answer. In fact, when the text's answer differed from Ms. Bryans' solution, students were quick to point out that one of them was wrong. By way of illustration, on one particular day, Sam questioned Ms. Bryans' solution, -3/2, to a worked problem on the overhead,

Sam: You said the answer was negative 1 _.

T: But if you have 3/2 that is ok too" (P10, p.4).

It was not made clear why both answers were valid, however. There was a similar incident on another day where students were confused because the text had a different answer for the circumference of a circle than the answer Ms. Bryans gave after working the exercise out on the overhead. Ms. Bryans replied that it was "cause the book probably used real pi." They (the class) had used 3.14, but real pi "goes on and on forever" (P16, p.3). Again, math work was perceived by students as having one correct answer. Multiple answers troubled them and had to be clarified.

There were three incidents that occurred during my observations, however, that reflected that Ms. Bryans' beliefs about math were less fixed than the previous examples reveal. One time, in response to students giving her different ways to solve a problem, she restated all of these ways, giving them validity by her acceptance of them: "So you could cross-multiply.... Someone else said you could reduce to lowest terms; someone else said (you could turn them into decimals.)" (P12, p.6). Another time, Ms. Bryans praised a student who used a school calendar to solve a problem instead of making a chart. Finally, Ms. Bryans told students that there was an alternate way to solve a problem than the way the student gave:

> Multiplying by negative four would be perfectly fine, but an alternate way to do this would be to multiply by negative four over five. That would be the easiest way to do this. (P13, p.4)

Even though these examples provide some evidence that math was not consistently presented as having one answer or only one solution, they still do not undermine the deeper belief that math was ultimately solvable and fixed. There may occasionally be multiple routes to a solution, but one way is usually better or easier, and, more importantly, there is always a way that can be figured out. Therefore, one of the benefits of procedural instruction is that it contributes to a certain worldview about mathematics that is prescriptive and optimistic: every problem has a solution that can be figured out if you know how to solve it. This belief contributes to a sense of efficacy for teachers and students that Smith III (1996) argued is the primary benefit of procedural-based instruction.

Efficacy

According to Smith's (1996) insightful paper on efficacy and procedural-based math instruction (what he calls "teaching by telling"), the belief that math is "a fixed set of facts and procedures" contributes to teachers' (and, I would add, students') sense of efficacy in five ways: (a) It "restricts the content that teachers must master to a manageable range;" (b) It "provides a relatively detailed model of what teachers should do in their teaching;" (c) It "accentuates the sense of knowledge transfer in teaching;" (d) It "defines what students should do to learn: listen, watch, and practice;" and (e) It "provides a rough outline of a typical day's instruction and simplifies issues of planning and classroom management" (pp. 392-393). In other words, this belief sets up clear expectations within which teachers, students, and schools can function. Teachers' sense of efficacy, or the expectancy that one's teaching will produce certain student outcomes (Ashton & Webb, 1986), is clearly delineated in this tradition of instruction. The problem with what I am calling inquiry-oriented math reform, according to Smith, is that it "undermines the base for teacher's sense or efficacy that teaching by telling provides" (p. 388) in two primary ways. First, the changing role of teachers as facilitators, rather than knowledge providers, is fuzzier to self-evaluate. Second, designing lessons is much more difficult and may be beyond the immediate capabilities of most traditionally trained math instructors. Additionally, as Sfard (1998) noted, not only is it difficult to find the appropriate real-life situations within which to carry out a PM-based curriculum, but ultimately, this metaphor leads to "a gradual disappearance of a well-defined subject matter" (p. 30).

Without a strong sense of efficacy, negative consequences for teachers, such as burnout, are likely to ensue (Ashton & Webb, 1986; Cherniss, 1993). Judging from my interviews and observations of Ms. Bryans, she had a strong sense of math efficacy and was far from being "burned out" even in this her 20th year of teaching. This was confirmed in our interviews where she told me that she liked students, she liked teaching math, and she has never experienced being "burned out." She said she thought it was because of her attitude—"You can't really tick me off (I2, p. 9)"—but it could also be because the math she teaches conforms to her view of how math should be learned, and her sense of efficacy is strengthened by this, as delineated by Smith's reasons above. Of course, this is a conjecture that should be investigated with further study before any claims could be made as to the generalizability of this statement.

According to this analysis then, the primary benefit of rule-based teaching is that it contributes to the participants' sense of efficacy. Next, I will examine some of the ways that Ms. Bryans' methods of instruction and classroom management support this conceptualization of efficacy.

Instruction

Hiebert and Carpenter (1992) argue, based on compelling research of their own and others in the field, that whether or not teaching is procedurally or conceptually-oriented, learners are still active in constructing mathematical knowledge. Furthermore, both skills and concepts are equally necessary components of mathematics learning. On the other hand, they claim that precedence should be given to helping children construct internal cognitive representations of important math concepts in order to promote understanding, even though procedural fluency is more efficient than the slower process of building such conceptual understanding. Nevertheless, a procedural approach to math instruction has instructional implications that relate to efficacy in ways that are not easily realized in a conceptual approach. I would argue that because progress in applying rules can be seen more quickly, it gives teachers immediate feedback upon which to judge their own effectiveness as well as concrete products for easily assessing students' work and with which students can judge their own progress. Furthermore, procedural knowledge has practical value and transfers to other subjects and contexts. Finally, it specifies how to scaffold students' knowledge of math procedures. I will explore each of these three aspects in turn with an analysis of incidents from my observation of Ms. Bryans' classroom.

Feedback/Assessment. Most of the days that I observed the Pre-Algebra class, answers to homework problems were reviewed, on average, for 19 minutes (24%) of the class period. Thus, almost daily, the teacher and students were given an opportunity to confirm their competence for about a quarter of the class time. A typical homework review session had the following format: Ms. Bryans passed back homework papers while students worked on the problem-of-the-day. Then, after reviewing the problem-of-the-day (or sometimes later in the class period), she read the answers to the homework (usually just the even ones because the answers to the odd numbered problems were given in the back of students' texts). Next, she recorded the number to the questions students wanted her "to work" through. Sometimes, she limited the amount of problems she would "promise" to cover. Finally, she reviewed, step-by-step, how to solve the homework exercise. Students received credit only for turning in their homework on time, not for the number correct.

The following example illustrated how the review portion of this process was enacted (P16, p.3):

> Problem: 5(3-k)=20
>
> T: What property do I use here?
>
> Carl: The distributive property.
>
> T (without missing a beat): Good, distributive property.

In this situation, Carl's answer confirmed to Ms. Bryans that students were following the lesson. For Carl, Ms. Bryans' answer may confirm that he was on the right track toward solving these exercises, plus he received direct verbal praise for figuring it out. Ms.

Bryans then told the class the next step of the process. Tania said quietly to herself, "Oh, I see that."

T: Some of the problems I see on your paper is (sic) this:

T writes the following down on the overhead:

$$5(3\text{-}k) = 20$$

$$15\text{-}5k = 20$$

$$\underline{-15 \qquad -15}$$

$$5k = 5$$

Then she asked: "What did I do [wrong]?" Without waiting for an answer, she pointed out that you have to bring down the negative sign in front of the 5. She continued to solve the problem, concluding by telling students you must factor out the fives to get k = -1. Several students called out "what?" Tonia asked Ms. Bryans to check it. When she did, Tonia replied, "Oh, Ok." Hannah asked, "Do we have to check it (on the test)?" Ms. Bryans replied, "Yes, you always have to check it."

In this teaching episode, Ms. Bryans was able to effectively answer students' questions, and students asking questions received immediate feedback on their responses. Both parties seemed satisfied by the exchange. The members of this class and the instructor did not had their efficacy beliefs challenged by this episode. Traditional roles were affirmed: Ms. Bryans was able to answer student questions, and students were able to answer Ms. Bryans' questions. I would even go so far as to suggest that what was going on was an unwritten contract between the two parties, each fulfilling the other's expectations so that a smooth transaction took place.

Regular feedback and assessment occurred through tests, quizzes, and projects as well. Projects, as mentioned previously, were assessed as homework grades on a completion basis. Tests and quizzes however were criterion- referenced and were weighted significantly higher than other assignments. Although the results of testing may lead to negative affect on the part of students, it still defines a norm of behavior within which students can get a sense of how they are doing in the class. For instance, these were some of the comments I overheard after students received their midterms: "You loser." "We don't care about your crappy grade." "Just six more points and I would've made A/B honor roll." "My Mom is gonna kill me!" Three students went up to the front of the room to ask Ms. Bryans questions about their grade, behavior she encouraged while handing out their papers, admitting she could have made a mistake in grading them. Ms. Bryans, with her beliefs that competition is a positive thing, was not unsettled by these comments.

