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ABSTRACT

The set of papers collected in this anthology were developed from presentations given at the 16th meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA), November 5-8, 1994, in Baton Rouge, Louisiana. These papers provide conceptual explorations of dimensions of teacher change such as the nature of teachers' mathematical knowledge and its role in informing their instructional options; the role that curricular and other material resources can play in the process of teacher change; and the effect of a new culture for teaching that legitimatizes curiosity, reflection, and critical colleagueship. Chapters contained in this report include: (1) "Learning Mathematics while Teaching" (Susan Jo Russell, Deborah Schifter, Virginia Bastable, Lisa Yaffee, Jill B. Lester, Sophia Cohen); (2) "Teachers' Changing Conceptions of the Nature of Mathematics: Enactment in the Classroom" (Deborah Schifter); (3) "Affective Issues in Developing Mathematics Teaching Practice" (Lynn T. Goldsmith and Linda Ruiz Davenport); (4) "Transforming Mathematics Teaching in Grades K-8: How Narrative Structures in Resource Materials Help Support Teacher Change" (Linda Ruiz Davenport and Annette Sassi); and (5) "Teacher Inquiry Groups: Collaborative Explorations of Changing Practice" (James K. Hammerman). (ASK)

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PAPER SERIES

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Edited by
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Inquiry and the Development of Teaching

Issues in the Transformation of Mathematics Teaching

Inquiry and the Development of Teaching: Issues in the Transformation of Mathematics Teaching

Chapter I: Introduction

Barbara Scott Nelson

Six years after the publication of the National Council of Teachers of Mathematics' (NCTM's) *Curriculum and Evaluation Standards for School Mathematics*, which set the course for a new era of mathematics education reform, professional development for mathematics teachers has moved to the center of the reform agenda. The argument has been successfully made that new curricula and educational policies alone will not adequately power the reforms (Cohen, 1990; Little, 1993; Lord, 1994). Rather, they depend on the transformation of teaching in the nation's many classrooms. Many teachers have embarked on the project of changing their teaching toward that envisioned in the *Standards*. Their work leads us to the following questions: Where are we in our understanding of the nature of this process? How can we help teachers in their efforts to invent a new form of teaching? and How can we continue to learn about what such invention entails?

The current mathematics education reform agenda requires not only that many teachers master new technical skills but, more importantly, that they reconceptualize and reinvent the overall nature of their teaching practice. Teaching needs to expand from a technical craft in which teachers arrange activities that lead students to having the "right" concepts, to a more broadly human endeavor that encompasses inquiry into the nature of mathematical thinking, learning, and

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teaching. Professional development, in turn, needs to provide teachers with not just skills training alone, but with opportunities to work with colleagues on the complex intellectual, emotional, and practical tasks of developing new pedagogical beliefs, knowledge, and practices (Little, 1993).

The movement toward conceiving of teaching as a significantly intellectual rather than a largely technical enterprise coincides with the emergence into the field of education of a set of ideas about the nature of knowledge itself—ideas that have long been percolating in the academic fields that buttress education. The epistemological position inherent in the NCTM *Standards* documents is a socioconstructivist one. Knowledge is considered to be the dynamic and conditional product of individuals working in intellectual communities, not a fixed body of immutable facts and procedures. For most teachers, developing a practice based on such a view of the nature of knowledge, learning, and teaching will not be accomplished merely by adding new techniques to their current repertoire. It will require a set of epistemological shifts—changing their beliefs about the nature of knowledge and learning, deepening their knowledge of mathematics, and reinventing their classroom practice from within the new conceptual framework.

Within the mathematics education community are several research programs that explore and offer theoretical explanations for the nature of the transformation that happens when teachers change beliefs, deepen their knowledge, and reinvent their practice. Carpenter and his colleagues (Carpenter et al., 1988; Fennema et al., in press; Peterson et al., 1989) suggest that teacher change is a matter of acquiring and using new knowledge about the evolution of children's mathematical thought and developing enriched and reorganized conceptual structures (Franke, 1991). In the Cognitively Guided Instruction (CGI) project, Carpenter et al. have been helping teachers build conceptual links between a research-based model of the development of children's mathematical thought and their own teaching practice by encouraging reflection about how the model could be interpreted in light of their own students and classrooms (Fennema et al., in press). For many participating CGI teachers, the process of focusing on their students' mathematical thinking in light of the new framework—and thinking hard about the mathematics problems that will stimulate those children to move further in their mathematical thinking—generates the context for integrating the research-based knowledge into their view of children's learning and moving their instructional practice forward.

Cobb and his colleagues posit that as teachers and their students renegotiate the norms of the classroom to legitimate students' construction of mathematical concepts and discussion of mathematical ideas, teachers encounter and resolve conflicts between their prior beliefs about learning and what they observe happening in their classrooms (Wood et al., 1991). This group worked with elementary teachers who, for the first time, were using mathematics curriculum materials that provided a problem-centered approach to learning mathematics and encouraged students to discuss mathematical ideas and construct their own knowledge of mathematics. They found that, throughout the year, the teachers encountered and resolved a series of conflicts between their prior beliefs and what they observed happening in their classrooms as they and their students renegotiated the norms of the classroom to include a valuing of students' construction of mathematical concepts and extended debate about mathematical ideas. It was the opportunity to encounter conflict in their own classrooms—coupled with the opportunity for reflection and resolution that was provided by the research team—that supported teacher change.

Schifter and her colleagues (Schifter, in press a & b; Schifter & Fosnot, 1993; Schifter & Simon, 1992) argue that change in teachers' ideas about the nature of learning requires a process of disequilibration of prior ideas and the reconstruction of more powerful ones. In this view, efforts to help teachers reconsider their ideas about the nature of learning require creating activities and events that stimulate cognitive reorganization on the part of participating teachers. One type of such activity involves teachers becoming mathematics students in lessons taught by project staff. Another involves examination and analysis of student thinking through videotape, live interviews, and transcriptions of dialogue from one's own and others' classrooms. In programs designed in accord with these principles, over the course of a year or more of seminars and classroom experimentation, teachers find their previous ideas challenged by what they see children do, and by what they and their colleagues do. They often then begin to construct more adequate explanations for what they see; develop new ideas about learning, teaching, and mathematics; and transform their practice in accordance with these new notions.

These bodies of work take teachers' own conceptual change as requisite and focus on the evolution of teachers' thinking about mathematics, children's learning, and the nature of teaching in relation to changes in their instructional practice. They focus on slightly different aspects of the phenomenon of teacher change—Carpenter's group on the nature of teachers' knowledge about children's mathematical thinking, Cobb's group on the process of social negotiation as the context for change, Schifter and her colleagues on the mechanism of conceptual change itself. These orientations are not mutually incompatible and, in fact, much could be learned by using each program's lens to look at the others' work.

Within this overall theoretical framework, if our understanding of the process by which teachers change their thinking and their practice of mathematics teaching is to be sufficiently articulated to guide the practice of professional development on a broad scale, we need further analysis and study of several fine-grained issues. Among these are the relationship between change in beliefs on the part of teachers and change in their instructional practice; elucidation of the several pathways through the conceptual change territory that teachers might take; analysis of the characteristics of teachers who move readily onto these pathways as distinguished from those who do not; the nature of facilitative teacher education; the characteristics of environments that support such learning on the part of teachers; the duration of the process; and so on. We also need to push beyond the psychological constructs currently being used to interpret and analyze teacher change and look to social and cultural constructs as well. Work is underway on a number of these issues (Ball, 1991; Barnett, 1991; Fennema et al., 1993; Franke et al., 1995; Russell & Corwin, 1991; Sassi & Goldsmith, in press; Schifter, in press a; Schifter, in press b; Schifter, 1993). The papers in this anthology contribute to this work by offering conceptual analyses of several of these features of teacher change as they emerge in programs designed to provide teachers with the opportunity to deepen their mathematics knowledge and critically examine the adequacy of their beliefs about mathematics, learning, and teaching.

These papers and the Epilogue are, themselves, positioned within a sociocultural framework like that argued in the NCTM *Standards*. This volume's very structure—papers and an Epilogue in which the authors and others discuss issues raised by the papers—argues the case for emergent, situated, mutable, and context-dependent knowledge. In the papers we present thinking in progress and invite colleagues, nationally, to comment. The Epilogue raises questions about particular points made in the papers, questions how best to represent our current knowledge of teacher change, and argues that the sociocultural perspective raises issues of consistency within the larger research and practitioner communities.

The Papers

The set of papers collected in this anthology were developed from presentations given at the sixteenth meeting of PME-NA,[1] November 5–8, 1994, in Baton Rouge, Louisiana. They are early reports of work conducted at the Center for the Development of Teaching at Education Development Center, Inc. (EDC) in several teacher education and research projects. Staff in each of these projects are working with teachers to help them examine their fundamental beliefs about learning, teaching, and the nature of mathematics; deepen their mathematics knowledge; and reconstruct their teaching practice from within a new conceptual framework. Specifically, all projects provide teachers with activities and settings that challenge the explanatory power of their prior beliefs and require the reorganization of their thinking about learning, teaching, and mathematics. Looking through new conceptual lenses, teachers see new aspects of their classrooms and can make space for new things to happen—challenging their interpretive lenses once again. And so the iterative process of change in beliefs and knowledge, and change in practice, proceeds.

All projects work with elementary school teachers on their mathematics teaching. Some teachers are at early stages of reflection on their teaching, others have been in similar projects before and are quite sophisticated in their views about students' mathematical thinking and are well underway in developing facilitative teaching practices. Each project also serves as a research site in which a set of questions about teacher change is being investigated.

The papers in this volume provide conceptual explorations of dimensions of teacher change—the nature of teachers' mathematical knowledge and its role in informing their instructional options; the role of affect in the process of teacher change; the role that curricular and other material resources can play in the process of teacher change; and the effect of a new culture for teaching that legitimates curiosity, reflection, and "critical colleagueship" (Lord, 1994). The papers reflect work in progress. In them, we see scholars at work—wrestling with constructs and data in a largely unmarked terrain.

The papers are as follows:

• Chapter II: Russell et al., in "Learning Mathematics While Teaching," address the issue of teachers' acquisition of the deep mathematical knowledge that can support teaching facilitative of student thought; they propose teachers' own classrooms as a site for their mathematics learning. Russell and her colleagues describe teachers who are learning mathematics while they teach it and consider what teachers must already understand in order to learn in this way. The authors propose three classroom contexts in which teachers learn mathematics: 1) exploring a mathematical problem or question embedded in the content they are teaching; 2) thinking through students' representations and strategies; and 3) looking underneath students' confusions or excitement to consider larger mathematical structures. They discuss what teachers need to know and know how to do in order to learn mathematics from each of these contexts.

• Chapter III: Schifter, in "Teachers' Changing Conceptions of the Nature of Mathematics," describes four conceptions of mathematics as enacted by teachers in their classrooms: 1) mathematics as an ad hoc accumulation of facts, definitions, and computational routines; 2) mathematics as student-centered activity, but with little or no systematic inquiry into issues of mathematical structure and validity; 3) mathematics as student-centered activity directed toward systemic inquiry into issues of mathematical structure and validity; and 4) systemic mathematical inquiry organized around investigation of "big" mathematical ideas. Schifter describes teaching that illustrates each of these orientations toward mathematics, asks if they might be four stages of a developmental trajectory, and raises theoretical questions related to the scheme's adequacy as the beginnings of a developmental theory.

• Chapter IV: Goldsmith and Davenport, in "Affective Issues in Developing Mathematics Teaching Practice," report on an exploration of teachers' affect in the process of changing their knowledge, beliefs, and practice. Arguing that understanding the affective side of change has both practical and theoretical implications, they suggest and give examples of three functional ways of thinking about the role of emotions in teacher change: 1) as motivators for development; 2) as indices of areas of practice ripe for change; and 3) as sources of decisionmaking in the classroom. Goldsmith and Davenport also raise methodological issues about the study of emotion.

• Chapter V: Davenport and Sassi, in "Transforming Mathematics Teaching in Grades K–8: How Narrative Structures in Resource Materials Help Support Teacher Change," report on a small empirical study that examines the role of material resources in helping teachers make fundamental changes in the way they think about mathematics, mathematics learning, and mathematics teaching. Participating teachers were newcomers to the project and the authors found that the resources they reported as significant—regardless of whether they were curricular materials, articles, or videotapes—contained stories from classrooms that conveyed images of how teachers and students might work together. Davenport and Sassi consider the relationship between this need for images and the "storied" forms of knowing characteristic of practitioner knowledge.

• Chapter VI: Hammerman, in "Teacher Inquiry Groups: Collaborative Explorations of Changing Practice," describes issues encountered in the development of a discourse community in which teachers meet regularly to critically examine their own and each others' mathematical and pedagogical knowledge, beliefs, and practice as they work to change their teaching. Hammerman gives examples of change in teachers' thinking and practice, describes the development of group norms and expectations, and analyzes the fundamental characteristics of this collegial form of professional development.

A number of issues embedded in this set of papers have larger significance since they characterize important features of the field as a whole. Most of the papers raise theoretical issues. In Chapters II and III, Russell et al. and Schifter puzzle about the nature of teachers' mathematical knowledge: What does it take to be a self-propelled mathematics learner? And what are the characteristics of teachers' mathematical knowledge at different points along the way? If one views knowledge as a complex web of ideas in which new connections get made and concepts get reconfigured, in interaction with experience, how is a teacher's mathematical knowledge to be characterized at any particular point? Might a teacher have very sophisticated ideas about the nature of a place value number system and very algorithmic ideas about geometry, depending on the nature of the opportunities he or she has had to think deeply in various mathematical domains? Is a stage theory, which implies homogeneity of any particular individual's knowledge, really appropriate? And do the qualitative differences observed in teachers' mathematical knowledge imply that changes in epistemology and in views of mathematics are interdependent? What would be the characteristics of a theoretical description of the development of teachers' mathematical knowledge?

Goldsmith and Davenport (Chapter IV) have theoretical questions about the relationship between affect and cognition. They propose to look at the interplay of thought, feeling, and action over long periods of time in order to consider how they, together, promote development. There also are fascinating problems of methodology in the work on affect: How does one reliably capture feelings, which are subjective, fleeting, and often either unnoticed or unremarked upon?

Davenport and Sassi (Chapter V) explore the consequences of considering narrative as a construct for thinking about teachers' need for images of new forms of teaching. They raise questions about the relationship between holding images in one's head and later enacting a "parallel" narrative in the classroom.

Some of the papers have policy relevance. Russell et al. raise the possibility of the classroom itself as a locus for teachers' learning of mathematics, thus speaking implicitly to the economic constraints of providing an adequate mathematics education for all practicing teachers, our lack of instructional capacity to provide such an education, and the number of years that it would take to do so. If teachers can learn mathematics while teaching, maybe we *can* get this reform movement underway on a large scale. Hammerman (Chapter VI), in focusing our attention on the cultural aspect of inquiry groups, reminds us that most of the time we view teacher development as an individual matter, one teacher at a time, and we study it in psychological terms. But how much work can the culture itself do if it carries the central norms of inquiry, respect for individuals and ideas, and reflective critique? What is the relationship between changing the intellectual culture of schools and change on the part of individual teachers? Is the unit of transformation, so to speak, the school, the individual, or both? And should we be using anthropological as well as psychological lenses when we look at teacher change?

These papers also illustrate the turns in education research toward qualitative studies based on interpretive paradigms which are well suited for understanding the changes in the *meanings* that teachers make for "learning," "teaching," and "mathematics," and toward research conducted by practitioners on their own practice. Three of the papers (Russell et al., Schifter, and Goldsmith & Davenport) are conceptual analyses grounded in work with teachers, where the data comes from the activities of the project itself and the constructs are emergent. Hammerman uses "practical inquiry" (Richardson, 1994) to understand the context and practice of a particular form of teacher education. This paper is in the tradition of research on practice done by practitioners (Cochran-Smith & Lytle, 1993) and researchers who are using their own teaching to provide descriptions and analyses of actual classroom practice (Ball, 1993; Hammer, 1995; Lampert, 1990; Richardson, 1994). In Chapter V, Davenport and Sassi explore "storied" forms of learning for teachers, tapping the rich literature on narrative and case study.

Finally, the papers represent the search for new vocabulary and conceptual structures for thinking about teaching itself. As our view of teaching evolves from passing on the facts and procedures that have accumulated over the years to the building of intellectually rich environments in which students construct their own knowledge—and our view of the teacher evolves from being the source of knowledge and authority in the classroom to being the guide to sound thinking and a collaborative inquirer into the nature of mathematical thinking—our view of the essential elements of teaching, itself, changes. For example, as Russell et al. illustrate, teachers will continue to learn mathematics while teaching it. This is a consequence not only of this transitional period, when many teachers lack the robust mathematical knowledge to support the kind of teaching they are trying to do, but also of an idea about teaching itself—that teachers continue to learn, with their students, in an intellectual community of which they are a part. Or, as Hammerman points out, as teachers work together in inquiry groups to analyze and understand each other's practice, they begin to see themselves as a community of professionals examining issues of teaching in order to improve it. That is, they are not only working to improve their own teaching, they are constructing a new culture for teaching, in which collaborative investigation is an essential part of the work. As Schifter notes in the Epilogue, part of the enterprise in papers such as those in this anthology is to identify and elaborate on the main components of the practice that the teachers are trying to create, particularly those that are different from traditional practice.

A new form of teaching is emergent, as teachers explore new ideas and classroom practices, and studies of teacher change require a context in which teachers are, indeed, changing. As a field we are now in the position to create "layered" communities of inquiry in which students investigate mathematics and what it means to understand it; teachers investigate students' mathematical thinking and the nature of teaching practice; and teacher educator/researchers investigate teachers' mathematical and pedagogical thinking, the process by which they change their practice, the contexts that support change, and their own practice. These communities of inquiry imply a new relationship between research and practice, in which all parties inhabit the same epistemological space and learn together. Such "layered" communities are characteristic of the work described in this volume and its implications are traced in the Epilogue.

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Notes

[1]The North American Chapter of the International Group for the Psychology of Mathematics Education.

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Chapter II: Learning Mathematics while Teaching

Susan Jo Russell, Deborah Schifter, Virginia Bastable, Lisa Yaffee, Jill B. Lester, and Sophia Cohen

This paper examines cases of elementary grade teachers learning mathematics in the context of their own teaching, as they explore mathematics content they are using with their students, consider student strategies and representations that are new to them, and try to understand how students are thinking about complex mathematical ideas. We consider what teachers must already understand in order to do this and discuss implications for teacher education.

