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ABSTRACT

By measuring the performance of 62 students enrolled in a community college introductory algebra course, this study challenges the generally accepted assumption among mathematics instructors that mastery of arithmetic is necessary for the learning of algebra. Study subjects were 35% male, 74% Hispanic, 16% Black, 8% white, and 2% other. A pretest, consisting of 10 items designed to indicate minimum competency in computational arithmetic, was administered at the beginning of the semester, and the final exam was used as a posttest. Using analysis of variance, analysis of covariance, and a Pearson r calculation, no significant differences were found between three groups that received different types of instruction: (1) computer-managed instruction combined with individualized instruction in a math lab; (2) computer-managed instruction used in the classroom; and (3) a traditional class, used as a control group. The study concludes that the ability to perform well in computational arithmetic has little, if anything, to do with the ability to perform well in beginning algebra. As a result of this finding, it is suggested that community colleges combine the topics of arithmetic and beginning algebra into one integrated course that uses a "spiral approach." Contains 3 tables and 11 references. (CAK)

Is Arithmetic Really Necessary for Algebra?

A Case for an Integrated Curriculum

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Running Head: Arithmetic vs Algebra

Is Arithmetic Really Necessary for Algebra? A Case for an Integrated Curriculum

Nearly everyone, mathematics instructors as well as laypeople, have assumed that one must master arithmetic before beginning the study of algebra. Is this assumption really true or has it seemed so obvious that no one has thought to question it? If it is false, the curriculum of both secondary and post secondary education could be radically changed. The two year college would especially be affected.

A computer search of the literature, using the Dialog data base, covered the past twenty years. This search revealed that no one has ever challenged the assumed relationship between arithmetic and algebra. The vast majority of investigators have examined the ability of success in arithmetic to predict success in algebra. These studies usually take the form of deriving a regression equation to predict success in algebra from variables such as grade point average, standardized test scores, college and/or high school ranking, gender, and many others. Unfortunately, none of these studies directly addresses the circumstances of the current investigation. However, they not only provide a background for the study, but in many instances support the results of the study.

In general, the literature indicates that some variable other than arithmetic is the best predictor of success in algebra. However, there is no agreement as to what that predictor is. Dykes (1980) found that the high school grade point average correlated better with grades earned in college algebra than scores on the ACT Mathematics test among junior college students in college algebra. Nejadsadeghi (1985) came to the same conclusion for university freshmen in intermediate algebra. Bloland (1984) continued the trend by showing that age was the best predictor for ninth and tenth grade secondary students. However, it should be pointed out that in those studies where a nonmathematical variable was the best predictor, some standardized test of mathematical ability usually was also found to be a significant predictor. Indeed, two studies (Siglin, 1978; Peteet 1978) indicated that some mathematically related ability may be the best predictor. The first study concluded that performance in eighth grade mathematics was the number one predictor of success in Algebra I in high school. The second found that the Stanford Test of Academic Skills was the best predictor in College Algebra for community college students. The importance of the above studies is that even though most included some form of arithmetic ability, none of them found a significant positive relationship between arithmetic and algebra. That is, these studies are relevant not because of what they found, but because of what they did not find.

Two studies offer indirect evidence that arithmetic and algebra are not related. Gray (1976) found that success in the high school courses General Mathematics I, General Mathematics II, and Business Mathematics had no significant relationship to success in college algebra. Clark (1982) confirmed this research when he found that grades earned in high school mathematics classes had no bearing on ability to succeed in beginning algebra at a community college. If it is assumed that terms such as "general mathematics" and "business mathematics" are other names for computational arithmetic and its applications, then it is reasonable to conclude, based on this research, that ability to do arithmetic has little to do with ability to do algebra.

Unlike the previous studies, two reports deal specifically with the relationship between ability to succeed in arithmetic and ability to succeed in higher mathematics. Mars (1970) compared arithmetic achievement with geometry achievement in the high school. His correlations showed that ability in arithmetic made no significant contribution to predicting ability in geometry. In another report on secondary students, Malinen (1971) concluded that the importance of numerical ability in predicting ability in algebra was insignificant.

