DOCUMENT RESUME

ED 415 940		JC 980 103
AUTHOR	DeMarois, Philip	
TITLE	Function as a Core Concept in Developmental Mathematics: A Research Report.	
PUB DATE	1997-11-15	
NOTE	10p.	
PUB TYPE	Reports - Research (143)	
EDRS PRICE	MF01/PC01 Plus Postage.	
DESCRIPTORS	Algebra; College Mathematics; Community Colleges; Computer Uses in Education; Concept Formation; Concept Teaching; Course Evaluation; *Developmental Studies Programs; Educational Assessment; *Functions (Mathematics); *High Risk Students; Mathematical Aptitude; *Mathematical Concepts; Outcomes of Education; Remedial Mathematics; Student Characteristics; Two Year College Students; Two Year Colleges	

ABSTRACT

Due to the large number of students requiring developmental college math courses, a study was conducted to determine if a beginning algebra course focusing on function and integrating technology as a tool to explore mathematics would aid students with previously debilitating experiences in math. The study evaluated 92 students enrolled in "pilot" sections of beginning algebra at 4 community colleges. Some students participated in interviews, and all completed written function surveys at the beginning and end of the course. Students' proficiency levels were measured for colloquial, symbolic, numeric, geometric, written, and notation facets for functions. Data analysis concluded that: the function concept is accessible to the developmental student; function machines are a reasonable entry point; students remained weak on the geometric facet; function notation was interpreted inconsistently; use of prototypes with the symbolic facet was common; constant functions caused confusion; requirement for exactly one output was applied inconsistently; it was very difficult to neutralize the effects of prior learning; and connecting facets proved difficult. Curriculum reforms should include more attention to the geometric facet and integration of facets, discussion of function as an object, a focus on best uses of each facet, and no graphing calculators. (YKH)

Function as a Core Concept in Developmental Mathematics: A Research Report

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FUNCTION AS A CORE CONCEPT IN DEVELOPMENTAL MATHEMATICS: A RESEARCH REPORT

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Introduction

U.S. college mathematics faculty encounter a sizable percentage of students who begin their college career in a non-credit mathematics course such as arithmetic, geometry, beginning algebra, or intermediate algebra. It is likely that many developmental algebra students have been severely debilitated by their previous exposure to mathematics. Succeeding with this population may require providing the students with a completely different educational experience, such as building a beginning algebra course around the function concept.

If college students are exposed to a beginning algebra course that focuses on function and integrates technology as a tool to explore mathematics, what will the outcome be? This research began as an effort to determine if the function concept was accessible to such students. The results suggest that students can acquire an understanding of function, but that there are pitfalls to beware of and that the understanding is often uneven.

Purpose of the Research

According to a CBMS (Loftsgaarden et al., 1997) survey in Fall, 1995, 53 percent of the nearly 1.5 million of community college students taking mathematics are enrolled in a developmental mathematics course. Algebra is not new to such students. The mean number of years of algebra prior to their enrollment in beginning algebra was 1.5 years for the students in this research study. With this background, none of the students could provide even a moderately acceptable definition for words such as variable, equation, or graph at the beginning of the course.

These students have been severely debilitated by their previous exposure to mathematics. Jim Kaput (1995) sums up algebra education in the United States when he writes:

> School algebra in the U. S. is institutionalized as two or more highly redundant courses, isolated from other subject matter, introduced abruptly to post-pubescent students and often repeated at great cost as remedial mathematics at the post secondary level. Their content has evolved historically into the manipulation of strings of alphanumeric characters guided by various syntactical principles and conventions, occasionally interrupted by "applications" in the form of short problems presented in brief chunks of highly stylized text. All these are carefully organized into small categories of very similar activities that are rehearsed by category before introduction of the next category, when the process is repeated. The net effect is a tragic alienation from mathematics for those who survive this filter and an even more tragic loss of life-opportunity for those who don't. p. 71

Kathryn Crawford suggests that school priorities are in conflict with work priorities when she writes: "The cognitive demands of learning mathematics at school and undergraduate level remain firmly focussed on capabilities to demonstrate operational knowledge of approved procedures and axioms....In the community the set procedures are the work of machines" (1997, p. 81). Working with students that have serious misconceptions about algebra may require a radically different course from the one they previously encountered. My two colleagues, Mercedes McGowen and Darlene Whitkanack, and I set out to create such an alternative curriculum in 1991. The result is a textbook (DeMarois, McGowen, & Whitkanack, 1998) that assumes access to powerful technology and that uses function as the organizing concept.