It is widely recognized that in order to teach mathematics for understanding, teachers, themselves, whose mathematical experiences have been limited to traditional instruction, need to understand the content more deeply than most currently do (Ball, 1991; Cohen et al., 1990; Schifter, 1993). Although some programs have provided opportunities for teachers to explore significant mathematics content (Lappan & Even, 1989; Russell & Corwin, 1993; Schifter & Fosnot, 1993;

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Simon & Schifter, 1991), it is unclear just what teachers need to learn to be able to support their students' constructions of rich mathematical concepts. In fact, it appears that the new mathematical understandings teachers must develop and the teaching situations they must negotiate are too varied, complex, and context-dependent to be anticipated in one or even several courses. Thus, teachers must become learners in their own classrooms (Ball, in press; Featherstone et al., 1993; Heaton, in press).[1]

In Teaching to the Big Ideas, a joint project of EDC, TERC, and SummerMath for Teachers, we are exploring the development of teachers' mathematical understandings and their effect on instruction (Schifter et al., in preparation). Data include classroom field notes, audiotaped interviews, and papers and journals written by the teachers which include reflections on episodes in their classrooms. The project began in the summer of 1993. Drawing from our work in Teaching to the Big Ideas during the 1993–1994 and 1994–1995 school years, this paper examines cases of teachers learning mathematics in the context of their own teaching and considers what teachers must already understand in order to do this.

What Do We Mean by Learning Mathematics?

When we first asked teachers to identify episodes in their classrooms during which they learned mathematics, they were stymied. To many of them, "to learn" seemed to mean either the acquisition of completely new knowledge about previously unfamiliar mathematics or, perhaps, an "Aha!" experience in which an idea is apprehended for the first time. We propose to extend our definition of learning mathematics to include a more ongoing and gradual process in which understanding of familiar content is deepened as one makes new connections and distinctions. A new representation of mathematical relationships may illuminate an aspect that was previously invisible even though that relationship was already "known" or "understood." An unfamiliar problem or context may highlight a mathematical idea in a new way, making one's thinking more problematic and causing one to think more explicitly about what was implicitly known.

It is in the context of this view of learning mathematics—a gradual building and deepening—that we offer glimpses of teachers engaged in learning mathematics in the course of teaching. As is the case when we observe student learning, we see only a slice and often do not know the whole story. However, we believe these episodes, taken together, provide evidence of what we mean by teachers becoming learners of mathematics in their classrooms. This paper considers episodes that illustrate teachers engaged in learning mathematics in three contexts: 1) exploring a mathematical problem or question embedded in content they are teaching; 2) thinking through students' representations and strategies; and 3) looking underneath students' confusions or excitement to consider mathematical structures.

1. Exploring Mathematics Content

One of the ways we see teachers learning mathematics is by engaging directly in the mathematical content they are teaching their students. Before they begin a unit, teachers might explore a mathematics topic by reviewing several resource books, solving some problems, or discussing the issues of content with colleagues. But sometimes it is only after plunging into work with students that teachers identify mathematical issues that they want to explore further for themselves.

Meg gave her second graders a word problem that she had not investigated ahead of time: What are some possible combinations of 12 marbles if each marble can be one of three colors? In other words, each combination of 12 marbles could include red and/or green and/or blue marbles. So, legitimate combinations might be 12 red marbles; 6 red marbles and 6 blue marbles; 3 green marbles, 8 green marbles, and 1 blue marble. Once she observed her students working on it, Meg realized the problem was more complex than it had first seemed and decided she needed to understand it better. At home, she worked on the problem with her husband. They began with 1 marble to find the number of combinations possible with 3 colors (3 combinations: 1 blue, 1 green, 1 red) then moved on to consider 2 marbles (6 combinations: 2 blue, 2 green, 2 red, 1 blue and 1 green, 1 blue and 1 red, 1 green and 1 red), 3 marbles

(10 combinations), and so forth up to 12 marbles. She noticed that the differences increased by 1 (with 2 marbles, there are 3 more combinations than with 1 marble; with 3 marbles, there are 4 more combinations than with 2 marbles; etc.), and wondered what would happen with four colors. Meg pursued this problem for 12 pages in her journal, and then commented:

> I have a *very* hazy, in-and-out picture of *why* I can make that number of combos with four colors of marbles and why it grows the way it does as you add a marble—because by adding 1, you're adding all these other possibilities. I could NEVER explain it at this point. AND, I *don't* get the relationship between what happens with two colors, three colors, five colors beyond that it grows REALLY FAST.... [emphasis as in original]

When Meg originally presented this problem to her class, she had not thought through the mathematical ideas embedded in the problem. She knew that it was a type of problem that calls for generating combinations systematically, but she did not know enough about the mathematics that arises in this problem to develop a mathematical goal for her students. As she worked on the problem for herself, she discovered a number of things, including the large number of combinations that can be generated, the need for systematicity in order to generate all the combinations, and the pattern of increases as marbles are added. She began to think through why the pattern of increases works as it does—that adding one marble adds "all these other possibilities"—and noticed that increasing the number of either marbles or colors leads to a rapid increase in the number of possible combinations.

Now that she had a deeper understanding of the complexity of the problem, Meg turned her attention back to the thinking of her students and the issue of how to proceed in her classroom:

> What are the implications for asking seven year-olds to work on a problem with 91 answers? . . . The kids who approached it randomly thought about 12-ness, and adding numbers, and three parts—and some kids went further. But now I want to ask them something more manageable to see if some "finish" and know they're done.

She chose to pose the following problem to her class: "There are 23 cupcakes for our good-bye show. Some have white frosting. The rest have green frosting. How many of each color frosting might there be?" This problem presents only two, instead of three, color possibilities, making it more possible for students at this age level to work on generation, comparison, and organization of combinations and to decide whether or not they have found all possible combinations. In working through the mathematics of the original problem, we surmise that Meg began to sort through the mathematical ideas that arise in a combinatorics problem and rejected some of these ideas as inappropriate for her second graders. She knew, for example, that for the marbles problem, her students would not be able to organize the many possibilities they generated and so would not be able to explore growth patterns or recognize whether or not they had all of the possible combinations. Because of her own thinking about why growth occurred in the way that it did, she realized that of the two questions—Do I have all the combinations? and Can I predict the number of combinations?—The first was the most accessible for her students. Meg then chose a mathematical goal that focused on systematicity rather than on growth patterns. The problem has actually been altered dramatically by moving from three colors to two colors. Now the problem focuses on the number of ways of breaking one quantity into two parts. While it involves work on combinations, it begins at a point that is very close to the work these second graders are doing as they become fluent in breaking numbers into manageable parts to solve addition and subtraction problems. By reducing the problem to two colors, Meg suspected that at least some of her second graders would be able to find a systematic way to keep track of the possible combinations of white and green frosting for 23 cupcakes and would be able to begin to look at patterns in their combinations (1 + 22, 2 + 21, 3 + 20, . . .). By learning more about the mathematics of combinatorics problems, she was able to devise a problem with an appropriate mathematical goal for her students.

This clearer view of the mathematics content also leads, we suspect, to the posing of more focused questions to students as they work. Choosing problems and linking them with clear mathematical goals is one of the key tasks of a teacher of mathematics. Learning more about the mathematical ideas embedded in a problem or class of problems supports the teacher in making choices about creating, choosing, or modifying problems for the students and in interactions with students' developing ideas.

2. Thinking through Students' Representations and Strategies

Another way in which we observe teachers learning mathematics is as they engage in thinking through students' approaches to solving problems. Learning takes place as teachers are confronted by student strategies or representations that are different from their own. In assessing the reasoning of students' responses, especially when they are unfamiliar and unexpected, teachers think through the mathematics again for themselves, seeing new aspects of familiar content, expanding their own understandings.

Denise, a second-grade teacher, expanded her view of the process of subtraction when she observed one of her students develop an algorithm that, she later wrote, "I had never thought of, or even imagined before." Denise was working with Ivan and his partner after he had made the familiar subtract-the-smaller-from-the-larger error in the problem 52 – 28, getting an answer of 36. Brandon said to Ivan, "But you can't take the 2 away from the 8; you have to take the 8 away from the 2." As Ivan began to rethink his solution and Brandon's comment, Denise expected him to work out something about "borrowing" a 10 from the 50 and adding it to the 2 to make 12, which would match her own representation of the problem. But, instead, Ivan invented a different method, "You take the 20 away from the 50 and get 30. Then you take the 8 away from the 2 which is minus 6. Then you take the minus 6 away from 30 and you get 24." Denise reported that she had to ask Ivan to repeat his solution several times before she understood his method. She commented in her journal:

> I've since read Connie Kamii's book in which she describes several common methods 2nd graders use to subtract in this situation and [Ivan's] method was one of them. It still feels new enough to me that I have to think it through each time. It is definitely a case of my learning some mathematics from my students.

Throughout their study of addition and subtraction, Denise encouraged students in her classroom to develop their own computation strategies. As part of this work, she emphasized pulling apart numbers in a variety of ways to make the addition or subtraction process more manageable. For example, methods in her classroom for this problem included strategies such as these:

$52 - 20 = 32; 32 - 2 = 30; 30 - 6 = 24$
OR $50 - 20 = 30; 30 - 8 = 22; 22 + 2 = 24$

So Denise was already comfortable with methods that involved breaking numbers into parts in a variety of ways and recombining them to solve the problem. What makes the problem 52 – 28 different is the embedded subproblem 2 – 8. Many of the strategies invented by Denise's students transform a problem like this one into a set of subproblems in a way that eliminates the larger-from-smaller subtraction (e.g, 2 – 8). For example, in the methods above, the 20 (from the 28) is subtracted first, then the 8 is subtracted from the result. In the first method, the 8 is broken into 2 and 6 in order to subtract easily from the 32, while in the second method, the 2 from the 52 is eliminated from the problem, then added back to the result in the final step. However, it had not occurred to Denise in her own thinking that these numbers could be pulled apart in such a way that the larger-from-smaller subproblem would be used as it is, leading to a negative result that would then be recombined with other parts of the problem. It also may not have occurred to her that one of her second graders would feel comfortable going "below zero" as part of the subtraction process. In coming to understand Ivan's method, Denise broke her own "below zero" barrier in thinking about what is allowed in solving a subtraction problem.

As another example, consider Ellen, who had been working on multiplication and division with her third-grade class: Multiplication was presented as repeated groups, while division was modeled as dealing out. These were models of the operations with which Ellen herself was comfortable. During one class, Ellen asked Kevon to illustrate 7 × 3 on the board. Kevon began by drawing seven circles. He then drew one mark inside each circle, paused, and drew a second mark in each circle. Ellen recognized these actions as what she and her students usually did for a division problem. She was about to have Kevon sit down to give someone else a chance, but she caught herself, hesitating as she studied the board. She asked the class, "What is Kevon doing? It looks like what we usually do for division but let's wait and see what he comes up with." Kevon then added a third line to each of his seven circles and wrote "21" to the right of the equals sign in his equation. Ellen asked Kevon how he came up with 21 and Kevon explained, "I put 1 of each, 2 of each, 3 of each." Ellen asked, "What does that mean?" Kevon responded, "I got 7 of them with 3 in each." Ellen thought about this for a long moment before answering. At this point in the interaction, she was challenged by Kevon's representation to expand her own model of multiplication. She said, almost to herself, "Seven groups of what? Seven groups of three. Should we consider what he did right?"

Once she and the children decided Kevon's representation fit the problem, Ellen asked the class if anyone had still other diagrams. The staff observer noted that Ellen's acceptance of alternative representations was unusual for her. In response, students offered a greater variety of solution methods than had previously been the case during the group's study of multiplication and division. We surmise that her recognition of Kevon's unfamiliar representation—beginning with 7 groups and then dealing out marks until each group had 3, rather than the iterating of threes she and her class had been using—acted as a catalyst to her invitation to students to show their own ways of thinking about multiplication.

In both of these episodes, teachers took time to understand students' approaches. In both cases, they not only learned about their students' thinking, but they actually expanded their own views of ways to model a whole number operation.

3. Delving Underneath Students' Reasoning: Looking at Mathematical Structure

In the third kind of example, teachers' reflections on what is problematic in students' reasoning leads them to rethink their own understanding of mathematical structures. By probing underneath students' confusions about mathematical ideas, they confront new mathematics themselves as they ask these questions: Why would a student think that? What is right about the solution from the student's point of view? What is difficult to understand about the mathematics here and why might that be the case? Often this process involves making explicit and reexamining what they have implicitly known.

For example, students in Sylvia's second-grade classroom were adding two-digit numbers, using base ten blocks to model "carrying." Most students easily undertook this task, following the conventions established in the class for using the blocks and recording the results. However, when some students were asked by a staff observer to determine *without trading* what quantity was represented by 4 tens sticks and 15 small cubes, they had difficulty counting this quantity. One student counted 10, 20, 30, 40 for the tens sticks, then continued counting the small cubes by tens, and was perfectly satisfied with her result of 190. Sylvia was quite surprised when she observed students counting ones as tens. She had assumed from their competent use of the base ten blocks to solve addition problems that they understood how to decompose two-digit numbers into tens and ones and how to recombine them. She was still thinking about this some months later when she described how various students in her classroom were solving two-digit addition problems, some breaking apart numbers into tens and ones, others counting on only by ones. She commented: "What goes on in someone's brain to make sense of tens and ones? Does a person need to construct a system of ones *and* a system of tens that sort of 'fits' like an overlay on the system of ones? How does this happen?"

The next year Sylvia again introduced rods trading but also used many word problems throughout the year. She did not insist that students use the rods to solve these problems but encouraged them to develop and discuss many strategies based on their own understanding of the number relationships. She wrote:

> Rods trading had no discernible impact on how the children thought about addition and subtraction problems. Some children, who could answer questions about how many tens, how many ones, where are the tens and ones, and so on, would still count on by ones when solving double-digit addition problems. [Other children] . . . added the way they always had: tens first, and then the ones.

Over two years, as Sylvia closely watched her students working with ones and tens, she began to refine and deepen her own ideas about the structure of the base ten system. Many of her students could easily learn how to manipulate the base ten blocks to come up with the correct solution to an addition problem. However, when operating in an addition or subtraction situation without the blocks, they still did not flexibly use what they seemed to "know" about tens and ones. As Sylvia pondered more deeply what is involved in constructing and simultaneously manipulating more than one unit (e.g., ones and tens), she began to consider how the conception of a unit does not inhere in a particular physical representation (such as base ten blocks) but in a complex mental model in which a larger unit (such as "ten") constructed out of smaller units (such as "ones") can exist simultaneously as a one and a collection of ones. It can be manipulated as a large unit that can be combined with, compared to, or separated from a collection of the same large units as if they were ones. Yet this "one" can also be decomposed into the smaller units of which it is composed, should the need arise.

Certainly, operating with ones and tens was not new to Sylvia. However, in watching her students, Sylvia began to think more deeply about the complexity of coordinating multiple units (in this case, tens and ones). Once the idea of coordinating multiple units—which had been obscured for Sylvia by her students' apparent competence in manipulating base ten blocks to solve computation problems—became explicit for her, she could see this same idea underlying many aspects of her students' mathematical activity. In skip counting, which she did frequently with her class, students say, for example, the numbers 2, 4, 6 to represent two, four, six objects while each number also represents one count (one unit). As she moved through the second-grade curriculum, Sylvia found that this now visible idea of multiple units comes up in multiplication and division as well as in problems related to time, money, and measuring. Sylvia's understanding of all these topics has been enriched by the new clarity with which she sees the common underlying idea of multiple units.

What Enables Teachers to Learn Mathematics while Teaching?

As we collect and analyze episodes such as those in this paper, we are beginning to identify some of the elements that appear to be necessary for teachers to learn mathematics while teaching. First of all, this kind of exploration of mathematics content requires teachers to see themselves as adult learners of mathematics and to see their own classrooms as contexts in which they learn. The teachers described here had participated in at least one year of experiences that emphasized adult mathematics learning, and they now share some assumptions: 1) learning about mathematics occurs when one is immersed in problem solving; 2) an important aspect of mathematical thinking is identifying, describing, and testing patterns and relationships; and 3) understanding the mathematics better has implications for their pedagogical decisions. Rather than expecting to acquire discrete bits of information, these teachers assume that the way to learn mathematics is to do mathematics.

In the first category of learning mathematics while teaching, teachers explore the mathematics content in which they engage their students. To do so, they must be curious about mathematics, know that they can pose their own mathematical questions, and assume that they can pursue those questions themselves.

In the second group of episodes, teachers expand their understanding of mathematical ideas by paying attention to student strategies and representations. This requires teachers to develop a classroom culture in which multiple strategies and unexpected responses are the norm, where the point is not to find the solution prescribed by teacher or textbook, but to *reason cogently* about mathematical relationships. In order to analyze student thinking, teachers must learn how to follow a mathematical argument and assess its validity. As they consider student strategies and representations that are different from their own, teachers become aware of new aspects of mathematical relationships. Revisiting familiar mathematical ideas in this way can lead to a deepening appreciation for their complexity.

In the third category of episodes, teachers delve beneath students' efforts at understanding to confront the underlying mathematical structures with which their students are grappling. By carefully listening to and observing their students, they begin to identify and describe the complexity of elementary mathematics—what mathematical ideas are central to student understanding and why these are hard for students. When teachers reflect on student learning in this way, they are doing more than understanding student thinking better; they are actually recognizing and articulating significant mathematical ideas and developing a deeper understanding of these ideas for themselves.

Do teachers know that they are learning mathematics as they teach? In some of the episodes recounted here, teachers seem to be aware that they are engaged in mathematics learning; in others, perhaps only the authors view the event in this way. Often, we have seen that the simplicity of the formulations brought about in such new learning situations leads teachers to discount or disparage their new knowledge: "Oh, I was so stupid not to see that," or "Of course, I'm sure it was obvious to everyone else, but . . ." On the other side of learning, the simple and elegant might appear trivial—not worth mentioning. Beginning to notice and appreciate the profundity of our own insights, as adults, is connected to appreciating the profundity of children's insights.

Learning in the context of one's own teaching is not simply a remedial measure. Rather, it is a component of the pedagogy itself and already requires of teachers a fairly sophisticated understanding of the discipline. In order for teachers to take advantage of the opportunities that present themselves in their own classrooms, they need an orientation to what such learning might be like. One of the objectives of teacher education programs can be to prepare teachers for the ways in which they can take advantage of their teaching as a site for their own ongoing learning of mathematics. Making explicit and validating new learning of this kind may be a critical function of teacher education.