Some parents, especially those of her honors students, support such comparisons as well and have asked Ms. Bryans to tell them their student's ranking in the class. This was a practice Ms. Bryans had been engaging for awhile without complaint until recently when one particular honors student with a low rank complained to her, as did his parent, causing Ms. Bryans to stop listing ranks on students' grade reports. However, she has since re-instituted it for certain students whose parents request it. These examples suggest that students, teachers, and even parents look to methods of assessment in order to gain a sense of how to judge their effectiveness (or, for parents, their child and the teacher's effectiveness). Traditional procedural assessment provides a common language with which to judge competence, thereby promoting a means for establishing one's sense of efficacy.

Transfer. Math skills are necessary to students' success in the school mathematics tradition. First, standardized assessments are still the norm. The eighth graders I observed were subject to two of these tests in math this year: the ITBS and the FCAT. Ms. Bryans taught specific test-taking skills for two weeks prior to the FCAT and for five days before the ITBS. Although the FCAT contained some problem-solving tasks and gave students partial credit for writing down correct explanations, the test was still taught to students by telling. In the following example, students have been working on a problem to find the area of a board with five holes drilled into it. Ms. Bryans has already restated the problem and has emphasized what facts were important to know, such as the length of the board and diameter of the drilled holes. After walking around the room giving students individual help on this, she went to the overhead and told the class:

> All right, so let's talk about what you should be doing. We know... if you take the holes out... How many holes are there? I know that all the holes would be 3 _ inches. That's the first step. I know the wood is nine. Six into nine goes one time with 3 left over. Which reduces down to 1 _. (P2, p.2)

After this explanation, she told students where to file this problem in their notebooks because they were finished with it. It has been successfully solved.

The practice for the ITBS involves teaching students direct strategies to use on the test via a workbook for this purpose. The areas reviewed with the Pre-Algebra students include "pacing yourself, good guessing, using estimation, data interpretation, and discovering equivalents." For the "pacing yourself" section, Ms. Bryans assigned practice problems and times the students with a stopwatch. Her aim was to teach students whether or not to skip certain problems in order to get as many problems done in the given time:

> You might have chosen to postpone #3, all those routes…

A boy said no out loud, as if he didn't choose to postpone this exercise. Ms. Bryans continued:

> #5 and #7 they say that those… you have to make a lot of comparisons. You may want to postpone it until later….6 is pretty straightforward. The answer is D. You may want to postpone on that one since you had a lot of comparing to do...11's not too bad either, the answer is…. 12. After you get through what all those symbols mean, then the answer is not too bad…. 13 and 14 take a little more time. If you had to postpone them that's ok, 14 is A and 15 is C (P13, p.2).

Therefore, knowing how and when to use math procedures, or the ability to transfer them to other contexts, is important to success on standardized tests. Furthermore, because schools and teachers are still judged on how their students perform on such tests, they are motivated to do what they can to ensure that their students test scores are the best they can be.

Another way that transfer of knowledge may contribute to teacher's sense of efficacy can be seen in the following exchange. Ms. Bryans was beginning a new lesson on metric measurement conversions. She told the class:

> What I'm going to do is to teach you a method that will help you in any system. When you get to chemistry or physics, this will help you.

She then added, pointing to all the possible metric prefixes:

> You won't use 'hecto' or 'deka' in real life. You'll use centimeters and millimeters. Now when you have to go from one unit to another, I won't make you memorize these, you'll just have to know how to do it. (P15, p.6)

As students worked on solving these conversion problems, they may increase their sense of efficacy by learning a procedure that has applicability to other fields. Correspondingly, Ms. Bryans might feel competent as an instructor when students successfully perform these metric conversions. The emphasis on procedural skill versus conceptual understanding was further revealed later in the lesson:

> T: Now what I am going to do is to write…a unit multiplier is any of those ratios…. It would say something like this: 1 km/1000 m. That would mean (writing on the overhead) 1 km = 1000 m…. These are called unit multipliers.
>
> Laura: Do we have to write this down?
>
> T: Not yet. Let's say that I wanted to change 8.9 km to meters. Write this down. Now write 8.9 km. Then write a times sign. Then write a fraction bar. Now, I don't want km, I want it to factor out, so I'm going to put it on the bottom. What facts convert meters to kilometers? Kara, you need to be copying this down. There's no place in this book where this is.

Ms. Bryans then wrote on the overhead:

$$8.9 \text{ km} \quad \times \quad \frac{1000 \text{ mm}}{1 \text{ km}} = 8900 \text{ m}$$

> T: What's the rule you're going to use?

Ashley calls out: What's a cm?

T (ignoring this comment): My unit multiplier says 10 mm = 1 cm. If you're confused, the reason it's (mm) on the bottom, is because I want this (mm in the denominator) to factor out with this (mm attached to the 9).

She wrote:

$$9 \text{ mm} \quad \times \quad \frac{1 \text{ cm}}{10 \text{ mm}} \quad = \quad \frac{9 \quad \text{cm}}{10 \text{ mm}}$$

Celia asked why she was using 10 instead of a 1000 (like in the previous example). Ms. Bryans replied that it had to do with the unit multiplier.

James: How are we supposed to know that 10 mm=1 cm?

T: Cause if it's on a test, I'll give it to you.

James: How are we supposed to know that right now?

T: It's in the book. 'Or you could write it down. You should probably write it down right now since the book doesn't give it to you in exactly this way.'

Each of these examples suggests that one of the useful aspects of procedures is that they transfer. If one believes that procedures can be successfully taught so that students correctly implement them, then it is both efficient and useful to teach them directly.

Scaffolding. For the purposes of this analysis, scaffolding refers to how teachers support students' learning of math concepts and skills[3]. Consequently, scaffolding in rule-based instruction will consist of cultivating students' proficiency with applying math procedures, such as algorithms; whereas in conceptually-oriented instruction, it will encompass aiding students in creating accurate internal conceptual representations

(Hiebert & Carpenter, 1992). Traditional instruction scaffolds students' ability to use procedures by direct explanation. Understanding why a procedure works is less important than knowing how to use it. The following passage, in which Ms. Bryans was teaching students how to divide numbers containing decimals, was representative of how such explanations were given. Ms. Bryans read the procedure out loud as students copied it into their notebooks:

> T: To divide decimals, multiply both the divisor and dividend by a power of ten so that the divisor is a whole number. Then divide. Ok? I'll show you an example you don't have to copy. All right. Now, that's the problem (2.25 divided by 2.5). You want to make the 2.5 a whole number. If I move it one place in the divisor, the divisor is the number 'on the outside,' so now you would just remember, dividing by 25 isn't too bad if you think of quarters....

She completed the division algorithm, writing 0.9 as her answer on the overhead projector.

> T: Ok, and that's your answer.
>
> Lara: How did you get 0.9?
>
> T (explaining each step of the procedure): ...so then you say how many 25's into 225?...If you're having trouble with division, you can say how many 2's in 22? If you don't understand it, we're going to work some examples after lunch.

In this lesson, understanding was something that occurred from watching and listening or from individual practice. It could be obtained by all through repeated practice.

Furthermore, there were a variety of tricks to make certain procedures easier to solve. In this example, Ms. Bryans suggested that for students who have a hard time figuring out how many 25's are contained in 225, they can think about how many 2's fit into 22, an easier conceptualization. This form of scaffolding, or breaking down procedures into smaller, more manageable parts, was the primary way that Ms. Bryans scaffolded math skills for students. This implied that once such procedures and rules were mastered, students would be good at math[4].

Moreover, such math tricks and rules turned out to be a notable portion of the data I collected that were subsumed under the category of "teacher explanations." Table 1 gives a representative list of these rules and tricks as they were presented in Ms. Bryans' Pre-Algebra class. The pedagogical emphasis of such rules supports my argument that they help to crystallize what is important to learn in Pre-Algebra, thus defining the parameters for discourse in this content area. Having parameters that are clear and known as opposed to more diffuse or controversial contributes to teachers' sense of competence, and hence, efficacy (Smith III, 1996).

Just because conceptual understanding was not emphasized, this does not mean the class was disjointed or merely focused on drill. Rather, Ms. Bryans took great care to ensure that students understood the purpose and applicability of what they were learning by using concrete examples and by connecting previous lessons to new ones. For instance, when teaching students how to solve two-step equations for the first time, Ms. Bryans told the class:

> T: We're going to take the knowledge you have and 'apply it to solving two-step equations.' (For example)'You have to check to oil in your car.'

Uncovering the overhead, she read the question she had prewritten up there:

T: What must you do to check the oil in your car?

A girl replied that you must raise the hood and check the dipstick. Some classmates laughed at this. Ms. Bryans agreed with her, then read the next question:

T: What do you do before you drive the car again?

Sam: Don't you have to (put the dipstick back and close the hood)?