This research focuses on the understanding of function that students acquire as a result of completing this technology-rich, "reform" beginning algebra curriculum. Can adult students who arrive at college having had debilitating prior experiences with algebra develop a rich concept image of "function" through appropriate instructional treatment?

Theoretical framework

The theoretical framework was initially described by DeMarois & Tall (1996) who suggest a structure for analyzing mathematical concepts along both breadth and depth dimensions. Schwingendorf et al. (1992) contrast the vertical development of function in which the process aspect is encapsulated as a function concept and the horizontal development relating different representations. They refer to these as depth and breadth respectively and investigate the way in which the student's concept image (Tall & Vinner, 1981) of function can be described in terms of these two dimensions.

The breadth dimension, with each dimension referred to as a facet (DeMarois & Tall, 1996), is conceived as consisting of various representations, including geometric, numeric, and symbolic. The facets of a mathematical entity refer to various ways of thinking about it and communicating to others, including verbal (spoken), written, kinesthetic (enactive), colloquial (informal or idiomatic), notational, numeric, symbolic, and geometric (visual) aspects. These are not intended to be independent or exhaustive, but provide a suitably broad framework to begin an analysis of the function concept.

DeMarois & Tall (1996) use the term layer to refer to various levels of the depth dimension. The pre-procedure layer assumes that the student is on the ground floor, so to speak, with respect to a concept. The procedure layer is indicated by the need for a specific algorithm while the process layer is not dependent on individual steps, but rather on the result produced from the original input. For example, the expressions $2x + 6$ and $2(x + 3)$ represent two different procedures. The results of applying each procedure to a given input are the same. Students who view these as different functions might be classified at the procedure layer while those who classify these as the same function might be placed at the process layer. The concept layer aligns closely with the ability to treat the mathematical idea as an object to which a procedure can be applied. After the concept layer, a procept layer is placed, to indicate the flexibility to move easily between process and object layers as required. Gray & Tall (1994) refer to a procept as an amalgam of three components: a process, an object, and a symbol that represents either the process or the object. Students reach the most depth (the procept layer) when they can demonstrate flexibility in viewing a facet of a function as either a process or an object, as required by the problem situation.

Method

The study was conducted on 92 students enrolled in "pilot" sections of beginning algebra at 4 different community colleges. The students completed written function surveys at the beginning (first day) and at the end (last day) of the course during the Fall Term, 1996. Subsequently, three students at each site participated in task-based interviews that were conducted one to two weeks after the end of the course. The interviews were video- and audio-taped. All questions on the pre-course survey were repeated on the post-course survey and on the interviews. Additional questions on the post-course surveys were also asked during the interviews.

The common questions on the pre- and post-course surveys were analyzed quantitatively by measuring the significance of the changes in responses from beginning to end of the instructional treatment. The data collected during the interviews along with the written surveys were analyzed qualitatively to create before and after snapshots of the depth of student understanding of function. Due to space confines, partial quantitative results on colloquial, symbolic, numeric, geometric, written, and notation facets are presented here.

Colloquial, symbolic, numeric, and geometric facets

Students were asked to find the output given the input (part a) and the input given the output (part b) thus assessing students' ability to apply a procedure (procedure layer) and reverse a procedure (process layer) for the colloquial (function machine), symbolic (equation in two variables), numeric (table), and geometric (graph) facets.

Question 1 involves a linear function expressed as a function machine. Figure 1 suggests that almost two-thirds of the students were able to find output and about half were able to find an input at the beginning of the course. This suggests that the function machine might be a natural vehicle for introducing function since students appear to have an inherent "feel" for what it is expressing. Note by the end of the course there was a significant improvement in the scores on both parts.