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Notes

[1]The classroom is also an important context for learning about teaching and about student thinking. While we in no way want to diminish the importance of these two components, in this paper we highlight how teachers learn *mathematics content* in their own classrooms.

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Chapter III:
Teachers' Changing Conceptions of the Nature of Mathematics: Enactment in the Classroom[1]

Deborah Schifter

This paper distinguishes four conceptions of mathematics as enacted in the classrooms of teachers working to transform their instruction along lines urged by the reform movement. Drawing on teachers' accounts of their own teaching, it proposes these enacted conceptions as four stages of a typical developmental trajectory. Questions concerning the implications of this model for both a theory and practice of mathematics teacher development are raised.

Over the past decade, thousands of pre- and in-service teachers have participated in projects designed to support the development of a practice consistent with the vision proposed by reformers (NCTM, 1989, 1991; National Research Council, 1989). Researchers associated with these variously conceived projects have been reporting on their results: many offer case studies of individual teacher participants (e.g., Featherstone et al., 1993; Fennema et al., 1993; Schifter & Fosnot, 1993; Wilcox et al., 1992; Wood et al., 1991); others identify those aspects of project participants'

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changing practice whose significance is thought to be generalizable (e.g., Hart, 1991; Russell & Corwin, 1993; Russell, et al., 1994); some investigate shifts in teachers' beliefs (e.g., Knapp & Peterson, 1995; Thompson, 1988); and still other researchers analyze changes in the performance of students whose teachers had earlier participated in their projects (e.g., Cobb et al., 1991, Carpenter et al., 1988, 1989).

Most of these studies share the assumption that the kind of instructional transformation being called for—a dramatic departure from traditional practice—extends beyond the acquisition and mastery of new teaching techniques and strategies. Rather, it involves the reconstitution of fundamental notions of teaching, learning, and the nature of mathematics as a discipline, and the creation of radically different classroom opportunities for learning. However, although the *results* of these teacher education projects are becoming available, the *process* by which teachers develop new practices is, as yet, little understood. It seems clear that for teacher development interventions to succeed in stimulating the profound and wide-scale changes envisioned by reformers, models of this process will be needed (Cooney, 1994; Goldsmith & Schifter, 1993, in press).

A few researchers have begun to address this issue, offering models designed to capture a putative developmental logic to the process of teacher change (e.g., Fennema et al., in press; Franke et al., 1992; Schifter & Simon, 1992; Schram & Wilcox, 1988; Schram et al., 1989; Simon & Schifter, 1991; Thompson, 1990). In the spirit of such research programs, this paper analytically isolates one "strand" of this complex process: changes in the conception of school mathematics enacted by teachers working to transform their practice along the lines of the reforms. The model proposed here is conceived as a framework for interpreting teacher development along this strand.[2]

Enacted Conceptions of Mathematics: A Model for Teacher Development

The model comprises four stages that have been derived from my own and colleagues' observations in classrooms of participants in professional development projects, as well as participants' writings about their practice. The backdrop of this effort is a series of projects,[3] extending from 1985 to the present, which have involved intensive work with over 250 teachers and less intensive work with many more. (The intensive work involved each teacher attending at least one summer institute or semester course and receiving weekly classroom support over the course of a year; though participation sometimes extends over a much longer period, even, in a few cases, for as long as seven years.) For this paper, I have selected illustrations from teachers' own accounts of their teaching and mathematical thinking.

I characterize conceptions of mathematics teachers enact in practice as 1) an ad hoc accumulation of facts, definitions, and computational routines; 2) student-centered activity, but with little or no systematic inquiry into issues of mathematical structure and validity; 3) student-centered activity directed toward systematic inquiry into issues of mathematical structure and validity; or 4) systematic mathematical inquiry organized around investigation of "big" mathematical ideas. Each conception, or stage, entails an understanding of what counts as "doing mathematics," of the extent to which mathematical results are interconnected, and where mathematical authority resides and how it is established.

The first of the four stages characterizes the mathematics of the typical contemporary U.S. classroom: facts and procedures, discrete and ungrounded, learned mechanically and mechanically applied. How authority over mathematical knowledge is bestowed is not an issue; truth is simply incarnate in textbook and instructor.

The introduction of innovative tools and strategies—use of manipulatives, group work, or electronic technology, for example—may be energetically promoted, being often mistaken for significant instructional change. But these are here used simply in support of traditional goals—as in the case of Ms. Murray,[4] who describes how she uses manipulatives in her mathematics instruction:

> With division, first I do teach them a formula. ... And I do tell them that I want them first to look at the problem, ask themselves what is the

> smallest number the divisor can go into, put a check over that column. Because otherwise, they don't line up numbers correctly and don't even realize how many digits the answer is expected to have. But then, in using the manipulatives, I would say, well, sometimes if you start in a tens column for that first digit, that means that we're going to have to do some regrouping. So we do that with blocks. And then eventually we actually get into doing the division on paper without using manipulatives.

Ms. Murray's division unit is organized to teach procedures—with manipulatives or on paper, it makes no difference—which she demonstrates and hopes her students remember.

School mathematics at the second stage is characteristic of classrooms whose instructors are committed to encouraging their students' cognitive autonomy. Now, finding patterns, solving problems, and making conjectures are all seen as central to the "doing of mathematics." Grounding these activities in familiar contexts—by using descriptions of real-world situations or introducing techniques of physical or diagrammatic representation—is understood to be an essential support for student initiative. Unfortunately, however, these instructional ambitions are compromised by a constricted view of the nature of the discipline and/or an inadequate grasp of disciplinary content. Mathematical actions and their results remain discrete when teachers are themselves either unable to recognize or frame deeper conceptual issues or don't see that this is where their teaching should lead.

For example, Mr. Smith describes how he used the *Geometric Supposer* with his high school geometry class. Students were asked to collect data and formulate conjectures about isosceles triangles. However, once their conjectures were listed on the board, he was unsure how to proceed. "Some of the conjectures were good, but others, I had to laugh. Like, all isosceles triangles are acute." When asked how he ended the lesson, he reports that he told the class which of their conjectures were correct, and which were incorrect.

Mr. Smith's colleague, Ms. Peterson, found herself in a similar situation when she experimented with the *Supposer*. Toward the end of a lesson, she, too, had a list of conjectures on the board—some of which were correct, others incorrect. However, since she did not want to discourage her students, she ended class without addressing the correctness of their conjectures and never referred to them again.

A dilemma facing teachers operating out of the disparity between their enlarged ambitions for their students and their own limited mathematical resources concerns how mathematical authority—validity—is to be established in their classrooms. The dilemma is this: If students are supposed to figure things out for themselves, what happens when they get it wrong? Rather than shift the locus of mathematical authority to the discursive process itself, teachers like Mr. Smith retain that authority. Other teachers, like Ms. Peterson, resolve the dilemma in an opposite, though equally mechanical way: if student autonomy is the goal, then all solutions are valid. Rejecting the tradition that accorded them cognitive omnipotence, but unable to envision for their classrooms a nonauthoritarian alternative by which mathematical validity might be assessed, they are left presiding over a kind of mathematical anarchy.

In the third stage, the disparities of the second stage have been rectified—teachers now possess an understanding of the nature of mathematics commensurate with their aspiration toward a practice emphasizing student construction. Solving problems is no longer an end in itself, but a means toward the exploration of mathematical connection, of underlying mathematical order. When students announce the discovery of a pattern, this is followed up with questions about whether the pattern will always hold and how that can be shown or what information the pattern conveys. And the formulation of conjectures leads to the investigation of their validity. Ms. Jackson provides an example of a teaching practice at the third stage.

Her sixth graders had determined that 11/16 of the class envision doctors as male, 5/16 as female, and she now asked how they could represent these fractions as percents. Through discussion for the remainder of the period, five methods of solution were proposed:

> 1. I see that 8/16 equals 50%. Half of that is 25% or 4/16. Half of 25% is 12 1/2% or 2/16. And finally, half of 12 1/2% is 6 1/4% which is

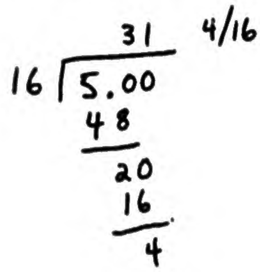
equal to 1/16. Since 1/16 + 4/16 = 5/16, then 25% + 6 1/4% = 31 1/4%.

2. You could try to divide 5 into 100 and it would come out with 20%.

3. I think you could divide 5 into 16 and you come out with 3 with a remainder of 1. That would be 31%.

4. You can divide 16 into 100 and come out with 6 4/16% or 6 1/4%. That would be for just 1/16. Then you could just multiply to figure out 5/16. 5 × 6 1/4% = 31 1/4%.

5. I would divide 16 into 5.



5/16 would be 31 1/4%.

At this point, Ms. Jackson is in the same position as that of Mr. Smith and Ms. Peterson. She solicited her students' ideas, which they volunteered, and has ended up with a list that includes both correct and incorrect solutions and methods. However, unlike Smith and Peterson, the list of ideas does not mark the end of the lesson. Rather, Ms. Jackson reports, "This was a starting point for my students and me to make sense of this process." The next day, the class returned to the five methods.

> As a group, we concluded that [the second] way, although easily understood, wouldn't work. Several students pointed out that if you divided 100 by the numerator, in this case 5, the percentage would always be the same no matter what the denominator was. For example, using this strategy, 5/6, 5/17, and 5/29 would all be equal to 20%, and that can't be. They aren't equal percents.
>
> Several students pointed out that [the third] way of dividing the numerator into the denominator accidentally came up with an approximate percent. When they applied it to other fractions, it didn't work. For example, 7/8 would be equal to 11% while 3/8 would be equal to 22%. It doesn't make sense. One student pointed out that when using this formula, the closer the numerator and denominator were to each other, the smaller the percent would become—like with the fractions just mentioned. This was another strategy for disproving [the third] procedure.
>
> The students agreed that [the first, fourth, and fifth] ways worked to find percentages. When asked which way they preferred, the majority responded that [the fifth] way seemed the easiest and would probably be the way they would find percentages. But some students found [the first and fourth] strategies most helpful.

Ms. Jackson's classroom resolves the dilemmas concerning mathematical authority. While offering ideas is encouraged, so is their challenge. As Ms. Jackson's students collectively weigh whether an answer is correct or debate the validity of a solution, they share authority over mathematical truth.

At the fourth stage, teaching is organized to allow students to confront the "big ideas" of the mathematics curriculum. By "big ideas" I mean central organizing principles of mathematics with which students must wrestle as they confront the limitations of their existing conceptions. Through our work as mathematics teachers and teacher educators, my colleagues and I (Schifter et al., in press) have found that there are particular themes—themes that embody critical mathematical concepts—that arise time after time with different groups of learners when instruction is organized around and responsive to student thinking. It is by listening to students, identifying common areas of confusion or questions that intrigue, and then analyzing the underlying issues that the big ideas emerge.

For example, one frequently hears questions and comments like the following from students engaged in mathematical explorations: "I solved the problem by adding [on] and he solved it by subtracting. We got the same answer, but he did it wrong." "How can that piece of cake be 1/2 and 1/4 at the same time?" "We just said that this rectangle is bigger than the other one. Now you say it's smaller. That doesn't make any sense!"

When statements such as these are repeatedly heard, indicating persistent difficulties and confusions, they likely flag crucial conceptual–developmental issues. These are points at which students need to step back and take time to ponder. The big ideas that may be implicated in the quandaries of these students include these:

- operations are systematically related and some problems might be solved using either addition or subtraction;
- the same quantity can be represented by different fractions, depending on their reference whole; and
- an object or process may have attributes that change at different rates—the area of a rectangle, say, may grow while its perimeter shrinks.

Teaching to the big ideas would orient the study of the topics of the mathematics curriculum toward the major conceptual unities that run through school mathematics. Developing such a practice would mean learning to recognize the specific and pervasive ways in which attempts to construct instances of these unities go wrong. And rather than explaining what the big ideas are, it would mean giving one's students opportunities to confront the limitations of their extant conceptions and time to work through their confusion to construct new, more inclusive understandings.

For example, with each expansion of the domain of number—include 0, or fractions and decimals, or negative or irrational numbers, and so on—the meanings of the basic operations must be rethought. So when Joanne Moynahan (in press) began a unit on fractions with her sixth graders, she knew they would need to reconsider generalizations they had made from their work with whole numbers. One instance would be to extend their definition of multiplication beyond repeated addition. To start, she gave them time to work on the following problems using whatever strategies they chose:

> *1. The Davis family attended a picnic. Their family made up 1/3 of the 15 people at the picnic. How many Davises were at the picnic?*
>
> *2. John ate 1/8 of the 16 hot dogs. How many hot dogs did John eat?*
>
> *3. One-fourth of the hot dogs were served without relish. How many were served without relish?*

After working in pairs for some time, the class came together to share their solutions. She describes what happened next:

> As we discussed each problem I recorded a shortened version on the dry-erase board.... At the end of sharing the board looked like this:
>
> 1/3 of 15 = 5
>
> 1/8 of 16 = 2
>
> 1/4 of 16 = 4
>
> We didn't have much time left before the recess bell, but I thought I would . . . give them something to think about and posed the following question:
>
> Does anyone know what they were doing with these numbers? (Long pause.) What operation did you use? Did you add, subtract, multiply, or divide? (Another long pause.) What symbol could we put in here instead of "of"?

And so began a two-day conversation in which students pondered the actions they had taken to solve 1/3 of 15 = 5, analyzed different ways of representing those actions, and considered whether any of the symbols "+," "–," "×," or "+" could meaningfully replace "of."

Questions Raised about This Model

The characterization of four enacted conceptions of mathematics as stages of development raises several questions, described below.

What Is the Evidence that the Four Conceptions Define a Developmental progression?

For this paper, I have chosen to describe four conceptions of mathematics enacted in classrooms, with accounts from different teachers to illustrate each. However, it is plausible to think of them as positions in a developmental sequence since teachers often offer evidence of transitions in their writing.

First Stage to Second

Ms. Collins, who had been a strong, conventional mathematics teacher, describes how her conceptions of an appropriate mathematics curriculum has changed during the course she is just completing.

> Mathematics . . . shouldn't be structured (as it so often is) so that students (and teachers) believe there is only one way to get to the "right answer." . . . Half the excitement, enjoyment, the learning is "getting there." The trip of exploring, manipulating, and connecting new

and old ideas is the most important part of math: not the finished puzzle or right answer.

Second Stage to Third

Ms. Rosen explains how she has recently identified an element missing from her practice.

> I feel like I am just beginning to grapple with the whole idea of mathematical reasoning and mathematical argument for myself. What does it mean for third graders to think and argue mathematically? ... In the past I have written and spoken about my desire for my students to communicate their mathematical ideas orally and through pictures, graphs, and writing. It has been a big and important step for both me and my students that they begin to do this, and while it has felt good, it has also felt incomplete. They didn't seem to be doing this the way I had seen Deborah Ball's students discuss their mathematical thinking in videos we have seen as part of this program. I think what has been different is that her students have been more engaged in mathematical argument. I'm not sure what this means, but I have a sense of wanting to move my students more in that direction.

Third Stage to Fourth

Ms. Norris explains how she has come to organize her teaching around big ideas—she calls them "dilemmas"—that her students must confront.

> I'm convinced that what was missing from [my teaching] last year was talking about numbers as a whole way of [thinking]. ... To engage in strategy alone, which is, I think, how I thought of math [last year] ... that's like a dead end. I mean, it's certainly worth doing and kids may try somebody else's, but at some point there's not much there. ... And so I think to have something to really mull over—a dilemma—is really much more interesting.

Does this model assume the progression of stages to be sequential and invariant?

While the quotations above provide evidence of movement from one position to the next, they do not necessarily demonstrate a sequential and invariant progression. In fact, the assignment of teacher to stage is often ambiguous. As teachers' mathematical understandings evolve in the context of their in-service work, boundaries between one stage or another may become very indistinct. Although some teachers may stand clearly for a time in one stage or another, many come to exemplify more than one—perhaps wavering back and forth as they confront new mathematical content, develop new beliefs about learning, hear surprising confusions among their students, work to establish new classroom structures, or deal with various and often conflicting administrative mandates.

Rather, I suggest that they indicate an "orderliness" in the transformation of pedagogical practice when viewed from the standpoint of the goals of the reform and of a set of ongoing efforts to achieve those goals (Goldsmith & Schifter, 1993, in press). While a teacher might enact different conceptions of mathematics on different days, there tends to be a "center of gravity," a conception that guides a teacher's major instructional goals over an extended period of time, an overarching agenda for student learning.

What is the use of such a model?

Although the development of a model to describe change in practice was initiated in order to assess program participants in the aggregate (which meant assigning teachers to stages at given points in time) (Schifter & Simon, 1992), my colleagues and I have found that its power is not so much as a formal assessment tool, but rather as a pedagogical heuristic. In identifying stages through which teachers may pass as they develop their practice, the model has helped us to clarify our long-range goals, to interpret what we see in teachers' practice, and to guide us in the overall design of teacher development projects as well as in on-the-spot interventions.

For example, consider Ellen, a third-grade teacher who feels that a major responsibility to her urban students is to prepare them for the annual metropolitan examination which emphasizes accuracy and speed of computation. Consistent with the first stage, a considerable amount of her program is dedicated to drilling conventional computational algorithms. At the same time, under the influence of several in-service programs she has attended, she periodically sets up activities in which students engage more actively, working together to solve problems. Her classroom frequently resembles those exemplified by the second stage. More recently, a regular visitor to Ellen's classroom observed a new classroom dynamic.[5] As a child stood at the board drawing his representation of a problem, Ellen saw that his representation differed from the one she expected. As was her wont, she began to ask him to sit down, but then stopped herself—as if remembering her resolution to break a habit—and instead asked him to explain. When she could not find fault with his reasoning, she asked the class to consider it. In this case, the validity of his solution method was determined by the correct logic of his reasoning rather than the teacher's preconceived notions, and the class was invited to join her in evaluating the reasoning. This event, if it were a normal classroom occurrence, would be typical of the third stage.

If it were necessary to assign Ellen to one of the four enacted conceptions presented in this paper, one would try to determine which most centrally guides the organization of her curricular goals. However, without assignment to one category or another, the framework highlights for us the central importance of the opening provided when Ellen began to attend to her student's reasoning and asked her class to do the same. It offers to her supervisor or in-service educator a way of thinking about the kinds of questions to ask Ellen, the kinds of feedback or suggestions to offer, and the kinds of issues to raise at the teacher seminar she attends.