Ms. Bryans agreed with this, emphasizing that the second part undid the first part.

T: We're going to solve equations… the second thing undoes the first one. It's just another form of?

Class: Working backwards.

T: So far, you're used to just working one step. Now it's going to take you two steps. In the first step you will either add or subtract, in the second step you will multiply or divide. (P13, p. 3)

In this situation, Ms. Bryans explicitly told students how this new skill related to a previous one by using a real life example to which they could relate. Along the same lines, in another lesson she told the class that "So we're reviewing positives and negatives because next week we're going to be multiplying and dividing." (P5, p.3) As Smith III (1996) claimed, telling provides a direct model for how to teach. These examples suggest that showing students how to solve math exercises, when it is done in a coherent way, engages the teacher and students in a choreographed dance where each person knows the rules for how to proceed; thereby creating a familiar and known environment. Such an environment is conducive to promoting a sense of competence, at least for the instructor and possibly for students, at least for those paying attention to the lesson.

This sense of instructional coherence was enhanced by the visible importance given to math in Ms. Bryans's classroom. Math posters containing math puzzles and problems adorned the walls. Some were student designed, others were professionally created—but they all asked questions (e.g. "What is the probability of flight?") or were otherwise visually compelling (e.g. brightly colored computer-generated fractal designs). One particularly fascinating poster contained pictures of such exotic polyhedra as a "rhombitruncated cuboctahedron." Examples of student-constructed polyhedra, built out of toothpicks and plastic connectors, hung from the ceiling above all of our heads. Student projects were likewise displayed on the walls, including the rotated tessellations they made at the end of January. Pictures and names of famous mathematicians (e.g. Pythagoras and Pascal) were displayed at various points around the room, along with examples of Escher's work, including a T-shirt and two shopping bags with his famous tessellations adorning them. Perhaps most representative of Ms. Bryans' beliefs, a large chart on the side wall comprehensively listed a variety of math skills along one axis and a diverse number of occupations on the other, noting what math skills were needed by which occupations. In sum, I am suggesting that if Ms. Bryans did not have such a strong sense of what math was, her classroom and instruction would reflect this by being less purposive and powerful. Interestingly, when I looked up "powerful" on my computer's thesaurus, I find that a synonym for it is "efficacious."

In conclusion, mathematics, as a body of knowledge and skills, is well-defined in procedurally-based instruction. Such explicitness provides opportunities for transfer to other domains, defines the parameters for concrete assessment of student achievement, and is easily scaffolded by explaining rules and tricks. Discourse-oriented (PM-based) teaching is fuzzier than this. As Sfard (1998) noted, inquiry as a curriculum may serve to subvert the whole notion of subject matter in it entirety. While some would probably not mind that outcome (Lave, 1990), others are rightly concerned about such extreme views of the nature of learning (Bereiter, 1997; Sfard, 1998). In addition, even though conceptual understanding is an essential goal of math reform (National Council of Teachers of Mathematics, 1989), some researchers have shown that discourse-oriented teaching may lead to more emphasis on constructing social norms than on scaffolding student understanding, so that conceptual understanding is sacrificed in favor of participation in rituals of discourse that have lost their meaning (Williams & Baxter, 1996). With procedural instruction, at least students are learning math skills that may be useful in their future careers (or at least allow them to pass the standardized tests that are the gateway to those careers!). Another main benefit of procedural instruction, as I will demonstrate in the next section, is that it provides certain advantages for effective classroom management.

Management

> G: What is an error that you think many beginning math teachers make when they first start teaching math?
>
> Ms. B: The problem is, they don't have the discipline….You can't teach if you don't have discipline.

This statement, taken from my interview with Ms. Bryans, reflects the importance of classroom management to Ms. Bryans' way of teaching. Interestingly, this same quote was recorded by Gregg in his interview of a traditional, rule-based teacher: "You know, you can't teach without discipline" (Gregg, 1995, p. 583). In addition, as Jones' (1996) review of the literature revealed, "as teachers move away from teacher-directed presentation and recitation, classroom management methods become more complex" (p.508). Unfortunately, there is not much research that reports how to successfully set up and manage classrooms based on math reforms such as inquiry or conceptual teaching. The research that does exist, however, informs us that the issue is quite problematic(Doyle, 1988; Gregg, 1995; Jones, 1996; McCaslin & Good, 1992; Williams & Baxter, 1996).

Successful classroom management in rule-based instruction is related to efficacy in several ways. First, it promotes a feeling of competence because students are doing what they are supposed to do in such instruction: listening, watching, and practicing skills (Smith III, 1996). Second, it yields a comfortable working environment where emotions are kept neutral because the content is unequivocal (Doyle, 1988). Third, it conforms to administrators' expectations of teachers (Gregg, 1995). As I will demonstrate, Ms. Bryans was quite proficient at classroom management, and her classroom promoted competence, yielded comfort, and conformed to administrator expectations because she was good at proactively and responsively managing <u>activity flow</u> (Evertson, Emmer, Clements, & Worsham, 1994). In their classroom management manual for secondary teachers, Evertson et al. (1994) summarized the research of Kounin and his colleagues on management behaviors that were related to high levels of student engagement and low levels of student misbehaviors. This outline provides a helpful conceptual organizer within which to discuss Ms. Bryans' effective management behaviors. In the discussion that follows, I will give examples from my field notes to illustrate that each of the three main types of teaching behaviors that Kounin and his colleagues (as cited in Evertson et al., 1994) found to be related to optimal activity flow was frequently exhibited by Ms. Bryans, and then I will discuss how they relate to the preceding three areas of efficacy.

The first teaching behavior cited by Evertson et al. (1994), preventing misbehavior, consists of two components: withitness and overlapping. Withit teachers have a heightened awareness of the classroom atmosphere; consequently, they are able to prevent student misbehaviors from escalating into full-blown conflicts. Ms. Bryans demonstrated withitness with both group and individual dynamics. For instance, students worked on individual seatwork for part of the day during every lesson I observed, and during this time, unless a test was being taken, inevitably students would start talking to each other after they completed the assignment. At first, Ms. Bryans would either ignore this talking or she would call out an individual student's name, saying "James, get busy please" (P2, p4). However, as soon as the noise began to escalate, Ms. Bryans would re-direct the whole class' attention with such focusing statements as "All right, are we ready?" (P16, p2) or "Ok, what I want you to do is…." (P16, p6). Such comments served to re-focus and quiet the class. She never let such noise build until the class was out of control. Similarly, when the off-task behavior was being perpetuated by just one student, Ms. Bryans was adept as preventing such misbehavior from escalating. For example, after handing out an assignment to her Pre-Algebra class, Ms. Bryans headed to her desk to talk privately with a student. Meanwhile, Tomas was not doing what he was supposed to be doing:

T: Tomas, if I say your name anymore, that's not good for you.

Tomas: Why not?

T: Because it's a dean referral.

Ms. Bryans then turned her attention immediately back to the class as they settled down after this quick exchange.

T: Don't forget if you're going to make up your test you have until Friday.

(P2, p3)

Each of these incidents demonstrates that Ms. Bryans was able to maintain a general awareness of the state of the classroom and, with minimal intervention, address and de-escalate potential obstacles to effective classroom functioning.

Overlapping, the ability to deal with two events at the same time, is another way to deal with student disruptions. Several examples from my observations will illustrate that Ms. Bryans was quite proficient at this management skill as well. For instance, during one problem-solving episode, Tony was up front by the overhead projector, distracting students who were copying notes from the screen. Ms. Bryans, helping Lara with a problem at Lara's desk, said "Tony, go back to your seat!" and then, immediately turning back to Lara, asked her, "How did you get your answer?" (P2, p2) On a different day, Ms. Bryans was at her desk talking with Celia and Krissy while the following exchange was occurring across the room:

Tony (to Ashley): Shut up blondie!

Ashley: Ooh, that really hurt. Tony, is there any time you are not talking?

Tony: Yes, while I'm sleeping.

Meanwhile, other students were vying for extra credit by calling out variations of "Ms. Bryans, this is really, really cool!" From her desk, Ms. Bryans ignored the students who were calling out, announced "Tony, let's be a little nicer please," then immediately returned to her students at the desk. (P5, p5). In both cases, Tony ceased engaging in the disruptive behaviors, and Ms. Bryans was able to quickly return to the task at hand. In the following section, I will present evidence that demonstrates how Ms. Bryans retained the class' focus and managed their activity flow by proactively and responsively controlling her own teaching behaviors.