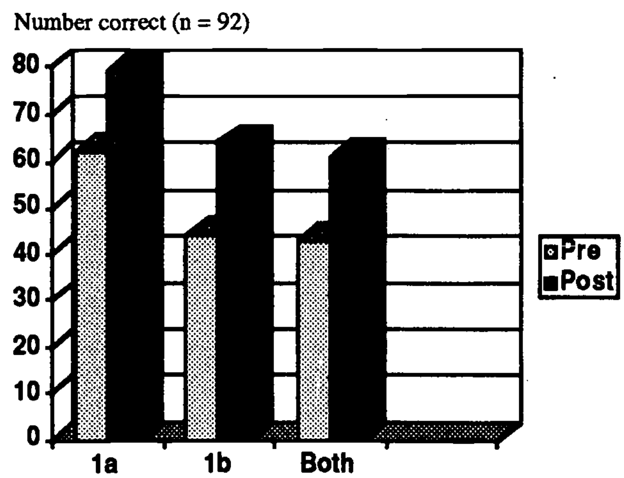

figure 1: Colloquial facet

Question 2 involves a linear function expressed as an equation in two variables. Figure 2 shows that more than two-thirds of the students were able to find the output at the beginning of the course, but very few were able to find the input which requires solving a linear equation. While there was significant improvement at the end of the course, the results were still disappointing.

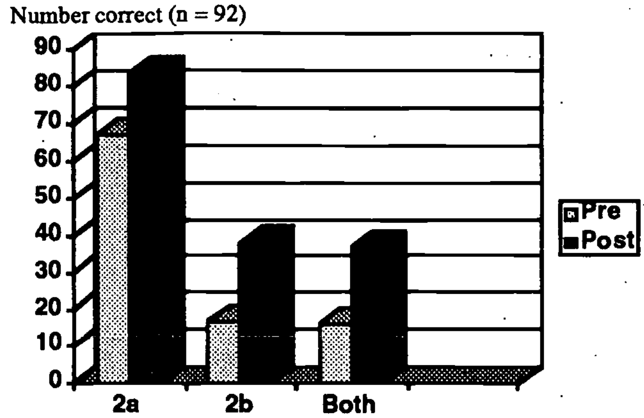

figure 2: Symbolic facet

Question 3 involves a quadratic function expressed as an input/output table. When finding the input for a given output, there were two answers. Thus figure 3 displays the results for part b indicating the number of students who gave only one answer as compared to the number of students who identified both answers. Students do quite well on this facet at the beginning of the course. However, while there was significant improvement from pre- to post-, the number of students who identify both answers to part b remains quite low.

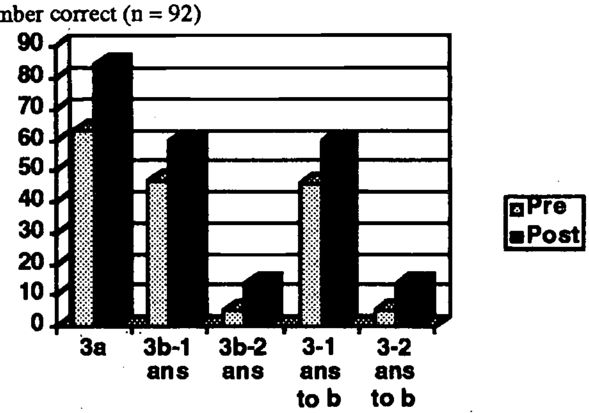

figure 3: Numeric facet

Question 4 involves a quadratic function expressed as a rectangular coordinate graph. Again, for part b, there are two inputs for the given output. Figure 4 shows that this facet causes the students much difficulty. Students' ability to answer this question improved markedly at the end of the semester, but the overall results remain low. Less than 25 percent of the students were able to answer the entire question correctly at the end of the semester. The hope was that the extensive use of technology would improve student understanding of graphs, but significant obstacles still appear to be present.

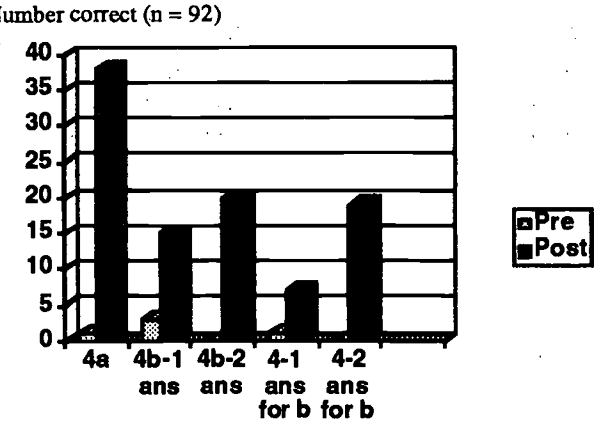

figure 4: Geometric facet

Written facet: function definition

Changes in the written facet were noted from pre- to post-course by asking students to respond to the question "What is a function?" These responses were divided into four main categories:

- blank: student did not respond to the questions
- pre-procedure: student displays little or no knowledge of the mathematical definition of function. For example, one student on the pre-course survey wrote: "an ability that something or someone is able to complete".
- procedure: student displays a procedural knowledge of the mathematical definition of function. For example, one student on the post-course survey wrote "an operation or a rule". Essentially, students who placed the emphasis on specific operations in their definition were classified at this layer.
- process: student displays a process-oriented knowledge of the mathematical definition of function. For example, one student on the post-course survey wrote: "a process that receives input and produces output". The process category could be further subdivided based on where the student placed the emphasis in the definition. There were 3 common subcategories: process; relationship; and, input-output. A process emphasis is reflected by the student who wrote: "the process that receives input and produces a unique output." A relationship emphasis is indicated by the statement: "a relationship between two quantities that change." Included in this category are those who emphasized the idea of a dependency between two variables. An input-output emphasis is sugested by the quote: "a list of inputs and outputs."

Figure 5 displays the number of students responding in each category. While only 2 percent of students responded at the process layer on the pre-course survey, 52 percent responded at the process layer on the post-course survey. Eighty-eight percent of the students indicated no knowledge of the written definition of function of the pre-course survey while sixty-two percent were at least at the procedure layer on the post-course survey.

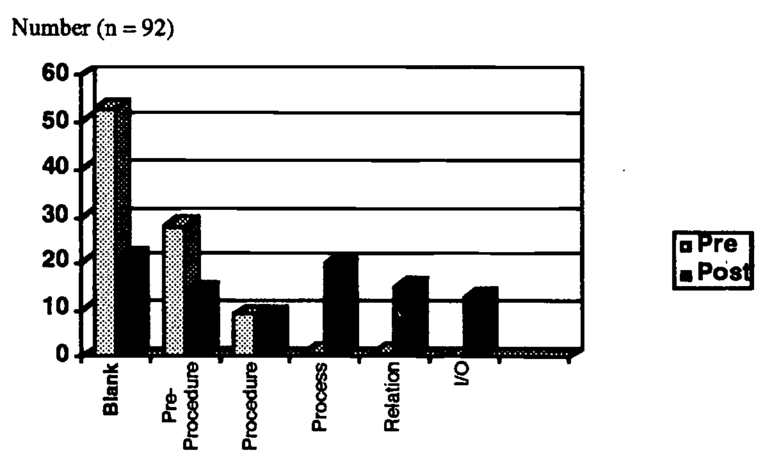

figure 5: Function definition

More importantly, how does individual student's ability to define "function" change as a result of the instructional treatment? Forty-three percent of the students evolved from either blank or pre-procedure to process. Only 8 students regressed and, of these, 7 regressed to blank on the post-course survey which may indicate that they just didn't take the time to answer the question.

Notation facet: function notation

Changes in the notation facet from pre- to post-course survey were measured using the following question on both surveys:

Briefly state what $f(x)$, $y(x) = 4$, and $a(b + c)$ mean to you.

Figure 6 displays the categorized responses for $f(x)$ at the beginning and at the end of the course. While 75 percent of the students interpreted the notation as multiplication on the pre-course survey, 62 percent interpreted the notation as function notation by the end of the course. A Chi-Square test indicated a significant shift toward a "function" interpretation of $f(x)$ from pre- to post-course survey ($\chi^2 = 87.7$, d. f. = 3, $p < 0.001$).