What are the implications of such a model for mathematics education reform?

The model I have presented here is not to be considered a comprehensive description of the development of conventional to reformed practice. Having isolated a single strand, I have necessarily neglected other significant aspects of practice. For example, one might apply a different lens to examine how practice changes in light of teachers' developing understandings of cognitive and social constructivist processes.

However, my concern is that pre- and in-service educators *must* attend to this strand of development, the nature of the mathematics teachers enact in the classroom, if the reforms are to succeed. The proposed pedagogy is not merely a matter of engaging students in fun and stimulating activity. Nor is it merely a matter of inviting students to solve problems, offer conjectures, or discover patterns. Nor is it a matter of listening to students' thinking without critique. If, as I propose, the second level is a stage in a developmental trajectory whose goal is far more ambitious, then teacher education programs, both pre- and in-service, must be designed, first, to help teachers move through that stage to a practice that invites students to inquire into mathematical structure and assess mathematical validity and, eventually, to help them conceptualize their mathematics programs in terms of big ideas.

Given that most teachers in the United States have not had sound mathematics experiences as students, we must think hard about the mathematics teachers need to learn and how we can support that learning. If programs are oriented to help teachers bring group work and manipulatives to a practice based on a conception of mathematics at the first, or if teacher educators feel successful when they observe that teaching shifts from the first to the second stage, but the course or program ends before teachers can move beyond—then surely the reforms will fail.

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Notes

[1]A longer version of this paper that presents another set of examples has been submitted for publication (Schifter, submitted).

[2]Researchers in the field of mathematics education are not in agreement about what constitutes a developmental model. These issues are touched upon toward the end of the paper.

[3]The Elementary Leaders in Mathematics Project (Simon & Schifter, 1991), the Mathematics Leadership Network (Schifter, 1993; Schifter & Fosnot, 1993), the Mathematics Process Writing Project (Schifter, 1994, in press a, in press b), and Teaching to the Big Idea (Russell et al., 1995; Schifter & Bastable, 1995; Schifter et al., in press).

[4]Teachers' names have been replaced by pseudonyms.

[5]This event is also discussed in Chapter II: "Learning Mathematics while Teaching," by Russell et al.

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Chapter IV:
Affective Issues in Developing Mathematics Teaching Practice

Lynn T. Goldsmith and Linda Ruiz Davenport

This paper explores the role of teachers' emotions in the process of developing a mathematics practice predicated on constructivist principles of learning and teaching. While most teachers and teacher educators would recognize from a clinical perspective that emotional responses to change processes are important, there has been little interest in engaging in systematic exploration of the roles that emotions might play in influencing the process of change itself. Below we consider possible roles that affect may play in this process in order to begin examining ways to use reflections on emotions to help promote growth in teaching.

The current reform movement in mathematics education asks a lot of teachers. With its focus shifting toward students' developing a deep, flexible, and usable understanding of mathematics, the teaching of mathematics must become more than just a matter of following a new and improved curriculum, mastering the management of cooperative group work, or introducing manipulatives into math lessons. Teachers are being asked to invent significantly new forms of mathematics

Nelson, B. S. (Ed.) 1995. *Inquiry and the Development of Teaching:Issues in the Transformation of Mathematics Teaching*. Newton, MA: Center for the Development of Teaching, Education Development Center, Inc.

practice—forms that emphasize careful assessment of students' conceptual grasp of mathematics, considered decisions about where (and how) to move within the curriculum, and the establishment of new classroom norms for working together. Teachers often find that this process challenges their fundamental beliefs about learning and teaching, the nature of mathematics, and the nature of mathematical understanding.

Making such changes in both belief and practice is no easy task, even for those who strongly wish to do so. Teachers often confront doubts about their professional effectiveness as they reflect on their own classroom practice. They must be willing to take risks in their classrooms, setting aside well-established classroom structures and activities in order to explore new ways of thinking and learning about mathematical ideas. Teachers rarely have the luxury of reconstructing their practice in an atmosphere that supports inquiry and reflection: they undertake this kind of examination and experimentation within the context of ongoing classroom responsibilities, where the desire to serve their students creates counter pressure for significant change (Cohen et al., 1990). Transforming teaching practice under these circumstances has been likened to redesigning an airplane while in flight.

Such circumstances are almost guaranteed to generate strong emotional responses. Observations of teachers as well as their own self-reports suggest that changing teaching practice invokes a variety of feelings, both positive and negative. Yet, despite acknowledgment from teachers, teacher educators, and researchers that there is, in fact, an affective side to teaching and to learning (Clandinin & Connelly, 1987; Kouba, 1993; McLeod & Adams, 1989; Schifter & Fosnot, 1993; Sowder, 1989), there has been little systematic study of this issue.

Both practical and theoretical benefits would derive from a more focused consideration of the affective aspects of teacher development. From the practical side, we will become better teacher educators if we can be more explicit about the kinds of roles that emotions play in the learning process, for we will be able to make pedagogical decisions that draw directly from this understanding. Certainly teachers know that learning (both their own and their students') is not simply a matter of "cold" cognition. Coming to understand something new (or to understand something old in new ways) requires a personal investment; once the learner has put his or her own sense of self squarely into the process, the learning experience cannot be emotionally neutral. Bringing the question of affect to the fore provides opportunities for teachers to reflect on their processes of change and for teacher educators to reflect on their own practice of facilitating this development. Hopefully, in so doing, the work of assisting teachers to reconstruct their mathematics practice will become stronger.

From a theoretical standpoint, this work can help to address questions about the nature of the relation between thinking and feeling. For much of the twentieth century the cognitive and affective aspects of experience have been treated as separable and distinct. Philosopher Martha Nussbaum traces the willingness to create this separation to the Platonic view of emotions as corrupters of rational thought (Nussbaum, 1990). She argues convincingly (and passionately) for the Aristotelian position, which holds emotions to be an absolutely essential aspect of understanding:

> And it isn't just that sometimes we need emotions to *get to* the right (intellectual) view of the situation; this is true, but not the entire story. Neither is it just that the emotions supply extra praiseworthy elements external to cognition but without which virtue is incomplete. The emotions are themselves modes of vision, or recognition. Their responses are part of what knowing, that is truly recognizing or acknowledging, *consists in*. (Nussbaum, 1990, p. 79; italics in original)

Nussbaum's recent argument for the recognition of emotion as an important aspect of practical decisionmaking has been mirrored in the psychological community where there has been both renewed attention to the study of emotion itself (Campos & Barrett, 1984) and to a consideration of how cognitive and affective systems function in relation to each other. Researchers have sought to characterize the nature of the emotional system, to investigate whether cognitive and affective systems are initially independent or interdependent, and to articulate the psycho-structural relations between the two (Clarke & Fiske, 1982; Izard et al., 1984; Mandler, 1980; Plutchick & Kellerman, 1980; Wozniak, 1986). Other research has focused on the behavior of the systems once interactions and interdependency between the two have been established. This work assumes a significant cognitive contribution to the experiencing of emotion and, likewise, assumes affective components to cognition.

The experiencing of emotions beyond earliest infancy, for example, is considered to rely significantly on cognitive interpretation (Kagan, 1984; Sroufe, 1984; Sroufe et al., 1984). The circumstances eliciting an internal feeling state are an important aspect of an emotional response. For example, Campos and Barrett (1984) reported that eight- and nine-month-old infants evidence different emotional responses to the presence of a stranger, depending on their mothers' emotional reactions to the unfamiliar person. Infants were observed to smile less and show more distress when their mothers were unfriendly and abrupt to a stranger in a laboratory setting, than when the mother is cheerful and smiling in greeting the stranger. The initial emotional arousal stimulated by the presence of the stranger is, presumably, the same in both of these circumstances. The difference lies in the infant's ability to read the tone of the mother's response to the event and to adjust the affective response accordingly.

The affective aspects of thinking, sometimes referred to as "hot" cognition, help to direct and shape perception, thought, and action. Researchers have determined, for example, that inducing positive feelings in experimental participants predisposes them to perceive positive aspects of subsequent situations, while inducing negative feelings yields more negative perceptions (Isen et al., 1982; Izard, 1984). Researchers have also described the intrusive and potentially maladaptive effect on productive thought of negative self-image or negative emotions such as fear or anxiety (Dweck, 1975; Ginsburg, 1988; Tobias, 1978), and the salutory effect of positive feelings (Csikszentmihalyi, 1990). The complexity of these interrelations between cognitive and affective systems leaves many issues about the influence of emotions on thinking and learning still unexplored.

One of these is the question of how cognitive and affective states interact over an extended period of time during learning. Relevant to this question are studies of motivational aspects of learning, which have described the important roles that noncognitive factors such as individual interest, self-image, or the desire for mastery play in the learning process (Csikszentmihalyi et al., 1993; Diener & Dweck, 1978; Dweck & Leggett, 1988; Renninger et al., 1992; White, 1959). There have also been several recent efforts to consider specifically the role of affect in learning mathematics (Goldin, 1988; McLeod, 1988; Schiefele & Csikszentmihalyi, 1995). The question motivating the present work—How do emotions influence teachers' efforts to develop their mathematics teaching?—directs attention to ways that feeling and thinking, together, might shape and regulate the development of professional practice.

The work described here sought evidence of naturally occurring expressions of emotion from teachers engaged in reformulating their mathematics teaching. These examples are used as the starting point for analysis of the roles emotions play in the development of teaching practice. The purpose of the work is to identify broad themes suggestive of the ways that feeling, thinking, and acting interact over extended periods of time to promote development. Several themes have emerged and suggest possible foci for future work. In addition, this work raises questions concerning the investigative approaches that are appropriate and productive for this kind of exploration.

Looking for the Emotions of Change

Exploration of emotional aspects of reformulating teaching practice needs to happen in the context of teachers working to change their teaching. We therefore studied four teachers who were participating in one of two extended professional development projects about teaching mathematics, Mathematics for Tomorrow (see Hammerman in Chapter VI of this anthology) and Teaching to the Big Ideas (see Russell et al. in Chapter II). These projects were both built on the principle that learners construct their understanding through inquiry and reflection. Both projects included opportunities for teachers to investigate mathematical ideas for themselves, inquire into the mathematical understanding of their students, and explore new ways to facilitate student understanding in the classroom. The projects also shared an emphasis on collegial collaboration and reflection as a means of professional growth.

Information about the affective aspects of teachers' efforts to change their practice came from artifacts produced by these four teachers as part of their participation in their respective programs. These included journal writings, directed writings assigned by project staff, observations of mathematics lessons, and individual interviews with the first author. With the exception of the interviews, none of these materials was specifically designed to study affect. The materials can, however, be examined for spontaneously occurring evidence of the emotions of change.

The data were coded for expressions of emotion and the eliciting context. Analysis focused on describing different roles that emotions play in the process of developing practice. Prior conceptual analysis of emotional concomitants of cognitive transitions (Walton & Goldsmith, 1987) guided some of aspects of the analysis.

Preliminary Framework and Findings

Expression of emotions were common among these teachers, although the extent to which participants made public their feelings varied from person to person. Data analysis allowed us to formulate three roles that emotions might play in changing teaching practice: 1) as motivators for development, 2) as indices of areas of practice ripe for change, and 3) as sources of decisionmaking within the classroom. These three are briefly described below. They should be taken neither as an exhaustive list nor as robust findings, but as a starting point for thinking about the functions of emotion in the development of new forms of teaching.

Motivators for development

It takes teachers a long time to rethink and reorganize their mathematics teaching in qualitatively different ways, on the order of at least two or three years (Cohen et al., 1990; Schifter & Fosnot, 1993). Given the substantial commitment of time, energy, and sense of self that seems necessary for such change, it is important to understand what motivates and sustains teachers through this long journey. We propose that a major impetus for change lies in teachers' feelings about themselves both as learners and as teachers. By looking to teachers' emotional responses to their practice (and to their participation in professional development experiences), we can further our understanding of the kind of energy involved in undertaking work of such scope and significance.

As a first pass at this issue we can ask broadly: How do teachers feel when trying to make substantial changes in their teaching, and how do emotions promote or hinder this process? In examining journals of our sample of teachers we found that they often included some kind of affective commentary in their writings. A host of different expressions of feeling appeared. Teachers variously described themselves as being frustrated, confused, scared, pained, uncomfortable, worried, guilty, panicked, lonely, uncertain, discouraged, overwhelmed, unsettled, dissatisfied, confident, eager, elated, thrilled, delighted, excited, and feeling good. For example, in the following journal entry Lynda[1] reflects on her responses to feeling poorly about her teaching.

> When I feel bad about my math teaching, one of the things I do is to figure out how to make sense of the rest of the week for the students, given the last few days' worth of math activities. I remind myself, without judgment, that many of these students are still very much in the concrete operations stage. . . . I try to refocus on what it is that I want them to be learning. I can see why selecting two or three main issues is important for me, too, because it keeps my guiding questions focused and the language specific. . . . I do this daily, anyway, but particularly when I'm feeling discouraged.

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Teachers who wrote about their feelings often focused on their frustrations, confusions, and discomfort. References to the negative end of the emotional spectrum were more common than allusions to the positive. The fact that teachers experience such emotions in these professional development programs indicates that the kind of reconstruction of practice that these programs offer challenges many participants' fundamental beliefs and practical knowledge of teaching. In being willing to explore and experiment with their practice, teachers may find that aspects of their teaching feel they are getting worse, for their experiments with practice interfere with the normal flow and rhythm of the class. Other aspects may feel suddenly open to serious question. It is difficult to experience such reversions. At best one might perceive them as frustrating and possibly intriguing and, at worst, they may call into question feelings of professional competence and personal worth.

Yet, what we typically think of as negative emotions may also serve important orienting and energizing functions in the change process. Lynda's journal entry alludes to the possibility that states of uncertainty and dissatisfaction or discomfort about one's teaching may stimulate reappraisal of current teaching practice.

Consideration of emotions' energizing and motivational functions can direct researchers to a number of questions about how emotions help to regulate the process of change. For example, in Lynda's passage of quoted above, we can ask what it is that allows her to use her discouragement to refocus her teaching in the positive way she describes? Why doesn't she instead resort to a catch lesson that will keep her students busy, or get annoyed with her students for not working hard enough? Why does one person relish the challenge of attacking a difficult mathematics problem while another gets sullen and withdrawn? To what extent are the answers to these questions to be found in understanding how individuals foster their emotional responses through the interpretive lenses of their own self-image? Understanding the motivational role of emotions may also involve examining the ways teachers construct images of themselves as mathematicians, learners, and teachers (Dweck & Leggett, 1988).

We can look to emotions, then, to help understand how teachers initiate and sustain their efforts to improve their practice. There is much to understand about the complex functions of negative emotions. Sometimes they seem, somewhat paradoxically, to have energizing effects which promote progress, and sometimes they seem to interfere with learning. Because teachers apparently acknowledge negative feelings often, it feels important to understand how these emotions enter into the process of change. And while teachers seem to focus less on positive feelings in the data sources we had available for study, it nonetheless feels important to explore how teachers invoke and maintain a positive, productive stance toward their task of changing practice.

Indices of development

It seems worthwhile to distinguish between emotions' broad energizing contributions to change—their motivational role—and emotions generated at particular times during the process of change. This process of developing new forms of practice involves reworking the overall structure and organization of teaching mathematics, but on any given day or week the degree of progress is hard to gauge. The phenomenology of change is most often modest, residing in daily challenges, struggles, victories. Efforts to improve teaching are realized incrementally; successes are interspersed with failures and teachers' experimentation with new forms of practice generate as many new questions as they offer solutions. Each lesson, each reflection and inquiry into one's teaching, each effort to solve a problem or understand a mathematical idea carries with it its own collection of emotions. For example, Lynda describes her reaction to a lesson where her normally disengaged fifth graders generated conjectures about even and odd numbers:

> I was so thrilled that the kids were thinking, "Well, maybe you can predict and maybe there's some use to a rule."... The whole conversation was generated by kids and I was so excited.

In contrast, Patricia recounts her own growing lack of engagement in her well-practiced, "traditional" ways of teaching mathematics to her second graders:

> It's very unsatisfying. It was also very boring. You watch the little faces and they're not turned on by what was maybe a keen lesson last year because you as the teacher thought it was nifty and got some excitement. And it means that you, as a teacher, you aren't growing as a person.

We can image Lynda's excitement stemming from her success in conducting a class embodying aspects of teaching that she values but cannot yet regularly create herself—students' intellectual engagement, investment in posing questions and searching for their own answers, participating in genuine mathematical reasoning. The lesson generated good feelings for Lynda because she was able to achieve something difficult. And having facilitated genuine student inquiry this time, she has presumably increased the likelihood that she will do so again. Her success is an indication that her teaching is progressing according to her aspirations. Patricia's comments suggest a different point in her development as a teacher. The boredom and dissatisfaction she reports are an indication to her of intellectual stagnation and serve as a signal for her to seek out professional development.

How can such feelings relate to local opportunities for learning and growth? One possibility is that they carry information about teachers' current understanding and readiness for further growth. Teachers may feel confused or uncomfortable when asked to think deeply or critically about ideas that are poorly formulated, but excited and energized by pushing on ideas that are reasonably well developed. In the former case, it may feel to them as if what little they know is being taken apart, while in the latter, that they are adding to their current understanding. They may experience pride, excitement, and joy when they succeed at something hard, while succeeding at something familiar and well understood may yield only mild pleasure, or perhaps even disdain.

This view assumes that the perception of intellectual challenge, and the affective response to this perception, differs depending on the individual's overall developmental status (Walton & Goldsmith, 1987). It further assumes that teachers' reformulations of their practice includes periods where their understanding and practice are relatively robust and applicable in different situations, interspersed with periods of reconstruction where understanding and practice are more fragile, fragmentary, and inconsistently applied. It is these cycles of reconstruction and consolidation of understanding that yield subsequent stages (or levels) of changing understanding (Case, 1985; Feldman, 1980; Fischer, 1980). We are proposing that teachers experience different collections of emotions depending on whether their understanding is robust or fragile, moving toward a more stable and consolidated state of knowing, or undergoing scrutiny and revision.

Some of the expressions of emotion we found in the data seem amenable to such an interpretation, although the feelings were not necessarily interpreted in this way by the teachers themselves. The conceptual framework for emotions as indicators of an individual's current developmental status can serve as a heuristic for exploring how teachers' emotions might help diagnose their receptivity for different kinds of learning experiences. It could be useful to teachers themselves for interpreting their professional development experiences. Before teachers would use emotions in this way, they would need to learn to reinterpret the meaning of emotional responses. Taking the perspective that emotions can provide clues to the status of one's understanding would involve learning to use emotions as information about the status of a *process* rather than as a judgment about the *self*.