The second teaching behavior, managing movement, is achieved through momentum and smoothness on the part of the teacher, according to Kounin and his associates (Evertson et al., 1994). Momentum refers to keeping the lesson moving at a continuous, quick pace. Ms. Bryans ensured that the class functioned at an engaging pace by both proactively arranging the classroom to deal with potential interruptions and by maintaining a steady pace during lessons. One of the first things I noticed on entering Ms. Bryans' classroom was that it was both colorful and well organized. On the back wall, a large, brightly-colored display of laminated construction paper provided the backdrop for a system of classroom organization in which the week's current group leader recorded homework assignments and answers to the daily problems. The front dry erase boards has a list of the day's objectives and homework assignments as well as Ms. Bryans' voice mail number for students to call in case they forget the homework assignment. On top of the long counter were crates for students to file late homework assignments or to search through papers without a name. In addition, each student had a notebook binder that was organized with sheets for them to record the answers to the problems of the day and bar graphs for recording the amount of homework exercises they answered correctly. The organization of the classroom allowed Ms. Bryans to reduce the amount of time she would normally have to spend bringing absent students up to speed.

As for pacing, there was a general pattern to the class period that included a variety of activities to promote student engagement as well provided a comfortable routine for students to follow. On a typical day, Ms. Bryans would get the students' attention and start them on problem solving (or similar warm up activity) while students passed their homework forward (10 minutes). As the class worked independently on the problem, Ms. Bryans recorded homework grades in her gradebook (10 minutes). After returning student's homework to them, she reviewed the problem of the day (5 minutes). At this point, she would often review the answers to the homework (8 minutes) then continue with the lesson of the day (8 minutes). Students would often work on some kind of assignment related to the new lesson (15 minutes), or they would take a test or quiz (30 minutes). At the end of the period, students would usually work on that night's homework (5 or more minutes). Similarly, regular classroom activities like problems of the day, homework checking, seatwork, test taking, test checking, and group work had a familiar pattern to them.[5]

Pacing was very important to Ms. Bryans. During one interview session, Ms. Jones approached Ms. Bryans to discuss the new eighth grade student at LMS who transferred here from another district. They couldn't believe that this girl was already several chapters ahead of their students. They were concerned about the possibility of another school being ahead of them. They tried to justify this, saying things like, 'Well there is one thing, she doesn't really know how to use a graphing calculator,' or 'We have more activities,' but they showed their unease by saying, 'Still, that's really far ahead. I didn't think anyone could be faster than us.' One said that she "still thinks we're gonna finish more than anyone in the county." This satisfied them. This incident revealed that it was important to both Ms. Bryans and her colleague that they were proceeding with their math lessons at an accelerated pace compared to other eighth grade classrooms and underscores the importance they gave to maintaining momentum.

. Ms. Bryans achieved such pacing using a variety of methods. Not only did she organize her classroom so that the rhythm of her class was undisturbed, she often chose to ignore certain student behaviors if addressing them would undermine the pacing she had established during a particular lesson. This was illustrated best in an excerpt from our second interview in which Ms. Bryans explained to me how "you can't really tick me off" by recounting the following incident from earlier in the year between her and James:

> T: … That one time when James ripped up the cards. We were playing this game, and, he just was, he had, he was not doing well that day. Anyway, they had to take these cards and they had to put them in order, and some of them were fractions, and some of 'em were decimals, and some of 'em were like pi, and they had to put 'em in order from smallest to largest in their group…. Well anyway, well James started taking the cards from his group, and he started tearing 'em up. So, and my <u>momentum</u> was going really well, so I took them away from him [emphasis added]. Ok, and I figured I would talk to him later. And then he got some other group's cards and started ripping <u>them</u> up. And finally, I just had to tell him to get out….And the kids kept on watching me because I wasn't getting mad. You know, I was taking the cards. Because, I mean, the whole thing was

> just going so well and if I had stopped, to write a referral [she says this word with annoyance], you know, I would've lost that whole thing [emphasis added]. Now my husband whole-heartedly disagrees with the whole thing. He thinks that I should've, you know, scolded him right then and there, written the referral, and gotten him out. And, um, I don't agree. So he got in trouble, he got suspended for a few days, and I was still able teach my class. (I2, p9)

This incident reflects how much emphasis Ms. Bryans gave to keeping the class on track, so much so that she sidestepped certain confrontations with students to focus on the learning of the majority of the class.

This type of selective ignoring was corroborated by my observations of her classroom. In the following incident, Ashley wanted Ms. Bryans' immediate attention:

> Ashley: Ms Teacher-lady!
>
> T (getting up from her desk): Ms. Teacher lady?
>
> Ashley: Well, I forgot your name.

Ms. Bryans didn't respond to this. Instead, she headed over to Ashley's desk and answered her question.

> Ashley (out loud): But you did it like that! It's not fair!

Again, Ms. Bryans ignored this outburst and proceeded to address the class about a paper without a name on it. Ashley continued to try and get Ms. Bryans attention by calling out complaints like "See, she helps her (another student) and not me. How rude!" and "You skipped me!" To which Ms. Bryans replied, "That's right." (P6, p2)

Other means of keeping the momentum of the class going included monitoring time and using quick question and answer recitation to teach skills. An example of the former occurred on a day when the initial activity was to review problems on the ITBS. As students walked into the room, they were greeted by instructions on the overhead that told them to get an ITBS booklet and to pass up their homework. As Ms. Bryans walked in the room, she told the class that they had three minutes to follow the instructions on the overhead. Less than three minutes later, she began the lesson. Examples of the latter can be found in the teaching exchanges previously cited, particularly in the problem solving episode.

Whereas momentum concerns the pace of the lessons, smoothness refers to the ability to avoid digressions that may cause student confusion. One way that Ms. Bryans accomplished this was to disregard irrelevant questions. For instance, during a review of calculator skills to practice for the FCAT, Carl calls out, "Can we rest for a few minutes?" Both teacher and students ignore him as the class solves the problem on the overhead. A similar incident occurred while Ms. Bryans was explaining the directions to a seatwork assignment:

> T: Turn to page 50. You are going to go on until--
>
> Felicia (interrupting her): What page?
>
> Jerald: Can we leave (when we're done)?
>
> T: You can leave when I tell you to leave.

As soon as this exchange ended, the class began their work. Ms. Bryans was not drawn into either repeating herself or taking up class time to chastise Jerald for his subversive remark. Another way Ms. Bryans achieved smoothness was to stick to her lesson plan when she had a specific purpose in mind that she wanted to accomplish. During one particular homework review session, instead of taking students questions like she normally did, Ms. Bryans chose to review those specific questions with which students had the most difficulty. When a student asked her to answer a different question, Ms. Bryans replied, "I don't have time, but I am going to do (number) ten." (P14, p7)

These examples are representative of the consistent implementation of "withit" and "smooth" behaviors on Ms. Bryans' part that occurred during each class I observed. This does not mean that the class was rigidly quiet or complacent, however. The noise level was allowed to escalate during group activities and sometimes during seatwork. Ms. Bryans smiled regularly, rarely raised her voice, and while she was stern when necessary, most of the time she seemed to be genuinely enjoying herself. She liked to joke with students, saying things like "Hey, you get to play with the broken tape" (P5, p4) or "This is not The Edge™ pizza. We are not cutting them up into squares" (P3, p5). Furthermore, Ms. Bryans never pretended to be infallible. In fact, she often emphasized that she was quite likely to be mistaken, and students seemed to enjoy pointing out her errors when they found them. For instance, once when a student pointed out an error Ms. Bryans had made on the overhead screen, she remarked in a teasing manner, "I've been making mistakes since you guys walked in"(P6, p4). Neither did she shy away from admitting her weaknesses as this remark about fractions reveals, "I don't know about you, but it wasn't my favorite 'when I was in school.'" (P7, p2). In addition, she praised students regularly (as documented in Table 2), which I will return to at the end of this section.

Whereas preventing misbehavior is concerned with student disruptions to the lesson and managing movement focuses on minimizing teacher interruptions to the lesson flow, the third teaching behavior, maintaining group focus, pertains to the behavior of the class as a single entity, much like a "conductor leading an orchestra" (Evertson et al., 1994, p. 98). There are three aspects to this task, group alerting, encouraging accountability, and high participation formats. Group alerting refers to engaging the entire class' attention while individual students are speaking. One way that Ms. Bryans accomplished this was to either call on students who were not paying attention or to remind students of the task at hand. The following exchange, during a homework review session, illustrates both types of group alerting:

> T: Lara, are you copying this and checking your work? John, could you tell me what number goes in that circle please?

John replied that he was writing something down.

> T: That's ok, you can still answer my question. (turning to another student) Steve, can you tell me what the final answer would be?
>
> Steve: No
>
> T: No? Really? That was easy. I would've thought you could have done that. (Moving on to the next problem) All right, now this one's tricky (P5, p2).

This exchange shows that one way Ms. Bryans kept students engaged during recitation was to ask questions of different students in quick succession as well as to redirect students who are not doing what they are supposed to do.