Perhaps these findings were best summarized in a report by Begle (1976). After reviewing 17 studies which attempted to predict success in algebra, he asserted that being able to understand and use mathematical concepts far outweighed being able to carry out arithmetic computation as a predictor of success in algebra. It would seem that there is a great deal of evidence in the literature which indicates that there may not be a relationship between arithmetic and algebra, once what to look for is known.

Sometimes in research a by-product result is obtained which proves to be more interesting than the original study. In 1980, I conducted an evaluation of a three-year grant received from the National Science Foundation. One of the objectives of the evaluation was to determine the effectiveness of computer managed instruction in beginning algebra. Three modes of instruction had been used in the project. They were the following: 1. computer managed instruction combined with individualized instruction in a math lab, 2. computer managed instruction used in the classroom, and 3. a traditional class as control group with no computer managed instruction. The final exam scores of these groups were compared using both analysis of variance and analysis of covariance.

As most students of mathematics do, I had always viewed algebra as a generalization of arithmetic. Therefore, I used the scores obtained from a pretest in arithmetic as the covariate in conducting the above analysis of covariance on final exam scores in beginning algebra. The results were that there was no significant difference between the treatments. That is, none of the three ways of teaching seemed to be superior to the others. However, in the analysis, the probabilities of the insignificant F ratios being due to chance seemed to be disproportionally high. This can be seen in Table 1 in the results section below. The question was: "Why were the probabilites of the insignificant F ratios so high?". If ability in arithmetic were closely related to ability in algebra, shouldn't the probabilities of the F ratios be closer to 0.05? Could it be that there is little or no relationship between ability to succeed in arithmetic and ability to succeed in algebra? In order to answer this question, I designed a follow up study which was conducted on the same data as the original study.

Method

Subjects

Sixty-two beginning algebra students from the urban campus of a large metropolitan community college participated in the study. There were twenty-two males and forty females. The ethnic distribution was the following: forty-six Hispanic, five white non-Hispanic, ten black non-Hispanic, and one Other. The subjects chosen for the study were those who completed both the pretest and posttest involved in the study. These students came from the daytime classes offered in beginning algebra at this campus. The classes chosen for the study accounted for 80% of the daytime sections offered at this campus. This sample of convenience was used because of the realities of the situation.

Materials and Design

A teacher-made test which had been developed by the department was administered as a pretest at the beginning of the semester. This pretest consisted of ten selected items designed to indicate minimum competency in computational arithmetic. The course final exam was used as a posttest. It consisted of forty items designed to test computational ability in beginning algebra. The same final was given to all beginning algebra classes.

Results

As mentioned in the introduction, an Analysis of Covariance, using the pretest as covariate, showed no significant difference between the three treatments CMI math lab, CMI classroom, and control group. Table 1 shows that the probability that the F ratio for the covariate (arithmetic pretest) was due to chance was 0.415. Also,

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Insert Table 1 about here

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the probability that the F ratio for the treatments was due to chance was 0.866. As noted earlier, these probabilities are suspiciously greater than the typical rejection value of 0.05. Table 2 shows that the Multiple Classification Analysis yielded a multiple correlation coefficient of 0.128

Insert Table 2 about here

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and a multiple correlation coefficient squared of 0.016. Since these were indirect measures of the relationship between arithmetic score and algebra score, a Pearson product-moment correlation was also computed. Table 3 reveals that the correlation between pretest and posttest was very, very weak, $r = 0.1069$, $p = 0.204$.

Insert Table 3 about here

Discussion and Implications

Discussion

Alone, a simple correlation has to be viewed with skepticism. Therefore, it is important to carefully examine the meaning of the multiple correlation coefficient in Table 2. This multiple R includes not only the effects of the treatment or independent variable, but the covariate as well (Nie, Hull, Jenkins, Steinbrenner, Bent, 1975, p. 418). In effect, this multiple R acts as a limit for the size of the individual correlations between independent variables and the dependent variable and also between the covariates and the dependent variable. Hence, a multiple R of 0.128 indicates that the correlation between arithmetic scores and algebra scores can be no larger than 0.128. Furthermore, the multiple R squared indicates that the covariate can account for no more than 1.6% of the variance. Therefore, the results of the Multiple Classication Analysis support the contention that the Pearson r value (0.1069) indicates that the two variables are independent or nearly so.