We have observed anecdotally, for example, that many teachers consider confusion and frustration to be debilitating emotions that signal failure. They work hard to minimize their students' encounters with these feelings and do not, themselves, relish the experience of uncertainty or unknowing. It appears that teachers often interpret feelings of confusion and frustration in terms of personal intellectual weakness. Once such a self-assessment has been made, it is not difficult to invoke subsequent feelings of shame, anxiety, sadness, or anger. However, if confusion were dissociated from the self and associated instead with a particular point in a learning process (one which *every* thinker invariably encountered regardless of their intellectual power), then it would take on a different meaning. Instead of perceiving confusion in terms of failure, it could be framed as an indication that understanding was still emergent but, in fact, well enough developed to command some intellectual power.

Taking this stance would involve learning to interpret emotional responses differently—to develop a new "language of emotions."[2] Rather than thinking about emotional responses as ancillary to the business of learning and growing, teachers could start thinking of feelings as another source of information about their own learning. The notion of emotions as indices of development, then, involves interpreting emotions as clues to the state of an individual's system of understanding. Teachers' affective responses can help to gauge whether it is time to examine critically certain ideas or classroom practices, introduce new possibilities, or ease up and give current ways of doing and thinking a chance to get settled and effective. Both teachers and teacher educators can begin to explore new meanings for the emotions that are necessarily generated as part of the process of change.

Sources of classroom decisionmaking

In the course of a school day teachers make a multiplicity of on-the-spot decisions about their classrooms. These run the gamut from decisions about the timing, pacing, and content of work in particular subject matter areas to decisions about the physical safety and emotional well-being of students. These kinds of momentary decisions require immediate action in response to events unfolding in the classroom in "real time."

Rachel, one of the teachers we studied, observed that she became interested in affective issues in developing practice because she felt that the majority of her daily decisions in class came from her gut rather than her head. While we question her unwillingness to attribute a cognitive component to her decisionmaking,[3] her observation did direct us to consider the ways that emotionality influences the development of good judgment in the classroom. Rachel's attention to her own emotional responsivity reinforces Nussbaum's (1990) observation that emotions provide important forms of recognition on which we base our actions.

Consider, for example, Rachel's journal entry describing her decision to leave unfinished a whole-class conversation in her kindergarten about classification, rather than "wrapping it up" at the end of math period.

> I was excited by the discourse but also felt that I wanted to interject some element of convention and confirm the discomfort that Alexis and Katherina seemed to be trying to express [about Craig's way of thinking about the problem].... Inside I was laughing but also feeling very torn as I remembered [colleague] Joan's strong reaction to being corrected [in their teacher seminar]. I didn't want Craig to feel his idea was incorrect or invalid nor did I want Katherina or Alexis to feel dismissed. I decided to ask the children if we could leave the graph on the rug and if we could be careful not to walk on it or disturb it while we went on to other work.

Rachel's decision to return to the conversation later on is based on a number of feelings: her own excitement with the ideas the children are developing; her reading of Alexis and Katherina's discomfort as indicating that the ideas could be extended yet further; her concern for Craig's sense of himself as a thinker. Had she felt less pleased with the success of the discussion, or had she felt that all of the children were reasonably satisfied with the work they had done, she might have been less likely to decide to continue the conversation later. This passage also illustrates how Rachel's decisions were sensitive to the emotional integrity of the students and to encouraging the intellectual liveliness of ideas.

As she considered her choices, Rachel responded to a variety of affective issues. She remembered the distress and embarrassment that a colleague felt in a project seminar and draws a parallel to the dynamic unfolding in her classroom. She is cognizant of her desire to keep Craig from feeling ashamed of his contribution to the discussion, and of also wanting Katherina and Alexis to feel satisfaction with theirs. Her "move" in the classroom is informed by her reading of both the affective and cognitive needs of the group. Her effort to make a good decision involves a sensitivity to emotional tone and signals in the group which informs her pedagogical and intellectual choices. Nussbaum (1990) maintains that this blending of emotion and reason is at the heart of making good practical judgments.

The cultivation of good judgment is, in itself, an aspect of developing teaching practice that merits attention. If developing such judgment proceeds in part by learning to read emotional responses and use them as guides for evaluating possible future action, then it is important to understand how teachers can learn to rely on their feelings mindfully and intentionally as they make decisions about the content and flow of classroom life.

The importance of attending to emotions will, in fact, increase as teachers construct classroom practice that is more focused on fostering deep mathematical understanding. Because these new forms intensify the nature and degree of intellectual engagement in mathematics, there will be greater personal investment in classroom work. This greater investment, coupled with greater emphasis on working through hard ideas, is likely to generate more occasion for emotional expression in the classroom. Having resources for interpreting and responding to emotional expression will help teachers to make good judgments and good "moves" in the classroom.

Studying the Emotions of Teacher Development

It is a complicated matter to try to understand the ways in which emotions impact the process of teacher change, partly because the emotional experiences of undertaking such work are primarily internal. This line of work, therefore, raises many methodological questions. For example, what kinds of situations or settings offer access to the emotional aspects of this process, and how do we recognize teachers' emotional responses? Teachers like Rachel above may report complex combinations of emotions in situations where there is little observable evidence of affect to an outsider.

Conversely, teachers may report one kind of affective response to a situation, while an observer would make a different interpretation. This raises the question of how much we rely on teachers' self reports and how much we rely on our own interpretations of their behavior. Finally, we must consider the need to account for individual variability in the expression (and possibly the recognition) of different feelings. While some teachers are quite forthcoming about their feelings, others are less comfortable sharing the emotional aspects of their experiences. How do we learn to recognize the important affective workings for particular individuals, and what kinds of commonalties can we hope to find among a number of different teachers?

Though methodological challenges are noteworthy, the question of how emotions impact the process of individual development seems intriguing enough to merit the effort to seek solutions. We believe that the most powerful practical models for facilitating change will be ones that take into account the emotional aspects of development as well as the intellectual ones. Thus, gaining a fuller understanding of how affective processes contribute to teachers' efforts to reconstruct their mathematics practice should help teacher educators and teachers alike to develop forms of mathematics teaching consistent with the current reform movement. As both teachers and students engage in thinking more deeply and more publicly about mathematical ideas, there will be more occasions for encountering and addressing the feelings of confusion, pride, frustration, and excitement that accompany genuine learning.

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Notes

[1]Pseudonyms are used for teachers in this paper.

[2]This term was coined by Margie Riddle, a teacher at the Bridge Street School, Northampton, MA.

[3]Our clinical experiences with teachers are that, when asked to explain their decisions at some later time, teachers do not describe their actions as based on emotional grounds only, but refer to a rich array of beliefs, feelings, memories, understanding (both pedagogical and content-specific), and sensitivity to the current context in elaborating upon the decisionmaking process. We wonder whether teachers experience this kind of decisionmaking as primarily emotional because the rapid mental processing required to make on-the-spot decisions leaves them without total awareness (in a metacognitive sense) of their own thinking. They may recognize the emotional aspects of the situation most readily because affective responses can have a strong somatic (or at least extralinguistic) component. The more cognitive aspects of decisionmaking, compressed in time, may not be equally available to recognition or articulation in the moment. Emotions might therefore serve as important *markers* for complex decisionmaking in the classroom because they are more recognizable and accessible, not because they are the sole sources of the decision. Being mindful of their emotional responses may help teachers to focus their attention on other important aspects of the situation that need to be considered in the decisionmaking process.

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Chapter V: Transforming Mathematics Teaching in Grades K–8: How Narrative Structures in Resource Materials Help Support Teacher Change

Linda Ruiz Davenport and Annette Sassi

This study examined the role of material resources in supporting teachers attempting to transform their mathematics teaching practice. Teachers in a larger funded project with regular access to a wide range of resources were asked to identify and discuss resources of significance to them. Most of the resources identified as helpful conveyed detailed information about other teachers' classrooms or contained numerous examples of student work. This information was often conveyed through narrative form. These findings suggest that resources using narrative structures to provide concrete images of teachers and students exploring this "new way" of doing mathematics can be of great value to teachers.

A vast number of resources materials is now available to teachers who seek to create the vision of reformed mathematics teaching contained in the recent NCTM *Standards* documents (NCTM, 1989, 1991, 1995). Many of these resources come from NCTM itself, such as the "Implementing the NCTM *Standards*" columns in their professional journals, the recent *Addenda Series*, and other publications

Nelson, B. S. (Ed.) 1995. *Inquiry and the Development of Teaching:Issues in the Transformation of Mathematics Teaching*. Newton, MA: Center for the Development of Teaching, Education Development Center, Inc.

addressing issues of relevance to reforming mathematics teaching practice. Additional resources include curriculum projects designed to reflect the NCTM *Standards*—many of them funded by the National Science Foundation—which are now being published and are increasingly available. Finally, a growing body of resources are being produced by a small number of teachers themselves, who, through videotapes of their practice and written descriptions of their classrooms, are beginning to publicly share their efforts to reform their mathematics teaching.

Although we still have a great deal to learn about the teacher change process, it is clear that the kind of change called for in the NCTM *Standards* involves more than assimilating new teaching techniques in an existing system of ideas. Rather, it requires the critical examination of long-standing beliefs about what it really means to teach mathematics, an epistemological shift involving the creation of a new vision of students as constructors of their own knowledge, and the reinvention of pedagogical strategies and approaches consistent with that vision (Cohen et al., 1993; Schifter & Fosnot, 1993; Simon & Schifter, 1991). Unfortunately, we know little about the role of material resources in prompting and supporting this kind of teacher change, and the kinds of resources that are proving to be helpful.

This small exploratory study was designed to examine the role of material resources in helping teachers make these fundamental changes in the way they think about mathematics, mathematics learning, and mathematics teaching. It was set in the context of the Teachers' Resources Network (TRN), a teacher change project funded by the DeWitt Wallace-Reader's Digest Fund. Directed by Linda Davenport, TRN was explicitly designed to provide teachers with opportunities to explore and discuss material resources in after-school inquiry group meetings and over an electronic network. Given the epistemological shift required in the teacher change process, questions centered around what ways the range of resources teachers chose to explore were helpful to them, and the extent to which these resources seemed to lead to the development of new ways of thinking about their mathematics teaching practice.

The Teachers and the Resources

The study involved 22 participating TRN teachers, with whom Davenport met regularly over a period of a year and a half. They came from three different districts—one urban, two suburban—in the metropolitan Boston area. They varied widely in their numbers of years of teaching experience, and included a first-year teacher as well as a veteran teacher of 33 years experience.[1] They brought with them a range of recent prior experiences in mathematics professional development—most had participated in occasional district workshops on problem solving or using manipulatives, but a few had also been involved in more extensive professional development efforts.[2] Although these teachers were a diverse group, they all shared a commitment to improving their mathematics teaching practice, a willingness to share and critically discuss aspects of that practice, and a curiosity about important underlying assumptions about mathematics teaching and learning.[3]

The TRN resource collection contained approximately 600 items including books, journals, articles, unpublished papers, videotapes, and samples of innovative curricula. These resources were selected because they addressed important aspects of mathematics education reform, including curriculum, instruction, assessment, and diversity and equity issues. Some of these contained examples of recommended practices, including classroom activities and descriptions of how they might be used. Others contained research information about students' mathematical thinking, drawn from student interviews or analyses of students' written work. Some described issues that teachers needed to think through as they attempted to reform their mathematics teaching practice, including discussions of the recommendations of the NCTM *Standards*. Many addressed combinations of this content.

Teachers were able to check out resources from our collection to explore on their own, between our biweekly inquiry group meetings. Inquiry group meetings themselves were often used to do mathematics, explore the mathematical thinking of students, and look critically at resources available; in many cases, resources from our collection were used as the basis for these activities. Inquiry group time was also used for teachers to discuss resources they had been exploring.

All teachers kept portfolios that included written reactions to resources, including discussions of why particular resources had been selected, issues the resources raised for them, and how they seemed to affect their thinking and their practice. In the cases where resources were explored collectively in our inquiry group, teachers were either expected to complete resource review forms or to discuss those resources (and our exploration of them) in their journals.

Twice a year, teachers were asked to reflect on the contents of their portfolios and respond to a set of focus questions about what they were learning in this project; one question required them to list the resources they had explored thus far, identify those that had been helpful to them, and discuss how they had helped.

This paper draws on the lists of resources that teachers identified as useful and what teachers wrote in their resource review forms, journals, and portfolio self-assessments during their first half year in the project. We offer one caution in interpreting the results of these data. While teachers identified and discussed specific resources, it is not our intention to specify a particular list of resources as being helpful in some way; rather, it is our intention to uncover in what ways the features of these resources resulted in their being helpful, and what this information suggests about the kinds of resources the mathematics education community might seek to cultivate and produce.

The Kinds of Resources Teachers Identified as Helpful

After being in the project for a half year, the 22 teachers had reviewed, altogether, a total of 99 resources and identified 29 of them on their first self-assessment as useful in some way. (These figures include some duplicate listings of resources; teachers eagerly shared resources they liked with each other, and sometimes those that were shared were identified as helpful by more than one teacher.) Some teachers identified several resources as helpful; a small number identified only 1 or 2. Teachers sampled widely from our resource collection, drawing on a mix of books, articles, papers, videotapes, and curricula. By no means did teachers sample exclusively from those resources that offered "something to do in the classroom tomorrow."

To organize the discussion of what teachers found helpful, we have grouped resources into several categories which capture the kind of content that the resource appears to address: curriculum, assessment, students' mathematical thinking, and the teacher change process,[4] Following a description of the resources themselves, we note what seem to be their significant features, and then add what the teachers themselves had to say.

Curriculum

The resources addressing curriculum identified by teachers as helpful included a set of middle school mathematics activities (Bennett et al., 1986) identified by two teachers;[5] a book of activities for integrating literature and mathematics (Whitin & Wilde, 1992) identified by four teachers; a "replacement unit" for teaching place value at the second-grade level, and a similar unit addressing geometry at the third-grade level and multiplication at the third-grade level (Burns, 1994a, 1994b, 1994c); a chapter about teaching place value to young children in a book about developmental issues related to curriculum content at the primary grades (Thompson, 1990); a book of stories about teachers engaging elementary school students in interesting mathematical investigations (Ohanian, 1992), identified by two teachers; and a set of puzzle problems for exploring area and perimeter (Dominguez & Laycock, 1986).

What is notable about many of these curriculum resources is that all but the last contain discussions of how students are likely to think about or respond to the suggested explorations. In fact, the literature and mathematics book and the replacement units contain stories of teachers using the activities and include samples of classroom dialogue as well as student work.

Below are some examples of teachers' writing about these resources. (Each passage represents the writing of one teacher about a single resource. The passage itself contains writing selected from teachers' self-assessment and their resource review forms.)

> This [resource] was the most significant for me for a number of reasons. First, I used it in my classroom. Whenever you do this, it immediately becomes much richer, more complex, and more relevant. I had explored the concepts of place value and the area model for multiplication but this was just so well organized it was joyful to use . . . I enjoyed watching the kids explore on many levels. I was struck by how they organized their information and I was struck by how little some students actually knew . . . Students enjoyed this activity very much and I definitely was awakened to the power of the base 10 blocks to develop place value, multiplication, and concepts of decimals.

> I was most interested in this [resource], as I am interested in children's literature with mathematical concepts. I have used many stories as an introduction or extension to math lessons but I was interested in finding new ones. I have *The King's Chessboard* [Birch, 1988] and I knew that it had a mathematical concept but I didn't want to use it unless I could accurately apply it to math... I was able to add to my list of stories that connect with math and thus make units on math interesting and relevant. I find math is so meaningful to many students when it is connected to literature or real situations. It was also helpful to read about how other teachers had used the books in their classrooms . . .

> As you know, I am interested in math through literature. This book contains many books to use to integrate math and literature. The book is geared towards K – 6 and the chapters are separated in sections K–2, 3–4, and 5–6. The suggested lessons are not very detailed but it does give *many* examples. I hope to be able to use some of these lessons.

> I found this resource *extremely* helpful. The entire unit is laid out for you with day-to-day sequencing, parent letters, worksheet masters, detailed materials lists, and explanations of how the lessons went in her classroom . . . I learned through reading and actually doing this unit that there was a significantly different, and I think more valuable, way to cover the same concept and achieve the same skill mastery as (the adopted curriculum) requires . . . This unit greatly influenced my classroom practice. The management of this unit uses a lot of . . . activities in which pairs of students work independently. This was something I was afraid to do too often with my group but I found it to be perfect for them. I then used the same management and structure of partner investigations for other lessons and other subjects . . . This resource was significant for me because it gave me a very specific structure and outline for my entry into drastically changing my way of covering the math curriculum. This book was a hand to hold on my journey into giving up mandated curriculum and replacing it with other mathematics . . .

For teachers who wanted to attempt new classroom practices, these resources seemed to provide guidance, structure, and useful images of teachers engaging students in mathematical investigations. In some cases, it was the detailed description of a single lesson that was powerful; in other cases, it was the multiplicity of examples, albeit less detailed, that teachers identified as helpful. The richness of the description seemed to allow teachers to envision the taking on of similar actions—and when they in fact took these on, they reported that their vision was further enriched.

What also emerged in teachers' reactions to these resources was the power of the images of students doing mathematics—including strategies they used to solve mathematical problems, insights they brought to bear on those problems, and mathematical ideas they found difficult. These descriptions seemed to pique teachers' curiosity about their *own* students' mathematical thinking, and, at least in some cases, provided additional incentive for teachers to take on these efforts.

Assessment

Here, the helpful resources consisted of a practical guide to alternative assessment (Stenmark, 1991) and an article in which a classroom teacher writes about her efforts to use portfolio assessment (Lambdin & Walker, 1994). Both resources contain examples of assessment tools and strategies that teachers might use to explore student thinking, as well as examples of student work itself. For example, the practical guide contains student logs of their mathematical investigations, journal entries in which students reflect on the growth in their mathematical understanding, and teacher notes from observations of students working cooperatively in small groups—all of which are highly suggestive of what life might be like in some mathematics classrooms. The article by Lambdin and Walker, one a university faculty member and the other a classroom teacher, is a personal account of how this teacher struggled with portfolio assessment and conveys a sense of the community that she and her students were creating.