Encouraging accountability requires the teacher to communicate her expectations for student involvement. Ms. Bryans accomplished this by regularly and clearly stating her expectations for student work and class participation. For example, while the class was working on creating their second tessellation drawing, Ms. Bryans brought out samples of students' work as a visual representation of what she wanted from them. Taking one particular sample in her hands, she told the class, "Don't just think you can color it in and get full credit for it" pointing out how the student sample in front of them was only going to receive 50 points out of 100 because it lacked such embellishments (P5, p4). In addition, Ms. Bryans had many other routine expectations for students that she communicated to them as necessary. Some of these academic expectations include the following: check your homework by writing out the check, do your homework on time, use correct spelling on written assignments, copy notes from the overhead screen when new material is presented, watch her demonstrate procedures before doing them yourself, show your work, and label the answers to word problems on your notebook sheets. Some of her behavioral expectations included: to work on seatwork quietly, to raise your hands when asking a question (this was not consistently enforced however), to keep "all eyes up here," to have your "mouths closed," to not sharpen pencils while she was talking, and to not talk when she was talking. Students seemed quite aware of these expectations and, for the most part, conformed to them, at least during my visits. Dr. Cannell confirmed that Ms. Bryans was adept at encouraging accountability when she told me that Ms. Bryans was "always focusing on what you're doing right or well, and (she uses) that as a link for stretching" (I3, p.2)

Finally, high participation formats are those that provide multiple opportunities for student involvement in the lesson. Generally, Ms. Bryans accomplished this by attending to individual student's needs as they arose while the rest of the class worked on their seatwork. In addition, unless students were required to work on the activity alone, such as with problem solving or tests (and sometimes she walked around even then), Ms. Bryans usually walked up and down the rows during seatwork to help individual students with questions. Moreover, during review of homework or test answers, students were required to check their own papers and keep track of their errors on a bar graph in their notebooks. Likewise, in problem solving, students kept track of the problem strategy and correct answer in their notebooks as well.

The evidence I have provided in this section suggests that Ms. Bryans' classroom management behaviors are effective in promoting what Kounin and his colleagues call managing activity flow (Evertson et al., 1994). This was corroborated by Dr. Cannell, Ms. Bryans, and myself. Dr. Cannell explained to me that Ms. Bryans was "exemplary" at classroom management because she was able to "orchestrate student movement" and didn't let "students pull her off task." (I3, p.2) In addition, when asked to rate herself on several teaching behaviors, Ms. Bryans gave the highest rating (10) to her classroom management skills, along with her creativity and effectiveness as an instructor. As a former eighth grade teacher, I think my own impressions are important to add here; especially because they differed from my initial expectations. I did not expect to find the classroom so well functioning nor students so engaged, not only because these were eighth graders (lower-tracked ones to boot) but also because the method of instruction was more traditional than I had expected. My overall sense was that students were respectful to Ms. Bryans, and they "bought into" her system of management for the most part. In my opinion, she was excellent at classroom management, defusing potentially volatile situations, and keeping student focused and engaged in the lessons, all the while communicating the importance of math. I will return to some of the implications of this for math reform in the final section of this paper.

For now, I want to conclude this section by suggesting that effectively managing the classroom promotes a sense of competence in an instructor. Not only is the school administrator reassured that things are under control, but students can feel safe from intellectual chaos and physical harm. Once students know a teacher's expectations for their behavior, and they trust a teacher to provide a consistent, fair, and engaging environment, it is likely that they will respond positively to instruction (Evertson et al., 1994). This may lead to general positive affect all around. As Doyle (1988) noted, "familiar work...often appears to be quite 'suitable' for managing the complexities of classroom environments...(by providing) a tangible structure and a clear program of action that is accessible to nearly all students" (p. 178).

As I indicated at the start of this section, competence, comfort, and conforming to administrators' expectations are all related to feelings of efficacy. Dunkin and Biddle's (1974) landmark study found that of this constellation of management behaviors, withitness, smoothness, momentum, and group alerting predicted work involvement during recitation, whereas withitness and momentum predicted "freedom from deviancy" during recitation (Dunkin & Biddle, 1974, p. 158). This supports my claim that there is something about effective classroom management that is important, and that classroom management in rule-based teaching is forthright, well-defined, and well-researched. We know what works in recitation-style classrooms, and Ms. Bryans is doing it. Classrooms that are "free from deviance" and show a high level of student involvement are powerful advantages of rule-based teaching, when it is done well.[6]

Problems with Procedural Teaching

As I mentioned previously, one of the major criticisms of traditional, rule-based math instruction is that it does not take into account developments in our understanding of the active, constructive nature of cognition; therefore, it does not promote optimal student understanding. From a political perspective, another criticism is that comparative studies between our nation and our competitors such as Japan reveals that U.S. elementary students' math ability is lower in all areas when compared to Japanese or Taiwanese students (Stigler & Fernandez, 1995). Other criticisms concern the social implications that this pedagogy engenders overly complacent, passive students who neither think critically nor learn self-regulatory behaviors (McCaslin & Good, 1992). Based on the data accrued in this study, I have found support for four obstacles to learning when procedural-based instruction is the predominant method of pedagogy in math classrooms. First, student misconceptions may flourish. Second, student achievement is not guaranteed. Third, student dependence is encouraged. Fourth, student motivation is primarily extrinsic. I will discuss each in turn, with references to examples culled from my observations of Ms. Bryans' classroom. However, due to a lack of student interviews, my discussion will be based on classroom observation and will lack the corroborating evidence that directly speaking with students would have provided.

Student Misunderstandings

Midway through my research, I came across Schoenfeld's (1988) influential case study of a tenth grade high school geometry class of a traditional "good" teacher whose students acquired significant misconceptions about not only specific geometry concepts, but about the nature of mathematics as a whole that were likely to interfere with learning math in the future. My ethnographic investigation of Ms. Bryans' Pre-Algebra classroom found that similar results were obtained with eighth graders: The effects of traditional, procedural instruction in this classroom yielded a number of negative side effects for students' overall conception of the nature of mathematics as well as specific procedural knowledge and skills.

One of the primary general misconceptions about math that students learned from rule-based Pre-Algebra instruction was that math consisted of a series of meaningless procedures that required that you just follow the steps to solve a certain problem without needing to understand why. For example, in describing how to divide 7 by 12, Ms. Bryans told the class, "I add two zeroes after my decimal point. I don't know why. I was just trained that way" (P8, p5). Because understanding why a procedure works is time consuming and perhaps seen as not very relevant, only the essentials are emphasized: how to implement the correct procedure. However, this often is not the best approach: our minds function by trying to understand what we are doing and why we are doing it, or as Hiebert and Carpenter (1992) phrased it, "Procedures in mathematics always depend upon principles represented conceptually" (p. 78). In the absence of such understanding, students will construct their own understandings of what they are learning, and such naïve theories will often be in conflict with correct understanding. Another instance that demonstrated how the meaninglessness of math was conveyed to students took place in the following interchange in which Ms. Bryans was explaining how to use the unit multiplier algorithm:

T: What number goes next to cm?

Class: 3

T: 3 times 100 is?

Class: 300

Felicia: This is confusing.

Hannah protested that she didn't understand why you multiplied 3 x 100 without using the "decimal way" (moving the decimal to the right or left based on the acronym MRDL, the turtle).

T: Can you multiply 3 x 100 in your head?

Hannah: Yes, but what if it's a different one? (meaning one you can't do in your head)

Ms. Bryans told her to just use a calculator then. Hannah protested that she lost her calculator. Ms. Bryans moved on to another example (P15, p8). In this incident, Hannah was confused about why Ms. Bryans had switched from one procedure to something different. Ms. Bryans had been teaching them how to move the decimal to the right or left when dividing or multiplying by a multiple of 10, but, because she had a deeper understanding, she was able to flexibly switch procedures and just use mental math to multiply 3 x 100. Some students wanted to be able to just stick with the new procedure however, but Ms. Bryans did not take the time to explain how the two procedures were related.

Lastly, general misconceptions about math were promoted during daily problem solving. Even though in our interview Ms. Bryans stated that the purpose of problem solving was to be able to solve an unknown problem, the following excerpt from our interview reveals that she did not expect students to meet this goal:

> G: What was the purpose of all this problem solving?
>
> T: To take a random problem, any problem, and by going through all the problem solving strategies, and doing them over and over and over again, just given any problem, can they go back on their previous knowledge and solve a new problem.
>
> G: For similar problems or for completely different kinds of problems?
>
> T: Well, eventually, for completely different problems, but I think in the short run, can you give 'em a problem where they would recognize the problem solving strategy and understand oh yeah, I need to draw a picture, or I need to guess and check on this one. And then eventually, yeah, can you carry it out later on in life? Cause a lot of algebra is draw a picture. You know, so if you find out "Gee it's ok I <u>can</u> draw a picture if I don't understand this," then that's a good approach to a problem if you don't understand it.