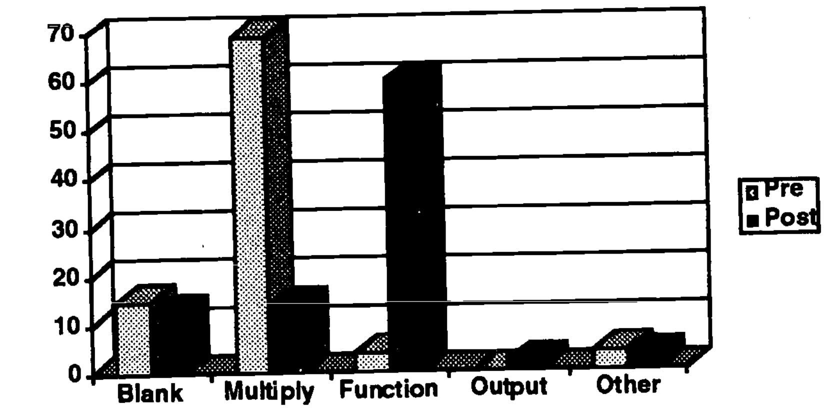

figure 6: Meaning of $f(x)$

Figure 7 displays the categorized responses for $y(x) = 4$ at the beginning and at the end of the course. Seventy-six percent of the students interpreted the notation as multiplication on the pre-course survey, including several who insisted that both x and y must be 2 in order for the product to be 4. Approximately 45 percent interpreted the notation correctly as either y of x equal to 4 or the constant function $y = 4$. A Chi-Square test indicated a significant shift toward a "function" interpretation of $y(x) = 4$ from pre- to post-course survey ($\chi^2 = 79.5$, d. f. = 4, $p < 0.001$).

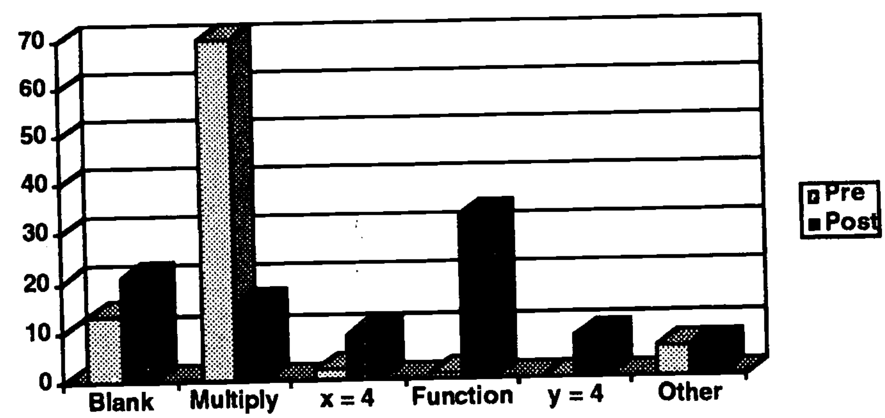

figure 7: Meaning of $y(x) = 4$

Finally, when asked about $a(b + c)$, no student interpreted the symbolism as function notation on either the pre- or post-course survey. The apparent familiarity of the symbolism eliminated any possible cognitive link to function notation.

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Conclusions

The above provides a piece of the quantitative data collected. In addition, a substantial amount of qualitative data was collected from the interviews. Some conclusions suggested by analyzing all data include:

- The function concept is accessible to this level student.
- Function machines are a reasonable entry point, but the understanding of this facet remains primitive and connections to other facets remain weak.
- Students remained disappointingly weak on the geometric facet.
- Function notation was interpreted inconsistently, even by the most capable.
- Use of prototypes with the symbolic facet was more common than with the geometric facet.
- Constant functions caused confusion and were interpreted inconsistently across facets.
- Requirement for exactly one output, given an input, was applied inconsistently.
- It appears more difficult than expected to neutralize the interferences of prior learning.
- Students are often good at "plug and chug" mathematics and use this ability to hide weaknesses in understanding.
- Connecting facets proved difficult, as expected.

Informing the curriculum

A primary purpose of the research is to help shape "reform" curricula in beginning algebra. The data analysis suggests that the curriculum used in this study could be revised to reflect the following:

- More attention must be paid to geometric facet.
- Increased attention related to moving between facets is necessary.
- Discussion of function as an object, in addition to a process, may deepen some students understanding.
- Focus on the best uses of each facet should be included.
- Graphing calculators introduce interferences that must be better addressed.

Reflection

The student population for this research is a high-risk group who have had little prior success with mathematics. Using "function" as a focal point of their beginning algebra course, the authors hope to provide students with a vehicle to build meaning into their work with algebra. While some common misunderstandings about function appear in the data, the in-depth analysis suggests that "function" is not beyond the conceptual grasp of students at this level. Continued research can help shape future "reform" curricula so that, eventually, developmental algebra courses can be not only brought more closely in line with various "standards" recommendations, but that student understanding in such courses has been carefully assessed to assure that the changes are indeed contributing to the students' intellectual and mathematical growth.

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