When one considers all of the evidence presented in this research, the review of the literature, the results of the Analysis of Variance, the results of the Multiple Classification Analysis, and the results of the Pearson r calculation, only one conclusion can be drawn. The ability to perform well in computational arithmetic has little, if anything, to do with the ability to perform well in computational algebra, that is, beginning algebra.

Implications

The implications for the curriculum of the two-year college are great. It is not necessary to require that students master arithmetic before taking algebra. The only reason for requiring arithmetic is to ensure that the student knows arithmetic for its own sake not as a prerequisite for algebra.

Most mathematics instructors will agree that to drop arithmetic from the curriculum would be unwise. Therefore, I propose that the topics of arithmetic and beginning algebra be combined and taught in an integrated course using a spiral approach. Some beginning algebra texts offer topics from algebra mixed with topics from arithmetic, but I have found none which weave the ideas together and use one discipline to reinforce concepts in the other.

For the two-year college, I propose a course which would meet five times a week, once each day. This course would start with integers and from these develop signed fractions. Thus, a signed arithmetic can be developed. For example, algebraic fractions can be developed from "arithmetic fractions".

The advantages of such a course are many. First, algebra can actually be taught as a generalization of arithmetic by the instructor guiding the necessary transfer of relationships from one subject to the other. Rather than parallel development of two subjects, there is one integrated discipline. Another, perhaps more important advantage is a psychological one. All students would rather study algebra than be relegated to repeating arithmetic. A course called "Integrated Algebra and Arithmetic" certainly has more attraction than "Basic Mathematics".

For the secondary school, in particular, the transition years, I propose that a course integrating arithmetic, pre-algebra topics, and algebraic topics be offered. This course would again be taught in a spiral approach with the title "Integrated Arithmetic and Algebra". Such a course would allow almost all eighth-graders to study "algebra" while reviewing topics in arithmetic from another point of view.

Summary

The entire mathematics curriculum design of the United States is based on the assumption that mastery of arithmetic is necessary for the learning of algebra. This study presents evidence which indicates that this presumed relationship does not exist. Of course, this study should be replicated with a randomly chosen sample. Other studies should be designed and executed to test this hypothesis. However, if the results of the above research holds true, then we should rethink how and when we teach algebra and arithmetic. Everybody can benefit from a restructuring of the curriculum, but our students can be the biggest winners.

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Author Notes

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Reprints of this paper may be obtained from the author by writing to the following address: William P. Palow, Department of Natural Science, Wolfson Campus, Miami-Dade Community College, 300 N.E. 2nd Avenue, Miami, Florida, 33132.

Table 1

Analysis of Covariance Table for Algebra Scores by Treatment with Arithmetic Scores

Source of Variance	Sum of Squares	DF	Mean Square	F	Probability
Arithmetic Score	200.656	1	200.656	0.674	0.415
Treatment	85.668	2	42.834	0.144	0.866
Explained	286.324	3	95.441	0.321	0.810
Residual	17259.770	58	297.582		
Total	17546.094	61	287.641		

Table 2

Multiple Classification Analysis Table for Algebra Scores by Treatment with Arithmetic Scores

Grand Mean = 59.35

Category	N	Unadjusted Deviation	Eta	Adjusted for Independents & Covariates Deviation	Beta
CMI Lab	5	2.45		2.28	
CMI Class	26	0.80		0.90	
Control	31	-1.06		-1.12	
			0.07		0.07

Multiple R squared	0.016
Multiple R	0.128

Table 3

Pearson Correlation Coefficient Table for Arithmetic vs Algebra Scores by Treatment

Treatment	Variable	N	Mean	r
CMI Lab	Arith	5	7.00	
	Alg	5	61.80	
				0.2676
CMI Class	Arith	26	6.69	
	Alg	26	60.15	
				0.0340
Control	Arith	31	6.87	
	Alg	31	58.29	
				0.1544
All	Arith	62	6.81	
	Alg	62	59.35	
				0.1069*

*p = 0.204

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