So, although Ms. Bryans wanted students to be able to solve more complex problems, her focus was on helping them identify certain strategies (e.g. draw a picture, make a table) that they could use to solve many problems. When I asked her if this was working, she replied:

> Um, each problem is different. They at least know where the list is to choose from the problem solving strategies. Um, and they know we are going to do <u>one</u> of those, so they, they've gotten better cause they know a little better how to approach it and some days are good days and some days are not so good days. Truthfully. (I5, p3)

Even Ms. Bryans realized that for most of her students, daily problem solving had mostly just taught them how to select a specific strategy from a list of previously taught heuristics without attaining her ultimate goals for them. In the nature of today's complex world, problems do not come in canned packages. They are complex and require that students are able to critically think about and break down problems on their own, test their solutions, and rework the problem when necessary. As the example of the problem solving incident cited earlier in this paper demonstrated, students are not given such an opportunity in Ms. Bryans' classroom—the difficult conceptual work of deciding how to even begin attacking the problem has already been done for them. These examples illustrate that math was presented as a discrete collection of procedures, useful for solving problems and exercises, but not connected to a deeper framework of thinking about and understanding mathematics as a whole.

Some might say that developing a deeper understanding of math is not the goal of school math instruction; as long as students can competently carry out the correct algorithms, that is enough[7]. While many researchers in the field would vehemently disagree with this statement (Bereiter & Scardamalia, 1996; Brown, 1997; Cobb et al., 1992; DeCorte, Greer, & Verschaffel, 1996; Hiebert & Carpenter, 1992; Lampert et al., 1996; Nickson, 1992; Schoenfeld, 1988; Schoenfeld, 1992; Stigler & Perry, 1990), it still is important to investigate whether or not traditional instruction succeeds at its original intent. Rule-based instruction has its goal students who are able to correctly use mathematical procedures to solve math problems. However, the following examples will show that even such specific procedural knowledge was often incorrectly learned. Although I have previously shown that Ms. Bryans' emphasis on pacing kept students engaged and focused, as well as allowed the eighth grade team to be ahead of "the rest of the county," an unfortunate side effect was that it did not allow time to correct student misunderstandings. I have already cited several instances when students' questions were ignored in favor of moving forward with the lesson (e.g. when Ashley called out "What's a centimeter?" during the lesson on unit multipliers). In another instance, James' misconception with two step equations was ignored so that the lesson could proceed:

> T: Who can tell me what 'x' represents? James?
>
> James: I don't know, I just wrote what it said. I got it right.

Ms. Bryans ignores James lack of understanding and moves on with the lesson.

> T: 'How would you represent three times the price of the apple?'
>
> Krissy: 3x.
>
> T: Yes, 3x. (Writing as she speaks:) 3x -.25 = 1.10.

James asked if the 3x and .25 could be switched (i.e. if the problem could be written as .25-3x).

> T: No.

James said that's what he did, and he got it right.

> T: Well you got lucky then.

Ms. Bryans then continued on to the next exercise. The pace was swift, and students did not have to sit through a lengthy explanation of why James was incorrect; nevertheless, James was left without understanding why his procedure worked, except that he was "lucky."

Other specific misconceptions about Pre-Algebra were less a result of pacing than due to the preponderance of rules that students needed to know to solve equations. The eighth grade math teachers were aware of these misconceptions, but seemed to blame students for having such buggy knowledge rather than the method if instruction they were using. This is corroborated by the following excerpt from my field notes of a conversation that took place in the team planning room among three of the math instructors:

> They are talking about students' misconceptions of certain concepts they are trying to teach them, such as perpendicular lines. Ms. Jones says that the students think they have drawn perpendicular lines correctly, even when they haven't. They justify their drawings saying things like, I got it but mine doesn't look like yours does. Ms. Jones continues to talk about students' responses. 'You'd think that if someone were walking down a street that had bully on it who beat them up, that person would learn to go down another street.' (In other words, you'd think kids would know that using the wrong method produces the wrong results, so they should change their method.) The conversation switches as Ms. Jones and Ms. Zaccaro talk about the problem kids have with estimating. One says, 'They think using a calculator is estimating.' And when students should

> round to an easier number to be able to divide evenly, one says that they 'round 16.8 to 17' which doesn't work out evenly and undermines the whole point of estimating. Ms. Jones wonders aloud about students' difficulty with estimation, especially because "It's supposed to be easier" (than performing the whole algorithm). (P12, p1)

This information was treated as something beyond the teachers' control: students just kept walking down that same bad street with the "bully" on it. A partial listing of other specific misconceptions held by students can be found in Table 3. These misconceptions reveal students' inability to make connections between school mathematics and applied, "real world" mathematics. Because these misunderstandings were held by different students at different times, they suggest that such bugs were part of the hidden curriculum of procedural instruction in Ms. Bryans' room, just as a virus is attached to certain software programs. Unless "debugged," these misconceptions can undermine students' procedural fluency as well as conceptual understanding of important math skills. Although Ms. Bryans occasionally attempted to "debug" her students by giving an example of why a certain misconception was not true (e.g. To correct the notion that _ / _ = 1/2, she gave a simpler example of 4 /4 =1), she mentioned such corrections quickly, in passing, by merely telling students that their original solutions were incorrect. Such explanations stayed in the realm of telling and thus were subject to being misconstrued themselves.

<u>Student Achievement</u>

I looked for evidence that perhaps Ms. Bryans' students were succeeding on traditional measures of assessment in order to assess whether students' misconceptions were merely an artifact of class participation and not representative of their performance. For if students were able to demonstrate accurate procedural skill when it counted (i.e. on tests), then perhaps these bugs were merely common problems along the way toward developing competence. Unfortunately, I could not gain access to standardized test scores for these students, but I was able to compile a list of scores for tests, quizzes and homework assignments given during the time period in which I was conducting my observation of Ms. Bryans' class. On average, the percentage of students who did not turn in their homework on time was about one-third. For tests, the average grade on a typical test (Chapter 6) was 85%, but the average grade for a notebook test (a test of student participation and organization) was 71%. Student averages for the Chapter 7 test were quite a bit higher (91%), but Ms. Bryans explained that this was because it was a multiple choice test on equations; it was easy for students to just substitute each answer in the equation to see which one worked. Finally, when asked about how her students compared to the other eighth grade Pre-Algebra students at LMS, Ms. Bryans told me that they were performing at comparable levels. While the evidence does not suggest that student achievement was guaranteed in this system, this conclusion would have been strengthened with more evidence to support it.

Student Dependence

McCaslin and Good's (1992) analysis claimed that the effects of classroom management policies that foster student obedience and dependence are likely to decrease student motivation for learning and ultimately negatively affect their future potential in a world that requires active, self-motivated thinkers. In other words, viewing the teacher as an authority figure may cause students to abdicate responsibility for critically thinking about mathematics and about knowledge in general (Nickson, 1992). Accordingly, the norms of student dependence and teacher authority, which I will demonstrate existed in Ms. Bryans' classroom, is another problem of rule-based instruction. Ms. Bryans, as Tables 1 and 2 and previously cited passages indicate, was the primary voice in the Pre-Algebra classroom. Students asked her how to solve classwork problems much more frequently than they asked each other. Ms. Bryans was the ultimate authority and judge of classroom events, both behavioral and cognitive. For instance, when students presented alternate solutions to problems, Ms. Bryans would often acknowledge their solution, but claimed that the one she showed them was better in some way as the following exchange illustrates. Jerald had just told Ms. Bryans his solution to a two-step equation. Ms. Bryans replied:

> No, I wouldn't do that....It takes twice as long....You can do it, but you have to know what you're doing" (P15, p4).

Another example of Ms. Bryans' authority can be seen in the previously cited example on unit multipliers where James asked "How are we supposed to know that 10 millimeters = 1 centimeter?" and Ms. Bryans answered, "Cause if it's on a test, I'll give it to you."

The student dependence that resulted sometimes led to situations, especially during seatwork, in which Ms. Bryans was in incessant demand, both for her help and her approval. For example, when students were working on their tessellations in cooperative groups, students sought out Ms. Bryans constantly:

> T (handing out supplies): Raise your hand if you need another folder.
>
> Several students: Ms. Bryans! Ms. Bryans! Ms. Bryans! Is this right?

T: Yes. Ok anyone else need a folder? (She helps Tony, telling him to cut out a certain shape, which he does.)

Student: Miss Miss Miss!

T: All right. I'm considering the fact that if you turn in both of them, you might get extra credit. Some of your drawings are terrific!

Felicia: 'Does mine look terrific?'

T: Yeah yours looks good, but I'm talking about the ones who've colored them in. Ok, where's the tape? Who needs more tape?

Several students cry out: "Me!"