Teachers' written remarks about these resources included the following:

> Even though I didn't read the complete packet, I think that this is a valuable resource and I want to spend more time [reading it]. There are so many ways for the child to assess what they have learned and how well they feel they understand what they are doing. Are they truly understanding or are they doing what the teacher tells them to do? Next year I would like to do a daily or a weekly journal with students to have them express their reaction to what they are learning. Interviewing may [also] be something I will try next year.

> A very good, clear, down-to-earth presentation on math portfolios. The article . . . was the best of the several articles that I have read on math assessment. This article was great because it was so math related. It listed successes and failures [about keeping portfolios]. This article . . . gave me more direction than others. After reading the article, I downsized my own portfolio project which I am planning to start up in September. I am only going with one class doing portfolios the first year (instead of the five I originally planned to go with).

Here, again, teachers' selection of resources as helpful appears to be connected to the images of new practices that they convey. In one case, the resource raised issues for a teacher about how quickly we assume that students understand what they are "taught" and prompted her to try these strategies to investigate her own students' learning. In the second case, the resource provided concrete images of a teacher's successful as well as failed efforts, leaving one teacher with a realistic sense of the magnitude of the change she was about to undertake, and causing her to scale back her initial intentions.

Students' mathematical thinking

Resources exploring student thinking included a book about the mathematical thinking of young children as it emerged through student interviews (Labinowicz, 1986) and an article describing how students invented algorithms for basic operations, which also provided numerous examples of these invented algorithms (Kamii et al., 1993). Each of these resources was identified by two teachers. The other three resources in this category included a discussion of the extent to which students' mathematical thinking is influenced by language difficulties (Miller, 1993); a discussion of student thinking about fraction ideas (Ball, 1990), which was not identified as helpful on a teacher's self-assessment but *was* described as helpful on her resource review form; and a videotape showing two teachers working with students as they invented algorithms for two-digit multiplication, which captured students demonstrating and explaining their invented algorithms at the chalkboard (Kamii, 1990). This videotape was also identified by another teacher as helpful, not on her self-assessment but on the resource review form itself.

Teachers wrote,

> [This resource] provided insight into the wealth of knowledge children actually have if allowed to show it. We, as teachers, so often stifle that creativity and don't allow children to be "smart."

> This resource was significant because I was about to begin to teach place value and started reading on page 241 . . . They relished counting as suggested . . . Everyone had a chance to show on the overhead how they would group a large number of objects. I liked that suggestion because they needed to listen carefully and watch what was happening. If they went up and repeated the same procedure as another person, I asked if they remembered who else had tried to group in that same way. To me, this relates to setting the stage for a "community of learners."

> The more I examine the problems I face in my classroom, the larger the language issue grows . . . Ana Maria recommended this reading to me with a comment that it is pretty "light" reading yet I found it insightful and it jogged me to relook at my class and my techniques . . . Maybe it is oversimplification but I found the hints or ideas very helpful, if nothing else than as a reminder . . . of what I am doing and assuming in class . . . I am the vehicle for the students to grasp the language . . . I have asked some students to write up an evaluation of the class. What I need to include is . . . what makes sense and what remains a puzzle. I know you have encouraged us to have students write about

> math and I've finally taken the plunge. I'm curious to see the results . . . It's sort of scary.

> I am planning to do a unit on teaching fractions and am looking for new and creative approaches . . . This article was most pertinent as the author focused on teaching an understanding of fractions to third graders . . . This paper was most helpful to me as it walked through every step the author used in her approach to teaching fractions . . . I am going to use this procedure, take my time and question the students as the author did and not overuse papers that had exercises.

> I was looking for some ideas to introduce multiplication of two-digit numbers . . . I was glad to see the teachers in the video presented multiplication . . . with story problems without giving them hints for a solution. I was most interested in the students' responses and I was very surprised at how they approached the solution. I will be interested in seeing how my students solve these kinds of problems . . . [This resource] is an interesting and valuable tool to use when preparing to teach multiplication of two-digit numbers. You could use the same problems in the video and compare your results with those in the video.

These resources provided teachers with powerful images of students as capable mathematicians, prompted teachers to wonder about the mathematical thinking of their own students, engaged teachers in thinking about how they might pose similar problems to their own students and what might happen as a consequence. In two cases, these resources allowed teachers to envision their practice somewhat differently, making connections to the notion of a classroom as a community of learners, and raising issues about assumptions that are made about what and how students learn. Noteworthy is the extent to which these resources about students' mathematical thinking, became, for many teachers, resources for thinking about their mathematics teaching practice.

Teacher change

These resources included an article about a teacher who thought she had "reformed" her mathematics teaching but whose practice still resembled the traditional paradigm in significant ways (Cohen, 1991), an article about the role of teacher reflection in the teacher change process (Feldt, 1993), and three papers by teachers engaged in the enterprise of transforming their mathematics classroom practice (Brown, in press; O'Brien, in press; Schott, 1992). Two additional papers written by teachers (Anderson, in press; Smith, 1992) were identified as helpful on the resource review form written by two different teachers, but neither was among those selected as helpful by those teachers during their self-assessment activity.

Four of these papers described, in vivid detail, features of classroom practice in transition—including descriptions of mathematical investigations, samples of dialogue among teachers and students, and reflections on these classroom events. In the case of Cohen, reflections were from an observer in the classroom exploring the match between a teacher's practice and a new curriculum framework. In the case of the papers written by Anderson, Brown, O'Brien, Schott, and Smith, reflections were from the teachers themselves.[6] All of these resources contain both samples of student dialogue and descriptions of student work as well as descriptions of teacher practice itself. While Feldt addressed the issue of teacher change more generally, her image of a new kind of professional development to support teacher change provides some sense of how teachers might begin to think about their practice differently.

Teachers' written reactions to these resources included the following:

> I was interested in this resource because I am in the process of changing my teaching practices. I thought that it would be helpful to read about someone else who was having a similar experience . . . . It helped me reflect on whether I too am using new materials and old mathematics. If I am, how do I change? . . . I found it reassuring that others are seeking to make changes (not only because they are mandated), and that it is one thing to embrace a doctrine of instruction and quite another to weave it deeply into one's practice . . . I feel everyone should read this article and discuss it in relation to our own practices . . . This article was important to me because it helped validate my change in teaching practices and the concerns that I have. While I am in the process of making the learning of mathematics more interesting by incorporating problem solving, it may be difficult to reconcile what is happening in one classroom with what may or may not be happening in

> another. Now I feel like I can stand alone, take risks, and enjoy what I feel comfortable doing.
>
> I really liked this article because the author approached algebra much like I do . . . She wanted students to explore concepts, discuss patterns, and form generalizations. She did not just give formulas and have students plug in numbers. She also explored and understood the resistance from students and parents . . .
>
> I took this resource because of its first sentence: "How do you know what to teach if you don't follow a book? . . . How do you know what to teach next?" That sentence sounds like me. I was hoping the article would answer that question . . . I found this very helpful. The author wrote clearly and I easily understood. Although her examples were of sixth graders I could easily adapt them to the fourth grade. The teacher takes the curriculum goals and listens . . . observes . . . listens . . . questions . . . listens. She is probably more comfortable with math than I am. Her lessons proceed from what she has observed in the previous lessons. I liked her use of cooperative/collaborative groups, since I use these in all subject areas. I hope to present the math problem she presented to my math group this week . . . and observe how my students solve it.

It is clear that these resources—a number of them papers carefully crafted by other teachers as part of a writing project—were powerful for TRN teachers. They related to the struggles and concerns of the teachers described in the resources and took courage from their struggles. They found these teachers to be a lot like themselves.

Resources Teachers Did Not Identify as Helpful

While it is informative to know what kinds of resources teachers selected as helpful, it is much more difficult to consider the question of resources that were not helpful. First of all, the fact that teachers did not identify a resource as helpful does not imply that it was not so; it just did not rise to the top of their list. Many of the resources identified as helpful by one teacher were explored but not identified as helpful by others. Teachers were never explicitly asked to identify resources that were not helpful.

Secondly, on the few occasions that teachers did remark on their resource review forms that a resource had not been particularly helpful, it was often because the grade level addressed did not seem appropriate for them, or the issue was not one they were interested in tackling. In a very few instances, teachers were critical of a resource because it seemed overly prescriptive or overly simplistic.

What Can We Learn from What Teachers Chose?

Generally, teachers explored a wide range of resources addressing a range of topics related to curriculum, assessment, students' mathematical thinking, and teacher change. Many of the resources that teachers reported as helpful contained stories from classrooms that conveyed vivid images of how teachers and students work together, and, often included numerous examples of students' mathematical thinking. Teachers reported that these resources gave them ideas for the kind of mathematics teaching and learning that might take place in their own classrooms, and, in several cases, reported that a resource helped them think about their own practice in a new way.

What is striking is the extent to which many of these resources seemed to invite teachers to take on similar kinds of practices to those described or discussed. Teachers wrote about being able to envision themselves undertaking the described mathematical investigation, and then proceeded to undertake it. Often, the descriptions of the student thinking contained in the resource prompted teachers to wonder about the thinking of their own students, and they then set off to explore that thinking using the strategies that had been described. Several teachers reported that reading the resource, *and then actually doing what was captured in the resource*, significantly enriched their thinking about mathematics teaching and learning.

We know from recent work on the use of teaching cases that they are powerful vehicles for professional development (Barnett, 1991a; Barnett et al., 1994; Shulman, 1992). Teaching cases embody principles and concepts of a theoretical nature and allow teachers to explore why certain actions are appropriate. They also provide precedents for working through problematic situations and, often, they capture moral and ethical principles. In addition, they convey dispositions and habits of mind that are valued by the profession. Even more importantly, they provide images of the possible. We also know that teachers often report trying out activities that had been captured in a case (Barnett, 1991b). However, it is not clear that we understand deeply how it is that the stories contained in teaching cases produce teacher learning.

Theoretical work on the role of narrative and storied forms of learning suggest that these are tools for teaching us about practical actions. They engage us in thinking deeply about a practical situation, allow us to consider the tacit assumptions that guide the actions described, as well as assumptions underlying alternative actions, and they often unveil the complexities with which practitioners must deal (Forester, 1993; Hummel, 1991; Mattingly, 1991.) The subtleties and complexities of practical actions are often difficult to capture in more prescriptive forms that suggest that certain actions are "appropriate" separate from the details of a particular context or the thoughts and feelings of the individuals involved.

The kind of learning claimed for teaching cases, specifically, and narrative, more generally, is borne out by what teachers wrote about in this small study. Teachers were indeed engaged by the rich descriptions and narratives they found in the resources they selected. They considered underlying assumptions of "reformed" mathematics teaching, as well as more traditional models, and they were able to envision themselves undertaking these new practices. They were also helped to consider the complexities of these new reform practices—an important learning as teachers attempt these reforms in their classrooms—and they identified overly simplistic or prescriptive resources as not helpful.

The teachers in this study described moving back and forth between the reading of a resource and the actions they took in their classroom as a consequence—suggesting that the coupling of narrative with the action provided much more than narrative itself. It is intriguing to think about the nature of this interaction. In what ways does the narrative become a backdrop against which teachers can reflect on the actions that they took, and the unfolding of those actions? In what ways does the narrative even help inform what teachers might profitably reflect on? To what extent do teachers construct yet another narrative for informing their own thinking and practice from reflecting on the actions they take in their own classrooms? These are questions that might be fruitfully explored as we continue to consider the kinds of resources that teachers find helpful, how these resources might be crafted and used, and how they ultimately play a role in helping teachers transform fundamental aspects of their mathematics teaching practice.

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Notes

[1]The mean number of years of teaching experience for the group was 13.6.

[2]These more extensive professional development efforts include week-long Marilyn Burns workshops, a week-long *Math: A Way of Thinking* workshop, and summer institutes associated with two EDC projects (Teachers, Time, and Transformation and Mathematics for Tomorrow).

[3]Commitment to improving mathematics teaching practice, willingness to share and discuss aspects of their practice, and curiosity about underlying assumptions of mathematics teaching and learning were assessed using teachers' applications to the TRN project. These application forms required them to describe (a) their current mathematics teaching practice and what they liked best and least about it; (b) the kinds of changes they would like to make in their classroom practice and how interested they are in

making those changes; and (c) a recent classroom event they found puzzling or challenging and what was interesting to them about this situation.

[4]It is actually quite difficult to categorize the content of the resources in our collection except superficially. For example, many of the curricular materials also implied certain instructional practices, including assessment strategies. Information about students' mathematical thinking often also included descriptions of rich and interesting tasks that could become part of a teacher's curriculum. Papers written by teachers about the change process also addressed curriculum, instruction, and assessment, as well as student thinking—issues teachers attended to as they attempted to reform their mathematics teaching practice.

[5]Unless otherwise noted, resources identified as helpful were selected by only one teacher. In the cases where resources were selected by more than one teacher, their number is given.

[6]These teachers were all part of the Math Process Writing Project directed by Deborah Schifter.

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Chapter VI: Teacher Inquiry Groups: Collaborative Explorations of Changing Practice

James K. Hammerman

This paper describes the collaborative inquiry group structure of the Mathematics for Tomorrow project, focusing primarily on the dynamic relationship between the community being formed and teachers' learning and growth, especially around issues of mathematics and pedagogy. It presents several vignettes of mathematical and pedagogical explorations in the inquiry group as well as teachers' own descriptions of the effects of participation in the group on their thinking and classroom practice. The paper also raises a variety of questions for further investigation and research.

The Mathematics for Tomorrow (MFT) project of the Center for the Development of Teaching is experimenting with the creation of a new form of collegial support for professional development in mathematics teaching. These "inquiry groups" form an ongoing discourse community of elementary and middle grades teachers who meet regularly to examine critically their own and each other's mathematical and pedagogical knowledge, beliefs, and practice as they work to make changes therein.

Nelson, B. S. (Ed.) 1995. *Inquiry and the Development of Teaching:Issues in the Transformation of Mathematics Teaching*. Newton, MA: Center for the Development of Teaching, Education Development Center, Inc.

Inquiry groups spring from an important set of propositions about the process of teacher change.

1. Change in the spirit of the vision of the reform documents (NCTM, 1989, 1991) requires substantial shifts in teachers' understanding, knowledge and beliefs, and the playing out of those in a complex, situated practice (Ball, 1994; Lampert, 1985; Nussbaum, 1990). It is not merely a set of "technical fixes" that can be learned relatively quickly.

2. Because of this depth and complexity, it is clear that change will take substantial time (Goldsmith & Davenport, 1995). It is an ongoing process with no clear endpoint. We envision teachers continuing to grapple with the dilemmas inherent in reform visions over the course of many years as they work out the implications for practice. For many teachers, ongoing dialogue with others struggling with similar issues can aid in exploring these implications.

3. Yet creating such a support structure for change requires new relationships among teachers—and long-term structures to support those relationships—as teachers continue to explore important issues over months and years (Little & McLaughlin, 1993).

We are inventing inquiry groups as a structure in which teachers can experiment with and build these supportive relationships—what Lord (1994) calls "communities for critical colleagueship." We hope to create an environment in which teachers can help one another challenge and develop their mathematical and pedagogical knowledge and beliefs, along with their teaching practices, *as* they continue to teach. Of course, this is not the first time that teacher educators have created group structures in which teachers improve knowledge and change practice by examining it carefully and deeply. Some have developed specific methods for promoting this examination (Carini, 1975, 1979; Watt & Watt, 1991). Others have begun to describe some of the characteristics of the work done in teacher groups. Lord, for example (p. 185), talks about the need for reciprocity in risk taking in the context of a resource-rich professional community. Kallick (1989) describes an "interpretive community" as one that takes time to think and reflect together on the variety of understandings that its members bring to an issue. Still others have described some of the effects of teachers meeting together in groups—how they can create shifts in the power structure of schools and schooling by helping teachers find, voice, and legitimate what they know (Cochran-Smith, 1991; Lytle & Cochran-Smith, 1992; MacDonald, 1986).

This paper will begin to examine what it means for teachers to be critical colleagues for each other in MFT inquiry groups. We will look at examples of inquiry group explorations of mathematics and pedagogy, and will consider how these may be affecting teachers' knowledge, beliefs, and classroom practice. We will then address issues arising from the change process itself. Our focus throughout is on how the community being formed supports teachers' learning in these realms, as well as how these different explorations build on one another to create a community of supportive but critical discourse. Clearly this is a dynamic process.

The Context

MFT is a systemically embedded, teacher development and research project involving 25 K–8 teachers in school-based teams from three Boston-area districts. The districts—Arlington, Brookline, and Cambridge—present a range of demographic characteristics: Arlington is small, suburban, white, working class; Brookline is medium-sized, linguistically diverse, urban-suburban, middle to upper-middle class; and Cambridge is medium-large, economically and racially mixed, urban.

For teachers, the program consists of two summer institutes, biweekly inquiry group meetings after school in schools during two academic years, and four classroom consultations and four day-long workshops each academic year. Teachers are asked to regularly reflect on their practice in writing and, starting in 1994, were provided the opportunity to consult with each other in classrooms. Teachers receive an annual stipend for their participation in the project, and districts contribute funds to cover release time for workshop days and peer consultations and for teachers to buy classroom materials.

In a second phase of the project, we will work with a different group of 36 to 40 teachers from the same districts plus Boston for another two years. Some currently participating teachers will work with staff in Phase II to facilitate inquiry groups, and then will continue facilitating groups independent of staff when the project shifts to district-based funding under the auspices of the Educational Collaborative of Greater Boston (EdCo). Under EdCo, summer institutes and inquiry groups will be made available to its 23 member districts, using MFT-trained teacher-facilitators as teacher leaders in new groups.

The Content of Inquiry Groups

Inquiry groups serve to ground the more general work we do in summer sessions and all-day workshops in the particulars of classroom practice. Inquiry groups focus on investigations of mathematics and pedagogy, with occasional discussions of issues of school culture and requirements, the role of parents and other community members, and the tensions between new ideas from the project and others that are prevalent in schools. In each of the sections below, we will describe how inquiry group interactions support teachers' own learning about content, as well as about what it means to be part of a professional development community.

Mathematics

Doing Mathematics

Many teachers recognize the need to broaden and deepen their own mathematical understanding so that they can make pedagogical decisions grounded in a clear conception of what's important about subject matter (Schifter & Fosnot, 1993). In inquiry groups, teachers explore mathematics together both to better understand mathematics and to learn what it means to *make* mathematics. By sharing a variety of ideas and hypotheses about a specific topic, and by challenging one another to clarify and elaborate on their ideas, teachers develop a real, working, mathematical community. There, they struggle together to understand mathematics as meaningful and inventable, rather than as a disparate collection of facts and procedures passed down, too often, from long-dead authorities. Like students in Ball's third-grade classroom (Ball, 1993), teachers in MFT inquiry groups learn mathematics from one another as they make conjectures, clarify, support, and revise their ideas.

For example, because teachers wanted students to learn traditional mathematical algorithms, we spent several sessions in the spring of the first year exploring multiplication algorithms. This began with our own attempts to model multiplication with manipulatives and to record our work. It became clear from discussions about this sometimes difficult process that, in general, the algorithms we created required finding ways to break problems into smaller pieces, somehow to multiply each piece, and then to put the pieces back together correctly.

As part of our exploration of multiplication, we also examined a number of algorithms from different cultures throughout history. These included Egyptian and "Russian Peasant" multiplication, which both involve keeping track of successive doubling and halving; Lattice/Gelosia multiplication, which involves keeping track of place value through use of a grid; and several variations on modern methods. Teachers learned the techniques involved in these methods, but spent most of their time trying to understand *why* they work. As they pushed themselves to talk through the reasoning behind each of these methods and the conceptual difficulties they had in understanding them, teachers came to some important insights about place value and number operations. Some noticed the connection between Egyptian multiplication and base 2 representations of number. Others were grappling with the structural strengths and conceptual difficulties associated with a system, such as our base 10 system, that uses powers of a base to represent number. Still others were pondering the connections between different physical representations of number operations—arrays versus discrete groups, for example—and the symbolic representations of those processes.

Clearly we didn't come to full resolution about these deep issues. Nonetheless, by raising the issues in the context of the group, teachers helped one another struggle with important mathematical ideas. They came to see themselves as people who could work together to make sense of mathematics, and perhaps, that mathematics is something about which communities can make sense. For some, this had an important impact on their own attitudes about math and thus, on their teaching (see "Attitudes about Math" below).

Yet questions still remain about what is needed to make these mathematical discussions fruitful. As the teacher educator facilitating these explorations and discussions,[1] I brought to the group knowledge about both mathematics content—which mathematical ideas are important and potentially generative—and mathematics as a discipline—what it means to work "mathematically," to look for patterns, make conjectures, and try to find reasoned arguments that will prove or disprove these conjectures. I used this knowledge to make more or less subtle judgments in selecting the materials and problems I brought to the group, as well as in the focus and challenges I provided in the course of group discussions. How much knowledge of mathematics—both its content and ways of working—is needed within a teacher group to keep the focus true to "mathematics"? How can teachers develop the knowledge, skills, and attitudes to be self-sufficient within such a group when the support of a teacher-educator is no longer available? How explicit do I need to be about my "moves"—the reasoned choices I make as group facilitator—to create a culture of mathematical investigation within the group, that will become institutionalized and maintained in the long term?

Attitudes about Math

Teachers have varied emotional reactions to the experience of participating in a group to do math and to come to really understand it. For some, math has always been interesting and exciting and this work only adds zeal to their previous energy:

> Math is powerful!! It has felt that way to me in my own math experiences and I have seen a similar effect on my students. My experiences teaching math this year have been entirely different from any other year.
>
> In trying to tease out just what has made the difference for me, a few things come to mind. The most important of these is my own inspiration about math. I have made discoveries and connections, and experienced the high of understanding math in new ways. This "math buzz" infected my whole classroom at times. The heightened awareness I brought to math allowed me to take risks and push kids into bigger ideas and projects than I'd attempted previously. I heard kids professing math to be their favorite subject, and they seemed to anticipate math with excitement. This was news!! (Kranz, 1994)

For other teachers, excitement about math is a newfound thing. For example, several teachers were both surprised and delighted to find themselves continuing mathematical explorations from the inquiry group in their free time. This excitement is especially important for the large number of elementary teachers who come to the program feeling "math-phobic"—i.e., confused by, or scared of math. For these teachers, the experience of making sense of mathematics in a community can be truly liberating. In a journal entry in the middle of the year, one teacher wrote

> Here's some math news you'll like. At one of my parent conferences my student's father was telling me how he'd been teaching his daughter about base 2. He and the daughter then began enthusiastically teaching me. We messed around doing math problems for a half hour. That would not have happened a year ago, I don't think. I would have said, "Base 2. Hmmm. Interesting. Henrietta's doing great in reading . . . " (King, 1994)

But not all teachers find this new way of viewing math exciting. For them, math was the last vestige of a clear-cut subject—there was always a single right answer, and getting that answer was all that mattered. If math, too, requires interpretation, arguing for a position, and justifying the reasonableness of an answer, then no field can be simple and clear-cut in a teacher's day. When knowing math meant memorizing facts and procedures, it was easier for teachers to check their own knowledge. If it requires continuously making sense of phenomena, then some teachers feel much *less* competent and confident. This reduced confidence can undermine a teacher's ability to teach.

How then can the group validate and acknowledge this range of emotional reactions—confusion, frustration, excitement, competence, fear—as an important concomitant of the process of learning something new? (Weissglass, 1994) How can it support teachers to become more comfortable *being* confused and dealing with the inherent ambiguities of this new view of mathematics teaching, while they work ideas out? Can groups develop images of *uncertainty* that enable teachers to maintain a sense of control?

On the other hand, how do teachers' own experiences making mathematics in a community of their peers relate to the kinds of community they create for their students in the classroom? Clearly having a viable image of such a community may make it possible, but this is not a simple translation. There's much to be studied here.

Pedagogy

Not only do teachers use the group to explore new ideas and develop new attitudes about mathematics, but also to address explicitly pedagogical issues—that is, questions about how students *learn* mathematics and how to help them grapple with important mathematical ideas. Some of these ideas come directly out of teachers' reflections on their own mathematical learning. In these reflections, teachers ponder the relationship between doing and understanding, and the role of a teacher in facilitating learning.

> [In past years] did my kids really understand what I was teaching them, or were they simply repeating steps I had taught them? The children left my room with some basic math skills, but I'm not sure exactly what they left understanding. I never gave much thought to their understanding. I always assumed that if they could perform the process, that they must understand what they were doing. I was quick to forget that, for myself, performance did not equate understanding. . . .
>
> A lot has changed for me and my students since I joined the program. They now have a teacher who is open to much more exploration, more questioning, and to mathematical integration within all subject areas. . . .(Cyr, 1994)

In addition, teachers use the group to sort through pedagogical issues that arise from their classroom practice. For example, late in the first year of inquiry group meetings, one teacher came to the group asking for help designing an activity that would engage the children in "a heated debate about the issues." The group began by probing into what she meant by "the issues." Through some discussion it became clear that the issues she wanted them to debate involved understanding the equivalence of multiple ways of representing a number within a place value framework—for example, seeing that 241 could be represented with 2 hundreds blocks, 4 tens blocks and 1 unit block; *or* with 1 hundred block, 12 tens blocks and 21 unit blocks; *or* in several other configurations. This teacher wanted students to grapple with how these representations were the same and how they were different—and consequently, with the nature of the rules in a place value system and when they can be intentionally broken—so that students could talk about when different representations are useful in performing calculations.

Based on their deep probe into the concepts that the teacher felt were important for students to grapple with, the group was able to generate several possibilities of what the teacher might do next. One of these possibilities was picked to develop further and together, the group then designed a classroom activity through which the children would explore these issues. The group used their analysis of the mathematics, data they had about prior students' understanding of these ideas, a process that was open to diverse approaches and hypotheses, and the need to clarify ideas in the group to develop a specific plan that met the teacher's initial goals. Interestingly, the teacher later reported that the activity hadn't engendered the heated debate she wanted, because the students found the equivalence of these representations much more obvious than she had expected. In this case, while the activity "flopped" in that it didn't meet her goals, it gave the teacher information about her students' understanding that was quite useful. In addition, the process of working together to deeply analyze an idea and to develop and discuss various hypotheses about how to teach it was important for *teachers'* learning about mathematics and pedagogy.

Another example may further illustrate the kind of discourse that we're hoping to promote. In pondering how to integrate a new curriculum into their ongoing practice, teachers were struggling with issues of pedagogical coherence. The curriculum was built on a spiral model, so that ideas addressed one day might not be addressed the next day, but were certain to resurface a few weeks later. Teachers were uneasy about this model. They wanted to use some of the more in-depth, long-term investigations of particular issues they had developed in the past because they thought those gave children a chance to think deeply about an idea over time. Yet persisting with a particular topic would mean cutting pieces from what they considered (and were told was) a very carefully crafted curriculum. Some asked, "How much can I take out [of the new curriculum] and still have it make sense?"

Others thought they might build coherence by helping students make connections *across* topics in the curriculum. Having the teacher "make the connections *for*" students didn't fit teachers' view of learning that required learners to make connections among ideas. Yet just "leaving room" for connections sounded quite laissez-faire—teachers were clear that random juxtaposition of ideas without a context in which to build connections was not enough. The question then turned on what it means to "leave students room to make connections," on how teachers can structure the environment to facilitate that, and on how they would know that these connections were being made. These are subtle questions, grounded in practice, but touching on deep philosophy.

As teachers work together to analyze and understand each other's practice, our goal is that they begin to see themselves as a community of professionals examining issues of teaching in order to improve it. Yet because such a vision is new to teachers, what this means and how it gets enacted in the group are potentially problematic. We're finding, for example, that it takes a fair amount of preparatory discussion to shape a question to bring to the group so that it embodies a good balance between specific detail and theoretical issues. What goes into this shaping process? What can we learn from others exploring case-based teacher education about the process of drawing theory out of specific classroom instances (Barnett, 1991; Shulman, 1992)? And how can teachers learn to support one another to delve deeply into the mathematical and pedagogical issues at hand; to engage *with* each other about substantive issues in teaching, bringing in data and stories (Carter, 1993) to support or refute points made by others, posing questions that challenge assumptions but are seen, nonetheless, as supportive rather than evaluative?

The Process of Change

In inquiry groups, teachers begin to work together to examine complex issues of teaching embedded in very particular situations. This kind of professional discourse is new and takes time to develop. It also feels very different to teachers from the evaluative reports of classroom activity and idea swapping that is more typical of teacher talk (Lortie, 1975). The discussion about curriculum coherence described above served to focus, but not answer, the questions raised. Shifting views of mathematics open up possibilities, but do not resolve them. How comfortable do teachers feel with this lack of closure? How does their level of comfort change over time, and what causes this change? In this section we will explore a bit about how teachers individually and in the context of the inquiry group grapple with their changing ideas and practice.

As we move into the second year of work with a single group of teachers, we're finding that conversation has changed. These changes enable the group discussion to probe deeper into mathematical and pedagogical issues. We think these changes may have several causes: 1) Many teachers have gained confidence in themselves as mathematical thinkers, along with new knowledge about mathematics, learning, and teaching. 2) Teachers can do different things in their classrooms—and thus bring different things to the group—because they're starting the second year with a new group of children and can invent a classroom culture and norms from the beginning of the year. 3) Teachers know the staff, the expectations of the project, and the other members of the inquiry group and feel more comfortable sharing aspects of their teaching that are potentially problematic. Finally, 4) for some teachers, the first year of MFT served to throw their thinking about mathematics and teaching into substantial disequilibrium, and much of the year was spent struggling to put the pieces back together. In the second year, teachers are starting on more solid ground.

Several teachers commented on these changes and this struggle to put things back together in their portfolio writings from the end of the first year of work together.

> While reading [this portfolio] please keep in mind that I am "a teacher (not a work) in progress." This portfolio has forced—yes, forced—me to reflect upon the inklings and nagging thoughts that have begun to assault me since the start of this journey. I am confident that one day I will reflect upon this early time and see its stage as necessary for change. Right now I feel piecemeal. I think, I experiment, and I read patiently. I feel hodgepodge. I haven't made any all-encompassing discoveries about how I view my teaching of mathematics. I have blurbs: "meaningful; teach for understanding; the why's not just the how's; use manipulatives; provide time for experimentation and discovery; discuss in groups of different sizes; cooperative learning groups; and write about mathematics." I have points, not philosophy. I hope to pull myself together and translate this mayhem into teaching that most prepares and excites my students to the world of mathematics—a world, which since high school, has excited and challenged me. I love math. I want my kids to feel and know that love for themselves. (Wilson-Callender, 1994)

Wilson-Callender was trying to find a coherent practice in the "hodgepodge" of ideas she was encountering. She was trying to balance cognitive and affective goals; philosophical and practical concerns. Cournoyer, too, acknowledged her changing goals and the challenge of putting them into practice.

> When I think about how what I am learning will impact my teaching style, I become somewhat frightened. I guess that's because I see great changes in store. I've never seen myself as a teacher with math strengths, but I've always been able to get to the answer procedurally and I'm good at memorizing. So, I fared well in my own math attempts and felt helping students to succeed in the same fashion would, in itself be successful. Wow, how my thinking has changed!
>
> I know too well now that math is more than numbers and procedures and memorization... I need to assist students with developing a thinking process that enables them to raise questions and discover answers without relying on memorized procedures or rote learning. In order to provide my students with all that I wish, I myself need to relearn, to question, to take activities I've been providing steps further. It's apparent to me what I want to do, the real question lies in "how." (Cournoyer, 1994)

In the inquiry groups, we are trying to support one another in grappling with this question: *How* can teachers teach given their new assumptions about mathematics, learning, and teaching? What structures and cultures in the classroom will support students' robust constructions of important mathematical ideas? In many ways we also need to apply our thinking about this question reflexively to our own learning in the group. How can we create structures and cultures of support in inquiry groups that build on our sense of the complexity of the subject matter, and of learning as an active, constructive, and social process? Clearly this is more than the familiar "sharing good ideas and activities." We've just begun to develop ways to investigate particular examples of teaching practice brought in by teachers as a vehicle for learning about essential issues in teaching.

Such inquiry into practice requires new norms for interaction. Examples of these norms might include a deep respect for teachers as learners and for the effort required to learn dramatically new ways of looking at and being in the world, a focus on judging ideas rather than individuals, a focus on intellectual rather than technical content, a respect for novel and diverse ideas, and clear expectations that we will grapple with new ideas even if these are difficult. How important are these and other norms? How do they get communicated, established, and developed?

Finally, as the project begins to pay explicit attention to the development of teacher leaders who will act as ongoing facilitators of inquiry groups under the EdCo part of the program, we wonder about issues of cultural conflict and expansion. What resources—in the form of knowledge and skills—will teaching facilitators, or other group members, need to continue our inquiry group experiment after funding for staff support is gone? How many people who are familiar with the "culture" of the group are needed to maintain that culture? We know that perpetuating the group's culture of inquiry depends in part on pressures that the group experiences from outside. We don't know enough yet about how inquiry groups, and the teachers who participate in them, are seen by their non-MFT colleagues. These will be important issues to consider as we imagine the inquiry group structure for professional development expanding throughout schools and districts. How, for example, will the norms and structures of inquiry groups support or clash with those in the broader school community?

Conclusion

In this paper we have begun to describe the interrelationship between teachers' collaborative learning of mathematics and pedagogy in inquiry groups and the creation of a community for professional development there. Our description of some of the mathematical and pedagogical explorations that take place in inquiry groups, as well as their meaning and importance to teachers in their own growth and development, has raised a variety of questions about the workings of the group and the process of teacher change. We have much work in store for us as we continue to develop our understanding of the nature of the inquiry group and its role in facilitating teacher learning via investigations of issues grounded in classroom practice.

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Notes

[1]Throughout this paper I use the plural voice to talk about findings. Here I switch to the singular "I" to describe my own role as teacher educator in the group itself.

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Chapter VII: Epilogue

Barbara Scott Nelson

Each of the papers in this anthology has focused on a dimension of the process entailed for teachers as they embark on the project of moving their teaching toward that envisioned in the NCTM *Standards*—the impact of the nature of teachers' mathematical knowledge on their visions for teaching, the role of affect in the process of change; the essential characteristics of helpful materials, and the issues to be addressed in developing a teacher community that supports investigation into practice. The existence of the *set* of papers invites "conversation" about the relationships between these elements: What role does affect play in teachers' developing mathematical sophistication? How can materials help teachers become better mathematicians? Does the presence of a supportive culture enhance both the expression of affect and mathematical growth? and so on.

Nelson, B. S. (Ed.) 1995. *Inquiry and the Development of Teaching:Issues in the Transformation of Mathematics Teaching*. Newton, MA: Center for the Development of Teaching, Education Development Center, Inc.

The collection also allows consideration of a broader range of issues than any one project or set of investigators can illuminate from within its own research agenda. For example, as work in mathematics education reform has progressed, it has become clear that the sociocultural perspective advocated in the NCTM *Standards* raises issues of consistency within the larger research and practitioner communities. To take a particular instance, are there boundaries, such as classroom walls, within which knowledge is considered sociocultural and outside of which the view of knowledge reverts to usable facts? So, should school administrators view the knowledge that students and teachers generate in classrooms as emergent and contingent, but view the knowledge produced by researchers differently? Further, the vision of the NCTM *Standards* has propelled many teachers toward the invention of new forms of teaching and there are questions about the best way to represent our knowledge about these emergent forms. What kind of data is most useful to the field at this time? What is the process by which new analytic constructs emerge from the construction and examination of cases? In what ways is teachers' knowledge, itself, held in narrative or "storied" form? These questions, and others, are important ones in the field at the current time, and also find resonance in the papers in this volume.

In order to explore these issues, and to make the papers themselves a dynamic part of our community rather than static, finished pieces on our bookshelves, we invited paper authors and other CDT colleagues[1] to examine critically the set of papers and offer questions to each other and reflections on issues raised by the collection as a whole. We then had a staff seminar at which we discussed the papers and subsequent comments. We invite readers of this anthology to do the same.

In the section below we have organized some of the questions, reflections, and discussion that occurred into an exploration of three themes: teachers learning mathematics; the relationship between research and practice in teacher development; and the relationship between analytic and narrative knowledge and how these two apply to research in our field. We have retained the voice of each commentator, but edited the conversation to make the focus on these three issues clear.

These are not meant to be exhaustive comments about the papers but rather provocative explorations of the issues explicitly or implicitly raised by them. Both the content of this section and its structure speak to the complexity of the phenomenon of teacher development and explore the consequences for our field of taking a sociocultural orientation toward knowledge.