In this example, students look to Ms. Bryans to tell them about their work, to get supplies, and to ask her questions. Although this might be fine in a tutoring setting, in a class of 31 students, it was impossible for Ms. Bryans to answer every students' question; hence students were sometimes left frustrated during the course of an assignment. This was demonstrated in the previously cited incident in which Ashley, trying to get Ms. Bryans' attention, ended up not satisfied with the quality of their interaction.

I looked for evidence of student autonomy in the classroom in order to check the validity of this claim, and although I found that Ms. Bryans was responsive to certain student requests, such as changing the thermostat upon student request, for the most part, a majority of the interactions were teacher-initiated and directed. Students occasionally resisted this by questioning the rules, but this resistance was infrequent and quickly redirected or silenced, in a firm, but not unkindly manner:

Lara: Why are we only allowed to use one? (Referring to the policy of being able to use index cards to record notes for a test)

T: Because that's the option I am giving you. (P3, p5)

Student Motivation

Before concluding this section, I want to briefly return to McCaslin and Good's (1992) statement concerning the declines in student motivation that traditional authoritarian teaching practices engender. Similarly, current research indicates that intrinsic motivation is related to learning environments that support autonomy; whereas controlling environments, or those that are primarily extrinsically motivating, are related to decreased intrinsic motivation and self-regulatory behavior (Deci, Ryan, & Williams, 1996). Although the lack of student data in this study makes it difficult to assess student motivation in Ms. Bryans' classroom, an examination of the type of rewards and punishments she used to motivate students will provide some evidence to support this claim that it was primarily authoritarian in nature.

In the previous section, I presented evidence demonstrating that student dependence on Ms. Bryans was fostered as a result of the rule-based pedagogy she practiced. Similarly, when student motivation was questionable, Ms. Bryans used rewards, punishments, and rationales as motivators to re-engage students in the lesson or stop certain misbehaviors. Ms. Bryans' primary means of rewarding students was to verbally praise their correct solutions to an exercise or problem. Examples of such praise are listed in Table 2. Once, however, she did tell students that "If I don't get to three (her system of warning), that homework assignment comes off the board" (P10, p6). Consequences for misbehavior included things like referral to the dean's office, calling parents, keeping students after class, getting an "inconduct," moving students who are talking, and verbal behavioral corrections. An example of the latter occurred when Tony was singing and Ms. Bryans replied, "Tony, that's more than enough" (P11, p5).

Rationales are similar to rewards and consequences in that they provide an extrinsic reason for engaging in a behavior. Some of the rationales Ms. Bryans used to persuade students to do their work included: to bring up their grades, to get partial credit on homework, to do well in high school, to avoid going to summer school, to get full credit, to let her "see how they did," and to ensure students will understand it. In addition, some the reasons Ms. Bryans gave to persuade students to pay attention to her included the following: because a test will be given on this material, to avoid not knowing the answer when called upon, because she was going to keep going until everyone could answer her questions, or because she will stop the review for the test otherwise. In conclusion, there is some evidence to suggest that Ms. Bryans used extrinsically motivating techniques, such as rewards, punishments, and rationales, to engage students in the lesson or correct behavior when necessary. Although such techniques are typical of the school mathematics tradition (Gregg, 1995), they are related to some negative outcomes for students' learning and ultimate well being (Gregg, 1995; McCaslin & Good, 1992).

In conclusion, I have argued that even though there are benefits to procedural math instruction, there is dark side as well, at least as it was practiced in Ms. Bryans' Pre-Algebra class. I have presented evidence to suggest that such instruction bred student misconceptions about the nature and scope of mathematics as well as specific misconceptions about Pre-Algebra concepts and skills while not being strongly related to student achievement in that class. Moreover, it fostered student dependence on Ms.

Bryans as the authority figure, and it created conditions where extrinsic rewards were emphasized over intrinsic learning. In the following section, I will discuss the implications of this for current math reform.

Analysis and Implications

Thus far, I have presented evidence demonstrating that Ms. Bryans' beliefs about math determined the standards for her assessment of efficacy, and that this influenced her pedagogy. Such beliefs were not created in a vacuum, but were encouraged by the school context itself, including student and administrator beliefs and expectations about what constitutes good math instruction. Furthermore, I have suggested that such beliefs caused her to assimilate such math reforms as problem solving, cooperative groups, and hands-on activities so that they still fit within a traditional, rule-based pedagogy. I then presented evidence to support the claim that such a method of instruction has significant benefits for instruction and classroom management, primarily by promoting a well defined, clear standard to emulate if one wants to be a good instructor. Subsequently, I documented some of the drawbacks of this approach. Next, I will discuss the implications of this analysis for reform of mathematics instruction.

Ms. Bryans would probably not agree that she teaches by telling[8]. Her classroom is decorated with student projects: Brightly colored lizard tessellations adorn the walls, complex toothpick polyhedra dangle from the ceiling, math puzzles greet you as you enter the front door. Students solve complex problems regularly and participate in hands-on activities more than in a traditional middle school math classroom. Still, as I have tried to show, her predominant pedagogy is rule-based. This is what is assessed, thus it is what is accorded value. Standardized tests are taken seriously, and a significant amount of class time is devoted to practicing for them. Cooperative groups are not truly collaborative, from my observations, nor are they used that frequently, hands-on activities supplement the curriculum and serve to reinforce specific concepts, but then it's time to "get down to the business" of learning the procedure. Problem solving activities, the focus of Ms. Bryans' reformed classroom, are taught through directive scaffolding that yields little cognitive autonomy for the students—they implement the procedure that Ms. B has already uncovered for them. This way of teaching fits in well with Ms. Bryans' beliefs about math as a concrete body of skills that students need to know to be successful in the world. As new reforms come down the pike, Ms. B has integrated them into her classroom in a way that increases her sense of efficacy without changing her beliefs about math. Hands-on activities and groups break up the curriculum and add interest when students' brains "are tapped out."

This method of instruction works for Ms. B. In her words: "A lot of kids, they like the class. You hear 'It's my favorite class'....I often hear, 'I've always hated math, and I really like math this year.'" She believes it's both due to the hands-on activities she provides and the fact that "you can't really tick me off." (I2, p.9). Her self-ratings reflect that she perceives her strengths to be that she is "organized, creative, and willing to listen." In addition, it works for Dr. Cannell, the principal of Latham Middle School, who remarked that Ms. Bryans was "oriented to students' needs" and "exemplary" in all areas of teaching. And for me personally, as a former middle school math instructor, it was encouraging to see most students engaged in creative math work at the eighth grade level. Additionally, I have seen too many poor math instructors who turn students off to math and who are just not good teachers. Ms. Bryans is good at what she does. There is a certain art form to traditional instruction and Ms. Bryans has mastered it, especially in her classroom management techniques. She feels competent in her job, and, in general, students seem to enjoy and feel comfortable in her class.

On the other hand, as I have just demonstrated, there are some problems with this approach. Although student dependence may not be considered a problem if one thinks that this is fitting for students, the other two issues are less easy to brush away. Student misconceptions result in serious misunderstandings about math for students. At a time when our nation's students are not mathematically competitive with the students of other nations (Stigler & Fernandez, 1995), this is not something that can be dismissed. Consequently, procedural-based instruction may not even be that effective in contributing to the traditional measures of success. Perhaps we could argue in favor of it if student achievement was really high, but it is not. Yet, I want to suggest that this method is the best we can get under the traditional system of schooling. Ms. Bryans is the one of the most highly motivated, engaging math teachers I have ever met. Students <u>are</u> interested. They don't hate math. They believe they can do it. These are significant effects. What is distressing is that students aren't more mathematically powerful, critical thinkers.

Many current reforms are trying to address this, but many of them are failing to produce true change (Ball, 1990; Cohen, 1990; Gregg, 1995; Williams & Baxter, 1996). It is difficult to implement significant, lasting changes without continuing support, organizational changes in the structure of schooling, and a sense of autonomy for those involved in the change (Richardson & Placier, in preparation). As Ms. Bryans remarked in our final interview, she recently threw away a curriculum guide that was so prescriptive it told her word for word what to say while teaching a lesson. Such control destroys intrinsic motivation and enjoyment in one's work (Deci, 1975), and may lead to eventual burnout (Burisch, 1993).

Besides loss of autonomy, another potential cause of burnout is the inability to feel competent or efficacious in one's work (Cherniss, 1993). As Smith III's (1996) analysis demonstrated, traditional methods of instruction and management exist because they contribute to teachers and schools' sense of efficacy. Not only do teachers need the support and time to make such changes, they need the autonomy, they need to be convinced that their own ways aren't working as effectively as other ways, they need models (videotapes of real teachers in real classrooms) and they need the technology or detailed training in how to teach in paradigmatically new ways (Pogrow & Londer, 1994). But most of all, they need to feel that implementing these deep changes will contribute to their sense of efficacy, not detract from it. Otherwise, teachers will continue with doing what works for them, and those that insist on heroically trying to change without such support (Bereiter, 1995) may be on the road to eventual burnout.