On Learning Mathematics

Sassi: The five papers together present a rather comprehensive view of the work to be done in achieving fundamental change in teaching practice. They ask of teachers to change what counts as mathematical knowledge, to change their own understanding of how they learn mathematics, to change how they engage with students in this learning process, to change how they collaborate with colleagues in fostering change, and to reconsider how they use resource materials as supports for teaching and learning. Taken independently, each change presents enormous challenges. Taken together, the task seems monumental.

Goldsmith: To start with just one of those points, having the opportunity to explore mathematics as an adult learner seems to be a critical factor in helping teachers to assume a more active role in evaluating mathematical ideas. Both the Russell et al. and Hammerman papers [Chapters II and VI] describe aspects of professional development programs that have teachers working on mathematics, and Hammerman explicitly notes that developing a "positive" attitude toward mathematics is an important change for some teachers. These teachers become freer to explore mathematical ideas both inside and outside the classroom. For some, this is the first time in their lives that doing mathematics hasn't brought with it a sense of dread and despair: never before had they imagined that they, themselves, could think critically and well about mathematics. Never had they considered that there was more to mathematics than using a given set of rules or procedures to get to the right answer. The chance to work on mathematics with colleagues in a setting that emphasizes genuine understanding offers teachers the opportunity to take an active role in their own learning, and to think about how it feels to really own one's ideas.

Davenport: The paper on learning mathematics in the context of one's own teaching [Chapter II] made me wonder whether one might also be able to learn mathematics from looking at narratives [cases] about the teaching of others. In the learning contexts that Russell et al. identify teachers are exploring content, thinking through students' representations and strategies, delving underneath students' reasoning to explore mathematical structure. Might these opportunities also be provided through the examination of a teaching case or episode? Might such cases provide an opportunity to learn to think deeply about teaching and learning—at a distance from one's own practice—which teachers could then use to look at their own practice?

Hammerman: Russell et al. argue that the new mathematical knowledge teachers need to teach mathematics for understanding comes in large part from grounded exploration of ideas in the context of the classroom. Their descriptions of teachers engaging in such explorations are rich and powerful. I am impressed and intrigued by their attempts to capture the complex thought processes that describe how this new knowledge of mathematics actually shapes teacher thinking about what to do in the classroom. This connection is hardly straightforward.

Meg, for example, having tried the combinations problem herself, comes to understand both the complexity of the problem she originally posed, and some of the sources of that complexity. By listening carefully to students' thinking in new ways, she may also be developing a richer picture of their mathematical understanding. She must use her new understanding of the mathematics, along with her growing knowledge of the ideas her students can use to tackle a problem, to devise a related problem that is challenging but doable. This process is neither simple nor clear-cut.

What does it take to make these connections, to build a mathematics teaching practice that incorporates these new views of mathematics with new views of learning and teaching practices? How do teachers integrate new views of mathematics with new epistemological and pedagogical perspectives? We might explore how teachers' changing math knowledge is related to their changing images of the nature of mathematics positing, as Schifter does, that it is these latter changes which are truly essential.

On the Relationship between Research and Practice

Kaplan: Russell et al.'s view of teachers learning mathematics while teaching has implications for teacher selection and evaluation, as well. If an administrator accepts the need for learning in the context of teaching, he/she will want to hire teachers with a genuine interest in mathematical ideas and in student thinking about them. The scenarios described in the paper might help them with how they review applications and conduct interviews. Teachers who are profoundly incurious about student thinking would probably be screened out in the selection process.

There also are implications for teacher evaluation. Russell et al. describe some teachers who effectively engage in learning in these contexts and some who are less effective. An administrator might be concerned about the possibility of misguided teachers misleading kids (like the geometry teachers in Schifter's paper [Chapter III] who don't know what to do with their students' conjectures). How does an administrator evaluate whether a teacher is effective at thinking through students' representations and delving underneath students' reasoning?

I thought about the Schifter developmental model from the point of view of someone who has some knowledge of reform in mathematics education, but is not an expert in the field. There's agreement that we're getting lousy results from the first stage. The second stage is maybe not much better. The third stage sounds pretty good; it sounds congruent with the NCTM *Standards*. Administrators may need some guidance on distinguishing between the second and third stages; again, the scenarios help. With regard to the fourth stage, an administrator might be asking why this is so powerful. Is it important for *every* teacher to reach it? From a policy point of view, is there a difference in cost between getting people to the third stage and the fourth stage? What's the cost/benefit in going from the third to the fourth stage? If there is a move toward merit pay schemes to replace the step raise structure for teachers' salary increases, administrators will be looking for models for distinguishing levels of proficiency. They might look to this kind of model for this purpose.

Schifter: One thing that I'm trying to do in the development of these models is to define the main components of the practice that we're trying to create. That doesn't mean that you attend to these components exclusively and ignore everything else. But these are components that are not well understood or are particularly unique to this practice, or certainly are different from a traditional practice. With regard to the model of enacted conceptions of mathematics, I'm really concerned about how many teachers and staff developers are interpreting the proposed mathematics education reforms in an inappropriate way. I don't intend that the model be used as a way to judge and categorize teachers and particularly wouldn't want to have their pay dependent on what slot their supervisor thinks, they fit in.

Goldsmith: As a research and development community we have one reason for thinking about these kinds of things, and the potential for having our constructs misunderstood or used in a way that's quite counter to their intention is something that needs to be marked directly. When I read the Schifter paper, I noted that the model is a heuristic. It helps us both think about what we want to be doing with teachers and it helps us think about what the nature of the domain is. If, indeed, we're going to be speaking to a community that needs to make judgments for different purposes and finds that making judgments by putting people in categories is efficient, then that point really does need to be very carefully made.

Schifter: I'm thinking of the model of enacted conceptions of mathematics more as a pedagogical tool for the teacher educator. As Linda Davenport said, if this is what you have in mind as you're working with teachers, then perhaps you would choose narratives that embodied a different conception of mathematics that would allow the teachers to at least be exposed to something else, even if they're not yet in the position to be able to explore it deeply.

Goldsmith: Proposing a set of stages implies that it's preferable to get to the end. The Schifter model of stages of enacted mathematics provides an opportunity for teachers to think about things that they don't ordinarily have a chance to think about. If you don't know that there's some possibility out there, then you can't move toward it. But if you do, then you can make some decisions about whether or not you value it. A teacher with a particular epistemology might think that drilling facts and teaching procedures is perfectly adequate and why would anyone want to go anywhere else. But without alternate visions the opportunity for taking on the personal task of change simply isn't there.

Hammer: The more general question is "How do the constructs of education research relate to educational practice?" And so, here's a construct of education research. It's a set of developmental stages for which you have lots of empirical evidence. How do we understand what that construct does? There are the many subcommunities of researchers who will understand what that construct does in a variety of ways. And there are the different communities of practitioners—teachers and administrators—who understand what these constructs do in still different ways. This scale is not the same scale to you as it might be to an administrator. I think it's very likely that someone might take it and say, this is a level-one teacher and this is a level three. The challenge is to articulate what we think the construct *does* do in some way that would be clear and so that as it gets taken up by these different communities, what they will construct from it will have more coherence, if that's possible.

Schifter: Perhaps as we present particular constructs we also should give some guidance about their use. So, rather than just present them, we should also have some discussion about these very issues—what is inappropriate use, what is a more appropriate use.

Hammer: You call this model a pedagogical tool. This is what I think research is constructing, and this is what I think teachers should be developing from research. We are developing conceptual tools for understanding education. And there should be an exchange and examination of conceptual tools among researchers and between the research and practitioner communities. As opposed to this other thing that people usually expect research to produce, which is generalizable, systematic statements that are taken to be true about some part of the world and can guide action.

Nelson: This also connects to the view that mathematics education reform is not only a matter of educating individual teachers, one-by-one, but is also a matter of building new cultures in schools, cultures in which intellectual inquiry is at the center and drives how the school is managed and run. Helping people think, or rethink, how to use theory is a critical part of helping to build such a culture. Just as viewing mathematics as a collection of facts and algorithms to be absorbed is a limited view of mathematical knowledge, so viewing pedagogical constructs as finished schema, ready to be applied, is a limited view of education research, more generally. Constructs need to be seen as things to think with, by practitioners as well as by researchers. If you're a thoughtful mathematics supervisor you may come to think that this set of constructs doesn't, in fact, discriminate important things that you're seeing in your observations of teachers. And you may come to see yourself as somebody who can contribute to the conversation about the constructs—about the ideas.

Schifter: And research, itself, has to make a concomitant change. It needs to become much more tentative, embedded.

Goldsmith: But then that makes us not be gods, and that's such a drag!

On Narrative as a Form of Knowledge Representation

Sassi: One of the themes that runs through all of the papers is that both mathematical and pedagogical knowledge are practical and experiential. Teachers will learn—and relearn—mathematics and teaching through doing, conversing, arguing, trying, writing, and reflecting. Indeed, four of the papers rely on vignettes, illustrations, or teacher quotes to situate their points in actual teacher practice. From this we may want to say tentatively that the notions of teacher change that are explicitly and implicitly laid out in the papers require a narrative and storied way of learning. We need to take this further to explore the implications of this more deeply. Some questions to explore include 1) Why do the four papers rely on narratives or case examples to make their points? 2) Is there something inherent in the nature of knowledge imbedded in these alternative conceptions of teaching that lead to narrative as natural mode of learning? 3) Could teachers ever "get it" (e.g., understand what Schifter or Russell mean by "mathematical understanding" or what Goldsmith means by the "affective aspects of teacher development") without seeing (and discussing) images of it in stories and accounts? and 4) Would teachers be more apt to explore mathematics if such explorations could be embedded in situational accounts of practice? These are just a few questions that could explore the complex relationship between narrative and learning in teacher change and development.

Nelson: In terms of how teachers learn from narrative, I'm reminded of Bruner's argument (Bruner, 1986) that there are two modes of cognitive functioning: the scientific one, which proceeds by logical argument, seeks general laws, and takes empirical truth as its ultimate verification; and the narrative mode, or story, which deals with human intentions and takes verisimilitude as its standard. Narrative's knowledge claim is not certainty but human plausibility, which is a very interesting thing to consider when you are talking about an educational system in reform—teachers see what fellow teachers write about as plausible, even though they may never actually have seen it.

Hammer: There are two different points. One is a pedagogical point: What's the most effective way for teachers to learn? The other is epistemological: What's the most effective way to materially represent the community's knowledge?

Schifter: A question that is currently before the mathematics education research community is, what's the difference between mathematics education research and inquiry into mathematics education? This is a challenge to a conception of research in which in order to count as research it has to be generalizable. Research has been defined in a portion of the community in a certain way that some of us are now finding irrelevant; it doesn't serve the kind of questions that we feel now need to be answered.

Nelson: Toulmin (1990) argues that the decontextualized, law-seeking apparatus of science is an historical and social artifact that grew out of more particular, concrete, and situated forms of knowledge.

Hammer: In mathematics, at least among mathematicians, you can communicate in very concise, precise terms. And it works!! Pretty well!!

Schifter: But what it doesn't communicate is how the idea developed. A lot gets left out. How they came up with that proof, how they thought about it, what the process was that they went through in order to get to that conclusion. This is one of the things that's actually very misleading about the way mathematics is taught. When mathematics is taught only as the presentation of findings, then the students never get a sense of what the process is to get that.

Hammerman: The fact that math is communicated in such a telegraphic way, is in part because of assumptions within that community about what counts as knowledge. That community has decided that it's not interested in the process of coming to a result but in the results themselves and making sure that those are solidly proven.

Nelson: Why does the research we do need a different form?

Schifter: Because of the contextual embeddedness of everything we're talking about. And because, at the same time that we are studying a phenomenon, we are also inventing it. I mean, at the same time that we are studying how teachers transform their practice from "conventional" to "reformed" pedagogy, we and the teachers—and our colleagues—are inventing just what a reformed pedagogy is. So, first, we need images—stories, cases, narratives in video and written form—in order to see if we are actually talking about the same thing as other teachers and researchers. And because new teaching practice is being invented, we can't predict how findings from the laboratory get transformed when put into classroom contexts. For example, the CGI folks[2] have reported that the way teachers used their new knowledge about the development of children's mathematical thought was quite different from what the researchers had initially envisioned.

Notes

[1]CDT staff who participated in these paper discussions include Linda Ruiz Davenport, Lynn T. Goldsmith, David Hammer, Jim Hammerman, Christine Kaplan, Barbara Scott Nelson, Annette Sassi, and Deborah Schifter.

[2]CGI stands for Cognitively Guided Instruction. Schifter is referring to the work of Carpenter, Fennema, and their colleagues.

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Virginia Bastable is the director of SummerMath for Teachers (SMT) at Mount Holyoke College. Committed to helping teachers examine the principles upon which they base their mathematics teaching, SMT offers summer institutes, academic year courses and seminars, and multi-year programs of educational research in which teachers are partners. Ms. Bastable has a BS in education with a minor in mathematics from the University of Massachusetts, a master's in Mathematics for Secondary School Teachers from Worcester Polytechnic Institute and a EdD with a focus on mathematics education from the University of Massachusetts. Before focusing her energies on working as a teacher educator, Ms. Bastable worked as a secondary school mathematics teacher for 20 years. Currently she is in engaged in a NSF-funded project, Teaching to the Big Ideas, which is a collaboration of Education Development Center, Inc., TERC, and SummerMath for Teachers.

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Sophia Cohen is a research associate at Education Development Center, Inc., Newton, MA. She is currently working on a teacher development project in elementary mathematics. She has a BA in psychology from Harvard University, and a PhD in developmental psychology from Stanford University. Her previous research has been on children's notational systems, language acquisition, and primary prevention of child abuse and neglect.

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Linda Ruiz Davenport is a project director in the Center for the Development of Teaching at Education Development Center, Inc., in Newton, Massachusetts. She has a BA in liberal arts, with a focus on the history of science, from the University of Texas, and an MEd and PhD in mathematics education from the University of Washington. She has been involved in preservice and inservice mathematics teacher education at the elementary, middle school, and secondary levels since 1979. She has also worked in bilingual education, focusing on issues of teaching mathematics in linguistically diverse classrooms. While a member of the faculty at Portland State University (OR), she codirected the Portland site of QUASAR, a national research and development project funded by The Ford Foundation and the Learning, Research, and Development Center at the University of Pittsburgh. QUASAR was designed to implement mathematics reform in middle schools serving economically disadvantaged communities. Now at EDC, she directs the Teachers' Resources Network (TRN), a project designed to explore the role of material resources in mathematics education reform at grades K–8.

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Lynn Goldsmith is a researcher and project director in the Center for the Development of Teaching at Education Development Center, Inc., in Newton, MA. Before studying teachers' professional development, she investigated formal and informal systems that support the development of extreme talent. She has recently become interested in exploring issues about development in complex intellectual and social domains, and in understanding the role played by affect in contributing to the process of development. This work makes her happy. Dr. Goldsmith has a BA in psychology from Yale University and a PhD in developmental psychology from the Institute of Child Development, University of Minnesota.

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Jim Hammerman is a senior research associate at the Center for Development of Teaching at EDC, and a doctoral candidate in Learning and Teaching at Harvard's Graduate School of Education where he is a Pforzheimer and Harvard Graduate National Fellow. He is primarily interested in collaborative models for teacher professional development that draw their strength from joint inquiry into pedagogical issues grounded in specific examples of teaching practice. Jim taught elementary and middle school for several years in what can most aptly be described as an "urban one room school." He has developed mathematics and geography curricula for elementary, middle and secondary levels, and has worked extensively as a teacher educator through SummerMath for Teachers and EDC. He is currently pursuing work focusing on how the culture and discourse of inquiry groups can support teacher change, and on how this culture can be established and maintained by the actions of group members, teacher leaders, and school adminstrators and structures. Jim is also contemplating what it might mean to turn the lens of inquiry reflexively back onto his own practice as a teacher educator.

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Jill Bodner Lester, the assistant director of SummerMath for Teachers at Mount Holyoke College, began her association with the SMT project as a participant in 1986 and became a member of the summer staff in 1990. She has a BS with a major in elementary education and a minor in psychology from Rhode Island College, an MS from Southern Connecticut State University with a focus on constructivist education and is currently a doctoral candidate at the University of Massachusetts, focusing on mathematics education. Prior to becoming a teacher educator, she worked as a public school teacher in urban, suburban and rural school systems and gained experience teaching grades one through six. She is currently engaged in an NSF project, Teaching to the Big Ideas, which is a collaboration of SummerMath for Teachers, TERC and Education Development Center.

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Barbara Scott Nelson is senior scientist and director of the Center for the Development of Teaching at Education Development Center, Inc. She has long been interested in the relationship between teachers' ideas about learning, teaching and subject matter, and the particular social and intellectual contexts in which their teaching is embedded. Currently she is working with school- and district-level administrators on the implications of mathematics education reform for the intellectual culture of school and for their own work. She holds the BA degree in philosophy from Mt. Holyoke College, the MAT from Johns Hopkins, and the EdD from Harvard University.

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Susan Jo Russell is a senior scientist at TERC, a nonprofit organization that works to improve mathematics and science education. She has a BA from Swarthmore College, an MS in early childhood education from Bank Street College, and an EdD in mathematics education from Boston University. After 10 years of classroom teaching and staff development in elementary schools, Dr. Russell became involved in research and development. At TERC, she currently directs Investigations in Number, Data and Space,which is developing a complete mathematics curriculum for grades K–5 and support materials for teachers engaged in changing their mathematics teaching. Her work focuses on the development of children's mathematical ideas and on understanding how practicing teachers can learn more about mathematics and about children's mathematical thinking.

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Deborah Schifter is a project director in the Center for the Development of Teaching at Education Development Center, Inc., in Newton, MA. She worked with the SummerMath programs at Mount Holyoke College from their inception in 1982 and directed SummerMath for Teachers from 1988 to 1993. She has also worked as an applied mathematician and has taught elementary, secondary, and college level mathematics. She has a BA in liberal arts from Saint John's College, Annapolis, an MA in applied mathematics from the University of Maryland, and an MS and a PhD in psychology from the University of Massachusetts. She coauthored, with Catherine Twomey Fosnot, *Reconstructing Mathematics Education: Stories of Teachers Meeting the Challenge of Reform*, and is the editor of a two-volume anthology of teacher narratives, *What's Happening in Math Class?*

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Lisa Yaffee is a project director at TERC, where she is helping to establish school-based math clubs for girls. She has worked on Investigations in Number, Data and Space, an innovative K–5 math curriculum, as well as on Teaching to the Big Ideas. An alumna of SummerMath for Teachers, Lisa has taught preschool, kindergarden, grades 2, 5, and 6, and remedial high school.

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