One important implication of this analysis is that the issues of math reform and classroom management are more complex that researchers such as Gregg (1995) have suggested. Whereas Gregg linked problems of discipline and control to traditional math instruction in his case study of a novice teacher, an ethnographic analysis of Ms. Bryans's classroom failed to support such claims in an expert traditional math instructor's classroom. This suggests that perhaps the effects Gregg noted were due to inexperience rather than beliefs. When rule-based, traditional math instruction is proficiently engaged in, the classroom seems to operate like a well-oiled wheel, rolling toward an expected outcome; hence, teachers, students, and schools may be resistant to truly restructuring their classrooms and overall organization.

Further research is needed in this area to provide scaffolding to support schools, teachers, and communities in restructuring their practices to take advantage of the research on the active nature of children's cognition and autonomy while avoiding the problems associated with traditional instruction. We know teaching by telling is not enough to produce the kinds of critical thinkers we want students to be. On the other hand, procedures are important because, according to Hiebert and Carpenter (1992), they "allow mathematical tasks to completed efficiently. Procedures can be executed quickly and with relatively little mental effort" (p.78); yet students need conceptual understanding as well. Inquiry-oriented instruction seems like one powerful way to encourage students to be autonomous, critical thinkers and learners. New visions and models of what such integrated classrooms would look like are important; further research in this area is critical. As McCaslin and Good (1992) remarked in their plea for structural support for student learning, "It is time to recognize that individual effort is insufficient. Effort is not merely an individual variable" (p. 10). I would add that the same goes for teachers and schools as well. Because the dynamics of the classroom are cognitively complex, a framework is necessary to give structure to this complexity so that math teachers, students, and administrators know their roles and how to be effective and competent within those roles while at the same time promoting conceptual understanding, increasing degrees of student autonomy, and mathematical discourse. A serious problem with many mathematics reforms is that they try to teach teachers by telling them how to change their practice through guidelines, initiatives, and standards (National Council of

Teachers of Mathematics, 1989); yet we know that teachers change what they learn and develop their own misconceptions just as students do (Cohen & Ball, 1990; Darling-Hammond, 1990). So, for the moment, let us appreciate Ms. Bryans for her motivated and dedicated attempt to translate mathematics reforms into the everyday realities of her classroom in a way that keeps the class engaged and informed. And then, let us think about ways that support her, and good teachers like her, to achieve even greater success in fostering students who are powerful math thinkers and learners without sacrificing her sense of efficacy or well-functioning classroom in the process.

Table 1

Domain of Math Rules and Tricks used by Ms. Bryans to teach Pre-Algebra Skills

Rule/Trick	Protocol/Page #
Find the Greatest Common Multiple by using "upside down division."	P2, p4
"Two negatives equals a positive."	P2, p5
"Secret 11 trick" (for multiplying by 11).	P2, p5
"The opposite of subtracting two numbers is adding two numbers."	P1, p2
"You have to line up your decimal points."	P5, p3
"Do the opposite of what it says" (for adding integers).	P14, p8
"There's always an imaginary plus sign" (in front of positive numbers).	P14, p7
"When the signs are different, you subtract and take the (sign of) the higher #.	P14, p7
"When we're adding, when the signs are the same you carry the signs."	P13, p4
"Leave the one that's the letter alone" (for solving two-step equations).	P13, p5
"Shortcut for multiplying fractions."	P11, p2
"Dividing by 12 is the same thing as multiplying by 1/12."	P11, p5
"Whenever you multiply a negative number, it makes (the inequality) flip."	P10, p4
"You flip the second one" (referring to multiplicative inverses).	P10, p4

"The knuckle trick" (for remembering what the number of days in a month).	P8, p2
"Do not put the decimal point in until after you have completely got the answer.	P8, p4
"You take off that zero" (referring to zeroes after the decimal point)	P8, p4
"The first number always goes under the division sign."	P8, p6
"Just take the whole number and tack it on the end" (for adding mixed fractions)	P7, p1
"Parentheses next to parentheses is a multiplication problem."	P7, p5
"The secret word is 'is.' 'Is' means it's going to be an inequality."	P15, p4
"MRDL, the turtle" (for moving the decimal with multiplication and division).	P15, p8
"Sum" means "add."	P16, p4
"We don't have repeating decimals in multiplication."	P15, p6

Table 2

Examples of Feedback and Praise that Ms. Bryans Gave to Students and to the Class

Feedback and praise given to the class as a whole
"I have to tell you everybody did a really good job but some people thought that was a division problem." (P7, p5)
"All right, most of you got this." (P15, p9)
"Most of you guys did 18 very well." (P1, p2)
"Most of you got the right answer." (P3, p3)
"I like these questions cause you're answering them yourselves." (P13, p2)
"I gave this problem to my Algebra 1 Honors students, and they couldn't solve this; not one of them. And you guys did a pretty good job." (P12, p4)
"Look how fast you are doing fractions in your head!" (P7, p2)
Public feedback and praise given to students
"Mr. Jackson is the only student who got this right, so 'be proud of it.'" (P10, p3)
Telling a female student that she "is bright too." (P13, p5)
"Thank you" (to a student who corrected Ms. Bryans' error) (P2, p5)
"That's a very good question." (P3, p3)
"Ooh, that's excellent! He noticed that was a repeating decimal…Excellent." (P3, p5)
(In response to a student answer) "Excellent….Let's try that." (P3, p5)
"Very good." (P5, p3)
"You're close." (P13, p5)

Table 3

Types of Misconceptions Held by Students in Ms. Bryans' Pre-Algebra class
The bigger number always comes first.
½ / ½ = ½
Not understanding why zeroes are used as lace holders in multi-digit multiplication.
Not knowing when numbers are positive or negative.
Not understanding that12 x ¼ = 12/4.
Not knowing where the numbers go when they are factored out of a fraction.
Not understanding why dividing by 7/8 = multiplying by 8/7.
Not realizing that 1 ½ = 3/2.
Not understanding the concept of "weighting" grades. Thinking homework grades are equivalent to test grades.
"If we forget the decimal, does that matter?"
Thinking $3^5 = 5^3$
Applying rules for multiplying and dividing fractions to adding and subtracting them.
Not understanding that x > some number means not inclusive of that number.
Thinking 14.4 can be a factor in multiplication expression.
Not knowing whether to move a decimal to the right or left when dividing or multiplying.

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Footnotes

[1] For the purposes of this paper, school mathematics will be used interchangeably with the terms procedural instruction, rule-based teaching, or teaching by telling. The math reforms referred to throughout the paper will refer to a general conception of inquiry-oriented math instruction, whether from a cognitive or discourse-oriented perspective.

[2] All names and some non-relevant details have been changed to protect the anonymity of the teacher, students, and the school in this study. In addition, all direct quotes are symbolized by double quotation marks (""), and indirect quotes by single quotation mark ('').

[3] In a classroom based on the PM metaphor, teachers are not solely responsible for this scaffolding. They provided the opportunity, but shared discussion and argument among peers would be the primary means of achieving this scaffolding (Lampert et al., 1996).

[4] When interviewed on this the question of what it means for her kids to understand math, Ms. Bryans said "They can help someone else, explain the concept to someone else." (I1, p.1) Thus, even though she spoke of <u>conceptual</u> understanding, she stated that it is still something that can be <u>told</u> to a student. Telling, versus active construction of knowledge, is what is emphasized.

[5] See methods of instruction for a description of the routine of checking homework. The numbers in parenthesis were taken from a typical lesson in which I had kept careful track of the times between each activity.

[6] Gregg's (1995) research on discipline and the school mathematics tradition found more negative effects of rule-based instruction, such as student boredom and control problems; however, because he studied a beginning high school teacher, he is probably confounding inexperience with type of instruction. The evidence in this study of an experienced teacher does not reveal any control problems in Ms. Bryans' classroom; moreover, the results of process product research have demonstrated that rule-based instruction, when perform by a competent, <u>withit</u> teacher, is related to more student involvement and less misbehaviors.

[7] This is an approach to math I call the back-to-basics approach in which the three 'R's (reading, writing, and arithmetic) form the foundation of instruction. When I was a public school teacher, I encountered this perspective in several of my students' parents.

[8] Ms. Bryans' potential disagreement with my assessment of her pedagogy as traditional was corroborated by my final meeting with her in May, 1998 after she had read the section of this paper entitled, Benefits of Procedural Teaching. Although she said had enjoyed reading the paper, she remarked that she disagreed with me on one point—she does not see herself as a traditional, or rule-based teacher.

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